Beam jets at small q_T

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(based on: T. Becher & MN, arXiv:1007.4005)

Drell-Yan production

- Drell-Yan processes such as pp→W, Z, H are of great importance to collider physics:
  - measurement of W-boson mass
  - determination of PDFs
  - discovery of Higgs boson
- Important kinematical situation:

$$M \gg q_T$$

with q_T either much larger than $\Lambda_{\rm QCD}$ or even comparable to it
→ classical two-scale problem with large Sudakov logarithms $\sim (\alpha_s \ln^2 M/q_T)^n$, which need to be resummed

Drell-Yan production

- Transverse momentum of Drell-Yan object (W, Z, H) due to initial-state radiation (ISR) off collinear partons
- Simple example of beam jets described by beam functions in SCET   Stewart, Tackmann, Waalewijn 2009
- Yet many surprises and subtleties arise (collinear anomaly, divergent expansions), which may be relevant also for other applications of beam functions in jet processes

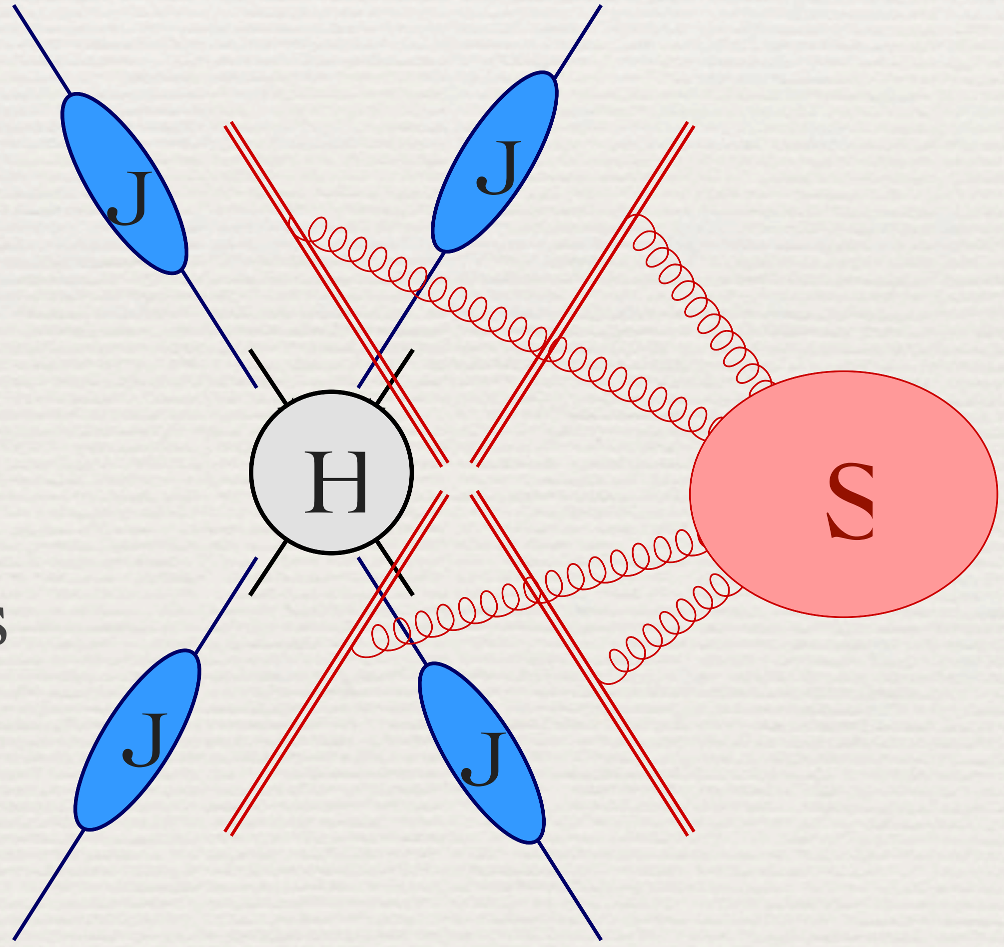
A tale of many scales

- Effective field theories provide an elegant approach to this problem, based on scale separation (factorization) and RG evolution
- Factorize cross sections:

$$\sigma \sim H(\mu_h) \prod_i J_i(\mu_i) \otimes S(\mu_s)$$

- Define components in terms of effective theory objects
- Resum large Sudakov logarithms directly in momentum space by solving RG equations

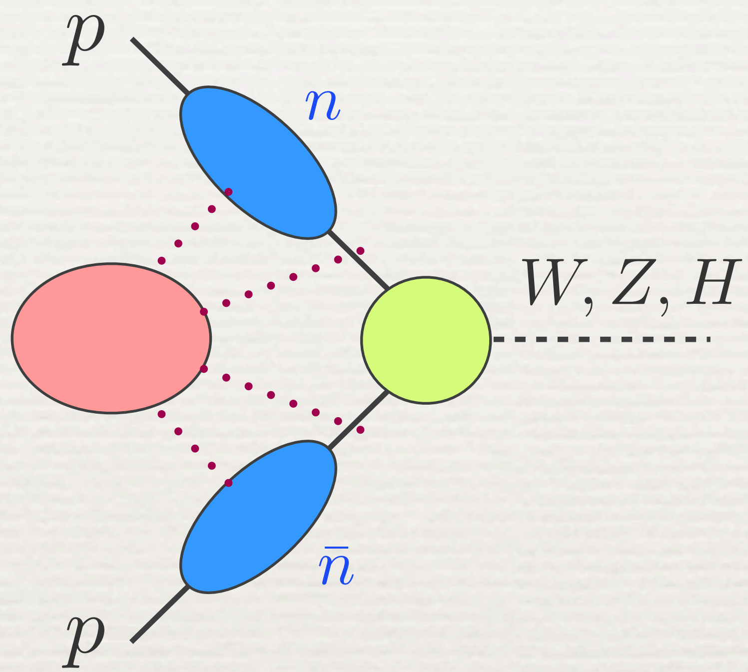
Becher, MN 2006

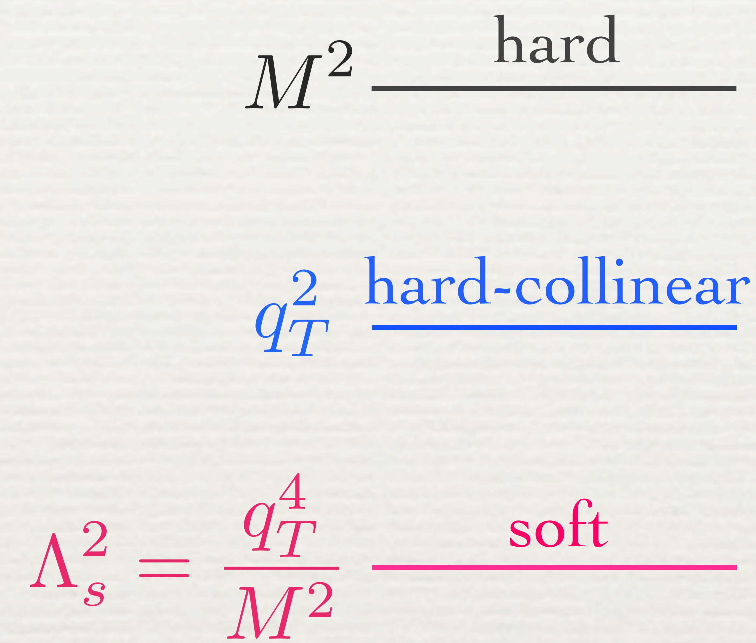


Sen 1983; Kidonakis, Oderda, Sterman 1998

Drell-Yan cross section in SCET

- Expect factorization theorem:





- Matching of the current onto SCET (integrate out **hard** quantum fluctuations):

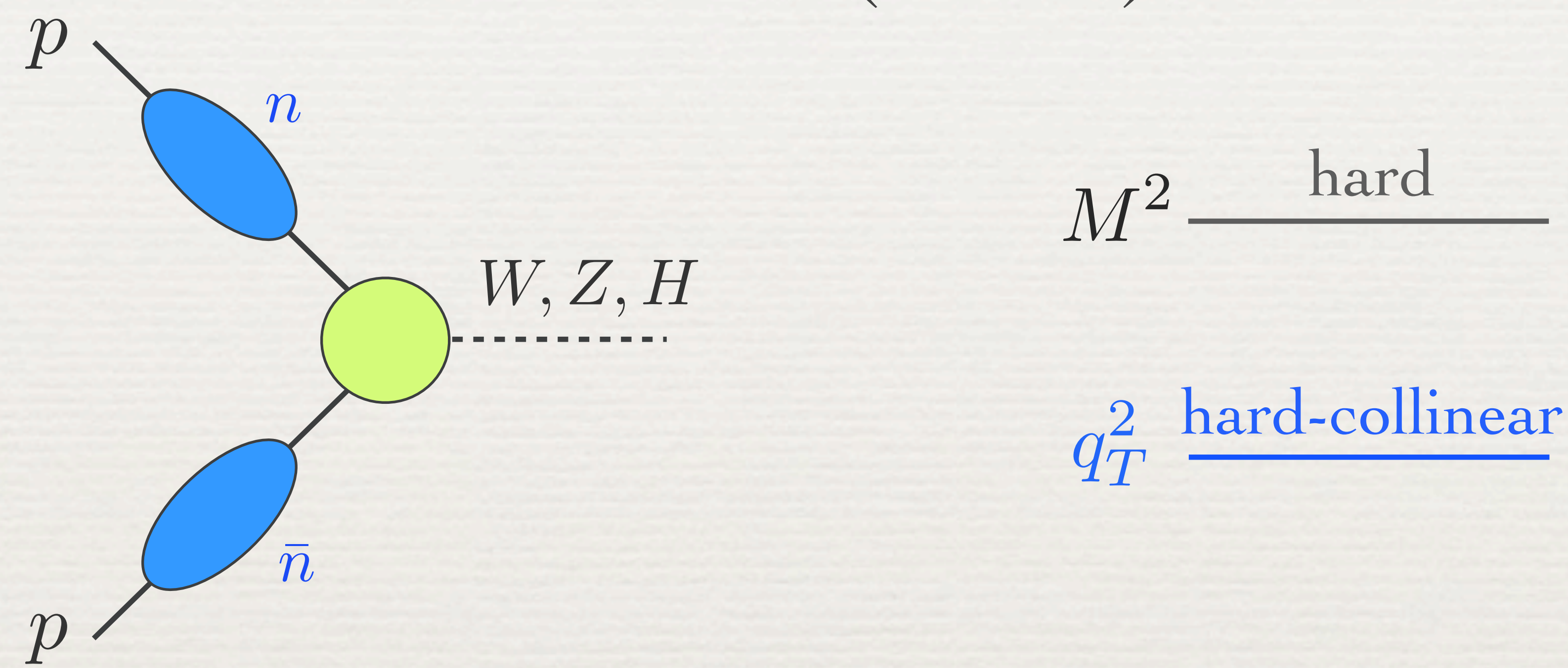
$$J^\mu = \sum_q \left(g_L^q\, \bar{q}\gamma^\mu \frac{1-\gamma_5}{2}\, q + g_R^q\, \bar{q}\gamma^\mu \frac{1+\gamma_5}{2}\, q \right)$$

soft Wilson lines

$$\to C_V(-q^2 - i\varepsilon, \mu) \sum_q \left(g_L^q\, \bar{\chi}_{hc} S_{\bar{n}}^\dagger \gamma^\mu \frac{1-\gamma_5}{2} S_n \chi_{hc} + g_R^q\, \bar{\chi}_{hc} S_{\bar{n}}^\dagger \gamma^\mu \frac{1+\gamma_5}{2} S_n \chi_{hc} \right)$$

Drell-Yan cross section in SCET

- But soft interactions cancel out (KLN)



- Result after multipole expansion:

$$d\sigma = \frac{4\pi\alpha^2}{3N_c q^2 s}\,\frac{d^4q}{(2\pi)^4}\int d^4x\, e^{-iq\cdot x}\,|C_V(-q^2,\mu)|^2 \sum_q \frac{|g_L^q|^2+|g_R^q|^2}{2}$$

$$\times \langle N_1(p)|\,\bar{\chi}_{hc}(x_+ + x_\perp)\,\frac{\not{\bar{n}}}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle \langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\,\chi_{\overline{hc}}(x_- + x_\perp)\,|N_2(\bar{p})\rangle$$

Drell-Yan cross section in SCET

A side remark:

- Absence of semi-soft contributions k~(λ,λ,λ) follows after proper multipole expansion using that x~(1,1, λ^{-1}), which implies:

$$(p-k)\cdot x = p\cdot x - k_\perp \cdot x_\perp + \mathcal{O}(\lambda)$$

- Relevant loops integrals such as

$$\int d^dk \, \frac{1}{(n\cdot k - i\epsilon)^{1+\alpha}} \, \frac{1}{(\bar{n}\cdot k - i\epsilon)^{1+\beta}} \, \delta(k^2)\, \theta(k^0)\, e^{ip\cdot x - ik_\perp \cdot x_\perp}$$

are scaleless and vanish in dimensional regularization → difference with: Mantry, Petriello 2009

Drell-Yan cross section in SCET

- Hadronic matrix elements define transverse position dependent (generalized) PDFs:

$$\phi_{q/N}(z,\mu) = \frac{1}{2\pi}\int dt\, e^{-izt\bar{n}\cdot p}\,\langle N(p)|\,\bar{\chi}(t\bar{n})\,\frac{\bar{\not{n}}}{2}\,\chi(0)\,|N(p)\rangle$$ ordinary PDF

$$\mathcal{B}_{q/N}(z,x_T^2,\mu) = \frac{1}{2\pi}\int dt\, e^{-izt\bar{n}\cdot p}\,\langle N(p)|\,\bar{\chi}(t\bar{n}+x_\perp)\,\frac{\bar{\not{n}}}{2}\,\chi(0)\,|N(p)\rangle$$ transverse PDF

- Differential cross section:

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\big|C_V(-M^2,\mu)\big|^2\,\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}$$
$$\times\sum_q e_q^2\,\Big[\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(\xi_2,x_T^2,\mu) + (q\leftrightarrow\bar{q})\Big] + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

where:

$$\xi_1 = \sqrt{\tau}\,e^{y}\,,\qquad \xi_2 = \sqrt{\tau}\,e^{-y}\,,\qquad \text{with}\quad \tau = \frac{m_\perp^2}{s} = \frac{M^2+q_T^2}{s}$$

Drell-Yan cross section in SCET

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- Resummation of large logarithms $(\alpha_s \ln^2 M/q_T)^n$ is accomplished by solving RGE $(q^2 = M^2)$:

$$\frac{d}{d\ln\mu}C_V(-q^2,\mu) = \left[\Gamma_{\rm cusp}^F(\alpha_s)\,\ln\frac{-q^2}{\mu^2} + 2\gamma^q(\alpha_s)\right]C_V(-q^2,\mu)$$

