Excited Hadron Channels in Hadronization

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Abstract. The proper treatment of hadronic resonances plays an important role for many aspects of heavy ion collisions. We expect this to be the case also for hadronization, due to the large degeneracies of excited states, and the abundant production of hadrons from their decays. We show how a comprehensive treatment of excited meson states can be incorporated into quark recombination, and in extension, into Hybrid Hadronization. We discuss in detail the quantum mechanics of forming excited states, utilizing the Wigner distribution functions of angular momentum eigenstates of isotropic 3-D harmonic oscillators. We describe how resonance decays can be handled, based on a set of minimal assumptions, by creating an extension of hadron decays in PYTHIA 8. Finally, we present a study of hadron production by jets using PYTHIA and Hybrid Hadronization with excited mesons up to orbital angular momentum $L = 4$. We find that states up to $L = 2$ are produced profusely by quark recombination.

Hybrid Hadronization (HH) [1–3] models the hadronization process by combining two distinct and well-established models, Lund string fragmentation [4, 5] and quark recombination [6–10]. The basic idea is that quarks close in phase space are allowed to recombine directly into mesons and baryons, governed by recombination probabilities $\mathcal{P}_h$. However, these probablities are small when a parton is far removed from other colored object. In that case the properties of the QCD vacuum and string dynamics become important. A colored parton needs to be part of a color neutral string system which then fragments into hadrons. Recombination probabilities play the role of a natural cutoff between the two domains.

In this write up we discuss the extension of the Hybrid Hadronization formalism from a rather schematic treatment of excited hadron states to a more realistic one that allows us to systematically include most resonances in the Particle Data Book [11], and even some additional ones that are predicted by the quark model but not yet confirmed. We focus here on mesons and leave baryons to a future work. The recombination probabilities can be factorized into contributions from SU(3) color, spin and the overlap of phase space distributions, $\mathcal{P}_h = \mathcal{P}_c\mathcal{P}_s\mathcal{P}_{kl}$, where k and l are the radial and orbital angular momentum quantum numbers of the meson h. We sum over the magnetic quantum number because we do not wish to consider polarized mesons here. Spin and color are usually treated purely statistical here, for

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exceptions see [12]. We will concern ourselves here mostly with the phase space coalescence probablity $\mathcal{P}_{kl}$.

We choose the simplest possible quantum mechanical model to describe the interaction between the quark and antiquark, which is a 3-D isotropic harmonic oscillator. We will denote the inverse length scale of the oscillator potential with ν. In practice, the values are determined by measured squared charge radii of the ground states, where available. The need for a phase space formalism comes from the unique requirements in $A + A$ collisions in which spatial information for both the medium and the particles that probe it, are believed to be important. The Wigner distributions of orbital angular momentum eigenstates in the 3-D harmonic oscillator potential are available in the literature [13]. They have been recomputed by us in convenient form in Ref. [14] by reducing the problem to the 1-D case [15]. For the lowest quantum numbers the phase space distributions as functions of position $\mathbf{r}$ and momentum $\mathbf{q}$ are

$$W_{00} = \frac{1}{\pi^3\hbar^3} e^{-\frac{q^2}{\hbar^2\nu^2}-\nu^2 r^2}\,, \tag{1}$$

$$W_{01} = W_{00}\left(-1+\frac{2}{3}\nu^2 r^2+\frac{2}{3}\frac{q^2}{\hbar^2\nu^2}\right), \tag{2}$$

$$W_{10} = W_{00}\left(1+\frac{2}{3}\nu^4 r^4-\frac{4}{3}\nu^2 r^2-\frac{4}{3}\frac{r^2 q^2}{\hbar^2}+\frac{8}{3}\frac{(\mathbf{r}\cdot\mathbf{q})^2}{\hbar^2}-\frac{4}{3}\frac{q^2}{\hbar^2\nu^2}+\frac{2}{3}\frac{q^4}{\hbar^4\nu^4}\right). \tag{3}$$

The distributions for $k = 1$, $l = 0$ are visualized in Fig. 1.

We assume that the quark and antiquark can be represented by Gaussian wave packets of a certain width. Certain parton shower Monte Carlos provide the average space-time position of partons but not their variance. We therefore fix the width of the parton wave packets to a value which makes the resulting formulas more compact. More general results are discussed in [14]. We interpret the space-time information given by shower Monte Carlos as the centroid positions of the wave packets. We can then proceed to compute the probabilities of a quark and an antiquark wave packet coalescing into a bound state of given quantum numbers k and l. The results for a few of the lowest energy states are

$$\mathcal{P}_{00} = e^{-u}\,, \tag{4}$$

$$\mathcal{P}_{01} = e^{-u}u\,, \tag{5}$$

$$\mathcal{P}_{10} = \frac{1}{2}e^{-u}\left(\frac{1}{3}u^2-\frac{1}{3}t\right) \tag{6}$$

