Sphaleron damping and effects on normal and anomalous charge transport in high-temperature QCD plasmas

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Abstract. We modify the hydrodynamic equations of a relativistic chiral plasma to account for dissipative effects due to QCD sphaleron transitions. By analyzing the linearized hydrodynamic equations, we show that sphaleron transitions lead to nontrivial effects on vector and axial charge transport phenomena in the presence of a magnetic field. Notably, dissipative effects of sphaleron transitions lead to the emergence of a wavenumber threshold that characterizes the onset of Chiral Magnetic Waves. Sphaleron damping also significantly impacts the time evolution of both vector and axial charge perturbations in the presence of a magnetic field. We further investigate the dependence of charge separation on the rate of sphaleron transitions, which may have implications for the experimental search for the Chiral Magnetic Effect in heavy ion collisions.

1 Introduction

QCD has a topologically non-trivial vacuum structure. Transitions between sectors can thermally activated by *sphaleron transitions* at sufficiently high temperatures, which is linked to the non-conservation of axial charge. Despite the expected importance of axial charge-changing processes in QCD, these have only been explored to a limited extent. In this proceeding, we present our primary results from [1]. We devise a macroscopic description of axial charge dynamics that includes damping due to sphaleron transitions. We then explore how this affects both vector and axial charge transport. Finally, we discuss the implications of our results for the experimental search for the Chiral Magnetic Effect (CME).

2 Sphaleron damping in QCD plasmas

Although chiral transport phenomena in high-temperature QCD plasmas are intrinsically non-equilibrium phenomena, their possible macroscopic manifestations emerge naturally within the framework of anomalous hydrodynamics [2]. If the process of axial charge equilibration is slow compared to the typical kinetic equilibration of the plasma, the axial currents $j^{\mu}_{A,f}$ represent additional slow variables whose dynamics can be described macroscopically by introducing additional axial chemical potentials μ^{f}_{A} associated with the residual deviations of the axial charge $j^{0}_{A,f}$ from the genuine equilibrium state. For a weakly-coupled $SU(N_c)$ plasma, this is indeed the case as the timescale of axial charge relaxation due to sphaleron transitions $\tau_{\rm sph} \approx \frac{\chi_A T}{\Gamma_{\rm sph}} \sim \alpha_S^{-5} T^3$ [3] is much larger than the timescale associated with the kinetic

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equilibration of the plasma, $\tau_{\rm kin} \approx \frac{4\pi\eta/s}{T} \sim \alpha_S^{-2}T^{-1}$ [4]. When considering the quark-gluon plasma (QGP) created in heavy ion collisions at RHIC and LHC energies, where temperatures typically range up to $\sim 4T_c$, one finds that with the estimate of $\Gamma_{\rm sph} \approx 0.1T^4$ from [5] $\tau_{\rm sph} \sim 10T^{-1}$ can be larger, but not significantly larger than $\tau_{\rm kin} \approx 2T^{-1}$fm/c for favorable values of the transport coefficient $\eta/s = 0.16$.

3 Hydrodynamic excitations

We consider a an $SU(N_c) \times U(1)$ gauge theory coupled to N_f flavors of massless Dirac fermions, which describes a high temperature QCD plasma in the presence of electromagnetic fields. In the presence of a slowly-varying, non-dynamical background electromagnetic field, the charge conservation laws take the form

$$\partial_\mu j^\mu_{V,f} = 0, \tag{1}$$

$$\partial_\mu j^\mu_{A,f} = (eq_f)^2 C E^\mu B_\mu - \frac{g^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a, \tag{2}$$

where the right hand side of Eq. (2) reflects the non-conservation of axial charge, where effects due to the Abelian chiral anomaly are described explicitly by the term $(eq_f)^2 C E^\mu B_\mu$ with the anomaly coefficient $C = N_c/2\pi^2$. Non-Abelian contributions to the axial anomaly are described by the last term in Eq. (2), which tend to erase any pre-existing axial charge imbalance. By following the arguments of Shaposnikov, McLerran, and Mottola [6], the expectation value of $\left\langle \frac{g^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \right\rangle$ can be expressed in terms of the sphaleron transition rate $\Gamma_{\rm sph}$ as

$$\left\langle \frac{g^2}{16\pi^2} G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \right\rangle = 4\Gamma_{\rm sph} \sum_f \frac{\mu_{f,A}}{T} \ , \tag{3}$$

