

Hydrodynamic initial conditions from non-linear causality

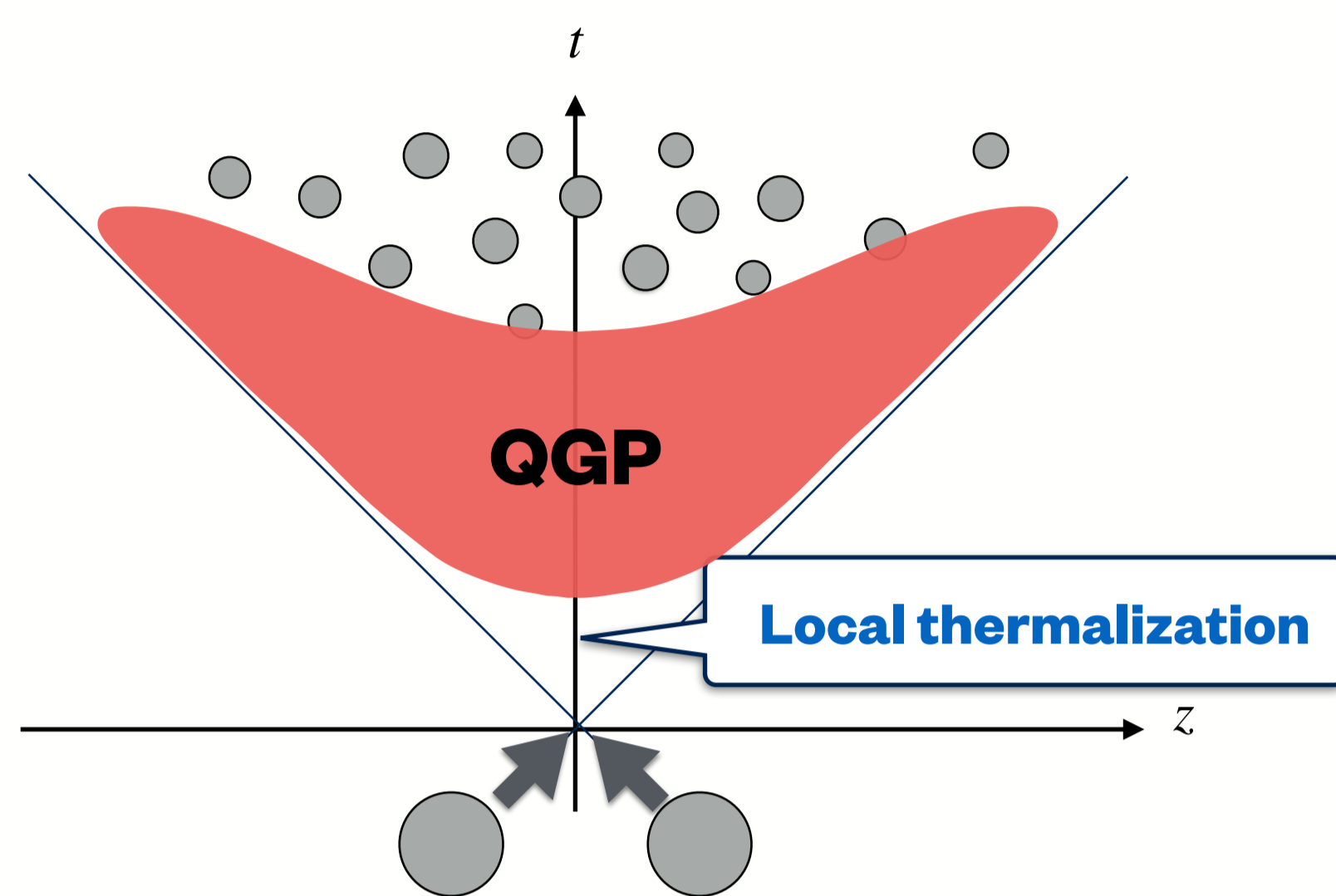


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Abstract: It is not at all trivial at which stage after the first contact the fluid picture can be applied. Whether non-linear hydrodynamic equations obey the causality depends on how far the system is away from local thermal equilibrium. Thus, for the system to be causal, initial conditions must be close to the equilibrium state. In this study, we apply the conditions obtained from causality to the conformal theory in a one-dimensionally expanding system, analyze how far the system can be away from local thermal equilibrium and constrain initial conditions so that the system can obey causality during the evolution.

1. Introduction



Press release
Discovery of QGP's perfect fluid behavior 04/18/2005

<https://www.bnl.gov/newsroom/news.php?a=110303>

Description by using relativistic hydrodynamics

At which stage after the first contact can the fluid picture be applied?