→ see SCET papers: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005

Drell-Yan cross section in SCET

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This formula is wrong!

where:

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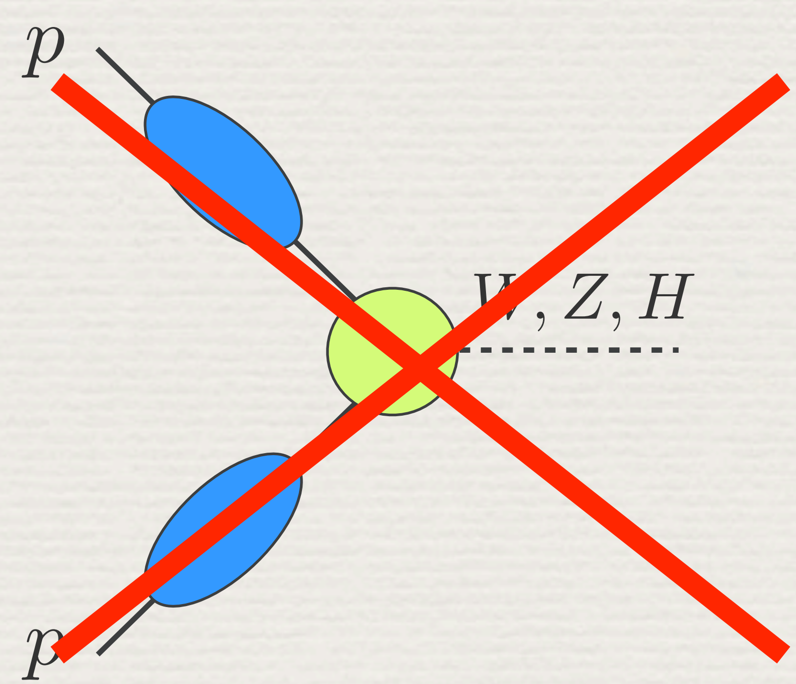
Collinear anomaly

- RG invariance of the cross section requires that the product $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ of generalized PDFs must carry an anomalous dependence on hard momentum transfer q^2:*)

$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu)$$

with:

$$\frac{dF_{q\bar{q}}(x_T^2, \mu)}{d\ln\mu} = 2\Gamma_{\text{cusp}}^F(\alpha_s)$$



*) Similar effect (in simpler setting) occurs for Sudakov form factor of a massive vector boson, see: Giu, Golf, Kelley, Manohar 2007

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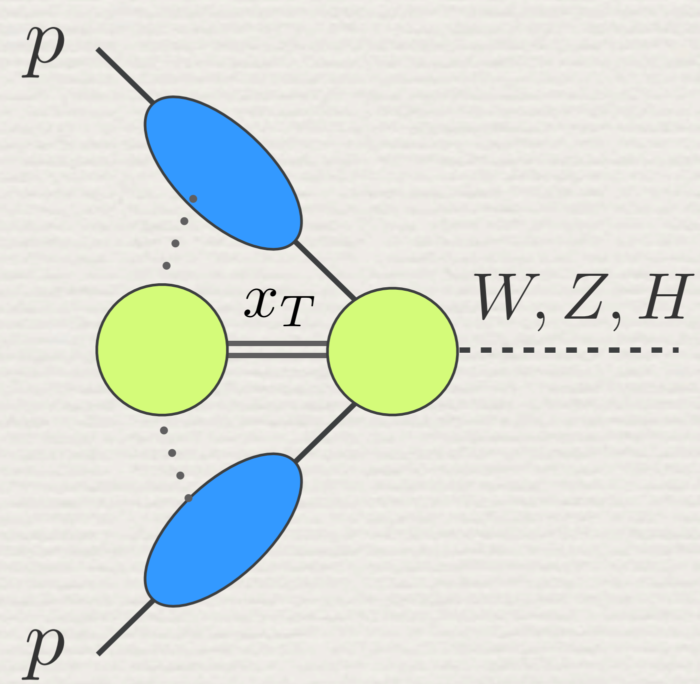
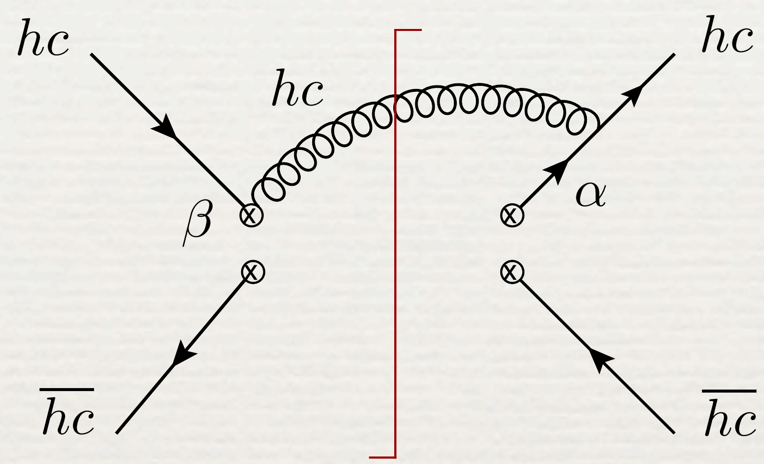


- Refactorization theorem:

Collinear anomaly

- Hard-collinear SCET loops graphs such as



  are not defined in dimensional regularization and require analytic regularization

- Not a new quantum anomaly of QCD, but a feature of the effective theory relevant to derivations of QCD factorization theorems

Collinear anomaly

- In SCET, a quantum anomaly in the usual sense, that a symmetry of the classical Lagrangian is broken by regularization:
  $\mathcal{L}_{hc}\ (\mathcal{L}_{\overline{hc}})$ invariant under: $\bar{p} \to \bar{\lambda}\bar{p} \quad (p \to \lambda p)$
- Regularization breaks this to subgroup $\lambda\bar{\lambda} = 1$, allowing for anomalous dependence on $q^2 = 2p \cdot \bar{p}$
- Consequence is that only the product of all generalized PDFs in a process is well defined, but individual transverse PDFs are not!

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- Regularization breaks this to subgroup $\lambda\bar{\lambda} = 1$, allowing for anomalous dependence on $q^2 = 2p \cdot \bar{p}$
- New functions $B_{i/N}$ obey well-defined RGEs:

$$\frac{d}{d\ln\mu} B_{q/N}(z, x_T^2, \mu) = \left[\Gamma_{\rm cusp}^F(\alpha_s)\, \ln\frac{x_T^2\mu^2}{4e^{-2\gamma_E}} - 2\gamma^q(\alpha_s)\right] B_{q/N}(z, x_T^2, \mu)$$

- Solves decade-old problem of how to make sense of transverse PDFs!

