

ALIGNMENT FROM SPIN-1 HYDRODYNAMICS

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I. Abstract

- We formulate a theory of hydrodynamics for a fluid consisting of massive spin-1 particles.
- The underlying kinetic theory features local and nonlocal collisions, which influence the transport coefficients.
- We compute an expression for the alignment of particles, which depends on the shear stress of the medium.

II. Quantum kinetic theory for spin-1 particles

- The starting point is given by the theory of interacting massive Proca fields

$$\mathcal{L} = -\hbar \left[\frac{1}{2} \left(\partial^\mu V^{\dagger\nu} - \partial^\nu V^{\dagger\mu} \right) (\partial_\mu V_\nu - \partial_\nu V_\mu) - \frac{m^2}{\hbar^2} V^{\dagger\mu} V_\mu \right] + \mathcal{L}_{\text{int}} . \quad (1)$$

- The suitable tool for formulating a quantum kinetic theory is the covariant Wigner function

$$W^{\mu\nu}(x, k) := -\frac{2}{(2\pi)^4 \hbar^5} \int d^4 v e^{-\frac{i}{\hbar} k^\alpha v_\alpha} \langle :V_+^{\dagger\mu} V_-^\nu: \rangle , \quad V_\pm^\mu := V^\mu \left(x \pm \frac{v}{2} \right) . \quad (2)$$

- It is convenient to *enlarge phase space* by introducing a “spin” variable \mathfrak{s}^μ to be able to work with a scalar distribution function

$$f(x, k, \mathfrak{s}) := \frac{1}{3} W_\mu^\mu + \frac{i}{2m} \epsilon^{\mu\nu\alpha\beta} \mathfrak{s}_\mu k_\nu W_{\alpha\beta} + \frac{5}{8} \left(\mathfrak{s}^\mu \mathfrak{s}^\nu + \frac{2}{3} g^{\mu\nu} \right) W_{\mu\nu} . \quad (3)$$

- We may take this distribution to be on the mass shell,

$$f(x, k, \mathfrak{s}) = 4\pi\hbar\delta(k^2 - m^2) f(x, k, \mathfrak{s}) . \quad (4)$$

- This function then fulfills a kinetic equation

$$k^\alpha \partial_\alpha f(x, k, \mathfrak{s}) = \int d\Gamma_1 d\Gamma_2 d\Gamma' d\bar{S}(k) \mathcal{W} [f(x + \Delta_1 - \Delta, k_1, \mathfrak{s}_1) f(x + \Delta_2 - \Delta, k_2, \mathfrak{s}_2) - f(x + \Delta' - \Delta, k', \mathfrak{s}') f(x, k, \bar{\mathfrak{s}})] , \quad (5)$$

with the measure of the enlarged phase space

$$d\Gamma := d^4 k \delta(k^\alpha k_\alpha - m^2) dS(k) , \quad dS(k) := \frac{3m}{2\sqrt{2}\pi} d^4 \mathfrak{s} \delta(\mathfrak{s}^\alpha \mathfrak{s}_\alpha + 2) \delta(k^\alpha \mathfrak{s}_\alpha) . \quad (6)$$

- Both local and **nonlocal** contributions are present

→ *Spin and orbital angular momentum can be exchanged during a collision!*

- The local-equilibrium distribution function f_{eq} that maximizes the entropy has to make the local collision term vanish $\mathfrak{C}_{\text{local}}[f_{\text{eq}}] = 0$. This implies that f_{eq} depends on the summational invariants, i.e.,

- the linear *four-momentum* k^μ , and
- the *total angular momentum* $J^{\mu\nu} := \Delta^\mu k^\nu - \Delta^\nu k^\mu + \hbar \Sigma_s^{\mu\nu}$,

where $\Sigma_s^{\mu\nu} := -\epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} \mathfrak{s}_\beta$. The local-equilibrium distribution function then reads

$$f_{\text{eq}}(x, k, \mathfrak{s}) = \exp \left[-\beta^\mu(x) k_\mu + \frac{\hbar}{2} \Omega_{\mu\nu}(x) \Sigma_s^{\mu\nu} \right] + \mathcal{O}(\hbar^2) , \quad (7)$$

where $\beta^\mu(x) = \beta(x) u^\mu(x)$ and $\Omega^{\mu\nu}(x)$ are *spacetime-dependent Lagrange multipliers*.

