Entanglement entropy measurements from p-p collisions at LHC energies

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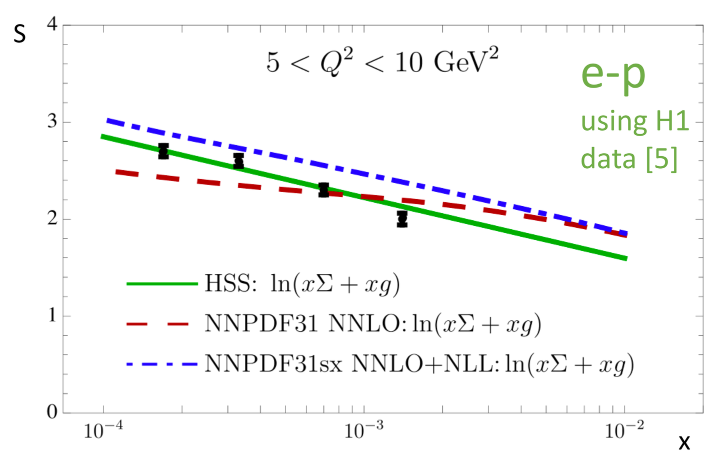
Introduction

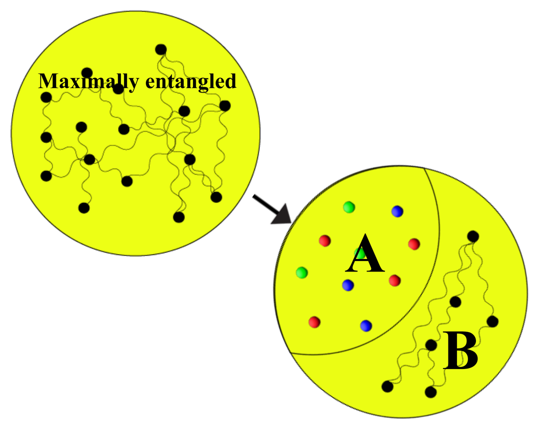
Gibbs Entropy (final-state)

- Gibbs entropy is a thermodynamic quantity that measures the degree of disorder, or randomness, in a system.
- This is calculated using a probability distribution of the number of microstates. In this case the number of hadrons
- The more hadrons produced the higher the entropy.
- While the Gibbs entropy is defined under different context than entanglement entropy, each of these definitions attempts to quantify the disorder of the system. And each is proportional to the number of particles.

$$S_{\text{hadron}} = -\sum \mathrm{P(N)} \ln \mathrm{P(N)}$$

We propose a new method for understanding particle production and thermal-like behavior in high energy collisions based on first principles in quantum mechanics. While it seems evident that there is a relation between entanglement entropy in the initial state and thermodynamic entropy in the final state in e-p systems. It is not yet clear whether this holds true in hadron collisions.



