BFKL Pomeron, high gluon densities and their imprint in exclusive vector meson production

(An half an hour lecture)

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based on:

- I. Bautista, Fernandez Tellez, MH, PRD 94 (2016) 5, 054002, arXiv:1607.05203
- A. Arroyo Garcia, MH, K.Kutak, PLB 795 (2019) 569-575, arXiv:1904.04394
- MH, E. Padron Molina, *Phys.Rev.D* 103 (2021) 7, 074008 arXiv:2011.02640
- Alcazar Peredo, MH, in preparation

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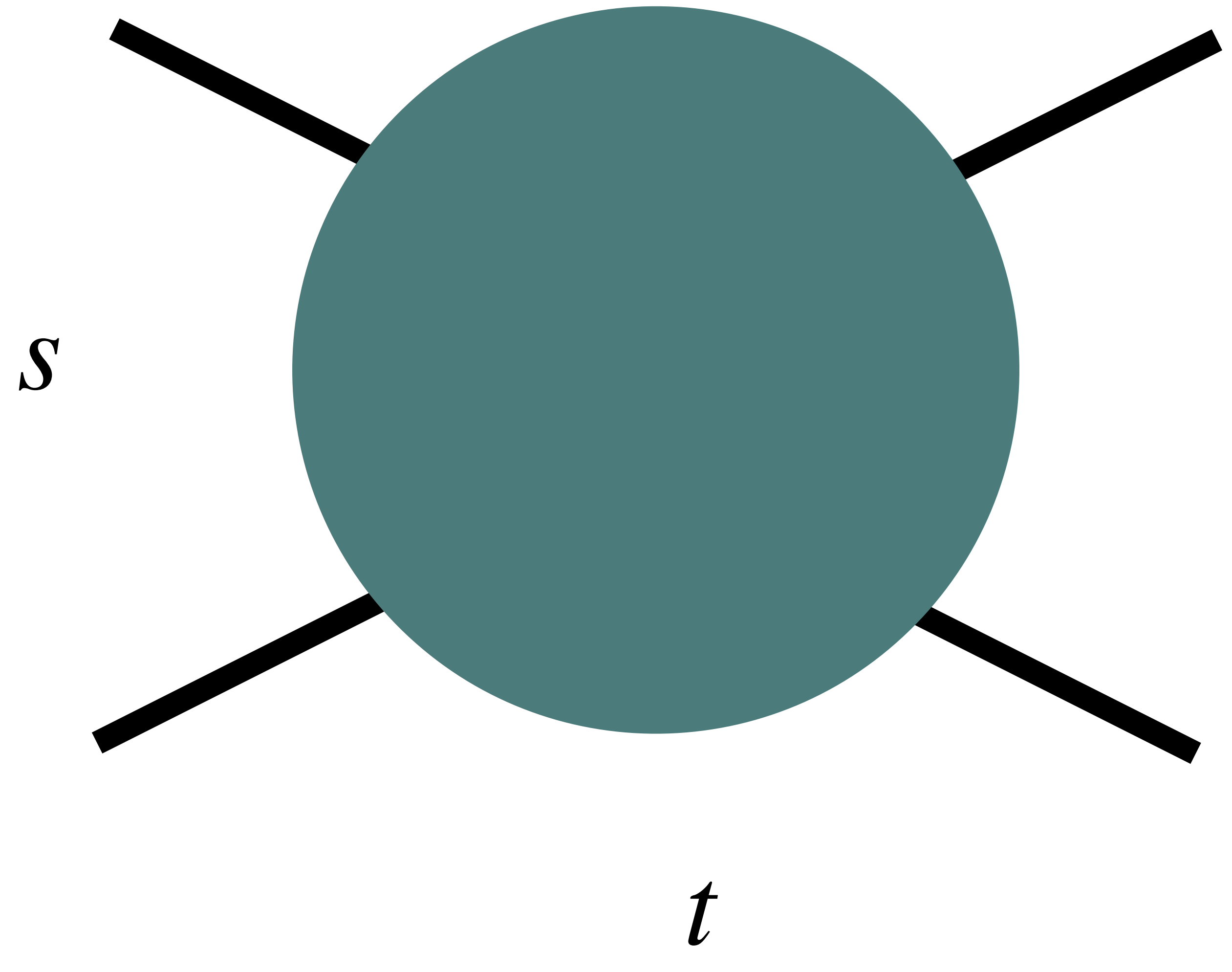
What is special about BFKL? What is it?

1975-1978: Baltisky, Fadin, Kuraev, Lipatov: study of QCD scattering amplitudes in the limit of high center of mass energies $\sqrt{s}$, $s \gg -t, m_i$



Actually: $SU(2)$ gauge theory + Higgs mechanism (infrared regularization)

Methodology:

- perturbation theory → studied scattering of gluons (and also quarks)
- sum up all terms in the perturbative series $\left(\alpha_s \ln s\right)^n$ i.e. rearrange perturbative series

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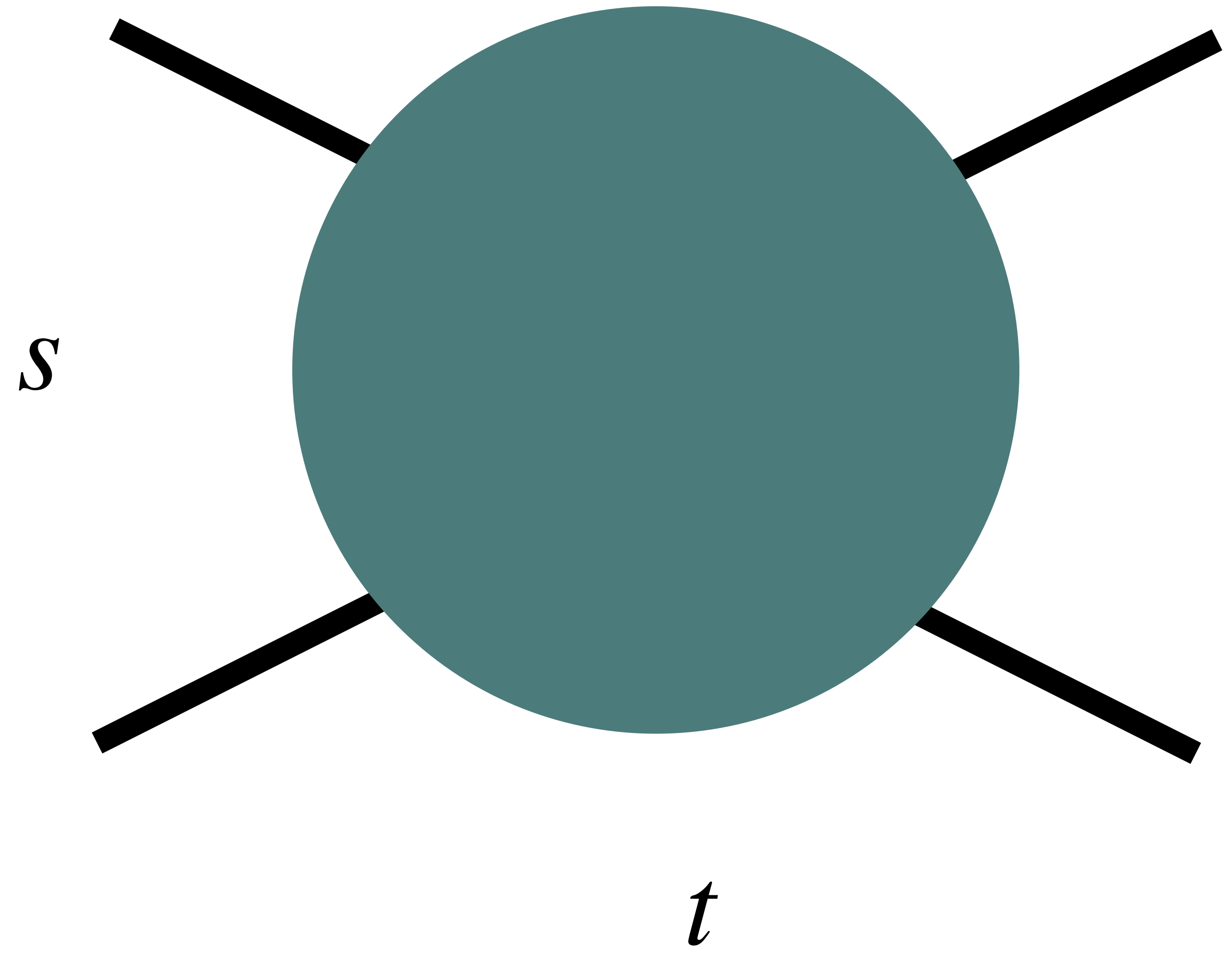


Result No 1:
Elastic scattering amplitude

$$\mathscr{A}_{gg\to gg}(s,t) = \mathscr{A}^{(0)}_{gg\to gg}(s,t) \cdot s^{\omega(t,\epsilon)}$$

Here:

- $\omega(t,\epsilon)$ = gluon Regge trajectory; right now know up to 2 loop
- It is IR divergent (regulator ϵ)
- Reggeization of the gluon

Extension to multi-particle production

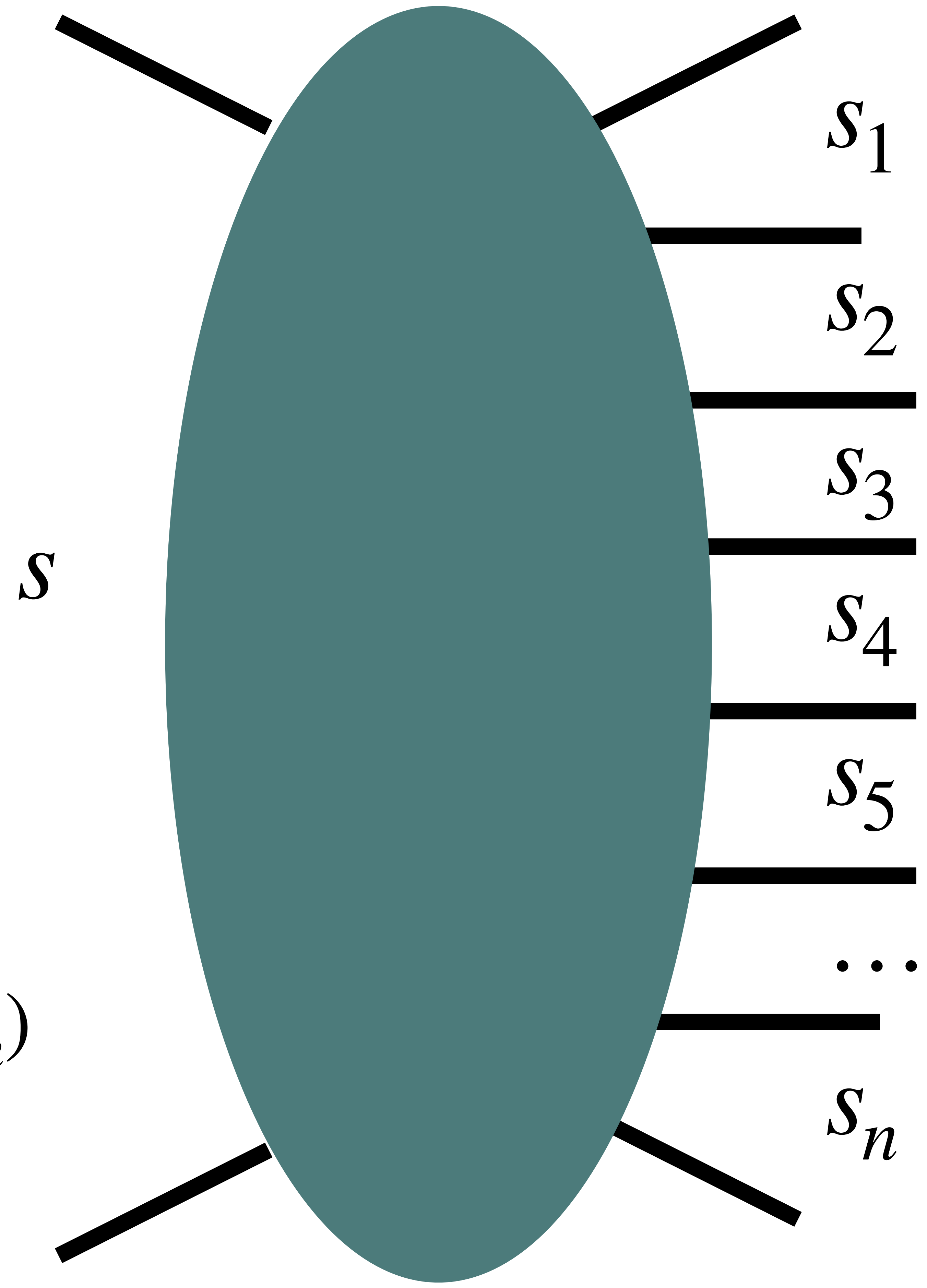
In Multi-Regge-Kinematics $s \gg s_i \gg -t_i, m_j$

Result:

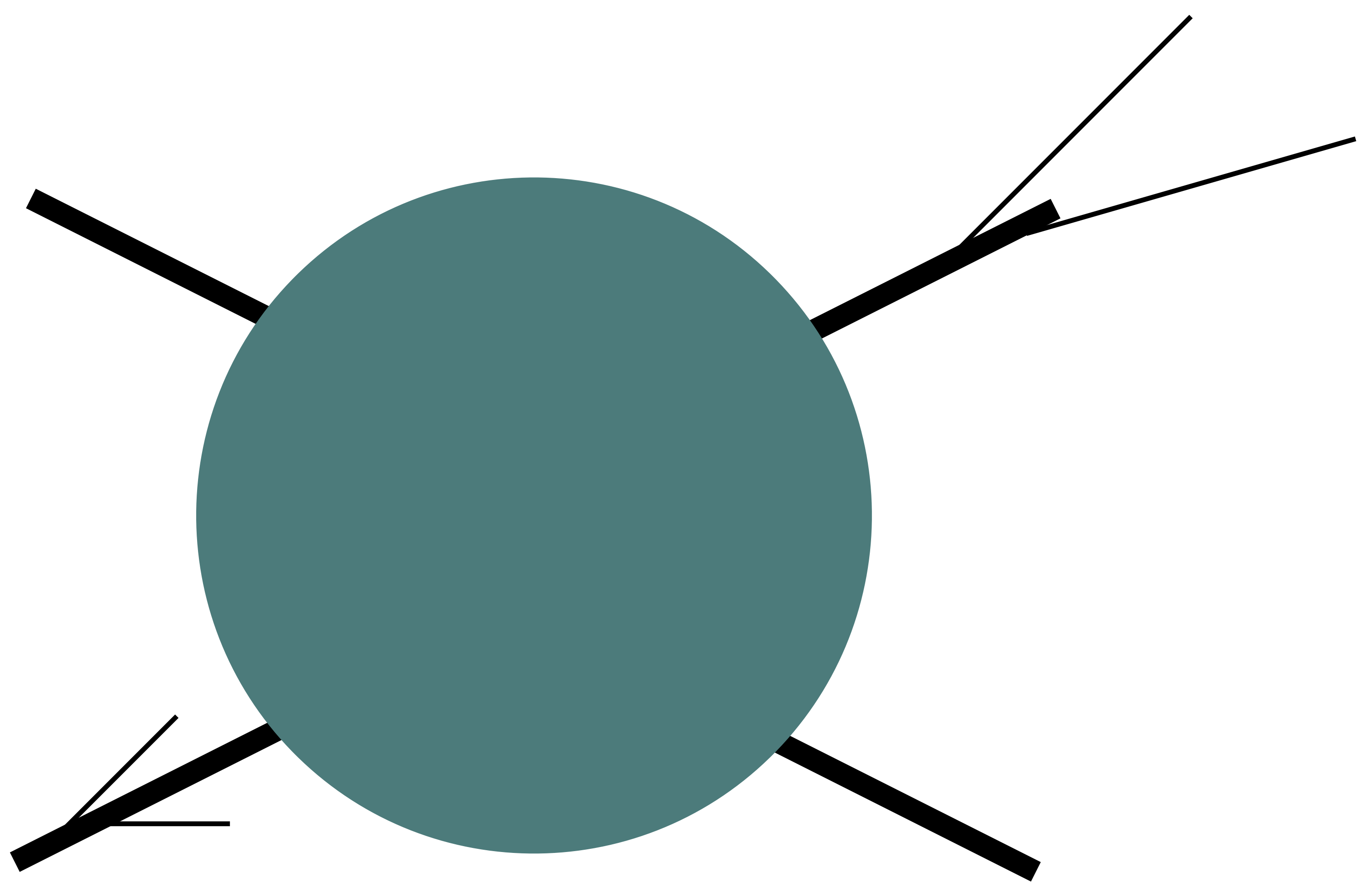
$$\mathscr{A}_{gg\to ng} = \mathscr{A}^{(0)}_{gg\to ng} \cdot s_1^{\omega(t_1,\epsilon)} \cdot s_2^{\omega(t_2,\epsilon)} \ldots \cdot s_n^{\omega(t_n,\epsilon)}$$

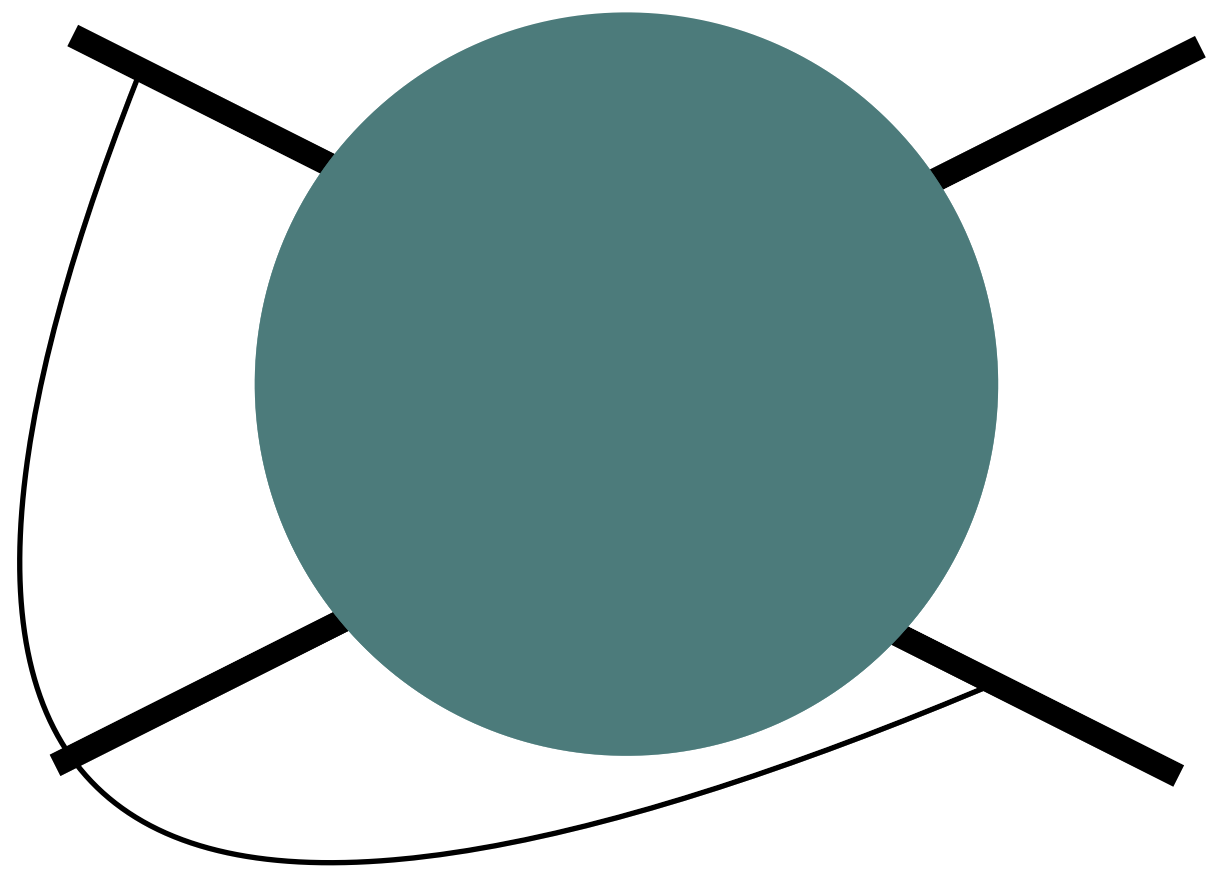
Production through gauge invariant Lipatov vertex C_μ:

$$\mathscr{A}_{gg\to ng} = s\Gamma(q_1)\frac{1}{q_1^2}C_\mu(q_1,q_2,k_1)\epsilon^\mu(k_1)\frac{1}{q_2^2}C_\mu(q_2,q_3,k_2)\epsilon^\mu(k_2)\ldots\Gamma(q_n)$$



Why this is remarkable?





