Transverse-momentum-dependent (TMD) factorization in reactions with nuclei: from Drell-Yan to hadron production

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Abstract. We study cold nuclear matter effects on Drell-Yan production at small and moderate p_T in proton/pion-nucleus collisions using a new transversemomentum dependent (TMD) factorization framework. Both collisional broadening and medium-induced radiative corrections in the initial state are considered in the soft-collinear effective theory with Glauber gluons (SCET_G) approach. We demonstrate that in-medium bremsstrahlung exhibits rapidity divergences as $x \to 1$ and collinear divergences at the endpoints x = 0, 1 of the medium-induced emission spectra. We further show that the rapidity divergences lead to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution of the collision kernel and can be resummed into the transverse momentum broadening of particle production. In turn, the endpoints divergences of in-medium radiation can be resummed through the collinear evolution of parton densities in nuclear matter. The TMD factorization framework is applied to understand the transverse-momentum spectra of Drell-Yan pair production in pA and πA collisions and provides calculations with improved accuracy for hadron production in cold QCD processes at RHIC and LHC.

1 Introduction

The study of parton propagation in cold nuclear matter effects is of great importance to the understanding of initial-state physics in heavy-ion collisions. Theoretically, the Drell-Yan (DY) pair production on a nuclear target is an ideal probe of such effects at hadronic colliders. The DY process is described by a well-established factorization formula in *pp* reactions. In the so-called transverse-momentum-dependent (TMD) limit, the differential production cross-section is factorized as

$$\frac{d\sigma_{h_1h_2 \to \gamma^*}}{dY dM^2 d^2 \mathbf{p}} = \sum_q H_{q\bar{q}}(M,\mu) \int d^2 \mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} B_{q/h_1}(x_1,\mathbf{b},\mu,\frac{\zeta_1}{\nu^2}) S(\mathbf{b},\mu,\nu) B_{\bar{q}/h_2}(x_2,\mathbf{b},\mu,\frac{\zeta_2}{\nu^2}) \\
+ \sum_q \left[q \leftrightarrow \bar{q}\right] + O\left(\frac{\mathbf{p}^2}{M^2}\right).$$
(1)

Here, $B_{\bar{q}/h_1}$ and $B_{\bar{q}/h_2}$ are the incoming quark beam functions from the projectile (h_1) and the target hadron (h_2) , $H_{q\bar{q}}$ is the hard matching coefficient, and S is the soft function. The TMD

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beam function is further matched onto the collinear parton distribution function,

$$B_{q/h_1}(x_1, \mathbf{b}, \mu, \frac{\zeta_1}{\nu^2}) = \sum_i \int_{x_1}^1 \frac{dy}{y} C_{qi}\left(z, \mathbf{b}, \mu, \frac{\zeta_1}{\nu^2}\right) f_{i/h_1}\left(\frac{x_1}{y}, \mu\right) + O(\mathbf{b}^2 \Lambda^2).$$
(2)

And a similar equation can be written down for $B_{\bar{q}/h_2}$. The scale and rapidity renormalization group equations evolve different sectors to common renormalization scale μ and rapidity scale ν .

Since the final-state particles do not participate in the strong interaction, the observed difference between pp and pA collisions is entirely attributed to the initial state. We distinguish 1) intrinsic non-perturbative nuclear effects that modify the (collinear and TMD) parton distribution function of the nucleus $f_{\bar{q}/h_2} \rightarrow f_{\bar{q}/A}$, and 2) dynamical modifications of proton beam function $B_{q/p}$ due to multiple collisions of the quark with other nucleons in the nucleus before it annihilates in the primary hard process. In these proceedings, we only used the collinear nuclear PDFs as nuclear NP inputs and focused on the second type of dynamical effects, which can be partly studied perturbatively. The rapidity and transverse-momentum-dependent cross-section provides direct information on the 3D dynamics of energetic patrons in cold nuclear matter.

2 Medium correction to beam and soft functions

The dominant interaction between the quark in the incident proton and the medium is mediated by the Glauber gluons — off-shell gluons with a large transverse momentum component. Using soft-collinear-effective-theory with Glauber gluons (SCET_G) [1], we compute medium correction to the one-loop matching coefficient *C* at the first order in opacity (double Glauber exchanges at the cross-section level). The matching coefficients are expanded to the leading power of a medium-related small parameter $v = \xi^2 L^+/E^+$. $E^+ = x_1 P_1^+$ is the light-cone momentum of the quark, L^+ is the nuclear path length that quark passes through, and ξ^2 is the scale of non-perturbative effects in reaction with the nucleus.

The medium contribution to the matching coefficient displays additional collinear and rapidity divergences. In Ref. [2], we have shown that the collinear divergence is a result of the Landau-Pomeranchuk-Migdal effects that qualitatively change the divergent structure of the parton splitting functions near x = 0 and x = 1. After renormalization, it leads to a partial-differential type RG evolution equation for the collinear proton PDF and encodes the medium-induced energy loss and collinear quark-gluon conversion when the quark traverses cold nuclear matter. Evolution is performed in the range from the CNM screening scale $\mu^2 = \xi^2$ to the semi-hard scale $\mu^2 = \min\{\mu_b^2, 2E^+/L^+\}$, where $\mu^2 = \mu_b^2 \sim 1/b^2$ is expected to work in the limit $p_T^2 \ll 2E^+/L^+$, while the latter choice is applicable when $2E^+/L^+ \gg p_T^2$.