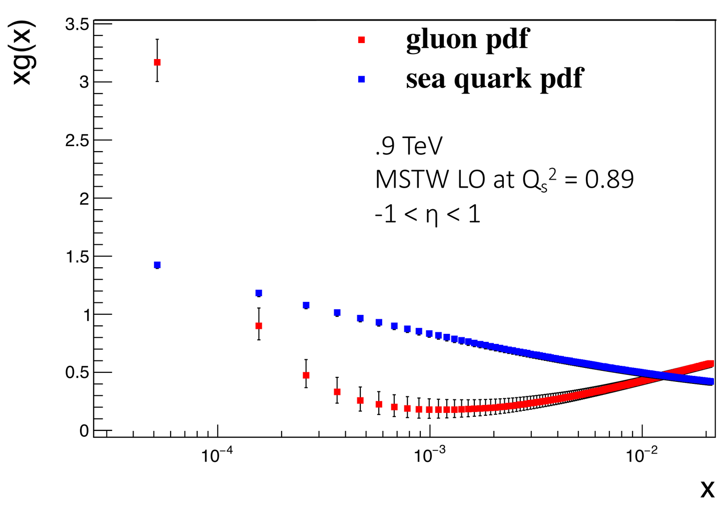


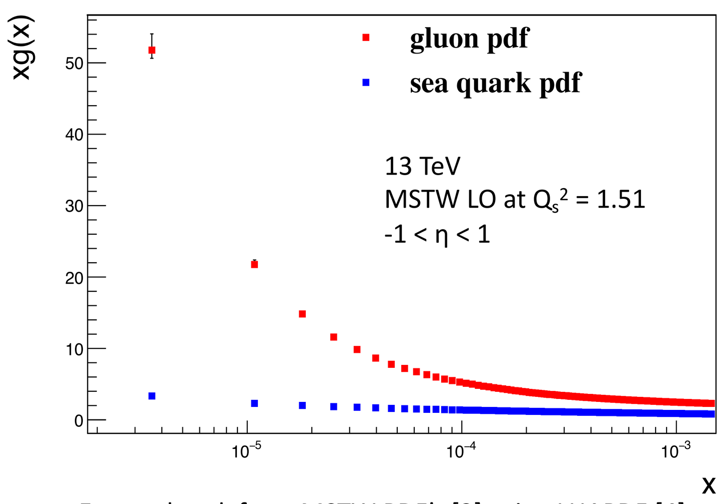
Entanglement Entropy (initial-state)

- Entanglement entropy is a metric for understanding the level of entanglement between two or more quantum systems.
- It quantifies the amount of information that is shared between particles (partons) and can be used to characterize their degree of correlation.
- A proton represents a pure state; meaning the partons are fully correlated and maximally entangled.
- The interacting proton can be segmented into an interaction region (A) and a region that remains unprobed (B). Entanglement entropy arises between these two regions.

$$S_A = -tr[\hat{\rho}_A \ln \hat{\rho}_A] = S_B = -tr[\hat{\rho}_B \ln \hat{\rho}_B]$$

Initial State



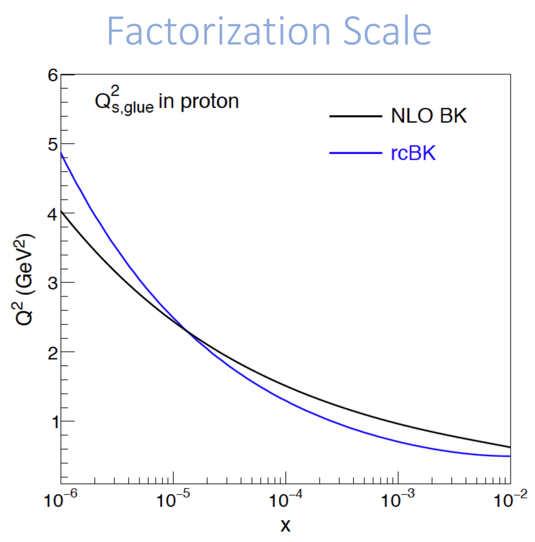


Extrapolated from MSTW PDF's [3] using LHAPDF [4]

In a kinematic regime in which gluons dominate, we can simplify the calculation of entropy to be proportional to the average number of partons.

$$S_{EE} = \ln(N_{parton})$$

Factorization Scale



For average p-p collisions this scale is characterized by the transverse momentum of partons, which is dependent on the density of partons. Parton density is said to saturate at low x so the corresponding scale is called the saturation scale Q_s^2. We map x to Q_s^2 using NLO BK calculations.

The number of partons is calculated by integrating over published PDF's extrapolated to the appropriate kinematic range using the LHAPDF framework.

