The Shear Viscosity of Parton Matter Under Two-body Scatterings

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Abstract

The shear viscosity η of a quark-gluon plasma in equilibrium can be calculated numerically using the Green-Kubo relation or analytically using several methods. We first confirm that the Chapman-Enskog (CE) method is the most accurate while correcting the typos in an earlier work. We then apply the CE method to study the shear viscosity of the parton matter from a multi-phase transport (AMPT) model. For the center cell of central and midcentral Au+Au collisions at 200AGeV and Pb+Pb collisions at 2.76ATeV, the average η/s of the parton matter is found to be very small, at (*1* to *3*)/(4π).

1) Introduction

The shear viscosity η is an important property of the quark–gluon plasma. η or η/s is an input function to viscous hydrodynamics. In kinetic theory, η is generated by interactions; therefore, it is important to know its relationship to interaction cross sections.

In this study [1], we consider a massless parton matter in thermal equilibrium under 2-to-2 elastic scatterings and the Boltzmann statistics. Both isotropic scattering and forward-angle scattering are considered, where for forward-angle scattering we take the form used in AMPT, ZPC and MPC transport models:

$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}(1+a)\frac{1}{(\hat{t}-\mu^2)^2}\ , \text{with } a \equiv \frac{\mu^2}{\hat{s}}\ . \qquad (1)$$

Here, μ is the screening mass, while $\hat{s}$ and $\hat{t}$ are Mandelstam variables for the two colliding partons. The above form, based on the pQCD *g*+*g* cross section, gives a $\hat{s}$-independent cross section $\sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$. One can also define a transport cross section as $\sigma_{tr} = \int d\sigma \sin^2\theta_{cm}$. For forward-angle scattering of Eq.(1), we get

$$\sigma_{tr} = 4a(1+a)\left[(1+2a)\ln\left(1+\frac{1}{a}\right)-2\right]\sigma \equiv h(a)\sigma\ . \qquad (2)$$

Function $h(a)$ can be called the *anisotropy function;* it increases from ~0 for very forward scatterings to 2/3 for isotropic scatterings.

2) Comparison of Various Analytical Methods

There are several methods that give an analytical expression of η, including the Israel–Stewart (IS) method [2], Navier–Stokes (NS) method [2], the improved relaxation time approximation (RTA*) [3], and Chapman–Enskog (CE) method [4,3]:

$$\eta^{IS} = \frac{4T}{5\langle\sigma_{tr}\rangle} \equiv \frac{4T}{5\sigma\, h_0(w)},\ \eta^{NS} \approx \frac{0.8436\,T}{\sigma\, h_0(w)},$$
$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr}v_{rel}\rangle} \equiv \frac{4T}{5\sigma\, h_1(w)},\qquad \eta^{CE} = \frac{4T}{5\sigma\, h_2(w)}. \qquad (3)$$

Here $\langle\sigma_{tr}\rangle$ and $h_{0,1,2}(w)$ are different "thermal" averages of the $h(a)$ function with $w \equiv \mu/T$ and v_{rel} being the relative velocity.

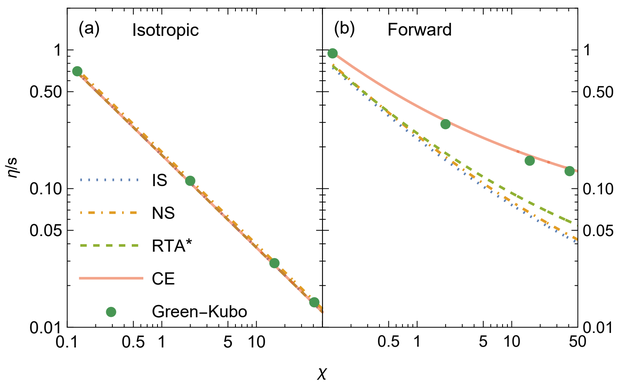

Fig.1 η/s results from four analytical methods and the Green-Kubo relation versus the opacity parameter χ of a gluon gas under **(a)** isotropic scatterings or **(b)** forward-angle scatterings.

From Fig.1, we see that all methods agree well for isotropic scatterings. For anisotropic scatterings, however, only the Chapman–Enskog results agree well with Green-Kubo. Note that each opacity parameter χ ($\equiv n\sqrt{\sigma^3/\pi}$) represents a gluon gas at a different temperature or density *n* in Fig.1. The fact that Green-Kubo results agree with CE (but not RTA*) has been shown earlier in Ref.[3], although there were typos in the expressions of both η^{RTA*} and η^{CE}, where $\int_1^\infty dy\, h(2zy\bar{a})$ ... in [3] should be $\int_1^\infty dy\, h(\frac{1}{(2zy\bar{a})^2})$ ...

