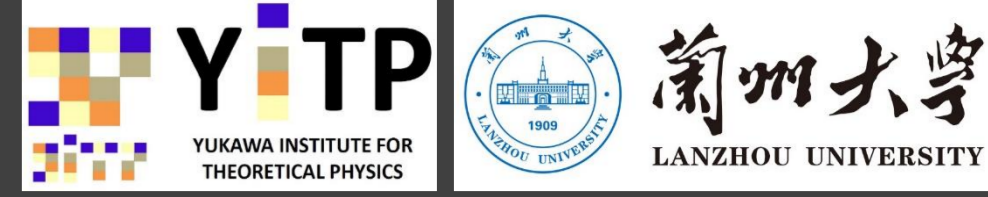


Understanding effect of event-by-event fluctuations on Light-Nuclei Yield Ratio

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Abstract: To understand how the *event-by-event fluctuations* of the final-state nucleon distributions $f(\mathbf{r}, \mathbf{p})$ affect the yield ratio of light nuclei $N_p N_t / N_d^2$ qualitatively, we model the ensemble of $f(\mathbf{r}, \mathbf{p})$ by the overlap of **random n -Gaussian hot spots** and obtain analytical formulae based on **the coalescence model**, which is used to establish qualitative understanding.

1. Background and Motivation

■ Critical-point search in heavy-ion collisions

The *critical point (CP) of quantum chromodynamics (QCD)* is predicted in the finite- μ_B (baryon chemical potential) region of the phase diagram based on effective theories. Heavy-ion collision experiments at lower collision energies, such as **RHIC BES programs**, **FAIR CBM**, etc. aim at finding the CP signal, yet its clear experimental evidence has not been established.

■ Signals of critical point

The collision energy $\sqrt{s_{NN}}$ roughly corresponds to μ_B through baryon stopping, which suggests us to see **non-monotonic $\sqrt{s_{NN}}$ dependence** of observables. Among many observables suggested for the CP signals, we focus on **the yield ratio of light nuclei** [K.-J. Sun et al (2017)]:

$$N_t N_p / N_d^2 \sim \int \langle n(x)n(0) \rangle / \langle n \rangle^2$$

N_t : triton yield, N_p : proton yield, N_d : deuteron yield

← Calculated by the *coalescence model* from the *nucleon distribution $f(\mathbf{r}, \mathbf{p})$* .

- ✓ Sensitive to local spatial correlations, i.e., *critical correlations*
- ✓ Cancels unwanted volume effect, etc. in the ratio
- ✓ Non-monotonic behaviors found in experimental data

■ Dynamical effects in realistic collisions

Previous analyses are based on *ideal setups* with Gaussian source or boost invariance. In realistic collisions, there are **various dynamical effects**:

- **Expansion** → coordinate-momentum correlation
- **Initial fluctuations** → non-critical correlations
- **Non-flow effects** → $f(x, p)$ not in equilibrium form
- **Finite size and time** (Kibble-Zurek) → correlation development

Question: How does the dynamical effects, especially the *event by event fluctuations*, affect $N_t N_p / N_d^2$?

Event-by-event

$$f^{(1)}(\mathbf{r}, \mathbf{p}), f^{(2)}(\mathbf{r}, \mathbf{p}), f^{(3)}(\mathbf{r}, \mathbf{p}), \dots$$

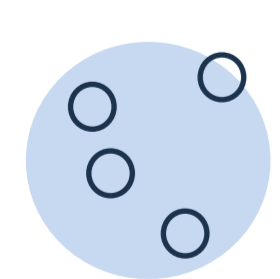
$$N_A^{(1)}, N_A^{(2)}, N_A^{(3)}, \dots \rightarrow \langle N_A^{(i)} \rangle$$

2. Model: fluctuating n -Gaussian hot spots

■ Coalescence model

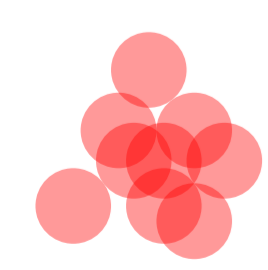
Nucleus of mass number A is formed by nucleons that are close in phase space.

