Finite volume effects near the chiral crossover

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Abstract. The effect of a finite volume presents itself both in heavy ion experiments as well as in recent model calculations. The magnitude is sensitive to the proximity of a nearby critical point. We calculate the finite volume effects at finite temperature in continuum QCD using lattice simulations. We focus on the vicinity of the chiral crossover. We investigate the impact of finite volumes at zero and small chemical potentials on the QCD transition though the chiral observables.

1 Introduction

It is well known that QCD exhibits a thermal transition which turned out to be an analytic crossover in the case of physical quark masses and vanishing baryon chemical potential $\mu_B = 0$ [1]. Finite size scaling using aspect ratios $LT = 4, 5, 6$ specified the transition as analytic since the peak of the chiral susceptibility shows basically no or a mild volume dependence. Further studies of the equation of state demonstrate that the main driver of uncertainties are not finite volume effects, but instead cut-off effects which lead to taste-violation in the case of staggered quarks [2]. These can be significantly reduced by employing tree-level corrections, stout-smearing methods or using the HISQ action [3, 4]. Nevertheless finite volume effects play a crucial role phenomenologically and theoretically. The fireball produced in heavy-ion collisions is of finite size and if the crossover turns into a real transition, volume effects get more and more severe. Hence we study the impact of finite volumes at vanishing chemical potential and at finite μ_B using the imaginary chemical potential Taylor method by setting the focus on the chiral observables.

2 Chiral observables

In the case of vanishing quark masses the chiral condensate $\langle\bar{\psi}\psi\rangle$ deals as a true order parameter to probe the spontaneous breaking of the underlying chiral symmetry. Since nature presents us small but finite quark masses, the symmetry is also explicitly broken which leads to a non-vanishing value of the condensate at high temperatures T although the spontaneous breaking is restored. We are interested in physical results and perform whenever it is possible

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a continuum extrapolation. Hence we use the following renormalization scheme to remove additive and multiplicative divergences

$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \log Z}{\partial m} \qquad \langle\bar{\psi}\psi\rangle_R = -\left[\langle\bar{\psi}\psi\rangle_T - \langle\bar{\psi}\psi\rangle_{T=0}\right]\frac{m}{f_\pi^4} \tag{1}$$

$$\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m^2} \qquad \chi_R = \left[\chi_T - \chi_{T=0}\right]\frac{m^2}{f_\pi^4}, \tag{2}$$

by subtracting the zero temperature part of the observable $\langle ... \rangle_{T=0}$ and multiplying with the light quark mass m in lattice units. To get a dimensionless quantity, the result is divided by the pion decay constant f_π.

3 Volume dependence of the chiral condensate

The key feature of a crossover transition is basically no or a very mild volume dependence of the observable and hence the absence of discontinuities or divergences up to the infinite volume limit. In the opposite direction, i.e. decreasing the volume, the behavior is not so clear. Chiral perturbation theory predicts an exponential dependence of the condensate as a function of the spatial extension N_x. The leading asymptotic behavior of the condensate at vanishing magnetic field and $T = 0$ takes on the form [6]

$$\langle\bar{\psi}\psi\rangle \sim \frac{\sqrt{m_\pi}}{F_\pi^2}\frac{e^{-m_\pi N_x}}{(2\pi N_x)^{3/2}}. \tag{3}$$

This can be compared with our lattice results if we pick a temperature below T_c as shown in Fig. 1. Here the chiral condensate is solved via a spline interpolation at fixed $T = 140$ MeV for all lattices with $N_t = 12$. The blue curve is the fit function $f(N_x)$ as shown in the

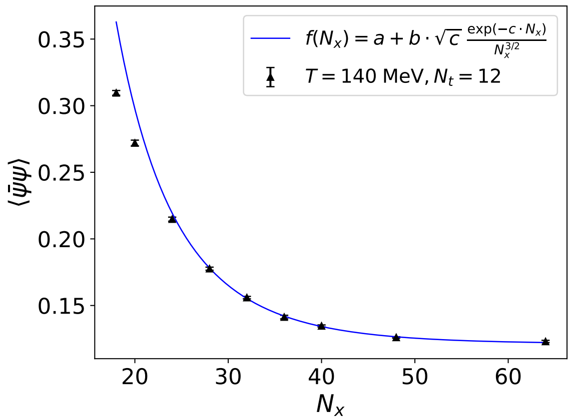

Figure 1. Chiral condensate solved at a fixed temperature $T = 140$ MeV for every lattice with $N_t = 12$ as a function of the spatial extension N_x. The blue curve is a fit inspired by chiral PT (Eq. (3)) in the range of $N_x \in [28, 64]$.

legend and provides $\chi^2/\mathrm{ndof} = 1.03$. The coefficient c is m_π according to Eq. (3) and reads $c = 131 \pm 10$ MeV. This remarkable agreement with the pion mass is only true for $N_x \geq 28$. One reason for this lies in the fact that the transition temperature for $18^3 \times 12$ and $20^3 \times 12$ is below $T = 140$ MeV as shown in Fig. 3 on the right panel. Hence the system tends to be deconfined and cannot be described by the chiral PT Eq. (3).