→ see e.g. review: Collins 2003

Factorized Drell-Yan cross section

- Correct factorization formula reads:

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \left|C_V(-M^2,\mu)\right|^2 \frac{1}{4\pi} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp} \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$
$$\times \sum_q e_q^2 \left[B_{q/N_1}(\xi_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q})\right] + \mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

- For $q_T \sim x_T^{-1} \sim \Lambda_{QCD}$ the functions $F_{q\bar{q}}$ and $B_{i/N}$ are genuinely non-perturbative objects, which must be extracted from data, e.g.:

$$\frac{9M^2 s}{4\pi\alpha^2} \int_0^\infty dq_T^2\, J_0(q_T x_T)\, \frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \left|C_V(-M^2,\mu)\right|^2 \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$
$$\times \sum_q e_q^2 \left[B_{q/N_1}(\xi_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q})\right] + \mathcal{O}\left(\frac{1}{x_T^2 M^2}\right)$$

Comparison with the CSS formula

- Classic result from Collins-Soper-Sterman: 1985

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2\sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2}$$

$$\times\exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$

$$\times\left[\overline{\mathcal{P}}_{q/N_1}(\xi_1,x_T,\mu_b)\,\overline{\mathcal{P}}_{\bar{q}/N_2}(\xi_2,x_T,\mu_b)+(q,i\leftrightarrow\bar{q},j)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- Disadvantages compared with our result:
  - $\bar{\mu}$ integral hits the Landau pole of running coupling and requires PDFs at arbitrarily low scales
  - practical calculations employ an x_T-space cutoff, which is model dependent and requires adding some ad hoc nonperturbative corrections

Comparison with the CSS formula

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$$\times\,\exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}\,A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$

$$\times\left[\overline{\mathcal{P}}_{q/N_1}(\xi_1,x_T,\mu_b)\,\overline{\mathcal{P}}_{\bar{q}/N_2}(\xi_2,x_T,\mu_b)+(q,i\leftrightarrow\bar{q},j)\right]$$

$$\mu_b = \frac{2e^{-\gamma_E}}{x_T}$$

- Equivalence to our result, once we identify:

$$A(\alpha_s) = \Gamma_{\text{cusp}}^F(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_1(\alpha_s)}{d\alpha_s}\,,\qquad g_1(\alpha_s) = F(0,\alpha_s)$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2}\frac{dg_2(\alpha_s)}{d\alpha_s}\,,\qquad g_2(\alpha_s) = \ln\left|C_V(-\mu^2,\mu)\right|^2$$

$$\overline{\mathcal{P}}_{i/N}(\xi,x_T) = \left|C_V(-\mu_b^2,\mu_b)\right|B_{i/N}(\xi,x_T^2,\mu_b)$$

anomaly contributions

- Important that $A(\alpha_s)\neq\Gamma_{\text{cusp}}^F(\alpha_s)$ in this case!

Comparison with the CSS formula

- Fact that $A(\alpha_s) \neq \Gamma^F_{\rm cusp}(\alpha_s)$ was missed by all previous SCET analyses! Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009
- From known expression for $B(\alpha_s)$ we can extract the two-loop result for $F(0,\alpha_s)$
- Can then predict the three-loop anomaly contribution to $A(\alpha_s)$ coefficient, which was unknown before but is numerically important:

$$A^{(3)} = \Gamma_2^F + 2\beta_0 d_2^q \qquad \Rightarrow \qquad \Gamma_2^F = 538.2 \text{ while } A^{(3)} = -930.8$$

→ ignoring extra term reduces cross section by 1% at q_T=4 GeV, raising to 2.6% at q_T=2 GeV (larger effect for Higgs prod.)

Simplifications for large q_T

Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

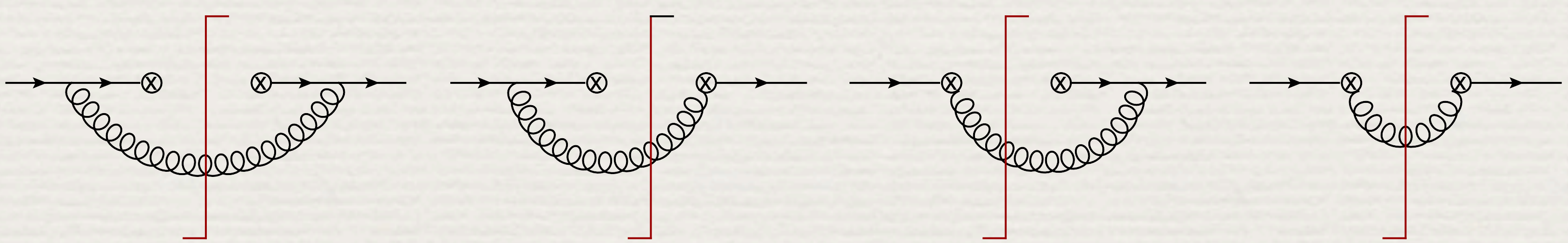
- Generalized PDFs at small transverse separation can be expanded in usual PDFs:

$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

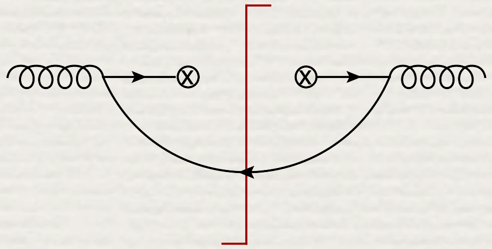
$$B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} I_{i\leftarrow j}(\xi/z, x_T^2, \mu)\, \phi_{j/N}(z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

- Expansion kernels are obtained from matching calculation

$\mathcal{I}_{q\leftarrow q}$:



$\mathcal{I}_{q\leftarrow g}$:

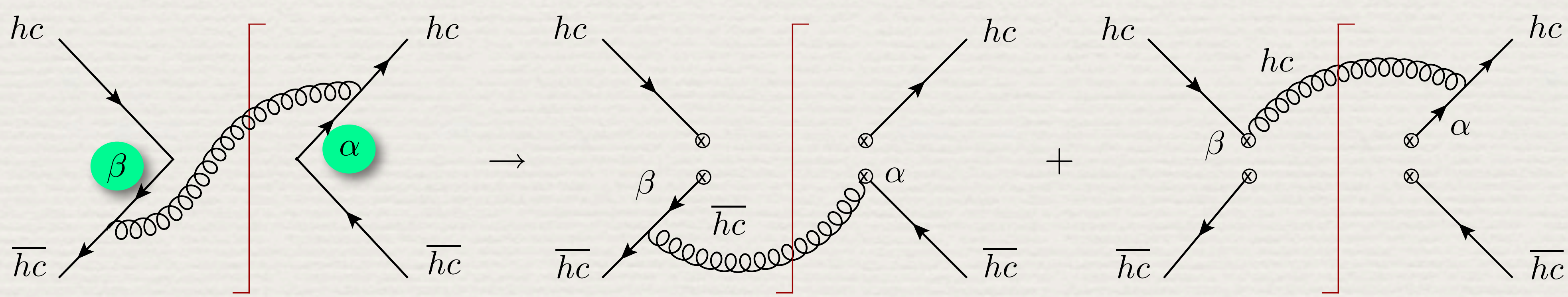


Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- Collinear loops are not defined and require a regulator beyond dimensional regularization
- Most economic possibility is to use analytic regularization scheme: Smirnov 1993

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- Adaption to SCET collinear propagators:



breaks rescaling invariance

$$\frac{n^\mu}{n\cdot k - i\varepsilon} \to \frac{\nu_1^{2\alpha}\, n^\mu\, \bar{n}\cdot p}{(n\cdot k\, \bar{n}\cdot p - i\varepsilon)^{1+\alpha}}$$

regularized Wilson lines

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

regularized propagator

Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- Introducing analogous regulator β in anti-collinear sector, we find:

$$L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}$$

$$\mathcal{I}_{q\leftarrow q}(z, x_T^2, \mu)\Big|_{\alpha\ \text{reg.}} = -\frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(\frac{2}{\alpha} - 2 \ln \frac{\mu^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] + \delta(1-z) \left(-\frac{2}{\epsilon^2} + L_\perp^2 + \frac{\pi^2}{6} \right) - (1-z) \right\} .$$

$$\mathcal{I}_{q\leftarrow q}(z, x_T^2, \mu)\Big|_{\beta\ \text{reg.}} = -\frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(-\frac{2}{\beta} + 2 \ln \frac{q^2}{\nu_2^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] - (1-z) \right\}$$