which in the presence of finite axial chemical potentials $\sum_f \mu_{f,A}$ is manifestly non-zero. The vector and axial currents are described by the constitutive relations:

$$j^\mu_{V,f} = n_{V,f} u^\mu + \nu^\mu_{V,f}, \ , \qquad j^\mu_{A,f} = n_{A,f} u^\mu + \nu^\mu_{A,f}, \tag{4}$$

where $\nu_{V/A,f}$ refers to viscous corrections to the currents for a given fermion flavor. Such viscous corrections incorporate transport coefficients associated with both ordinary and anomalous charge transport, which include the CME, the Chiral Separation Effect, and the chiral vortical effects. This is described in more detail in Ref. [1].

We collect the conservation laws, constitutive equations, and the transport coefficients to obtain the collection of hydrodynamic equations. Then, we linearize the equations around a static equilibrium background, characterized by a fluid velocity field $u^\mu = (1, \mathbf{0})$, temperature T, and vanishing vector/axial charge chemical potentials $\mu_{V_f} = \mu_{A_f} = 0$, which is typical in high energy heavy ion collisions.

Denoting the evolution equation in the form $\partial_t \phi_a + M_{ab}\phi_b = 0$, one find that for a single fermion flavor ($N_f = 1$), for the fields $\phi_a = (\delta n_V, \delta n_A)$ the evolution takes the form

$$M^{N_f=1}_{ab} = \begin{pmatrix} D\mathbf{k}^2 & ieq_f C\chi_A^{-1}\mathbf{k}\cdot\mathbf{B} \\ ieq_f C\chi_V^{-1}\mathbf{k}\cdot\mathbf{B} & D\mathbf{k}^2 + \gamma_{\rm sph} \end{pmatrix} \tag{5}$$

where for illustrative purposes, we will use the equation of state $c_s^2 = 1/3$ as well as $\chi_V = \chi_A = T^2$ and $D = (2\pi T)^{-1}$.

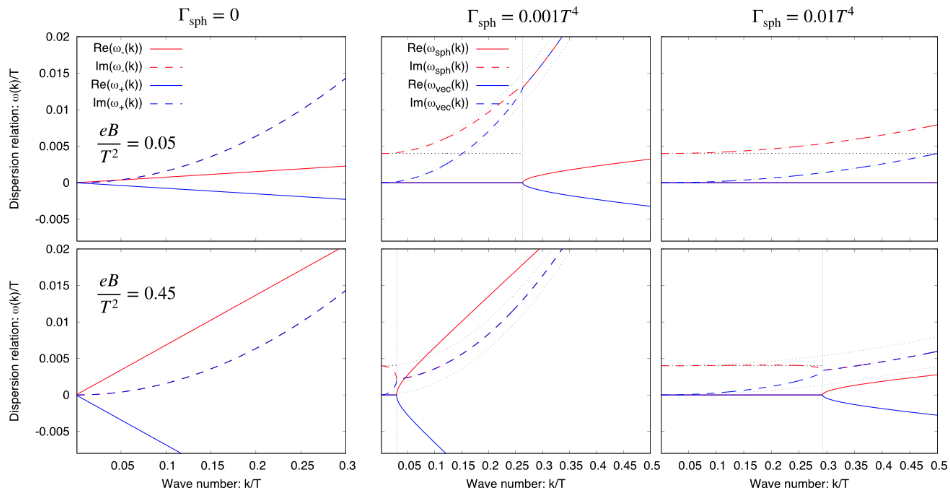

Figure 1. The dispersion relations for an $N_f = 1$ QCD plasma. The first column shows the dispersion relations in the absence of sphaleron transitions. Propagation of the Chiral Magnetic Wave depends on the magnetic field strength; at lower magnetic field strengths the propagation is dominated by charge diffusion. In the second and third column, a wavenumber threshold $k_{\rm CMW}$ emerges that indicates the on-set of the formation of the CMW. Below $k_{\rm CMW}$, the modes are purely dissipative due to charge diffusion and dissipative effects from sphaleron damping. Aboce $k_{\rm CMW}$, the behavior of modes depends on both the magnetic field strength (as in ordinary CMWs) and the sphaleron transition rate.

