PNJL equation of state with off-shell mesonic excitations

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1. Introduction

- Polyakov-loop improved NJL (PNJL) model is a common tool for a phenomenological description of the chiral and deconfinement phase transitions, and bound states formation and dissociation in hot and dense baryonic medium
- The contribution of mesonic correlations to the pressure is given by the generalized Beth-Uhlenbeck formula following from the corresponding Luttinger-Ward functional [1, 2]
- The Lorentz-invariant approximation is commonly used for treating mesonic excitations, with leaving out of consideration the spacelike off-shell mesonic excitations, i.e. Landau damping

In this contribution we demonstrate the importance of including such excitations into calculation of thermodynamic quantities within the simplest "mean-field + fluctuations" formulation of the PNJL model [3].

2. PNJL model at mean-field (MF) level

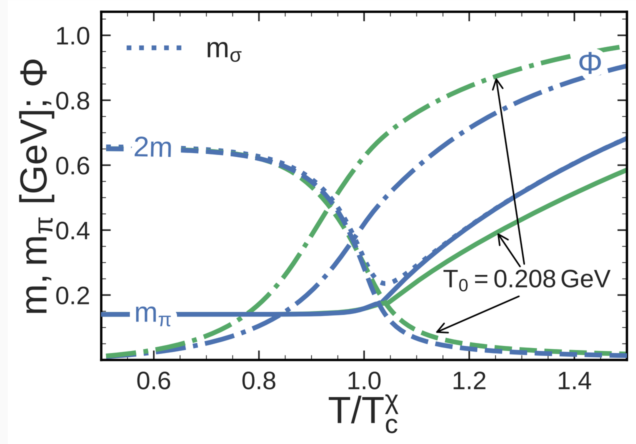
We use the PNJL model with $N_f = 2$ quark flavors and $N_c = 3$ colors at the baryon chemical potential μ and temperature T described by

$$\mathcal{L}_{\rm PNJL} = \bar q(i\not{D} - m_0)q + G_s\left[(\bar q q)^2 + (\bar q i\gamma^5\vec\tau q)^2\right], \quad D_\mu = \partial^\mu - i\delta_0^\mu(A^0+\mu), \quad m_0 = 5.5\,\mathrm{MeV}, \quad G_s = 5.04\,\mathrm{GeV}^{-2} \qquad (1)$$

The gluon contribution to the thermodynamics is modeled by the effective potential $\mathcal{U}(\Phi,\bar\Phi,T)$ in terms of the traced Polyakov loop fitted to describe the lattice data on pure SU(3) Yang-Mill thermodynamics. The total grand canonical thermodynamic potential of the model reads

$$\Omega(T;\Phi,\bar\Phi,m) = \mathcal{U}(\Phi,\bar\Phi,T) + \frac{(m-m_0)^2}{4G_s} - 2N_f\Big\{\int\limits_{|\vec p|<\Lambda}\frac{d^3p}{(2\pi)^3}N_c\varepsilon_p + T\int\frac{d^3p}{(2\pi)^3}[\mathrm{Tr}_c\ln[1+Ly] + \mathrm{Tr}_c\ln[1+L^\dagger y]]\Big\}, \quad \varepsilon_p = \sqrt{m^2+p^2}.$$

The MF values of $\Phi, \bar\Phi, m$ follow from $\frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar\Phi} = \frac{\partial\Omega}{\partial m} = 0$



- $T \lesssim T_c^\chi$: pions exist as stable bound states
- $T \gtrsim T_c^\chi$: $m_\pi > 2m(T)$ finite width of π, Mott dissociation

3. Beyond-mean-field mesonic fluctuations

The π- and σ-meson 1PI polarization operators read

$$\Pi_M(\omega,\vec q) = T\sum\!\!\!\!\!\!\int_P G(\omega_n,\vec p)\Gamma_M G(\omega_n-\varepsilon_k,\vec p-\vec q)\Gamma_M, \quad M=\{\pi,\sigma\}, \quad \Gamma_\pi = i\gamma^5, \Gamma_\sigma = 1. \qquad (2)$$

The RPA-resummed propagator of a quasi-meson M is then determined as

$$D_M(\omega,q) = -\frac{2G_s}{1+2G_s\Pi_M(\omega,q)}. \qquad (3)$$

The mass m_M of a quasi-meson M is defined either as a solution of $1 + 2G_s\,\mathrm{Re}\,\Pi_M(\omega = m_M, q = 0) = 0$ or as the position of the maximum of the spectral function $\rho_M(\omega,q) = -2\,\mathrm{Im}\,D_M(\omega,q)$ at zero momentum in the case of finite meson width.