- The product of two such functions is regulator independent:

anomalous hard logarithm

$$\left[\mathcal{I}_{q\leftarrow q}(z_1, x_T^2, \mu)\, \mathcal{I}_{\bar{q}\leftarrow \bar{q}}(z_2, x_T^2, \mu) \right]_{q^2}$$
$$= \delta(1-z_1)\, \delta(1-z_2) \left[1 - \frac{C_F \alpha_s}{2\pi} \left(2L_\perp \ln \frac{q^2}{\mu^2} + L_\perp^2 - 3L_\perp + \frac{\pi^2}{6} \right) \right]$$
$$- \frac{C_F \alpha_s}{2\pi} \left\{ \delta(1-z_1) \left[L_\perp \left(\frac{1+z_2^2}{1-z_2} \right)_+ - (1-z_2) \right] + (z_1 \leftrightarrow z_2) \right\} + \mathcal{O}(\alpha_s^2)$$

Short-distance expansion for $x_T \ll \Lambda_{QCD}^{-1}$

- From previous result we read off:

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2)$$

$$\begin{aligned} I_{q\leftarrow q}(z, L_\perp, \alpha_s) &= \delta(1-z)\left[1 + \frac{C_F\alpha_s}{4\pi}\left(L_\perp^2 + 3L_\perp - \frac{\pi^2}{6}\right)\right] \\ &\quad - \frac{C_F\alpha_s}{2\pi}\left[L_\perp P_{q\leftarrow q}(z) - (1-z)\right] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

$$I_{q\leftarrow g}(z, L_\perp, \alpha_s) = -\frac{T_F\alpha_s}{2\pi}\left[L_\perp P_{q\leftarrow g}(z) - 2z(1-z)\right] + \mathcal{O}(\alpha_s^2)$$

Altarelli-Parisi splitting functions

- Two-loop result for $F_{q\bar{q}}(L_\perp, \alpha_s) = \sum_{n=1}^{\infty} d_n^q(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$:

$$d_2^q(L_\perp) = \frac{\Gamma_0^F \beta_0}{2} L_\perp^2 + \Gamma_1^F L_\perp + d_2^q\,, \quad d_2^q = C_F C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{27} C_F T_F n_f$$

Factorized Drell-Yan cross section

- Final factorization formula reads:

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2}$$

$$\times\left[C_{q\bar{q}\to ij}\left(\frac{\xi_1}{z_1},\frac{\xi_2}{z_2},q_T^2,M^2,\mu\right)\phi_{i/N_1}(z_1,\mu)\,\phi_{j/N_2}(z_2,\mu)+(q,i\leftrightarrow\bar{q},j)\right]$$

- Hard-scattering kernels:

$$C_{q\bar{q}\to ij}(z_1,z_2,q_T^2,M^2,\mu) = \left|C_V(-M^2,\mu)\right|^2\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$

$$\times\, I_{q\leftarrow i}(z_1,x_T^2,\mu)\,I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)$$

- Final task is to perform the Fourier transform, which can be done either numerically or in quasi-closed form

Subtleties and surprises

Asymptotic divergence

- Leading behavior follows from ($\ell = L_\perp$):

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp - \frac{1}{4}aL_\perp^2} = \frac{e^{-2\gamma_E}}{\mu^2}\int_{-\infty}^{\infty} d\ell\, J_0\!\left(e^{\ell/2}\, b_0\, \frac{q_T}{\mu}\right) e^{(1-\eta)\ell - \frac{1}{4}a\ell^2}$$

where $\eta = \dfrac{C_F\alpha_s}{\pi}\ln\dfrac{M^2}{\mu^2}$ and $a \sim \alpha_s$

- With proper choice of scale $L_\perp = \mathcal{O}(1)$ it looks like one could expand the quadratic term in ℓ, but this generates strong factorial growth

$$\sum_{n=0}^{\infty}\frac{(2n)!}{n!}\left(-\frac{a}{4}\right)^n\left[\frac{1}{(1-\eta)^{2n+1}} - e^{-2\gamma_E}\right]$$

first noted by:
Frixione, Nason, Ridolfi 1999

Asymptotic divergence

- Series is Borel summable (just keep quadratic term in exponent)
- Gives rise to highly non-trivial dependence on a:

$$\sqrt{\frac{\pi}{a}}\left\{e^{\frac{(1-\eta)^2}{a}}\left[1-\mathrm{Erf}\left(\frac{1-\eta}{\sqrt{a}}\right)\right]-e^{-2\gamma_E+\frac{1}{a}}\left[1-\mathrm{Erf}\left(\frac{1}{\sqrt{a}}\right)\right]\right\}$$

- Perturbative expansion of this result has zero radius if convergence
- Hints at important non-perturbative effect of short-distance nature! Precise meaning?

Asymptotically large M^2

- Careful analysis shows that the appropriate choice of μ eliminating large logarithms from integral is $\mu \sim \langle x_T^{-1} \rangle \sim \max(q_T, q_*)$, where:

$$q_* = M \, \exp\left(\frac{\pi}{2C_F \alpha_s(q_*)}\right)$$

  corresponding to $\eta = 1$

- For M=m$_Z$, one finds that $q_* \approx 2\,\mathrm{GeV}$ is in the perturbative domain

→ spectrum can be calculated down to q$_T$=0 using short-distance methods !

Intercept at q_T=0

- Dedicated analysis of $q_T \to 0$ limit yields:

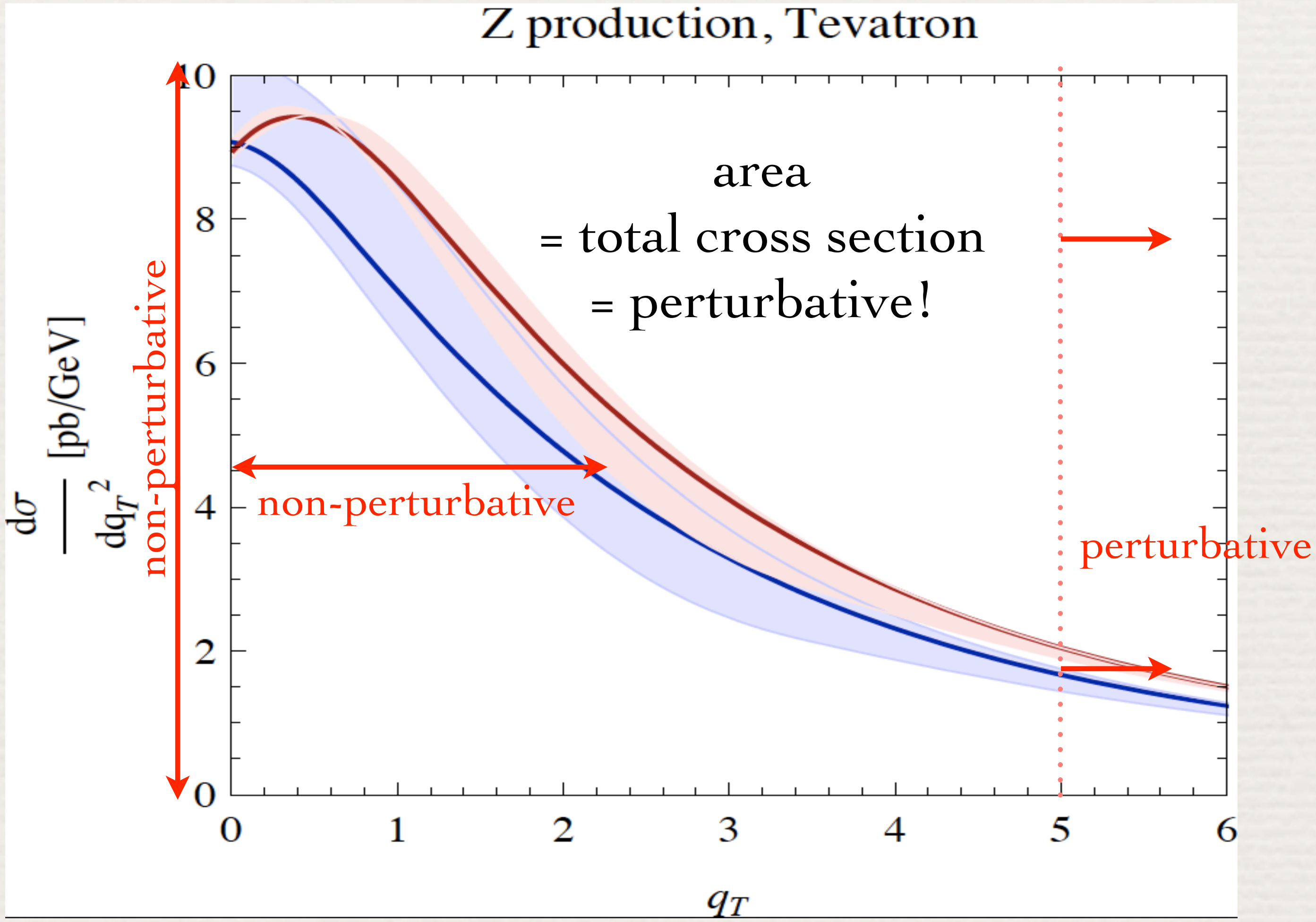
$$\frac{d\sigma}{dq_T^2} \sim \frac{\mathcal{N}}{\sqrt{\alpha_s}}\, e^{-\#/\alpha_s} \left(1 + c_1\alpha_s + \ldots\right)$$

Parisi, Petronzio 1979;
Collins, Soper, Sterman 1985; Ellis, Veseli 1998

- Were for the first time we are able to compute the normalization $\mathcal{N}$ and NLO coefficient c_1
- Expression cannot be expanded about $\alpha_s = 0$ (essential singularity)

The big picture

- Borel resummation at moderate q_T interpolates between the non-perturbative result at q_T=0 and the perturbative result at large q_T



- Essential features are non-perturbative!

More surprises

- Once we can calculate the intercept at $q_T=0$, what about derivatives w.r.t. q_T^2 (i.e., entire spectrum at very small q_T)?
- Analyzing once again the leading behavior

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp-\frac{1}{4}aL_\perp^2} = \frac{e^{-2\gamma_E}}{\mu^2}\int_{-\infty}^{\infty} d\ell\, J_0\!\left(e^{\ell/2}\, b_0\, \frac{q_T}{\mu}\right) e^{(1-\eta)\ell-\frac{1}{4}a\ell^2}$$

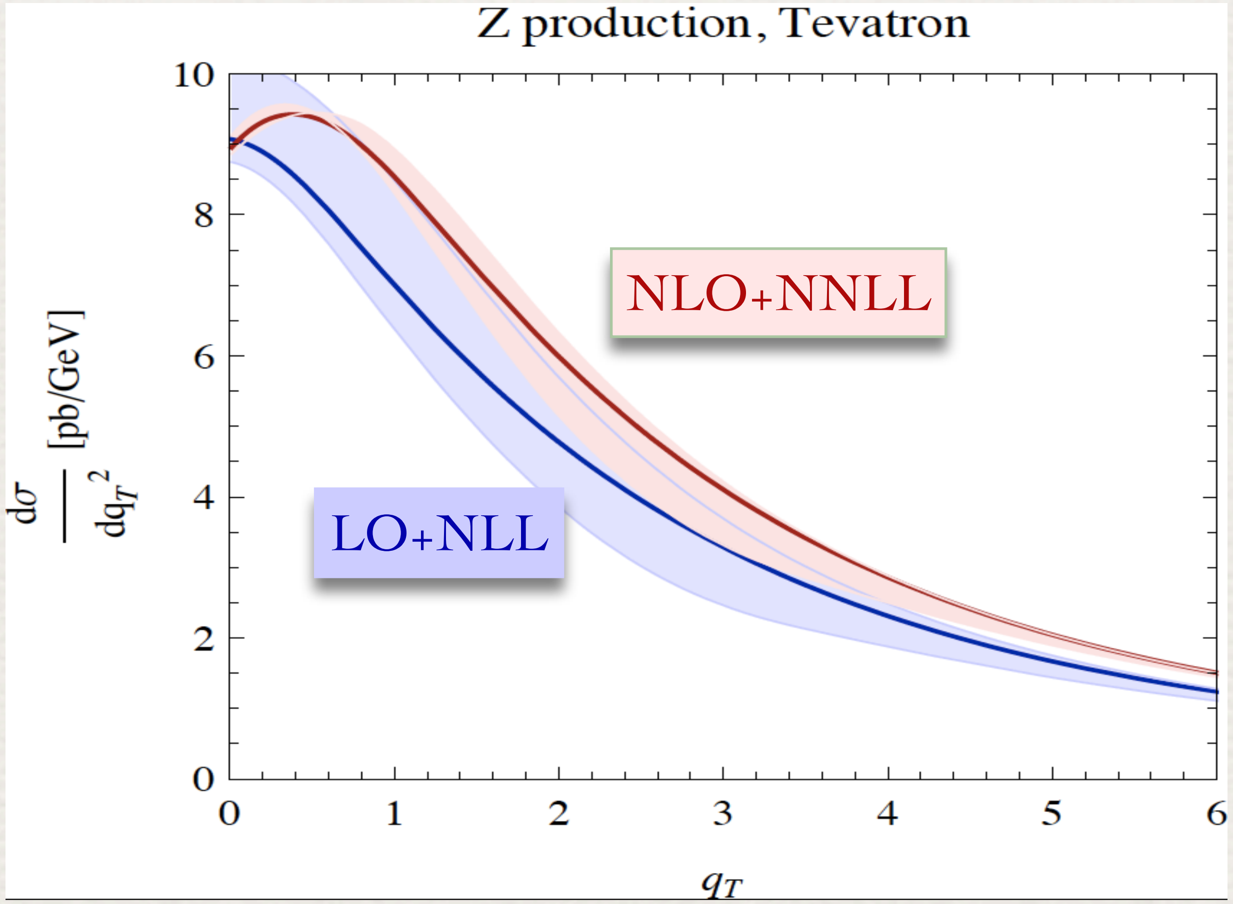
we find an **extremely strong** divergent behavior:

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\, e^{\# n^2/\alpha_s}\left(\frac{q_T^2}{q_*^2}\right)^n$$

incredibly violent divergence!

