# Finite volume effects on the Chiral Magnetic Effect (CME)

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## Motivations

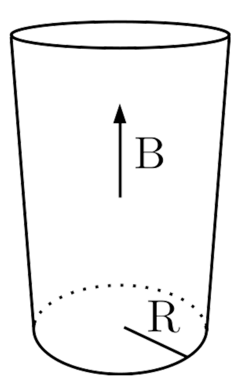
- The CME in heavy-ion collisions probe the expected local parity violations in QCD.
- The CME is usually calculated in a system with infinite volume.
- Instead, the QGP plasma has a small volume.

Are there any finite volume effects on the CME?

## Model

- We consider a non-interacting, massless Dirac field with charge $q$ in magnetic field $B$.
- The system is confined in a cylinder with finite radius $R$.
- We use **MIT Boundary condition**, *i.e.* no fluxes of energy-momentum and currents outside the cylinder:

$$\mathrm{i}\gamma^r\psi(R) = -\psi(R)$$




Relevant parameter: ratio between radius and magnetic length

$$\rho_R = \frac{|qB|R^2}{2}$$

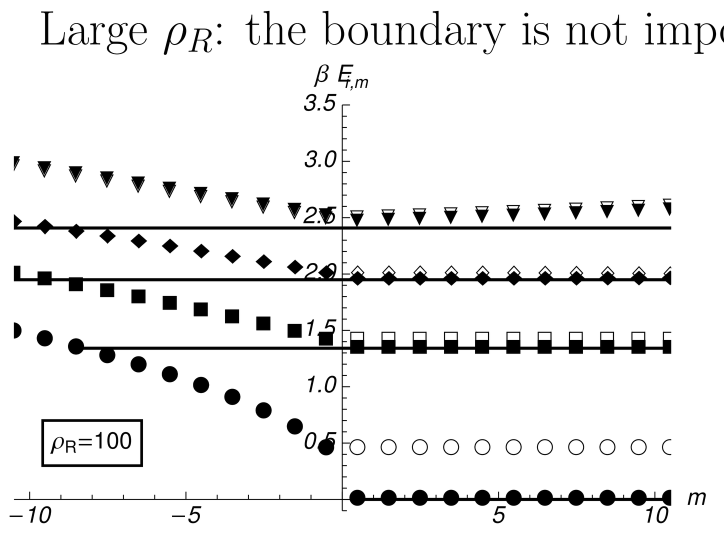
- We obtain the analytical solution of the Dirac eq.
- and compute the exact CME current from Statistical Mechanics

$$\langle\hat{j}^\mu(x)\rangle = \mathrm{tr}\left[\hat{\rho}\,\hat{j}^\mu(x)\right] = \frac{1}{\mathcal{Z}}\mathrm{tr}\left[\exp\left(-\beta\hat{H}+\beta\mu_A\hat{Q}_A\right)\hat{j}^\mu(x)\right]$$

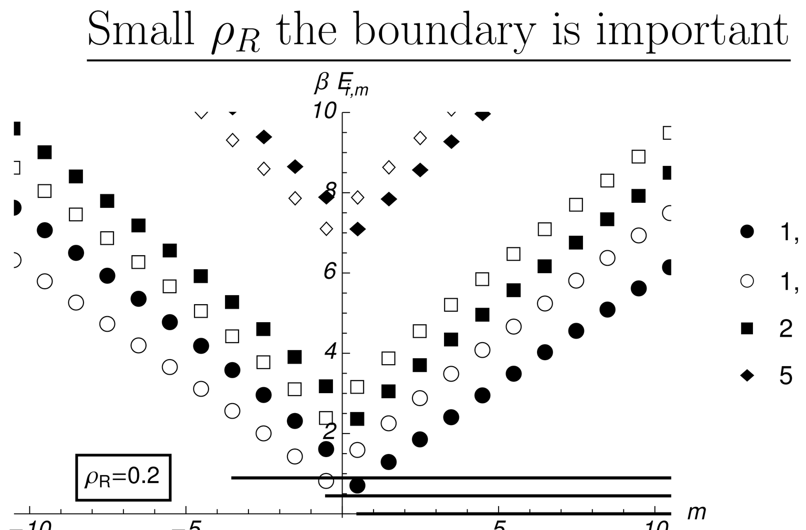
## Bounded Landau Levels

The boundary lifts the degeneracy in angular momentum $m$ and in polarization $\zeta$. The Landau level quantum number $\lambda$ becomes a real number instead of an integer.