The meson contribution to the pressure is given by generalized Beth-Uhlenbeck formula [1, 2]

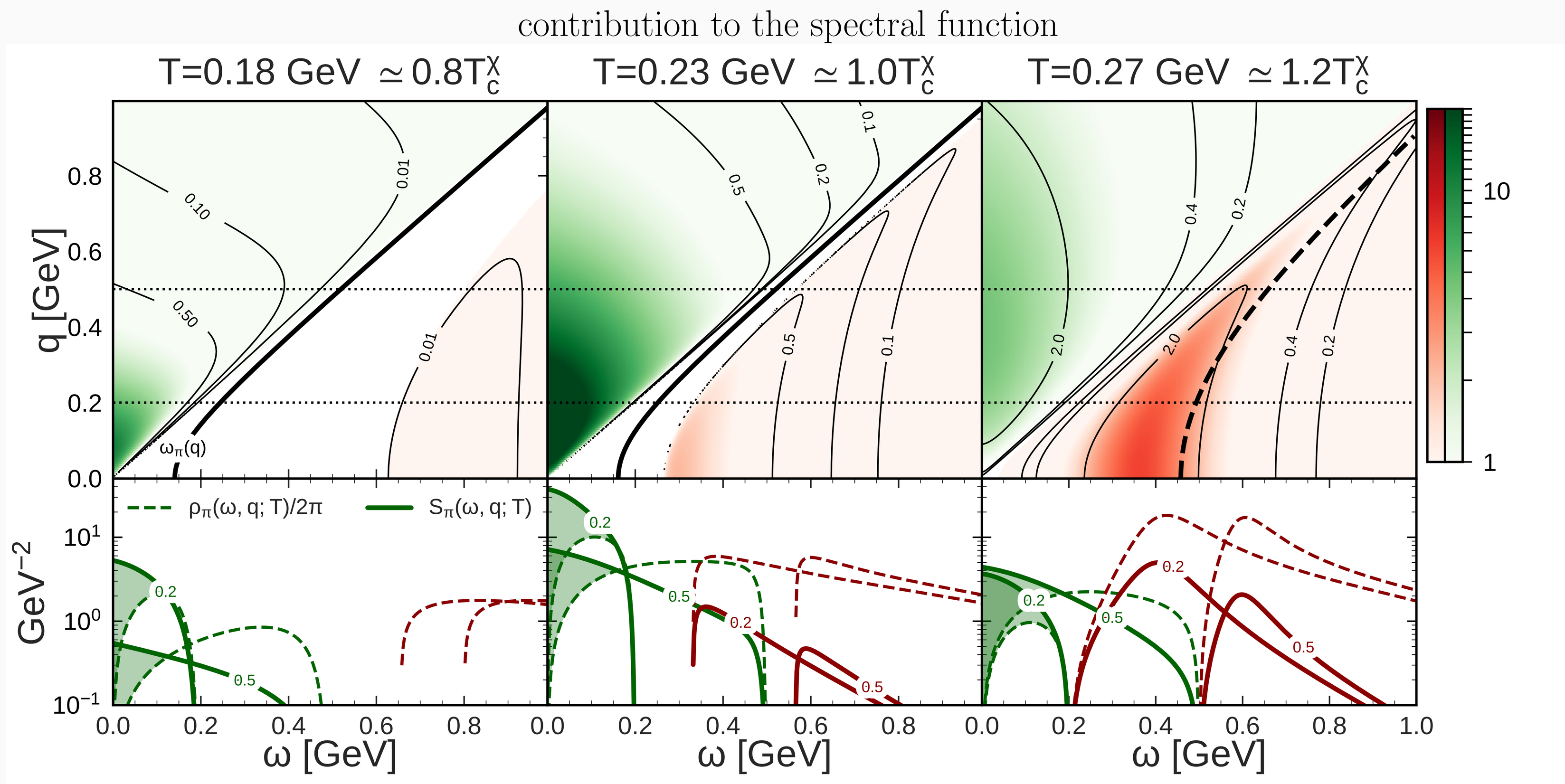
$$P_M = d_M\sum_{k=\mathrm{QP,LD}}\int\limits_{|\vec q|<\Lambda^k}\frac{d^3q}{(2\pi)^3}w_M^k(q,T), \quad w_M^{\rm QP} \equiv \int\limits_q^\infty\frac{d\omega}{\pi}\frac{\delta_M(\omega,q,T)}{e^{\omega/T}-1}, \quad w_M^{\rm LD} \equiv \int\limits_0^q\frac{d\omega}{\pi}\frac{\delta_M(\omega,q,T)}{e^{\omega/T}-1},$$

