September 19th 2022,
Student Day @ Photon Vistas, Krakow

Deutsche Forschungsgemeinschaft
Research Unit FOR 2783

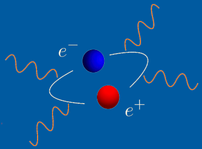
FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

HI JENA
Helmholtz Institute Jena

Physics of strong electromagnetic fields

Felix Karbstein

(f.karbstein@hi-jena.gsi.de)



Abstract

In this lecture,

- I will sketch how **quantum vacuum fluctuations** affect the physics of strong electromagnetic fields in the vacuum and how we analyze them theoretically → "Heisenberg-Euler".

- I will focus on **photonic signatures** of the resulting nonlinear couplings in strong electromagnetic (→ laser) **fields**:

  → vacuum birefringence,

  → photon merging.

- I will argue that their experimental verification is feasible with **state-of-the-art technology**.

Outline

I. The quantum vacuum

II. (Strong) Electromagnetic fields in the quantum vacuum

III. Photonic signatures of quantum vacuum nonlinearity

IV. Examples

(i) Vacuum birefringence

(ii) Photon merging

V. Conclusions and outlook

I. The quantum vacuum

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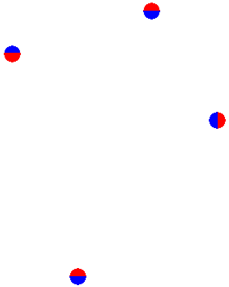
Classical description versus quantum vacuum



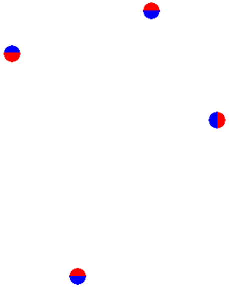
vacuum fluctuations

I. The quantum vacuum

Classical description versus quantum vacuum

- empty space = absence of particles and fields

→ boring!



vacuum fluctuations

I. The quantum vacuum

The vacuum in relativistic quantum field theory:

→ c and $\hbar$ relevant

- vacuum fluctuations = virtual processes
- not on the mass-shell, i.e. $E^2 \neq \vec{p}^{\,2} c^2 + m^2 c^4$
- amount to Feynman diagrams without external lines

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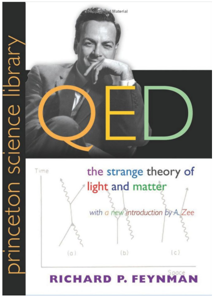
Consider quantum electrodynamics (QED):

= relativistic quantum field theory

- describes interaction of light (=photons) and matter (=electrons/positrons)

→ parameters: m and e.



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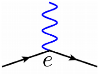
Consider quantum electrodynamics (QED): $(c = \hbar = 1)$

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e\bar{\psi}\gamma^\mu \psi A_\mu$$





where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

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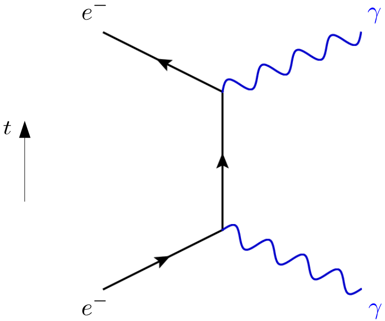

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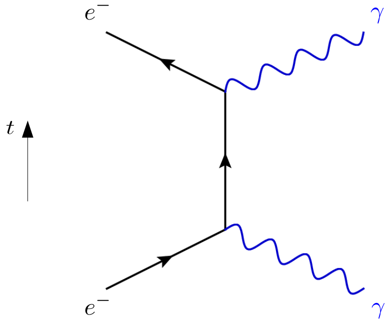

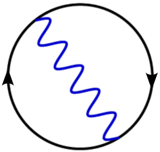

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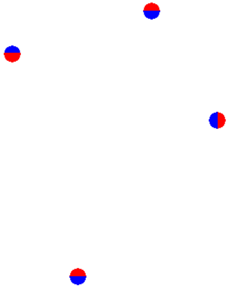

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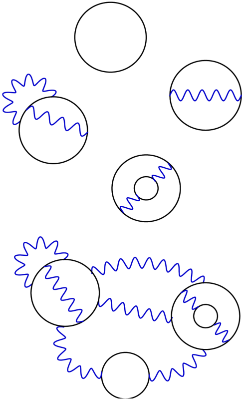

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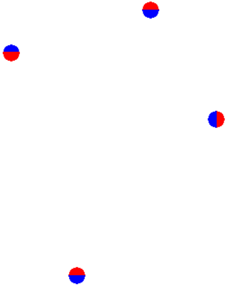

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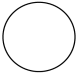

→ leading contribution in a loop-expansion:

$$l\text{-loop diagram} \sim \left(\frac{\alpha}{\pi}\right)^{l-1}$$

$$\frac{\alpha}{\pi} \approx \frac{1}{137}\frac{1}{\pi} \approx \frac{1}{430}$$

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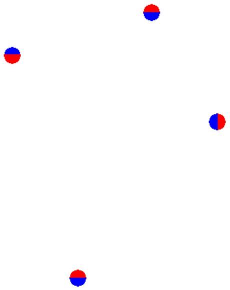

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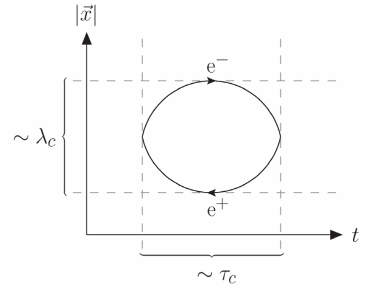

with $\lambda_c = \dfrac{\hbar}{mc} \simeq 3.9 \cdot 10^{-13}\,\mathrm{m}$

$$\tau_c = \frac{\lambda_c}{c} = 1.3 \cdot 10^{-21}\,\mathrm{s}$$

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Quantum vacuum + external electromagnetic fields:

→ e.m. fields couple to charges and can thus couple directly to quantum vacuum fluctuations of charged particles

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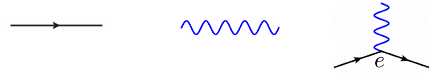

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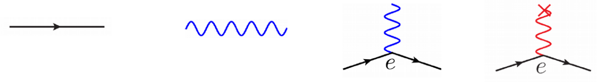
Quantum vacuum + external electromagnetic fields:

→ e.m. fields couple to charges and can thus couple directly to quantum vacuum fluctuations of charged particles

Consider QED in an external e.m. field A_μ :

$$\mathcal{L}_{\mathrm{sfQED}} = \bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu e(A_\mu + A_\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

cf., e.g. [Gies, FK: JHEP **03** 108 (2017)]

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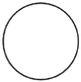
1-loop diagram:



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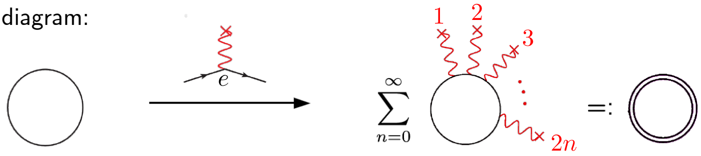

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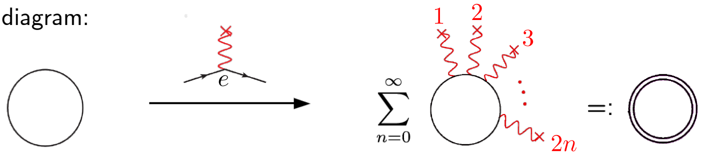

→ vacuum fluctuations induce nonlinear interactions between electromagnetic fields.

