

Anomalous magnetic and electric dipole moments of tau lepton and spin correlations in the $e^-e^+ \rightarrow \tau^-\tau^+$ reaction

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- Current status of anomalous magnetic dipole moment (AMDM) and electric dipole moment (EDM) of τ lepton
- Features of the Monte Carlo program KKMC for $e^-e^+ \rightarrow f\bar{f}$ processes
- Spin-correlations in the $e^-e^+ \rightarrow \tau^-\tau^+$ reaction
- Test of spin-correlations in the Standard Model for the τ -decay channels $\tau^- \rightarrow \pi^-\nu_\tau$ and $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$
- MC modeling of distributions sensitive to AMDM and EDM for the decays $\tau^- \rightarrow \rho^-\nu_\tau \rightarrow \pi^-\pi^0\nu_\tau$ and $\tau^+ \rightarrow \rho^+\bar{\nu}_\tau \rightarrow \pi^+\pi^0\bar{\nu}_\tau$
- Conclusions

The talk is based on the recent preprint: e-Print: 2209.06047 [hep-ph], in collaboration with Swagato Banerjee (University of Louisville, Kentucky, USA) and Zbigniew Was (Institute of Nuclear Physics PAN, Krakow, Poland)

AMDM and EDM of the τ lepton

A few definitions of e.m. dipole moments of τ lepton (actually any fermion) with magnetic field \vec{B} and electric field \vec{E} :

$$H_M = -\vec{\mu}_\tau \vec{B} = -\frac{g_\tau}{2} \mu_B \vec{\sigma} \vec{B} \equiv (1 + a_\tau) \mu_B \vec{\sigma} \vec{B}$$

$$H_E = -\vec{d}_\tau \vec{E} = -\left(\frac{2m_\tau}{e} d_\tau\right) \mu_B \vec{\sigma} \vec{E}$$

where $\mu_B = e/(2m_\tau)$ is the Bohr magneton (for $\hbar = c = 1$).

The e.m. vertex of the lepton in the covariant form is

$$\Gamma^\mu = F_1(s) \gamma^\mu + \frac{\sigma^{\mu\nu} q_\nu}{2m_\tau} [i F_2(s) + \gamma_5 F_3(s)]$$

in terms of the Dirac (F_1), Pauli (F_2) and electric dipole (F_3) form factors. Here q is the photon four-momentum and $s = q^2$.

The form factors are normalized at $s = 0$:

$$F_1(0) = 1, \quad F_2(0) = \frac{1}{2}(g_\tau - 2) \equiv a_\tau, \quad F_3(0) = \frac{2m_\tau}{e} d_\tau$$

Magnetic dipole moment of the τ : theory

What are the motivations to study a_τ ?

In the Standard Model (SM) a_τ is calculated with high accuracy [S. Eidelman, M. Passera 2007]:

$$\begin{aligned}a_{QED}(3 \text{ loops}) &= 117324(2) \times 10^{-8}, \\a_{EW}(1 \text{ loop}) &= 47.4(5) \times 10^{-8}, \\a_{Hadr+LbL} &= 350.1(4.8) \times 10^{-8} \\a_\tau^{(SM)} &= a_{QED} + a_{EW} + a_{Hadr+LbL} = 117721(5) \times 10^{-8}\end{aligned}$$

The largest theoretical uncertainty from the **hadron contribution** is a limiting factor in the SM.

Effects of New Physics typically are expected to be $a^{(NP)} = \mathcal{C}(m_\tau/\Lambda)^2$, where Λ is the scale of NP, and $\mathcal{C} \sim 1$, $\mathcal{O}(\alpha/\pi)$ or $\mathcal{O}((\alpha/\pi)^2)$.

Compared to muon, effects of NP can be enhanced by a factor of $m_\tau^2/m_\mu^2 \approx 280$.

Suppose, e.g. that NP effects are of the order of $a_{EW} \approx 50 \times 10^{-8}$, then

$$a^{(NP)} \gg \sigma(a_{Hadr+LbL}) \sim 5 \times 10^{-8}$$

Magnetic dipole moment of the τ : experiment

No direct measurement has been performed because of its very short lifetime, $\tau = 2.903(5) \times 10^{-13} \text{ s}$ with $c\tau = 87.03 \mu\text{m}$. This does not allow applying methods used in the electron and muon $g - 2$ experiments.