where $\delta_M(\omega,q,T) = -\arctan\frac{\mathrm{Im}\,D_M}{\mathrm{Re}\,D_M}$ is the quark-antiquark scattering phase shift in channel $M = \pi,\sigma$

$\Lambda^{\rm QP} \to \infty$, $\quad\Lambda^{\rm LD} = (1-2)\Lambda$ encountered in the literature [4, 5, 6]

4. Pion dynamical structure factor $S_\pi(\omega,q) = \frac{1}{2\pi}\frac{\rho_\pi(\omega,q)}{e^{\omega/T}-1}$

Low-frequency spacelike excitations are enhanced by the thermal distribution despite their relatively small contribution to the spectral function



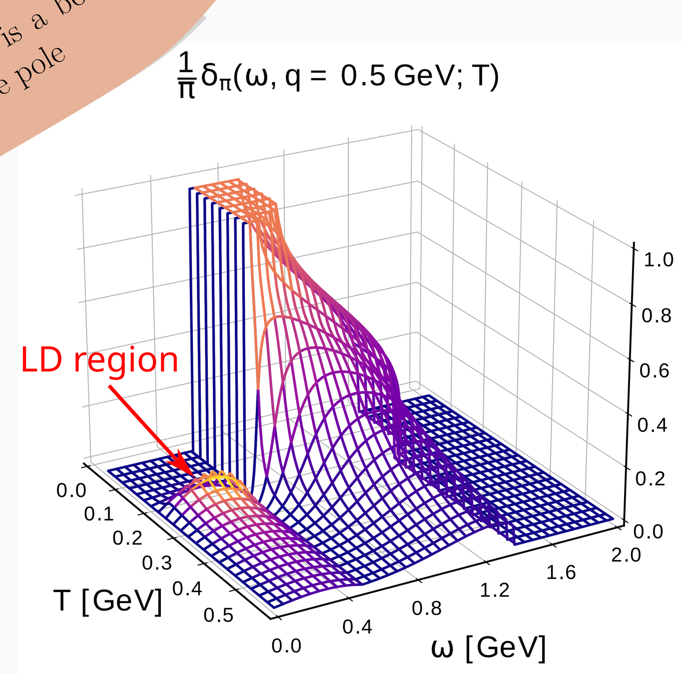
$T < T_c^\chi$: Most of spectral strength is in the pion pole $\omega_\pi(q)$

$T \simeq T_c^\chi$: LD region significantly enhanced

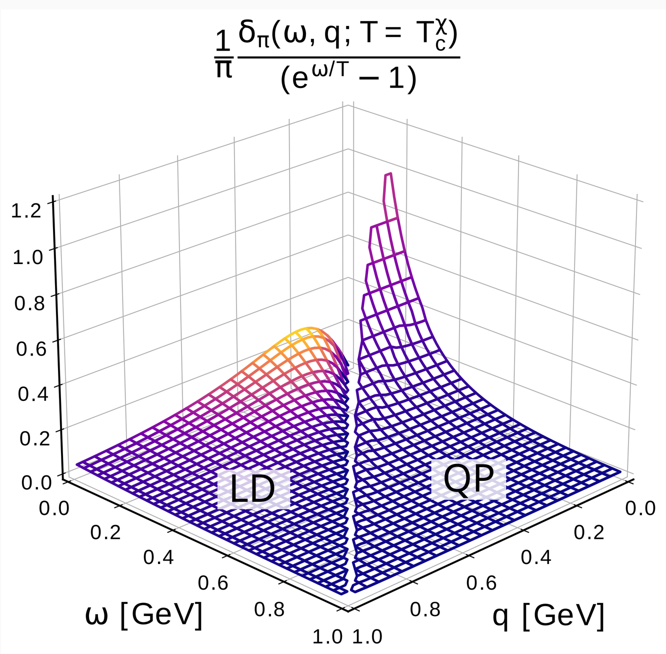
$T > T_c^\chi$ – LD contribution becomes comparable to QP one

