Effective Theory Approach to Jet Propagation in Dense QCD Matter



Boston Jet Physics Workshop January 12, 2011

Monday, January 10, 2011

Effective Theory Approach to Jet Propagation in Dense QCD Matter



In collaboration with Ivan Vitev

Boston Jet Physics Workshop January 12, 2011

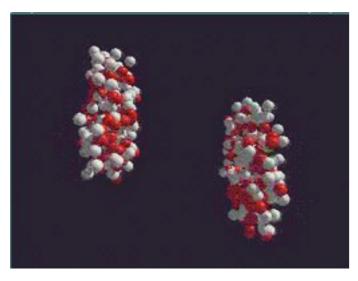
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Outline

- Introduction
- Effective theory for jets in the medium
- Results for jet broadening and jet energy loss
- Conclusions

Introduction

Motivation to study heavy-ion collisions



AA at I0GeV

Picture from http://www-subatech.in2p3.fr/~theo/qmd/hic/hic3.html

- Plenty of experimental data available
- To study the properties of Quark Gluon Plasma, predicted by QCD
- Connection to Early Universe (a few microseconds after the Big Bang)

Existing Facilities

Existing Facilities



RHIC E_{NN}=20-200GeV

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Existing Facilities

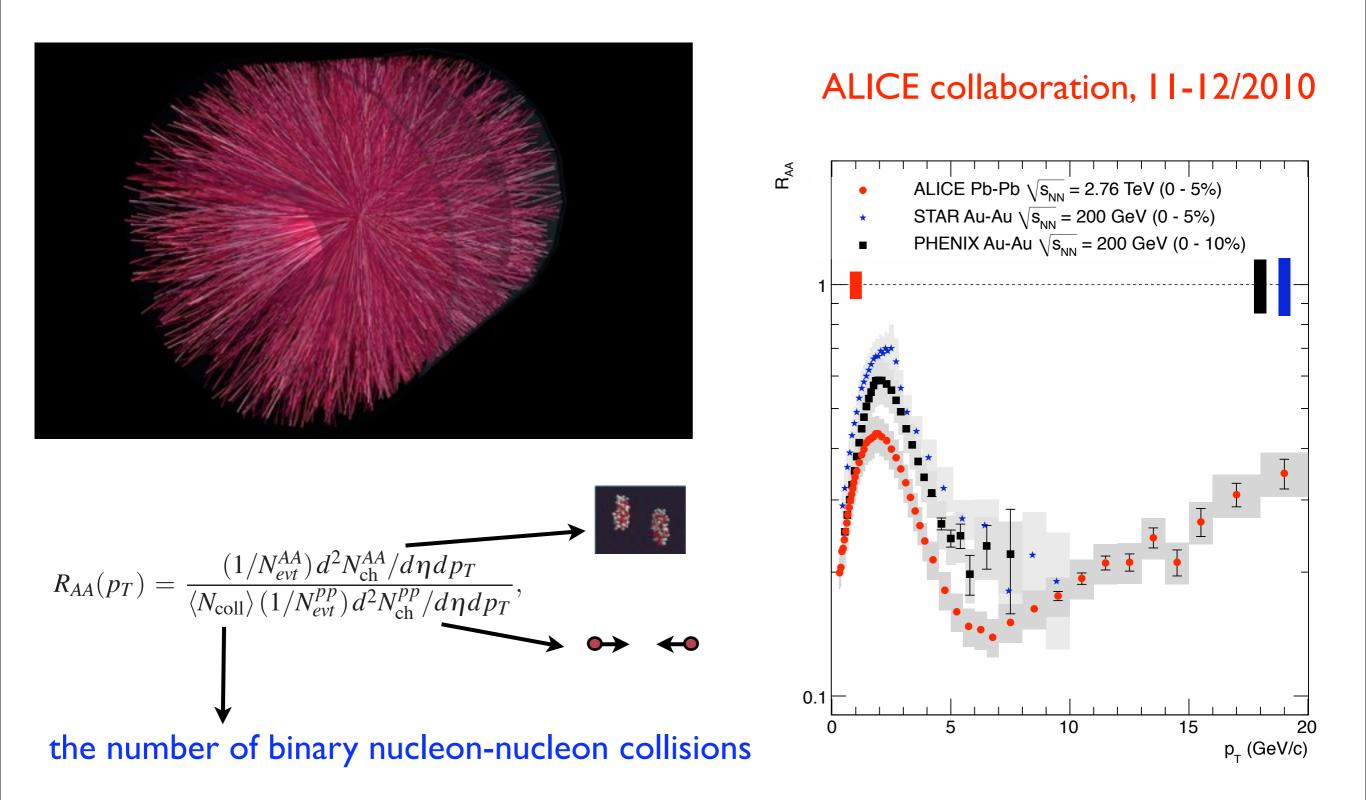


E_{NN}=2.76TeV LHC (1 month) 5.5TeV in AA 8.8TeV in pA

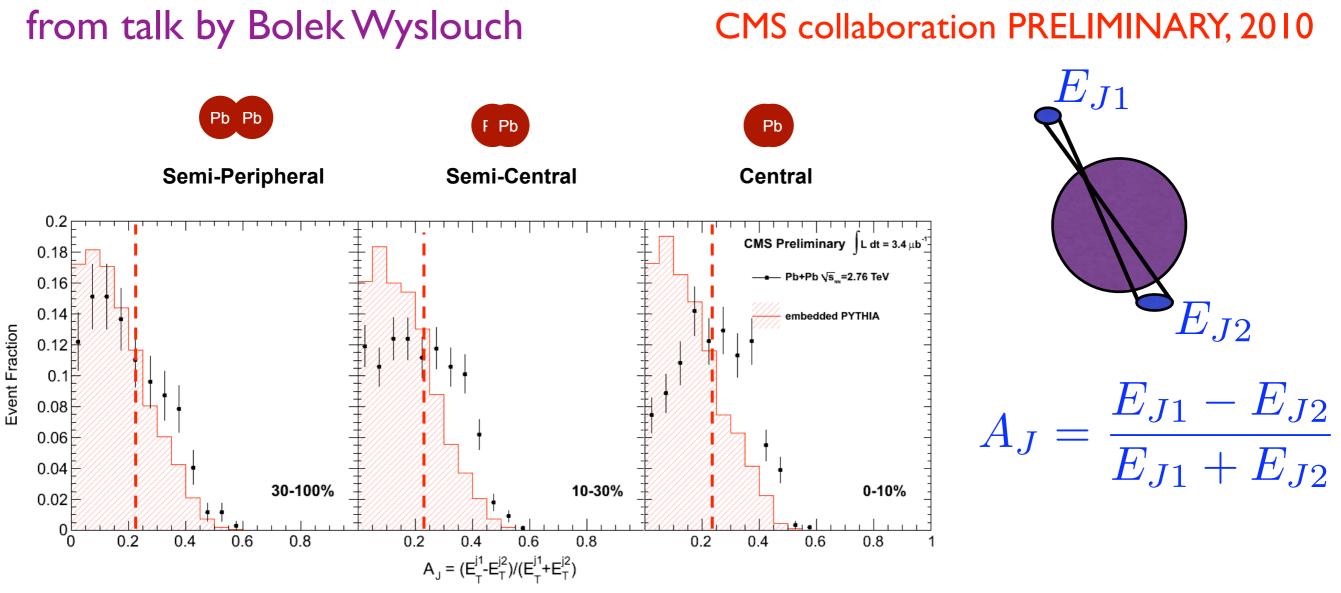
RHIC E_{NN}=20-200GeV



New LHC heavy ion data!