At present, a_τ is known to an accuracy of only about 10^{-2} . This limit was obtained by the DELPHI collaboration (2003) from the $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ total cross section at LEP2. At 95 % CL, the confidence interval is

$$-0.052 < a_\tau < 0.013$$

It is also given in the form $a_\tau = -0.018(17)$.

There is an analysis of Gonzalez-Sprinberg et al. (2000), based on LEP1, SLD and LEP2 experiments on the τ production, which gives stricter 2σ bounds

$$-0.007 < a_\tau < 0.005$$

The experimental uncertainties are much larger than the SM predictions.

Electric dipole moment of the τ : theory and experiment

EDM of the τ lepton, d_τ , can take nonzero values if the parity P , time reversal T , and CP symmetries are violated. This causes the great interest to the EDM of leptons.

In the SM, the lepton EDM are extremely small as being originated from the 4-loop diagrams and additionally because of the smallness of CP violation in the Cabibbo-Kobayashi-Maskawa matrix.

The SM estimation for electron is $d_e^{(SM)} \lesssim 10^{-38}$ e·cm, while the τ -lepton EDM can be larger by a factor $m_\tau/m_e \approx 3500$

$$d_\tau^{(SM)} \lesssim 3.5 \cdot 10^{-35} \text{ e} \cdot \text{cm}$$

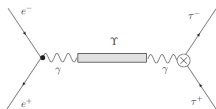
Experimental information comes from Belle collaboration at KEK collider (2002), based on 29.5 fb^{-1} of data:

$$\begin{aligned} -0.22 < \text{Re } d_\tau < 0.45 (10^{-16} \text{ e} \cdot \text{cm}), \\ -0.25 < \text{Im } d_\tau < 0.008 (10^{-16} \text{ e} \cdot \text{cm}) \end{aligned}$$

which is also written as

$$\text{Re } d_\tau = (1.15 \pm 1.70) \times 10^{-17} \text{ e} \cdot \text{cm}, \quad \text{Im } d_\tau = (-0.83 \pm 0.86) \times 10^{-17} \text{ e} \cdot \text{cm}$$

Electric dipole moment of the τ : experiment



Recently new constraints on EDM have been obtained with Belle detector at the KEKB e^+e^- collider (2021), after collecting 833 fb^{-1} of data:

$$\text{Re } d_\tau = (-0.62 \pm 0.63) \times 10^{-17} e \cdot \text{cm}, \quad \text{Im } d_\tau = (-0.40 \pm 0.32) \times 10^{-17} e \cdot \text{cm}$$

which are 17 orders of magnitude worse than the SM prediction.

Note that obtained at Belle values are not EDM, but electric dipole form factor $F_3(s)$ at the energy of KEKB collider $\sqrt{s} = 10.58 \text{ GeV}$. The difference between $F_3(s)$ and $F_3(0)$ can be sizable.

Clearly the SM value is hardly reachable in experiments and therefore observation of the τ EDM will be indication of CP violation beyond the SM.

Any additional sources of CP violation can be very important in solutions of the problem of matter-antimatter asymmetry in the Universe.

An incomplete list of publications on anomalous electromagnetic moments of the τ lepton:

- CP violating effects in $e^-e^+ \rightarrow \tau^-\tau^+$: Bernreuther et al. 1993, B. Ananthanarayan et al. 1993
- role of longitudinal polarization of electron beam in CP violating effects: B. Ananthanarayan et al. 1993, J. Bernabeu et al. 2004, 2007, 2008, 2009
- in peripheral collisions and $\gamma\gamma$ processes at the LHC: M. Dyndal, M. Klusek-Gawenda, M. Schott, A. Szczurek 2020, P. Buhler, N. Burmasov, R. Lavicka, E. Kryshen 2022
- use of τ -spin precession in bent crystals at the LHC: A.S. Fomin, A.Yu. Korchin, A. Stocchi et al. 2019, J. Fu, M.A. Giorgi, L. Henry et al. 2019
- in γp collisions at the LHC: M. Köksal, S.C. İnan, A.A. Billur et al. 2018
- in $e^-e^+ \rightarrow \tau^-\tau^+$ reaction: W. Bernreuther, Long Chen, O. Nachtmann 2021, X. Chen, Y. Wu 2019
- in many experimental searches, including the latest observations at the LHC: ATLAS (submitted to PRL), CMS (submitted to PRL), and at KEKB: K. Belous 2014, K. Inami et al. 2022, and other.