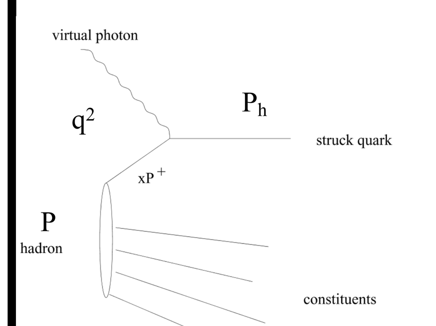
The limits of integration when calculating the parton number in the initial state, are set by the range in η over which we measure in the final state.

$$S_{EE} = \ln\left[\int \frac{xg(x)}{x}dx + \int \frac{xq(x)}{x}dx\right] + \ln\left[\frac{2}{3}\right]$$

Gluons — Quarks — Neutral factor

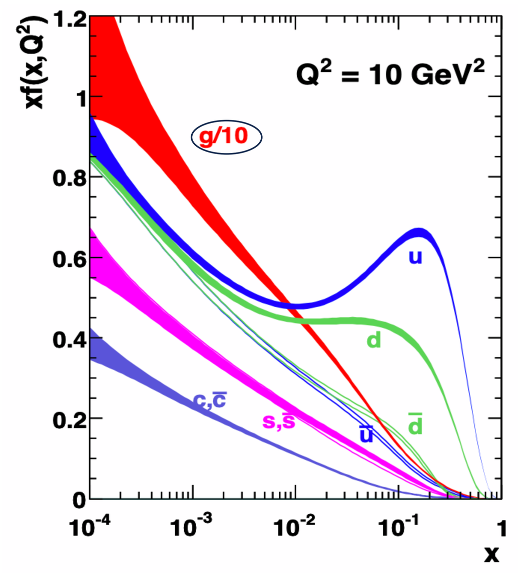
Bjorken-x

The formalism that allows us to relate x (not measurable in our system) to pseudorapidity (a measurable quantity) we will borrow the formalism of DIS.



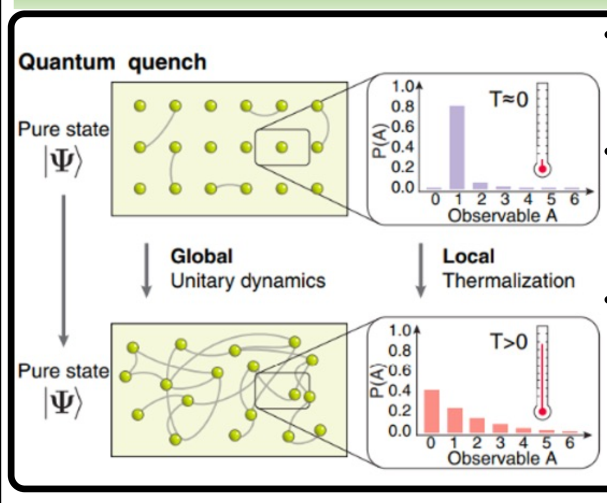
It can be shown that, in the limit of massless hadrons, x can be related the the rapidity of the proton (y) and the pseudorapidity of the produced hadron (η).

$$x = e^{-y}\, e^{\eta}$$



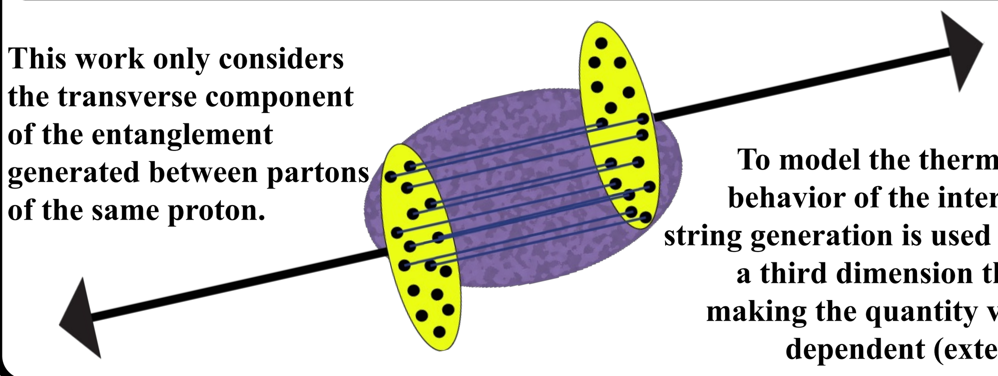
As shown in the above figure [3], at low-x the protons wavefunction becomes dominated by gluons.

