Power Counting and Jet Algorithm Parameters

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Overview

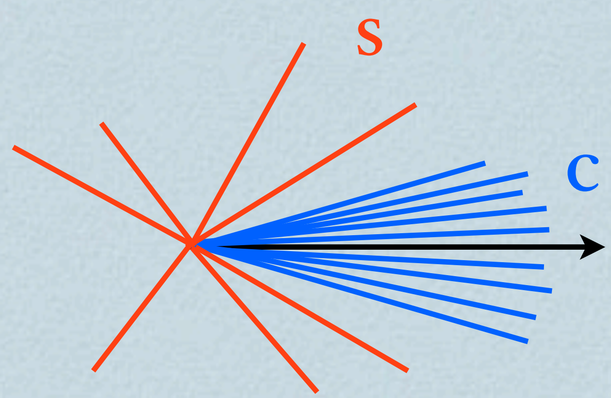
- Power counting in soft - collinear effective theory (SCET)
- Factorization and properties of jets through power counting
- Applications to jet substructure

Small Parameters and Jets

- Jet observables often defined with small parameters:
  - Jet size R, jet energy cut Λ/Q
  - Jet substructure methods introduce small parameters to separate out dynamics in jets (e.g. pileup from FSR)
  - These parameters will generate large logarithms
- Resummation of these parameters key to correctly predicting behavior of jet substructure and more complex observables for jets

Brief SCET Overview

- Modes in SCET :



  - collinear: $p_c \sim Q(1, \lambda^2, \lambda)$
  - soft: $p_s \sim Q(\lambda^2, \lambda^2, \lambda^2)$

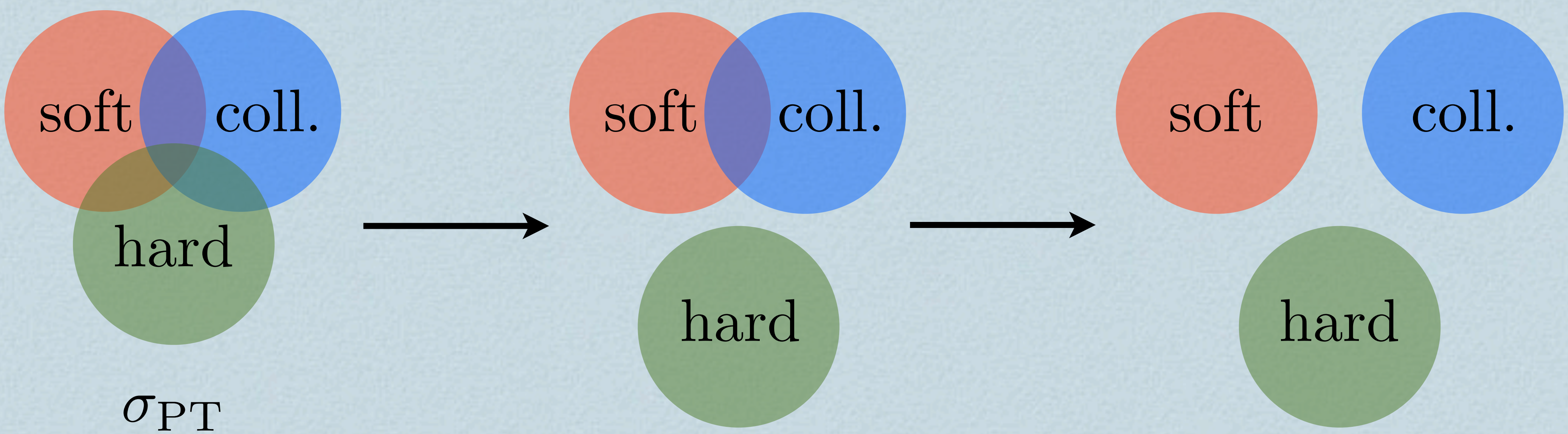
$$\begin{aligned} p &= (E + p_z,\, E - p_z,\, p_\perp) \\ &= (p^-, p^+, p_\perp) \end{aligned}$$ - light cone coordinates

- Collinear modes describe jet evolution, soft modes the global soft radiation in/between jets

- Factorization allows us to separate contributions of soft and collinear modes (live at different scales), resum large logarithms

