Exploring Neutron Stars with Three Conserved Charges in a Newly Optimized C++ Chiral Mean Field Code

Nikolas Cruz-Camacho, Jacquelyn Noronha-Hostler
Department of Physics, University of Illinois at Urbana-Champaign

Rajesh Kumar, Mateus Pelicer, Veronica Dexheimer
Department of Physics, Kent State University

Andrew Manning, Roland Haas
NCSA, University of Illinois at Urbana-Champaign

Introduction

- To comprehensively investigate the interior of a neutron star, one must take into account an equation of state that encompasses the entire baryon octet, decuplet, and quarks, along with their intricate interactions.
- The Chiral Mean Field (CMF) is a relativistic mean field model with quarks and hadrons where interactions are mediated via meson exchange (σ, ζ, δ, ω, ϕ, and ρ). [1]
- CMF is a non-linear $SU(3)$ extension of the linear Sigma model that has been fitted to agree with low- and high-energy physics data. [2]
- CMF uses a Polyakov-inspired loop (Φ) as an order parameter for the deconfinement phase transition (Chiral symmetry restoration). [3]
- At finite temperature, CMF has a critical point (T_c = 167 [MeV], $\mu_{B,c}$ = 354 [MeV]) and a chiral first-order phase transition. [3]
- CMF spans over μ_B, μ_S, μ_Q, T and magnetic field (B), allowing simulations of heavy-ion collisions and neutron stars. [4]
- **A full runtime with all effects in the Fortran legacy version takes a couple of months.**

Lagrangian

The mean field Lagrangian is written as

$$\mathcal{L}_{\rm CMF} = \mathcal{L}_{kin} + \mathcal{L}_{int} + \mathcal{L}_{self} + \mathcal{L}_{SB} - U_\Phi,$$

where $\mathcal{L}_{kin}$ stands for kinetic, $\mathcal{L}_{int}$ for interaction, $\mathcal{L}_{self}$ for self interactions, $\mathcal{L}_{SB}$ for symmetry breaking, and U_Φ is a Polyakov-inspired induced potential.

Equations of Motion

Upon applying the Euler-Lagrange equation to each mean field variable, we have derived the subsequent algebraic system of equations

$$\sigma: \quad \sum_i g_{i\sigma}\rho_{s,i} = -k_0\chi_0^2\sigma + 4k_1(\sigma^2+\zeta^2+\delta^2)\sigma + 2k_2(\sigma^2+3\delta^2)\sigma + 2k_3\chi_0\sigma\zeta + \frac{2\epsilon}{3}\chi_0^4\frac{\sigma}{\sigma^2-\delta^2} - m_\pi^2 f_\pi,$$

$$\delta: \quad \sum_i g_{i\delta}\rho_{s,i} = -k_0\chi_0^2\delta + 4k_1(\sigma^2+\zeta^2+\delta^2)\delta + 2k_2(3\sigma^2+\delta^2)\delta - 2k_3\chi_0\delta\zeta - \frac{2\epsilon}{3}\chi_0^4\frac{\delta}{\sigma^2-\delta^2},$$

$$\zeta: \quad \sum_i g_{i\zeta}\rho_{s,i} = -k_0\chi_0^2\zeta + 4k_1(\sigma^2+\zeta^2+\delta^2)\zeta + 4k_2\zeta^3 + k_3\chi_0(\sigma^2-\delta^2) + \frac{\epsilon}{3\zeta}\chi_0^4 - \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right),$$

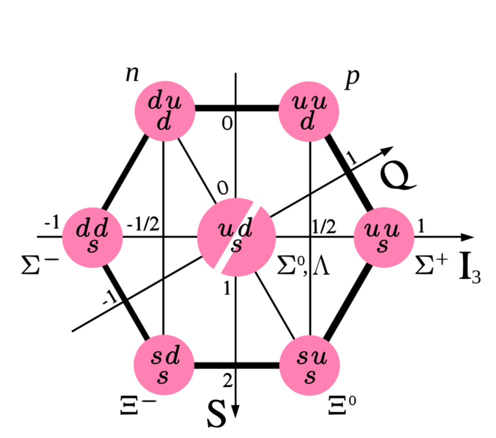
$$\omega: \quad \sum_i g_{i\omega}n_{B,i} = m_\omega^2\omega - 2g_4\begin{cases} \text{C1: } 2\omega\left(\omega^2+3\rho^2\right), \\ \text{C2: } \omega\left(2\omega^2+3\phi^2\right), \\ \text{C3: } 2\omega\left(\omega^2+\rho^2+\phi^2\right), \\ \text{C4: } \left(2\omega^3+3\phi^2\omega+3\sqrt{2}\phi\omega^2+\frac{\phi^3}{\sqrt{2}}\right), \end{cases}$$