5. Pionic phase shifts and momentum distributions

$\delta_\pi(\omega)$ jumps from 0 to π if there is a bound state pole



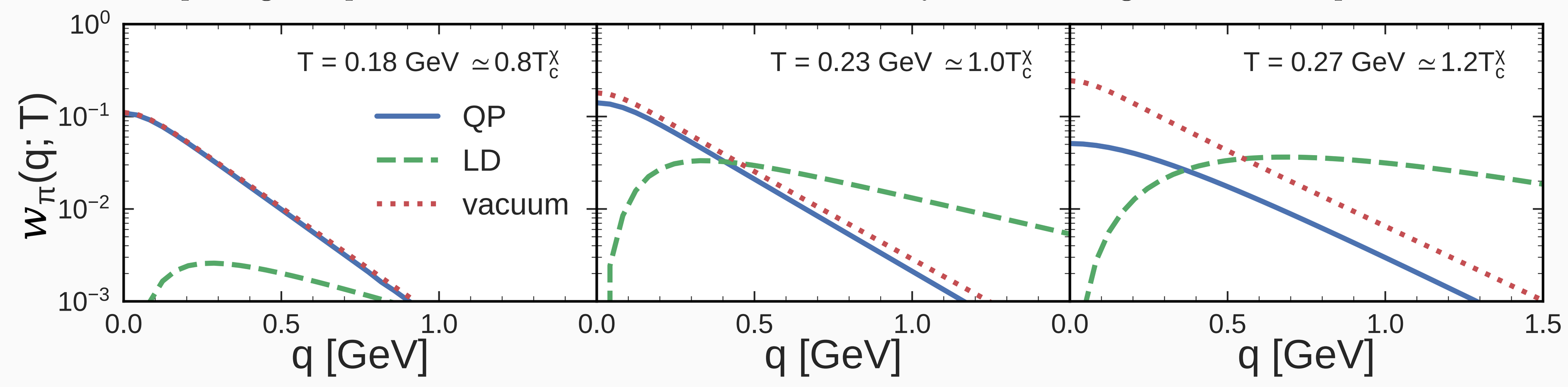
Pion phase shift shows a dissociation of bound state and noticeable peak due to LD



Thermal distribution weight ⇒ LD exceeds QP for $q \gtrsim 0.2\,\mathrm{GeV}$ near T_c^χ

6. Pion pressure momentum integrand

Comparing the pion momentum "distributions" from QP and LD region with free pion case



Most of spectral strength is in the pion pole $\omega_\pi(q)$

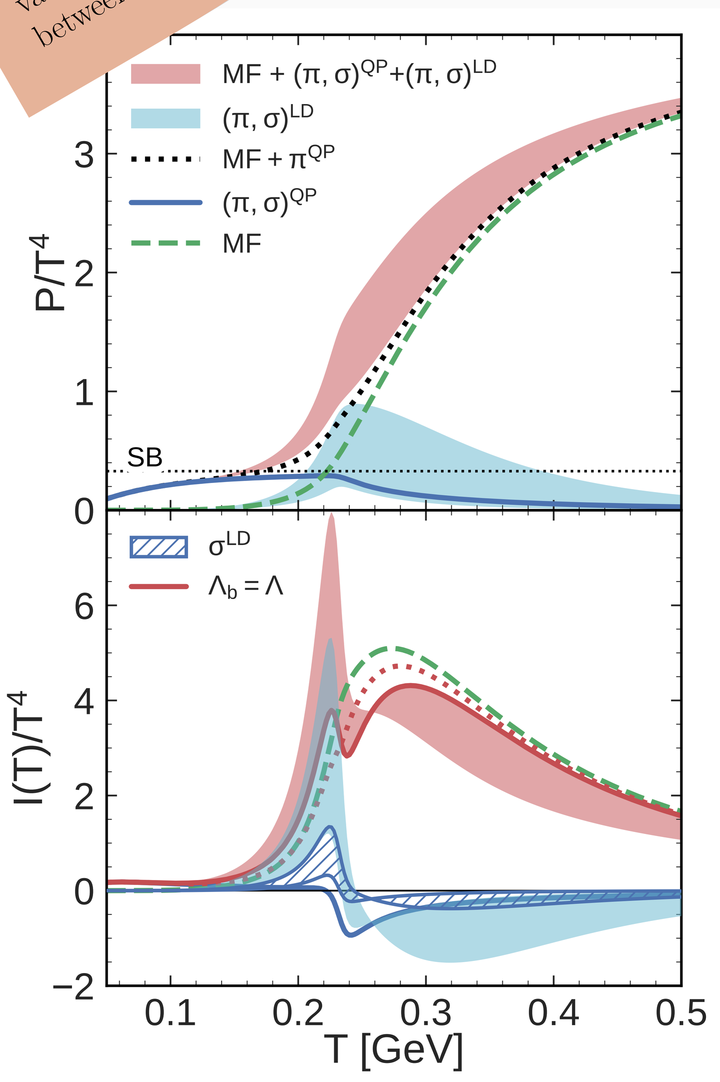
LD region significantly increases for $T \to T_c^\chi$