More surprises

- Spectrum can be calculated numerically, even though power expansion in q_T^2 is absolutely meaningless (not even Borel summable)!
- Find smooth behavior down to very small q_T



More surprises

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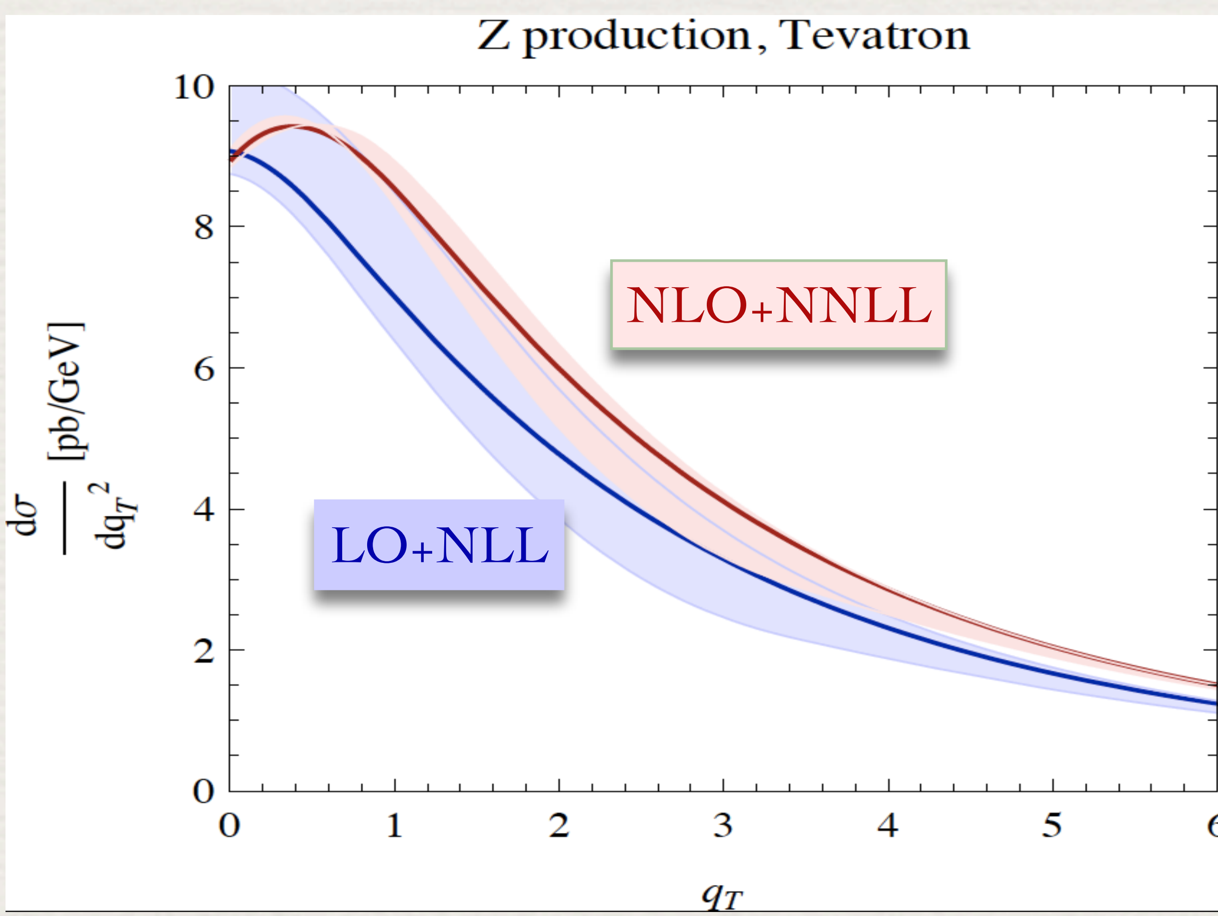
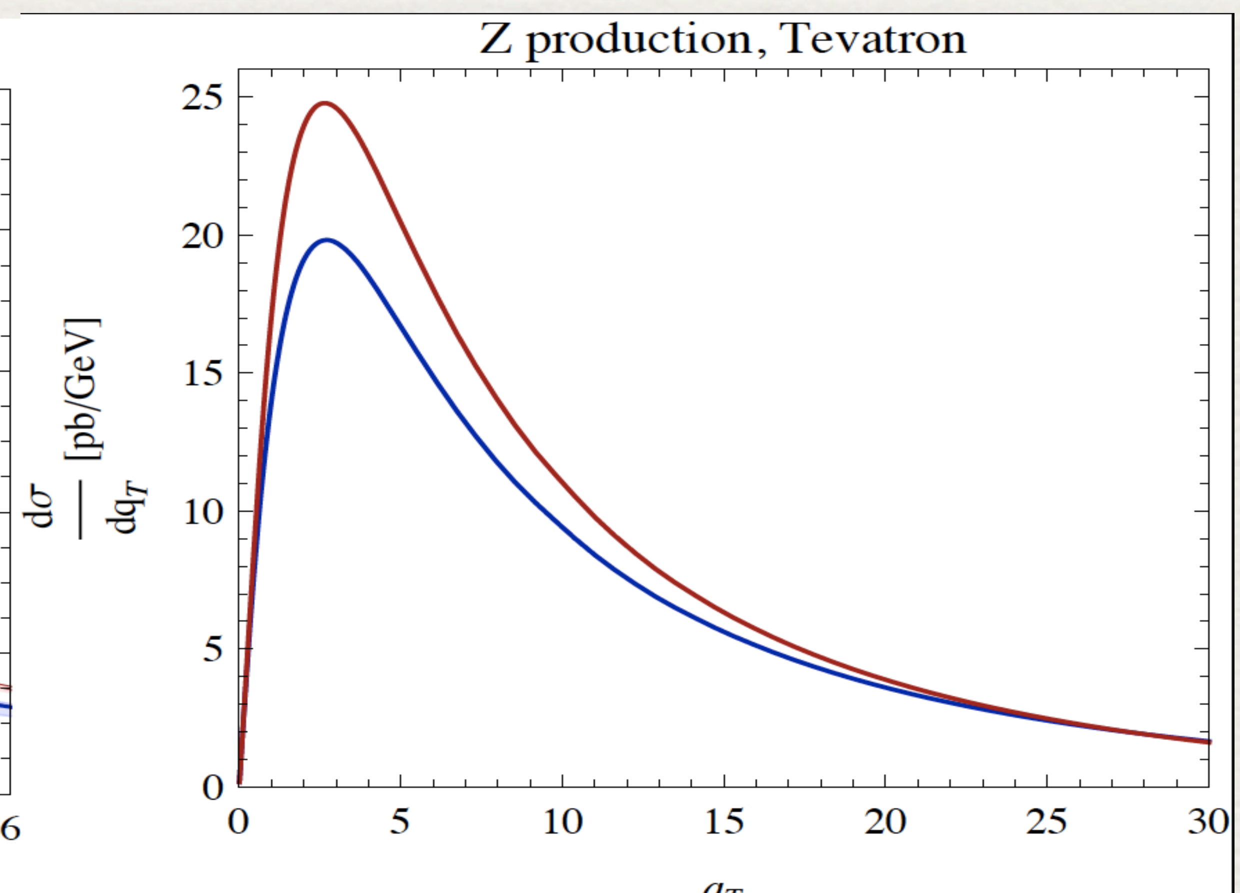




Yet more surprises

- Related question is that about the impact of long-distance power correction in matching relation