Factorization

Divides the cross section into pieces
depending on separate scales



σ_{PT}

$$\sigma = H_N \otimes |O_N|^2 \qquad \sigma = H_N \otimes J^N \otimes S_N$$

Factorization of N-jet operators in SCET shown at leading power
(Bauer, Pirjol, Stewart)

Factorization of Observables

Start with basic SCET distribution $\frac{d\sigma}{d\tau} = H_N \langle O_N^\dagger \, \hat{\mathcal{R}}(\tau) \, O_N \rangle$

The restriction operator specifies the phase space cuts and measurement of the observable

O_N factorizes into jet and soft operators $O_N = O_J^N O_{S_N}$

Need to show the restriction operator factorizes: $\hat{\mathcal{R}} = \hat{\mathcal{R}}_c + \hat{\mathcal{R}}_s$

Why is Factorization Important?

- Small parameters in the algorithm definition ($\hat{\mathcal{R}}$) will create large logarithms in cross sections
  - Without factorization, these logs not resummed

$$\begin{matrix} \alpha L^2 & \alpha L & \alpha \\ \alpha^2 L^4 & \alpha^2 L^3 & \cdots \\ \alpha^3 L^6 & \cdots & \\ \vdots & & \end{matrix}$$

without factorization,
higher order leading logarithms
may be unrelated to
lowest order leading log term
(e.g. JADE)

- The problem of non-global logarithms in jet observables is much more challenging (outside this scope)

Power Counting in SCET

- Power counting can determine the dominant physics for an observable or algorithm

- We will use power counting to test a necessary condition for factorization: $\hat{\mathcal{R}} = \hat{\mathcal{R}}_c + \hat{\mathcal{R}}_s$

- This simple power counting can extract properties of jets and place constraints on jet substructure methods

Power Counting Kinematics in SCET

- Energies: $E_c \sim \lambda^0$, $E_s \sim \lambda^2$
- Angles:
  - collinear - collinear: $\theta_{cc} \sim \lambda$
  - soft - soft: $\theta_{ss} \sim \lambda^0$
  - collinear - soft: $p_c \cdot p_s = 2E_c E_s (1 - \cos\theta_{cs})$

independent of p_c

$$\frac{p_c \cdot p_s}{E_c E_s} = 2\frac{p_c^- p_s^+}{p_c^- (p_s^+ + p_s^-)} + \mathcal{O}(\lambda) = \boxed{\frac{2p_s^+}{p_s^+ + p_s^-}} + \mathcal{O}(\lambda)$$

$\longrightarrow$ can write θ_{cs} as θ_{ns}

soft Wilson line Y_n depends only on label direction

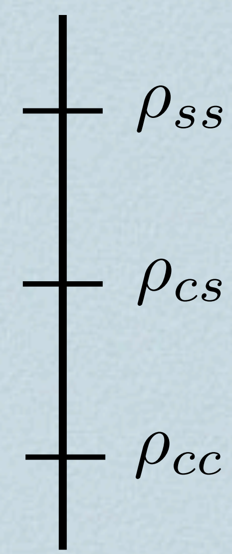
Power Counting for Jet Algorithms: anti-kT

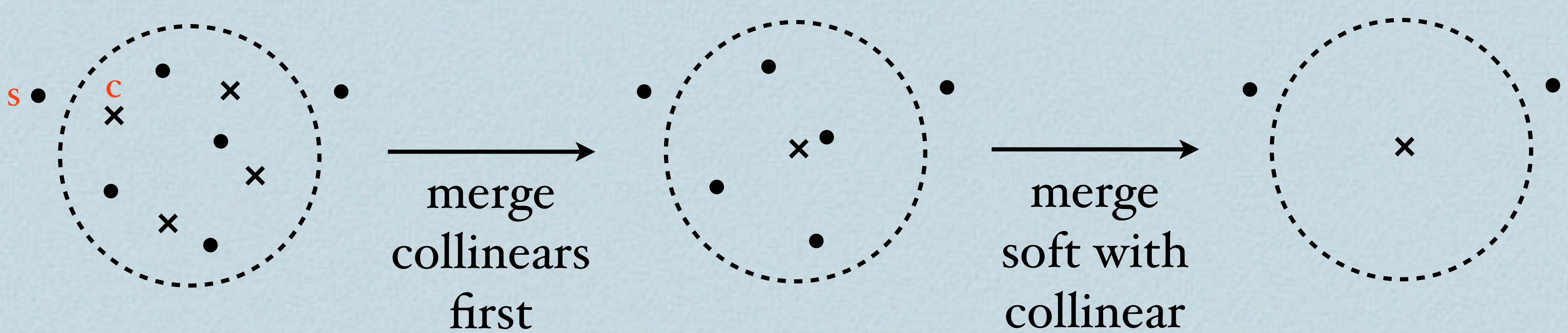
algorithm: $\rho_{ij} = \min\left(\frac{1}{E_i}, \frac{1}{E_j}\right)\frac{\theta_{ij}}{R}\,, \quad \rho_i = \frac{1}{E_i}$

