Controlling large K-factors in processes with jets

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in collaboration with Gavin Salam and Mathieu Rubin 1

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$$E_T^{\text{miss}} + \geq 4 \text{jets (+ leptons)}$$

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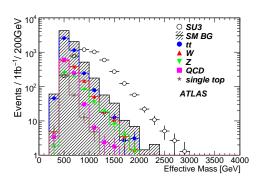
$$M_{\text{eff}} = \sum_{i=1}^{4} p_{T,i}^{\text{jet}} + \sum_{i=1}^{4} p_{T,i}^{\text{lepton}} + E_{T}^{\text{miss}}$$

Example: no-lepton search mode

[ATLAS NOTE '08, CERN-OPEN-2008-020]

Cuts:

- $ightharpoonup p_T^{\text{lepton}} < 20 \text{ GeV}$
- 4 jets with $|\eta| < 2.5$ and $p_T > 50$ GeV
- ▶ hardest jet $p_T > 100 \text{ GeV}$
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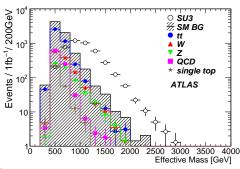
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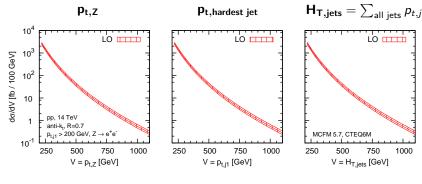
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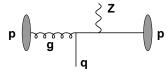
Is it really safe? Let us look at a simpler process...

The problem of giant K factors

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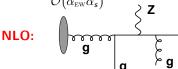


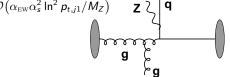
LO:



The problem of giant K factors

► Z+i at the LHC $\mu_{R,0} = \mu_{F,0} = \sqrt{m_Z^2 + p_{t, ext{hardest jet}}^2}$ $\mathbf{H}_{\mathsf{T},\mathsf{jets}} = \sum_{\mathsf{all} \; \mathsf{iets}} p_{t,j}$ $p_{t,Z}$ Pt,hardest jet 10⁴ NLO WWW NLO WW NLO WWW 10³ do/dV [fb / 100 GeV] 10² 10 anti-k,, R=0.7 $p_{k+1} > 200 \text{ GeV}, Z \rightarrow e^+e^-$ MCFM 5.7, CTEQ6M 10⁻¹ 500 750 1000 250 500 750 1000 250 500 750 1000 250 $V = p_{t,Z} [GeV]$ $V = p_{t,i1}$ [GeV] $V = H_{T,jets} [GeV]$ $\mathcal{O} \left(lpha_{\scriptscriptstyle \mathrm{EW}} lpha_s^2 \ln^2 p_{t,j1}/M_Z
ight)$ $\mathcal{O}(\alpha_{\scriptscriptstyle \mathrm{EW}}\alpha_s^2)$



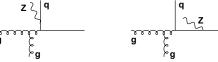


► The large K factor for the Z+jet comes from the new "dijet type" topologies that appear at NLO 72 | q



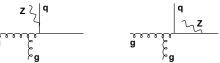


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- ▶ though formally NLO diagrams for Z+jet, these are in fact leading contributions to $p_{t,j1}$ and H_T spectra
- this raises doubts about the accuracy of these predictions
- need for subleading contributions for Z+jet, in this case NNLO

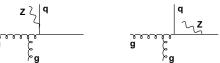
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 at NNLO = $Z+3j$ tree + $Z+2j$ 1-loop + $Z+j$ 2-loop $Z+2j$ at NLO

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▶ 2-loop part

- ▶ we need it to cancel IR and collinear divergences from Z+2j at NLO result
- it will have the topology of Z+j at LO so it will not contribute much to the cross sections with giant K-factor

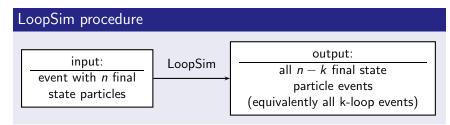
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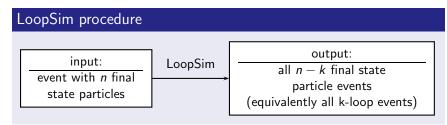
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notation:

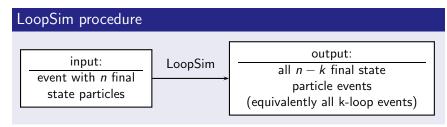
*n***LO** − simulated 1-loop

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this will work very well for the processes with large K factors e.g.