Large $\rho_R$: the boundary is not important




Small $\rho_R$ the boundary is important

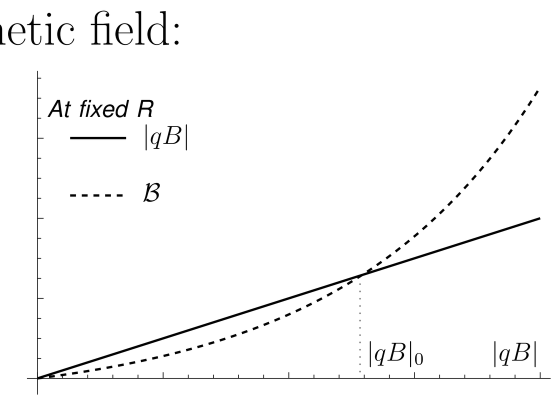



The Landau levels as a function of angular momentum $m$ at $\beta^2|qB| = 0.1$, and $p_z = 0$. Different markers correspond to the $i$'th root of the MIT boundary conditions equation as indicated in the legends. Solid lines show the unbound case.

The lowest Landau level (LLL) quantum number $\lambda_{\rm LLL}$ defines a characteristic value for the magnetic field:

$$\beta^2\mathcal{B}(\rho_R) = \frac{1}{2\lambda_{\rm LLL}(\rho_R)}$$

$\mathcal{B}$ roughly grows exponentially with $\rho_R$.




Comparison between $|qB|$ and $\mathcal{B}$ at fixed cylinder radius $R$.

## Chiral imbalance $\mu_A$

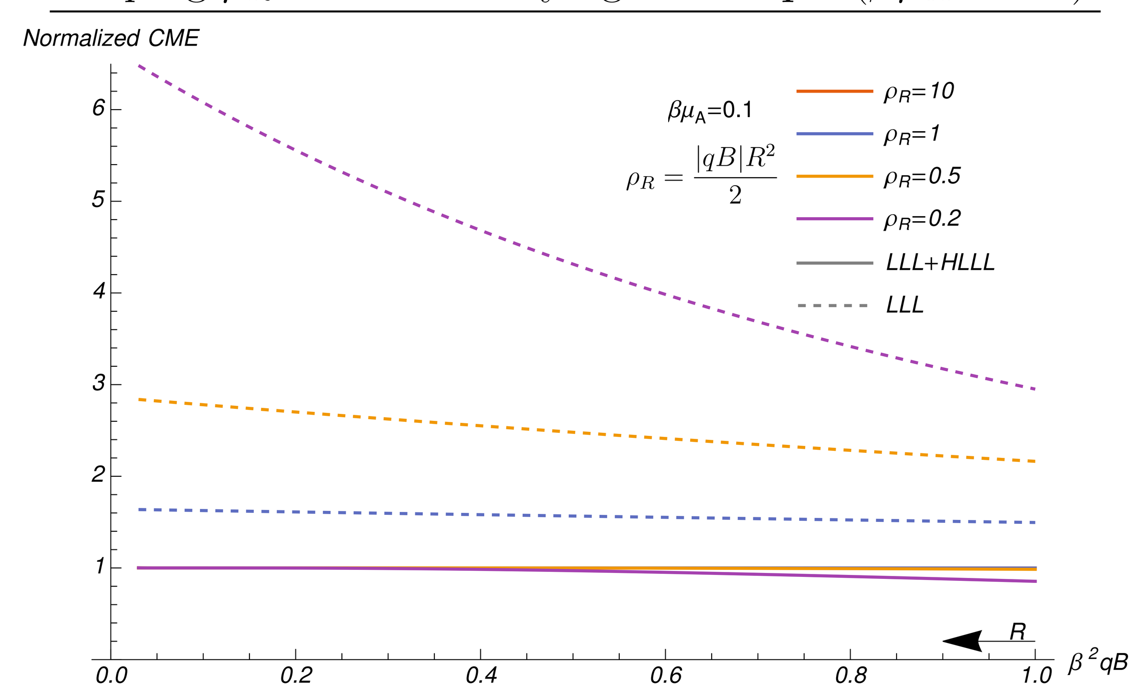
- $\mu_A$ is constant in our calculation
- indeed, at times smaller than $\mu_A$ relaxation time, we can consider it decoupled to the system (we have no interactions).

