Far-off-equilibrium early-stage dynamics in high-energy nuclear collisions

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Abstract. We explore the far-off-equilibrium aspects of the (1+1)-dimensional early-stage evolution of a weakly-coupled quark-gluon plasma using kinetic theory and hydrodynamics. For a large set of far-off-equilibrium initial conditions the system exhibits a peculiar phenomenon where its total equilibrium entropy decreases with time. Using a non-equilibrium definition of entropy based on Boltzmann's H-function, we demonstrate how this apparently anomalous behavior is consistent with the second law of thermodynamics. We also use the H-function to formulate 'maximum-entropy' hydrodynamics, a far-off-equilibrium macroscopic theory that can describe both free-streaming and near-equilibrium regimes of quark-gluon plasma in a single framework.

1 Introduction

Precise determination of transport coefficients like the specific shear and bulk viscosities, η/s and ζ/s, of the quark-gluon plasma formed in high-energy nucleus-nucleus collisions hinges upon accurately modeling the stress tensor ($T^{\mu\nu}$) evolution during the system's early stage. This stage is characterised by far-off-equilibrium dynamics which may be modeled by weakly coupled kinetic theory until $\mathcal{O}(1)$ fm/c [1, 2]. This approach is, however, numerically daunting as solving kinetic theory amounts to tackling a 7-dimensional problem in phase-space. Moreover, if one is only interested in the evolution of macroscopic quantities like $T^{\mu\nu}$, solving for the full kinetic distribution is likely unnecessary. It is thus desirable to have a macroscopic framework which can model the far-off-equilibrium evolution of $T^{\mu\nu}$ both physically accurately and numerically efficiently. In this work, we first explore the sensitivity of the $T^{\mu\nu}$ evolution in kinetic theory to initial state momentum anisotropies of the plasma. By considering extreme off-equilibrium initial conditions for a quark-gluon gas undergoing Bjorken expansion [3], we point out non-intuitive out-of-equilibrium effects arising in kinetic theory. In the second part we formulate a new macroscopic theory (ME-hydrodynamics) which can be used to describe in a single framework both the far-off-equilibrium pre-hydrodynamic and the near-equilibrium dissipative hydrodynamic regimes of the plasma.

2 Kinetic theory of a massive quark-gluon gas

For a weakly interacting gas of quarks, anti-quarks, and gluons undergoing boost-invariant Bjorken expansion along the beam axis, we solve the Boltzmann equation in a relaxation-time

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approximation,

$$\frac{\partial f^i}{\partial \tau} = -\frac{1}{\tau_R(T)} \left(f^i - f^i_{\rm eq}\right). \tag{1}$$

Here τ is Milne time, τ_R is the microscopic relaxation time, and the superscript $i \in \{q, \bar{q}, g\}$ on the kinetic distributions distinguishes between particle species. $f^i_{\rm eq}$ are given by Fermi-Dirac (for quarks and anti-quarks) or Bose-Einstein (for gluons) distributions which involve the Landau matched effective temperature and quark chemical potential (T, μ). Symmetries of Bjorken flow imply vanishing net-quark diffusion, i.e. $n(\tau) \propto 1/\tau$ and $T^{\mu\nu} = \sum_i \int_{p_i} p^\mu_i p^\nu_i f^i =$ diag(e, P_T, P_T, P_L), where e is energy density and P_T and P_L are effective transverse and longitudinal pressures. An important physical quantity is the non-equilibrium entropy density (in the rest frame of a fluid having velocity u^μ), obtained from Boltzmann's H-function:

$$s = -\sum_i \int_{p_i} (u \cdot p_i) \left[f^i \ln f^i - \frac{1 + a_i f^i}{a_i} \ln\left(1 + a_i f^i\right)\right], \tag{2}$$

where $a_{q,\bar{q}} = -1$ and $a_g = 1$. In equilibrium $s \to s_{\rm eq} = (e + P - \mu n)/T$. In Fig. 1 we show solutions of kinetic theory for two sets of extreme far-off-equilibrium initial conditions (see figure caption) which were set up using a Romatschke-Strickland (RS) distribution [5, 6]. Although all curves start with the same effective (T, μ_B), the phase trajectories are quite sensitive to the choice of initial momentum space anisotropy. In Bjorken flow, Navier-Stokes hydrodynamics predicts that the ratio $s_{\rm eq}/n$ must increase over time due to viscous heating. While this is indeed the case for panel (a) (see dotted lines for $s_{\rm eq}/n$ evolution in (b)), this expectation is not borne out for the trajectories in panel (c). Here, s_{eq}/n decreases for a certain duration of time. However, this does not imply a violation of the second-law of thermodynamics as the total entropy per baryon which includes non-equilibrium effects never decreases. The feature of decreasing equilibrium entropy per baryon density results in a peculiar phenomena which we call 'non-equilibrium cooling' (see Fig. 2). Here, the effective temperature falls even faster than what is expected for an ideal (inviscid) fluid.