Consider **infinitely extended, constant** e.m. fields:

[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

($\mathfrak{E}, \mathfrak{B}$ Kraft auf das Elektron.
$|\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\frac{e}{(e^2/mc^2)^2} =$ „Kritische Feldstärke“.)

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

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The Nobel Prize in Physics 1932
Werner Heisenberg

Werner Heisenberg - Facts

Werner Karl Heisenberg

Born: 5 December 1901, Würzburg, Germany

Died: 1 February 1976, Munich, West Germany (now Germany)

Affiliation at the time of the award: Leipzig University, Leipzig, Germany

Prize motivation: "for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"

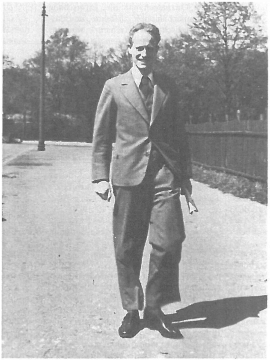
Field: quantum mechanics

Werner Heisenberg received his Nobel Prize one year later, in 1933.

Kriegsschicksale Phys. Bl. 45 (1989) Nr. 9

Hans Euler (1909–1941)

Von D. Hoffmann, Berlin*)



Hans Euler, um 1935 (Foto: H. Wergeland, Norwegen/Hurdal)

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[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

- for $F^{\mu\nu} = \mathrm{const.}$
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- to all orders in $eF^{\mu\nu}$

cf. also [Schwinger: Phys. **82** 664 (1951)]

II. Electromagnetic fields in the quantum vacuum

Modern formal **definition** of the all-loop Heisenberg-Euler effective action.

$$\mathcal{L}_{\text{sfQED}} = \bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu e(A_\mu + A_\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

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→ integrate out the quantum fields:

tree-level current $j_\mu = -e\bar{\psi}\gamma_\mu\psi$

$$\mathrm{e}^{\mathrm{i}\int \mathcal{L}_{\rm HE}[F]} = \int \mathcal{D}A \int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi \, \mathrm{e}^{\mathrm{i}\int \mathcal{L}_{\rm sfQED}} = \left\langle \mathrm{e}^{\mathrm{i}\int(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu)} \right\rangle_{\rm QED}$$

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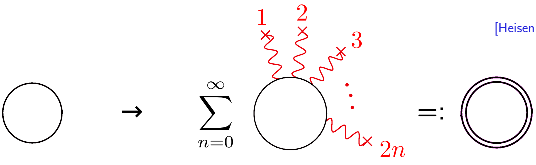


[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

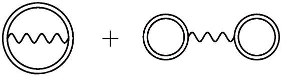
cf. also [Schwinger: Phys. **82** 664 (1951)]

→ higher loop contributions scale as $\sim (\frac{\alpha}{\pi})^{\ell-1}$; with ℓ number of loops

→ at two loops: $(\frac{\alpha}{\pi} \approx \frac{1}{137}\frac{1}{\pi} \approx \frac{1}{430})$

[Ritus: Sov. Phys. JETP **42**, 774 (1975)]

[Ritus: Sov. Phys. JETP **46**, 423 (1977)]

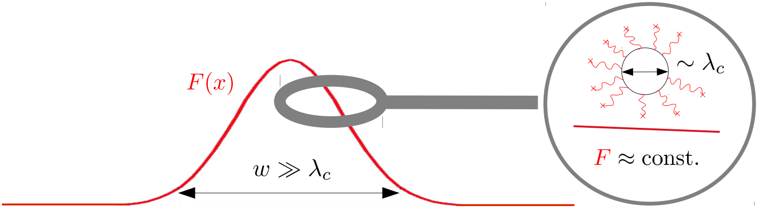


[Gies, FK: JHEP **03** 108 (2017)]

II. Electromagnetic fields in the quantum vacuum

The Heisenberg-Euler effective action (in const. fields) [Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

→ also allows for insights into inhomogeneous e.m. fields, fulfilling $w \gg \lambda_c$:



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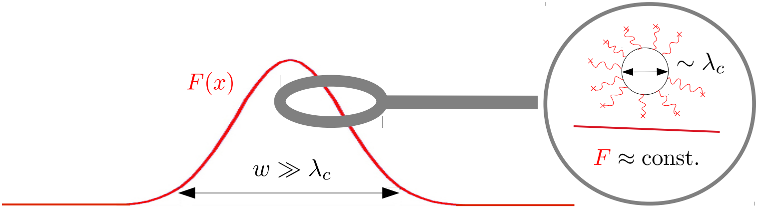

$\mathcal{L}_{\rm HE}(F) \;\to\; \mathcal{L}_{\rm HE}(F(x))$ $\qquad$ $(\lambda_c = \frac{h}{mc} \simeq 3.9 \cdot 10^{-13}\,\mathrm{m})$

→ all macroscopic e.m. fields available in the laboratory fulfill this criterion

→ LCFA (= Locally Constant Field Approximation) : $\mathcal{L}_{\rm HE}(F(x)) = \mathcal{L}_{\rm true}(F(x), \partial F(x), \ldots) + \mathcal{O}((\frac{\lambda_c}{w})^2)$

II. Electromagnetic fields in the quantum vacuum

A closer look on the result of Heisenberg and Euler:

[Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

Es ergibt sich für das Feld eine Lagrange-Funktion:

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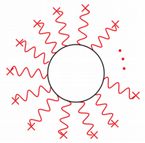

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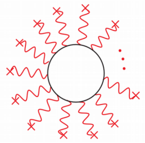

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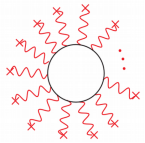

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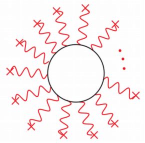

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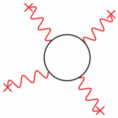
- In the limit of "weak" e.m. fields we obtain

$$\mathcal{L}_{\mathrm{HE}}(F) = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m^4}\left[(\vec{E}^2-\vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right] + \dots$$



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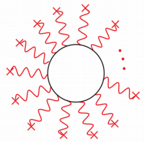

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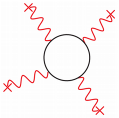
$$\mathcal{L}_{\mathrm{HE}}(F) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{5760\pi^2 m^4}\left[4\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + 7\left(F_{\mu\nu}{}^{*}F^{\mu\nu}\right)^2\right] + \ldots$$



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A closer look on the result of Heisenberg and Euler:

→ now, the parameters of the theory are $c, \hbar, m, e$ and $\vec{B}, \vec{E}$.