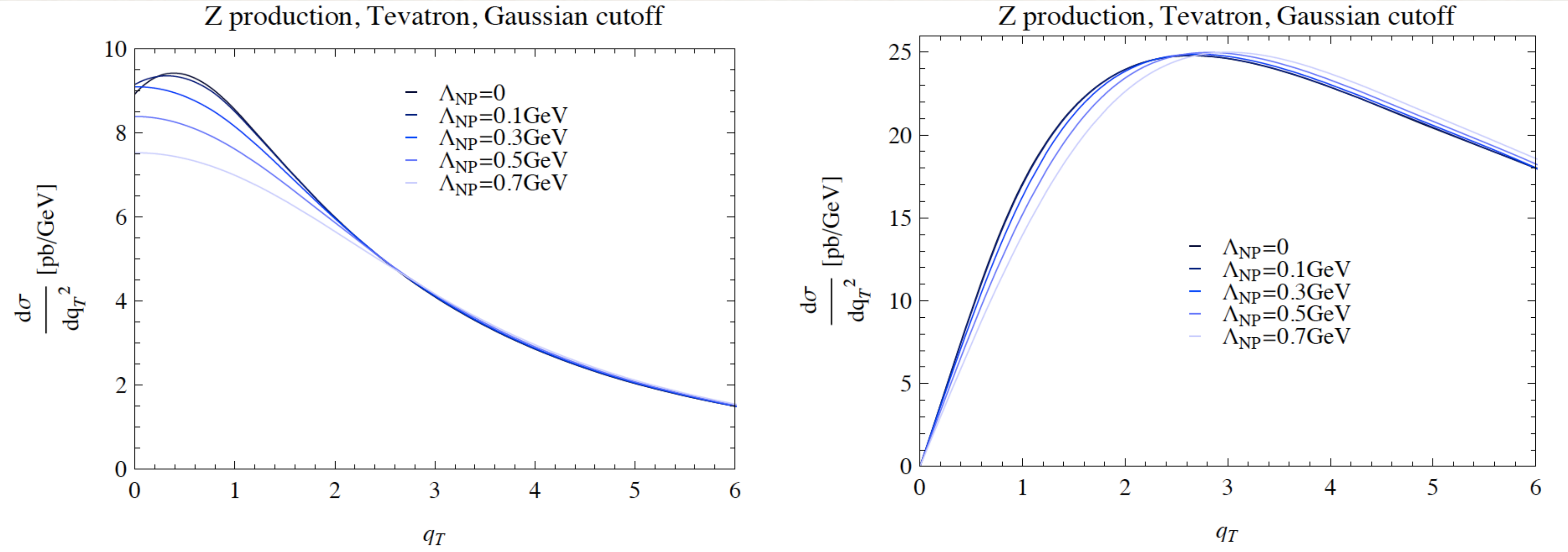
$$B_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z} \, I_{i\leftarrow j}(z, x_T^2, \mu) \, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2 \, x_T^2)$$

- Find that these cannot be analyzed order by order, but only numerically using functions that vanish at large x_T^2, such as $e^{-\Lambda^2 x_T^2}$ or $\theta(1 - \Lambda^2 x_T^2)$

- Fixed-order OPE in x_T^2 is again extremely divergent

Yet more surprises

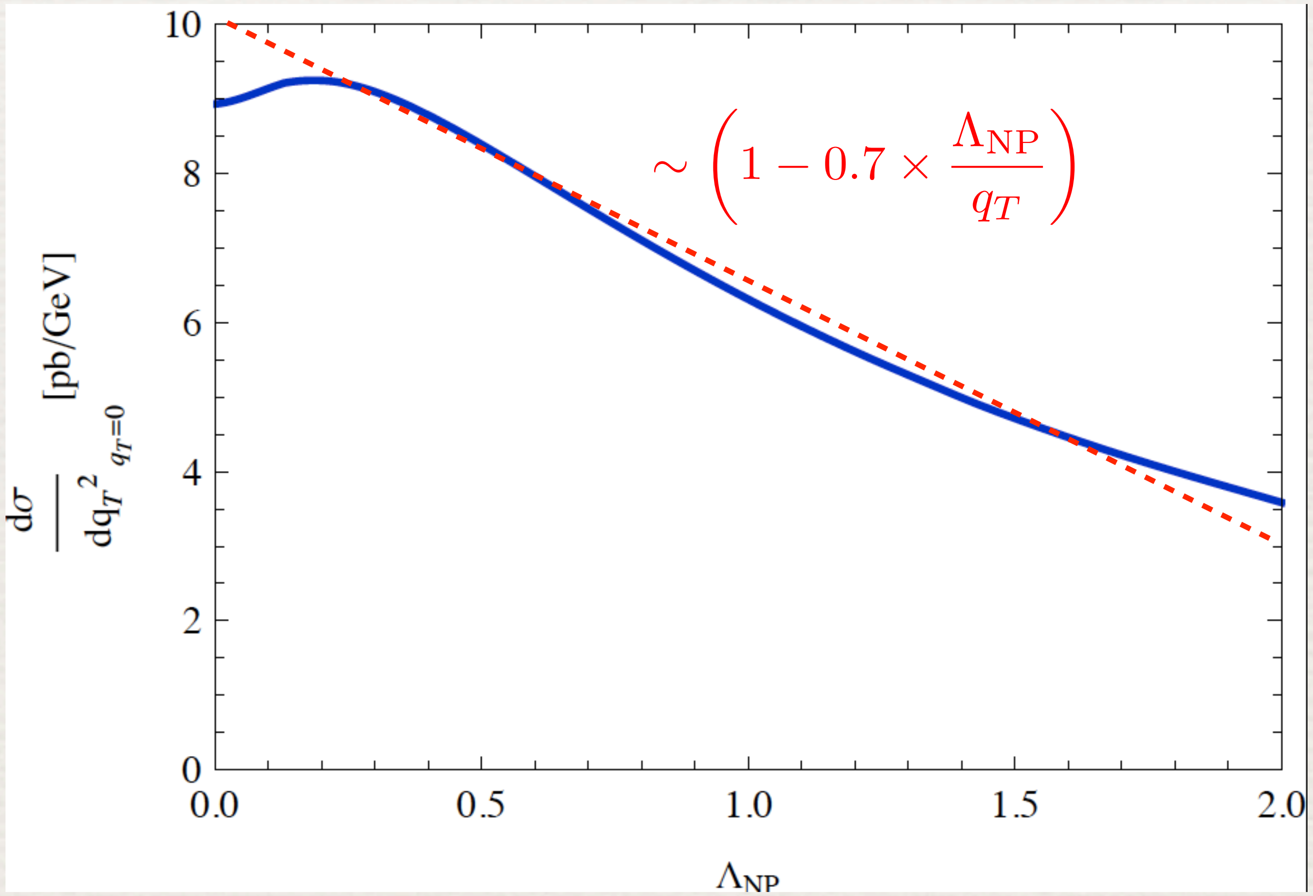
- Yet resummed behavior is smooth and rather insensitive to the way in which the cutoff is introduced:



- Indications that long-distance effects are very small already above q_T=2 GeV

Yet more surprises

- Resulting power correction has a complicated shape, but is approximately linear:



- Cannot be described in terms of a single operator matrix element

Conclusions

- Effective field theory provides efficient tools for addressing difficult collider-physics problems
- Systematic “derivation” of factorization theorems and simple, transparent resummation techniques
- Detailed applications exist for Drell-Yan, Higgs, and top-quark pair production; first result for jets at hadron colliders emerging recently

Conclusions

- Correct SCET analysis reproduces CSS formula with a nontrivial relation between A and Γ_{cusp}
- Transverse PDFs do not exist as individual objects,*) but only products are well defined
- Such products carry anomalous dependence on hard momentum transfer q^2
- Implications for phenomenology of transverse momentum-dependent PDFs under study

*) They are gauge dependent in the standard treatment and affected by (dim. unregularized) “rapidity divergences”