

BFKL Pomeron, high gluon densities and their imprint in exclusive vector meson production
Martin Hentschinski

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- MH, E. Padron Molina, *Phys.Rev.D* 103 (2021) 7, 074008 arXiv:2011.02640
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based on:

(An half an hour lecture)

What is special about BFKL? What is it?

1975-1978: Baltisky, Fadin, Kuraev, Lipatov: study of QCD scattering amplitudes in the limit of high center of mass energies \sqrt{s} , $s \gg -t$, m_i

Actually: $SU(2)$ gauge theory + Higgs mechanism (infrared regularization)

Methodology:

- perturbation theory → studied scattering of gluons (and also quarks)
- sum up all terms in the perturbative series $(\alpha_s \ln s)$ " i.e. rearrange perturbative series *n*

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Here:

- $\omega(t, \epsilon)$ = gluon Regge trajectory; right now know up to 2 loop
- $-$ It is IR divergent (regulator ϵ)
- Reggeization of the gluon

Result No 1: Elastic scattering amplitude

$$
\mathcal{A}_{gg \to gg}(s,t) = \mathcal{A}_{gg \to gg}^{(0)}(s,t) \cdot s^{\omega(t,\epsilon)}
$$

Extension to multi-particle production

In Multi-Regge-Kinematics s ≫ s_i ≫ $-t_i$, m_j

$$
\mathscr{A}_{gg\to ng} = \mathscr{A}_{gg\to ng}^{(0)} \cdot s_1^{\omega(t_1,\epsilon)} \cdot s_2^{\omega(t_2,\epsilon)} \cdots s_n^{\omega(t_n,\epsilon)}
$$

Result:

Production through gauge invariant Lipatov vertex *C* : *^μ*

$$
\mathcal{A}_{gg \to ng} = s\Gamma(q_1) \frac{1}{q_1^2} C_\mu(q_1, q_2, k_1) \epsilon^\mu(k_1) \frac{1}{q_2^2}
$$

- not only a correction to external legs (collinear radiation)
- Not only a soft correction
- Need to "break up" scattering amplitudes for such resummation \rightarrow deal with internal off-shell states ("reggeized gluons")

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- confirmed by exact calculations (*e.g*. anomalous DGLAP dimension to 3-loop etc., N=4 SYM amplitudes etc., exact QCD scattering amplitudes)
- Reveals beautiful mathematical structure (conformal symmetry, integrability) in certain setups

Phenomenology

Observe cross-sections, not amplitudes

n

$$
d\sigma = \sum |\mathcal{A}_{2\to n}|^2 d\Phi^{(n)}
$$

- Yields perturbative, hard, or BFKL Pomeron
- Predicts in principle a power-like rise of the total cross-section *σ* ∼ *s^λ*
- In general more complicated:

Pomeron = t-channel exchange with quantum numbers of the vacuum; responsible for the rise of the total QCD cross-section

Phenomenology

Observe cross-sections, not amplitudes

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$\Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b) \Phi_B(\mathbf{k}_b, Q_B)$

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- In general more complicated:

$$
\sigma_{AB}(s, Q_A, Q_B) = \int \frac{d^2 \mathbf{k}_a}{\pi} \int \frac{d^2 \mathbf{k}_b}{\pi}
$$

 f_{BFK} (ln *s*, \mathbf{k}_a , \mathbf{k}_b) universal BFKL Green's function

 $\Phi_I({\bf k}, {\cal Q})$: impact factors = describe coupling of BFKL Green's function to external scattering particles

Pomeron = t-channel exchange with quantum numbers of the vacuum; responsible for the rise of the total QCD cross-section

(potential) Issues with this expression

π ∫ $d^2\mathbf{k}_b$ *π* $\Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b) \Phi_B(\mathbf{k}_b, Q_B)$

- $β_0 \ln(\mu^2/\mathbf{k}^2)$
-

 $\alpha_s s^{2\alpha_s \omega_0} \gg s^{\alpha_s \omega_0}$ possible etc.

$$
\sigma_{AB}(s, Q_A, Q_B) = \int \frac{d^2 \mathbf{k}_a}{\pi}
$$

- expression derived in perturbation theory \rightarrow need some hard scale $\mathcal{Q}_a, \mathcal{Q}_b \gg \Lambda_{QCD}$
- expression derived in perturbation theory \rightarrow small $\alpha_{s}(\mu)$, yet integrated over all transverse momenta