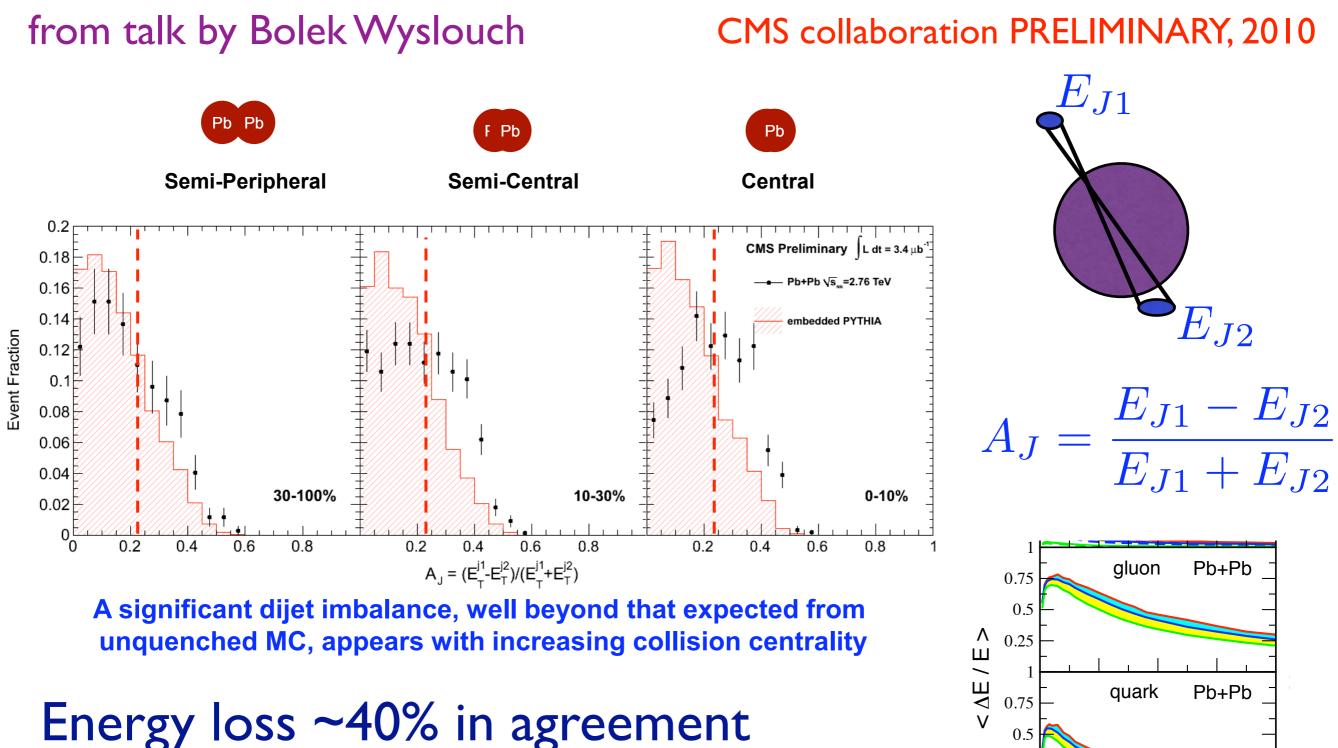


New LHC heavy ion data!



A significant dijet imbalance, well beyond that expected from unquenched MC, appears with increasing collision centrality

New LHC heavy ion data!



0.25

0

50

100

E_{iet} [GeV]

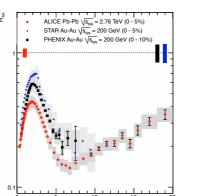
150

200

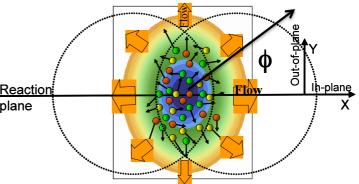
with prediction in Vitev, 06

Strong indications of QGP production

R_{AA} suppression (RHIC, LHC)



- Azimuthal angle di-hadron(Jet) correlations IAA (RHIC, LHC)
- Elliptic flow v₂ (RHIC, LHC)

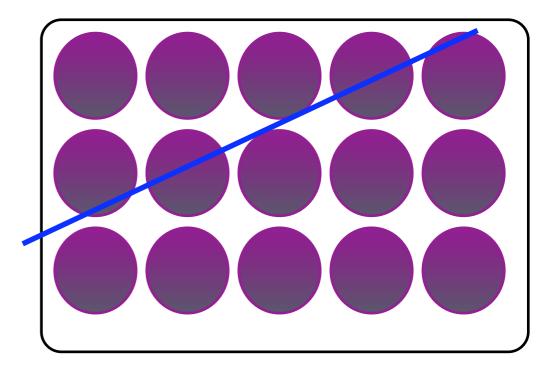


So far R_{AA} and I_{AA} have been analyzed using leading particle approach

Using jets is a new promising direction in heavy ion collisions

Theoretical Approaches

- PQCD
- Thermal Field Theory
- Lattice QCD
- Hydrodynamics
- AdS/CFT symmetry



Gyulassy-Wang model Gyulassy, Wang, 94

 The medium is modeled with a finite number of scattering centers with static Debyescreened potential

 $H = \sum_{n=1}^{N} H(q; x_n) = 2\pi\delta(q^0) v(q) \sum_{n=1}^{N} e^{iqx_n} T^a(R) \otimes T^a(n)$ $v(q) = \frac{4\pi\alpha_s}{q_z^2 + q^2 + \mu^2}$