$$\phi: \quad \sum_i g_{i\phi}n_{B,i} = m_\phi^2\phi - 2g_4\begin{cases} \text{C1: } 4\phi^3, \\ \text{C2: } \phi\left(\phi^2+3\left(\omega^2+\rho^2\right)\right), \\ \text{C3: } 2\phi\left(\omega^2+\phi^2+\rho^2\right), \\ \text{C4: } \frac{\phi^3}{2}+3\omega^2\phi+\sqrt{2}\omega^3+\frac{3}{\sqrt{2}}\omega\phi^2, \end{cases}$$

$$\rho: \quad \sum_i g_{i\rho}n_{B,i} = m_\rho^2\rho - 2g_4\begin{cases} \text{C1: } 2\rho(3\omega^2+\rho^2), \\ \text{C2: } \rho(3\phi^2+2\rho^2), \\ \text{C3: } 2\rho(\omega^2+\phi^2+\rho^2), \\ \text{C4: } 0, \end{cases}$$

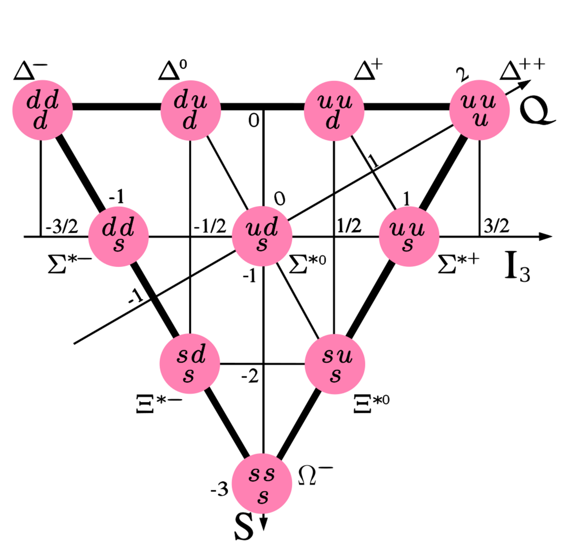
$$\Phi: \quad \sum_i g_{i\Phi}\rho_{s,i} = 2(a_1\mu_B^4)\Phi + a_3T_0^4\frac{12\Phi}{3\Phi^2-2\Phi-1},$$