 \rightarrow not necessarily a problem (do the same in loop calculations, but $\beta_0 \ln(\mu^2/\mathbf{k}^2)$ can lead to complications with Landau pole of running coupling etc.; Appears at NLO …

- expression derived in perturbation theory \rightarrow it's the dominant term at any order in perturbation theory; not necessarily true, once summed up

→ diffusion in transverse momentum ("Bartels's cigar")

BFKL Pomeron in conjugate Mellin space

 $\sigma_{AB}(s, Q_A, Q_B) =$ $d^2\mathbf{k}_a$ *π* ∫ d^2 **k**_{*b*} *π*

> Similar to moments for DGLAP evolution, Fourier transform: convolutions in transverse momenta turn into products for conjugate Mellin space

$$
\sigma_{AB}(s, Q_A, Q_B) = \int_{1/2 - i\infty}^{1/2 + i\infty} \frac{1}{Q_A} \frac{2}{2\pi i} \left(\frac{Q_A^2}{Q_B^2} \right)^{\gamma} \Phi_A(\gamma) \Phi_B(\gamma) s^{\chi(\gamma)} (1 + \alpha_s^2 \ln(s) f(\gamma) + ...)
$$

With $\chi(\gamma) = \frac{\alpha_s N_c}{\pi} \chi_0(\gamma) + \left(\frac{\alpha_s N_c}{\pi} \right)^2 \chi_1(\gamma) + ...$ BFKL eigenvalue

$$
\int_{-\infty}^{\infty} \frac{1}{Q_A} \frac{2}{2\pi i} \left(\frac{Q_A^2}{Q_B^2} \right)^{\gamma} \Phi_A(\gamma) \Phi_B(\gamma) s^{\chi(\gamma)} (1 + \alpha_s^2 \ln(s) f(\gamma) + ...)
$$

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$\Phi_A(\mathbf{k}_a, Q_A) f_{\text{BFKL}}(\ln s, \mathbf{k}_a, \mathbf{k}_b) \Phi_B(\mathbf{k}_b, Q_B)$

Hard vs. Soft Pomeron

Approximate solution (saddle point approximation $\lim_{\alpha_s} \sin s \to \infty, \quad \bar{\alpha}_s = \frac{\sin s}{\pi}$: $\alpha_{s}N_{c}$ *π*

- Idea: existence of 2 Pomerons s^{λ} (soft with $\lambda \simeq 0.1$ and hard with $\lambda \simeq 0.5$ - Hard Pomeron in above approximation problematic:
	- Intercept is very large
	- HERA: intercept increases with hard scale; seems to indicate the opposite
	- BFKL wrong?

$\sigma_{AB} \sim s^{\bar{\alpha}_s 2.77259} \simeq s^{0.52}, \quad \alpha_s = 0.2$

Complete description:

Data: [H1 & ZEUS collab. 0911.0884] Theory: [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

- Description uses complete Mellin integral + NLO corrections + collinear resummation of NLO BFKL + BLM scale setting for running coupling
- Tendency even there for LO BFKL with
fixed coupling

$$
\lambda(Q^2) = \left\langle \frac{d \ln F_2(x, Q^2)}{d \ln 1/x} \right\rangle_x
$$

Note: this is expect: BFKL and DGLAP agree in the double log approximation

Effective Pomeron intercept in DIS $x = Q^2/s$

Unitarity & the BFKL Pomeron

- Non-perturbative Froissart theorem: total QCD cross-section grows asymptotically at most as $\sigma_{tot} \leq c_0 \ln^2 s$
- Derived from unitarity (and finite range of strong interactions?)

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Quite ironically, the original BFKL deviation uses heavily unitarity as well - But naturally $\ln(-s) = \ln(s) - i\pi \simeq \ln(s)$ etc

-
- Keeping track of $i\pi$'s reveals other terms which belong to multiple reggeized gluon exchange (Pomeron = "bound state" of 2 reggeized gluons);
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 $c_{i>1}$ subleasing in α_s , but lead to unitarization of the result

How can this help? Schematically

With $z = s^{\lambda}$, multiple (Pomeron) exchange can yield something like $z = s^{\lambda}$, $\sigma \sim c_1 z - c_2 z^2 + c_3 z^3 - c_4 z^4 + \dots$

$$
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$$

An illustrative example: dipole models

Lipatov (some DESY seminar 2009): "Exponential is a very nice function but it is not always the correct function"