3) η and η/s of the parton matter in the AMPT model

We now study high energy AA collisions with the string melting AMPT model. For the center cell, we have extracted the effective temperatures from the parton energy density ε or mean p_T:

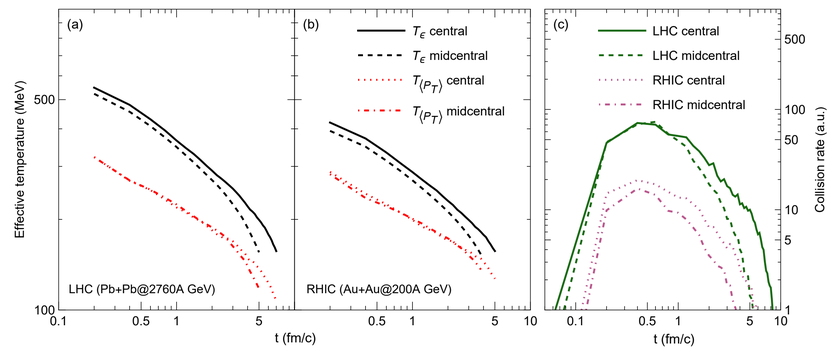

Fig.2 Time evolutions of effective temperatures T_ε and $T_{\langle pT\rangle}$ for the center cell in **(a)** Pb+Pb collisions at 2760A GeV and **(b)** Au+Au collisions at 200A GeV. **(c)** shows time evolutions of the collision rates for the center cell.

When treating the parton matter as a QGP in full equilibrium, we use temperature T_ε to calculate both η and *s* and get Fig.3: the time-averaged <η/s> is found to be quite small, at about *3*/(4π).

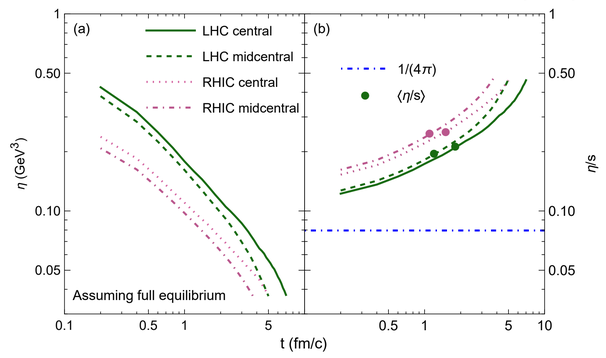

Fig.3 Time evolutions of **(a)** the CE shear viscosity and **(b)** *η/s* of the center cell. Here the parton matter is treated as a QGP in full equilibrium at temperature T_ε. Circle on each curve represents the time-averaged *η/s* value weighed by the collision rate.

Since the parton matter in AMPT is not in full equilibrium, e.g., $T_{\langle pT\rangle} < T_\varepsilon$ in Fig.2, we may treat the parton matter as a QGP in partial equilibrium. Then we use temperature $T_{\langle pT\rangle}$ to calculate η but use T_ε to calculate *s*. Fig.4 shows that <η/s> is even smaller at (1 to 2)/(4π); since the lower $T_{\langle pT\rangle}$ makes scatterings more isotropic and effective.

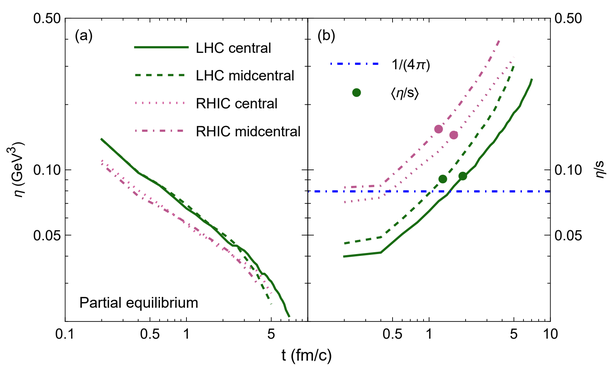

Fig.4 Same as Fig.3, but the parton matter is treated as a QGP in partial equilibrium, where the momentum distribution corresponds to temperature $T_{\langle pT\rangle}$ but energy density corresponds to temperature T_ε.

4) Conclusions

- We confirm that the Chapman-Enskog method is the most accurate for calculating shear viscosity η for non-isotropic scatterings; we also correct typos in an earlier work on η.
- The average *η/s* value for the center cell of parton matter in high energy AA collisions from AMPT model is very small at (1-3)/(4π).
- T- or time-dependence of *η/s* in AMPT is opposite to pQCD expectation; this problem can be resolved by adopting $\mu \propto gT$ based on finite temperature pQCD or the AMY formulation.

References **[1]** N. MacKay & Z.W. Lin, Eur. Phys. J. C 82, 918 (2022).
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[3] S. Plumari, A. Puglisi, F. Scardina & V. Greco, Phys. Rev. C 86, 054902 (2012).
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