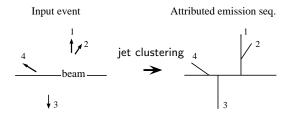
$$\sigma_{\bar{n}\mathsf{NLO}} = \sigma_{\mathsf{NNLO}} \left(1 + \mathcal{O}\left(\frac{\alpha_{\mathsf{s}}^2}{\mathsf{K}_{\mathsf{NNLO}}}\right) \right) \,, \quad \mathsf{K}_{\mathsf{NNLO}} \gtrsim \mathsf{K}_{\mathsf{NLO}} \gg 1$$

Input event

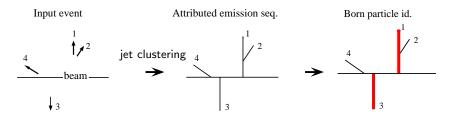
1

2

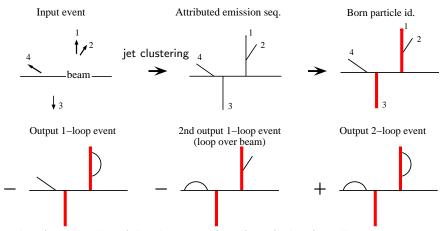
beam—



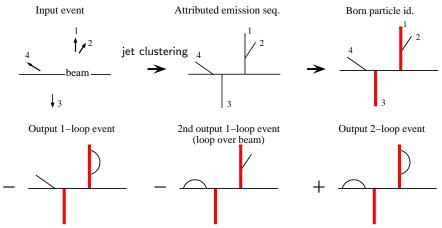
lacktriangledown jet clustering ij o k is reinterpreted as the splitting k o ij



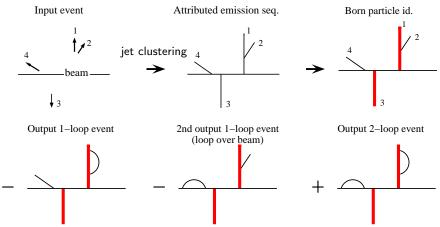
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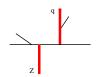
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- weight of an event $\sim (-1)^{\text{number of loops}}$



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- beware: the loops above are just a shortcut notation!



weight = 1	p_x	р_у	p_z	E
particle 0: Z	-21.01	7.90	-1561.72	1564.54
particle 1: q	19.40	-10.65	1006.92	1007.16
particle 2: q	-5.75	-0.15	-150.64	150.75
particle 3: q	7.36	2.90	418.96	419.03
total momentum :	0	0	-286.48	3141.48

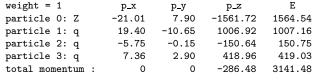




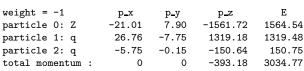
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total momentum :	0	0	-393.18	3034.77







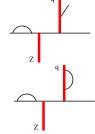




weight = -1	p_x	р_у	p_z	E
particle 0: Z	-24.03	7.82	-1573.56	1576.40
particle 1: q	17.47	-10.71	931.93	932.15
particle 2: q	6.56	2.88	379.16	379.23
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particle 1: q		24.03	-7.82	1196.14	1196.4
total momentum	:	0	0	-377.42	2772.8

The LoopSim method: some more details

The principle of the method is simple. There is, however, a number of issues that need to be addressed to fully specify the procedure and make it usable:

The LoopSim method: some more details

The principle of the method is simple. There is, however, a number of issues that need to be addressed to fully specify the procedure and make it usable:

- ▶ infrared and collinear safety
- conservation of four-momentum
- choice of jet definition (algorithm, value of R)
 - ightharpoonup in what follows: Cambridge/Aachen with R = 1
- treatment of flavour (e.g. for processes with vector bosons)
 - Z boson can be emitted only from quarks and never itself emits
- extension to input events with exact loops