$$N_A = g_A \int \left[\prod_i d^3 \mathbf{r}_i d^3 \mathbf{p}_i f_i(\mathbf{r}_i, \mathbf{p}_i) \right] W_A(\{\mathbf{r}_i, \mathbf{p}_i\}_{i=1}^A)$$



$W_A()$... Wigner fn. for nucleus wave fn. (assumed to be **harmonic oscillator**)
 $f_i()$... phase space distribution of i th nucleon
 g_A ... statistical factor from spin

■ Single-event $f(\mathbf{r}, \mathbf{p})$ by n -Gaussian hot spots



$$f(\mathbf{z}) = \frac{1}{n} \sum_{h=1}^n c_h G(\mathbf{z} - \mathbf{a}_h; \mathcal{C}_2^{\text{hs}})$$

$\mathbf{z} \sim \sqrt{2}(\mathbf{r}/\sigma_A, \mathbf{p}/\sigma_A)$

σ_A : nucleus size

G : Gaussian profile

n Number of hot spots

$\mathcal{C}_2^{\text{hs}}$ Covariance; hot-spot size/shape in \mathbb{R}^6

\mathbf{a}_h Center of hot spot h

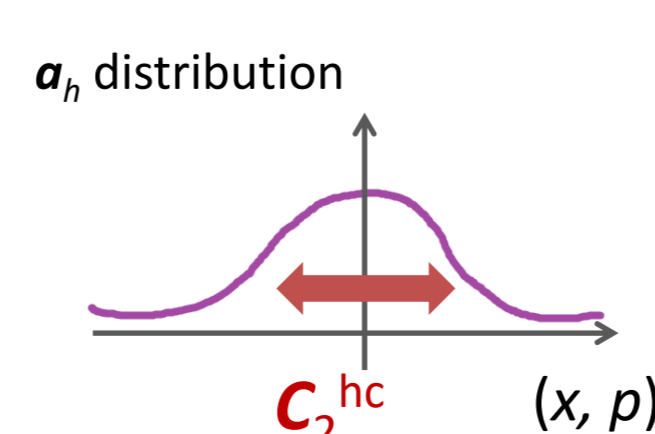
c_h Magnitude of hot spot h

■ Ensemble of $\{f(\mathbf{r}, \mathbf{p})\}$ by randomizing (\mathbf{a}_h, c_h)

Gaussian distribution of hot-spot centers \mathbf{a}_h

$$\text{Pr}(\{c_h, \mathbf{a}_h\}_{h=1}^n) = \prod_{h=1}^n p(c_h) \frac{e^{-\frac{1}{2} \mathbf{a}_h^T (\mathcal{C}_2^{\text{hc}})^{-1} \mathbf{a}_h}}{\sqrt{\det(2\pi \mathcal{C}_2^{\text{hc}})}}$$

$\mathcal{C}_2^{\text{hc}}$ Covariance of hot-spot centers



3. Results of n -Gaussian hot-spot model

■ Analytic result of yield of light nuclei (event averaged)

$$N_A = g_A 8^{A-1} \det(\mathcal{C}_2^{\text{hs}} + 1)^{-(A-1)/2}$$

$$\times \sum_{\substack{m_1 \geq \dots \geq m_n \geq 0 \\ \sum_{h=1}^n m_h = A}} \frac{n! A!}{n^A S_{m_1, \dots, m_n}} \frac{I_{m_1} \dots I_{m_n}}{\prod_{\lambda} (\sum_{h=1}^n \frac{m_h}{A} \frac{1}{1+m_h \lambda})^{1/2}},$$

$$\left\{ \begin{array}{l} \sum_{\substack{m_1 \geq \dots \geq m_n \geq 0 \\ \sum_{h=1}^n m_h = A}} \text{sum for classification of } A \text{ items into } n \\ S_{m_1, \dots, m_n} = \prod_m a_m! (m!)^{a_m} \text{ symmetry factor} \\ I_m = \frac{1}{\prod_{\lambda} (1+m\lambda)^{1/2}} \int d\mathbf{c} p(\mathbf{c}) c^m \text{ moments of } c_h \\ \lambda \text{ eigenvalues of } \mathcal{C}_2^{\text{hc}} (\mathcal{C}_2^{\text{hs}} + 1)^{-1} \end{array} \right.$$

yield is mainly determined by these factors

$$\lambda \sim \frac{\text{Cov}[\text{fireball}]}{\text{Cov}[\text{nucleus}] + \text{Cov}[\text{hotspot}]}$$