- not only a correction to external legs (collinear radiation)
- Not only a soft correction
- Need to “break up” scattering amplitudes for such resummation → deal with internal off-shell states (“reggeized gluons”)

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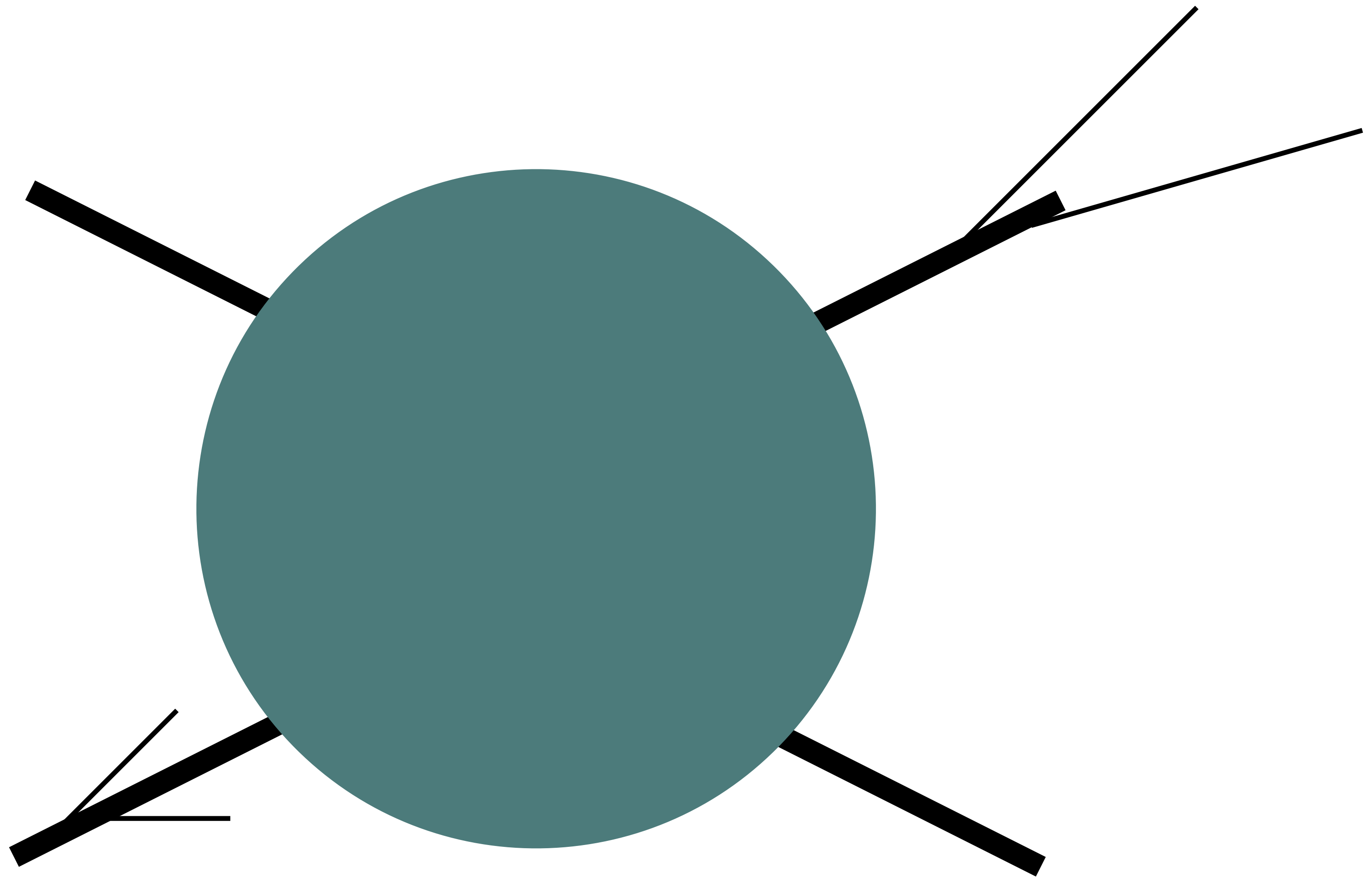

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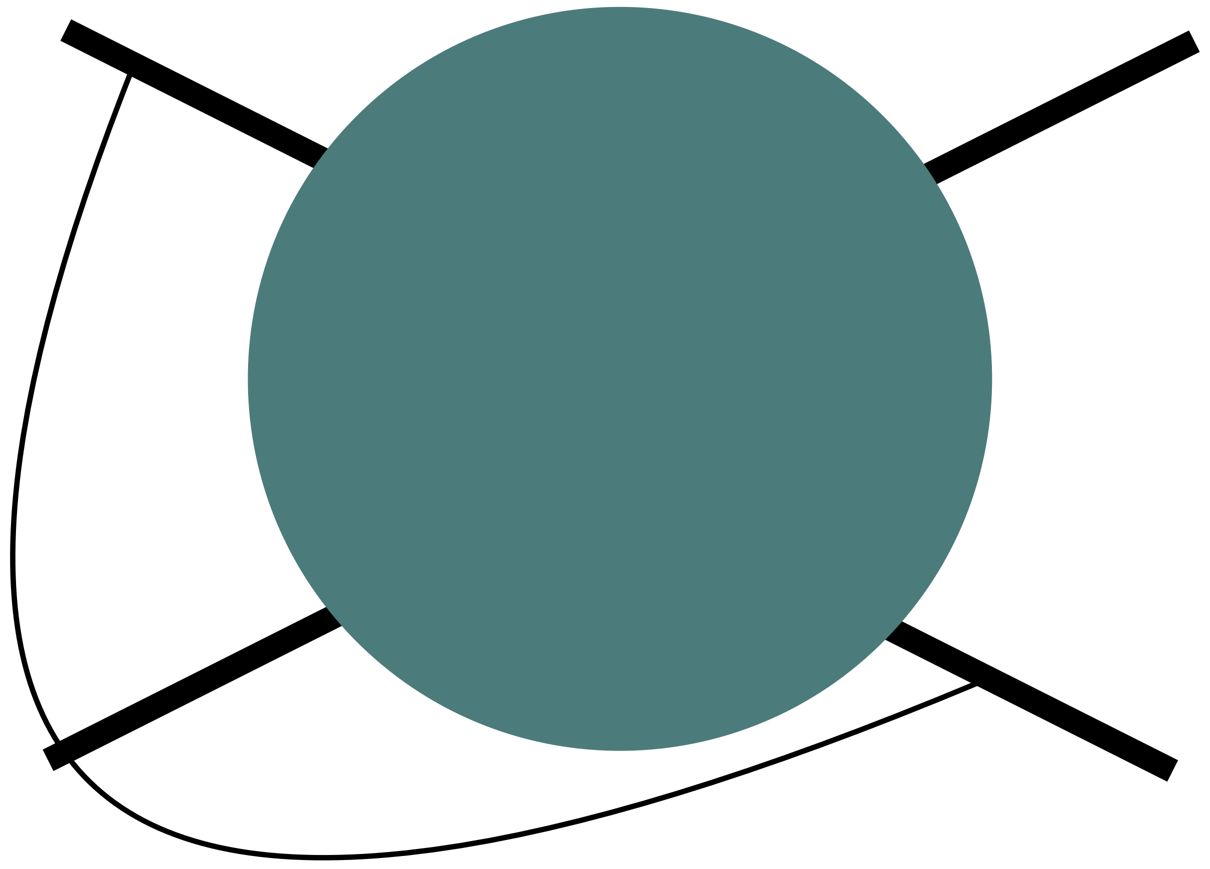

- confirmed by exact calculations (*e.g.* anomalous DGLAP dimension to 3-loop etc., N=4 SYM amplitudes etc., exact QCD scattering amplitudes)
- Reveals beautiful mathematical structure (conformal symmetry, integrability) in certain setups

Phenomenology

Observe cross-sections, not amplitudes ….

$$d\sigma = \sum_n |\mathscr{A}_{2\to n}|^2 d\Phi^{(n)}$$

Pomeron = t-channel exchange with quantum numbers of the vacuum; responsible for the rise of the total QCD cross-section

- Yields perturbative, hard, or BFKL Pomeron
- Predicts in principle a power-like rise of the total cross-section $\sigma \sim s^\lambda$
- In general more complicated:

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$$\sigma_{AB}(s, Q_A, Q_B) = \int \frac{d^2\mathbf{k}_a}{\pi} \int \frac{d^2\mathbf{k}_b}{\pi} \Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b)\, \Phi_B(\mathbf{k}_b, Q_B)$$

$f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b)$ universal BFKL Green's function

$\Phi_I(\mathbf{k}, Q)$: impact factors = describe coupling of BFKL Green's function to external scattering particles

(potential) Issues with this expression

$$\sigma_{AB}(s, Q_A, Q_B) = \int \frac{d^2\mathbf{k}_a}{\pi} \int \frac{d^2\mathbf{k}_b}{\pi} \Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b)\, \Phi_B(\mathbf{k}_b, Q_B)$$

- expression derived in perturbation theory → need some hard scale $Q_a, Q_b \gg \Lambda_{QCD}$
- expression derived in perturbation theory → small $\alpha_s(\mu)$, yet integrated over all transverse momenta
  → not necessarily a problem (do the same in loop calculations, but $\beta_0 \ln(\mu^2/\mathbf{k}^2)$ can lead to complications with Landau pole of running coupling etc.;
  Appears at NLO …
  → diffusion in transverse momentum ("Bartels's cigar")
- expression derived in perturbation theory → it's the dominant term at any order in perturbation theory; not necessarily true, once summed up

$\alpha_s s^{2\alpha_s\omega_0} \gg s^{\alpha_s\omega_0}$ possible etc.

BFKL Pomeron in conjugate Mellin space

$$\sigma_{AB}(s, Q_A, Q_B) = \int \frac{d^2\mathbf{k}_a}{\pi} \int \frac{d^2\mathbf{k}_b}{\pi} \Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b)\, \Phi_B(\mathbf{k}_b, Q_B)$$

Similar to moments for DGLAP evolution, Fourier transform: convolutions in transverse momenta turn into products for conjugate Mellin space

$$\sigma_{AB}(s, Q_A, Q_B) = \int_{1/2-i\infty}^{1/2+i\infty} \frac{1}{Q_A}^{2} \frac{d\gamma}{2\pi i} \left(\frac{Q_A^2}{Q_B^2}\right)^{\gamma} \Phi_A(\gamma)\Phi_B(\gamma) s^{\chi(\gamma)}(1 + \alpha_s^2 \ln(s) f(\gamma) + \ldots)$$

With $\chi(\gamma) = \frac{\alpha_s N_c}{\pi}\chi_0(\gamma) + \left(\frac{\alpha_s N_c}{\pi}\right)^2 \chi_1(\gamma) + \ldots$ BFKL eigenvalue

Hard vs. Soft Pomeron

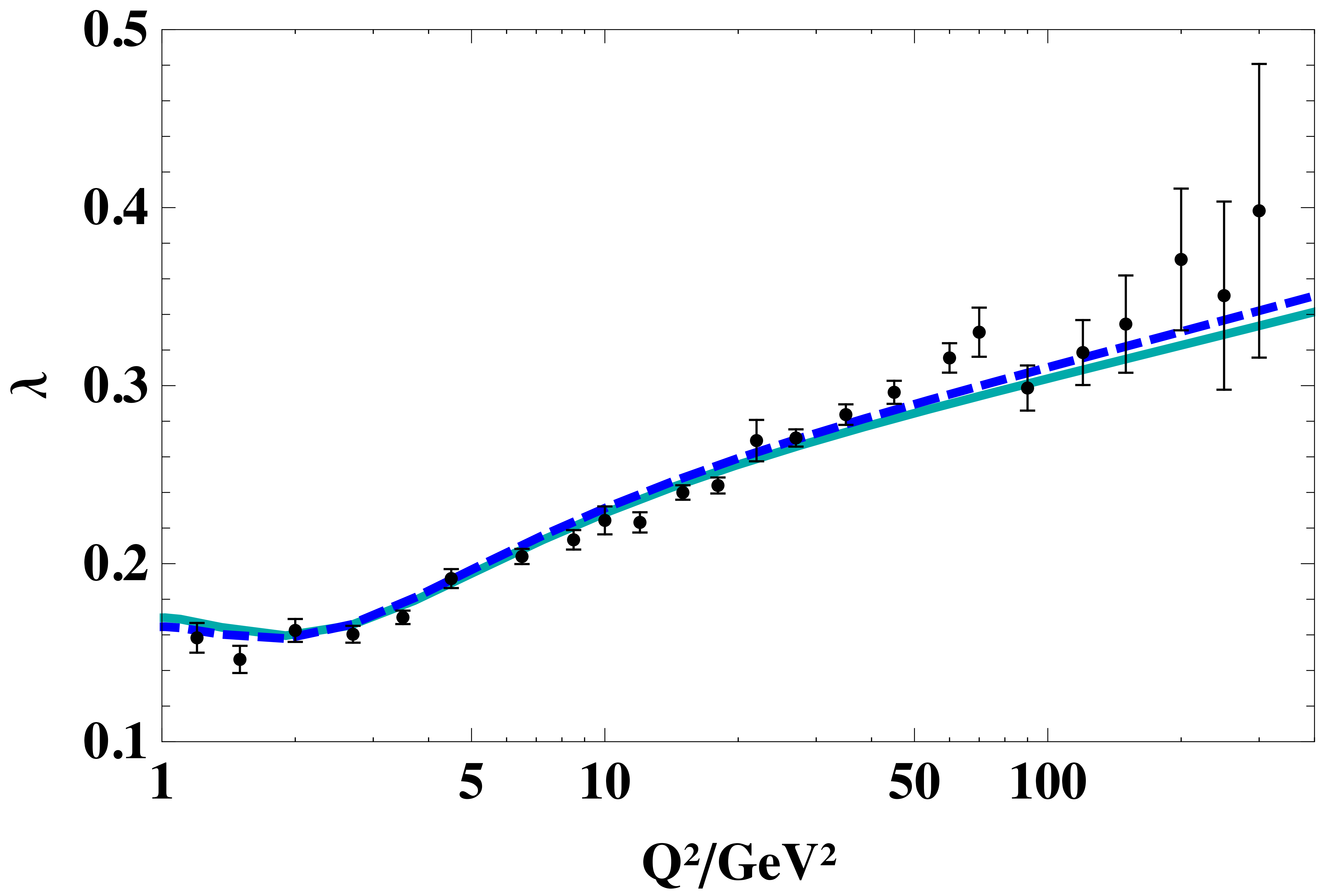
Approximate solution (saddle point approximation limit $\bar{\alpha}_s \ln s \to \infty, \quad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$):

$$\sigma_{AB} \sim s^{\bar{\alpha}_s 2.77259} \simeq s^{0.52}, \quad \alpha_s = 0.2$$

- Idea: existence of 2 Pomerons s^λ (soft with $\lambda \simeq 0.1$ and hard with $\lambda \simeq 0.5$
- Hard Pomeron in above approximation problematic:
  - Intercept is very large
  - HERA: intercept increases with hard scale; seems to indicate the opposite
  - BFKL wrong?

Complete description:

Effective Pomeron intercept in DIS $x = Q^2/s$



$$\lambda(Q^2) = \left\langle \frac{d \ln F_2(x, Q^2)}{d \ln 1/x} \right\rangle_x$$

Data: [H1 & ZEUS collab. 0911.0884]
Theory: [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

- Description uses complete Mellin integral + NLO corrections + collinear resummation of NLO BFKL + BLM scale setting for running coupling
- Tendency even there for LO BFKL with fixed coupling

Note: this is expect: BFKL and DGLAP agree in the double log approximation

Unitarity & the BFKL Pomeron

- Non-perturbative Froissart theorem: total QCD cross-section grows asymptotically at most as $\sigma_{tot} \leq c_0 \ln^2 s$
- Derived from unitarity (and finite range of strong interactions?)

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Quite ironically, the original BFKL deviation uses heavily unitarity as well ….

- But naturally $\ln(-s) = \ln(s) - i\pi \simeq ln(s)$ etc
- Keeping track of $i\pi$'s reveals other terms which belong to multiple reggeized gluon exchange (Pomeron = "bound state" of 2 reggeized gluons);
- Yields so-called Triple Pomeron Vertex → non-linear term in BK equation

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How can this help? Schematically

With $z = s^\lambda$,
multiple (Pomeron) exchange can yield something like

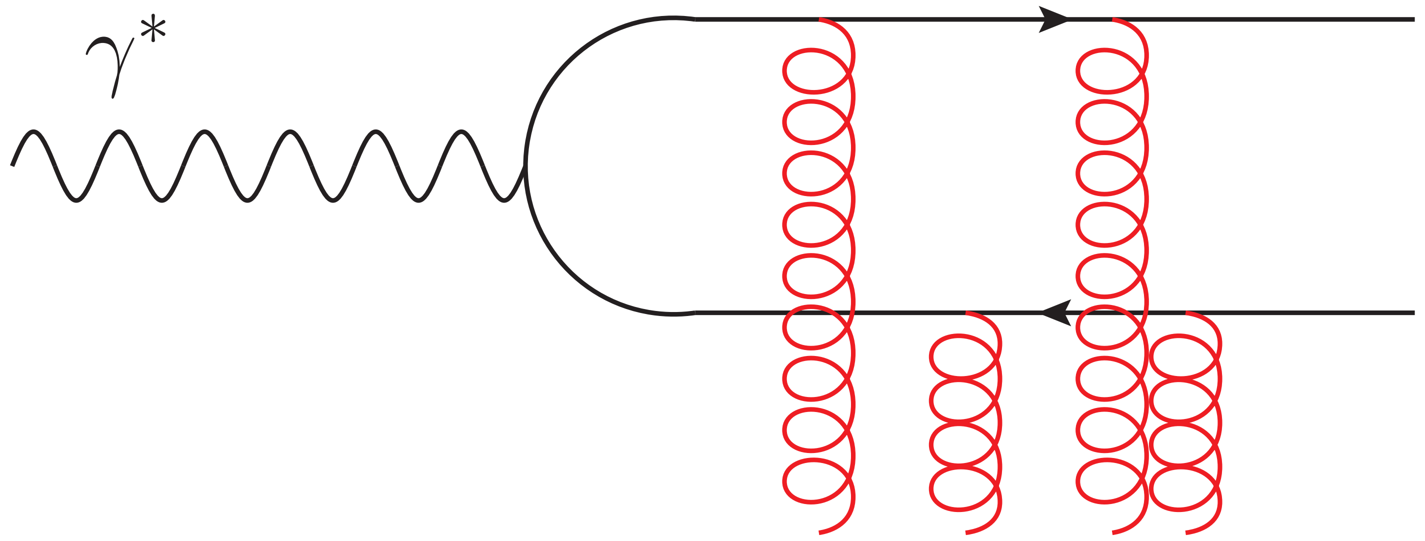
$$\sigma \sim c_1 z - c_2 z^2 + c_3 z^3 - c_4 z^4 + \ldots$$

$c_{i>1}$ subleasing in α_s, but lead to unitarization of the result

An illustrative example: dipole models

Lipatov (some DESY seminar 2009): “Exponential is a very nice function but it is not always the correct function”

Cross-section of a color dipole (quark -antiquark pair with transverse separation r in configuration space)