$$\rho_{c_1c_2} = \min\left(\frac{1}{E_{c_1}}, \frac{1}{E_{c_2}}\right)\frac{\theta_{c_1c_2}}{R} \sim \frac{\lambda}{R}$$

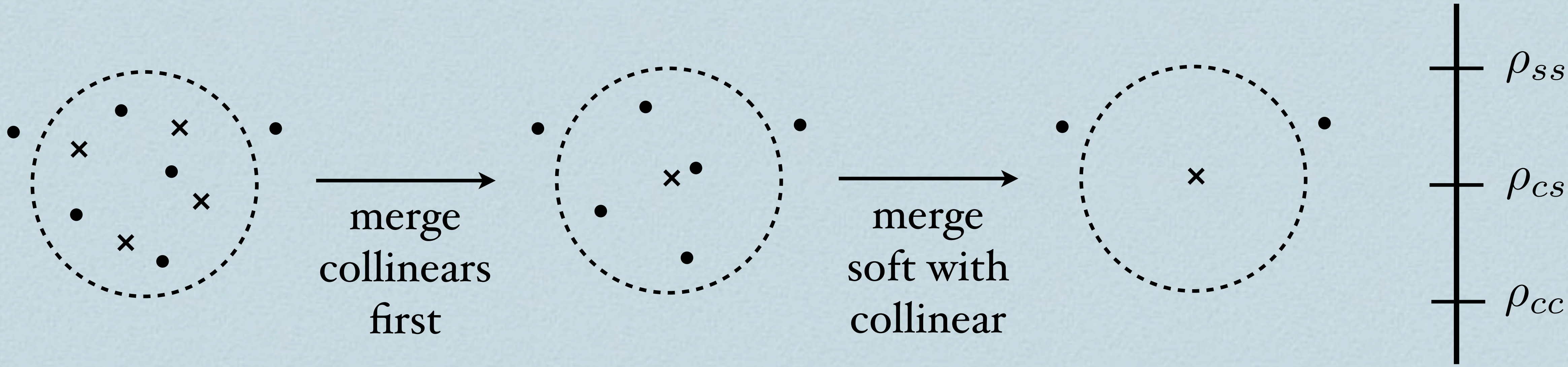
$$\rho_{cs} = \frac{1}{E_c}\frac{\theta_{ns}}{R} \sim \frac{\lambda^0}{R}$$

$$\rho_{s_1s_2} = \min\left(\frac{1}{E_{s_1}}, \frac{1}{E_{s_2}}\right)\frac{\theta_{s_1s_2}}{R} \sim \frac{\lambda^{-2}}{R}$$





Power Counting for Jet Algorithms: anti-kT



soft - collinear phase space constraints decouple: $\widehat{\mathcal{R}} = \widehat{\mathcal{R}}_c + \widehat{\mathcal{R}}_s$