Particles considered



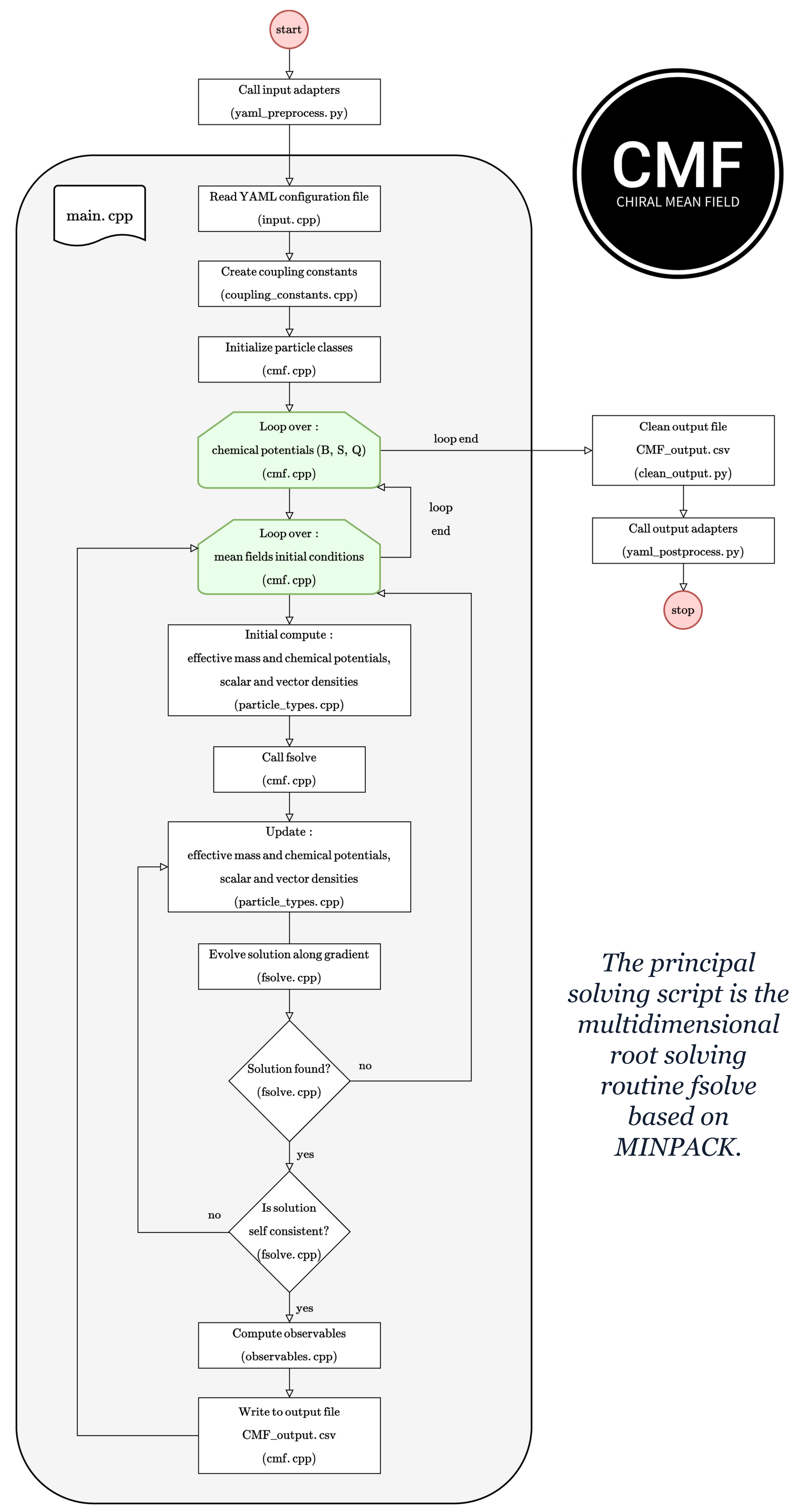
Baryon octet

SU(3) quarks



Baryon decuplet

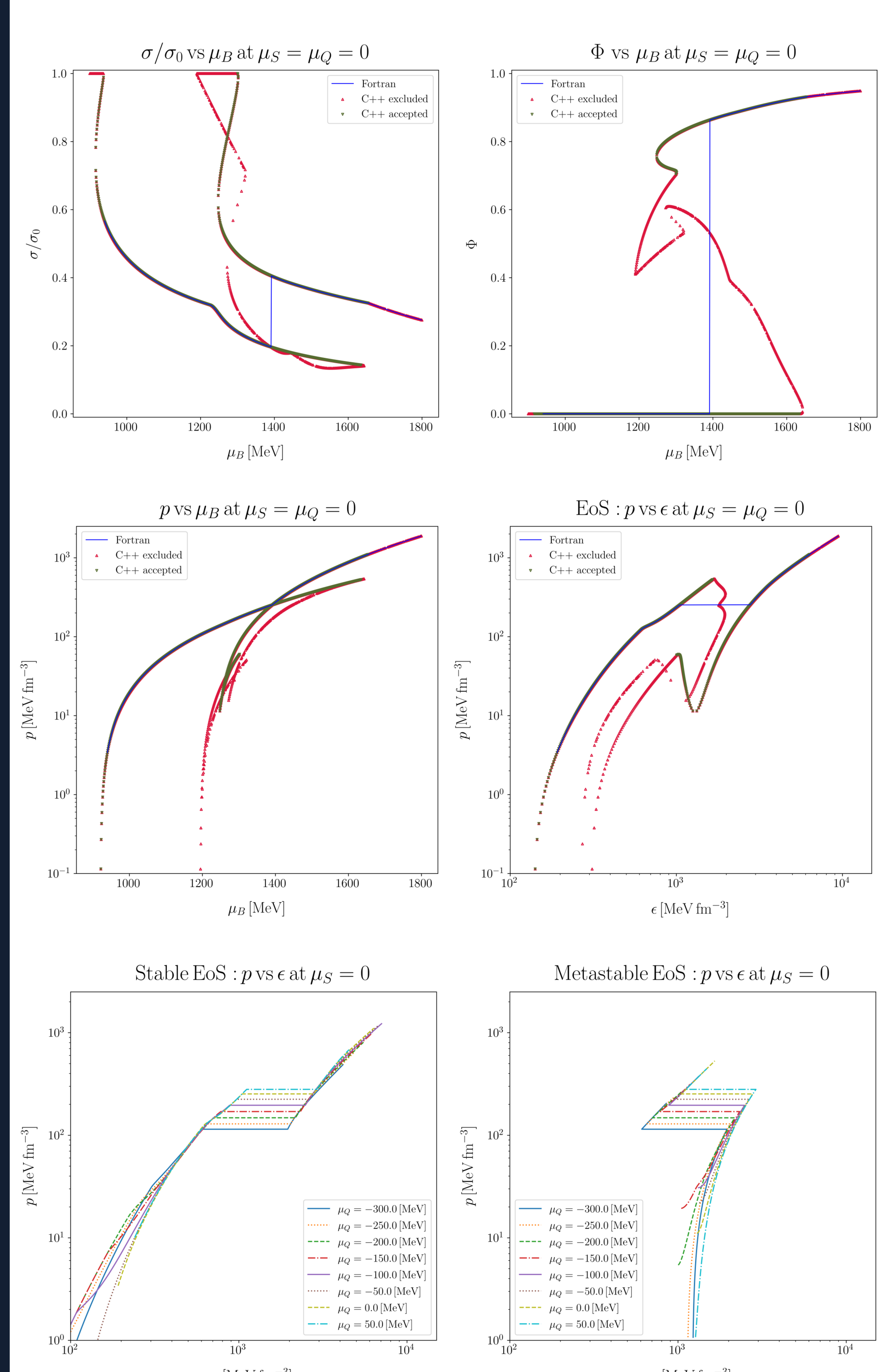
Simplified C++ and Python flowchart

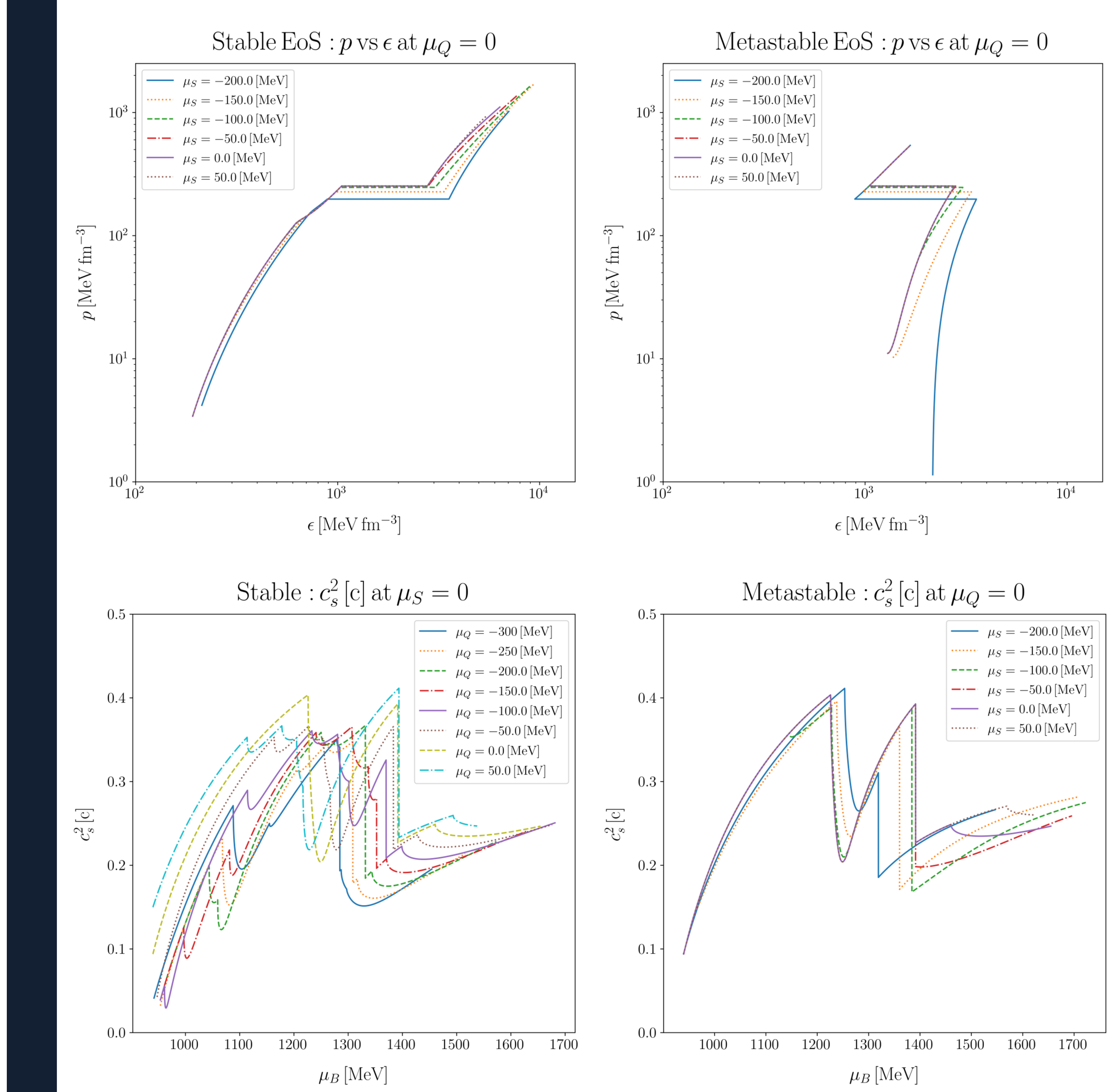


The principal solving script is the multidimensional root solving routine fsolve based on MINPACK.

Simplified flowchart for a standard CMF execution. Enclosed are described the details of the C++ main routine.

Main results





Conclusions

- The MUSES collaboration has recently introduced a novel C++20 implementation of the Chiral Mean Field model at zero temperature.
