Harmonics of Parton Saturation in inclusive and diffractive Lepton-jet correlation at EIC

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Abstract. We study the harmonic coefficient of both inclusive and diffractive azimuthal angle dependent lepton-jet correlations in Hadron-Electron Ring Accelerator and the future electron-ion collider. Numerical calculations for inclusive and diffractive harmonics and the ratio of harmonics in $e+\text{Au}$ and $e+p$ reveal their strong discriminating power for non-saturation model and saturation model. Moreover, we demonstrate that the t-dependent diffractive harmonics are innovative observables for nuclear density profile.

1 Introduction

Gluon saturation, as one of the three pressing questions the future electron-ion collider(EIC) will address, has attracted much efforts on constructing observables to probe it. Two particle correlations, such as dijet, dihadron and jet plus color-neutral particle, are promising observables under intense study. In this manuscripts, we will introduce a new type two-particle correlation, lepton-jet correlation[1, 2] as probe for gluon saturation phenomenon.

The typical configuration for two particle correlation in the transverse plane is back-to-back, where the imbalance momentum $|\vec{q}_\perp| = |\vec{k}_{1\perp} + \vec{k}_{2\perp}|$ is much smaller than the relative momomentum $|\vec{P}_\perp| = |(\vec{k}_{1\perp} - \vec{k}_{2\perp})/2|$. The soft gluon radiation of the final jet favors the jet direction, which causes the azimuthal angle anisotropy of the lepton-jet correlation[3, 4]. The gluon saturation determine the intial quark TMD distribution in dipole picture. The transverse momentum of initial quark do not have preferred angle. Therefore, gluon saturation will suppress the anisotropy.

We calculate the fourier coefficients or the harmonics of the azimuthal angle dependent cross section, for both inclusive and diffractive lepton-jet production processes, in the EIC kinematics. The suppression of harmonics by gluon saturation will be demonstrated by the nuclear modification factor, the ratio of the harmonics in $e + \text{Au}$ and $e + p$ collisions. The t-dependent diffractive harmonics are found to be sensitive to different nuclear density profiles.

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2 Lepton-jet correlation

In deeply inelastic scattering(DIS), an energetic lepton scatters off a proton or nucleus target. We detect the scattered lepton and final jet, and measure the azimuthal angle between final lepton and jet.

The leading order cross section do not have azimuthal angle dependence. Soft gluon radiations introduce the azimuthal angle dependence, and can be resummed into Sudakov factor. The Sudakov factor in $b_\perp$ space is expressed as $\mathrm{Sud}(b_\perp) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \frac{\alpha_s(\mu) C_F}{\pi} \Big[\ln \frac{Q^2}{\mu^2} + \ln \frac{Q^2}{P_\perp^2} + c_0(R) \Big]$. The azimuthal angle dependent lepton-jet correlation reads

$$\frac{d^5\sigma}{dy_\ell d^2P_\perp d^2q_\perp} = \sigma_0 \int \frac{b_\perp db_\perp}{2\pi} x f_q(x, b_\perp) e^{-\mathrm{Sud}(b_\perp)} \Big[J_0(q_\perp b_\perp) + \sum_{n=1}^{\infty} 2\cos(n\phi) \frac{\alpha_s(\mu_b) C_F c_n(R)}{n\pi} J_n(q_\perp b_\perp) \Big] . \quad (1)$$

The harmonics are defined as renomalized Fourier coefficient of the azimuthal angle dependent lepton-jet correlation

$$\langle \cos n\phi \rangle = \frac{\sigma_0 \int b_\perp db_\perp J_n(q_\perp b_\perp) W(x, b_\perp) \frac{\alpha_s(\mu_b) C_F c_n(R)}{n\pi}}{\sigma_0 \int b_\perp db_\perp J_0(q_\perp b_\perp) W(x, b_\perp)} . \quad (2)$$

W function is defined as $W(x, b_\perp) = x f_q(x, b_\perp) e^{-\mathrm{Sud}(b_\perp)}$.

We calculate the harmonics for saturation model, with two different parameterization for dipole scattering matrix $S_x(r_\perp)$. The first parameterization is GBW model[5], the other parameterization is solution of the running-coupling Balitsky-Kovchegov (rcBK) equation, with the modified McLerran-Venugopalan model[6, 7] as the initial condition.