necessary condition for factorization satisfied

soft phase space for collinear jets factorizes: $\theta_{ns} < R$

anti-kT has circular jets

anti-kT is an ideal algorithm for SCET

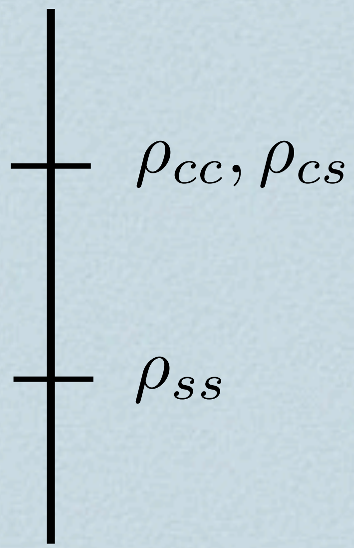
Power Counting for Jet Algorithms: JADE

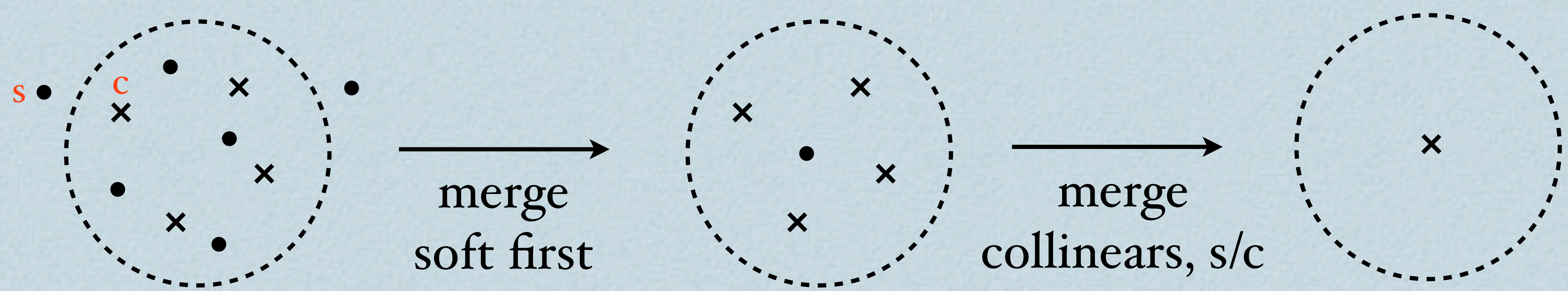
algorithm: $\rho_{ij} = E_i E_j \theta_{ij}^2\,, \quad y_{\rm cut} \sim \lambda^2$

$$\rho_{c_1 c_2} = E_{c_1} E_{c_2} \theta_{c_1 c_2}^2 \sim \lambda^2$$

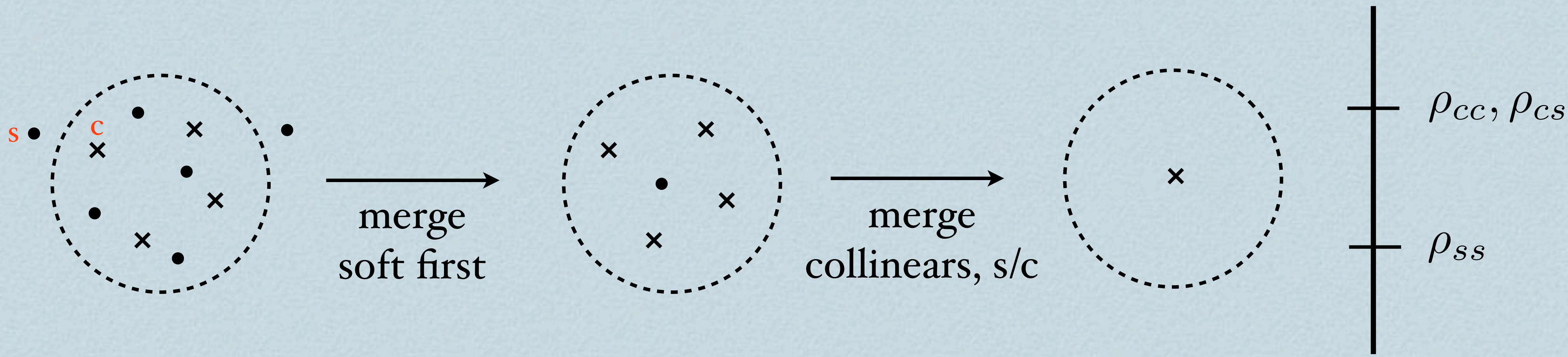
$$\rho_{cs} = E_s E_c \theta_{ns}^2 \sim \lambda^2$$

$$\rho_{s_1 s_2} = E_{s_1} E_{s_2} \theta_{s_1 s_2}^2 \sim \lambda^4$$





Power Counting for Jet Algorithms: JADE



soft - collinear phase space constraints do not decouple

$\widehat{\mathcal{R}} \neq \widehat{\mathcal{R}}_c + \widehat{\mathcal{R}}_s$ ⟶ does not satisfy necessary condition for factorization