 $E_{n,l}$ – input event with n final state particles and l loops

 U_l^b – operator producing event with b Born particles and l loops

 $U^b_orall$ — operator generating all necessary loop diagrams at given order

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How to introduce exact loop contributions?

$$U_{\forall}^{b}(E_{n,0})$$

generate all diagrams from the tree level event

 $E_{n,l}$ – input event with n final state particles and l loops

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$$U^b_{\forall}(E_{n,0}) + U^b_{\forall}(E_{n-1,1})$$

- generate all diagrams from the tree level event
- generate all diagrams from the 1-loop event

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$$U_{\forall}^{b}(E_{n,0}) + U_{\forall}^{b}(E_{n-1,1}) - U_{\forall}^{b}(U_{1}^{b}(E_{n,0}))$$

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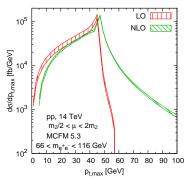
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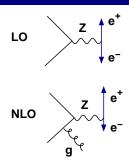
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- generate all diagrams from the 1-loop event
- remove all approximate diagrams from $U_{\forall}^b(E_{n,0})$ that have exact counterparts provided by $U_{\forall}^b(E_{n-1,1})$
- ▶ inclusion of exact loops helps reducing scale uncertainties
- straightforward generalization to arbitrary number of exact loops



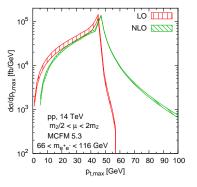
Validation

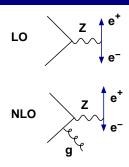
Drell-Yan at NNLO: spectrum of harder lepton



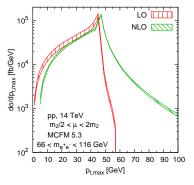


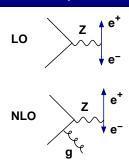
▶ giant K factor due to a boost caused by initial state radiation





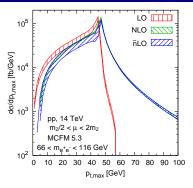
- giant K factor due to a boost caused by initial state radiation
- ▶ the agreement between NLO and \bar{n} LO may serve as a indication whether the method works for a given observable, $Z@\bar{n}$ LO = $Z@LO+LoopSim \circ (Z+j@LO)$

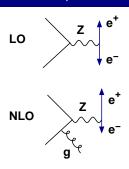




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- three regions of $p_{t,max}$:

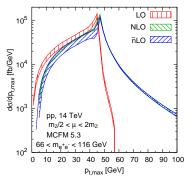
$$\lesssim \frac{1}{2}M_Z$$

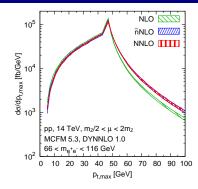




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	•		•	,
•	three regions of $p_{t, \max}$:	$\lesssim rac{1}{2} M_Z$	$[\frac{1}{2}M_Z, 58{ m GeV}]$	$>58\mathrm{GeV}$
	īnLO vs NLO	very good	excellent	perfect
		(not guaranteed)	(expected)	(expected)

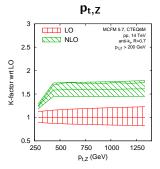


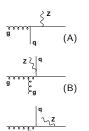


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- three regions of $p_{t,\text{max}}$: $\lesssim \frac{1}{2} M_Z$ $\left[\frac{1}{2} M_Z, 58 \, \text{GeV}\right] > 58 \, \text{GeV}$ $\bar{n} \text{LO vs NLO}$ very good excellent perfect and $\bar{n} \text{NLO vs NNLO}$ (not guaranteed) (expected) (expected)

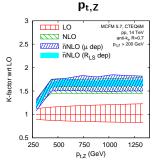
\bar{n} NLO predictions for LHC



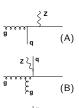




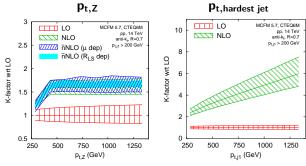
 $\mu = \sqrt{m_Z^2 + p_{t, ext{hardest je}}^2}$