Gyulassy-Wang model Gyulassy, Wang, 94

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XI

Xi

•••

Gyulassy-Wang model Gyulassy, Wang, 94

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$$v(q) = \frac{4\pi \alpha_s}{q_z^2 + \mathbf{q}^2 + \mu^2}$$
• The momentum scaling of the exchange gluon is that of the Glauber gluon:
$$q(\lambda^2, \lambda^2, \lambda)$$

XI

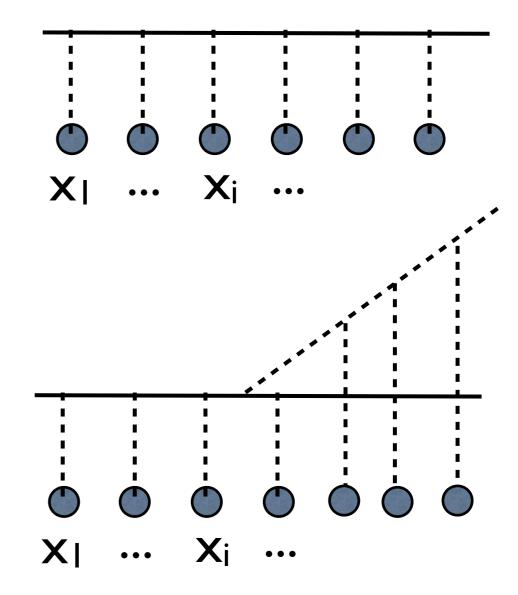
Xi

Gyulassy-Levai-Vitev reaction operator Gyulassy, Levai, Vitev, 00

 $\hat{R}_n = \hat{D}_n^\dagger \hat{D}_n + \hat{V}_n + \hat{V}_n^\dagger$

Jet broadening

Radiative energy loss



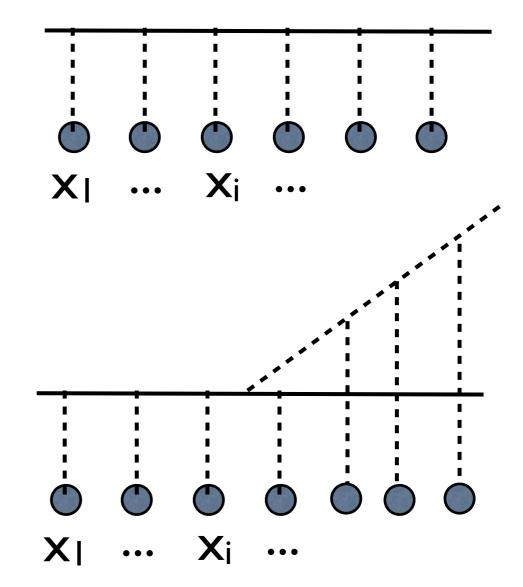
Gyulassy-Levai-Vitev reaction operator Gyulassy, Levai, Vitev, 00

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Other applications: meson dissociation, electromagnetic energy loss



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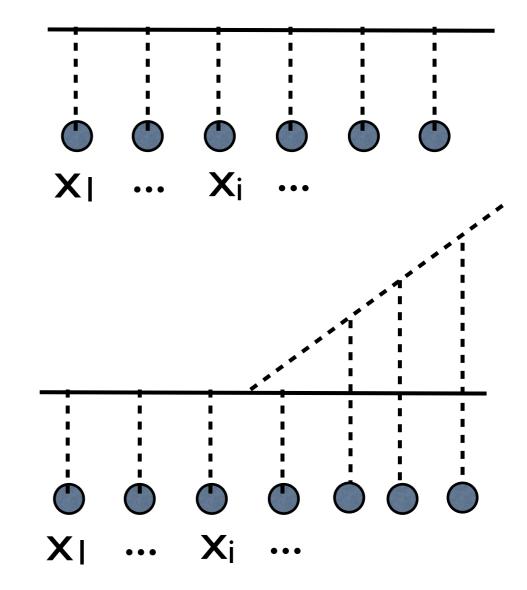
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Jet broadening

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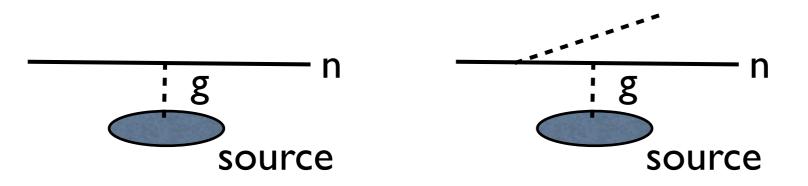
Other applications: meson dissociation, electromagnetic energy loss

This talk: How to derive Jet broadening and Radiative energy loss using Effective Field Theory?

