Analysis of Static Wilson Line Correlators from Lattice QCD at Finite Temperature with T-matrix Approach

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Abstract. The thermodynamic T-matrix approach is used to study Wilson line correlators (WLCs) for a static quark-antiquark pair in the quark-gluon plasma (QGP). Selfconsistent results that incorporate constraints from the QGP equation of state can approximately reproduce WLCs computed in 2+1-flavor lattice-QCD (lQCD), provided the input potential exhibits less screening than in previous studies. Utilizing the updated potential to calculate pertinent heavy-light T-matrices we evaluate thermal relaxation rates of heavy quarks in the QGP. We find a more pronounced temperature dependence for low-momentum quarks than in our previous results (with larger screening), which turns into a weaker temperature dependence of the (temperature-scaled) spatial diffusion coefficient, in fair agreement with the most recent lQCD data.

1 Introduction

Heavy-flavor (HF) particles are versatile probes for investigating the properties of the quark-gluon plasma (QGP) in ultrarelativistic heavy-ion collisions (URHICs): their large masses enable potential approximations and a Brownian motion description, and lead to a prolonged thermalization time that entails a sensitivity to their interaction history [1–3]. On the other hand, lattice-QCD (lQCD) computations provide first-principles information about in-medium properties of quarkonia, such as heavy-quark (HQ) free energies and Euclidean correlators [4, 5], although their interpretation is usually not straightforward. The thermodynamic T-matrix approach has been developed to combine input from lQCD with rigorous many-body theory to connect the open and hidden HF sectors and enable applications to URHICs phenomenology [6, 7].

In the present paper we focus on Wilson line correlators (WLCs) for a static quark-antiquark pair at finite temperature. Recent 2+1-flavor lQCD computations with realistic pion masses have found that the results cannot be described by hard-thermal loop perturbation theory predictions [8], while the use of various fit functions suggested a rather weak screening of the underlying potential along with large spectral widths. Here, we deploy the T-matrix approach to investigate these findings in a microscopic calculation based on nonperturbative input interactions.

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2 T-matrix Approach

The thermodynamic T-matrix is a quantum many-body formalism to compute 1- and 2-body correlation functions selfconsistently. It is a potential-based approach (energy-transfer is suppressed in heavy-particle scattering) for a 3D reduced Bethe-Salpeter equation [9], well suited for examining bound and scattering states in a strongly coupled medium [10, 11],

$$T_{ij}^{L,a}(z,p,p') = V_{ij}^{L,a}(p,p') + \frac{2}{\pi}\int_0^\infty k^2 dk V_{ij}^{L,a}(p,k) G_{ij}^0(z,k) T_{ij}^{L,a}(z,k,p') \,. \tag{1}$$

Here, $V_{ij}^{L,a}$ denotes the in-medium potential between partons i and j in color (a) and angular-momentum (L) channels, G_{ij}^0 the 2-parton propagator, and p (p') are the incoming (outgoing) CM momenta. In the color-singlet channel, we use the ansatz $\widetilde{V}(r,T) = -\frac{4}{3}\alpha_s[\frac{e^{-m_d r}}{r} + m_d] - \frac{\sigma}{m_s}[e^{-m_s r-(c_b m_s r)^2} - 1]$, with in-medium screening masses, $m_{d,s}$, and string breaking parameter, c_b, determined through constraints from thermal lQCD. In vacuum one recovers the Cornell potential, with coupling constant, $\alpha_s = 0.27$, and string tension, $\sigma = 0.225$ GeV^2. The Fourier transform of $\widetilde{V}(r,T)$, which figures in Eq. (1), contains relativistic corrections for finite-mass partons which renders the potential Poincare-invariant [6].