References

- [1] DW, N. Weickgenannt, E. Speranza, arXiv: 2306.05936
- [2] DW, N. Weickgenannt, E. Speranza, Phys.Rev.Res. 5 1, 013187 (2023)
- [3] DW, N. Weickgenannt, D. H. Rischke, Phys.Rev.D 106 11, 116021 (2022)
- [4] DW, A. Palermo, V. E. Ambruš, Phys.Rev.D 106 1, 016013 (2022)
- [5] N. Weickgenannt, DW, E. Speranza, D. H. Rischke, Phys.Rev.D 106 9, 096014 (2022)

III. Moment expansion

- Split the distribution function into a local-equilibrium and a nonequilibrium part,

$$f(x, k, \mathfrak{s}) = f_{\text{eq}}(x, k, \mathfrak{s}) + \delta f_{\mathfrak{k}\mathfrak{s}} . \quad (8)$$

- Expand the nonequilibrium part in a complete orthogonal basis of irreducible tensors,

$$1, k^{\langle\mu\rangle}, k^{\langle\mu k^\nu\rangle}, \dots , \text{ where } k^{\langle\mu_1 \dots k^{\mu_\ell}\rangle} := \Delta_{\nu_1 \dots \nu_\ell}^{\mu_1 \dots \mu_\ell} k^{\nu_1} \dots k^{\nu_\ell} . \quad (9)$$

- Knowing the time evolution of the irreducible moments

$$\rho_r^{\mu_1 \dots \mu_\ell} := \int d\Gamma E_k^r k^{\langle\mu_1 \dots k^{\mu_\ell}\rangle} \delta f_{\mathfrak{k}\mathfrak{s}} , \quad (10a)$$

$$\tau_r^{\mu, \mu_1 \dots \mu_\ell} := \int d\Gamma E_k^r \mathfrak{s}^\mu k^{\mu, \langle\mu_1 \dots k^{\mu_\ell}\rangle} \delta f_{\mathfrak{k}\mathfrak{s}} , \quad (10b)$$

$$\psi_r^{\mu\nu, \mu_1 \dots \mu_\ell} := \int d\Gamma E_k^r K_{\alpha\beta}^{\mu\nu} \mathfrak{s}^\alpha \mathfrak{s}^\beta k^{\langle\mu_1 \dots k^{\mu_\ell}\rangle} \delta f_{\mathfrak{k}\mathfrak{s}} \quad (10c)$$

is equivalent to solving the complete Boltzmann equation.

→ Needs to be truncated!

- Truncation specified by sets $\mathbb{S}_\ell^{(j)}$, $j = 0, 1, 2, \ell \in \mathbb{N}_0$.

IV. Alignment

- Goal of this work: Study the effects of hydrodynamics on the **tensor polarization**,

$$\bar{\Theta}^{\mu\nu} = \int dK N(k) \Theta^{\mu\nu}(k) = \frac{1}{2} \sqrt{\frac{3}{2}} \int d\Sigma_\lambda \left(u^\lambda \psi_1^{\mu\nu} + \psi_0^{\mu\nu, \lambda} \right) . \quad (11)$$

- Connected to (global) **alignment** through

$$\rho_{00} - \frac{1}{3} = -\sqrt{\frac{2}{3}} \epsilon_\mu^{(0)} \epsilon_\nu^{(0)} \bar{\Theta}^{\mu\nu} . \quad (12)$$

- Genuine spin-1 effect, does not exist for spin-1/2 particles.

- Choose the simplest truncation: $\mathbb{S}_2^{(0)} = \{0\}$, $\mathbb{S}_0^{(2)} = \{1\}$, all others disregarded.