Perturbative result: power-like growth of cross-section $\sigma_{q\bar{q}}^{lin.} = \sigma_0 r^2 Q_0^2 x^{-\lambda} = \sigma_0$

Cross-section of a color dipole (quark -antiquark pair with transverse separation r in configuration space)

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Unitarized version: $\sigma_{q\bar{q}} = \sigma_0 \left(1 - e^{-t}\right)$

Cross-section of a color dipole (quark -antiquark pair with transverse separation r in configuration space)

> Exponential (=eikonal) correct in QED, most likely not in QCD \rightarrow a model (here GBW model)

$$
-r^{2}Q_{s}^{2}(x)/4\bigg) = \sigma_{0}\sum_{k}\frac{(-1)^{k+1}}{k!}\left(\frac{r^{2}Q_{s}^{2}(x)}{4}\right)^{k}
$$

$$
\frac{v^*}{\sqrt{2}} = \frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \right)^{-\lambda} = \frac{1}{2} \left(\frac{1}{2} \frac{1}{2} \right)^{-\lambda} = \frac{1}{2} \left(\frac{1}{2} \right)^{-\lambda} = \frac{1}{2} \
$$

[Golec-Biernat, Wüsthoff, 1998-1999]

[related to gluon distribution]

resulting *x*-dependence of the underlying gluon distribution. While this initial *x*-distribution Complete picture: non-linear QCD evolution

BK evolution for dipole amplitude *N(x,r)*∈ [0,1] HSS gluon provides a very good description of both ⌥ and *J/* photo-production data,

> $complete$ \sum \sum \sum \sum linear BFKL evolution = subset of complete BK

Derivation: assumes presence of strong color field $A^+ \sim 1/g + 1/g$ use of renormalization group wrt. Rapidity cut-off

 $X = \ln 1/x$ $Y = \ln 1/x$

 $Grihov$ *2 avin Ruskin* **1083** in the current case of the current to account that DGC evolution is that DGC evolution is that DGC evolution is the current of the c large *x* international ability of the mere ability of DGLAP fits the mere ability of DGClare ability of DGClare fits to \mathbf{x} in the mere ability of DGClare fits to \mathbf{x} above to \mathbf{x} above to \mathbf{x} above to \math accommodate low *x J/ photo-production data, tool*
I lalilian-Marian lancul McLerran Weigert Leonidov Konvex sizable non-linear e↵ects for the data points at highest *W*-values. 1996-2000][Gribov, Levin, Ryskin; 1983] [McLerran, Venugopalan; 1993] [Balitsky, 1995], [Kovchegov, 1997] [Jalilian-Marian,Iancu, McLerran, Weigert, Leonidov, Konvex,

$$
= \int d^2 \bm{r}_1 K(\bm{r}, \bm{r}_1) \bigg[N(x, r_1) + N(x, r_2) - N(x, r) \bigg] - \bigg[N(x, r_1) N(x, r_2) \bigg]
$$

non-linear term relevant for N~1 (=high density)

 $X \vdash \mathsf{In}$ 1/x

 $= \ln 1/x$

Complete picture: non-linear QCD evolution

 $d^2\bm{r}_1K(\bm{r},\bm{r}_1)\left[N(x,r_1)+N(x,r_2)-N(x,r)-N(x,r_1)N(x,r_2)\right]$

- Non-linear term: bring growth to hold $(N = 1)$ is solution) - linear terms (LO BFKL): power-like growth solution)
- $\overline{\text{erized}}$ by saturation scale $\mathcal{Q}_{\mathcal{S}}(x)$, $\overline{\text{g}}$ - Transition between linear & non-linear regime characterized by saturation scale $Q_{s}(x)$, growing with

How to provide evidence for such physics?

- Observables with 1 hard scale $M \rightarrow$ construct dimensionless function which scales with saturation scale Can search for such scaling pattern *e.g.* [Praszalowicz, Stebel, 2013] $f(M^2, x) = g(M^2/Q_s^2(x))$
- Imprints of the saturation scale in transverse momentum spectra (*e.g.* decorrelation of back-to-back dijets/dihadrons)
- Investigate dependence of cross-sections on center-of-mass energy Serves both as further tests of BFKL evolution +search for deviations from BFKL at highest center of mass energies

photo induced exclusive photo-production of J/*Ψs* and $\Psi(2s)$

- hard scale: charm mass (small, but perturbative)
- reach up to x≳.5・10-6
- perturbative cross-check: ϒ (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)
- Enormous range in center of-mass energies