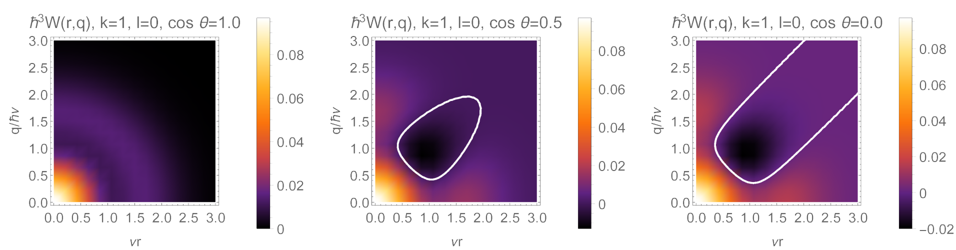

Figure 1. Phase-space distribution W_{10} for quantum numbers $k = 1$, $l = 0$ as functions of $r = |\mathbf{r}|$ and $q = |\mathbf{q}|$. The white lines indicate nodes where $W_{10} = 0$. Distributions are shown for several values of the angle θ given by $\cos\theta = \mathbf{r}\cdot\mathbf{q}/rq$.

where $u = \nu^2 x^2/2 + p^2/(2\hbar^2\nu^2)$ and $t = (\mathbf{x}\times\mathbf{p})^2/\hbar^2$ are the dimensionless squared distance of the initial partons in phase space, and their dimensionless squared total angular momentum, respectively. $\mathbf{x}$ and $\mathbf{p}$ are the relative distances of the centroids in coordinate and momentum space. It is instructive to analyze the mapping of the initial angular momentum $L^2 = t\hbar^2$ onto the orbital angular momentum quantum number l which is discussed in [14]. Illustrations of the coalescence probabilities $\mathcal{P}_{10}$ are shown in Fig. 2

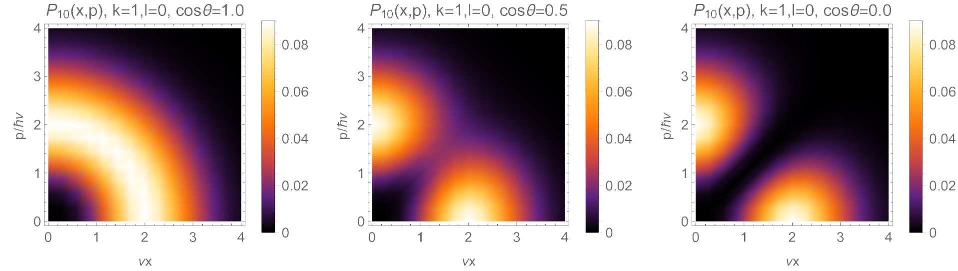

Figure 2. Coalescence probabilities $\mathcal{P}_{10}$ for quantum numbers $k = 1$, $l = 0$ for two Gaussian wave packets being mutually attracted by an isotropic 3-D harmonic oscillator potential. Probabilities are shown as functions of relative coordinates $x = |\mathbf{x}|$ and $p = |\mathbf{p}|$ for several values of the angle $\cos\theta = \mathbf{x}\cdot\mathbf{p}/xp$ between the vectors.

In the following study, we include excited meson states up to $N = 2k + l = 4$. Only about half of these states are considered confirmed in the Particle Data Book. We assume here that these states exist and we have assembled the necessary particle data, in a format compatible with PYTHIA 8 [12]. We estimate masses of mesons with higher quantum numbers using the well-established scaling laws with k and l [16]. We also provide branching ratios for strong decays into ground state mesons applying simple phase space estimates [1] and isospin algebra.

As an important preliminary result, we discuss here the importance of exited meson states as channels in quark recombination. To this end, we generated $e^+ + e^-$ collisions at 91.2 GeV with PYTHIA 8 [5] and handed the partons to Hybrid Hadronization. Fig. 3 shows the distribution of different quantum numbers for the recombined hadrons before decays. The total spin j of the meson is determined by l and the spin state of the quarks as usual. Note that the relative contribution of recombined hadrons to the total yield is relatively modest in a system like $e^+ + e^-$, only a few percent. We clearly see that P- and D-wave mesons are as important as S-wave mesons in this example. Overall, mesons up to spin-3 make a significant contribution to recombination yields.

In summary, we have extended the Hybrid Hadronization model to include the full spectrum of excited mesons. It is also possible to include states "missing" from the Particle Data Book. To this end, we have computed the Wigner phase space distributions of angular momentum eigenstates in the 3-D isotropic harmonic oscillator. We have then considered the coalescence of wave packets into such states. We find that P-wave and D-wave mesons are very important channels for hadronization. In the future we plan to extend our study to baryons and heavy flavor states.

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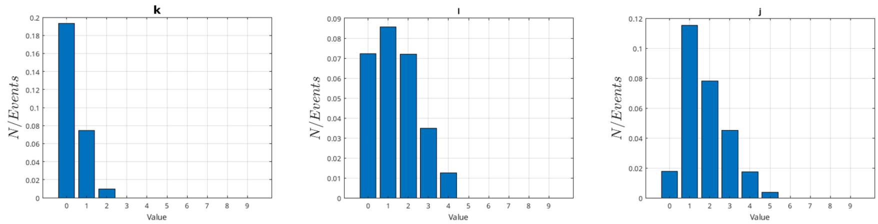

Figure 3. Yields of mesons of different radial quantum number k (left panel), orbital angular momentum quantum number l (center panel) and total angular momentum quantum number j (right panel) in e^+e^- collisions at $\sqrt{s} = 91.2$ GeV using PYTHIA 8 and Hybrid Hadronization. Only mesons from recombination are shown before decays of excited states.

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