$T > T_c^\chi$ – LD contribution exceeds the QP one

LD: large-momentum tail scales as $\exp(-\sqrt{(2m)^2+q^2}/2T)$
⇒ effective temperature $2T$ for LD part

7. Thermodynamic quantities

Bands stand for variation of $\Lambda_{\rm LD}$ between $(1-2)\Lambda$



Pressure

QP contribution:

- π and σ quasiparticle gas toward the SB limit at small T
- decreasing for $T > T_{\rm Mott} \simeq T_c^\chi$ due to Mott transition

LD contribution:

- Growing similarly to MF pressure as the quark mass $m(T)$ decreases
- Peaks around T_c^χ with magnitude sensitive to the 3-momentum cutoff

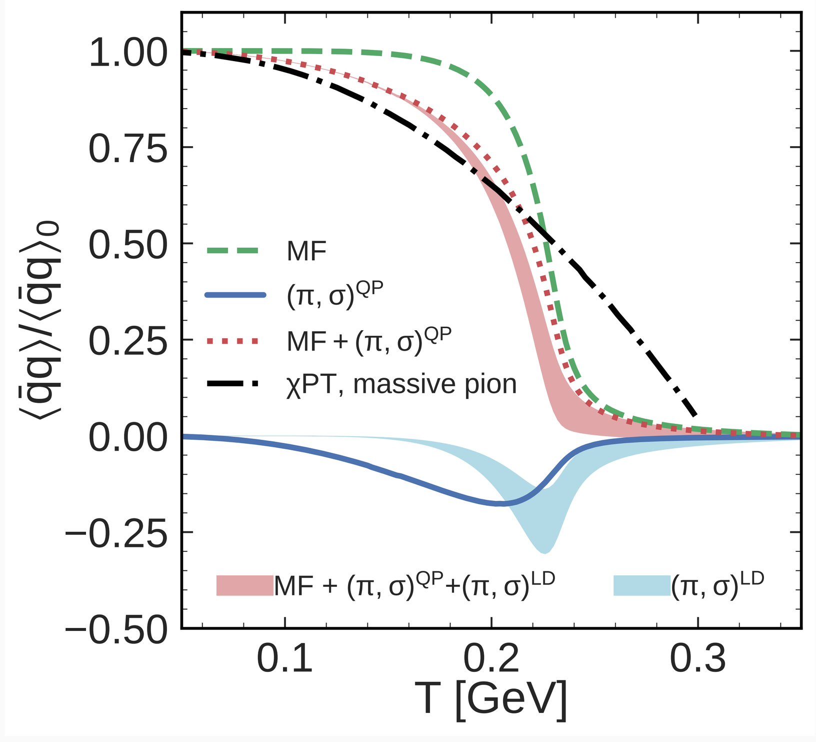
Trace anomaly

- More sensitive to the LD contribution
- Peak position shifts to lower T

8. Estimate of the effect on $\langle\bar q q\rangle$

"Perturbative" estimate using Hellmann-Feynman theorem:

$$\langle\bar q q\rangle_M = -\frac{\partial P_M(T;m)}{\partial m_0}$$



- QP contribution does not affect T_c^χ
- LD contribution shifts T_c^χ to lower values

9. Conclusion

- Presence of the Landau cut in the meson propagators leads to a significant enhancement and threshold dependence of the total pressure
- This contribution will arise in any model where the Hartree propagators for quarks are used, e.g. NLO $1/N_c$ expansion
- A self-consistent calculation of meson and quark spectral properties is necessary

References

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