- the fundamental QED parameters can be combined to

$$E_{\rm cr} = \frac{m^2 c^3}{\hbar e} \simeq 1.3 \cdot 10^{18}\,{\rm V/m}$$

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→ consider equations of motion: ↔ superposition principle

$$\partial_\mu \left(F^{\mu\nu} - \frac{e^4}{720\pi^2 m^4} \left[4(F_{\alpha\beta}F^{\alpha\beta})\,F^{\mu\nu} + 7(F_{\alpha\beta}{}^*F^{\alpha\beta})\,{}^*F^{\mu\nu} \right] + \ldots \right) = 0$$

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$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

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$$\partial_\mu \left(F^{\mu\nu} - \frac{4\alpha}{45\pi} \frac{1}{2} \left(\frac{\vec{B}^2}{B_{\rm cr}^2} - \frac{\vec{E}^2}{E_{\rm cr}^2} \right) F^{\mu\nu} + \frac{7\alpha}{45\pi} \frac{\vec{E} \cdot \vec{B}}{E_{\rm cr} B_{\rm cr}} {}^*F^{\mu\nu} + \ldots \right) = 0$$

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How strong are the e.m. fields available in the laboratory?

	field strength	length scale
- magnetic field of the earth	$5 \cdot 10^{-5}\mathrm{T}$	$\mathcal{O}(1000\mathrm{km})$
- permanent magnets	$\lesssim 1.6\mathrm{T}$	$\mathcal{O}(\mathrm{cm})$
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- dipole magnets @ CERN	$8.6\mathrm{T}$	$\mathcal{O}(10\mathrm{m})$
- capacitors	$\lesssim 10^{7}\mathrm{V/m}$	
- high-field laboratory Dresden	$\lesssim 100\mathrm{T}$	$\mathcal{O}(\mathrm{cm})$
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- strongly charged heavy ions, U^{91+}	$\gtrsim E_{\mathrm{cr}}$	$\mathcal{O}(100\mathrm{fm})$
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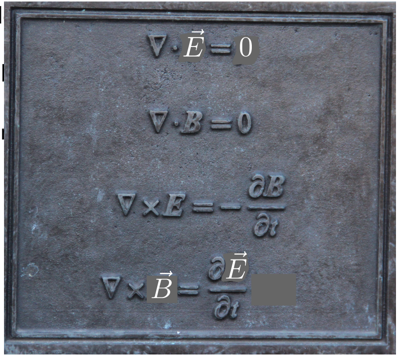

[from Wikimedia Commons/Wikipedia]

James Clerk Maxwell 1831-1879

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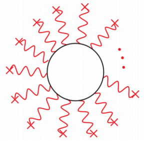
→ quantum corrections to Maxwell's theory of electrodynamics

become relevant for $\frac{|\vec{E}|}{E_{\mathrm{cr}}} \to \mathcal{O}(1)\,, \quad \frac{|\vec{B}|}{B_{\mathrm{cr}}} \to \mathcal{O}(1)\,.$

Intermediate summary

So far we have limited ourselves on the effective interactions among classical e.m. fields induced by vacuum fluctuations.



→ modified Maxwell-equations in the quantum vacuum:

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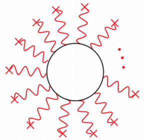

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→ experimental verification very challenging.

On the other hand: Rather than measuring small changes in e.m. fields we "only" need to detect real photons!

→ single-photon detection schemes, precision polarimetry, ...

III. Photonic signatures of quantum vacuum nonlinearity

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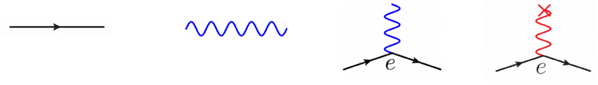
Quantum vacuum + external electromagnetic fields:

→ e.m. fields couple to charges and can thus directly couple to quantum

quantum fluctuations of charged particles

Consider QED in an external e.m. field A_μ :

$$\mathcal{L}_{\mathrm{sfQED}} = \bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu e(A_\mu + A_\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

cf., e.g. [Gies, FK: JHEP **03** 108 (2017)]

High-intensity lasers constitute a promising tool to probe quantum vacuum nonlinearities:

[Marklund, Lundin: Eur. Phys. J. D **55** 319 (2009)]

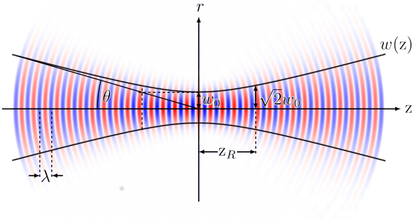
[Di Piazza, Müller, Hatsagortsyan, Keitel: Rev. Mod. Phys. **84** 1177 (2012)]

- current high-intensity lasers reach peak field strengths of

  $B \sim \mathcal{O}(10^5 - 10^6)\mathrm{T},$

  $E \sim \mathcal{O}(10^{13} - 10^{14})\mathrm{V/m}$

  in focus spots of radius $w_0 \sim \mathcal{O}(\mu\mathrm{m})$.



- important ratios $\frac{B}{B_{\rm cr}} = \frac{B[T]}{4\times10^9} \ll 1, \quad \frac{E}{E_{\rm cr}} = \frac{E[\mathrm{V/m}]}{1.3\times10^{18}} \ll 1.$

III. Photonic signatures of quantum vacuum nonlinearity

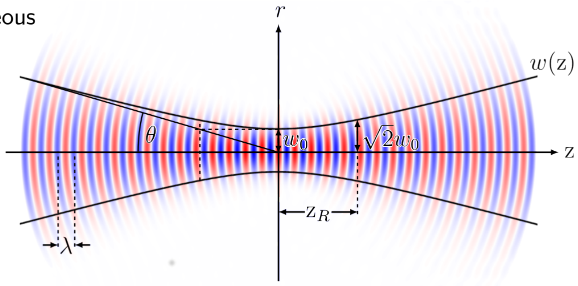
Problem: Most analytical calculations have been performed either for uniform, constant or plane wave (null-field) backgrounds.