$I - 2C$ Important: not a contest with DGLAP evolution - ask different questions

ahout saturation (easily overseen) and BFKL (titted) and the second of the second statement of the \sim → constrain pdfs, but don't learn about saturation (easily overseen) and BFKL (fitted)

 $I/NU(2A)C_0/21D Y(22A)C_0/21$ $\sqrt{2}$ (i.e. $\sqrt{2}$ studies highly value $\sqrt{2}$ at $\sqrt{2}$ at $\sqrt{2}$ at $\sqrt{2}$ at $\sqrt{2}$ at $\sqrt{2}$ at $\sqrt{2}$

DGLAP:

- fit x-dependence + evolve from J/Ψ (2.4 GeV²) to Y (22.4 GeV²)
- DGLAP shifts large x input (low scales) to low x (high scales) + higher twist dies away fast in evolution

What did we find so far?

Can BFKL fit [MH, Salas, Sabio Vera; 1209.1353; 1301.5283] describe *J*/Ψ and Υ data? YES. [Bautista, Fernando Tellez, MH; 1607.05203]

At highest W , BFKL fit unstable (NLO>LO)

BUT:

- resulting growth too strong for *J*/Ψproduction
- classical sign for onset of high density effects/transition towards saturated regime?

Next step:

- Cannot really distinguish between linear vs nonlinear - Note: normalization is fitted. [MH, E. Padron Molina, 2021]

- - uncertainties
-
-

- Refined wave function + include $\Psi(2s)$ + renormalization scale

Observation:

- very similar energy dependence predicted by linear and non-linear QCD evolution for total photo-production cross-section of *J*/Ψ and $\Psi(2s)$ - But differs for the ratio *σ*(*J*/Ψ)/*σ*(Ψ(2*s*))

- non-linear KS gluon (subject to BK evolution): growing ratio
- Linear HSS gluon (subject to NLO BFKL evolution): approximately constant ratio
- also: unstable fixed scale HSS gives decaying ratio: related to enhanced IR contribution for the $\Psi(2s)$

What causes the difference for Ψ(2*s*) and *J*/Ψ?

- Node of the 2s state - Makes this state (somehow counter-intuitively) more perturbative (cancellation) - Noted before [J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov V.R. Zoller; J. Exp. Theor. Phys. 86, 1054 (1998)] and [Cepila, Nemchik, Krelina, Pasechnik; 1901.02664]

Here:

- Gaussian model, next slide: numerical solution to Schrödinger equation etc.
- In common: position of node somehow constraint through charm mass

Wave function overlap for Ψ(2*s*) and *J*/Ψ?

23

- Need to produce VM from photon
- Reduces size of node, but enhanced, once multiplied with dipole cross-section

Buchmüller-Tye Potential: Coulomb-like béfravior at small r and a string-like behavior at large r [Buchmüller, Tye; PRD24, 132 (1981)]

Here: use wave function overlap as provided by [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](https://arxiv.org/abs/1812.03001); [1901.02664\]](https://arxiv.org/abs/1901.02664)

- includes relativistic spin rotation effects + (more) realistic $c\bar{c}$ potential
- •Obtained from numerical solution to nonrelativistic Schrödinger equation & boosted
- Also seen for simple boasted Gaussian

The role of the node for slope λ where $\sigma_{q\bar{q}} \sim x^{-\lambda}$

- small, but relevant where linear and non-linear differ
- Recall: slope of linear GBW $=$ a line at 0.248

-
-
-

Perturbative dipole build on conventional PDF

- Use NNPDF NLO fit with NLO small x resummation
- Non-trivial energy dependence + does not really describe cross-section (within our framework, misses of course NLO corrections etc)
- Ratio of cross-sections is approximately

$$
\text{here: } \sigma_{q\bar{q}}^{lin}(x, r) = \frac{\alpha_s(\mu(r))\pi^2}{3} r^2 x g(x, \mu(r))
$$

Conclusion

- Energy dependence of exclusive vector meson production (charmonium, bottomium) is a good place to investigate QCD high energy evolution
- Both learn about BFKL and to search for signs of non-linear effects
- There is a chance to learn something from the ratio of $\Psi(2s)$ and J/Ψ about the relevance of non-linear effects and/or the size of the saturation scale [study in progress]