Using the η -regulator [3], we isolate the rapidity divergence from the matching coefficient. One can demonstrate that it is a BFKL type of rapidity divergence, and earlier studies [4–6] have shown that the soft function related to the Glauber gluon exchange shows the same type of singular behavior. We explicitly verified that the rapidity divergences cancel between the medium-corrected part of the collinear matching coefficient and the soft radiation induced by the Glauber gluon exchange. For the medium correction, the natural rapidity renormalization scale of the proton-collinear sector is $(\zeta_1^{(1)})^{1/2} = \min\{x_1P_1^+, L^+\mu_b^2\}$. As for the soft sector and collinear color sources, these natural scales are μ_b and $(\zeta_2^{(1)})^{1/2} \sim P^-$, where P^- is the large momentum component of the color source parton in the nucleus.

The LO momentum broadening factor is taken as the Fourier transform of a screened Coulomb scattering cross-section, the same as the one used in the previous study of semi-



Figure 1. Left: the exponentiation of the nuclear-broadening factor that undergoes the BFKL evolution. The evolution restricted by the LPM effect is shown by the red dashed line. Right: modifications of the dynamical nuclear effects on the proton-collinear TMD beam function.

inclusive DIS process in eA collisions [2],

$$\Sigma(\mathbf{b}) = g_s^2 C_F \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{1}{(\mathbf{q}^2 + \xi^2)^2} \left(e^{-i\mathbf{b}\cdot\mathbf{q}} - 1 \right).$$
(3)

The contribution from multiple collisions can be resumed into an exponential form $e^{\rho_G L\Sigma(b)}$, where ρ_G is an effective density of Glauber gluon also determined in Ref. [2]. This is regarded as the initial condition of the BKFL evolution equation. The evolution then resume soft emissions from y = 0 to $y_{\text{max}} = \ln\left(\sqrt{\zeta_1^{(1)}}\sqrt{\zeta_2^{(1)}}/\mu_b^2}\right)$. For example, when $L^+\mu_b^2 < x_1P_1^+$, $y_{\text{max}} = \ln(L^+P^-)$. The effect of medium-induced soft radiation is to enhance the momentum broadening as compared to LO calculation. Since the measurement is inclusive over the final states of the nuclear target under multiple collisions with the energetic partons, there is an ambiguity in choosing P^- . We make the following assumption that if we express quantities in the nuclear rest frame, $y_{\text{max}} = \ln(L^+P^-) \sim \ln(r_0A^{1/3}m_T)$. Where $m_T \sim \mu_b$ is the transverse mass of the collinear color source at scale μ_b . This choice is used in the estimation of y_{max} in the phenomenological calculation.

Thus, our final formula to compute TMD DY production includes 1) an in-medium evolved collinear parton density of the proton when passing through the nuclear medium, and 2) multiple-collision broadening of the TMD distribution, where the broadening factor of a single collision is evolved from the LO initial condition by the BFKL equation with a limited evolution range

$$f_{i/p}\left(\frac{x_1}{y},\mu_b\right)S(\mathbf{b},\mu,\nu) \to f_{i/p,\mathrm{med}}\left(\frac{x_1}{y},\mu_b,\min\left\{\mu_b,\sqrt{\frac{2x_1P^+}{L^+}}\right\}\right)S(\mathbf{b},\mu,\nu)e^{\rho_G L\Sigma(\mathbf{b},y_{\mathrm{max}})}.$$
 (4)

3 Results

The transport parameters related to the medium properties are taken to be the same as those describing the cold nuclear matter effects of hadron fragmentation function in SIDIS with



Figure 2. Left: the p_T -dependent nuclear modification factor $R_{W/Be}$. The calculation with only LO transverse momentum broadening disagrees with the E866 [7] experimental measurement. The inclusion of radiative corrections improves the agreement. Right: the calculation explains the p_T modification at both forward and backward rapidity measured by the PHENIX Collaboration [8], [9].

a nuclear target [2]. We compare our calculation to the nuclear modification factor of TMD Drell-Yan cross-sections in both fixed target experiments and in collider experiments. We find that the LO collisional broadening formula alone does not adequately explain the observed transverse momentum broadening, if one requires the CNM properties to be universal in both DY and SIDIS processes. With the radiative corrections included (the radiative energy loss and broadening), one can obtain correction description of TMD DY production using *the same* set of CNM parameters as those used for SIDIS.

4 Summary

To conclude, within $SCET_G$ we computed the medium correction at first order in opacity to the transverse-momentum-dependent production of Drell-Yan pairs on a nuclear target. We found that for the projectile-collinear beam function (proton, pion), a renormalization of the NLO medium correction leads to parton energy loss and a BFKL evolution of the collisional broadening factor. The NLO calculation significantly improves the comparison with experimental data and the same set of CNM transport parameters can simultaneously explain hadron fragmentation in SIDIS and TMD DY production in reactions with nuclei.

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