JADE does not factorize:
soft phase space cuts depend on details of collinear splittings

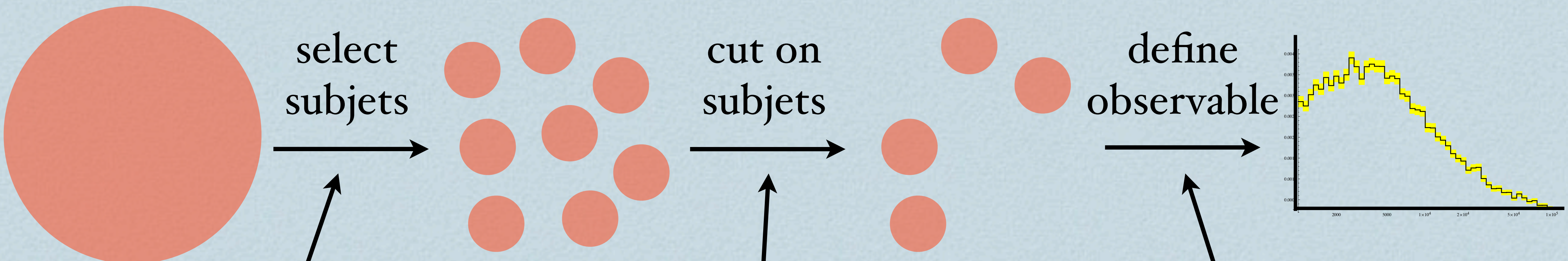
$$\rho_{cs} = E_s E_c \theta_{ns}^2 \sim \lambda^2$$

Power Counting Results for Jet Algorithms

- Key (already known) results:
  - anti-kT: circular jets
    - Soft PS factorizes into single particle PS
  - JADE: does not factorize
    - No two loop calculation needed
  - kT: soft phase space non-circular
    - No single particle PS

Power Counting for Jet Substructure

Essential steps in a generic substructure method:



Assume subjet select defined with factorizable algorithm (true in known cases)

Additional power counting analysis can be applied to the observable

Focus on kinematic cuts used to select subjets
Needs to factorize to be able to factorize observable

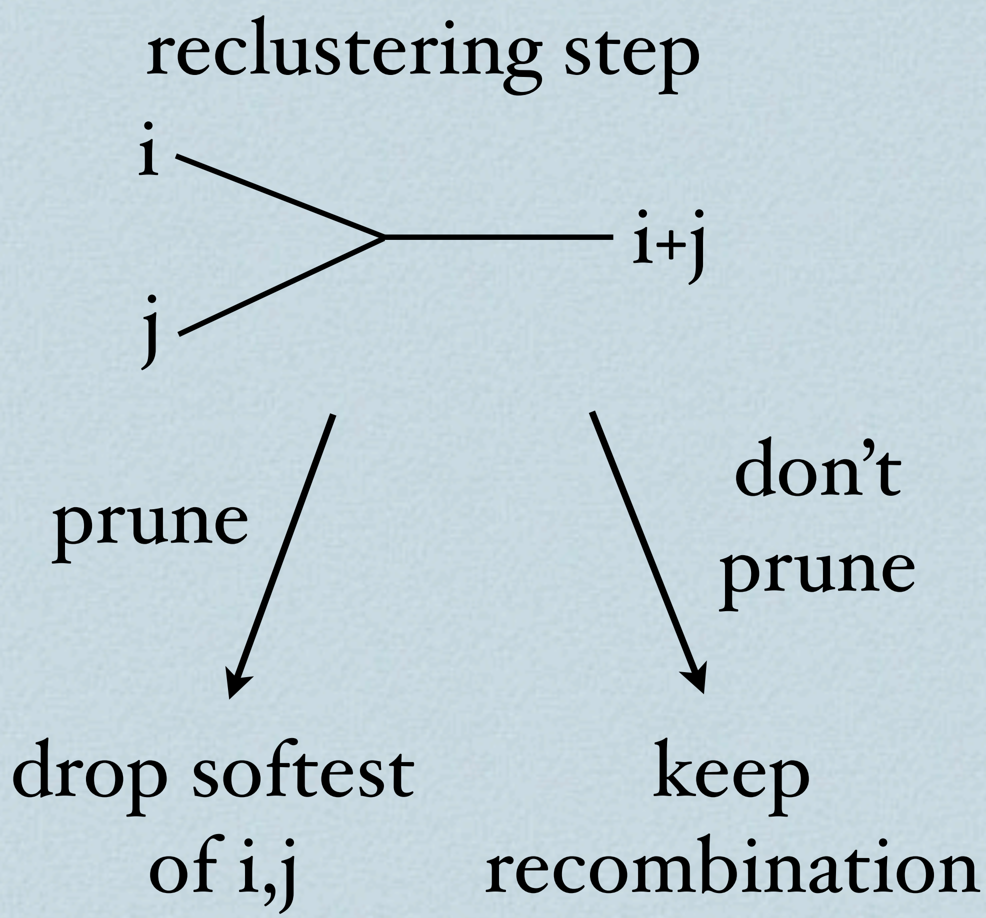
Test soft - collinear factorization for subjet selection