However, the model can describe a dynamical $\mu_A$.

## Results: CME at the center of cylinder

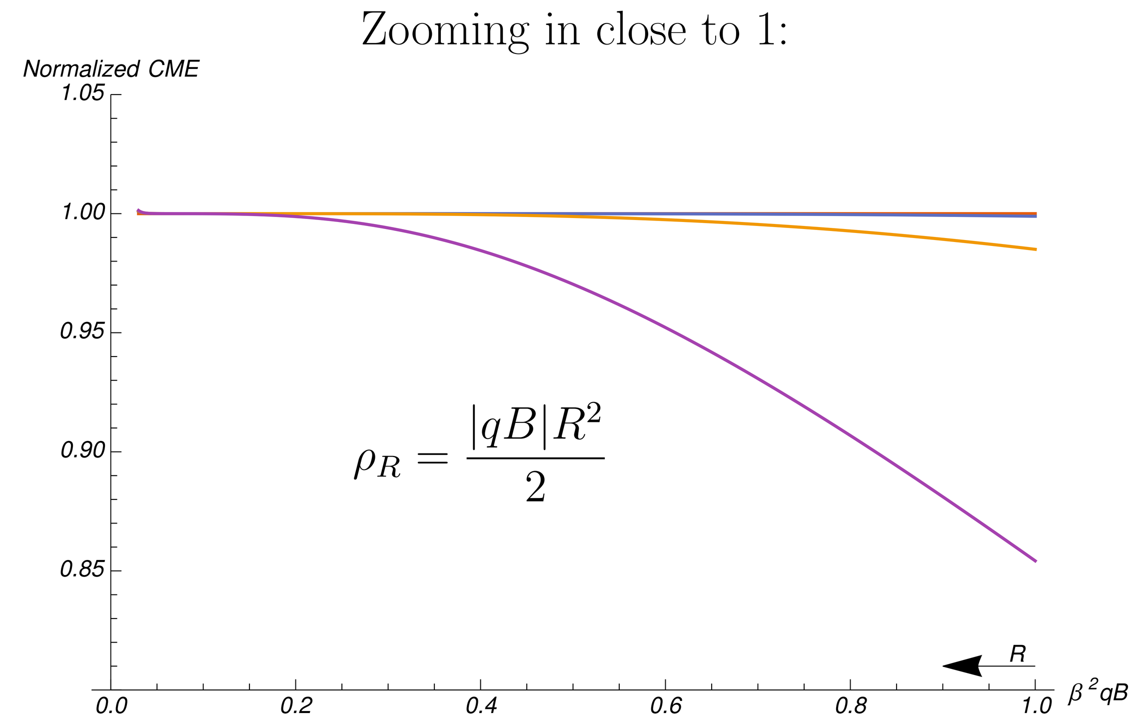
$$\text{Normalized CME} = \langle\hat{j}^z(x)\rangle / \left(\frac{q^2B}{2\pi^2}\mu_A\right)$$

Keeping $\rho_R$ fixed and varying $R$ and $qB$ ($\beta\mu_A = 0.1$).