$e^- + e^+ \rightarrow \tau^- + \tau^+$ process

Complications of extracting AMDM and EDM from the $e^-e^+ \rightarrow \tau^-\tau^+$ experiments:

- final state particles from decays of τ leptons ($\tau^- \rightarrow \nu_\tau + \dots$, $\tau^+ \rightarrow \bar{\nu}_\tau + \dots$) are not observed due to the presence of neutrinos,
- kinematic constraints from energy-momentum conservation have to account for initial-state bremsstrahlung photons, some of which are often lost in the beam pipe,
- measurable signatures need to be combined from many τ -decay channels.

In the present work we develop an algorithm for the calculation of event weights embedding effects of AMDM and EDM in simulation of $e^- + e^+ \rightarrow \tau^- + \tau^+ + (n\gamma)$ events, and the subsequent decay of the produced τ leptons.

The impact of AMDM and EDM on the τ -lepton spin-correlations and the total cross-section is taken into account. The algorithm is prepared to work with the Monte Carlo event generator *KKMC* developed earlier.

Some features of the KKMC and TAUOLA programs

At present, the KKMC Monte Carlo event generation program [S. Jadach, B.F.L. Ward, Z. Was 2000, and updates A. Arbuzov, S. Jadach, Z. Was et al. 2021, S. Jadach, B.F.L. Ward, Z. Was 2022] for the $e^-e^+ \rightarrow f\bar{f}$ (where $f = \mu, \tau, u, d, s, c, b$) is widely used in Belle II at KEKB.

It features:

- all major τ -decay channels via TAUOLA,
- higher order QED corrections, and EW corrections,
- spin effects, including longitudinal and transverse spin correlations between the two τ -leptons,
- capability of assuring that the $e^- + e^+ \rightarrow \tau^- + \tau^+ (+n\gamma)$ cross-sections reach precisions at the per-mille level [Sw. Banerjee et al. 2007].

Development of tools, which include effects beyond the SM, in particular, AMDM and EDM, and also assure high precision of SM prediction, is at present of renewed interest. We worked out the corresponding tools and give some predictions for observables.

Formalism and spin correlations

Consider electron-positron annihilation to a pair of polarized τ leptons

$$e^-(k_-) + e^+(k_+) \rightarrow \tau^-(p_-) + \tau^+(p_+)$$

with polarization four-vectors in their corresponding rest frames

$$s^- = (0, \vec{s}^-), \quad s^+ = (0, \vec{s}^+)$$

In the center-of-mass frame, where all momenta take the form $p_- = (E, \vec{p})$ and $p_+ = (E, -\vec{p})$, $k_- = (E, \vec{k})$ and $k_+ = (E, -\vec{k})$, these polarizations take the form

$$s^- = \left(\frac{\vec{p}\vec{s}^-}{m_\tau}, \vec{s}^- + \frac{\vec{p}(\vec{p}\vec{s}^-)}{m_\tau(m_\tau + E)} \right), \quad s^+ = \left(-\frac{\vec{p}\vec{s}^+}{m_\tau}, \vec{s}^+ + \frac{\vec{p}(\vec{p}\vec{s}^+)}{m_\tau(m_\tau + E)} \right)$$

Define the coordinate system with OZ axis parallel to \vec{p} , plane XOZ is spanned on \vec{p} and \vec{k} , and OY is along $\vec{p} \times \vec{k}$.

Then $\vec{p} = (0, 0, p)$ and $\vec{k} = (E \sin(\theta), 0, E \cos(\theta))$, where θ is the scattering angle.

This is sufficient to obtain dependence of the cross section on the spin-correlations.

Formalism and spin correlations

Let us denote

$$F_2(s) \equiv a = a^{QED}(s) + a^{NP}(s), \quad F_3(s) \equiv b = b^{NP}(s)$$

with

$$a^{QED}(s) = \frac{\alpha m^2}{\pi s \beta} \left(\log \frac{1-\beta}{1+\beta} + i\pi \right) = -0.000244 + i 0.000219$$

For comparison:

$$F_2(s=0) = a = \frac{\alpha}{2\pi} = 0.00116141 \quad \text{Schwinger}$$

Here $\beta = \sqrt{1 - 4m_\tau^2/s}$ is the tau velocity. We neglect tiny contribution to b in the SM.