Backup

0.2

0.6

1.0

 F_2 (**x**, Q^2)

1.4

0.2

0.6

1.0

 F_2 (**x**, Q^2)

1.4

0.2

0.6

1.0

 F_2 (**x**, Q^2)

1.4

0.2

0.6

1.0

 F_2 (**x**, Q^2)

linear low x evolution as benchmark \rightarrow requires precision (updated version desirable, work has started; not expected too soon)

use: HSS NLO BFKL fit

1.4

• initial kT distribution from fit to combined HERA data

0.6

1.0

 F_2 (**x**, Q^2)

1.4

• uses NLO BFKL kernel [Fadin, Lipatov; PLB 429 (1998) 127] + resummation of collinear logarithms

[MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

- based on unified (leading order) DGLAP+BFKL framework [Kwiecínski, Martin, Stasto, PRD 56(1997) 3991]
- combined with leading order BK evolution [Kutak, Kwiecinski;hep-ph/0303209][Kutak, Stasto; hep-ph/0408117]
- initial conditions: fit to combined HERA data [H1 & ZEUS collab. 0911.0884]
- both non-linear and linear version available (= non-linearity switched off)

gluon with non-linear terms: KS gluon [Kutak, Sapeta; 1205.5035]

⇠ *^eBD*(*x*)*|t[|]* t of standard procedure for comparing inclusive gluon to exclusive data) normalization of the cross-section with a mild logarithmic dependence on the energy. To I Standard procedure for comparing inclusive giuon to exclusive data) *b* (sort of standard procedure for comparing inclusive gluon to exclusive data) total cross-section for vector meson production is therefore obtained as *i* + tan y inclusiv *drW*(*r*) .
י *i* + tan lusive da

a) analytic properties of scattering amplitude \rightarrow real part 0 by its imaginary part. Corrections due to the real part of the scattering amplitude can be α scattering amplitude \rightarrow real part

t = 0 (which can be expressed in terms of the inclusive gluon distribution); in a second step

correction to the *W* dependence of the complete cross-section. We therefore do not assume

b) differential Xsection at t=0:

c) from experiment: $d\sigma$ \int f_{reco} \in 0.40 σ σ is numerical values $d\sigma$ \int I_{reco} \in $B_{\text{D}}(W)$. *b* \overline{dt} ^{(γp})

 $d\sigma$

 $\sigma^{\gamma p\to V p}(W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt} \left(\gamma\right)$ *<i>z D*_{*D*}(*W*) $\frac{a}{\sqrt{a}}$ $\overline{\gamma^{N,p}(W^2)} = \frac{1}{B_D(W)} \frac{d\sigma}{dt}$ $B_D(W)$ $d\sigma$

how to compare to experiment? *A*
P p(*x*) α *i* + tan (*x*)⇡ ◆ and the state of the Z *drW*(*r*) ✓ *i* + tan (*x*)⇡ *p BD*(*W*) $\overline{}$ *dt d d d i s i d j n i s i d j n i s i n i n i n i n i n i n i n i n i n i n i n i n i n i n i n i n i n i n i* description, Sec. 3 is dedicated to a discussion of the large perturbative corrections of the large perturbativ
The large perturbative corrections of the large perturbative corrections of the large perturbative corrections NLO BFKL gluon in the large *W* region while in Sec. 4 we present our conclusions. Following [21, 22], we use for the numerical values ↵⁰ = 0*.*06 GeV² where *^A*(*W*2*, t*) denotes the scattering amplitude for the reaction *^p* ! *V p* for color singlet exchange in the *t*-channel, with an overall factor *W*² already extracted. For a more detailed discussion of the kinematics we refer to [25].

 Γ ean energy dependence noming the $B_{\mathcal{D}}(W)$ $\frac{1}{2}$ = $\frac{1}{2}$ = photons. The photon photons is provided a photon of photons. The photons of photons is provided a photons. *r*2, while *f* = *c, b* denotes the flavor of the $B_D(W) = \left\lvert b_0 + 4\alpha' \ln \frac{1}{W_0} \right\rvert \text{ GeV}^{-2},$ a naramatar ϵ boranneren ϵ ⇠ *^eBD*(*x*)*|t[|]* nergy dependence from
arameter $D_D(VV) =$ $B_D(W) = \left\lfloor b_0 + 4\alpha' \right\rfloor$ $\frac{V}{T}$ \overline{a} eV ⁻ The uncertainty introduced by the modeling of the *t*-dependence mainly a↵ects the overall wear energy dependence norm $B_D(W) = \left| b_0 + 4\alpha' \ln \frac{W}{W} \right| \text{ GeV}^{-2}$ siope parametermine the state \mathfrak{c}_1 $\sqrt{ }$ $b_0 + 4\alpha' \ln \frac{W}{W}$ *W*⁰ $\overline{}$ \rm{GeV}^{-2} . weak energy dependence from slope parameter