For comparison, we also calculate the harmonics for non-saturation model. The expression of harmonics is the same as Eq. (2), but with different $\widetilde{W} = \sum_q e_q^2 x f_q(x, \mu_b) e^{-\widetilde{\mathrm{Sud}}(b_\perp)}$ function, which can be derived from collinear factorization framework[3, 4]. The $f_q(x, \mu_b)$ represents the collinear quark distribution: we utilize the NLO PDF sets of CT18A for proton, the EPPS21 PDF sets for the gold nucleus. The Sudakov factor for non-saturation model is $\widetilde{\mathrm{Sud}}(b_\perp) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \frac{\alpha_s(\mu) C_F}{\pi} \Big[\ln \frac{Q^2}{\mu^2} + \ln \frac{Q^2}{P_\perp^2} - \frac{3}{2} + c_0(R) \Big]$. There is a $-3/2$ single logarithmic term difference between saturation model sudakov factor $\mathrm{Sud}(b_\perp)$ and $\widetilde{\mathrm{Sud}}(b_\perp)$, which corresponds to the collinear divergence in the collinear factorization framework[3, 4].

In the numerical calculation, we introduce the non-perturbative Sudakov factor[8, 9]: for saturation model $\mathrm{Sud}(b_\perp) \to \mathrm{Sud}(b_*)$, for non-saturation model $\widetilde{\mathrm{Sud}}(b_\perp) \to \widetilde{\mathrm{Sud}}(b_*) + \widetilde{\mathrm{Sud}}^q_{\mathrm{NP}}(b_\perp)$. The b_*-prescription is $b_* = b_\perp / \sqrt{1 + b_\perp^2 / b_{\max}^2}$, with $b_{\max} = 1.5\ \mathrm{GeV}^{-1}$. Fig. 1 presents the $q_\perp$-distribution of $\langle \cos n\phi \rangle$ for different models, using both proton and nucleus targets with a jet cone size $R = 0.4$. The EIC kinematics are $\sqrt{s_{eN}} = 89$ GeV, $y_\ell = 2.41$, $0.008 \le x \le 0.01$, 4 GeV$\le P_\perp \le 4.4$ GeV, 5.6 GeV$\le Q \le 5.9$ GeV. The harmonics of the saturation model show a sizable decrease from the proton to the gold nucleus target. The QED radiation in the QED sector of lepton-jet scattering will contribute to the harmonics, the largest contribution is about 10% (see [11, 12]). We include the QED correction in Fig. 1 and the subsequent calculations.

To quantify the suppression of harmonics in e + Au collisions compared to $e + p$ collisions, we define nuclear modification factor $R_{eA}^{(n)} = \langle \cos n\phi \rangle_{eA} / \langle \cos n\phi \rangle_{ep}$. Figure of $R_{eA}^{(n)}$ shows the striking difference between saturation and non-saturation framework, one can find in papers[11, 12]. This justify the harmonics and nuclear modification factor are robust probe for gluon saturation.

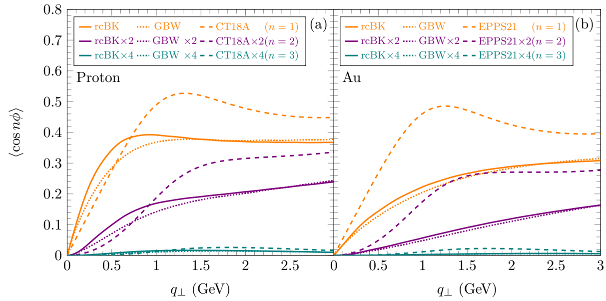

Figure 1. First three harmonics of inclusive lepton-jet production in (a) $e + p$ (b) $e + \mathrm{Au}$ collisions using inputs from the rcBK solution, GBW model, and CT18A PDFs. The QED corrections are included.

3 Diffractive lepton-jet correlation

In high energy ep and eA collisions, in the diffractive lepton-jet process we observe a large rapidity gap Y_{IP} between the hard interaction part and the remnant proton/nucleus, in addition to measuring the scattered lepton and one jet.

The momentum transfer in the diffraction is denoted as $t = (p' - p)^2 = \Delta^2 \approx -\vec{\Delta}_\perp^2$. The momentum fraction carried by these colorless gluons from the incoming nucleon is $x_{\mathrm{IP}} = n \cdot (p - p')/n \cdot p$, where $n = (0, 1, 0_\perp)$. The longitudinal momentum fraction carried by the quark from these colorless gluon is $\beta = x/x_{\mathrm{IP}}$. The rapidity gap of the diffractive lepton-jet process is the same as rapidity gap for semi-inclusive diffractive DIS (SIDDIS), $Y_{\mathrm{IP}} \sim \ln 1/x_{\mathrm{IP}}$, since the Lorentz rotation between these two frames is nearly identity matrix.