Thermal-like behavior



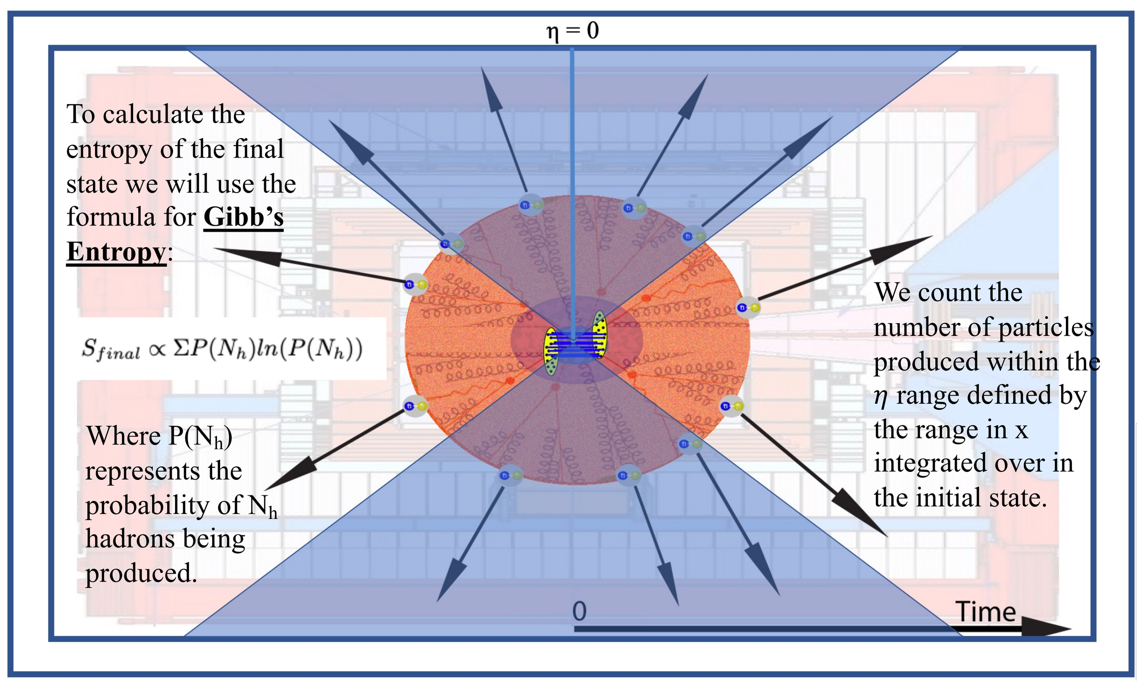
- Currently it is not understood how the Quark Plasma thermalizes at such early times.
- A recent study done at Harvard University [6] has shown that it is possible for a system to thermalize through entanglement.
- If shown that entanglement survives the evolution of the collision, this would provide further evidence that the thermal-like behavior of the collision is due to this quantum effect.

This work only considers the transverse component of the entanglement generated between partons of the same proton.



To model the thermal-like behavior of the interaction string generation is used to add a third dimension thereby making the quantity volume dependent (extensive).