$$
A^{\gamma p \to Vp}(x, t=0) = \left(i + \tan \frac{\lambda(x)\pi}{2} \right) \cdot \Im \mathbf{m} A^{\gamma p \to Vp}(x, t=0)
$$

with intercept

$$
\lambda(x) = \frac{d \ln \Im \mathbf{m} A(x, t)}{d \ln 1/x}.
$$

$$
\frac{d\sigma}{dt} \left(\gamma p \to Vp \right) \bigg|_{t=0} = \frac{1}{16\pi} \left| \mathcal{A}^{\gamma p \to Vp} (W^2, t=0) \right|^2
$$

$$
\frac{d\sigma}{dt}(\gamma p \to Vp) = e^{-B_D(W) \cdot |t|} \cdot \frac{d\sigma}{dt}(\gamma p \to Vp)\Big|_{t=0}
$$

$$
\sigma^{\gamma p \to V p} (W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0}
$$
 extracted from data

 \overline{Q}

determine the scattering amplitude, we first note that the dominant contribution is provided *drW*(*r*)*qq*¯(*x, r*)

Why is this happening?

GBW model: [Golec-Biernat, Wusthoff, hep-ph/9807513]

linearized version:

$$
\sigma_{q\bar{q}}(x,r) = \sigma_0 \left(1 - \exp\left(-\frac{r^2 Q_s^2(x)}{4}\right) \text{ with saturation scale } Q_s^2(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{\lambda}
$$

$$
\sigma_{q\bar{q}}^{lin.}(x,r) = \sigma_0 \frac{r^2 Q_s^2(x)}{4}
$$

recent fit [Golec-Biernat, Sapeta, 1711.11360] to combined HERA data with $\mathcal{Q}^2 \leq 10$ GeV 2 and $\chi^2/N_{dof} = 352/219 = 1.61$

Very clear for the GBW model

$\sigma_0[mb]$ *l* λ $x_0/10^{-4}$

27.43±0.35 0.248±0.002 0.40±0.04

$$
\mathfrak{Im}\mathscr{A}^{lin}\left(x\right)\sim\mathcal{Q}_s^2(x)\cdot\int dr...
$$

⌃(*i*)

^T (*r*)=ˆ*e^f*

The ratio for the GBW model ⁼m*A*(*W*2*, t*) = tan α for the GBW model $\sqrt{ }$ $\sqrt{ }$ ⁰ = 4*.*63 GeV² for ⌥ production. The

From scattering amplitude:
\n
$$
\Im \mathbf{m} \mathcal{A}_T(W^2, t=0) = \int d^2 \mathbf{r} \left[\sigma_{q\bar{q}} \left(\frac{M_V^2}{W^2}, r \right) \overline{\Sigma}_T^{(1)}(r) + \frac{d \sigma_{q\bar{q}} \left(\frac{M_V^2}{W^2}, r \right)}{dr} \overline{\Sigma}_T^{(2)}(r) \right]
$$

Recall: *i* + tan *dt* (*^p* ! *V p*)

$$
\sigma_{q\bar{q}}^{GBW}(x,r) = \sigma_0 \left(1 - \exp(-\frac{r^2 Q_s^2(x)}{4})\right)
$$

$$
\sigma^{\gamma p \to V p}(W^2) = \frac{1}{B_D(W)} \frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0}
$$
\nAnd

\n
$$
\frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0} = \frac{1}{16\pi} |A^{\gamma p \to V p}(W^2, t=0)|^2
$$
\nAnd

\n
$$
\frac{d\sigma}{dt} (\gamma p \to V p) \Big|_{t=0} = \frac{1}{16\pi} |A^{\gamma p \to V p}(W^2, t=0)|^2
$$

with *^r* ⁼ *[|]r|*. The functions ⌃(1*,*2) *^T* describe the transition of a transverse polarized photon For **LINEAR** GBW

$$
Q_s(x) = Q_s(M_V^2/W^2)
$$
 cancels for the ratio

²⇡² *^K*0(*m^f ^r*) ⌅(*i*) *Priergy depert
Aifferent VM Z
<i>T z*(*z*) *z m^T* + *m^L ^V* (*z, |p|*)*,* Complete GBW: non-trivial rdependence **→** different energy dependence for different VM