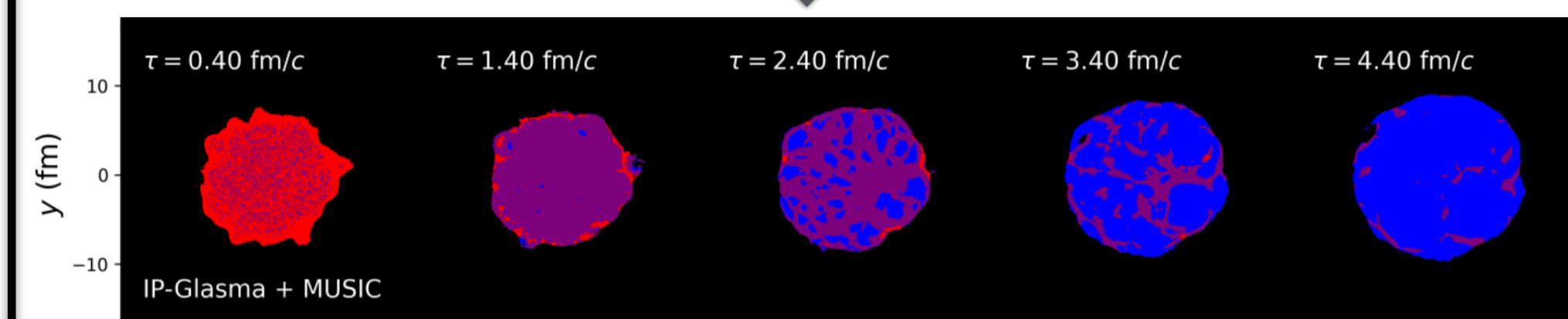
Necessary/sufficient conditions for causality in relativistic hydrodynamic equation

$$F_i(e, P, \Pi, \pi^{\mu\nu}, \dots) \geq 0 \quad (i = 1, 2, \dots, n)$$

Equilibrium variables Non-equilibrium variables

F.S. Bemfica et al., Phys. Rev. Lett. 126, 222301 (2021).

Apply to numerical simulation



Red: acausal \approx Away from equilibrium
Blue: causal \approx Close to equilibrium
Purple: indeterminate

Evolution in the transverse plane

C. Plumberg et al., Phys. Rev. C 105, L061901 (2022)

Causality violates in the area far away from equilibrium (?)

Purpose of this study Constrain initial conditions in a one-dimensionally expanding conformal system from a view point of causality

2. Model

Hydrodynamic model for conformal fluids

Hydrodynamic equations under Bjorken expansion J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

$$\frac{d}{d\tau} e = -\frac{4e}{3\tau} + \frac{\phi}{\tau}$$

$$\left(1 + \tau \frac{d}{d\tau}\right) \phi = \frac{4\eta}{3\tau} - \frac{4\tau_\pi}{3\tau} \phi - \frac{1}{2\eta} \frac{C_{\lambda_1}}{T} \phi^2$$

R. Baier et al., JHEP 0804, 100 (2008).

e : energy density $\phi = \pi^{00} - \pi^{33}$
 $\tau = \sqrt{t^2 - z^2}$: proper time
 τ_π : relaxation time η : shear viscosity
 C_{λ_1} : dimensionless constants
 T : temperature

Can we really describe the system far from equilibrium as a fluid?

Reynolds number

$$Re_\pi^{-1} = \sqrt{\frac{6\pi_{\mu\nu}\pi^{\mu\nu}}{e^2}} = \frac{3|\phi|}{e}$$

V.E. Ambrus et al., Phys. Rev. Lett. 130, 152301 (2023).

Conditions from causality

Conditions for conformal and Bjorken system

Violate necessary condition: **acausal**
Satisfy sufficient condition: **causal**

Example

One specific necessary condition:

$$e + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi} (2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_\pi\Pi}{4\tau_\pi} (\Lambda_d + \Lambda_a) \geq 0 \quad \Lambda_a: \text{An eigenvalue of } \pi^{\mu\nu}$$