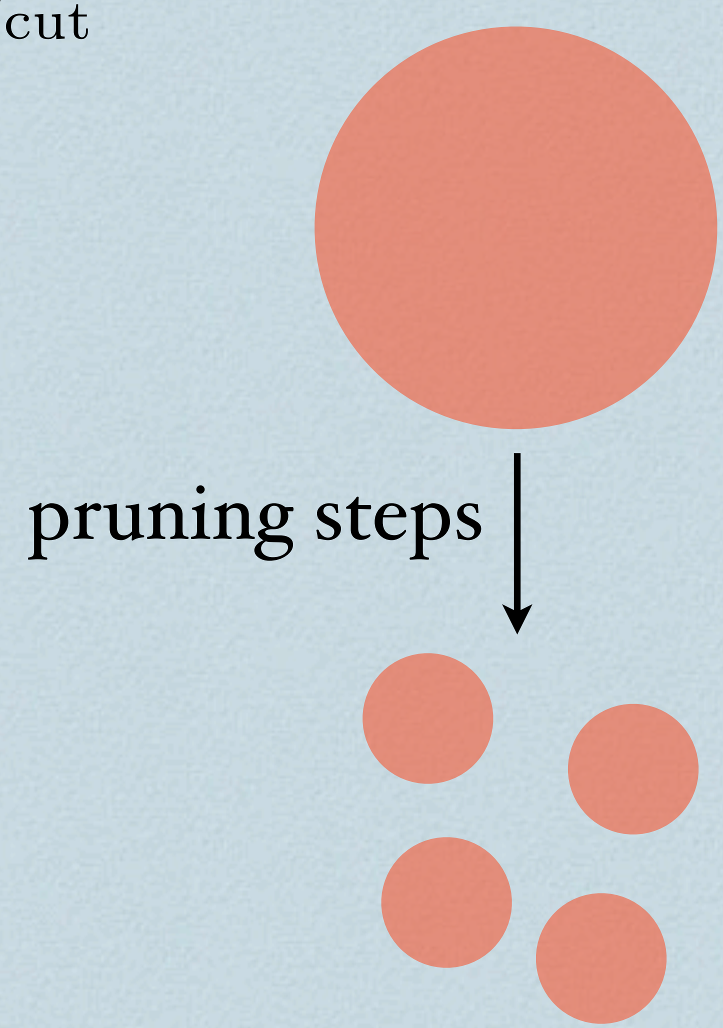
$$\widehat{\mathcal{R}} = \widehat{\mathcal{R}}_c + \widehat{\mathcal{R}}_s$$

Power Counting for Pruning

- Pruning: recluster found jets and prune recombinations with

$$\frac{\min(E_i, E_j)}{E_{i+j}} < z_{\rm cut} \quad \text{and} \quad \theta_{ij} > D_{\rm cut}$$