Final State

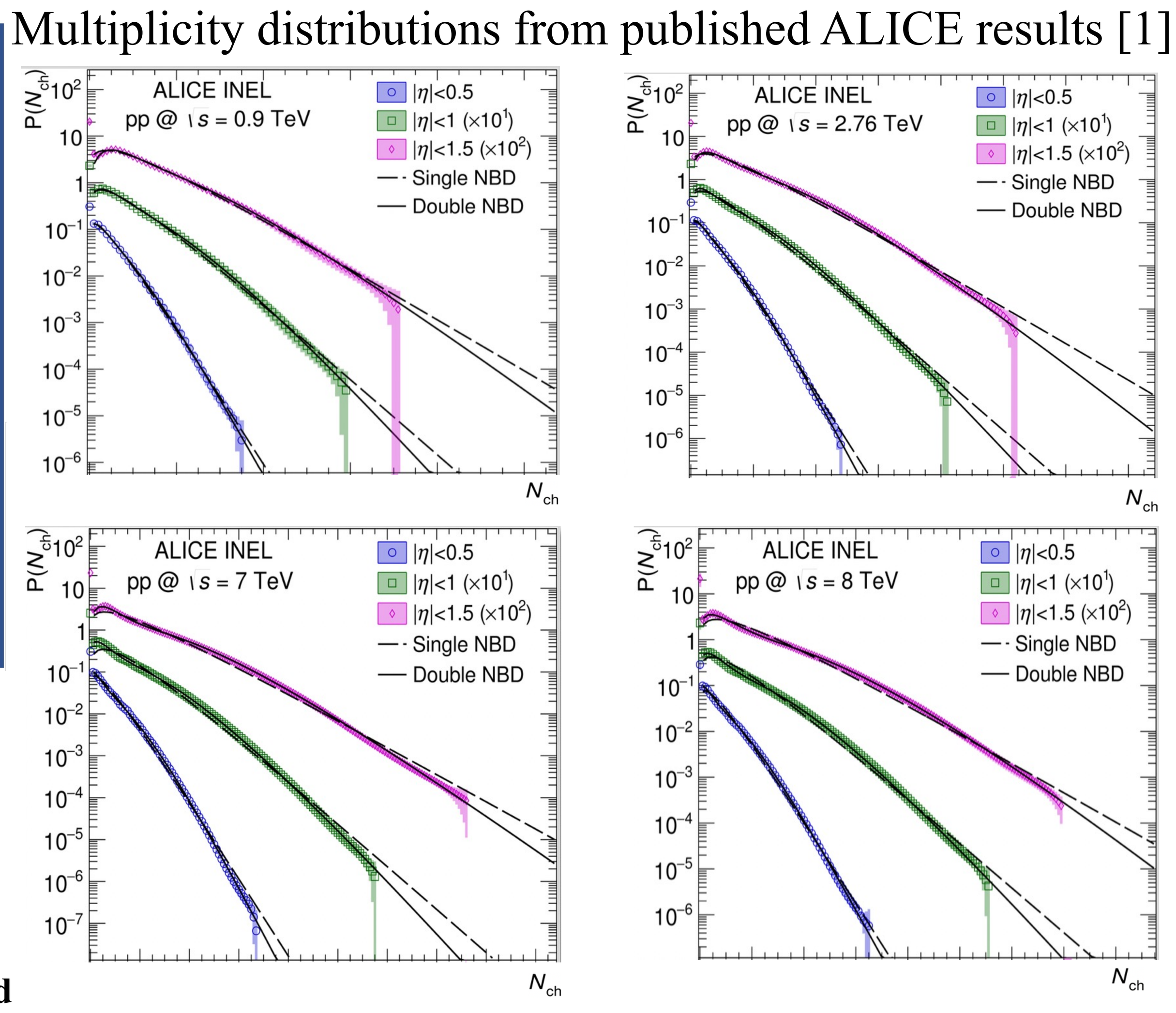


Multiplicity distributions for INEL events are used to calculate final state entropy.