Perturbative result: $\sigma_{q\bar{q}}^{lin.} = \sigma_0 r^2 Q_0^2 x^{-\lambda} = \sigma_0 \frac{r^2 Q_s^2(x)}{4}, \qquad x = Q^2/s, \quad Q_s^2 = Q_0^2 x^{-\lambda}$

power-like growth of cross-section

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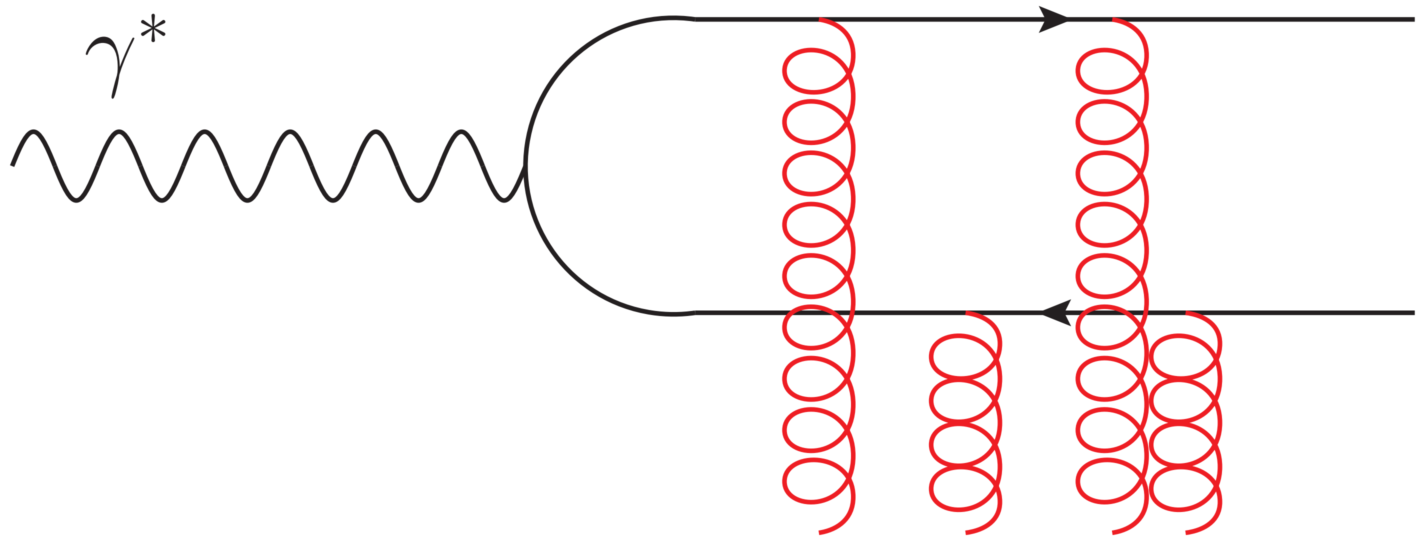

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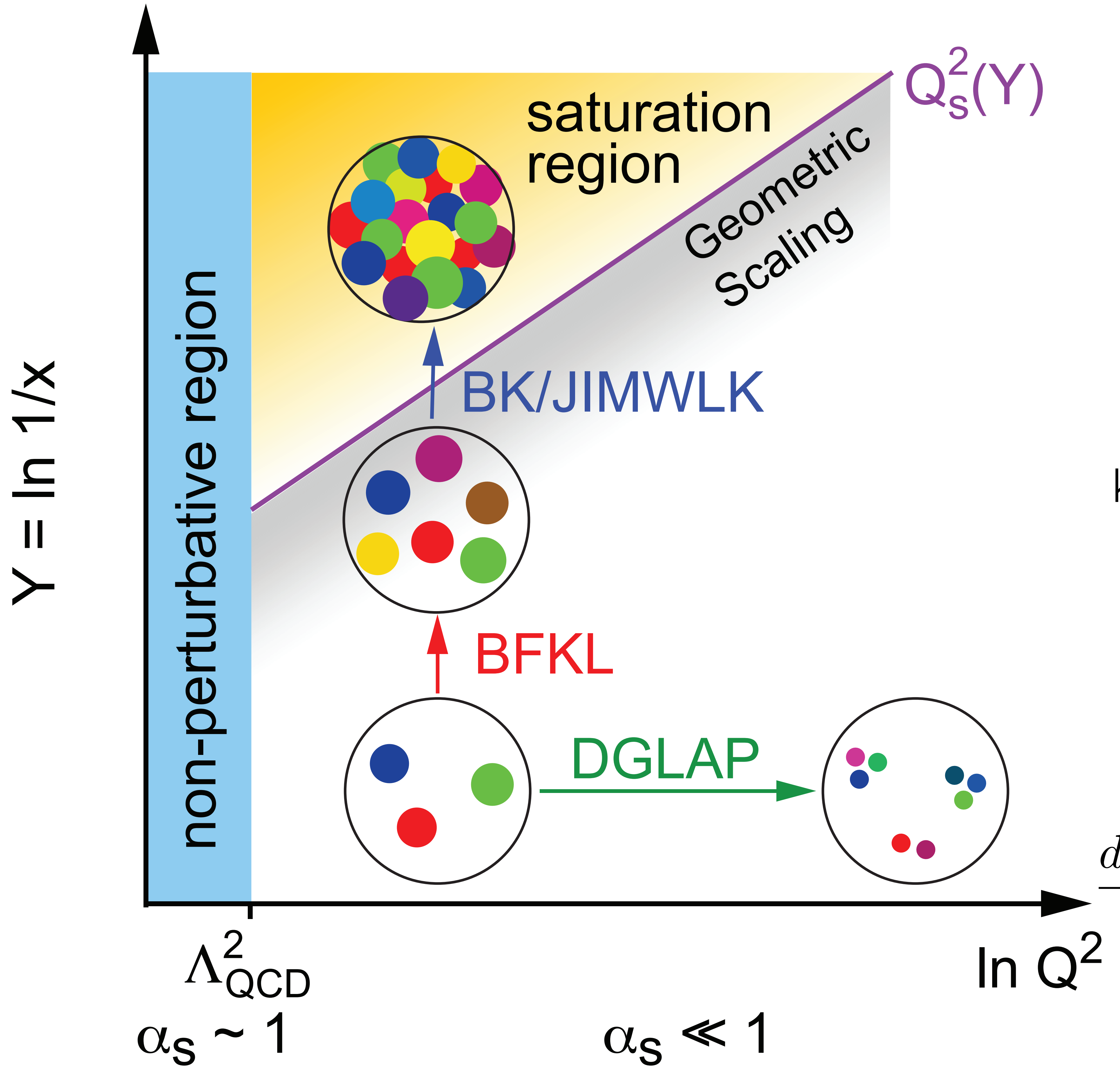
power-like growth of cross-section

Unitarized version: $\sigma_{q\bar{q}} = \sigma_0 \left(1 - e^{-r^2 Q_s^2(x)/4}\right) = \sigma_0 \sum_k \frac{(-1)^{k+1}}{k!} \left(\frac{r^2 Q_s^2(x)}{4}\right)^k$

Exponential (=eikonal) correct in QED, most likely not in QCD → a model (here GBW model)

[Golec-Biernat, Wüsthoff, 1998-1999]

Complete picture: non-linear QCD evolution



[Gribov, Levin, Ryskin; 1983]
[McLerran, Venugopalan; 1993]
[Balitsky, 1995], [Kovchegov, 1997]
[Jalilian-Marian,Iancu, McLerran, Weigert, Leonidov, Konvex, 1996-2000]

BK evolution for dipole amplitude $N(x,r) \in [0,1]$

[related to gluon distribution]

kernel calculated in pQCD

$$\frac{dN(x,r)}{d\ln\frac{1}{x}} = \int d^2\boldsymbol{r}_1 K(\boldsymbol{r},\boldsymbol{r}_1)\left[N(x,r_1)+N(x,r_2)-N(x,r) - N(x,r_1)N(x,r_2)\right]$$

linear BFKL evolution = subset of complete BK

non-linear term relevant for N~1 (=high density)

Derivation: assumes presence of strong color field $A^+ \sim 1/g$ + use of renormalization group wrt. Rapidity cut-off

Complete picture: non-linear QCD evolution

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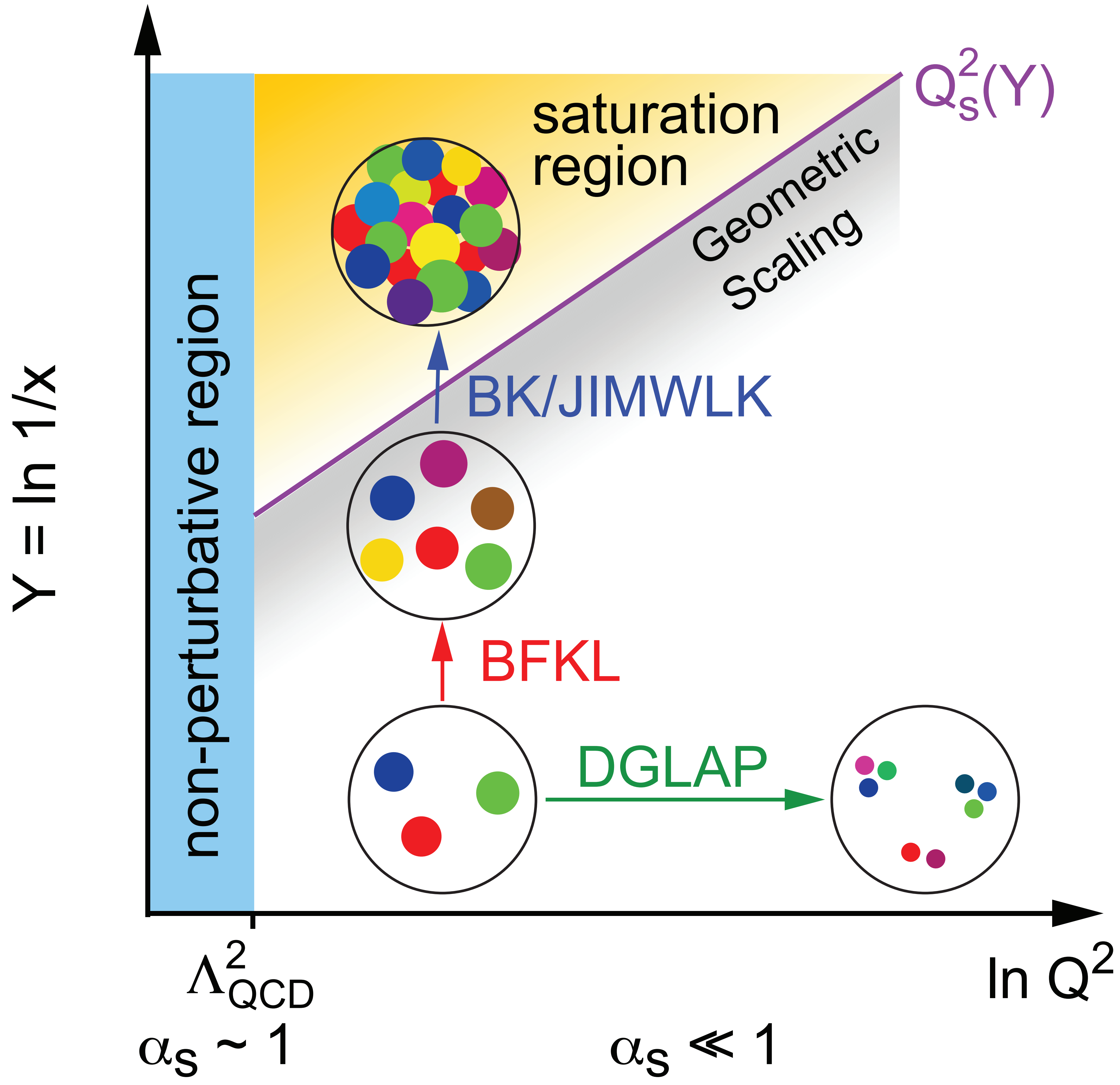
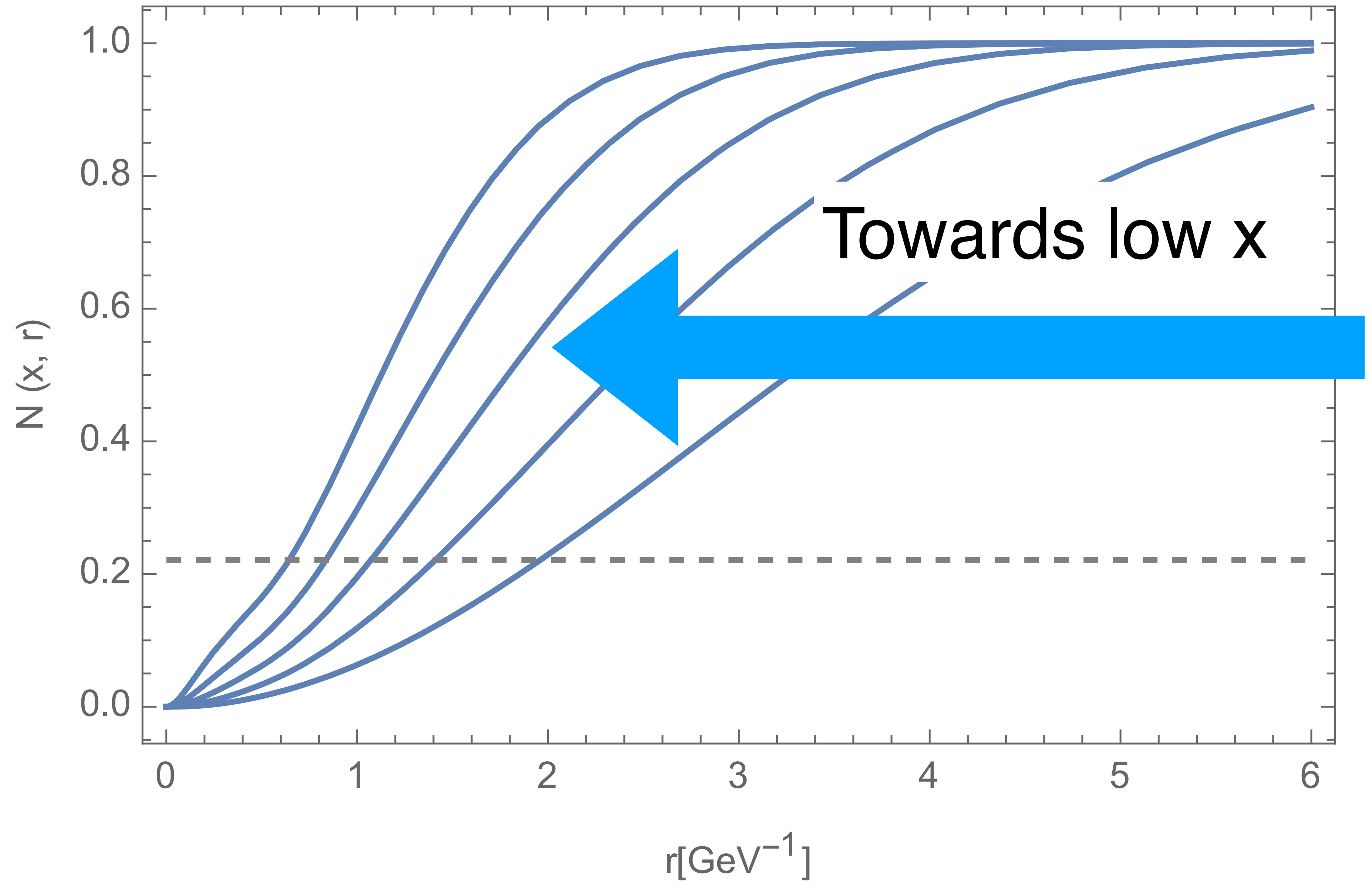


- linear terms (LO BFKL): power-like growth
- Non-linear term: bring growth to hold ($N = 1$ is solution)
- Transition between linear & non-linear regime characterized by saturation scale $Q_s(x)$, growing with energy



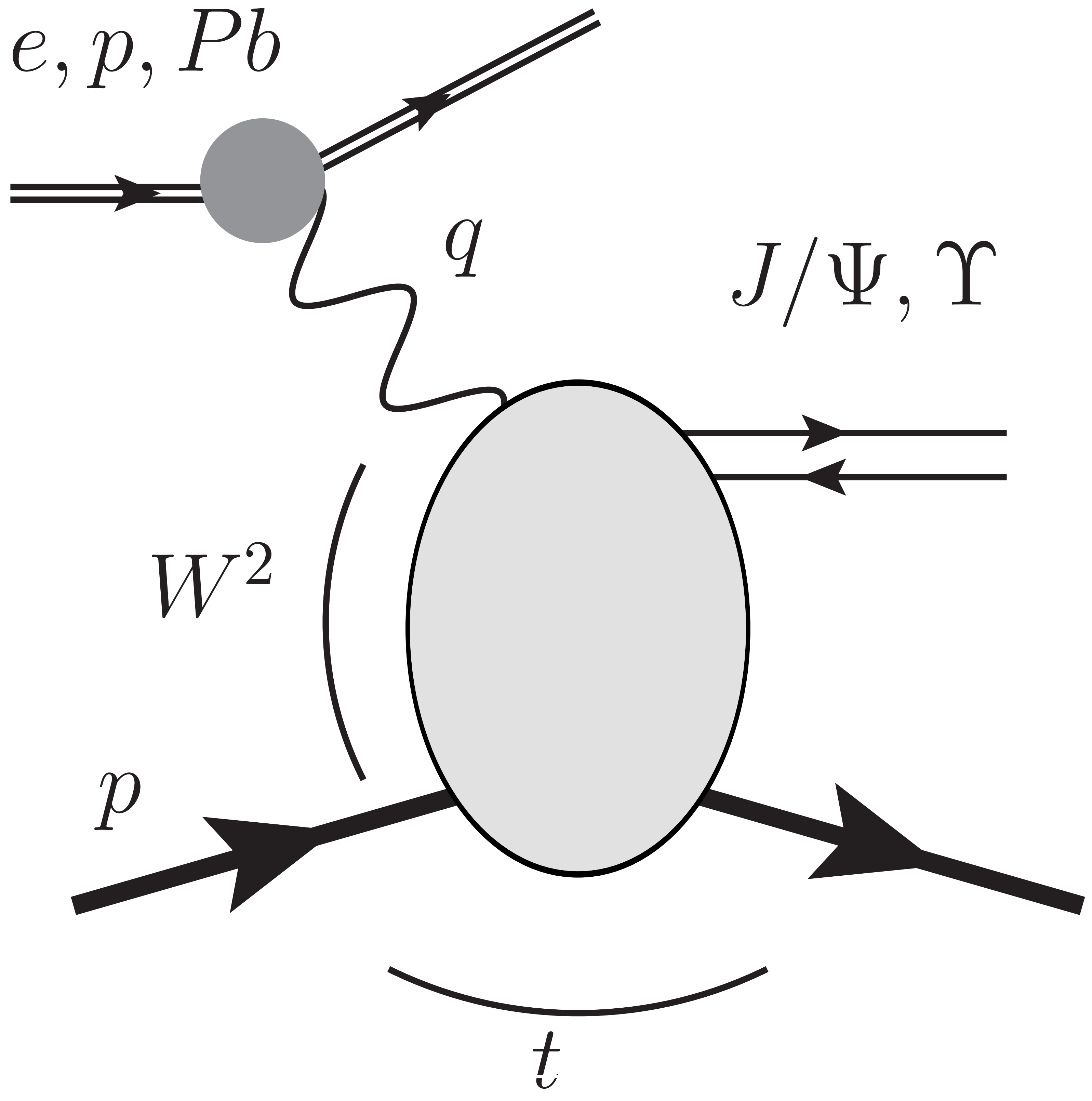
How to provide evidence for such physics?