Power Counting for Pruning

- Pruning: recluster found jets and prune recombinations with

$$\frac{\min(E_i, E_j)}{E_{i+j}} < z_{\rm cut} \quad \text{and} \quad \theta_{ij} > D_{\rm cut}$$

↖ angles: no dependence on both soft and collinear particles

- Consider soft - collinear candidate recombination

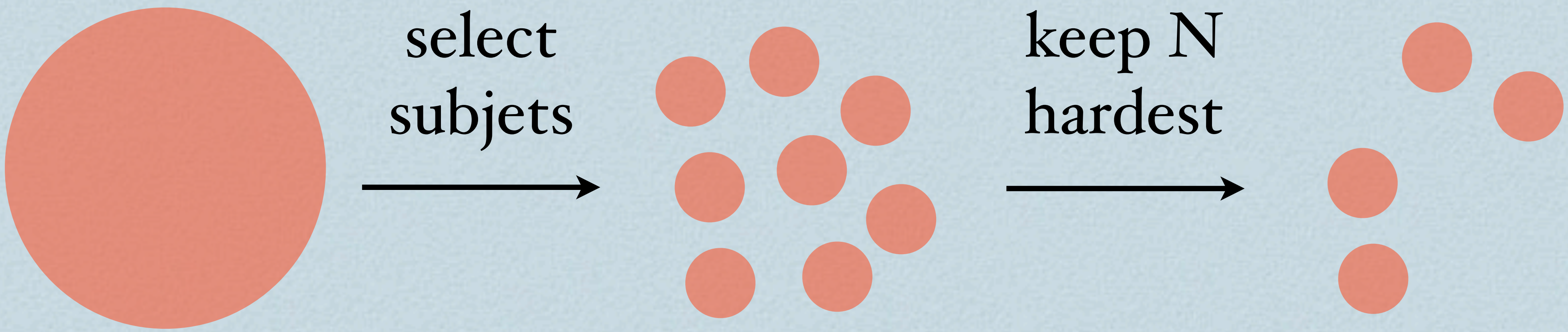
comparison: $\frac{E_s}{E_c} \sim \lambda^2 < z_{\rm cut}$

- Factorization fails if $z_{\rm cut} \sim \lambda^2$: suggests $z_{\rm cut} \sim \lambda$ (or higher)