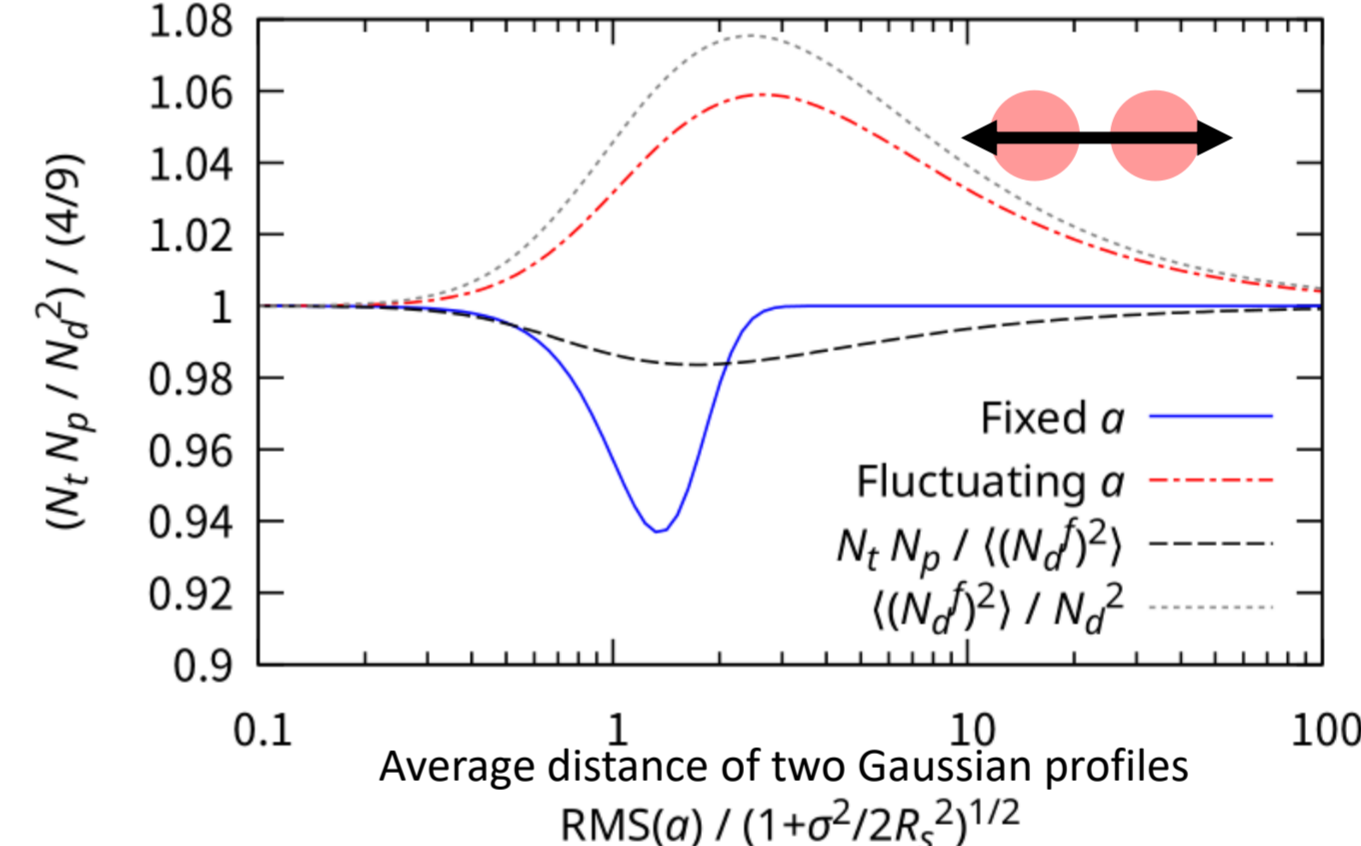
■ Yield ratio in limits $n = 1$ and $n \rightarrow \infty$

$n=1$: single Gaussian, **Ratio = 4/9** (ideal value)

$n=\infty$: infinite number of hot spots **Ratio = 4/9** (fluctuations smeared out)