- e.g. photon polarization tensor

[Narozhnyi: Sov. Phys. JETP **28** 371 (1969)]
[Batalin, Shabad: Sov. Phys. JETP **33** 483 (1971)]
[Baier, Milshtein, Strakhovenko: Sov. Phys. JETP **42** 961 (1976)]
[Becker, Mitter: J. Phys. A **8** 1638 (1975)]

↔ the electromagnetic fields delivered by focused high-intensity lasers are highly inhomogeneous

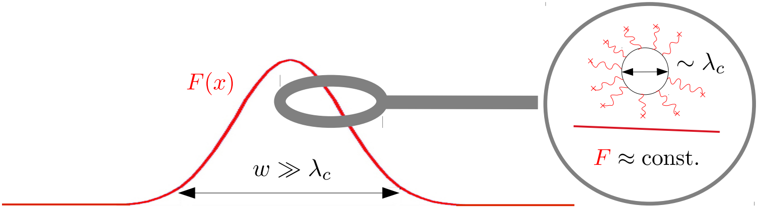
- pulsed, focused Gaussian beams



III. Photonic signatures of quantum vacuum nonlinearity

The Heisenberg-Euler effective action (in const. fields) [Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

→ also allows for insights into inhomogeneous e.m. fields, fulfilling $w \gg \lambda_c$:



$\mathcal{L}_{\rm HE}(F) \;\to\; \mathcal{L}_{\rm HE}(F(x))$ $\quad$ $(\lambda_c = \frac{h}{mc} \simeq 3.9 \cdot 10^{-13}\,{\rm m})$

→ all macroscopic e.m. fields available in the laboratory fulfill this criterion

→ LCFA (= Locally Constant Field Approximation) : $\mathcal{L}_{\rm HE}(F(x)) = \mathcal{L}_{\rm true}(F(x), \partial F(x), \ldots) + \mathcal{O}((\frac{\lambda_c}{w})^2)$

III. Photonic signatures of quantum vacuum nonlinearity

The Heisenberg-Euler effective action (in const. fields) [Heisenberg, Euler: Z. Phys. **98** 714 (1936)]

→ allows to study eff. ints. of e.m. field $(w \gg \lambda_c)$ & in/out photon field $(k^\mu \ll m)$:

(1) $F^{\mu\nu} \to F^{\mu\nu}(x) + f^{\mu\nu}(x)\,,$ with $f^{\mu\nu}(x) = \partial^\mu a^\nu(x) - \partial^\nu a^\mu(x)$

[Bialynicka-Birula, Bialynicki-Birula: Phys. Rev. D **2** 2341 (1970)]

[FK, Shaisultanov: Phys. Rev. D **91** 085027 (2015)]

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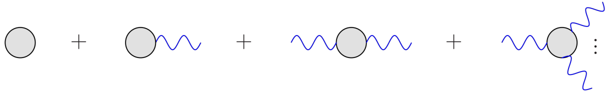
(2) expand in eff. n-photon interactions [Gies, FK: JHEP **03** 108 (2017)]

$\Gamma_{\rm int} =$



$$s_{(n)}^{\sigma_1\ldots\sigma_n}(k_1,\ldots,k_n) \sim \int_x e^{{\rm i}x\sum_{j=1}^n k_j} \prod_{j=1}^n \left(k_j^{\mu_j} g^{\nu_j\sigma_j} \frac{\partial}{\partial F^{\mu_j\nu_j}(x)} \right) \mathcal{L}_{\rm HE}\big(F(x)\big)$$

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$\Gamma_{\text{int}} =$ at one loop:

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III. Photonic signatures of quantum vacuum nonlinearity

For the theoretical study photonic signatures of quantum vacuum nonlinearity,

we advocate the "Vacuum Emission Picture": [FK, Shaisultanov: Phys. Rev. D **91** 113002 (2015)]

→ all prescribed e.m. fields are considered as "pump" $= |0\rangle$

↔ no distinction between pump and probe fields

→ the signature of vacuum nonlinearity is encoded in signal photons

$$= |\gamma_{p_1}(\vec{k}_1)\ldots\gamma_{p_n}(\vec{k}_n)\rangle$$

III. Photonic signatures of quantum vacuum nonlinearity

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we advocate the "Vacuum Emission Picture": [FK, Shaisultanov: Phys. Rev. D **91** 113002 (2015)]

→ all prescribed e.m. fields are considered as "pump" $= |0\rangle$

↔ no distinction between pump and probe fields

→ the signature of vacuum nonlinearity is encoded in signal photons

$$= |\gamma_{p_1}(\vec{k}_1)\ldots\gamma_{p_n}(\vec{k}_n)\rangle$$

→ calculate transition amplitudes:

$$\mathcal{S}_{p_1\ldots p_n}(\vec{k}_1,\ldots,\vec{k}_n) = \langle\gamma_{p_1}(\vec{k}_1)\ldots\gamma_{p_n}(\vec{k}_n)|\,\hat{H}_{\rm int}(\hat{a},F)\,|0\rangle$$

→ applicable for small signals (beam depletion effects, etc. neglected)

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$$\sim \epsilon^{(p_1)}_{\sigma_1}(\vec{k}_1)\ldots\epsilon^{(p_n)}_{\sigma_n}(\vec{k}_n)\;s^{\sigma_1\ldots\sigma_n}_{(n)}(k_1,\ldots,k_n)\big|_{k_i^0=|\vec{k}_i|}$$

with $s^{\sigma_1\ldots\sigma_n}_{(n)}(k_1,\ldots,k_n) \sim \int_x e^{{\rm i}x\sum_{j=1}^n k_j}\prod_{j=1}^n\left(k_j^{\mu_j}g^{\nu_j\sigma_j}\frac{\partial}{\partial F^{\mu_j\nu_j}(x)}\right)\mathcal{L}_{\rm HE}\big(F(x)\big)$

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→ differential number of signal photons in far field:

$$\mathrm{d}^{3n}N_{p_1\ldots p_n}(\vec{k}_1,\ldots,\vec{k}_n) = \frac{\mathrm{d}^3k_1}{(2\pi)^3}\ldots\frac{\mathrm{d}^3k_n}{(2\pi)^3}\,\big|\mathcal{S}_{p_1\ldots p_n}(\vec{k}_1,\ldots,\vec{k}_n)\big|^2$$

IV. Examples

IV. Examples: (i) Vacuum birefringence

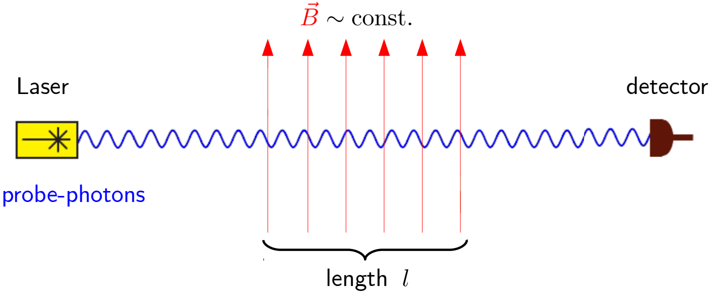
Conventional scenario:

→ there are experiments!