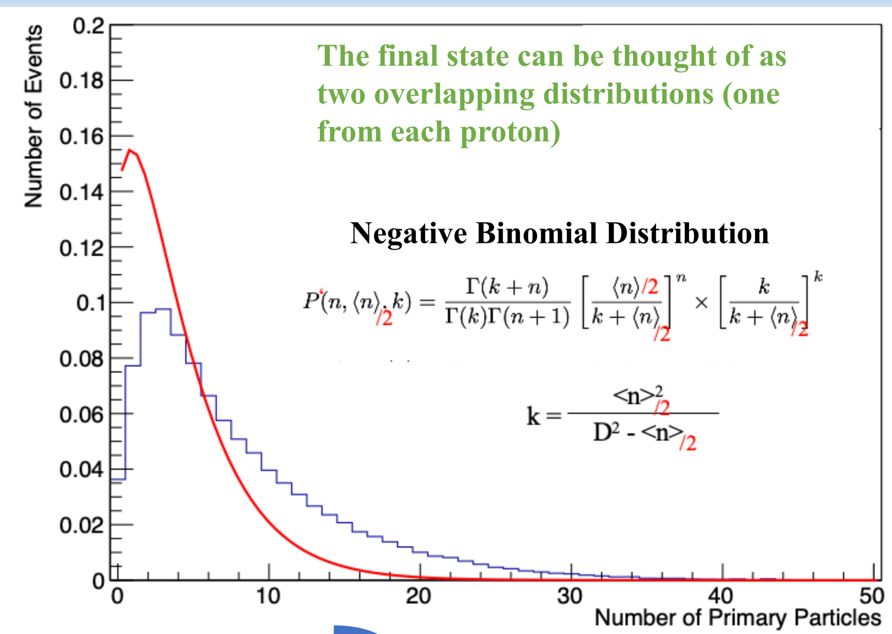
Collision Energy	Mean	Standard Deviation
$\sqrt{s} = 0.9$ TeV, $\lvert\eta\rvert < 0.5$, INEL	3.05 ± 0.03	3.57 ± 0.05
$\sqrt{s} = 2.76$ TeV, $\lvert\eta\rvert < 0.5$, INEL	3.80 ± 0.02	4.53 ± 0.03
$\sqrt{s} = 7$ TeV, $\lvert\eta\rvert < 0.5$, INEL	4.54 ± 0.02	5.71 ± 0.03
$\sqrt{s} = 8$ TeV, $\lvert\eta\rvert < 0.5$, INEL	4.86 ± 0.05	5.96 ± 0.07

The final-state entropy increases with the mean of the distribution and the energy of the collision.

Multiplicity distributions from published ALICE results [1]