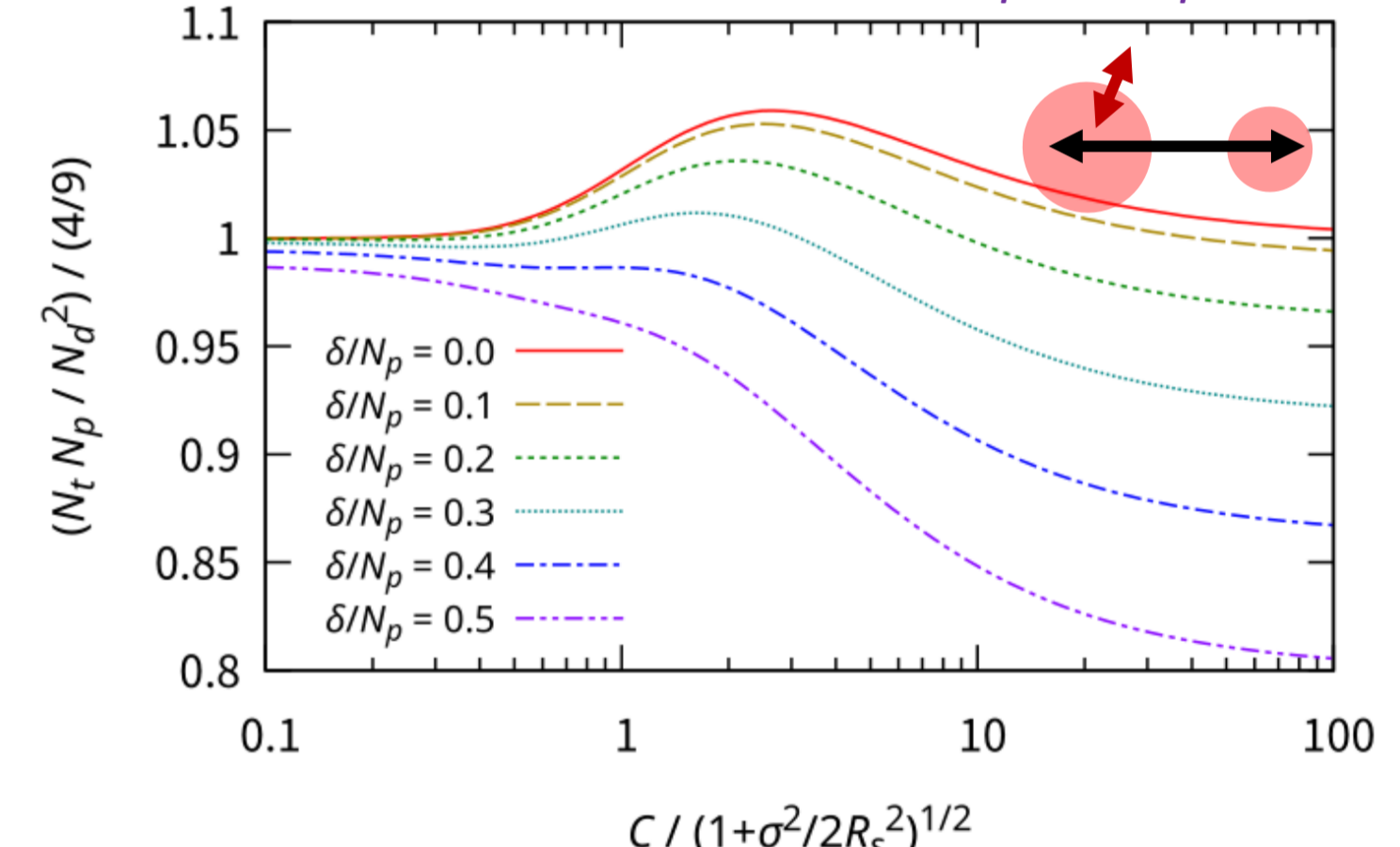
■ Yield ratio for 2-Gaussian case ($n = 2$)