- Observables with 1 hard scale M → construct dimensionless function which scales with saturation scale
  $f(M^2, x) = g(M^2/Q_s^2(x))$
  Can search for such scaling pattern *e.g.* [Praszalowicz, Stebel, 2013]

- Imprints of the saturation scale in transverse momentum spectra (*e.g.* decorrelation of back-to-back dijets/dihadrons)

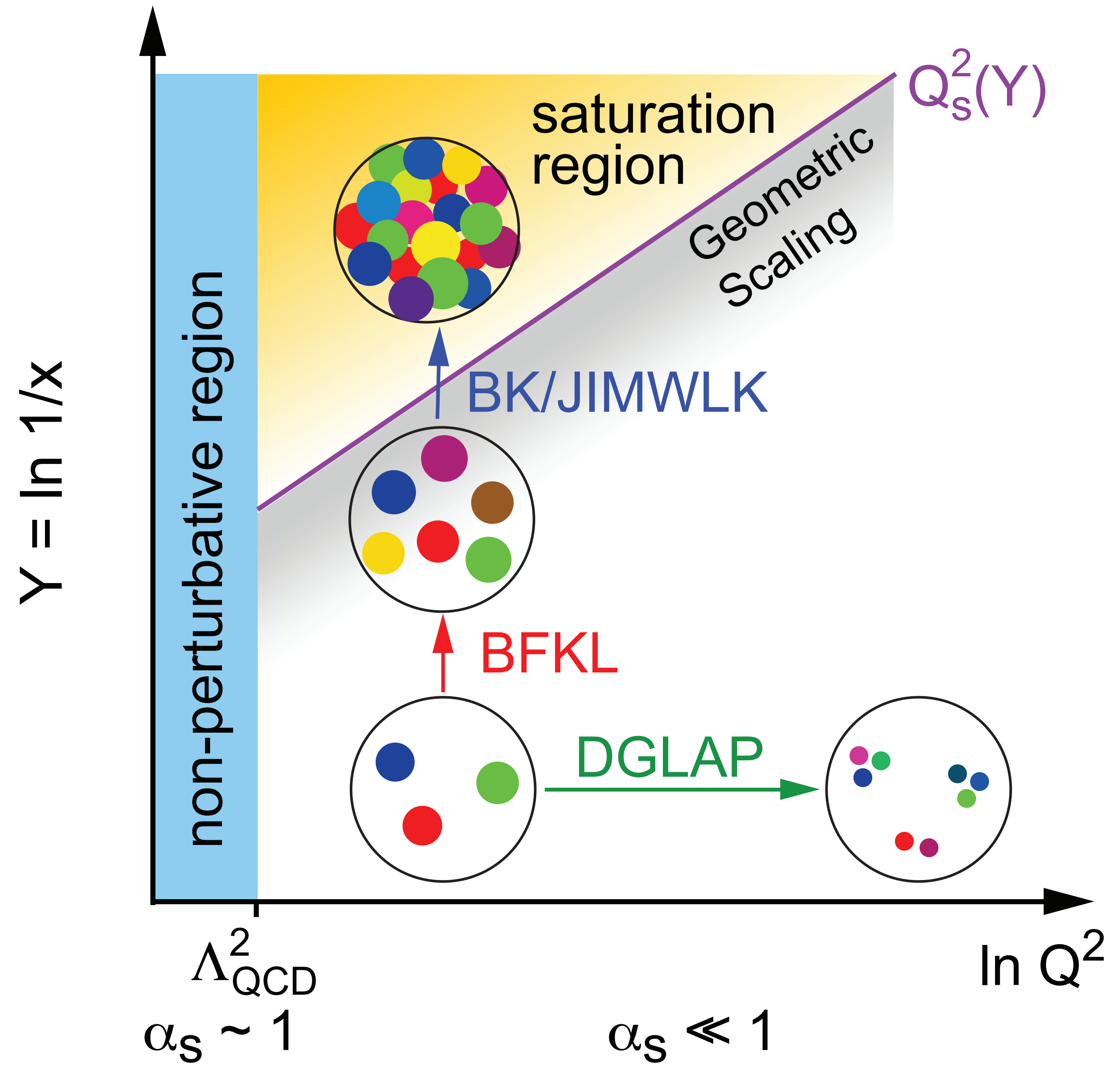
- Investigate dependence of cross-sections on center-of-mass energy
  Serves both as further tests of BFKL evolution
  +search for deviations from BFKL at highest center of mass energies

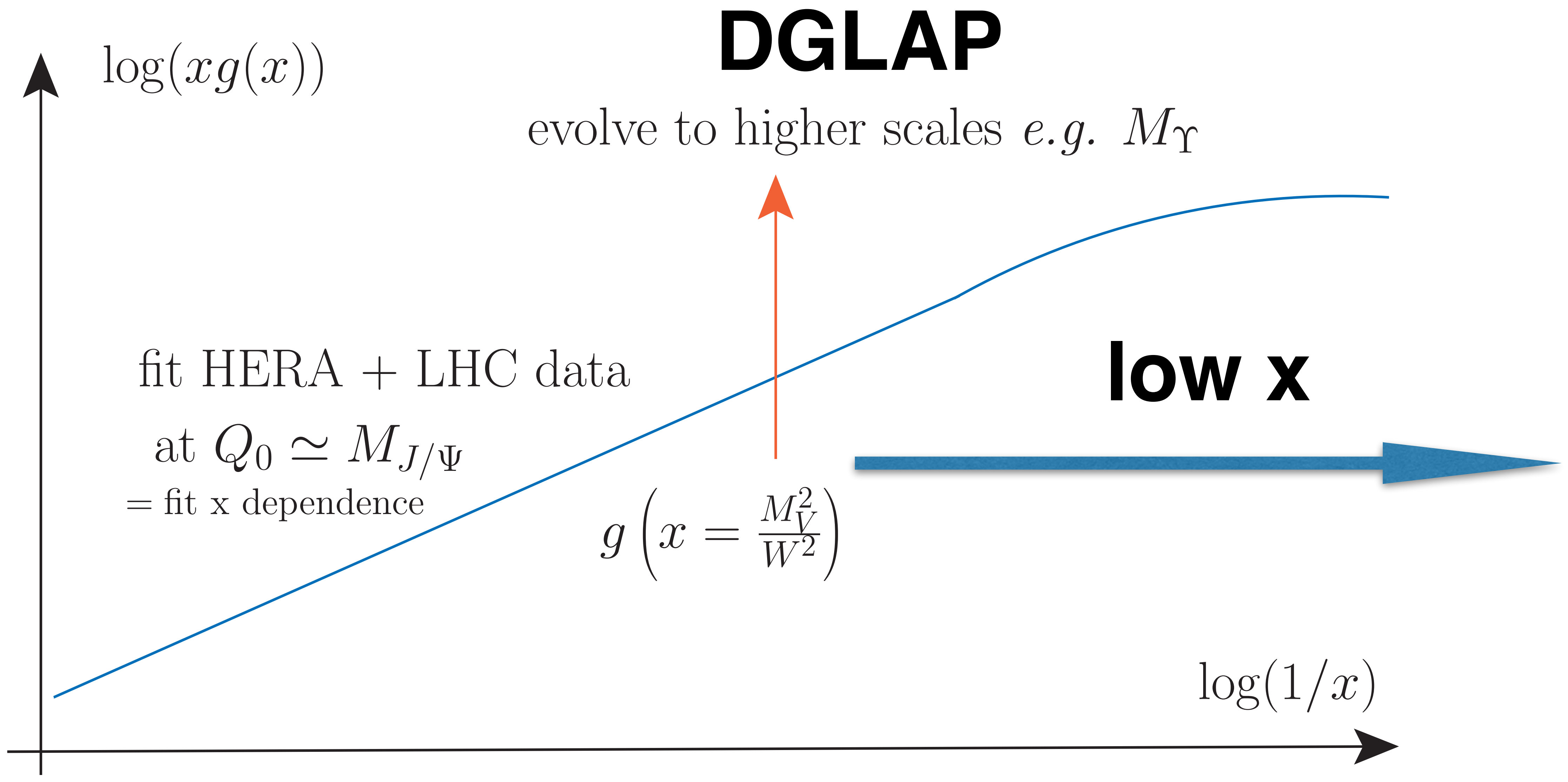
photo induced exclusive photo-production of J/Ψs and $\Psi(2s)$



- hard scale: charm mass
  (small, but perturbative)
- reach up to $x \gtrsim .5 \cdot 10^{-6}$
- perturbative cross-check:
  Υ (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)
- Enormous range in center of-mass energies

Important: not a contest with DGLAP evolution - ask different questions



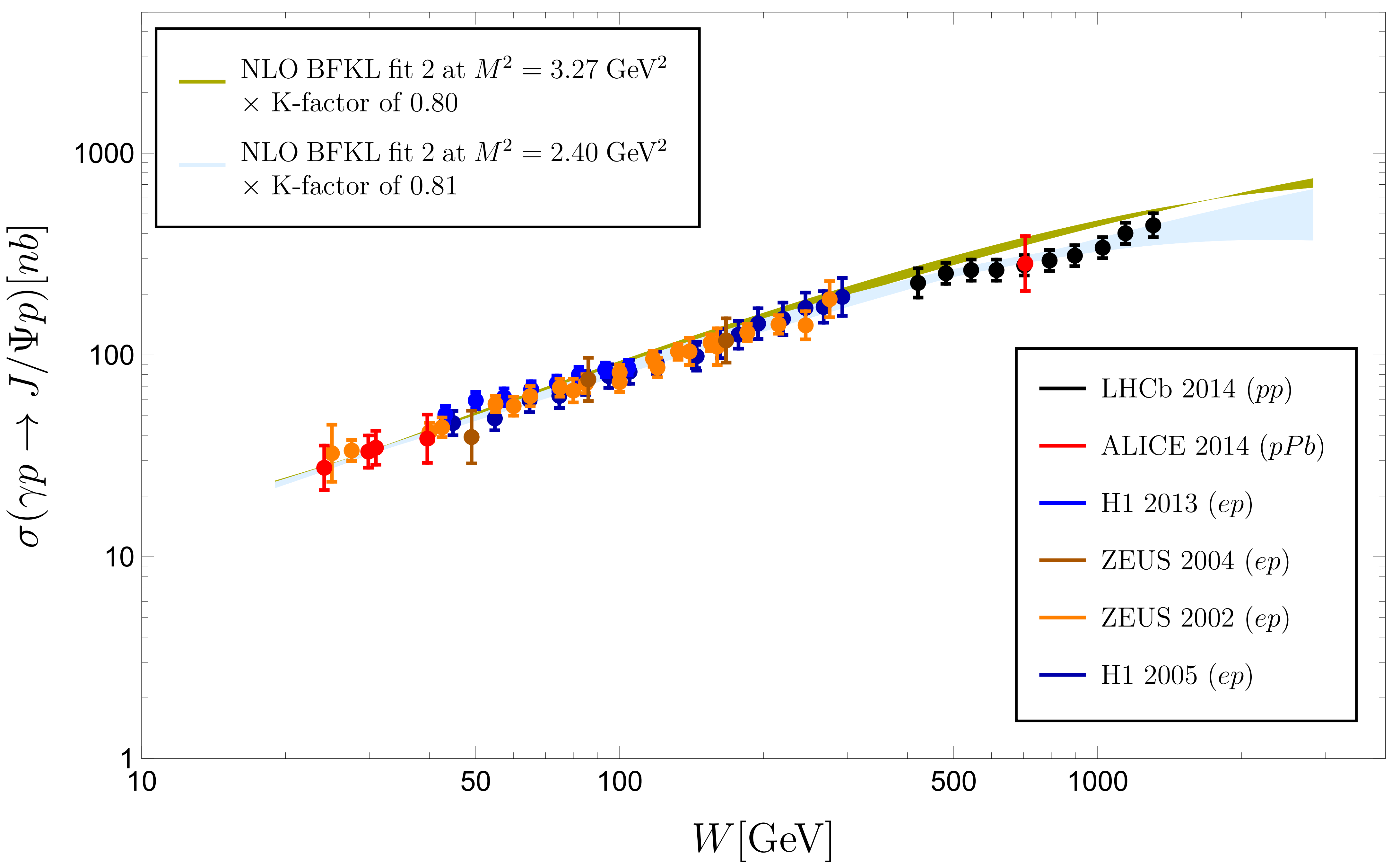


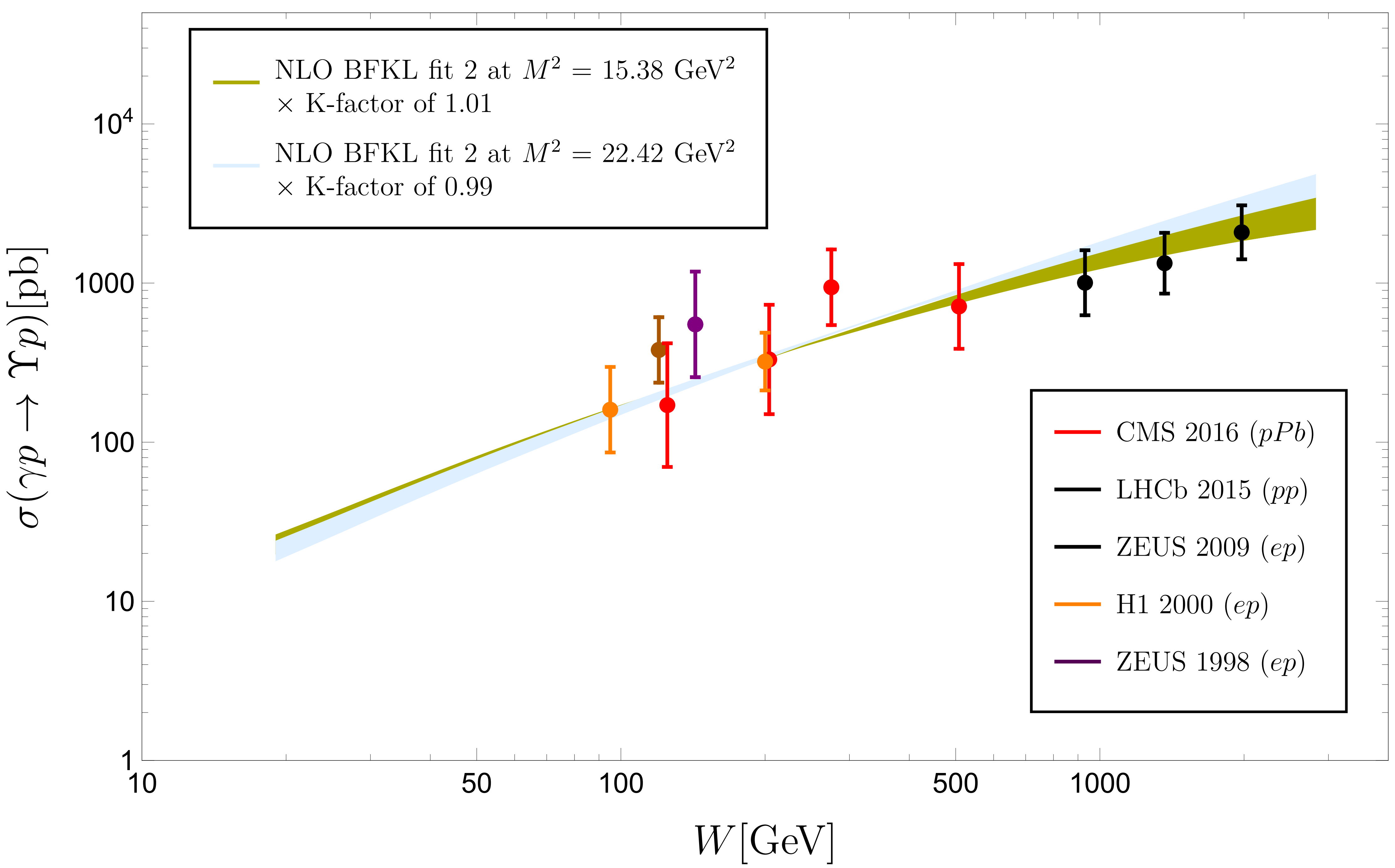
DGLAP:

- fit x-dependence + evolve from J/Ψ (2.4 GeV^2) to Υ (22.4 GeV^2)
- DGLAP shifts large x input (low scales) to low x (high scales)
  + higher twist dies away fast in evolution

→ constrain pdfs, but don't learn about saturation (easily overseen) and BFKL (fitted)

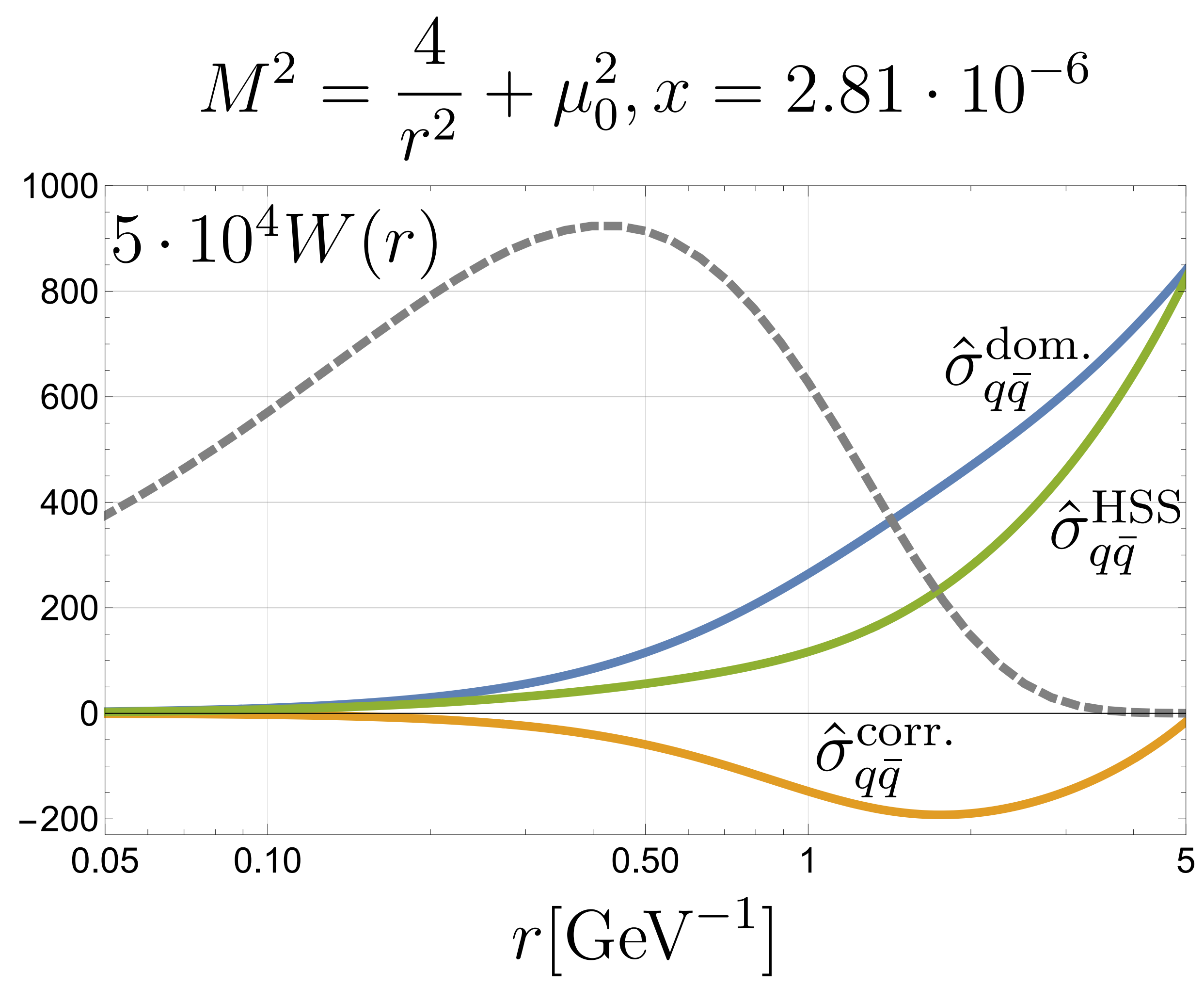
What did we find so far?

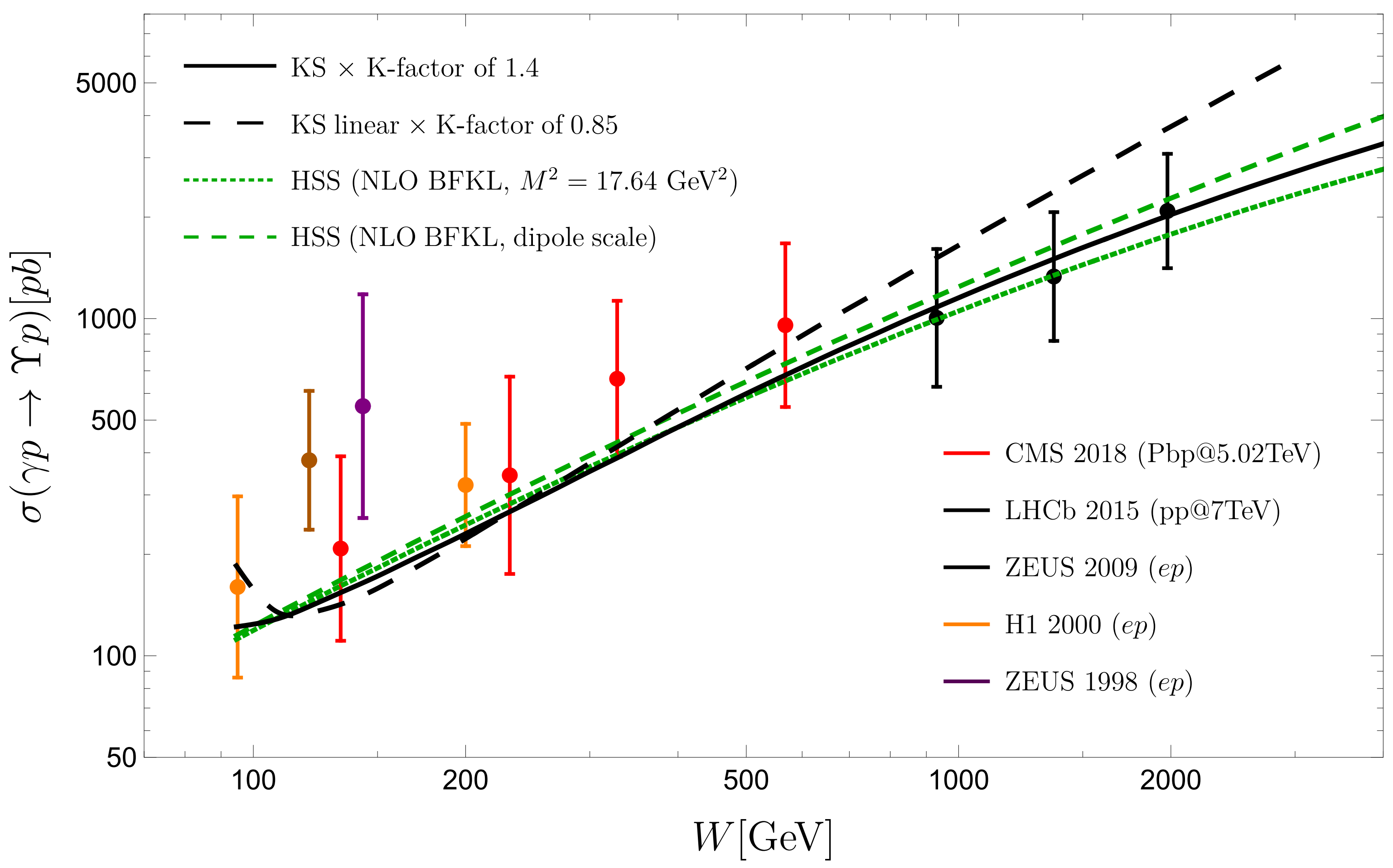




Can BFKL fit [MH, Salas, Sabio Vera; 1209.1353; 1301.5283] describe J/Ψ and Υ data? YES.