- The observable will set λ - tells us the size of $z_{\rm cut}$ needed

Power Counting for Filtering

- Filtering: select subjets and keep the N hardest



Power Counting for Filtering

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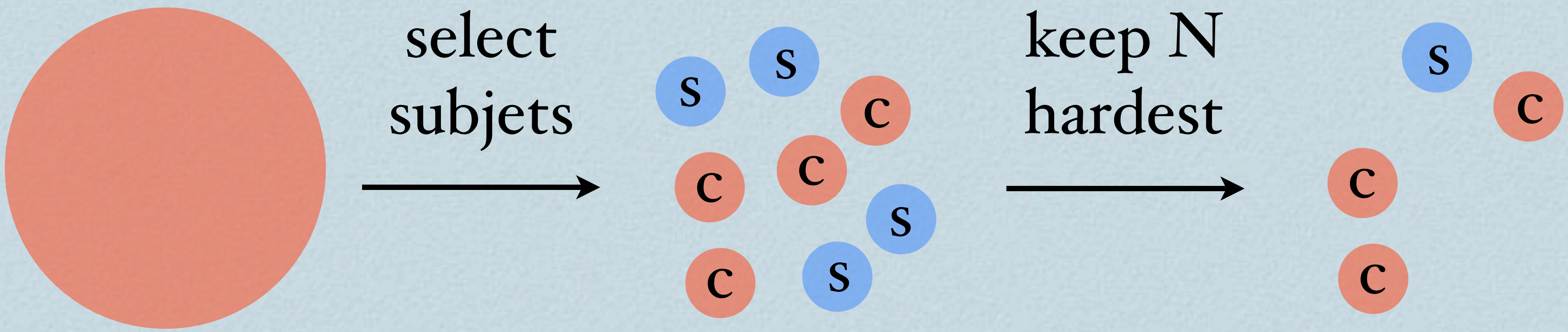
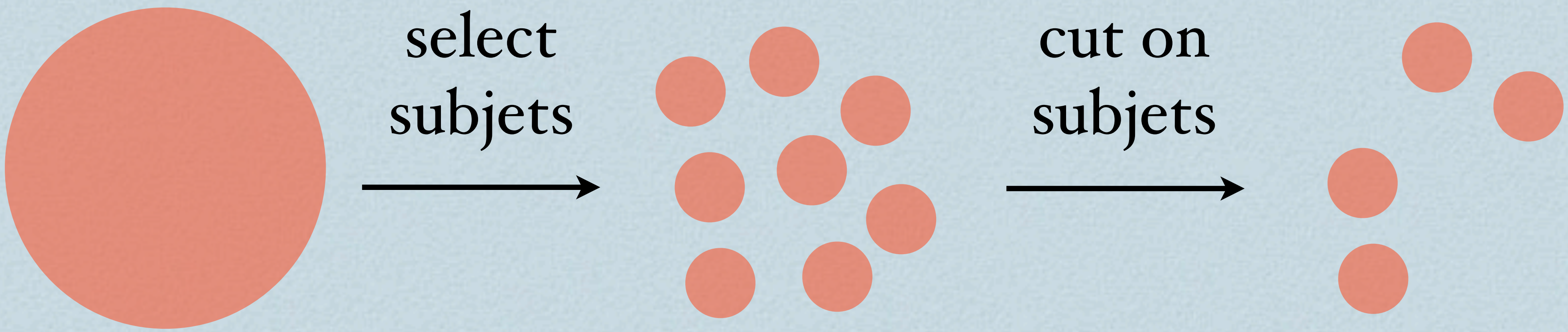


- If there are "soft subjets", whether or not they pass the cut depends on the number of collinear subjets

- Filtering does not factorize unless there are no soft subjets: constrains the algorithm used to find subjets (e.g. MD-F)

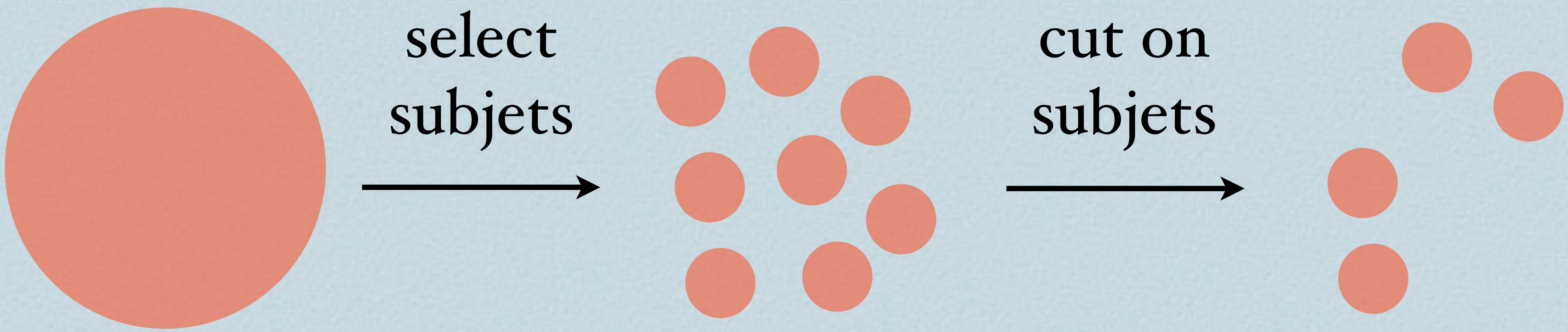
Power Counting for Trimming

- Trimming: select subjets and keep a subset by a cut



Power Counting for Trimming

- Trimming: select subjets and keep a subset by a cut



- If the cut is on individual subjets - e.g. a jet - wide pT cut, then trimming could be factorized

- Factorization of trimming seems most straightforward

Conclusions

- Use SCET power counting to analyze factorizability of jet algorithms/observables
- Simple properties of jets and constraints on jet substructure can be extracted
- Worthwhile exercise when designing jet substructure methods
  - Can tell you whether the behavior can be reliably predicted