a_h : Gaussian distribution in one direction



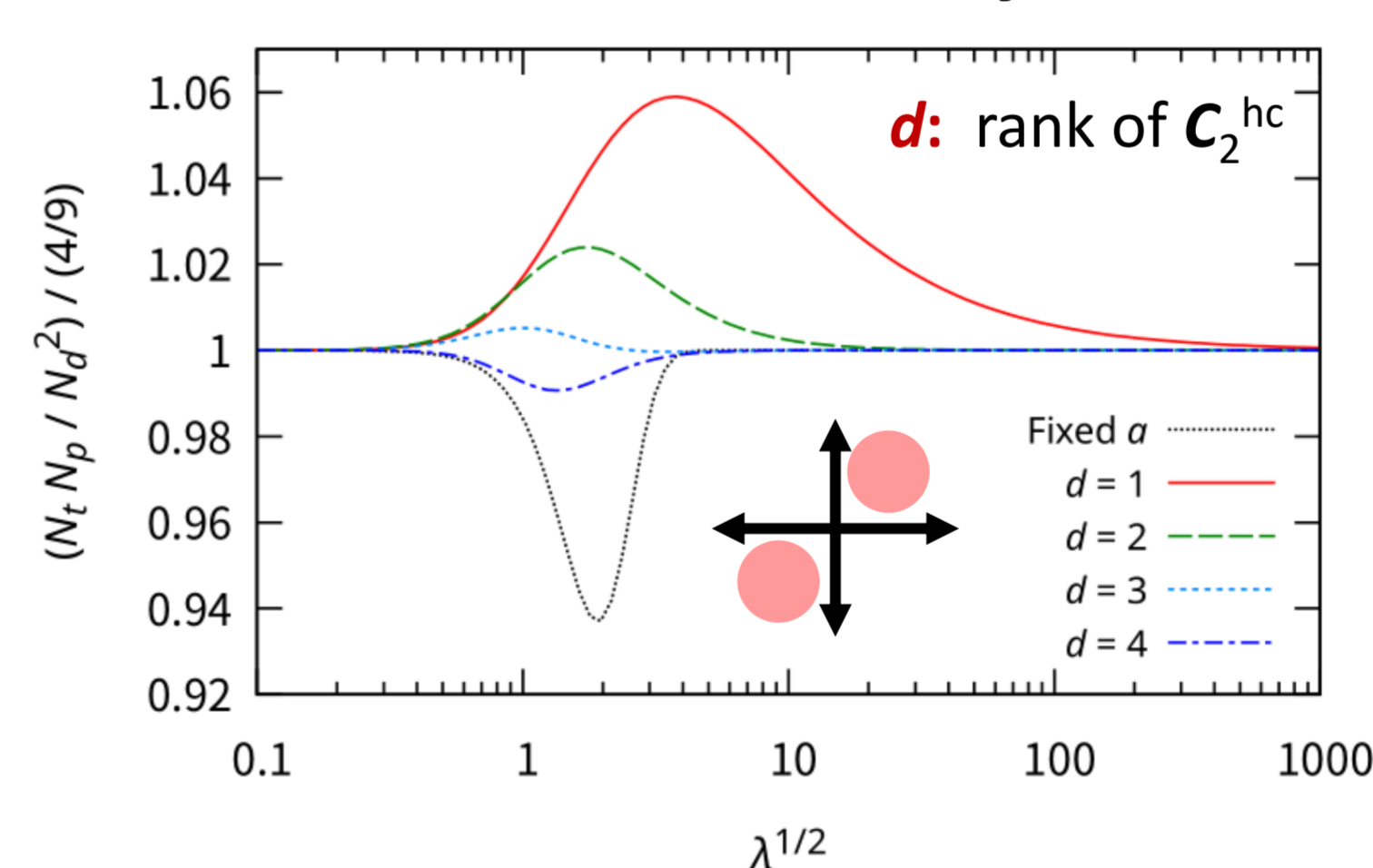
✓ Position fluctuations increase the ratio