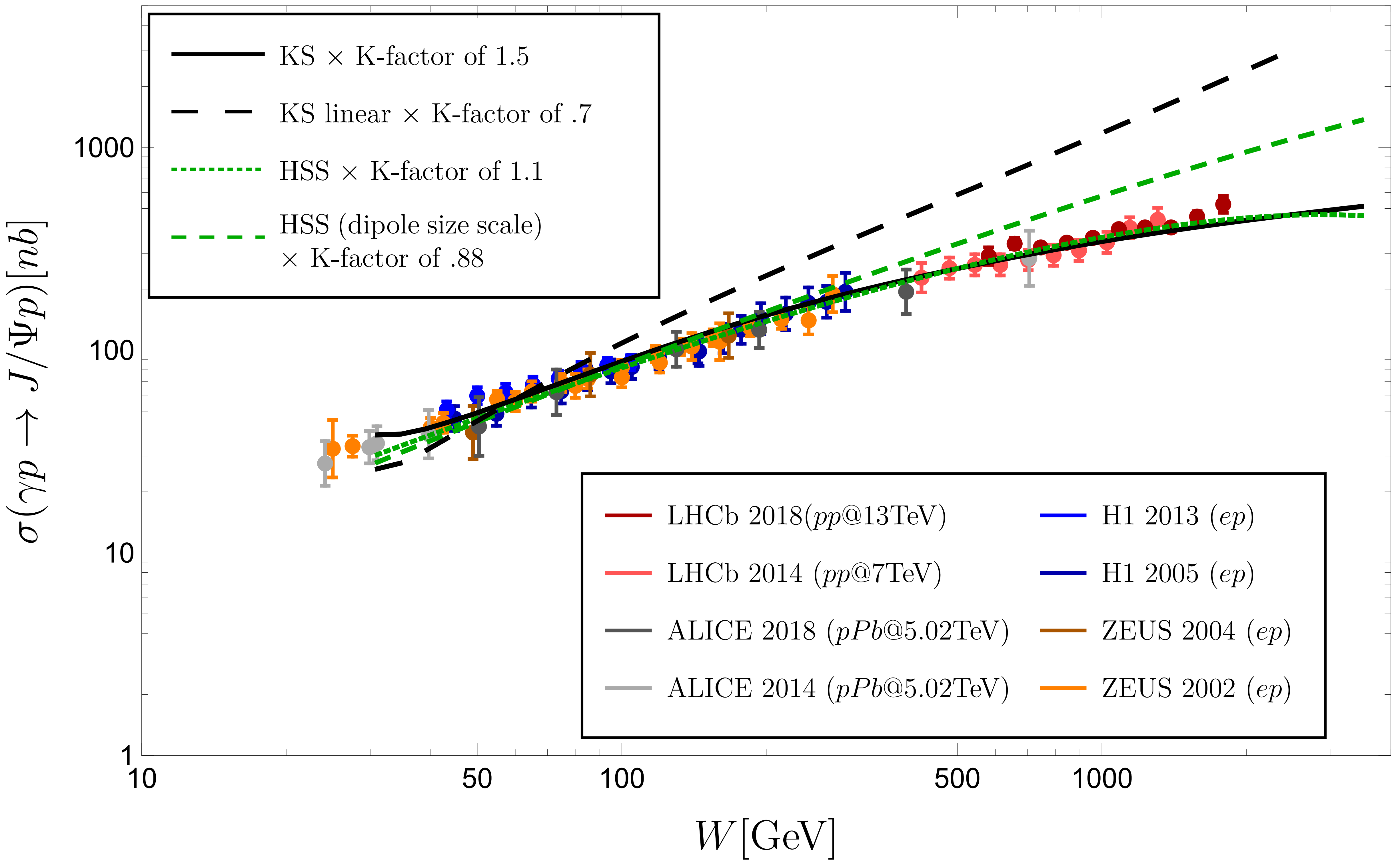
[Bautista, Fernando Tellez, MH; 1607.05203]





- still describe Υ production
  → perturbative cross-check
- not true for high precision HERA data

At highest W, BFKL fit unstable (NLO>LO)

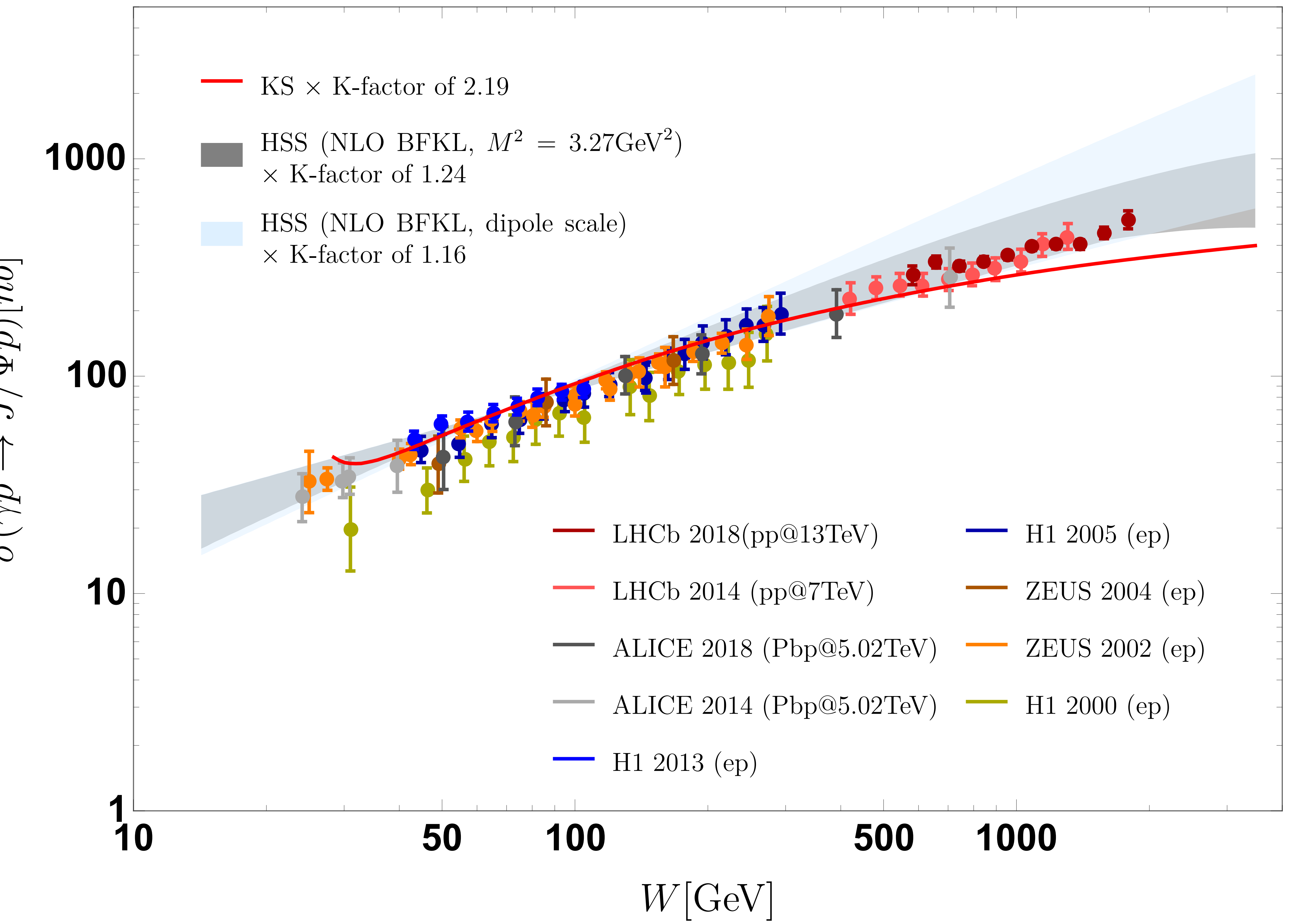


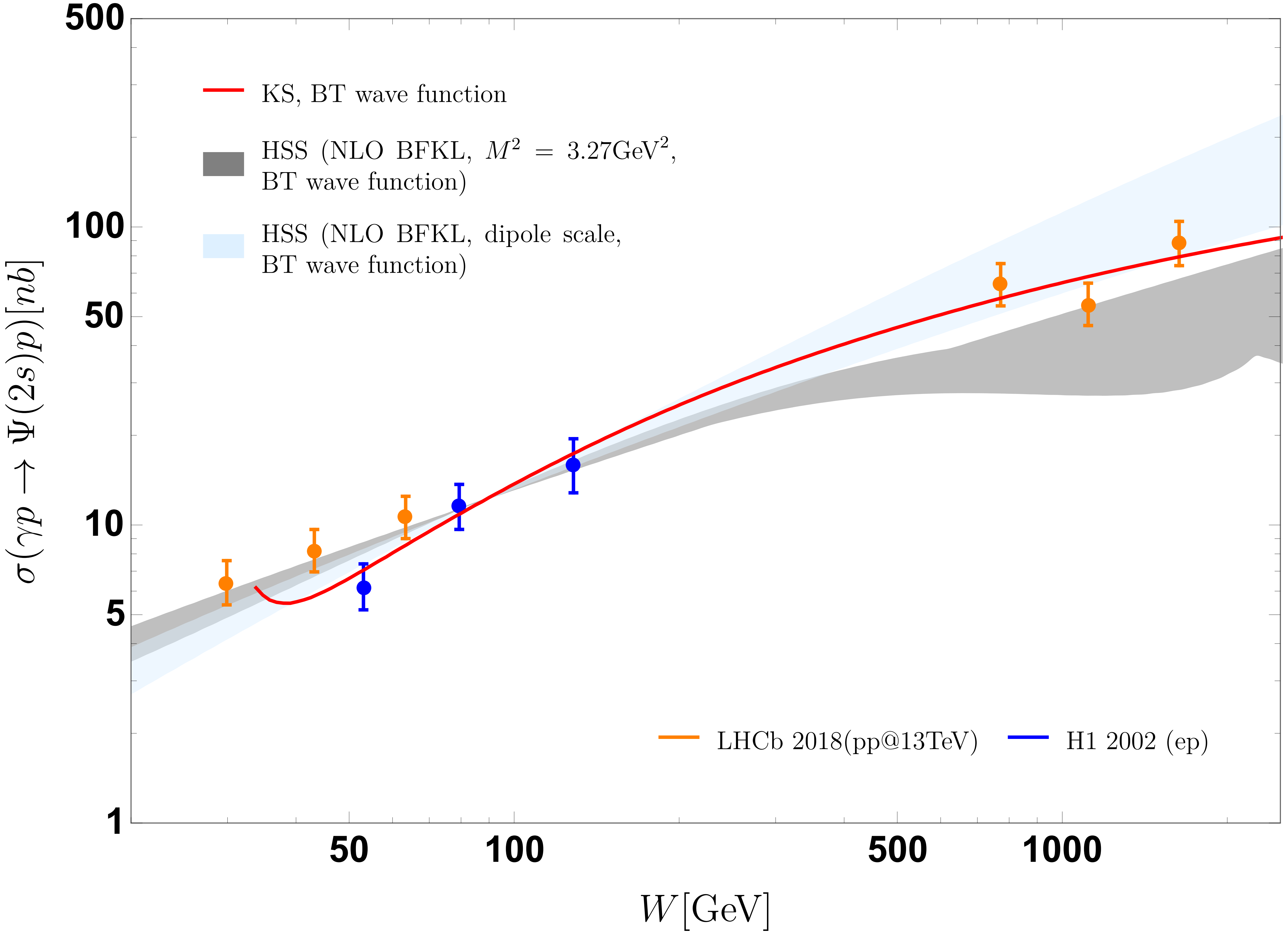
BUT:

- resulting growth too strong for J/Ψ production
- classical sign for onset of high density effects/transition towards saturated regime?

Next step:

- Refined wave function + include $\Psi(2s)$ + renormalization scale uncertainties
- Cannot really distinguish between linear vs nonlinear
- Note: normalization is fitted. [MH, E. Padron Molina, 2021]

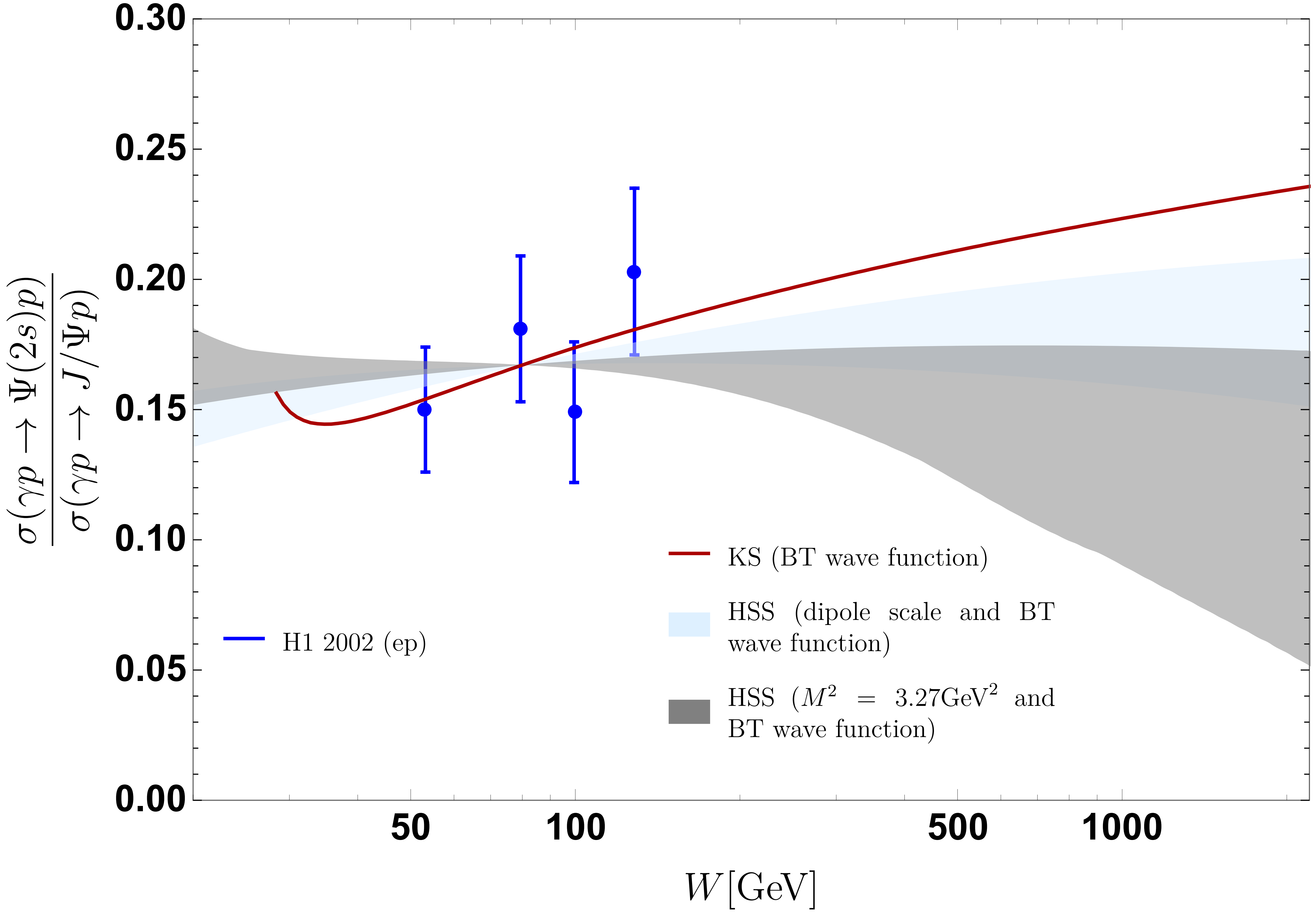




Shown: linear NLO BFKL (HSS) [MH, Salas, Sabio Vera; 1209.1353; 1301.5283] and non-linear BK (KS) [Kutak, Sapeta; 1205.5035]

Observation:

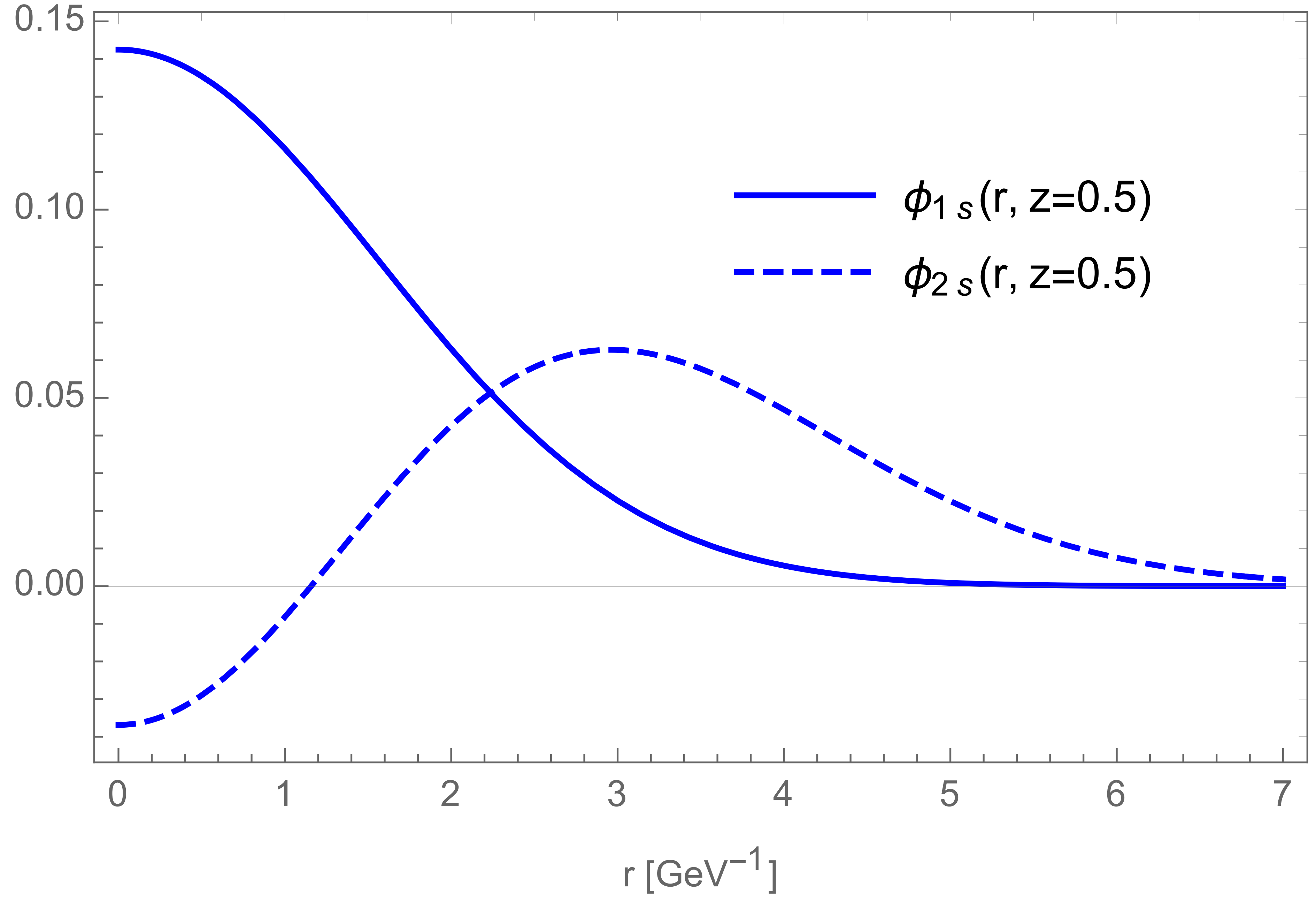
- very similar energy dependence predicted by linear and non-linear QCD evolution for total photo-production cross-section of J/Ψ and $\Psi(2s)$
- But differs for the ratio $\sigma(J/\Psi)/\sigma(\Psi(2s))$



- non-linear KS gluon (subject to BK evolution): growing ratio
- Linear HSS gluon (subject to NLO BFKL evolution): approximately constant ratio

- also: unstable fixed scale HSS gives decaying ratio: related to enhanced IR contribution for the $\Psi(2s)$

What causes the difference for $\Psi(2s)$ and J/Ψ?