3 Maximum-entropy truncation of the Boltzmann equation

The Boltzmann equation can be expressed as an infinite hierarchy of equations for momentum moments of $f(x, p)$ [7] where low-order moments corresponding to components of $T^{\mu\nu}$

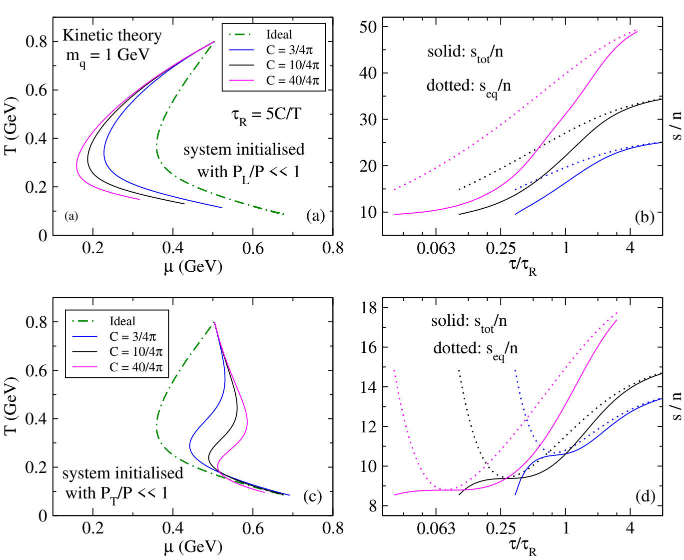

Figure 1. Phase trajectories and corresponding entropy evolution of a quark-gluon gas initialized near the *stable* fixed point of early-time Bjorken dynamics, i.e., $P_L/P \ll 1$ [4] (upper panels), and the *unstable* fixed point $P_T/P \ll 1$ (lower panels). The magenta curves depict a very weakly interacting gas whereas the green curve ($\tau_R \to 0$) is for a strongly interacting system (perfect fluid).

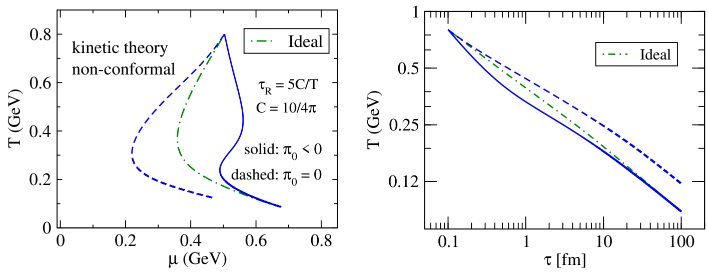

Figure 2. Non-equilibrium cooling: the solid (dashed) blue curve, initialised with non-equilibrium (equilibrium) initial conditions, cools more rapidly (slowly) than a dissipationless system.

are coupled to higher-order 'non-hydrodynamic' moments. To obtain a macroscopic description solely in terms of $T^{\mu\nu}$, the infinite hierarchy has to be truncated by expressing the non-hydrodynamic moments in terms of an approximate kinetic distribution using only information contained in $T^{\mu\nu}$. Based on Jaynes's insights on the connections between statistical mechanics and information theory [8], Everett *et al.* [9] recently proposed a novel way of reconstructing a kinetic distribution from the energy-momentum tensor using the maximum entropy principle. The idea is to find an $f(x,p)$ that maximizes the non-equilibrium entropy density (2), subject to the information (constraint) that it reproduces the given 10 components of $T^{\mu\nu}$. For a single component gas the maximum entropy distribution is [9]

$$f_{\rm ME}(x,p) = \left[\exp\left(\frac{\Lambda_{\mu\nu}p^\mu p^\nu}{u\cdot p}\right) - a\right]^{-1}, \tag{3}$$

where $\Lambda_{\mu\nu}$ are Lagrange multipliers corresponding to $T^{\mu\nu}$. Landau matching conditions further simplify the argument of the exponential [10]. Unlike the commonly used distributions for Grad or Chapman-Enskog (CE) truncation, $f_{\rm ME}$ is positive definite for all momenta and allows for non-equilibrium matching to conserved currents for a wide range of non-equilibrium stresses. It also ensures that the resulting macroscopic framework, which we call ME-hydro, has a non-negative entropy production rate [11] and that in the limit of small viscous stresses ME-hydro reduces to second-order Chapman-Enskog fluid dynamics [9].