3.1 Dispersion relations

The dynamics of vector and axial charges in the absence of sphaleron transitions can be studied by setting $\gamma_{\rm sph} = 0$ in Eq. (5). The resulting dispersion relations are the known dispersion relations associated with the CMW up to $\mathcal{O}(\mathbf{k}^2)$ [7], as shown in the first column of Fig. 1. We observe that the dispersion relations have two distinct, competing parts, namely a diffusive imaginary part and a propagating real part. Since the diffusion constant D is fixed, the mechanism dominating the behavior of the excitations depends primarily on the magnitude and orientation of the wavevector $\mathbf{k}$ of the perturbation and on the strength of the magnetic field. In the presence of a weak magnetic field, the dynamics of charge modes will be governed by diffusion. As the magnetic field increases in strength, the low $\mathbf{k}$ modes oriented along the magnetic field will propagate with decreasing influence from diffusion.

The inclusion of sphaleron transitions associated with the term $\gamma_{\rm sph}$ leads to the emergence of a wavenumber threshold that provides the minimum wavenumber $k_{\rm CMW} = \sqrt{\frac{\chi_V}{\chi_A}\frac{2\Gamma_{\rm sph}}{e|q_f|C|\mathbf{B}|}}$, above which a propagating CMW can form for a given magnetic field strength. In the second and third column of Fig. 1, one readily observes that below $k_{\rm CMW}$, the modes are dominated by dissipation due to sphaleron damping and charge diffusion. Above $k_{\rm CMW}$, modes are highly diffusive unless the sphaleron transition rate is low and the magnetic field strength is sufficiently high.

We note that in [1] we have also performed a similar analysis for a two flavor $N_f = 2$ scenario, where sphaleron transitions induce a non-trivial coupling between the two flavors.

3.2 Charge separation

Vector charge separation along the direction of the magnetic field has been suggested as an experimental signature of the CME in heavy-ion collisions [8]. We consider vector charge separation as the result of an initial axial charge perturbation. We can quantify charge separation via the electric dipole moment:

$$D(B,t) = \int d^3x \frac{x \cdot B}{|B|} \sum_f e q_f n_{V,f}(t,x). \tag{6}$$

In Fig. 2, we present the dependence of the dipole moment $D(\mathbf{B}, t)$ on the sphaleron transition rate $\Gamma_{\rm sph}$. By normalizing the dipole moment to its value for $\Gamma_{\rm sph} = 0$, the quantity $D(\Gamma_{\rm sph})/D(\Gamma_{\rm sph} = 0)$ becomes independent of the magnetic field strength and can be viewed an overall suppression factor of the charge separation signal due to sphaleron transitions. When the sphaleron transition rate is large, the charge separation is proportional to $1/\Gamma_{\rm sph}$. By inspecting the results in Fig. 2 one finds that after an evolution for $10\text{fm}/c$, the suppression for sphaleron rates $\Gamma_{\rm sph}/T^4 \lesssim 0.01$ is still rather modest. However, for values on the order of the (quenched) lattice QCD estimates [5] $\Gamma_{\rm sph}/T^4 \gtrsim 0.02$ there is in a significant suppression of the signal, as well as a strong sensitivity of the result to the actual value of the sphaleron transition rate. While such a suppression may make it harder to detect possible signatures of the CME and CMW in heavy-ion collisions, the strong sensitivity to the sphaleron rate also suggests a possible experimental avenue for constraining the sphaleron rate using charge separation measurements associated with chiral phenomena such as the CME and CMW.

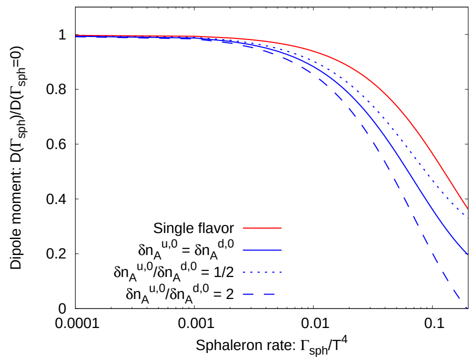

Figure 2. Electric charge separation, quantified by the electric dipole moment D for an initial axial charge distribution as a function of $\Gamma_{\rm sph}$ for single- and two-flavor configurations of various initial charge ratios at $t = 10$ fm$/c$.

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