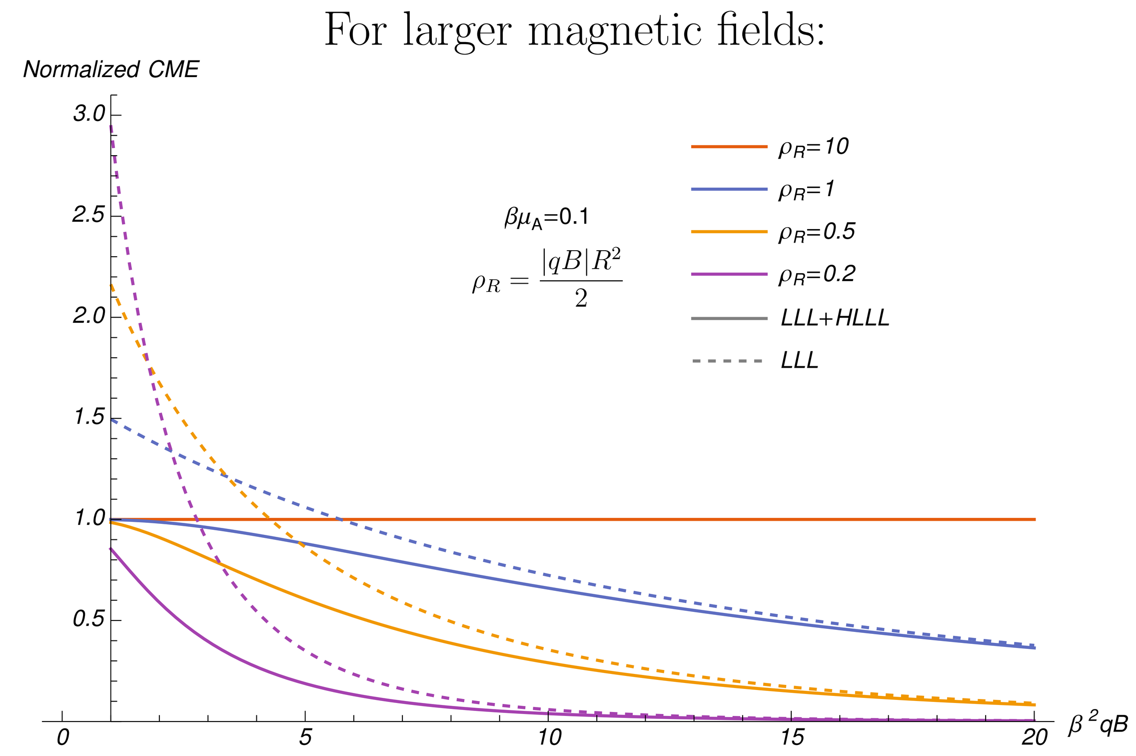



LLL = Lowest Landau level, HLL= higher Landau levels

Zooming in close to 1:



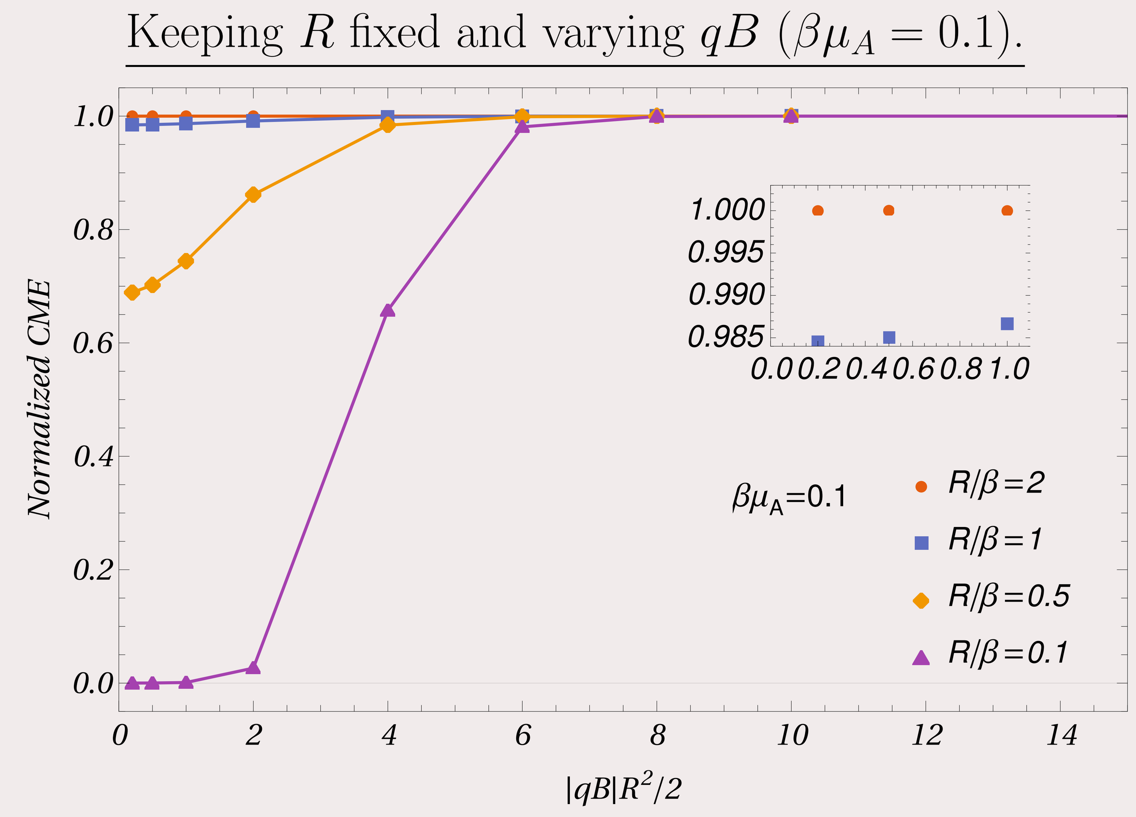

For larger magnetic fields:




## Main Result: Suppression of the CME

We found suppression when $|qB|/\mathcal{B}(\rho_R) > 1$.

Keeping $R$ fixed and varying $qB$ ($\beta\mu_A = 0.1$).

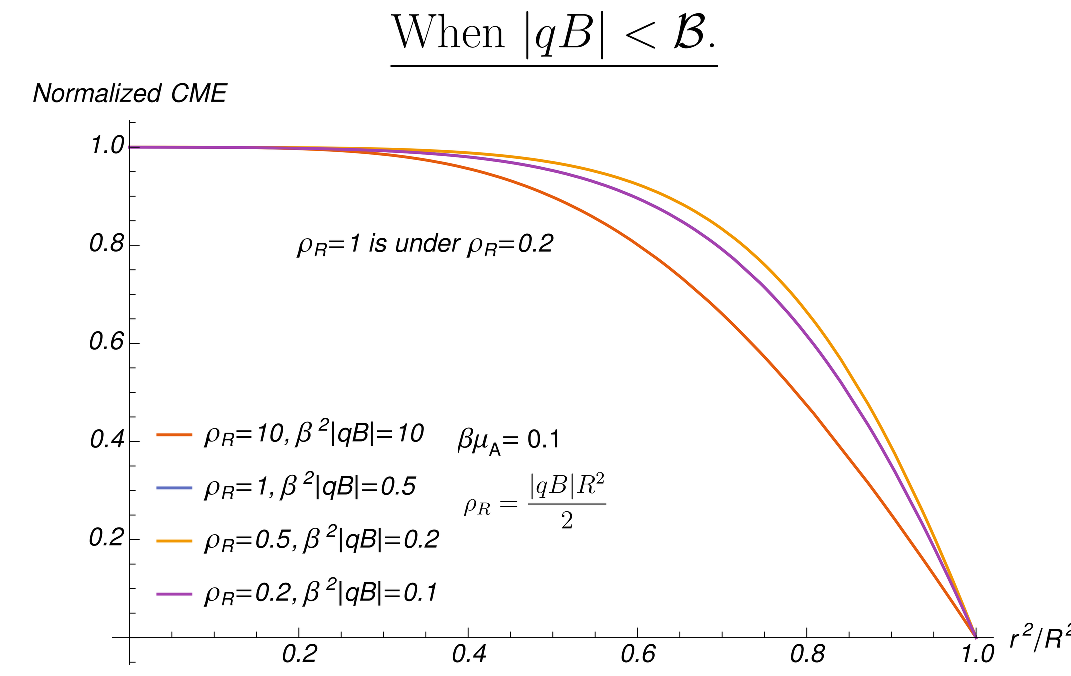



At fixed radius we found suppression for weak magnetic fields.