Add the 4-th components of the polarization vectors in the rest frames of τ^- and τ^+ :

$$s_i^- = (\vec{s}^-, 1), \quad s_j^+ = (\vec{s}^+, 1), \quad i, j = 1, 2, 3, 4$$

Then the cross section is

$$d\sigma/d\Omega = \frac{\beta}{64\pi^2 s} |\mathcal{M}|^2 = \frac{\beta}{64\pi^2 s} \sum_{i,j=1}^4 R_{ij} s_i^- s_j^+$$

The spin-correlation coefficients R_{ij} depend on the energy s and the scattering angle θ .

Spin-correlation matrix R_{ij} with anomalous moments

Below are a few elements of the matrix R_{ij} (the notation: $1, 2, 3 \leftrightarrow x, y, z$):

$$R_{11} = \frac{e^4}{4\gamma^2} (4\gamma^2 \operatorname{Re}(a) + \gamma^2 + 1) \sin^2(\theta),$$

$$R_{12} = -R_{21} = \frac{e^4}{2} \beta \sin^2(\theta) \operatorname{Re}(b),$$

$$R_{13} = R_{31} = \frac{e^4}{4\gamma} [(\gamma^2 + 1)\operatorname{Re}(a) + 1] \sin(2\theta),$$

$$R_{14} = -R_{41} = \frac{e^4}{4} \beta \gamma \sin(2\theta) \operatorname{Im}(b),$$

$$R_{24} = R_{42} = \frac{e^4}{4} \beta^2 \gamma \sin(2\theta) \operatorname{Im}(a).$$

...

where $\gamma = \sqrt{s}/(2m_\tau)$ is the Lorentz factor.

Note that imaginary parts of a and b lead to nonzero polarizations of the τ^\pm leptons: the **transverse polarization** along OX axis and the **longitudinal polarization** along OZ axis, induced by $\operatorname{Im}(b)$, are opposite in sign for τ^- and τ^+ , while the **normal to plane polarization** along OY axis, induced by $\operatorname{Im}(a)$, have the same sign.

Test of spin correlations

If we switch off e.m. dipole moments we obtain

$$\begin{aligned} |\mathcal{M}|_{a=b=0}^2 &= \frac{e^4}{4\gamma^2} \{ \gamma^2 + 1 + \beta^2 \gamma^2 \cos^2(\theta) \\ &+ s_3^- s_3^+ [\beta^2 \gamma^2 + (\gamma^2 + 1) \cos^2(\theta)] \\ &+ [s_1^- s_1^+ (\gamma^2 + 1) - s_2^- s_2^+ \beta^2 \gamma^2] \sin^2(\theta) \\ &+ (s_1^- s_3^+ + s_3^- s_1^+) \gamma \sin(2\theta) \}, \end{aligned}$$

which agrees with well-known result of Tsai, Yung-Su (1971). Only the spin-spin correlations between components $z - z$, $x - x$, $y - y$, $x - z$, $z - x$ remain.

Now check if conventions of our code for calculation of weights, and orientation of reference frame, match what is used in KKMC and TAUOLA. For this we calculate the following weights for events generated with KKMC+TAUOLA:

$$\begin{aligned} wt_{spin}^{SM} &= R_{ij}^{SM} h_i^- h_j^+ / R_{44}^{SM}, \\ wt_{spin} &= R_{ij} h_i^- h_j^+ / R_{44} / wt_{spin}^{SM}, \\ wt &= R_{44} / R_{44}^{SM} \end{aligned}$$

Here $R_{ij}^{SM} \equiv R_{ij}|_{a=b=0}$ is defined.

The so-called polarimetric vectors h_i^- and h_j^+ depend on the τ^- - and τ^+ -decay products, and are known for the main decays.

Test in the SM for decays $\tau^- \rightarrow \pi^- \nu_\tau$, $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$

First we check in the SM if obtained spin correlations match those in KKMC.