4 Volume dependence of the transition temperature T_c

To obtain the transition temperature T_c we follow a similar strategy as described in [5]. For a broad range of aspect ratios we can now perform a continuum extrapolation as examplary demonstrated in the left panel of Fig. 2. The continuum extrapolotated results of the transition temperature for each aspect ratio are shown on the right panel. Again we observe an

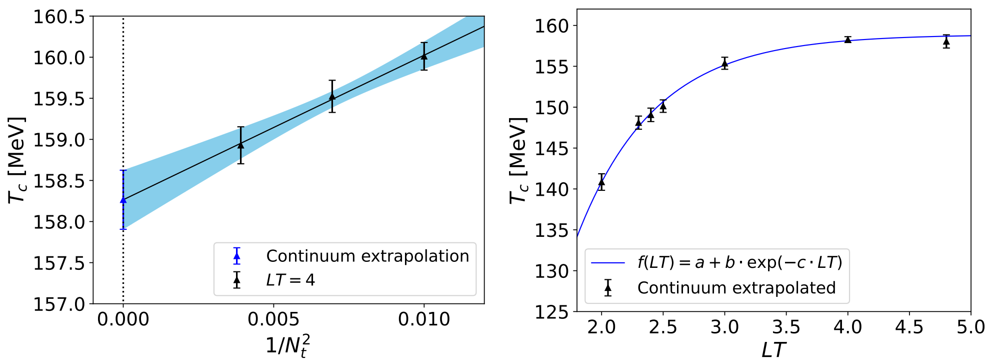

Figure 2. Left: Exemplary continuum extrapolation of T_c at aspect ratio $LT = 4$. Right: Continuum extrapolated T_c as a function of the aspect ratio LT and additional infinite volume extrapolation via an exponential fit.

exponential dependence which allows us to obtain the infinite volume limit of the continuum extrapolated transition temperatures

$$T_c(N_t \to \infty, LT \to \infty) = 158.9 \pm 0.6 \text{ MeV}. \tag{4}$$

The exponential dependence on the volume is not limited to T_c. As demonstrated in Fig. 3, the peak of the susceptibility $\chi_{\max}$ indicates a similar behavior (left). It decreases and stays

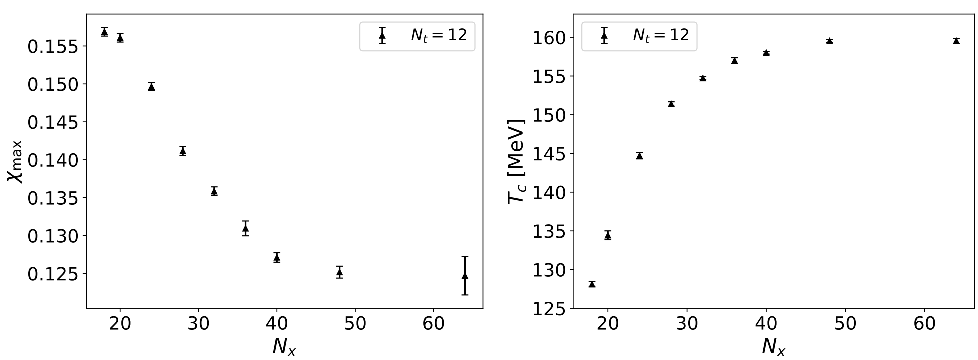

Figure 3. Volume dependence of $\chi_{\max}$ (left), and T_c (right) as functions of N_x.

nearly constant if $N_x \gtrsim 40$ ($LT \gtrsim 3.3$) which is a clear sign of a crossover. It confirms the common standard to use $LT = 4$ in QCD thermodynamics to be close to the infinite volume limit. In the opposite direction, the peak increases significantly as the volume is further decreased.

5 Volume dependence of T_c at finite and real μ_B

So far we set the focus on vanishing chemical potential. Let us extend our results to finite density and investigate the volume dependence. To circumvent the sign problem, we performed simulations at purely imaginary and vanishing chemical potentials. These runs deal as a lever arm to extrapolate to finite and real $\hat{\mu}_B$. Given these runs, we extrapolate T_c to real μ_B according to $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2$ up to leading order.

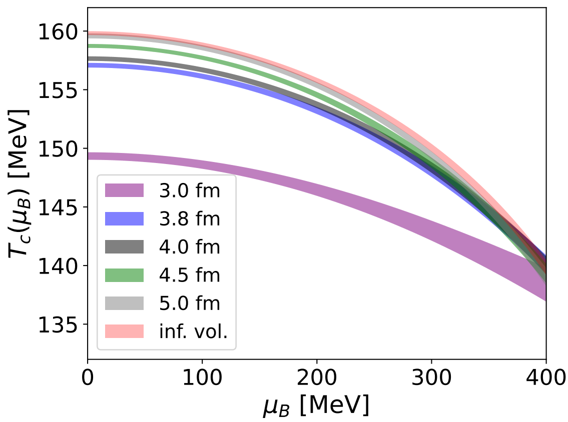

Figure 4. T_c extrapolated to finite and real μ_B for various box sizes converted in fm for $N_t = 12$.

The results calculated in a finite box of a length given in fm are shown in Fig. 4. Here we can conclude that a box size of 5 fm agrees with the infinite volume extrapolated result.

Acknowledgments

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References

[1] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz, K. K. Szabó Nature 443 (2006)

[2] S. Borsányi, G. Endrody, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti, K. K. Szabó JHEP 11 (2010)

[3] S. Borsányi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krief, K. K. Szabo Phys. Lett. B 730 (2014)

[4] hotQCD collaboration Phys. Rev. D 90 (2014)

[5] S. Borsányi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pásztor, C. Ratti and K. K. Szabó Phys. Rev. Lett. 125, 052001 (2020)

[6] P. Adhikari and B.C. Tiburzi Phys. Rev. D 107, 094504 (2023)