Bulk Viscosity of Hadronic Matter from a Microscopic Transport Model

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Abstract

Ultra-relativistic heavy-ion collisions at RHIC are thought to have created a Quark-Gluon-Plasma (QGP) with a very low shear viscosity in the deconfined phase. However, as the QGP hadronizes it will evolve through a hadronic phase with rapidly increasing shear viscosity. In order to fully characterize the QGP state, one has to separately determine the viscosity of the hadronic phase. Although many approaches have been used to determine the shear viscosity coefficient and the associated shear viscosity to entropy density ratio η/s in the hadronic phase, much is unknown regarding the bulk viscosity to shear viscosity coefficient ζ/s in the hadronic phase. We present preliminary results of an estimate of the bulk viscosity ζ for a hot hadronic gas of massive pions interacting through an elastic cross section. The Ultrarelativistic Quantum Molecular Dynamics (UrQMD) model is used to simulate the hadronic medium and periodic boundary conditions are used to simulate infinite hot equilibriated hadronic matter. The Green-Kubo formalism is employed where we work in the microcanonical ensemble and a comparison is made with the results of the bulk viscosity calculation from the Simulating Many Accelerated Strongly Interacting Hadrons (SMASH) model .

Introduction

Ultrarelativistic heavy-ion collision experiments allow physicists to study the behavior of nuclear matter at extremely high temperatures. In such experiments, high-energy collisions of heavy nuclei are used to heat a volume of matter up to temperatures that exceed the critical temperature (Tc ~ 155 MeV) necessary to create a quark-gluon plasma(QGP). The QGP is a state of matter in which quarks and gluons are deconfined, commonly known as a "quark soup." The early universe was expected to be in a QGP microseconds after the Big Bang, in the form of a "near perfect fluid". [1] Naturally, whenever near perfect fluid behavior is mentioned, it becomes crucial to characterize the viscosity coefficients. One should note that in any attempt to quantify the transport coefficients of the QGP, the post-QGP hadronic effects need to be unmasked when interpreting the final state observables. [2] As such, it is necessary to determine separately the viscosity of the hadronic phase. Several different approaches have been used to calculate the hadronic bulk viscosity, such as the Chapman-Enskog method [3]. A very interesting estimate of the bulk viscosity from the Simulated Many Accelerated Strong Interacting Hadrons (SMASH) model was performed, which motivates the need for a separate comparison with a different hadronic microscopic transport model, which is one goal of this work. We use the Ultrarelativistic Quantum Molecular Dynamics (UrQMD) model with periodic boundary conditions to simulate infinite equilibriated hadronic matter, and employ the Green-Kubo method to estimate the bulk viscosity coefficient.

Objectives

- **Use UrQMD to simulate infinite hadronic matter.**
- **Use the Green-Kubo method to estimate the bulk viscosity coefficient.**
- **Compare our results for the bulk viscosity with those from SMASH for a simple test system.**

Methodology

Part I: Simulating the Hadronic Medium: UrQMD

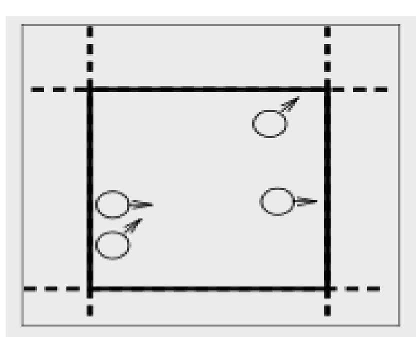
UrQMD is a microscopic transport model based upon the Boltzmann equation:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{E}\cdot\nabla_r\right) f = \mathcal{C}_{coll}$$

$$\mathcal{C}_{coll} = N \int \sigma d\Omega \int d\vec{p}_2 |\vec{v}_1 - \vec{v}_2| (f(\vec{p}_1')f(\vec{p}_2') - f(\vec{p}_1)f(\vec{p}_2))$$

-Collision criterion: $d_{min} \leq \sqrt{\frac{\sigma_{tot}}{\pi}}$

- Use periodic boundary conditions to force system into equilibrium!



Test system: Gas of massive pions ($m_\pi = 138$ MeV) interacting with elastic cross section of $\sigma_{elastic} = 20$ mb, confined to a box of size L=20 fm.

Methodology (cont.)

Part II: Green Kubo Formalism

The Green-Kubo method is a systematic way of calculating linear transport coefficients for a system near equilibrium. The transport coefficient is computed by calculating the relevant correlation of fluctuations of the relevant function near equilibrium. In the case of the bulk viscosity coefficient, the relevant Green-Kubo relation is

$$\zeta = \frac{1}{T}\int d^3x \int dt < \Delta\Pi(\vec{0},0)\Delta\Pi(\vec{r},t) >$$

where $\Pi \equiv P - \left(\frac{\partial P}{\partial \varepsilon}\right)_n \varepsilon - \left(\frac{\partial P}{\partial n}\right)_\varepsilon n$ is the pressure current [4,5].