Transport coefficients from AdS/CFT

$$\tau_\pi T = \frac{2 - \ln 2}{2\pi}, \quad \frac{\eta}{s} = \frac{1}{4\pi}$$

R. Baier et al., JHEP 0804, 100 (2008).
P. Kovtun et al., Phys. Rev. Lett. 94, 11601 (2005).

$$\text{Local rest frame: } \pi_{\text{Bj,LRF}}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\phi}{2} & 0 & 0 \\ 0 & 0 & \frac{\phi}{2} & 0 \\ 0 & 0 & 0 & -\phi \end{pmatrix}$$

$$(e + P) \left[1 - \frac{1}{2(2 - \ln 2)} \right] + \Lambda_a \geq 0$$

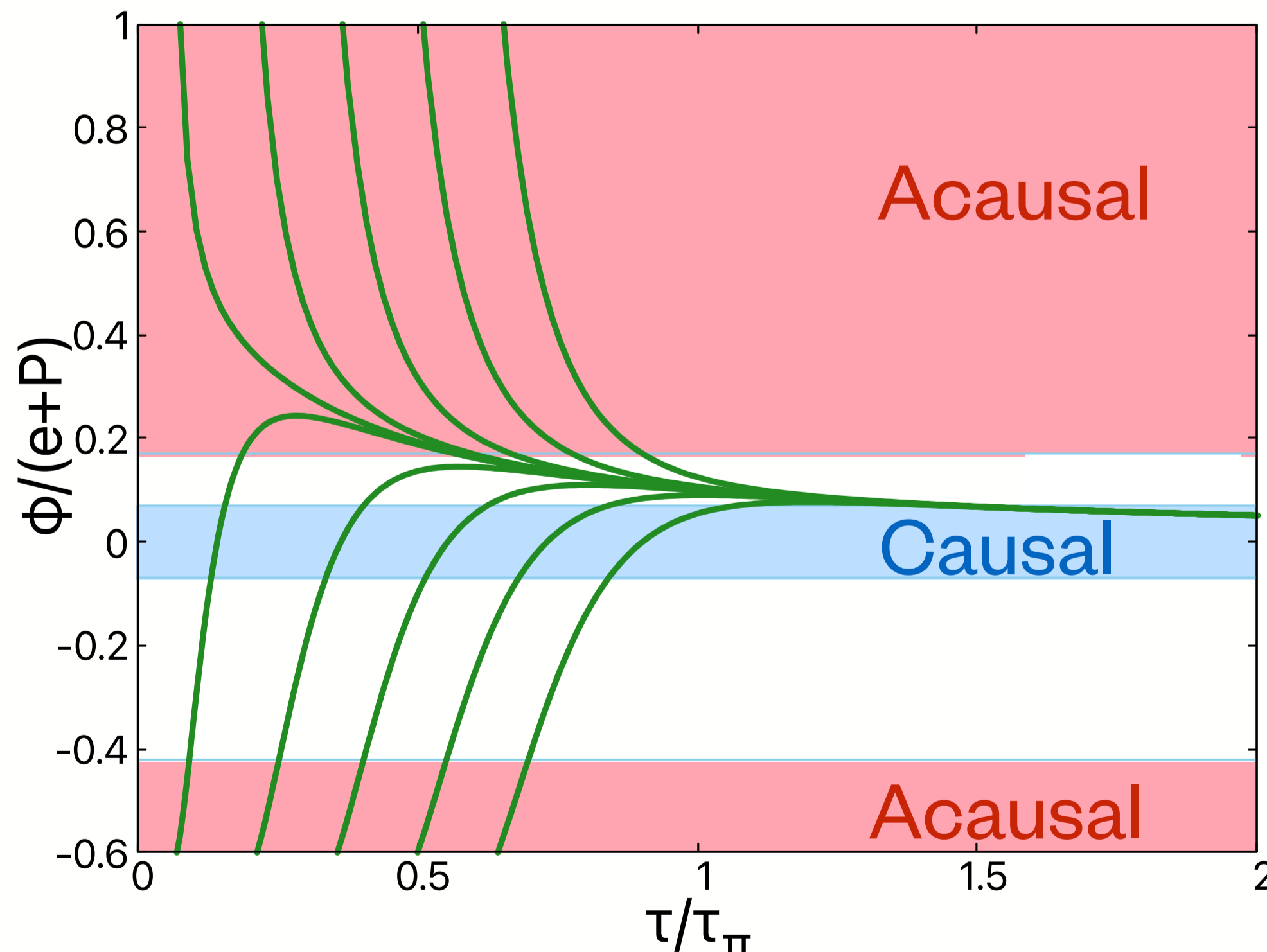
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Conditions for dissipative quantity

in conformal theory under Bjorken expansion

3. Results

Causal and acausal areas



Acausal:

$$\frac{\phi}{e + P} < -0.42, \quad \frac{\phi}{e + P} > 0.17$$

$$\Rightarrow Re_\pi^{-1} > 0.68$$

Causal:

$$-0.07 \leq \frac{\phi}{e + P} \leq 0.07$$

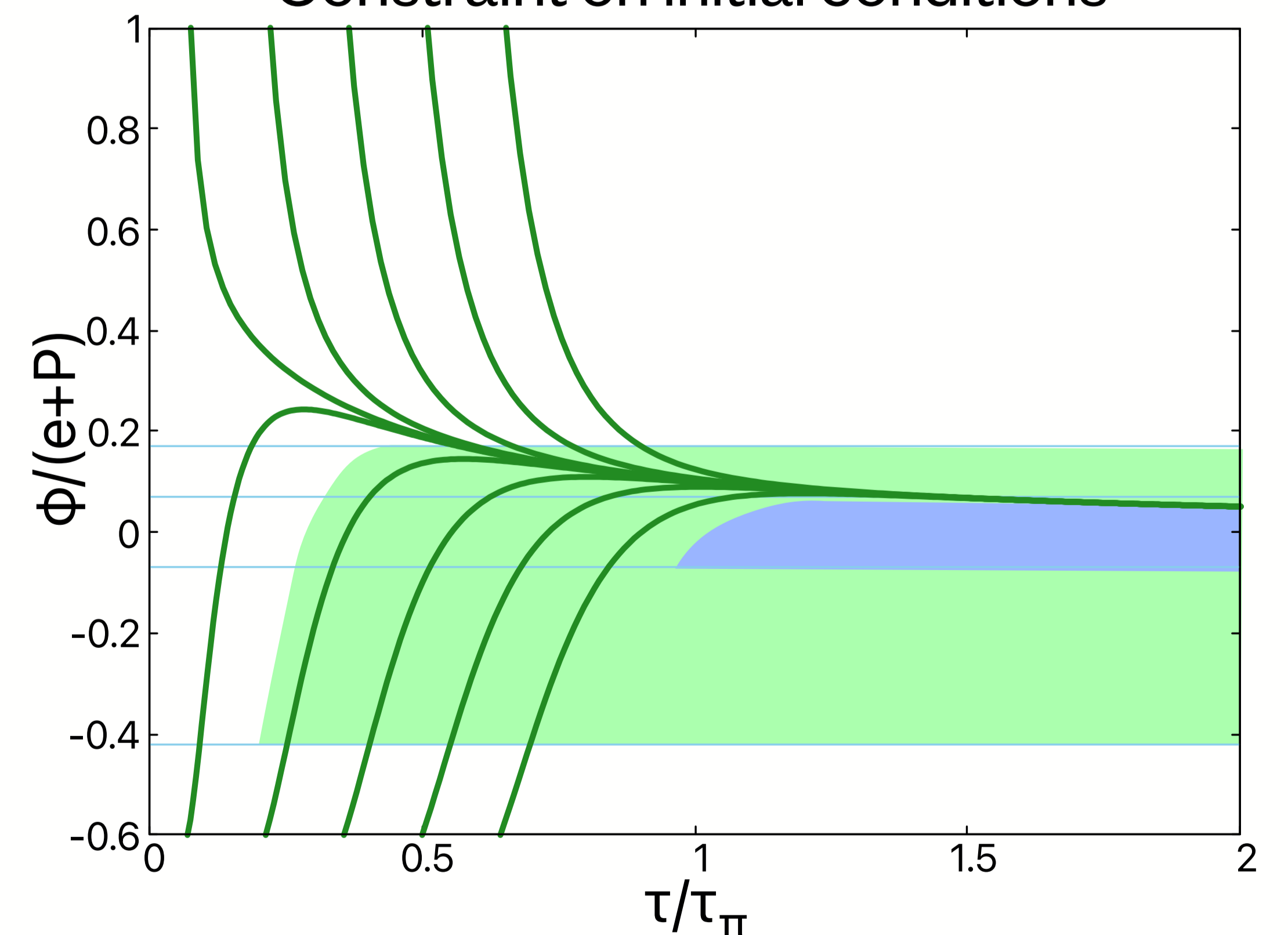
$$\Rightarrow Re_\pi^{-1} < 0.28$$

Others:

May or may not be acausal

- **Acausal** when the system is far from local equilibrium
- Any solutions in the acausal area are **not** acceptable.

Constraint on initial conditions



Solutions do **not** pass through **acausal** area

Always **causal**

4. Summary

- We analyze how far the one-dimensionally expanding system can be away from local thermal equilibrium from the causality.
- We **constrain the initial condition** of thermodynamic and dissipative variables in conformal theory under Bjorken expansion.
 - At least, one should take initial conditions which do not pass through the acausal area.
 - There is little room of initial conditions for the system to strictly obey the causality.