3 Static Wilson Line Correlators from the T-matrix

The Euclidean-time static WLC can be expressed through the static quarkonium spectral function, $\rho_{Q\bar{Q}}$, via a Laplace transform,

$$W(r,\tau,T) = \int_{-\infty}^{\infty} dE e^{-E\tau} \rho_{Q\bar{Q}}(E,r,T) \ ; \tag{2}$$

(r: distance between Q and $\bar{Q}$; E: total energy relative to the HQ threshold, $2M_Q^0$). Within the T-matrix formalism, the static $Q\bar{Q}$ spectral function can be expressed as [10, 12]

$$\rho_{Q\bar{Q}}(E,r,T) = \frac{-1}{\pi}\text{Im}\left[\frac{1}{E-\widetilde{V}(r,T)-\Phi(r,T)\Sigma_{Q\bar{Q}}(E,T)}\right] , \tag{3}$$

where $\widetilde{V}(r,T)$ is the static in-medium potential introduced in Sec. 2. The two-body selfenergy, $\Sigma_{Q\bar{Q}}$, contains the single HQ selfenergies and an r-dependent interference function, $\Phi(r,T)$, which mimics 3-body diagrams from thermal-parton scattering off Q and $\bar{Q}$ [10, 12]. We focus on the the first-order cumulant of the WLCs, defined as $m_1(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T)$ [8], which can be interpreted as an effective mass that is commonly used in lattice QCD.

Our goal is now to constrain the in-medium potentials by the static WLCs while maintaining a description of QGP equation of state within a selfconsistent calculation [12]. For simplicity, the interference function $\phi(r,T)$ is adapted from previous work [10], but allowing for $\pm 10\%$ variations. Figure 1 shows our fit results for the cumulants of the WLCs, where we used a constant string-screening mass of m_s=0.2 GeV, while m_d and c_b increase less with temperature than before [12]. A fair agreement with the lQCD results can be achieved, albeit with some deviations at higher temperatures. Different from previous constraints based on HQ free energies [10, 11], we infer less screening for the input potential, especially at higher temperatures and small and intermediate distances. Previous lQCD results and/or extractions for the complex potential have resulted in somewhat conflicting conclusions [8, 13–16], likely due to the ill-posed nature of the spectral reconstruction needed to obtain the potential. In the T-matrix approach, the potential, defined as the input driving kernel to the scattering equation does not necessarily have to agree with the potential extracted from lQCD. Rather, we directly compare the results of the T-matrix calculations to lQCD data in terms of the Wilson line correlators, thus avoiding a spectral reconstruction.

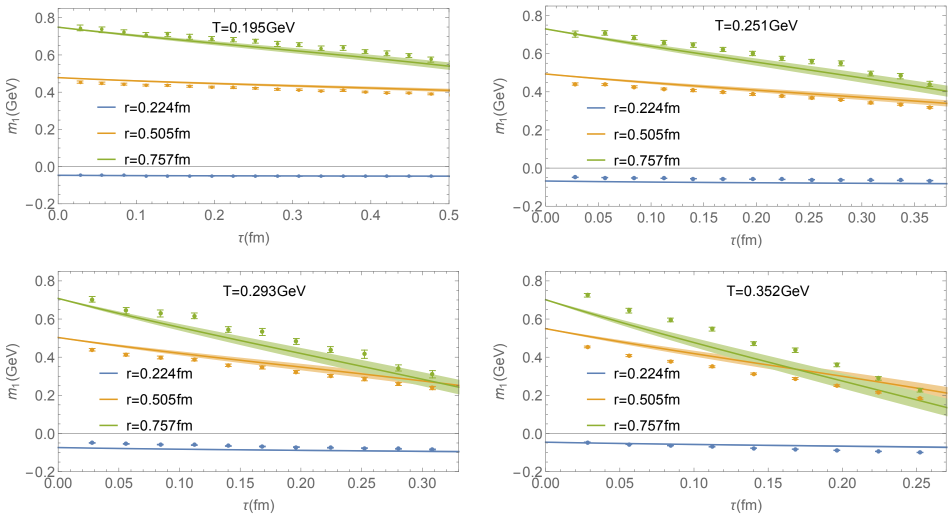

Figure 1. T-matrix results for the first cumulant of static euclidean-time WLCs (lines) for three r-values (color coded) at different temperature in each panel, compared to 2+1-flavor lQCD data [8].