For point particles uniformly distributed in real space, this becomes

$$\zeta = \frac{V}{T}\int dt < \Delta\Pi(0)\Delta\Pi(t) >$$

For our test system, total particle number and energy are conserved implying the system is in the microcanonical ensemble! → $\Delta n(t) = \Delta\varepsilon(t) = 0$

Hence $\zeta = \frac{V}{T}\int dt < \Delta P(0)\Delta P(t) >$

$T^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x,p)$ is the energy momentum tensor. For a system of point particles uniformly distributed in real space, $f(x,p) = \frac{1}{V}\sum_{j=1}^{N_{part}} \delta(\vec{p} - \vec{p_j})$

implying the pressure is $P = \frac{1}{V}\sum_{j=1}^{N_{part}} \frac{|\vec{p_j}|^2}{3E_j}$

For our test system, we assume the correlation function of the pressure-pressure current has an exponential ansatz:

$$< \Delta P(0)\Delta P(t) > = Ae^{-t/\tau_\zeta} \text{ implying that } \zeta = \tau_\zeta \frac{VA}{T}$$

Preliminary Results and Comparison

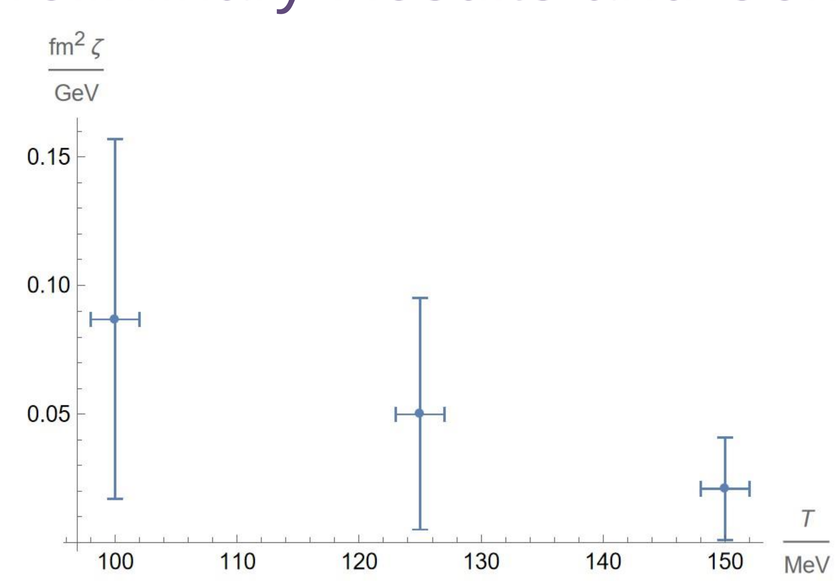


Fig 1: Preliiminary results of bulk viscosity coefficient from UrQMD for a gas of massive pions elastically scattering.

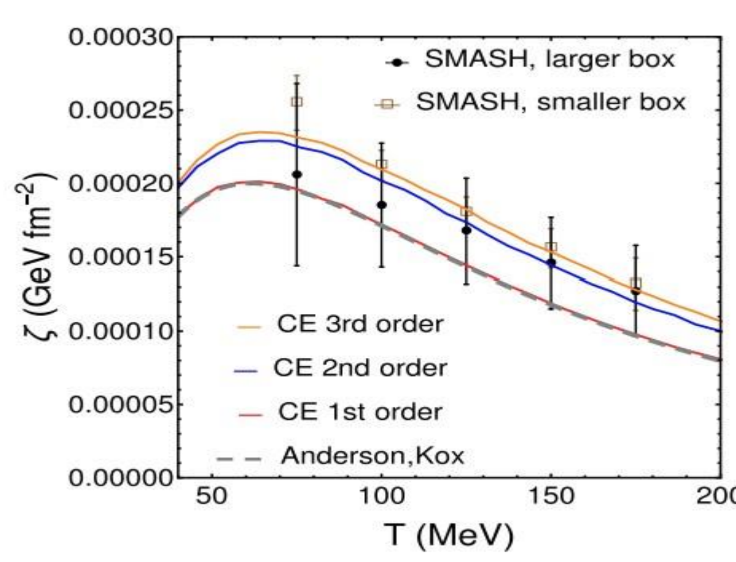


Fig 2: Results of bulk viscosity coefficient from [4] with comparisons for an gas of massive pions elastically scattering. L=20 fm for smaller box for T=100-175 MeV.

Conclusions/Outlook

-Used UrQMD with Green Kubo formalism to estimate bulk viscosity and compared with the Chapman-Enskog method and the result from SMASH.

- Uncertainties very large : need to reduce them and investigate validity of exponential ansatz .
- Extend calculation to include pion pion scattering through intermediate rho resonance: note that a simple exponential ansatz was found to not be sufficient when resonances are included in [4].

References

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Acknowledgements

N. Demir thanks the KU Research Sector for travel support and Duke University Physics Department for access to its computer servers to perform simulations remotely.

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