c_h : uniform distribution in $[N_p - \delta, N_p + \delta]$



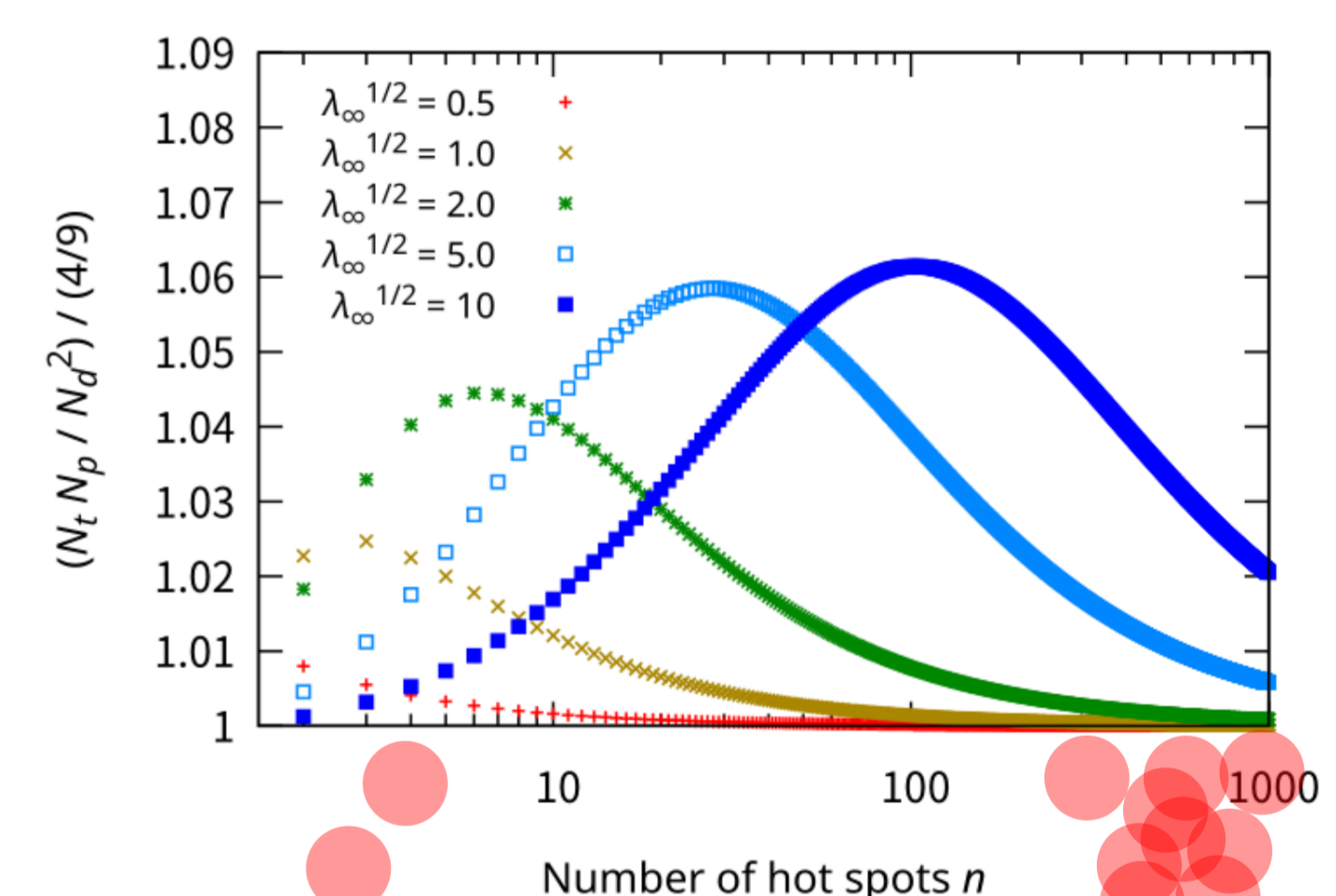
✓ Magnitude fluctuations decrease the ratio

■ Fluctuation dimensionality



✓ The increase largely depends on the dimensionality

■ Many Gaussians n -dependence



✓ A peak at a certain hot-spot number "n"