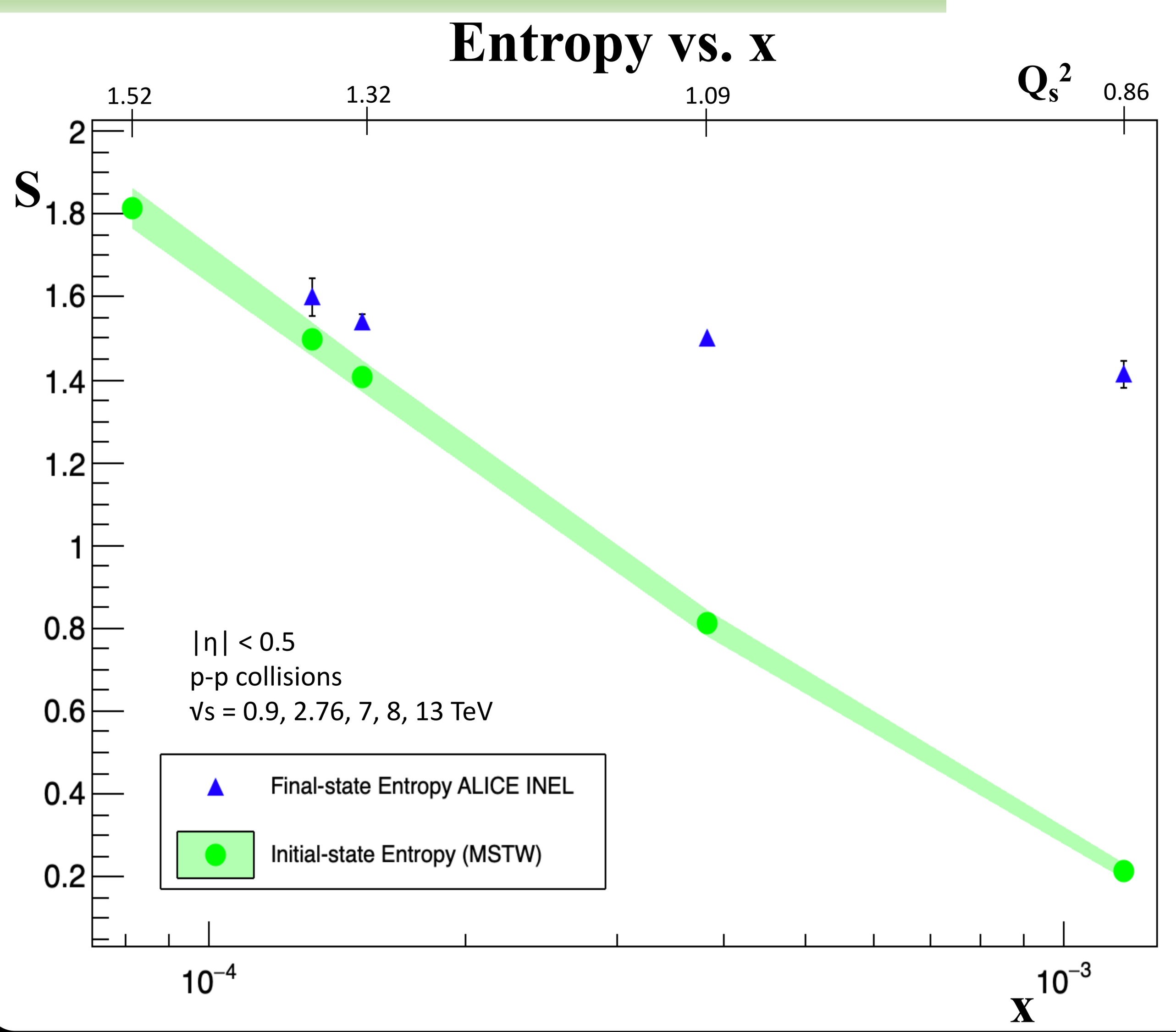
Extrapolating to single proton



To make a meaningful comparison to the single distribution of partons quantified by PDF's it is necessary to make a transformation on the final-state distribution.

1. **Assume a negative binomial distribution**
2. **Reduce the mean by half**
3. **Renormalize**

Conclusions and future work



As we approach x=0 (gluon dominate) the initial and final state entropies are increasing and closer in value.

The significant difference between the initial and final state at high-x may be attributed to an increase in quarks contributing to the wavefunction.

A better understanding of how to extrapolate our data to that of a single proton, may help further studies. We will continue to explore other considerations in this extrapolation.

There may also be other sources of entropy generation in the collision and a more robust theory may be necessary to demonstrate this equivalence. Particularly an understanding of the longitudinal components of entanglement may be necessary to explain fluctuations and other phenomena associated with particle production. In the future we would like to measure this in different kinematic regimes and systems. It would be particularly interesting to see these measurements done in systems which have been shown to generate QGP.

Refrences

[1] ALICE Collaboration, "*Charged-particle multiplicities in proton-proton collisions at √s = 0.9 to 8 TeV*", The European Physical Journal C, vol. 77, no. 1, January 2017

[2] Zhoudunming Tu, Dmitri E. Kharzeev, Thomas Ullrich, "*Einstein-Podolsky-Rosen Paradox and Quantum Entanglement at Subnucleonic Scales*", Physical Review Letters, vol. 124, no. 6, February 2019

[3] A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, "*Parton distributions for the LHC*", The European Physical Journal C, vol. 63, no. 2, pp. 189-285, July 2009.

[4] Buckley, Andy, et al. "*{LHAPDF}6: Parton Density Access in the {LHC} Precision Era.*" The European Physical Journal C 75.3 (2015): DOI: 10.1140/epjc/s10052-015-3318-8.

[5] Hentschinski, Martin, Krzysztof Kutak, and Robert Straka. "*Maximally Entangled Proton and Charged Hadron Multiplicity in Deep Inelastic Scattering.*" The European Physical Journal C 82.12 (2022): DOI: 10.1140/epjc/s10052-022-11122-1.

[6] Kaufman, Adam M., et al. "*Quantum Thermalization through Entanglement in an Isolated Many-Body System.*" Science, vol. 353, no. 6301, 2016, pp. 794-800, DOI: 10.1126/science.aaf6725.