4 ME-hydro vs. RTA kinetic theory in Bjorken and Gubser flows

The exact evolution equations for the 3 independent components of $T^{\mu\nu} = \mathrm{diag}(e, P_T, P_T, P_L)$ in Bjorken flow are given by

$$\frac{de}{d\tau} = -\frac{e+P_L}{\tau}, \quad \frac{dP_T}{d\tau} = -\frac{P_T-P}{\tau_R} - \frac{P_T}{\tau} + \frac{\zeta_T}{\tau}, \quad \frac{dP_L}{d\tau} = -\frac{P_L-P}{\tau_R} - \frac{3P_L}{\tau} + \frac{\zeta_L}{\tau}. \tag{4}$$

The terms (ζ_T, ζ_L) introduce couplings to 'non-hydrodynamic' moments of $f(\tau, p_T, p_z)$; for example, $\zeta_L = \int_p E_p^{-2}\, p_z^4\, f$. To truncate we replace $f \mapsto f_{\rm ME}$ where $f_{\rm ME}$ is constructed using the instantaneous values of (e, P_T, P_L) [12]. In Gubser flow [13] the exact evolution equations

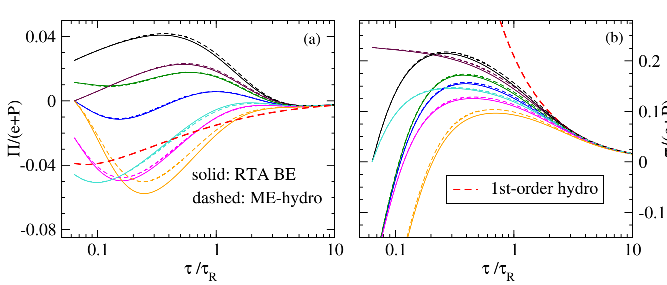

Figure 3. Comparison of ME-hydro results with non-conformal RTA BE for the scaled bulk viscous pressure $\Pi = (P_L + 2P_T - 3P)/3$ (left panel) and shear stress tensor compo- nent $\pi = 2(P_T - P_L)/3$ (right).

for the two independent (dimensionless) variables $(\hat{e}, \hat{P}_T)$ as functions of de-Sitter 'time' ρ are similarly truncated using f_{ME} [12]. Figures 3-4 show that ME-hydro is in excellent agreement with the underlying kinetic theory for both of these profiles even when the system is far-off-equilibrium.

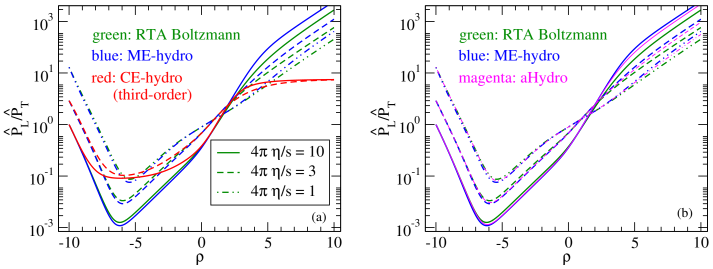

Figure 4. Evolution of the pressure anisotropy in Gubser flow. At early and late times the system approaches longitudinal and transverse free-streaming regimes, respectively.

Figure 4a shows that Chapman-Enskog hydrodynamics [14] fail to capture the late-time transverse free-streaming regime of Gubser flow. The only framework that performs slightly better than ME-hydro is anisotropic hydrodynamics [15, 16] (shown in panel (b)) which uses the RS ansatz as a truncation distribution.

4.1 Summary

Non-equilibrium effects during the early stages of QGP evolution can substantially alter its phase trajectories as compared to near-equilibrium predictions. ME-hydrodynamics, a macroscopic theory based on a simple physical principle, holds promise in describing such far-off-equilibrium effects. Further numerical analysis is required to test this expectation.

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