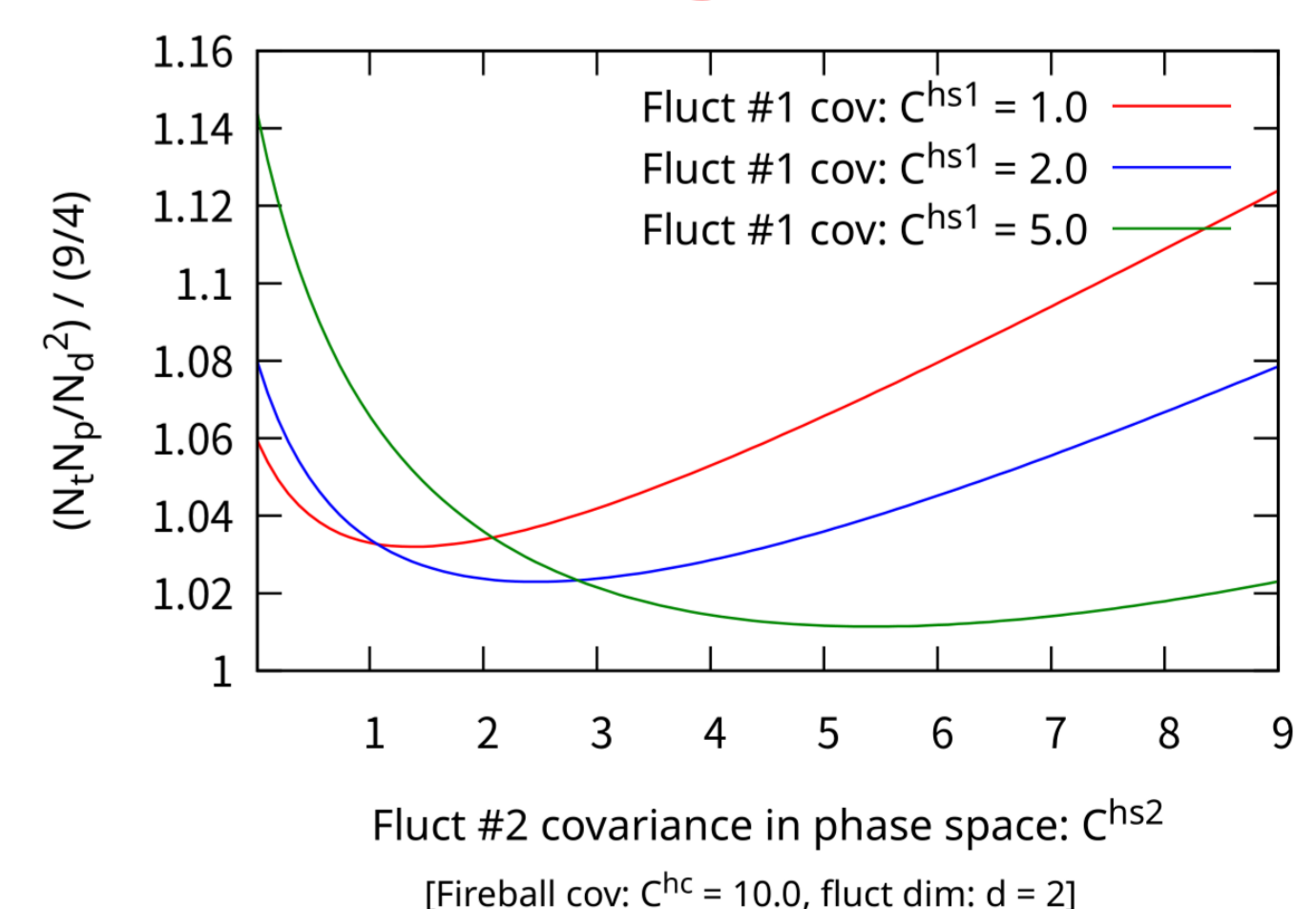
4. Two fluctuation sources

■ Extended Model

$f(\mathbf{r}, \mathbf{p})$: sum of two fluctuations sources with different hot-spot sizes \mathcal{C}^{hs1} & \mathcal{C}^{hs2} . (E.g. **critical fluctuations** vs shorter-scale **thermal fluctuations**).

Question: When scale separation of two fluctuations are not sufficient in HIC, can we differentiate them?

■ Result



✓ The effect becomes larger when two sources have different sizes

5. Summary

Introduced "*fluctuating n -Gaussian hot spots*" for coalescence $f(\mathbf{r}, \mathbf{p})$

- **Yield ratio of light nuclei is largely affected by e-by-e fluctuations:** Position/magnitude fluctuations increase/decrease the ratio. The dimensionality and # of hot spots also matter.
- **Scale separation of two fluctuation sources make effect larger**