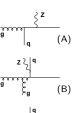
▶ $p_{t,Z}$: no correction; topology (A) dominant at high $p_{t,Z}$ (extra loops w.r.t. NLO do not change much)



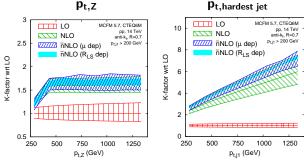
$z=\sqrt{m_Z^2+p_{t, ext{hardest jet}}^2}$



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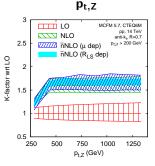
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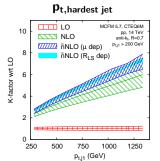


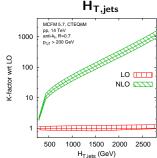




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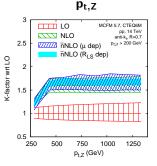
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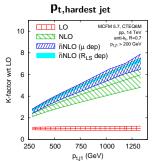


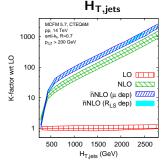




$\mu = \sqrt{m_Z^2 + p_{t, ext{hardest jet}}^2}$







- ▶ $p_{t,Z}$: no correction; topology (A) dominant at high $p_{t,Z}$ (extra loops w.r.t. NLO do not change much)
- ▶ $p_{t,j}$: small correction; \bar{n} NLO is like NLO for the dominant (B) and (C) configurations and it behaves like healthy NLO
- ► H_{T, jets}: significant correction; K factor ~ 2; given that it is more like going from LO to NLO this may happen sometimes, especially for nontrivial observables like H_T; can we understand it here?





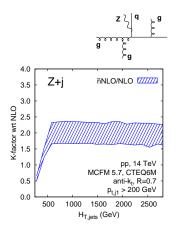


► Z+jet at NNLO like dijets at NLO (same topology, Z only provides the enhancement $\mathcal{O}(\alpha_{\text{EW}} \ln^2 p_{t,j1}/m_{\text{Z}})$)



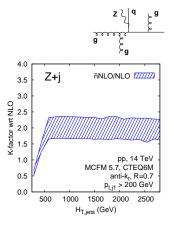


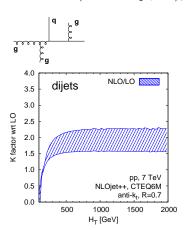
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► Z+jet at NNLO like dijets at NLO (same topology, Z only provides the enhancement $\mathcal{O}(\alpha_{\text{EW}} \ln^2 p_{t,j1}/m_{\text{Z}})$)

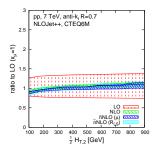




- \blacktriangleright H_T for dijets receives large contributions at NLO!
 - caused by appearance of the third jet from initial state radiation

Dijets at \bar{n} NLO

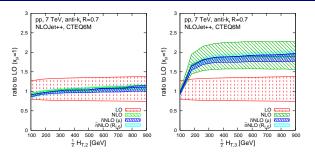
$\overline{H_{T,n}} = \overline{\sum_{n \text{ hardest jets}} p_{t, \text{jet}}}$



H_{T,2}: central value and scale uncertainties stay the same: adding NNLO corrections without proper finite part cannot improve the result

Dijets at $\bar{n}NLO$

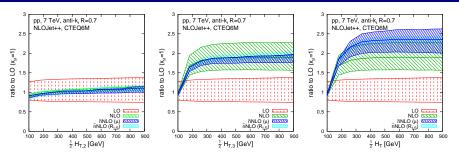
$H_{T,n} = \sum_{n \text{ hardest jets}} p_{t, \text{jet}}$



- H_{T,2}: central value and scale uncertainties stay the same: adding NNLO corrections without proper finite part cannot improve the result
- ▶ $H_{T,3}$ converges, significant reduction of scale uncertainty: the observable comes under control at $\bar{n}NLO$