[BMV (Biréfringence Magnétique du Vide) experiment, Toulouse]

[PVLAS (Polarizzazione del Vuoto con Laser) experiment, Padova/Ferrara]

[OVAL (Observing Vacuum with Laser) experiment, Tokyo]



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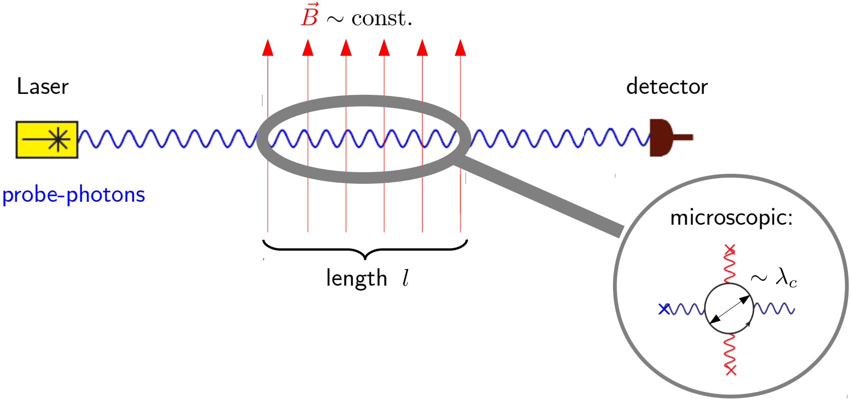

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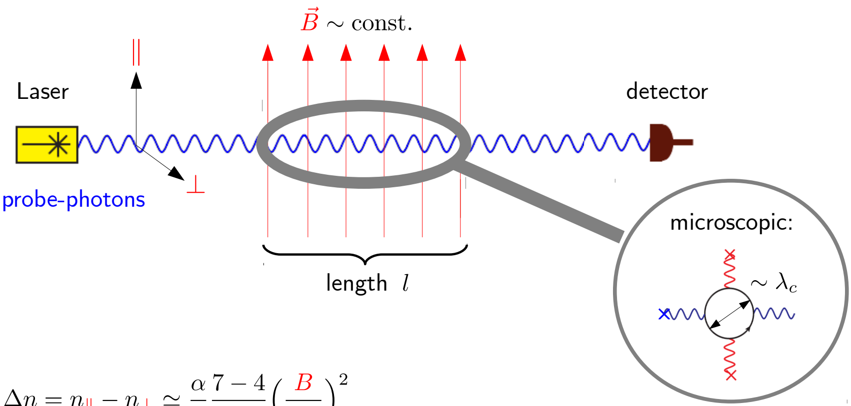

$$\Delta n = n_{\parallel} - n_{\perp} \simeq \frac{\alpha}{\pi} \frac{7-4}{90} \left(\frac{B}{B_{\rm cr}} \right)^2$$

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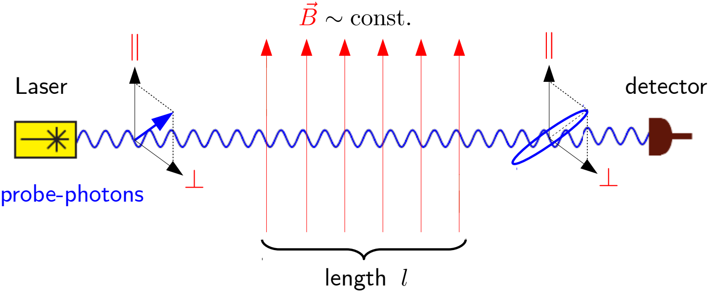

vacuum birefringence → ellipticity:

$$\Delta\phi \sim \alpha \frac{l}{\lambda}\left(\frac{B}{B_{cr}}\right)^2$$

← relative phase shift between $\parallel$ and $\perp$ modes

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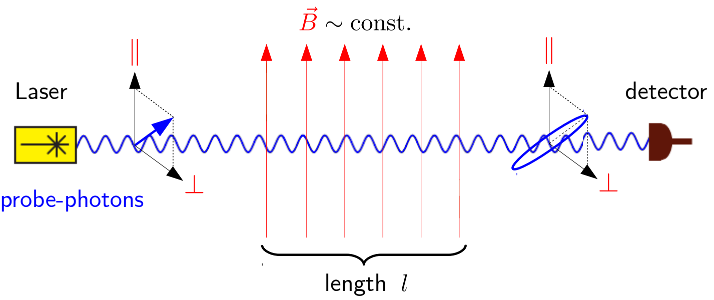


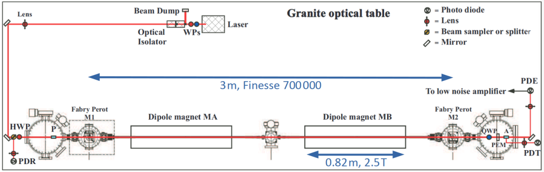
vacuum birefringence → ellipticity: $$\Delta\phi \sim \alpha \, \frac{l}{\lambda} \left(\frac{B}{B_{\rm cr}}\right)^2 \simeq \frac{1}{137} \frac{l}{\lambda} \left(\frac{B}{4 \cdot 10^9\ \mathrm{T}}\right)^2$$

↔ Induced photons in $\perp$ polarization mode: $$N_\perp \simeq \left(\frac{\Delta\phi}{2}\right)^2 N_{\rm in}$$

IV. Examples: (i) Vacuum birefringence

Conventi

→ there

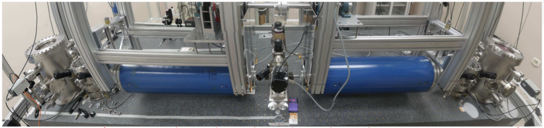


:, Toulouse]

ent, Padova/Ferrara]

okyo]

Laser



letector

probe-photons

length l

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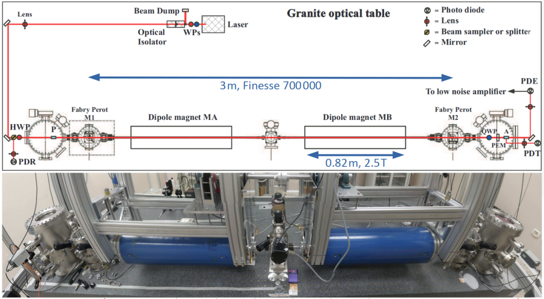