- Equations of motion for the moment $\psi_1^{\mu\nu}$ give in the long-time (Navier-Stokes) limit

$$\psi_1^{\mu\nu} = \xi \beta \pi^{\mu\nu} , \quad (13)$$

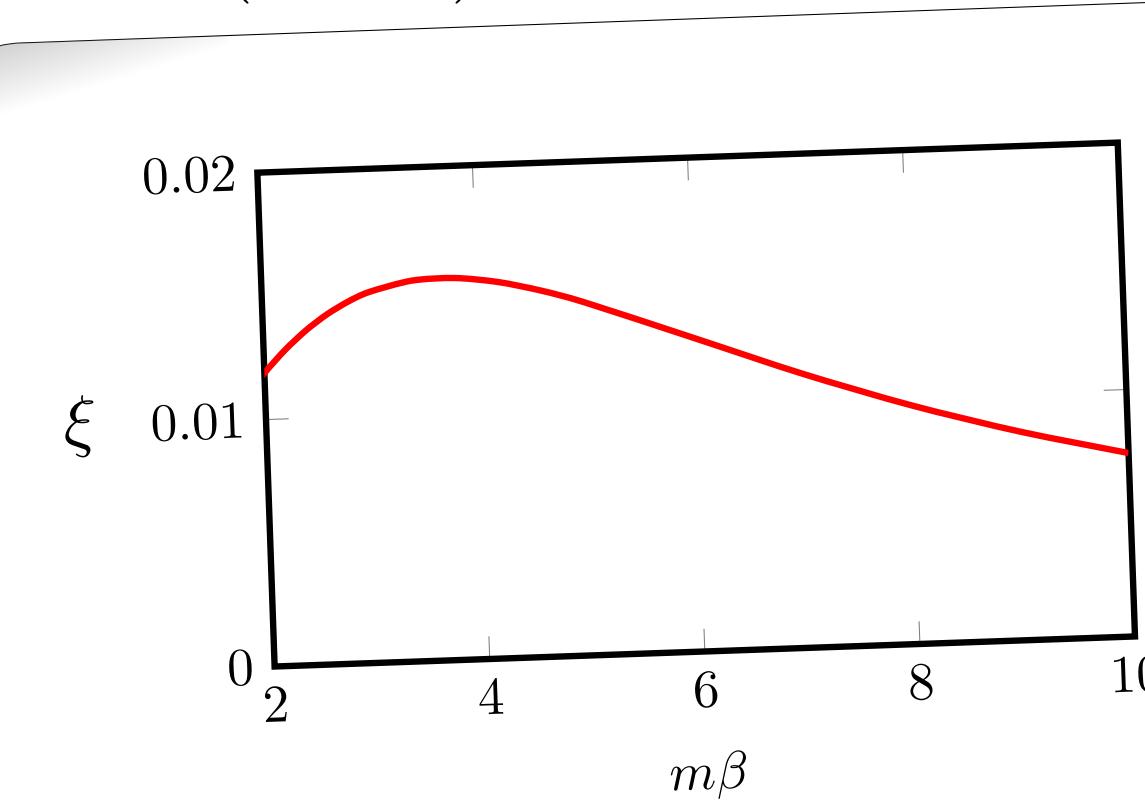
where ξ depends on the details of the interaction.

→ *Tensor polarization of particles is sourced by shear stress of the medium!*

- Reminder: Vector polarization of particles is sourced through the vorticity of the medium.

- The tensor polarization depends only on the *local* collision term, while the Navier-Stokes value of the vector polarization depends on the *nonlocal* collisions.

- For a four-point interaction, $\mathcal{L}_{\text{int}} = (V^\mu V_\mu)^2$, the coefficient ξ is on the order of a percent.



V. Outlook

- Perform simulations to compare with data

- Remove truncation and resum all transport coefficients [4]

→ Tensor polarization will depend on other dissipative quantities as well, e.g. on the bulk viscous pressure.

- Explore the effects on the standard transport coefficients, e.g. shear viscosity, which also receive corrections from the moments $\psi_r^{\mu_1 \dots \mu_\ell}$.

- Couple to electromagnetism to derive spin magnetohydrodynamics.

- Extend to massless spin-1 fields, i.e., gauge fields.