Two samples of events are generated and compared: one with the spin effects included and the other with spin effects not included. For the latter case, spin effects are instead implemented with the event weights for

$$e^- e^+ \rightarrow \tau^- \tau^+ (n\gamma), \tau^- \rightarrow \pi^- \nu_\tau, \tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$$

The polarimetric vectors for $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$ have the simplest form $\vec{h}^\pm = \alpha_\pm \hat{q}_\pm$, where \hat{q}_\pm is the unit vector in the direction of π^\pm momentum \vec{q}_\pm . Also $\alpha_- = 1$, $\alpha_+ = -1$ are asymmetry parameters in the weak τ decays.

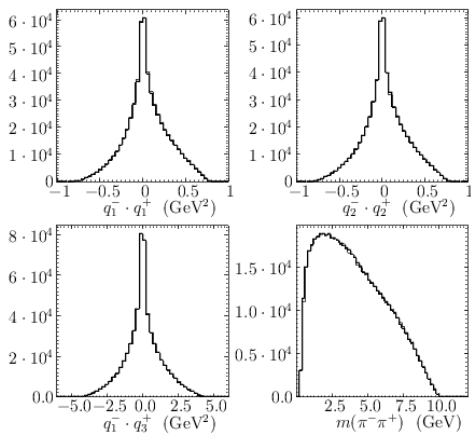
The test are performed for events in which

- a) no bremsstrahlung photons are present,
- b) soft/collinear photons are present, and
- c) hard photons are present.

Results for a) and b): there is no differences for spin correlations from KKMC and from the weights, both in the case without photons and if the “soft” photons, determined by constraint $m^2(\tau^- \tau^+) > 0.98 s$, are allowed.

Test in the SM for $\tau^- \rightarrow \pi^- \nu_\tau$, $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$

c) Events in which “hard” photons are present, i.e. for $\tau^- \tau^+ (n\gamma)$, and constraint $0.2 \text{ s} < m^2(\tau^- \tau^+) < 0.98 \text{ s}$. Most of these events include hard photons collinear to the beams.



Correlation of various components of the π^- and π^+ momenta: $x - x$, $y - y$, $x - z$ and distribution over $\pi^- \pi^+$ invariant mass. The energy $\sqrt{s} = 10.58 \text{ GeV}$ corresponds to Belle II conditions.

Results with anomalous dipole moments

How to find appropriate distributions sensitive to AMDM and EDM?

The neutrino momenta are not observable, though partially they can be inferred through reconstruction of τ -decay vertex position.

Of course, the $\tau^\pm \rightarrow \pi^\pm \nu_\tau$ decay mode provides signatures which are the easiest to interpret. However, the precision of the τ -decay vertex position reconstruction (and direction of the τ^\pm) is the main ambiguity for this channel, and perhaps for any 2-particle decay channel $\tau^\pm \rightarrow h^\pm \nu_\tau$ ($h = \pi, \rho, \rho', a_1$).

For these reasons, we choose the channel $\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^\pm \pi^0 \nu_\tau$, and rely on the kinematic of secondary ρ^\pm decays.

The idea of method is due to [G.R. Bower, T. Pierzchala, Z. Was, M. Worek 2002, K. Desch, A. Imhof, Z. Was, M. Worek 2004]. For constructing observables sensitive to the CP parity of the Higgs boson, it was suggested to measure **acoplanarity angle φ between the planes spanned on the decays $\rho^- \rightarrow \pi^- \pi^0$ and $\rho^+ \rightarrow \pi^+ \pi^0$ and defined in the $\rho^- \rho^+$ rest-frame.**

The momenta of all 4 pions are to be measured, which gives the ρ^- and ρ^+ momenta, which are then boosted to the $\rho^- \rho^+$ rest-frame.

Decay channel $\tau^\pm \rightarrow \rho^\pm \nu_\tau \rightarrow \pi^\pm \pi^0 \nu_\tau$

Thus we consider the cascade process:

$$e^- e^+ \rightarrow \tau^- \tau^+ (n\gamma), \quad \tau^- \rightarrow \rho^- \nu_\tau \rightarrow \pi^- \pi^0 \nu_\tau, \quad \tau^+ \rightarrow \rho^+ \bar{\nu}_\tau \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$$

Moreover, to be sensitive to the spin correlations, additional cuts need to be applied, which is the constraint on the sign of the product $y_1 y_2 > 0$, where