0

4
4 (1941)
4 (1941)

 $($

^V)*^T N* (*x, r, b*) (6)

=m*Ap*!*V p ^T* (*W, t* = 0) = 2 ^Z *d*2*r* 9) ^{....}.... *d*2*b* •Ratio constant with energy for **linear GBW**

2.1 The theoretical setup of our study of our
The theoretical setup of our study of our stu

The ratio: GBW model

- for linear model x-dependence in $Q_s^2(x) = Q_s^2(x)$
- Non-trivial r -dependence for complete GBW

$$
b_0^2 \left(\frac{x}{x_0}\right)^{\lambda}
$$
 we have
$$
\frac{d \ln \sigma_{q\bar{q}}}{\ln 1/x} = \lambda = \text{const.}
$$

model \rightarrow rise of the ratio

[Bartels, Golec-Biernat, Kowalski; hep-ph/0203258] 2.2 Fits with the DGCLAP in the DGCLAP i [Bartels, Golec-Biernat, Kowalski; hep-ph/02

Essentially the GBW model with DGLAP evolution The DGLAP improved saturation model [27, 28] implements the dipole cross section given $\begin{array}{c} \text{c} \\ \text{c} \end{array}$ by the contract of the contrac a
P evolution n

Factorization scale originally: $\mu^2 =$ \mathbb{C} \overline{C} above C and also be obtained for a slightly be obtained for a slightly section can also be obtained for a slightly section can also be obtained for a slightly section can also be obtained for a slightly section r actorization scale originally.

∂g(x, µ2)

 \sim

$$
\frac{C}{r^2}+\mu_0^2.
$$

A less trivial model: The DGLAP improved saturation model A lace trivial model: The DCI AD improved eaturation model scale

 μ_0^2 and μ_0^2 are $\mu \to \mu_0$ giluonic sector.
Sector sector Recent fit: apola, 1711.11000] $\mu^2 =$ [Golec-Biernat, Sapeta; 1711.11360]

In common: $\frac{2}{0}$. \ln common: In common:

- for large dipole sizes r,

35 eters are given by MINOS from the MINUIT package. We no longer restrict the data to

Saturation scale becomes r-deponding the initial conditions in the initial condition of the initial condition o 0, for a choice, the fit of the fit of the fit of the such a complete for small r Saturation scale becomes r-dependent \rightarrow includes correct DGLAF infilt for Saturation scale becomes *r*-dependent → includes correct DGLAP limit for small *r*

$$
\sigma_{\text{dip}}(r,x) = \sigma_0 \left\{ 1 - \exp\left(-\frac{\pi^2 r^2 \alpha_s(\mu^2) x g(x,\mu^2)}{3\sigma_0}\right) \right\} ,
$$

$$
\mu^2 = \frac{\mu_0^2}{1 - \exp(-\mu_0^2 r^2/C)}
$$

$$
\mu \to \mu_0
$$
 \n
$$
\mu_0 \sim \mu_0
$$
 \nGolec-Biernat Saneta: 1711 11360

$$
\mu \rightarrow \mu_0
$$

 r^{u} r^{u} r^{u} - Otherwise $\sim C/r^2$

Complementary to BFKL/BK study
35 \overline{u} (2.112) (2.1212)

Discussion

0.6 $100 \cdot \Sigma_1^{J/\Psi}$: 0.5 $100 \cdot \Sigma^{\Psi(2s)}_1$: 0.4 $\int d\ln\sigma^{BGK}_{q\bar{q}}(x,r)$ $d\ln 1/x$ 0.3 0.2 0.1 0.0 0.01 0.10 0.50 0.05 r/fm

Towards smaller x

"Slope" for complete BGK

- Difference between J/Ψ and $\Psi(2s)$ at relative large dipole size r
- Full non-linear model: non-trivial x-dependence in this region
- Linear model with factorization scale frozen at large dipole size r , there is not much happening \rightarrow constant ratio
- Trivial for GBW model; also seen for BFKL vs BK (QCD low x evolution)
- Prediction depends on VM wave function $=$ the position of the node

"Slope" for linear BGK