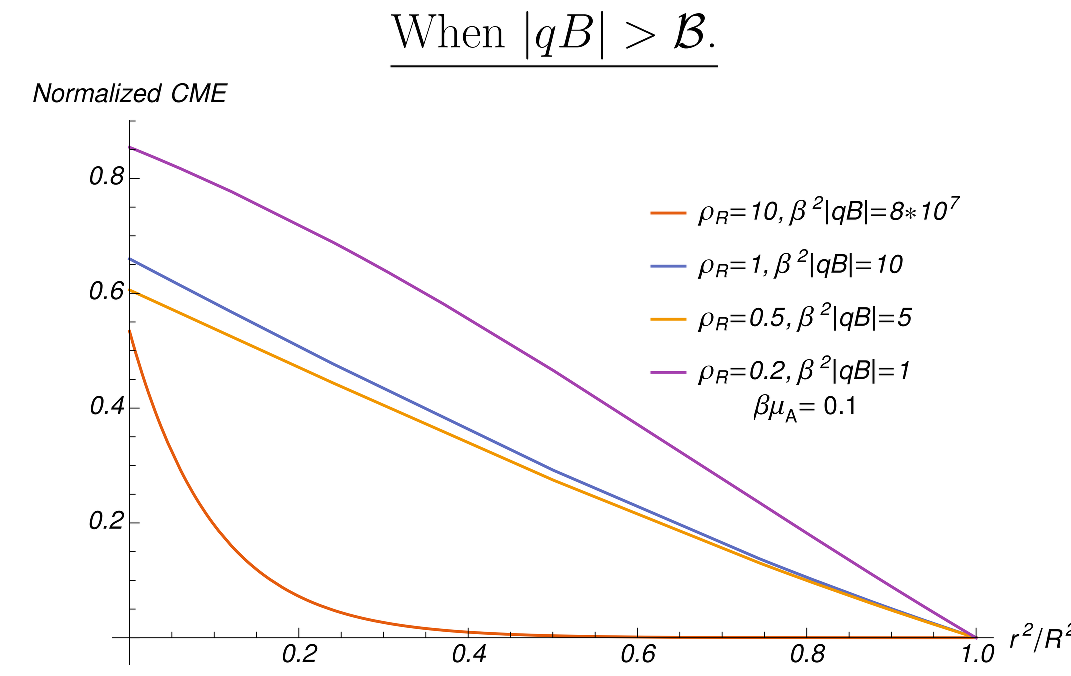
$$|qB| > \mathcal{B}(\rho_R)|_{\text{fixed } R} \Rightarrow |qB| < |qB|_0$$

## Radial profile of the CME current

When $|qB| < \mathcal{B}$.