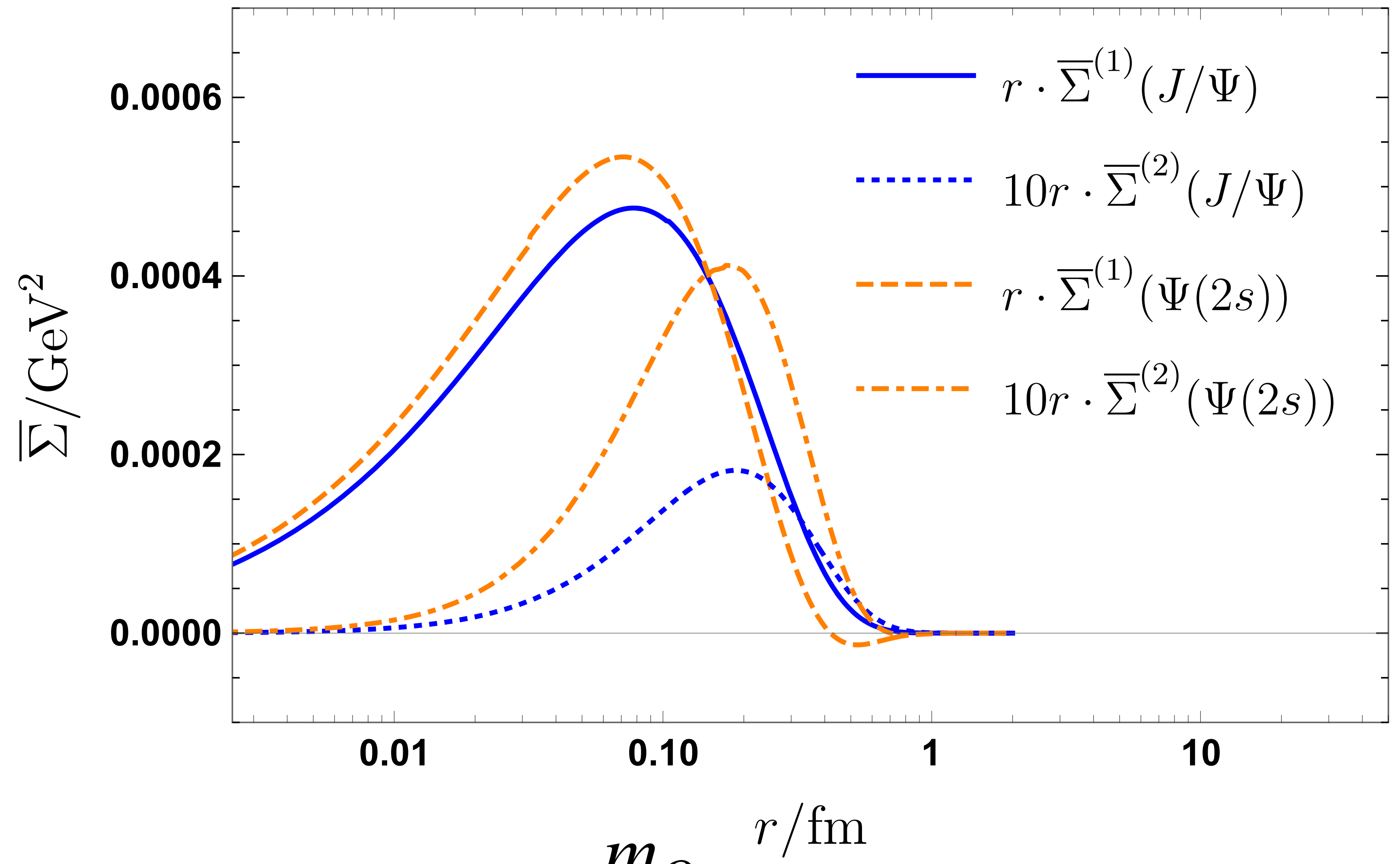
- Node of the 2s state
- Makes this state (somehow counter-intuitively) more perturbative (cancellation)
- Noted before [J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov V.R. Zoller; J. Exp. Theor. Phys. 86, 1054 (1998)] and [Cepila, Nemchik, Krelina, Pasechnik; 1901.02664]

Here:

- Gaussian model, next slide: numerical solution to Schrödinger equation etc.
- In common: position of node somehow constraint through charm mass

Wave function overlap for $\Psi(2s)$ and J/Ψ?

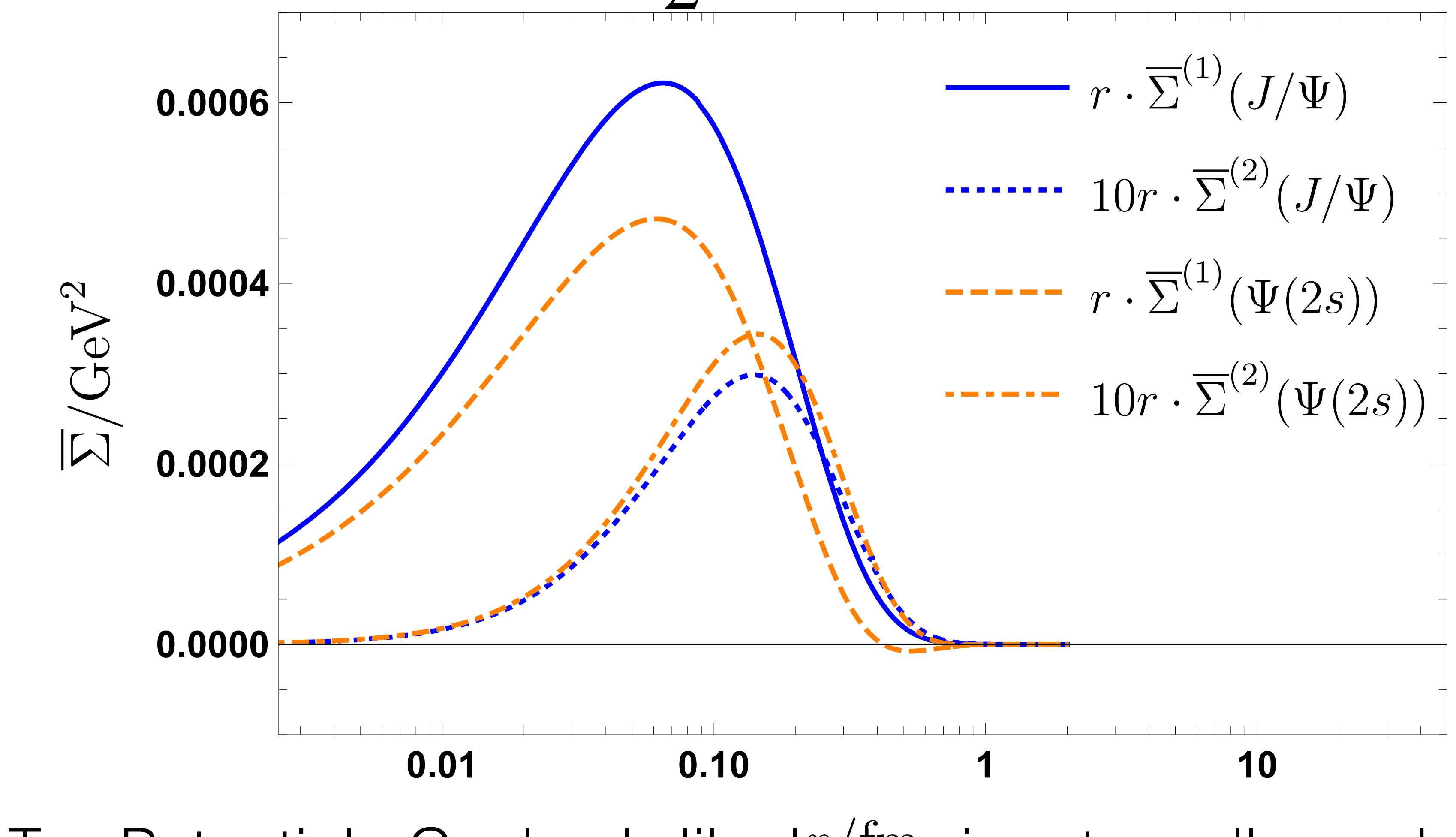
- Need to produce VM from photon
- Reduces size of node, but enhanced, once multiplied with dipole cross-section



harmonic oscillator (HO): $U(r) = \frac{m_Q}{2}\omega^2 r^2$

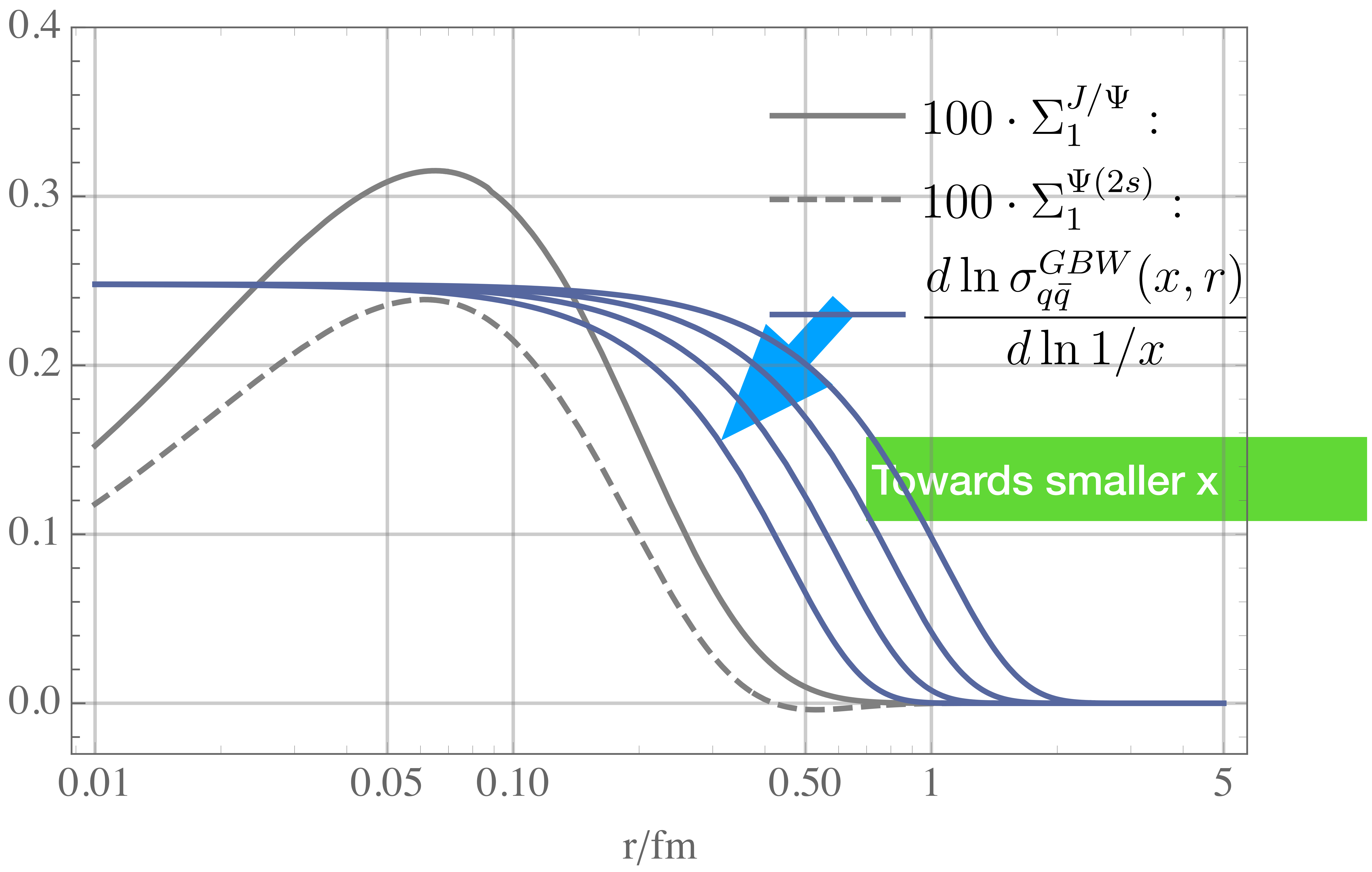
Here: use wave function overlap as provided by [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1812.03001; 1901.02664]

- includes relativistic spin rotation effects + (more) realistic $c\bar{c}$ potential
- Obtained from numerical solution to non-relativistic Schrödinger equation & boosted
- Also seen for simple boasted Gaussian

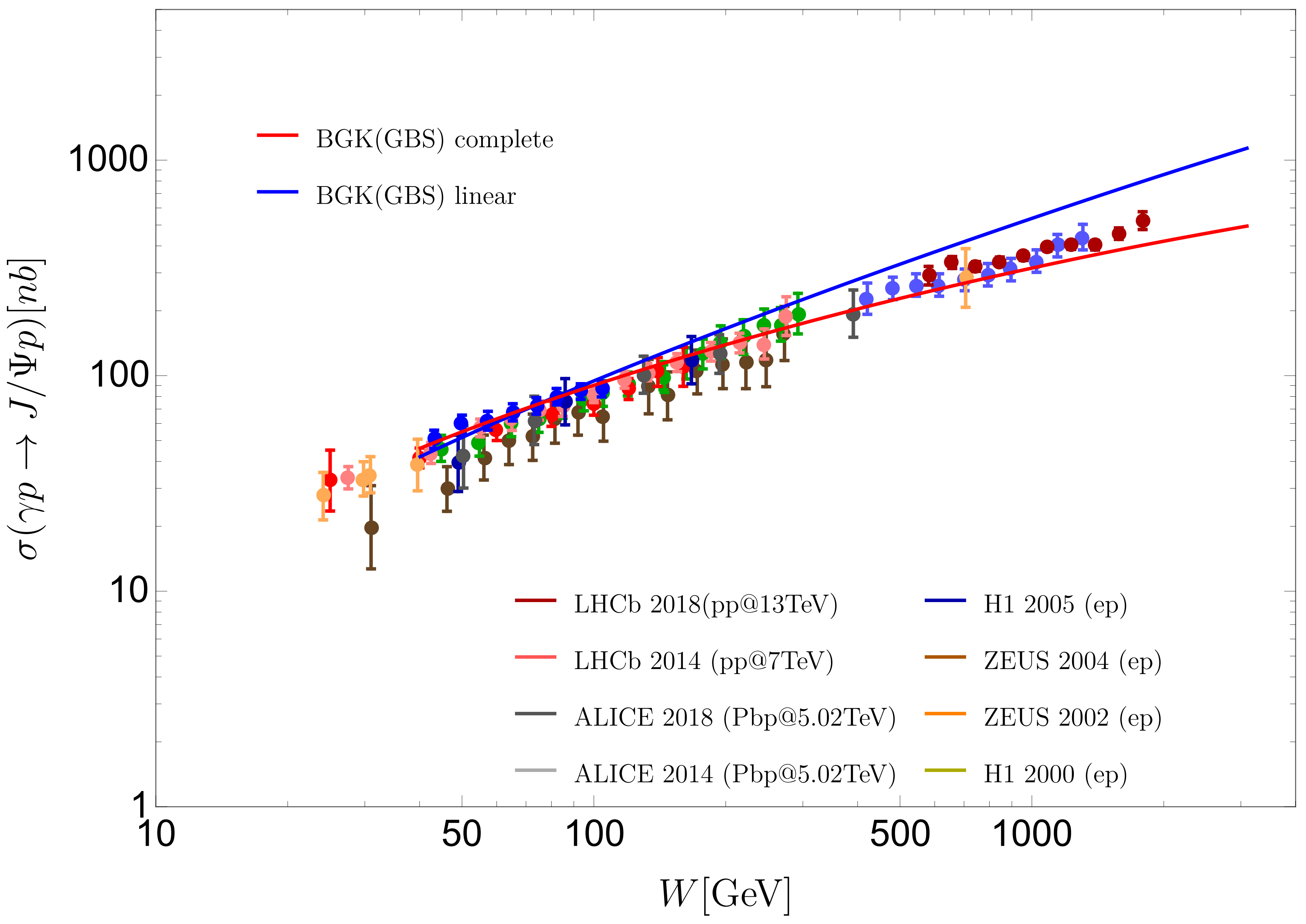


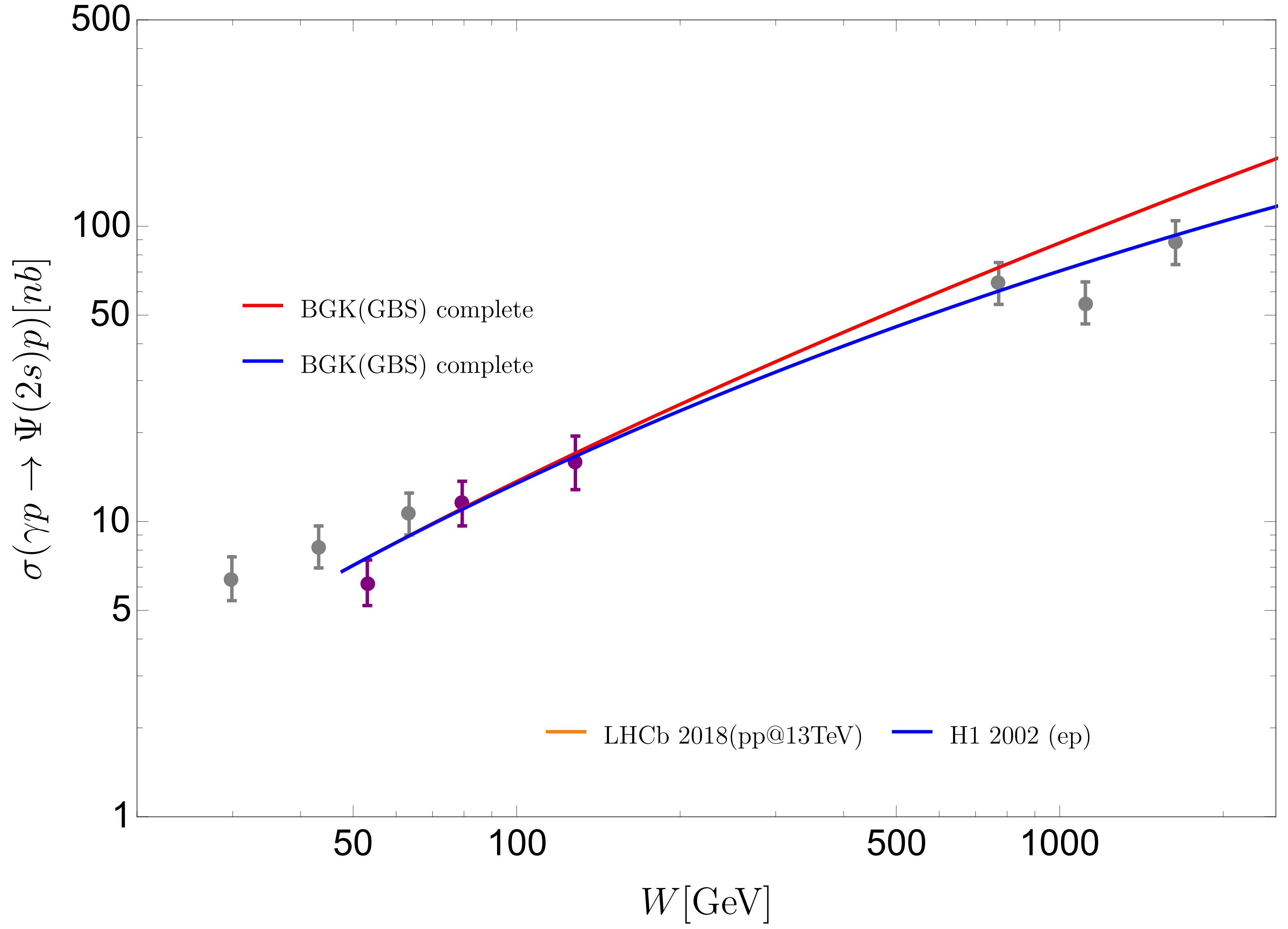
Buchmüller-Tye Potential: Coulomb-like behavior at small r and a string-like behavior at large r [Buchmüller, Tye; PRD24, 132 (1981)]