Inspired by SIDDIS process, we assume the diffractive lepton-jet production can also be factorized in terms of quark TMD diffractive PDF $df_q^D(\beta, k_\perp, t; x_{\mathrm{IP}})/(dY_{\mathrm{IP}}dt)$[10]. The azimuthal angle dependent cross section and harmonics of diffractive lepton-jet process have similar definition as Eq. (1) and Eq.(2), with W function as $W_{\mathrm{diff}}(x, \beta, b_\perp; x_{\mathrm{IP}}) = e^{-\mathrm{Sud}(b_\perp)} \int d^2k_\perp e^{i\vec{k}_\perp \cdot \vec{b}_\perp} x \frac{df_q^D(\beta, k_\perp, t; x_{\mathrm{IP}})}{dY_{\mathrm{IP}}dt}$.

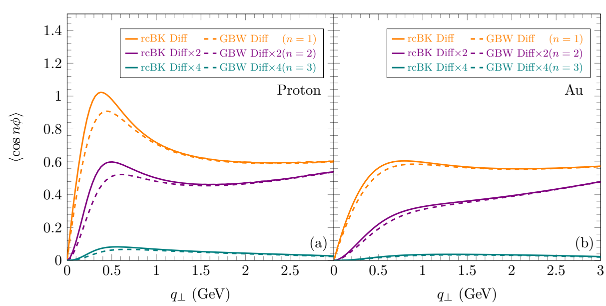

Figure 2. Harmonics of diffractive lepton-jet production in (a) $e + p$ collisions and (b) $e + \mathrm{Au}$ collisions with the inputs from the rcBK solution, GBW model.

In the numerical calculation, we first neglect the impact parameter $b_\perp$ dependence of the dipole S-matrix, and untilize GBW and rcBK solution as two dipole S-matrix parameterizations. Fig. 2 plot the harmonics of diffractive lepton-jet production for saturation models, considering both proton and gold nucleus target, with jet cone size $R = 0.4$. The kinematics bin of diffractive lepton-jet production at the future EIC is defined as follows: $\sqrt{s_{eN}} = 89$ GeV, $y_\ell = 2.41$, $0.008 \le x \le 0.0094, \beta = 0.94, x_{\rm IP} = x/\beta$, 4 GeV$\le P_\perp \le 4.32$ GeV, 5.6 GeV$\le Q \le 5.89$ GeV. The decrease of harmonics from proton to gold nucleus target are also observed in Fig. 2. Notably, the harmonics of the diffractive process are nearly two times the value of the harmonics of the inclusive lepton-jet process. The nuclear modification factors are nearly the same as inclusive process. These make them even better observables for studying saturation phenomenon.

We investigate the t dependence of the harmonics, by restoring the impact factor $b_\perp$ dependence of the quark diffractive PDF. We choose two different density profiles for proton(nucleus): one being a uniform cylinder, the other a uniform sphere. Fig. 3 shows the

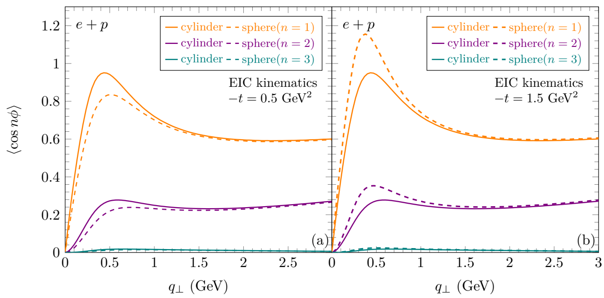

Figure 3. The comparison of diffractive harmonics of lepton-jet production in $e + p$ collisions in the EIC kinematics, for cylinder and sphere proton shape. For (a) $-t = 0.5$ GeV^2(b) $-t = 1.5$ GeV^2.

diffractive lepton-jet harmonics for $e + p$ collisions in the EIC kinematics: $\sqrt{s_{eN}} = 89$ GeV, $x = 0.008$, $y_l = 2.41$ with $\beta = 0.94, x_{\rm IP} < 0.01, R = 0.4$.The sizable difference between cylinder and sphere proton(nucleus) suggests harmonics as new probes for the density profile of the target. One can find the t-distributions of diffractive harmonics of $e + p$ collisions in paper[11]. They resembles the diffraction pattern in optics, with its minima determined by zeros of Bessel functions.

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