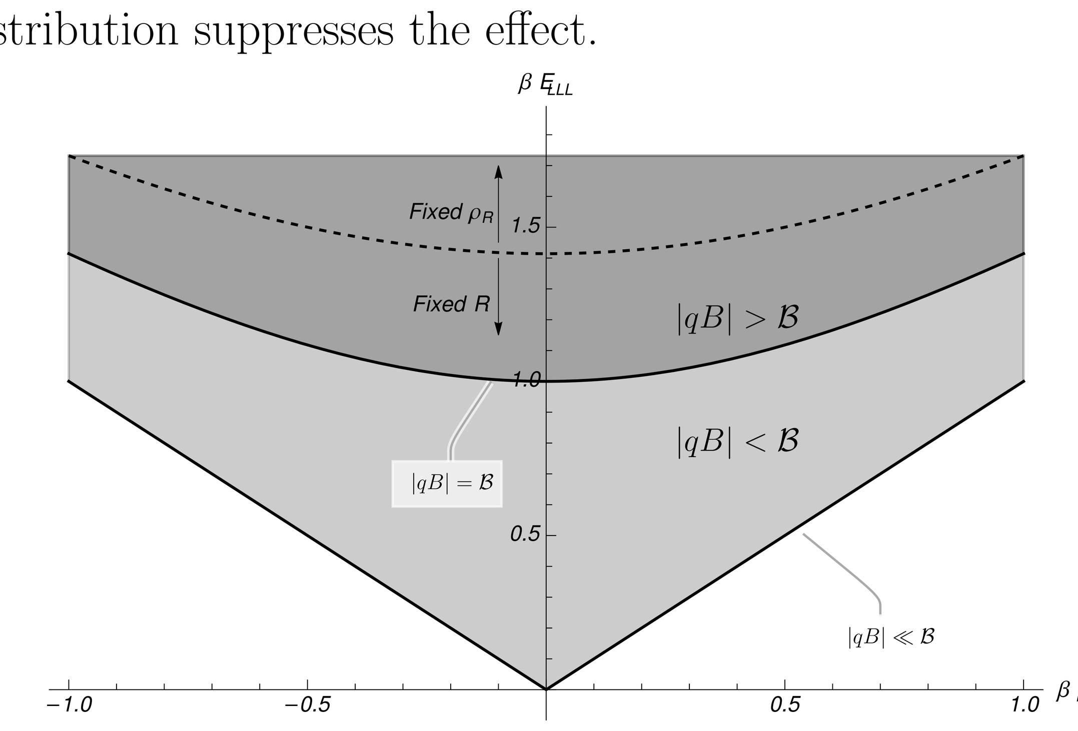

When $|qB| > \mathcal{B}$.




$r$: radial distance, $R$: cylinder radius.

## Qualitative explanation

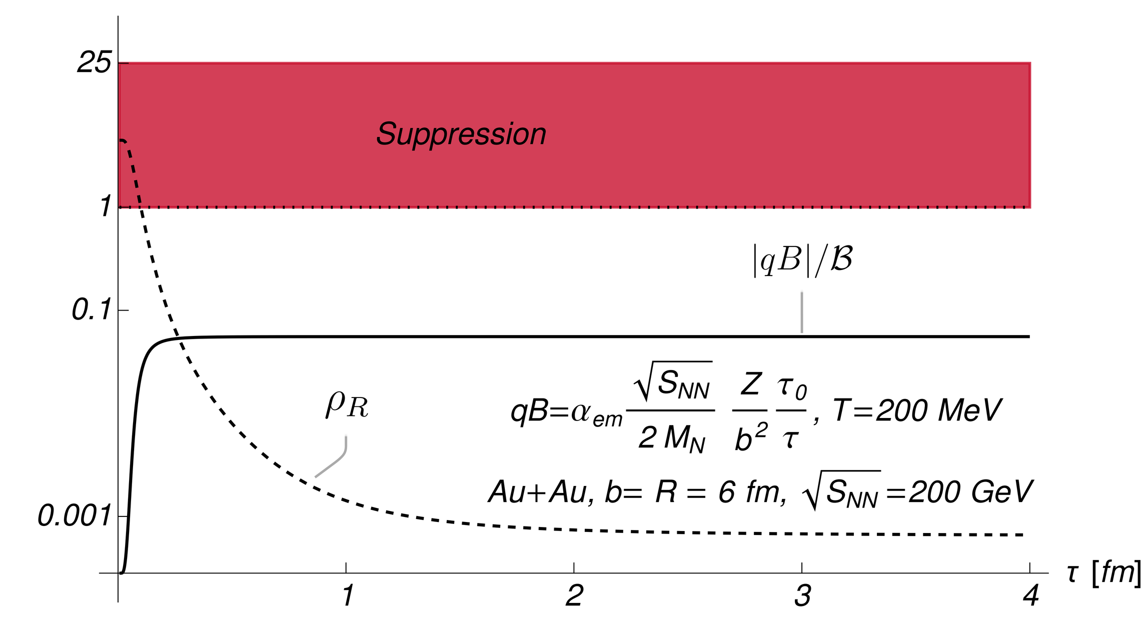
The CME is proportional to the Fermi-Dirac thermal distribution function. At higher energies the thermal distribution suppresses the effect.




The lowest Landau level (LLL) energy $E_{\rm LLL}$. When $|qB| < \mathcal{B}(\rho_R)$ the LLL lays in the light shaded area and for $|qB| \ll \mathcal{B}(\rho_R)$ it has negligible transverse momentum $2|qB|\lambda \approx 0$ (the lowest solid line). When $|qB| > \mathcal{B}(\rho_R)$ the LLL lays in the dark shaded area. The arrows indicate the shift of the LLL energy if we increase $|qB|$ at fixed $\rho_R$ or at fixed $R$.

## Conclusion

Consider the time evolution of a mid centrality collision




We estimate that $|qB|/\mathcal{B} < 1$:

**The volume effects do NOT suppress the CME in heavy-ion collisions.**

However, we estimate that possible contribution can be detected in Weyl/Dirac semi-metals.

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