Laser

letector

probe-photons

	BMV	PVLAS	OVAL
Magnet type	pulsed	rotating permanent	pulsed
Maximum field B_{max} [T]	6.5	2.5	9.0
Field length L [m]	0.14	0.82 + 0.82	0.17
B^2L [T^{2}m]	5.8	5.06 + 5.06	13.8
"Filtered" B^2L [T^{2}m]	2.7	5.06 + 5.06	4.1
Wavelength λ [nm]	1064	1064	1064
Cavity finesse $\mathcal{F}$	445 000	700 000	320 000
Integration time [s]	0.3	$2 \cdot 10^6$	0.6
Repetition rate [Hz]	0.0017	continuous	0.17
Uncertainty in Δn [10^{-23}/T^2]	270	2.7	110 000

$$\Delta\phi \sim \alpha \frac{l}{\lambda}\left(\frac{B}{B_{cr}}\right)^2 \simeq \frac{1}{137}\frac{l}{\lambda}\left(\frac{B}{4\cdot 10^9\,\mathrm{T}}\right)^2$$

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[Battesti *et al.*: Phys. Rept. **765** (2018)]

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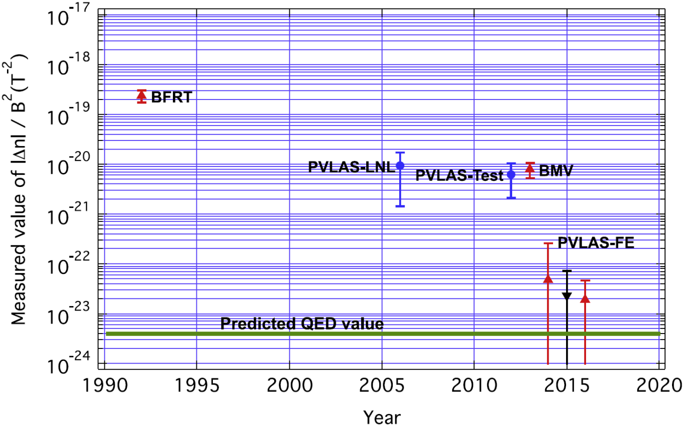

[Battesti *et al.*: Phys. Rept. **765** (2018)]

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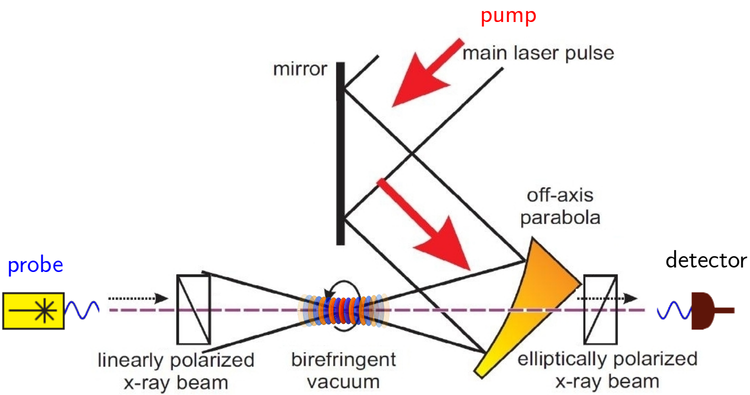
Scenario with pump = high-intensity laser:

→ experiments proposed [Heinzl, Liesfeld, Amthor, Schwoerer, Sauerbrey, Wipf: Opt. Comm. **267** 318 (2006)] [Schlenvoigt, Heinzl, Schramm, Cowan, Sauerbrey: Phys. Scr. **91** 023010 (2016)]



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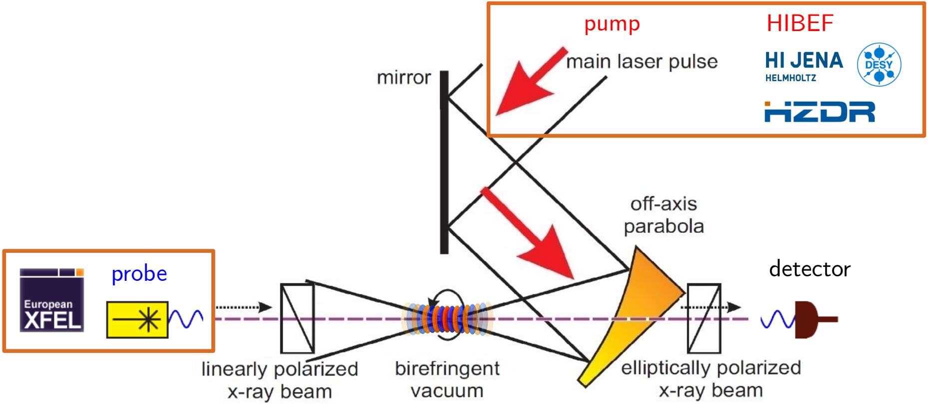

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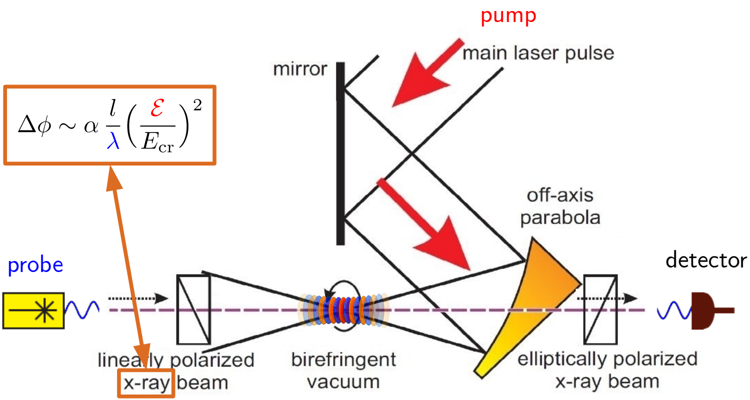

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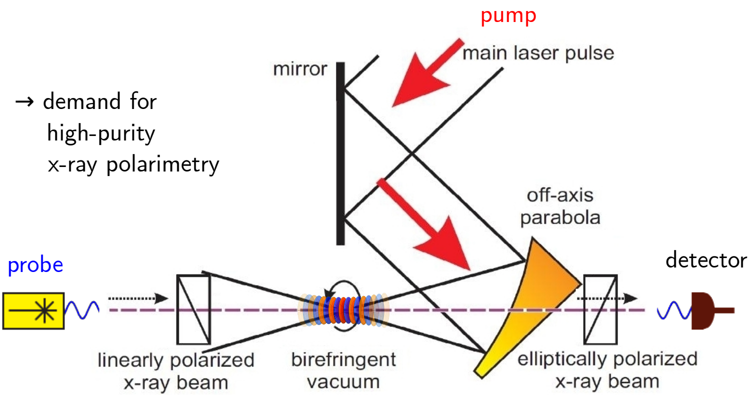
→ demand for high-purity x-ray polarimetry



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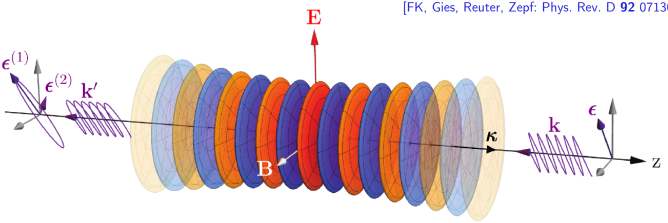
Analysis of this scenario, accounting for the full inhomogeneous field profile of a linearly polarized, pulsed laser beam (Gaussian beam).