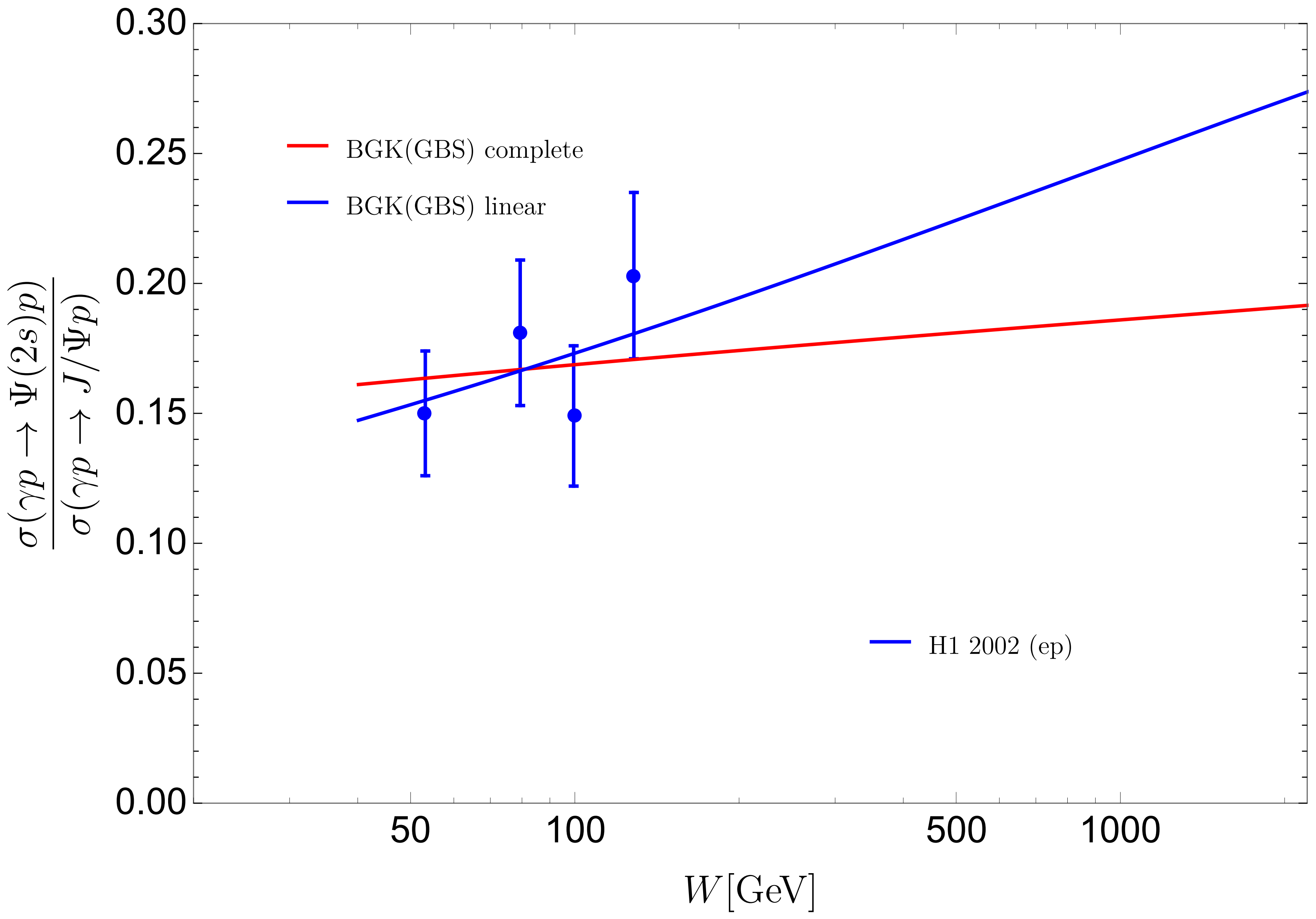
The role of the node for slope λ where $\sigma_{q\bar{q}} \sim x^{-\lambda}$



- small, but relevant where linear and non-linear differ
- Recall: slope of linear GBW = a line at 0.248

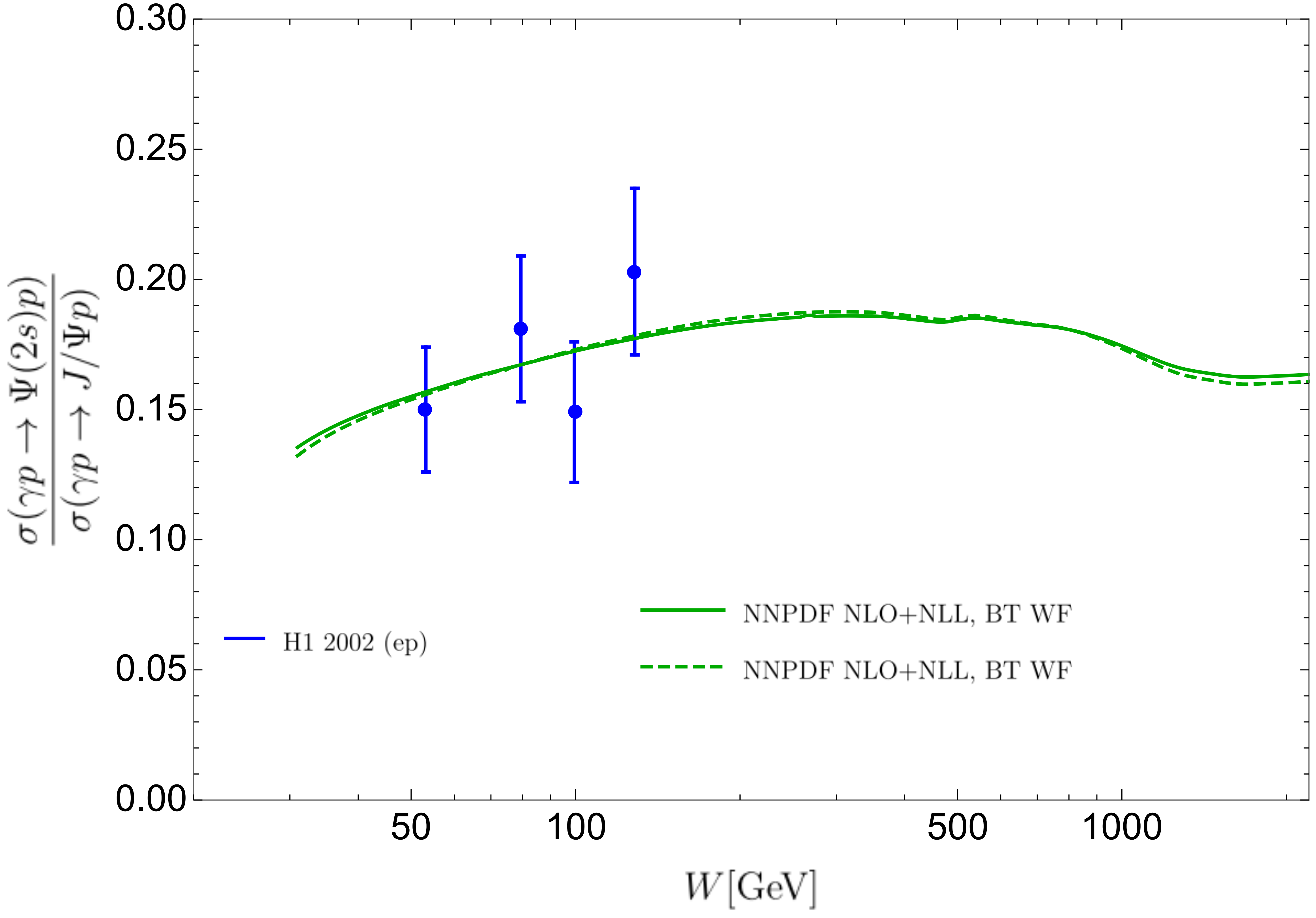






- DGLAP improved saturation model [Bartels, Golec-Biernat, Kowalski; hep-ph/0203258], fit [Golec-Biernat, Sapeta; 1711.11360] (GBW in backup)
- Complete vs. linearized version; issue: uncertainties ….
- Need for data (low energy to fix normalization, high energy to see which scenario is realized)

Perturbative dipole build on conventional PDF



- here: $\sigma_{q\bar{q}}^{lin}(x, r) = \frac{\alpha_s(\mu(r))\pi^2}{3} r^2 xg(x, \mu(r))$
- Use NNPDF NLO fit with NLO small x resummation
- Non-trivial energy dependence + does not really describe cross-section (within our framework, misses of course NLO corrections etc)
- Ratio of cross-sections is approximately constant with center of mass energy W

Conclusion

- Energy dependence of exclusive vector meson production (charmonium, bottomium) is a good place to investigate QCD high energy evolution
- Both learn about BFKL and to search for signs of non-linear effects
- There is a chance to learn something from the ratio of $\Psi(2s)$ and J/Ψ about the relevance of non-linear effects and/or the size of the saturation scale [study in progress]

Backup

linear low x evolution as benchmark →requires precision

(updated version desirable, work has started; not expected too soon)

use: HSS NLO BFKL fit [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

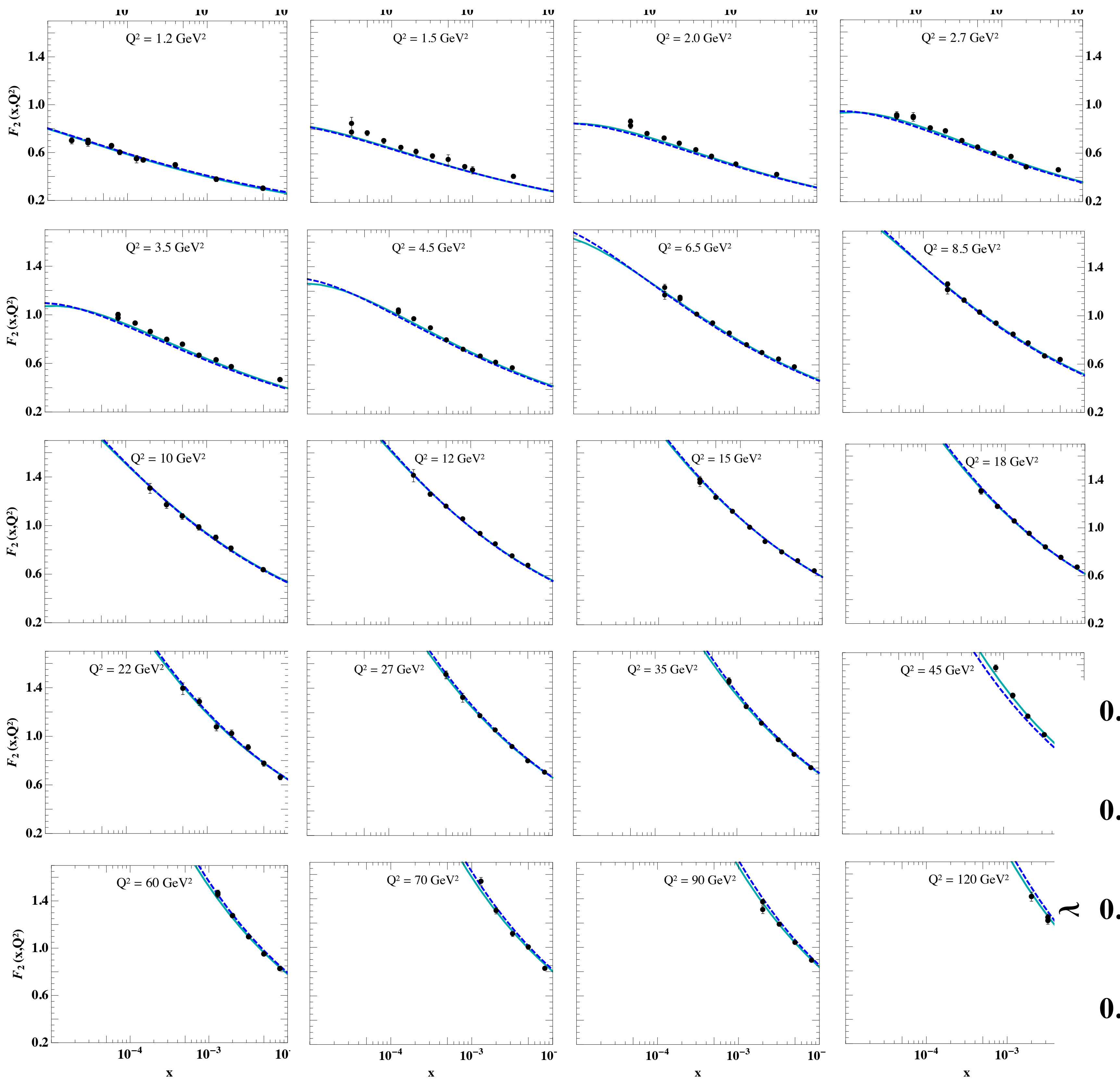
- uses NLO BFKL kernel

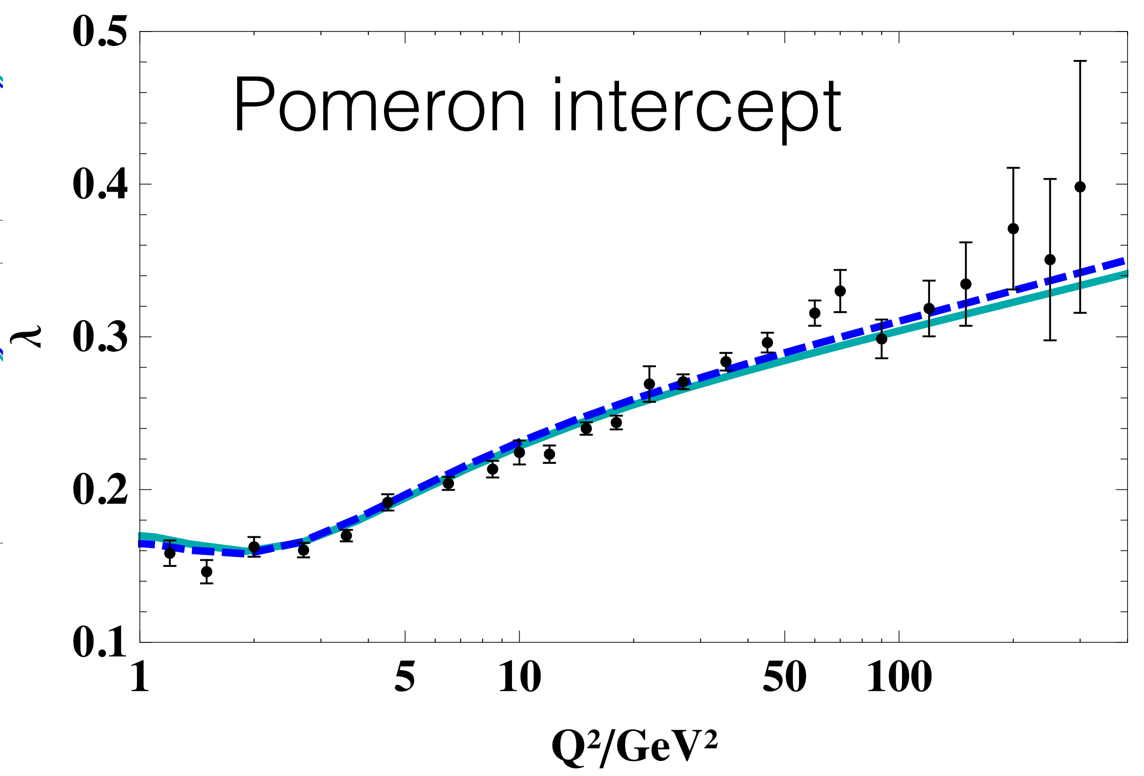
  [Fadin, Lipatov; PLB 429 (1998) 127]

  + resummation of collinear logarithms

- initial kT distribution from fit to combined HERA data

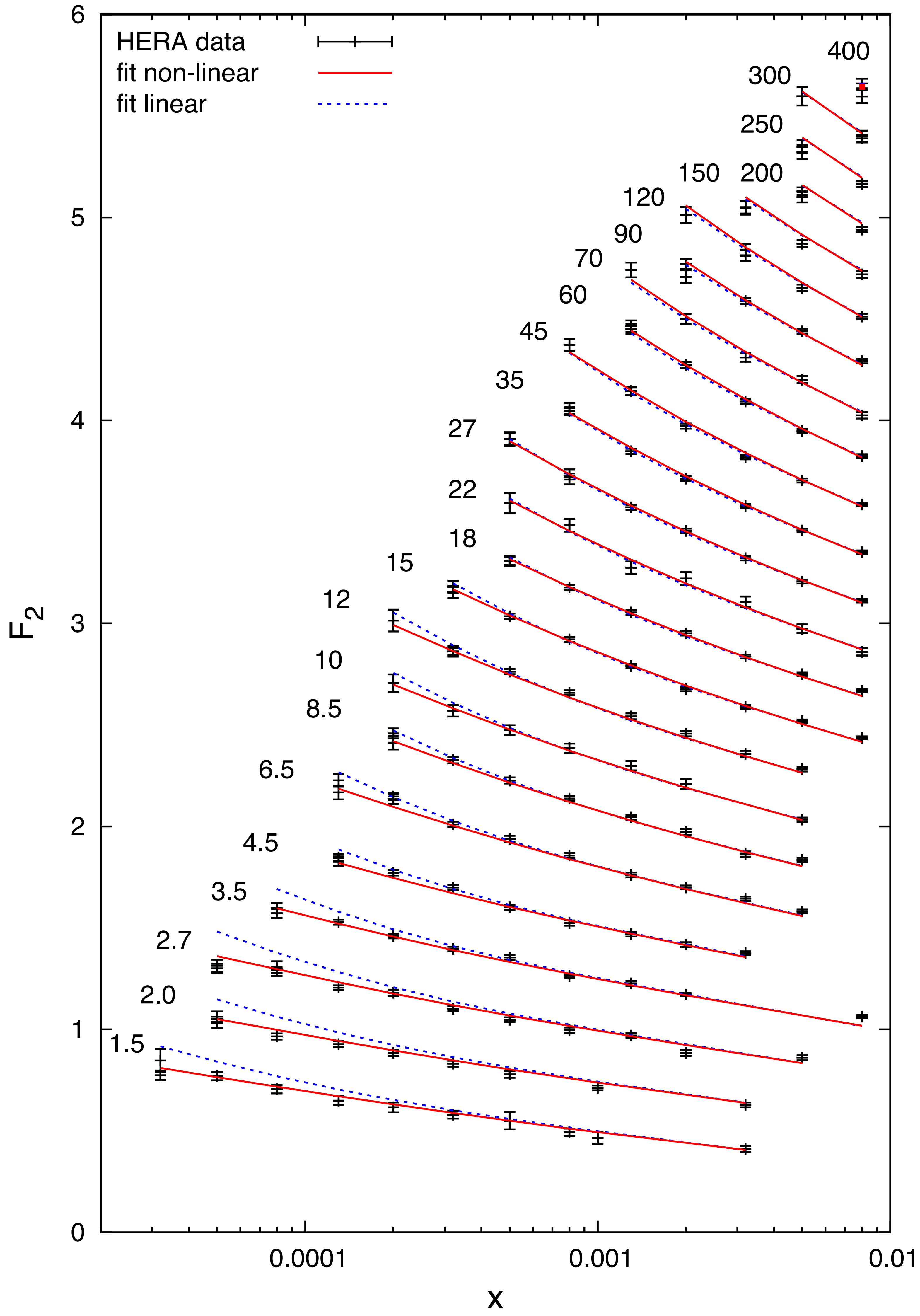
  [H1 & ZEUS collab. 0911.0884]





gluon with non-linear terms: KS gluon [Kutak, Sapeta; 1205.5035]

- based on unified (leading order) DGLAP+BFKL framework [Kwiecínski, Martin, Stasto, PRD 56(1997) 3991]
- combined with leading order BK evolution [Kutak, Kwiecinski;hep-ph/0303209][Kutak, Stasto; hep-ph/0408117]
- initial conditions: fit to combined HERA data [H1 & ZEUS collab. 0911.0884]
- both non-linear and linear version available (= non-linearity switched off)



how to compare to experiment?