Dijets at *n*NLO

$H_{T,n} = \overline{\sum_{n \; ext{hardest}} \overline{\sum_{j ext{ets}} p_{t,j ext{et}}}}$



- ► H_{T,2}: central value and scale uncertainties stay the same: adding NNLO corrections without proper finite part cannot improve the result
- ▶ $H_{T,3}$ converges, significant reduction of scale uncertainty: the observable comes under control at $\bar{n}NLO$
- ▶ H_T does not converge: again caused by the initial state radiation, this time a second emission which shifts the distribution of H_T to higher values and causes no effect for the $H_{T,3}$ distribution

Summary

- several cases of observables with giant NLO K factor exist
- those large corrections arise due to appearance of new topologies at NLO
- we developed a method, called LoopSim, which allows one to obtain approximate NNLO corrections for such processes
- the method is based on unitarity and makes use of combining NLO results for different multiplicities
- ▶ we gave arguments why the method should produce meaningful results and we validated it against NNLO Drell-Yan and also NLO Z+j and NLO dijets
- we computed approximated NNLO corrections to Z+j and dijets at the LHC finding, depending on observable, either indication of convergence of the perturbative series or further corrections
- ▶ the latter has been understood and attributed to the initial state radiation

Outlook

processes with W, multibosons, heavy quarks, . . .

BACKUP SLIDES

What can we compute with LoopSim?

A few examples...

Drell-Yan

```
Z@\bar{n}LO = Z@LO + LoopSim \circ (Z+j@LO)

Z@\bar{n}NLO = Z@NLO + LoopSim \circ (Z+j@NLO_{only})
```

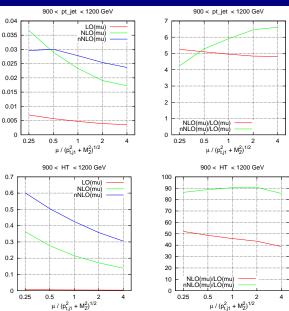
Z+j

dijets

```
2j@\bar{n}NLO = 2j@NLO + LoopSim \circ (3j@NLO_{only})
```

► mZ+nj@NLO_{only} means only the highest order contribution

Scale dependence: Z + jet



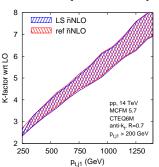
Reference-observable method

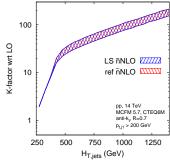
Take a reference observable identical at LO to the observable A

$$\begin{split} \sigma_{\text{Z+j@NNLO}}^{(A)} &= \sigma_{\text{Z+j@NNLO}}^{(\text{ref})} + (\sigma^{(A)} - \sigma^{(\text{ref})})_{\text{Z+j@NNLO}} \\ &= \sigma_{\text{Z+j@NNLO}}^{(\text{ref})} + (\sigma^{(A)} - \sigma^{(\text{ref})})_{\text{Z+2j@NLO}} \end{split}$$

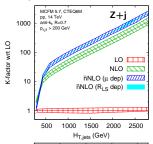
If the reference observable converges well

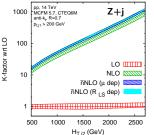
$$\sigma_{\mathsf{Z}+\mathsf{j@NNLO}}^{(A)} \simeq \sigma_{\mathsf{Z}+\mathsf{j@NLO}}^{(\mathrm{ref})} + (\sigma^{(A)} - \sigma^{(\mathrm{ref})})_{\mathsf{Z}+2\mathsf{j@NLO}}$$

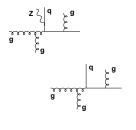


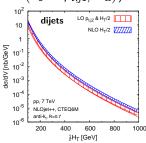


▶ Z+jet at NNLO like dijets at NLO (same topology, Z only provides the enhancement $\mathcal{O}(\alpha_s \ln^2 p_{t,j1}/m_{\rm Z})$)









- \blacktriangleright H_T for dijets receives large contributions at NLO!
 - caused by appearance of the third jet from initial state radiation
- ▶ if the same is valid for Z + j we should see only small correction for $H_{T,j2} = \sum_{i=1}^{2} p_{t,ji}$
 - and indeed it is small!