[FK, Gies, Reuter, Zepf: Phys. Rev. D **92** 071301(R) (2015)]



- pump: 1PW class laser $(W = 10\text{J},\ \tau = 30\text{fs},\ \lambda = 800\text{nm}, w_0 = 1\mu\text{m})$
- probe: x-ray beam of free electron laser $(\omega_{\text{probe}} = 12914\text{eV},\ N_{\text{in}} \simeq 10^{12})$

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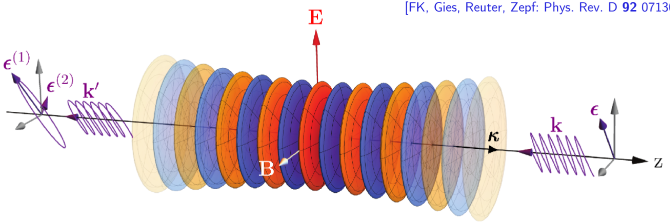

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polarization purity record: $\mathcal{P} = 5.7 \cdot 10^{-10}$

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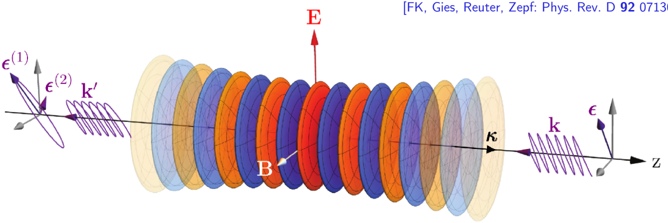

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Goal: $\frac{N_\perp}{N_{\text{in}}} > \mathcal{P}$

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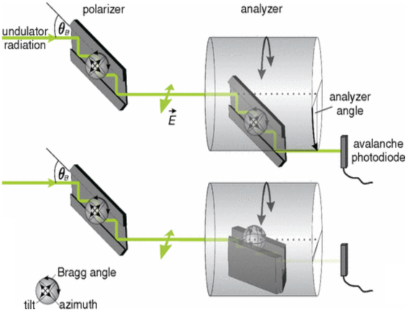

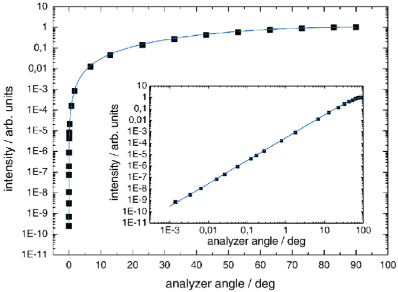

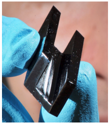

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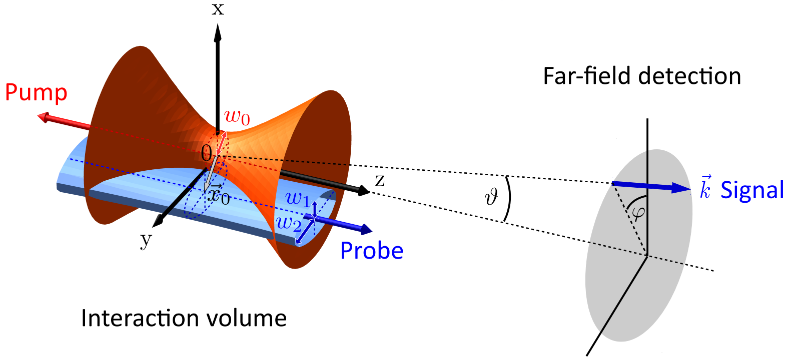
[Mosman, FK: Phys. Rev. D **104** 013006 (2021)]



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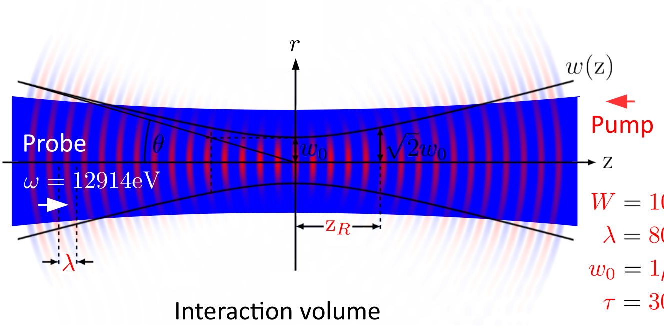

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Interaction volume

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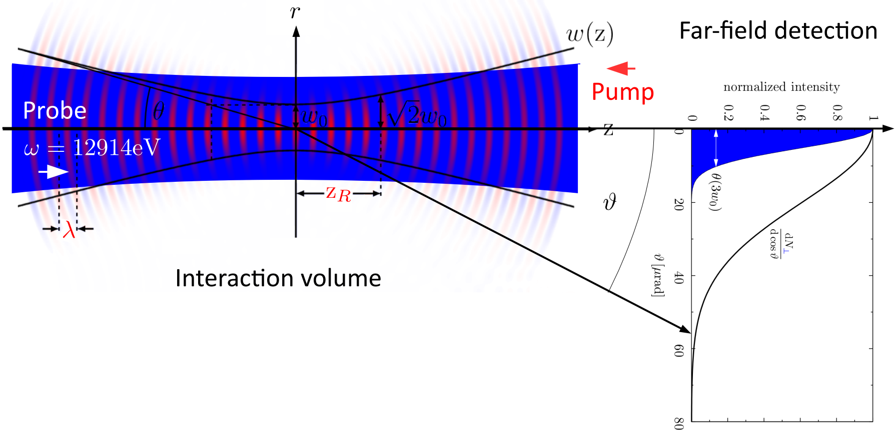

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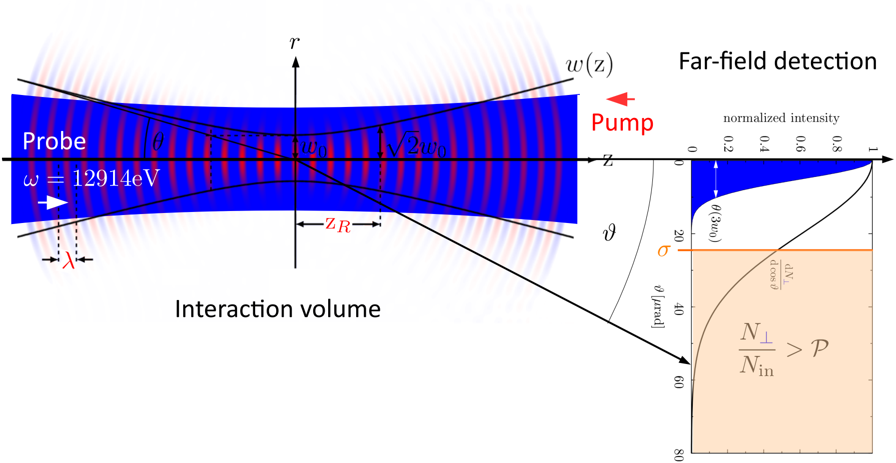
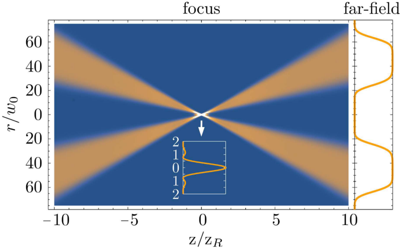


IV. Examples: (i) Vacuum birefringence

Further improvement of the signal-to-background separation in experiment.