- The C++ code matches up to high numerical precision with the legacy Fortran version with an improved runtime of an order of magnitude and an easier way to handle exotic particles.
- The C++ code allows studying the spinodal solutions around the quark deconfinement phase transition with resolution unreachable by the legacy code.
- Observables, like the speed of sound or susceptibilities, can be computed natively within the code framework.

Outlook

- Incorporate antiparticles and temperature dependence with the correspondent mesonic background.
- Add magnetic field and anomalous magnetic moments effects.
- Couple it with numerical relativity codes, e.g., heavy-ion hydrodynamic simulations.

References

[1] Papazoglou, P., et all. "Nuclei in a chiral SU(3) model,“ Phys. Rev. C **59**, 411–427 (1999). DOI: 10.1103/PhysRevC.59.411

[2] Dexheimer, V. and Schramm, S. "Proto-Neutron and Neutron Stars in a Chiral SU(3) Model," Astrophys. J. **683**, 943 (2008). DOI: 10.1086/589735

[3] Dexheimer, V. A. and Schramm, S. "Novel approach to modeling hybrid stars," Phys. Rev. C **81**, 045201 (2010). DOI: 10.1103/PhysRevC.81.045201

[4] Dexheimer, V., Negreiros, R., and Schramm, S. "Hybrid stars in a strong magnetic field," Eur. Phys. J. A **48**, 189 (2012). DOI: 10.1140/epja/i2012-12189-y

Acknowledgments

RH acknowledges funding through NSF grants OAC-2004879, OAC-2103680, and OAC-1238993.