(sort of standard procedure for comparing inclusive gluon to exclusive data)

a) analytic properties of scattering amplitude → real part

$$\mathcal{A}^{\gamma p\to Vp}(x,t=0) = \left(i + \tan\frac{\lambda(x)\pi}{2}\right)\cdot \Im\mathrm{m}\mathcal{A}^{\gamma p\to Vp}(x,t=0)$$

with intercept

$$\lambda(x) = \frac{d\ln \Im\mathrm{m}\mathcal{A}(x,t)}{d\ln 1/x}.$$

b) differential Xsection at t=0:

$$\frac{d\sigma}{dt}(\gamma p\to Vp)\bigg|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p\to Vp}(W^2,t=0)\right|^2$$

c) from experiment:

$$\frac{d\sigma}{dt}(\gamma p\to Vp) = e^{-B_D(W)\cdot|t|}\cdot\frac{d\sigma}{dt}(\gamma p\to Vp)\bigg|_{t=0}$$

$$\sigma^{\gamma p\to Vp}(W^2) = \frac{1}{B_D(W)}\frac{d\sigma}{dt}(\gamma p\to Vp)\bigg|_{t=0}$$

extracted from data

weak energy dependence from slope parameter

$$B_D(W) = \left[b_0 + 4\alpha'\ln\frac{W}{W_0}\right]\mathrm{GeV}^{-2}.$$

Why is this happening?

Very clear for the GBW model

GBW model: [Golec-Biernat, Wusthoff, hep-ph/9807513]

$$\sigma_{q\bar{q}}(x,r) = \sigma_0 \left(1 - \exp(-\frac{r^2 Q_s^2(x)}{4} \right) \text{ with saturation scale } Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0} \right)^{\lambda}$$

linearized version: $$\sigma_{q\bar{q}}^{lin.}(x,r) = \sigma_0 \frac{r^2 Q_s^2(x)}{4}$$

recent fit [Golec-Biernat, Sapeta, 1711.11360] to combined HERA data with $Q^2 \leq 10$GeV^2 and $\chi^2/N_{dof} = 352/219 = 1.61$

$\sigma_0[mb]$	λ	$x_0/10^{-4}$
27.43±0.35	0.248±0.002	0.40±0.04

The ratio for the GBW model

Cross-section:

$$\sigma^{\gamma p\to Vp}(W^2) = \frac{1}{B_D(W)}\frac{d\sigma}{dt}(\gamma p\to Vp)\Big|_{t=0}$$

And

$$\frac{d\sigma}{dt}(\gamma p\to Vp)\Big|_{t=0} = \frac{1}{16\pi}\left|\mathcal{A}^{\gamma p\to Vp}(W^2, t=0)\right|^2$$

$$\sigma_{q\bar{q}}^{GBW}(x,r) = \sigma_0\left(1-\exp(-\frac{r^2Q_s^2(x)}{4}\right)$$

From scattering amplitude:

$$\Im\mathrm{m}\mathcal{A}_T(W^2, t=0) = \int d^2\boldsymbol{r}\left[\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2}, r\right)\overline{\Sigma}_T^{(1)}(r) + \frac{d\sigma_{q\bar{q}}\left(\frac{M_V^2}{W^2}, r\right)}{dr}\overline{\Sigma}_T^{(2)}(r)\right]$$

Recall:

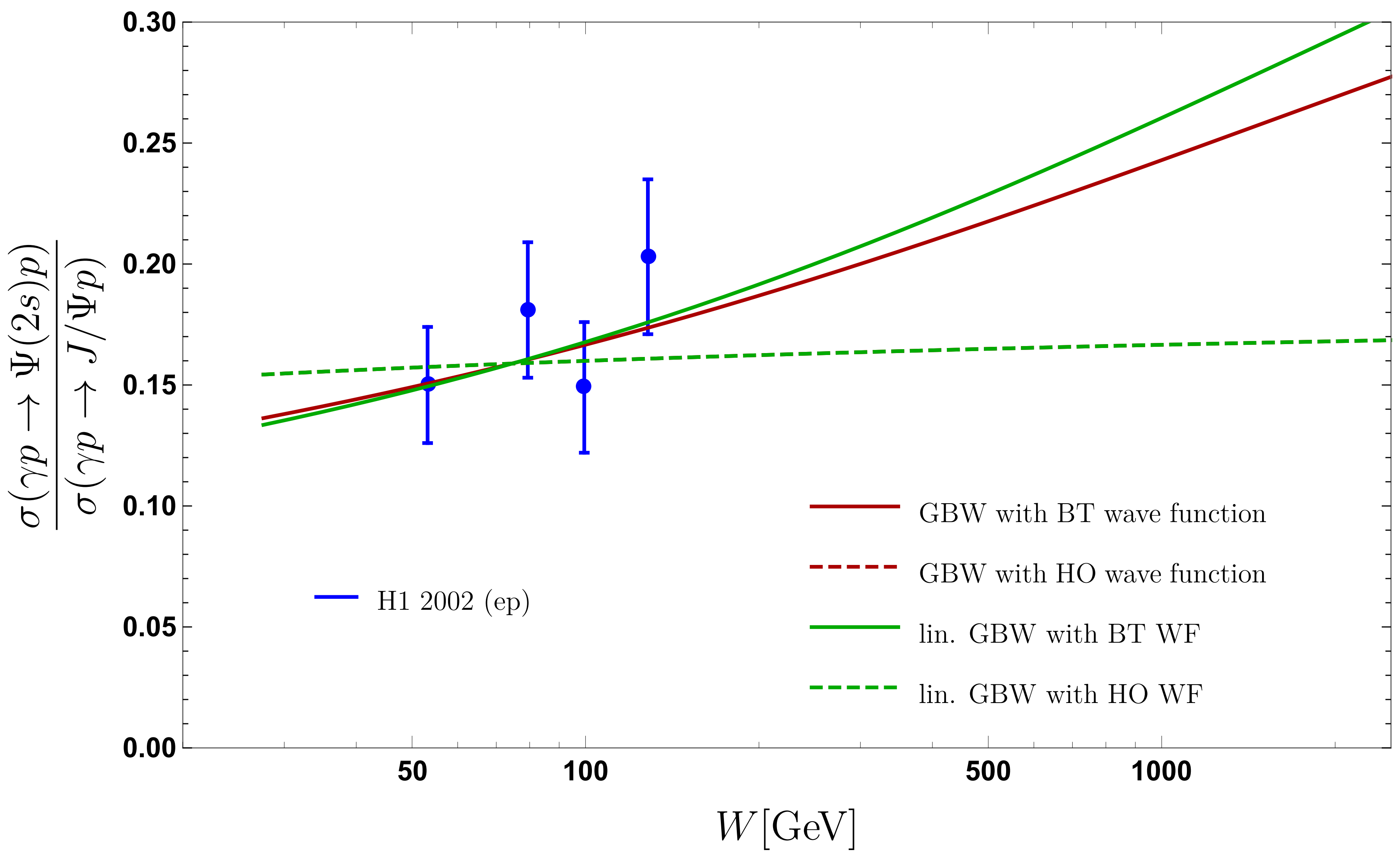
For **LINEAR** GBW

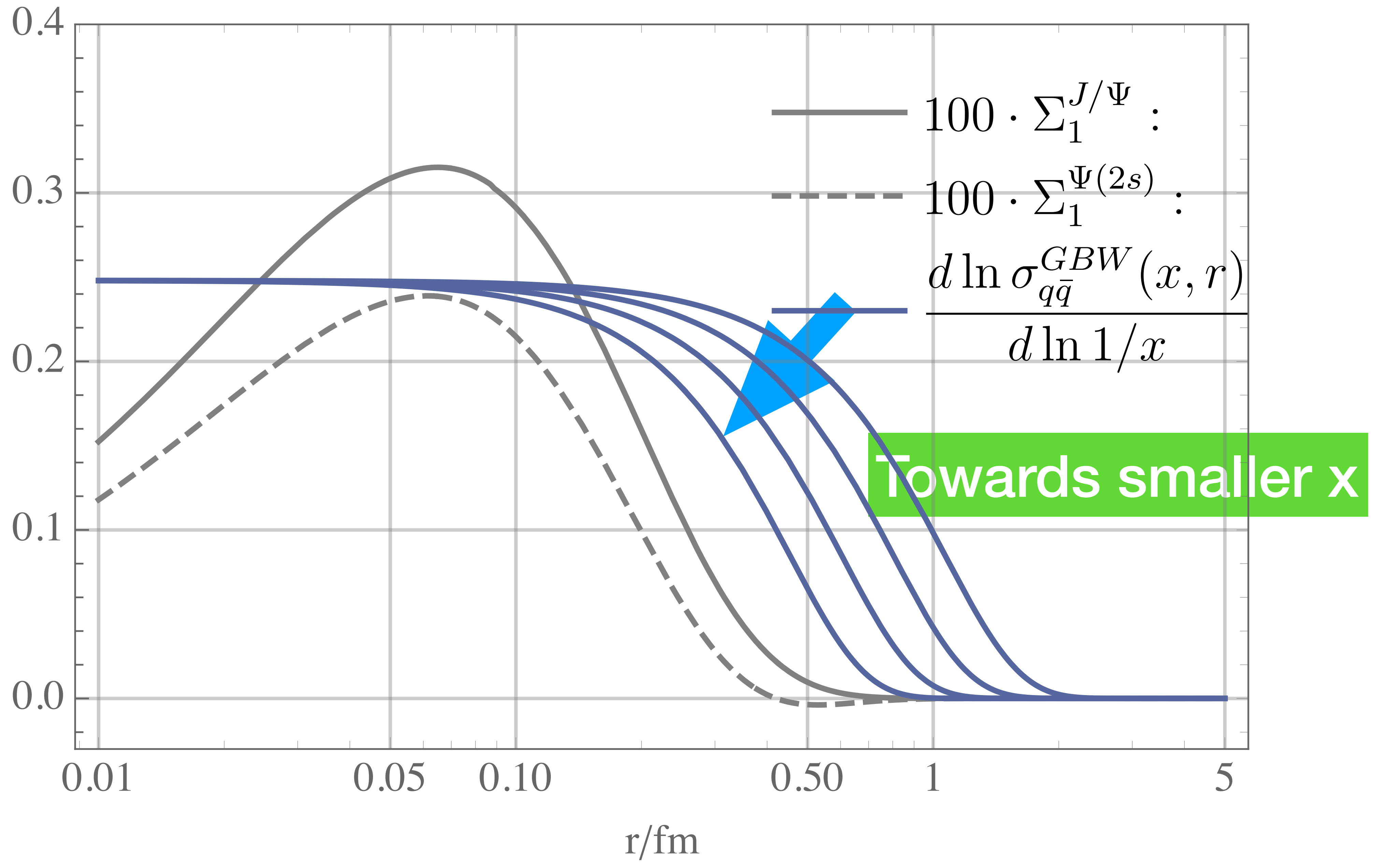
$$\Im\mathrm{m}\mathcal{A}^{lin.}(x) \sim Q_s^2(x)\cdot\int dr\ldots$$

- $Q_s(x) = Q_s(M_V^2/W^2)$ cancels for the ratio
- Ratio constant with energy for **linear GBW**

Complete GBW: non-trivial r-dependence → different energy dependence for different VM

The ratio: GBW model





r-dependence of the "slope" $\frac{d \ln \sigma_{q\bar{q}}}{\ln 1/x}$

- for linear model x-dependence in $Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0}\right)^\lambda$ we have $\frac{d \ln \sigma_{q\bar{q}}}{\ln 1/x} = \lambda$=const.
- Non-trivial r-dependence for complete GBW model → rise of the ratio

A less trivial model: The DGLAP improved saturation model

[Bartels, Golec-Biernat, Kowalski; hep-ph/0203258]

Essentially the GBW model with DGLAP evolution

$$\sigma_{\mathrm{dip}}(r,x) = \sigma_0 \left\{ 1 - \exp\left(-\frac{\pi^2 r^2 \, \alpha_s(\mu^2)\, xg(x,\mu^2)}{3\sigma_0} \right) \right\} \, ,$$

Factorization scale originally: $$\mu^2 = \frac{C}{r^2} + \mu_0^2 \, .$$

Recent fit:
[Golec-Biernat, Sapeta; 1711.11360]

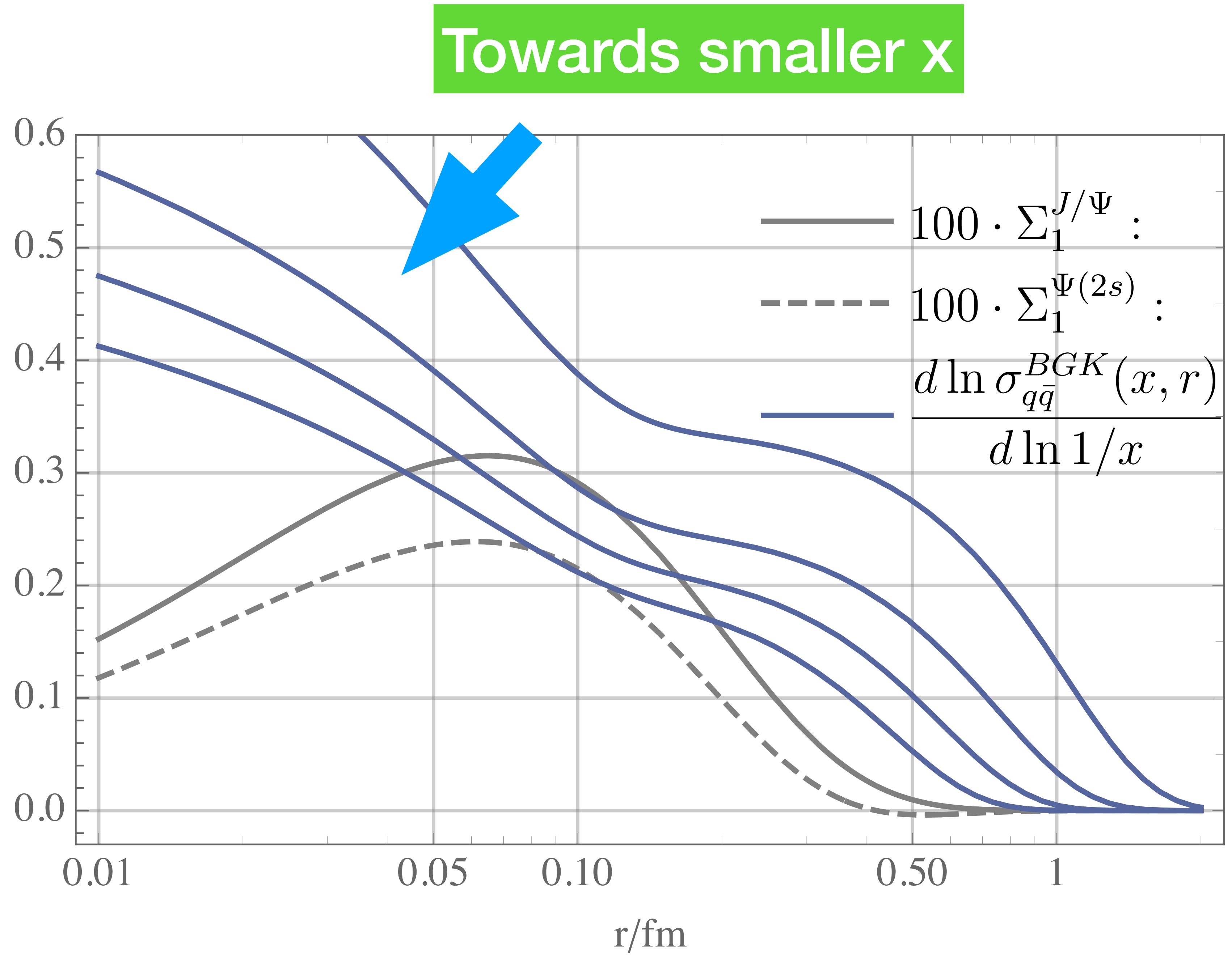
$$\mu^2 = \frac{\mu_0^2}{1 - \exp(-\mu_0^2\, r^2 / C)}$$

In common:
- for large dipole sizes r, $\mu \to \mu_0$
- Otherwise $\sim C/r^2$

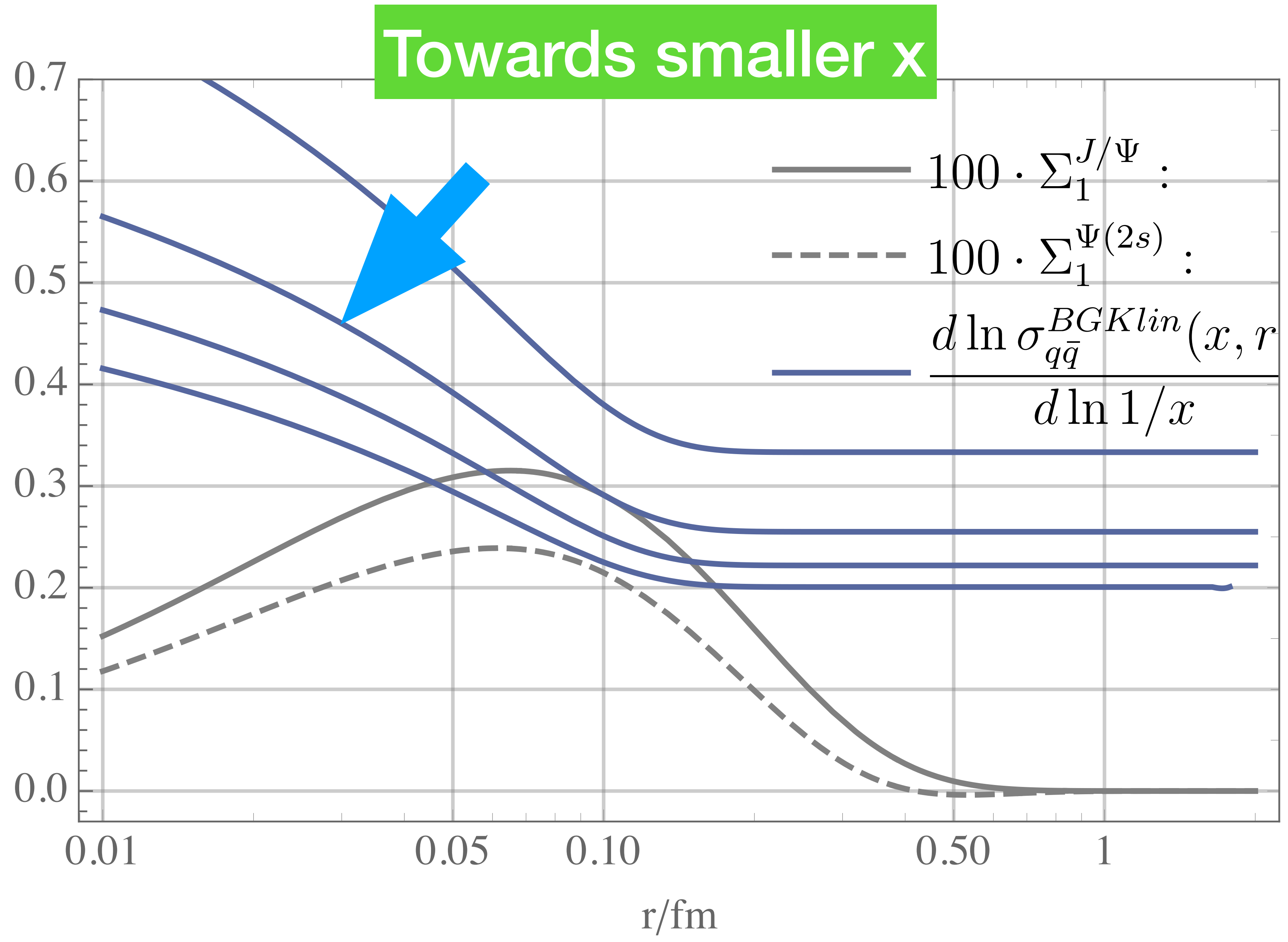
Saturation scale becomes r-dependent → includes correct DGLAP limit for small r

Complementary to BFKL/BK study

Discussion



“Slope" for complete BGK



"Slope" for linear BGK

$$\lambda = \frac{d \ln \sigma_{q\bar{q}}}{\ln 1/x}$$

- Difference between J/Ψ and $\Psi(2s)$ at relative large dipole size r
- Full non-linear model: non-trivial x-dependence in this region
- Linear model with factorization scale frozen at large dipole size r, there is not much happening → constant ratio
- Trivial for GBW model; also seen for BFKL vs BK (QCD low x evolution)
- Prediction depends on VM wave function = the position of the node