4 Heavy-Quark Transport Coefficients

The updated potential as obtained in the previous section is Fourier-transformed and augmented with relativistic corrections for finite heavy- and light-parton masses [6] to calculate the pertinent scattering amplitudes which are then used to evaluate HQ transport in the QGP, *i.e.*, the thermal relaxation rate of charm (c) quarks, expressed as $A(p) = \left\langle (1 - \frac{\mathbf{p}\cdot\mathbf{p}'}{\mathbf{p}^2})\rho_i\rho_i\rho_c \right\rangle$; here, it is important to include off-shell integrations over the (broad) spectral functions, $\rho_{i,c}$, of the thermal-light and outgoing-charm quarks to access the bound-state peaks in the scattering amplitude below threshold [17, 18]. The relaxation rate has a more gradual dependence on temperature than previously [10, 11], mostly due to larger values at the higher temperatures, see Fig. 2 left. Consequently, the spatial diffusion coefficient, $D_s = T/(M_c A(p = 0))$, scaled by $2\pi T$, has a flatter dependence on temperature, see Fig. 2 right. In the static limit, the prediction is in reasonable agreement with recent lQCD data [19]. The ramifications of these results on HF phenomenology observables in URHICs will be studied in the near future.

5 Conclusions

We have conducted a microscopic calculation of static $Q\bar{Q}$ Wilson line correlators employing the in-medium T-matrix formalism. With a moderately modified input potential, which features less screening at higher temperatures than previously, we have been able to obtain a fair description of pertinent lQCD data while maintaining a description of the EoS in the light-parton sector. This has been challenging for (resummed) perturbative approaches and thus reiterates the prevalence of nonperturbative forces in the sQGP. When deploying the updated potential to evaluate HQ transport properties, we find that the increase in interaction strength at higher temperatures leads to a weaker temperature dependence for spatial HQ diffusion coefficient, which improves the agreement with recent 2+1-flavor lQCD data in the static limit. Applications of these results to the phenomenology of HF observables in URHICs are in progress.

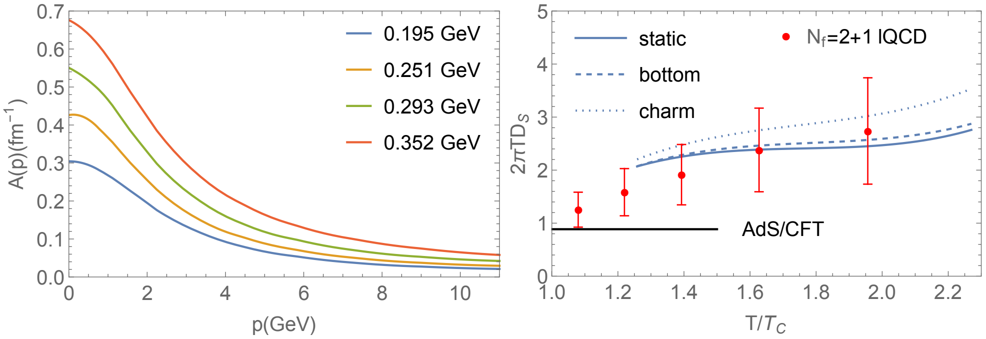

Figure 2. Left: thermal relaxation rate of *c*-quarks vs. 3-momentum at different temperatures. Right: spatial diffusion coefficient for static (blue solid line), bottom (blue dashed line) and charm (blue dotted line) quarks vs. temperature (scaled by T_c=155 MeV) in comparison to the 2+1-flavor lQCD data [19] (red dots with error bars) and an AdS/CFT estimate [20] (black line). The bare quark masses are 1.359 GeV (charm), 4.681 GeV (bottom), and 10 GeV to approximate the static limit.

Acknowledgments

This work has been supported by the U.S. National Science Foundation under grant no. PHY-2209335 and by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics through contract No. DE-SC0012704 and the Topical Collaboration in Nuclear Theory on *Heavy-Flavor Theory (HEFTY) for QCD Matter* under award no. DE-SC0023547.

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