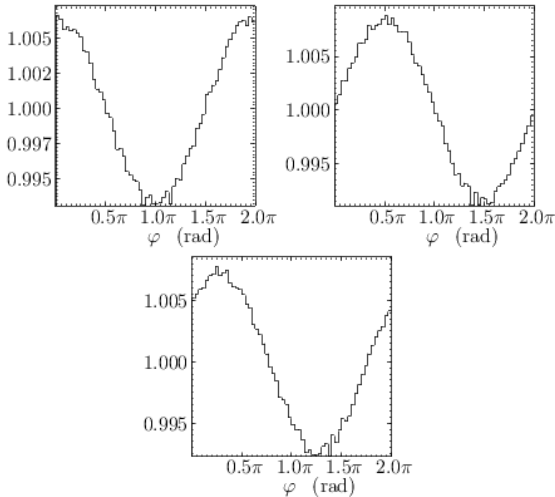
$$y_1 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}, \quad y_2 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}.$$

Actually, factors $E_{\pi^-} - E_{\pi^0}$ and $E_{\pi^+} - E_{\pi^0}$ are connected with the polarimetric vectors for this channel.

We present below results of the MC simulation of the acoplanarity angle distribution using weight method, for a few choices of the couplings $a(s)$ and $b(s)$, which are assumed to be real.

Only events without the photons, or with the soft photons fixed by the constraint $m^2(\tau^- \tau^+) \geq 0.98 s$, are selected.

Distribution sensitive to AMDM and EDM



Distribution of the acoplanarity angle φ at $\sqrt{s} = 10.58$ GeV with the constraint $y_1 y_2 > 0$. Top left: for $\text{Re}(a) = 0.04$, top right: for $\text{Re}(b) = 0.04$; bottom: for $\text{Re}(a) = 0.04 \cos(\pi/4)$ and $\text{Re}(b) = 0.04 \sin(\pi/4)$.

Results with AMDM and EDM

- Effect of AMDM or/and EDM can reach about 0.005, but this is related to the chosen absolute value 0.04 of the couplings, which is too large to be a realistic value (it is also much larger than a_{τ}^{QED});
- distributions with AMDM or EDM are shifted with respect to each other by the angle $\approx 90^\circ$, and distribution with both AMDM and EDM is shifted by the angle $\approx 45^\circ$;
- we can combine $\text{Re}(a)$ and $\text{Re}(b)$ in one complex coupling

$$c \equiv \text{Re}(a) + i \text{Re}(b) = \sqrt{(\text{Re}(a))^2 + (\text{Re}(b))^2} \exp(i\psi),$$
$$\tan(\psi) = \text{Re}(b)/\text{Re}(a),$$

then the angle ψ describes the shift of the observed distributions; it gives a measure of CP violation in the photon interaction with the τ leptons.

In measurement of this observable there is no need to use τ -decay vertex position reconstruction. Only the entirely visible particles are detected.

Conclusions

- 1 A simple algorithm for the calculation of event weights embedding effects of the dipole anomalous magnetic and electric moments in simulations of $e^-e^+ \rightarrow \tau^-\tau^+(n\gamma)$ events with the τ decays is presented. Impacts on the spin effects and on the cross section are taken into account.
- 2 The algorithm is prepared to work with KKMC Monte Carlo, and solution is ready for use with the Belle II software KKMC installation.
- 3 We calculated the effect of AMDM and EDM on acoplanarity of two planes build with $\pi^+\pi^0$ and $\pi^-\pi^0$ momenta in the $\pi^+\pi^0\pi^-\pi^0$ system rest-frame of decay products in $e^-e^+ \rightarrow \tau^-\tau^+n\gamma$; $\tau^\pm \rightarrow \pi^\pm\pi^0\nu_\tau$.
- 4 Experimental data can be used to confirm or rule out the strength of dipole moments predicted by various New Physics models [e.g., W. Bernreuther et al. 1997, T. Huang et al. 1997].
- 5 Finally, this method can be extended to the case of a polarized electron. This is important in view of the planned upgrade of the SuperKEKB e^-e^+ collider with polarized electron beam
[Sw. Banerjee, J.M. Roney, US Belle II Group and Belle II/SuperKEKB e-Polarization Upgrade Working Group 2022].

Thank you for attention!

I am also thankful to Organizers of the Workshop for opportunity to present our results!