→ use "tailored probe beam":

[FK, Mosman: PRD **101** (2020)],
[FK, Ullmann, Mosman, Zepf: PRL **129** (2022)]



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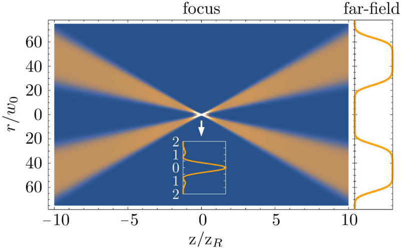

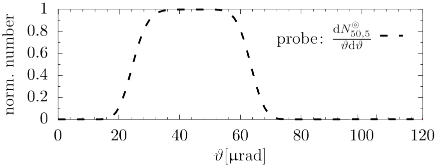

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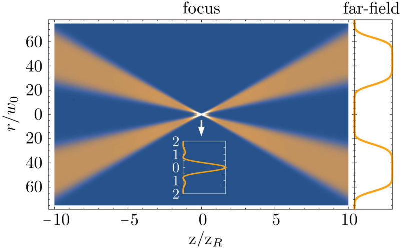

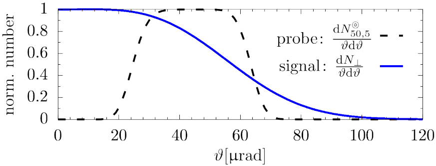

$N_{\rm signal} \gtrsim 8$ /shot in the field-free hole

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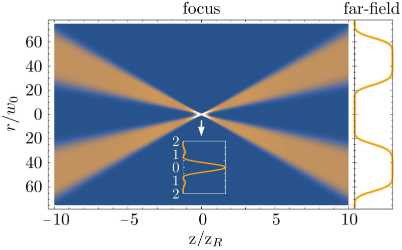

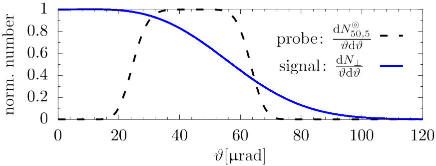

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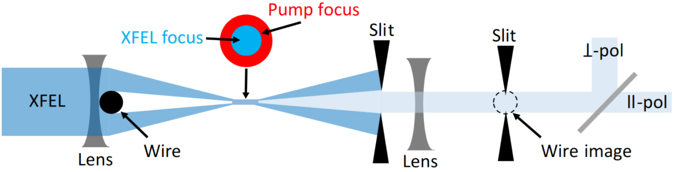

IV. Examples: (ii) Photon merging

Goal: solely high-intensity laser based setup

[Gies, FK, Seegert: Phys. Rev. D **93** 085034 (2016)]
[Gies, FK, Kohlfürst: Phys. Rev. D **97** 036022 (2018)]
[Gies, FK, Kohlfürst, Seegert: Phys. Rev. D **97** 076002 (2018)]
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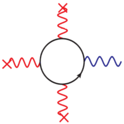

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cf. also [Lundstrom et. al.: Phys. Rev. Lett. **96** 083602 & Phys. Rev. A **74** 043821 (2006)]

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Goal: solely high-intensity laser based setup

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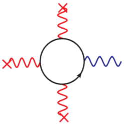

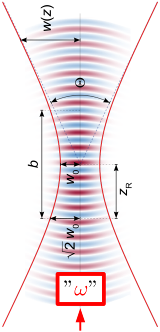

$W = 25\,\mathrm{J}, \ \lambda = 800\,\mathrm{nm}$

$$\tau_{\mathrm{pulse}} = 25\,\mathrm{fs}$$
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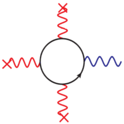
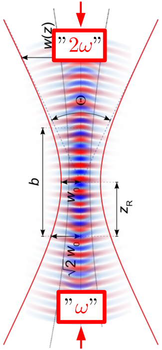


$W = \frac{25}{4}\,\mathrm{J},\ \ \lambda = 400\,\mathrm{nm}$



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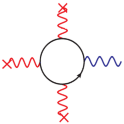

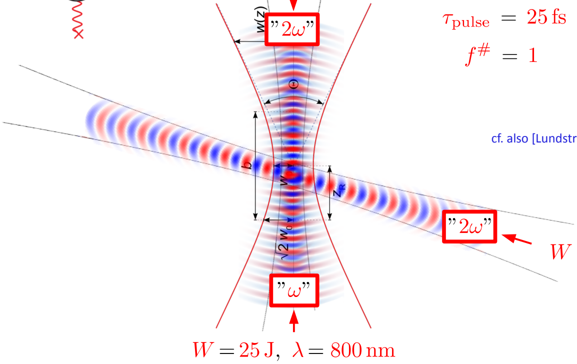

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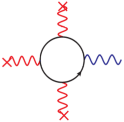

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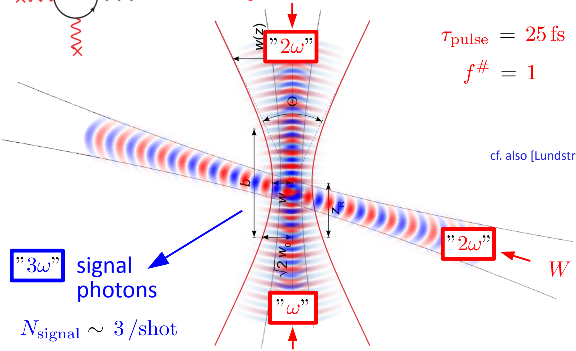

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$W = 25\,\mathrm{J},\ \lambda = 800\,\mathrm{nm}$

"3ω" signal photons

$N_{\mathrm{signal}} \sim 3\,/\mathrm{shot}$

$\tau_{\mathrm{pulse}} = 25\,\mathrm{fs}$

$f^{\#} = 1$

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Review: [Fedotov, Ilderton, FK, King, Seipt, Taya, Torgrimsson: "Advances in QED with intense background fields", arXiv:2203.00019 (2022)]

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Research Unit FOR 2783
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Thank you very much for your attention!