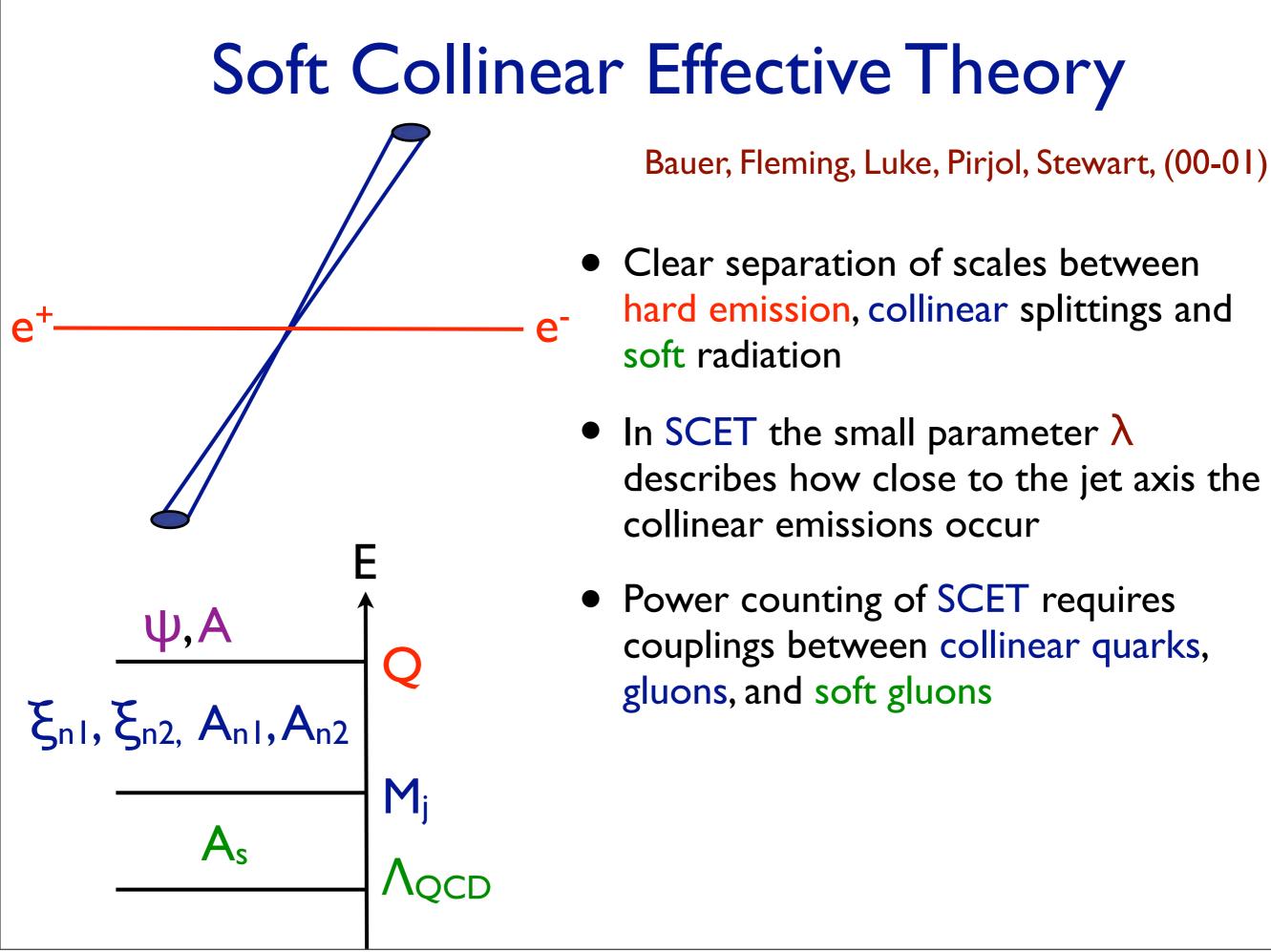


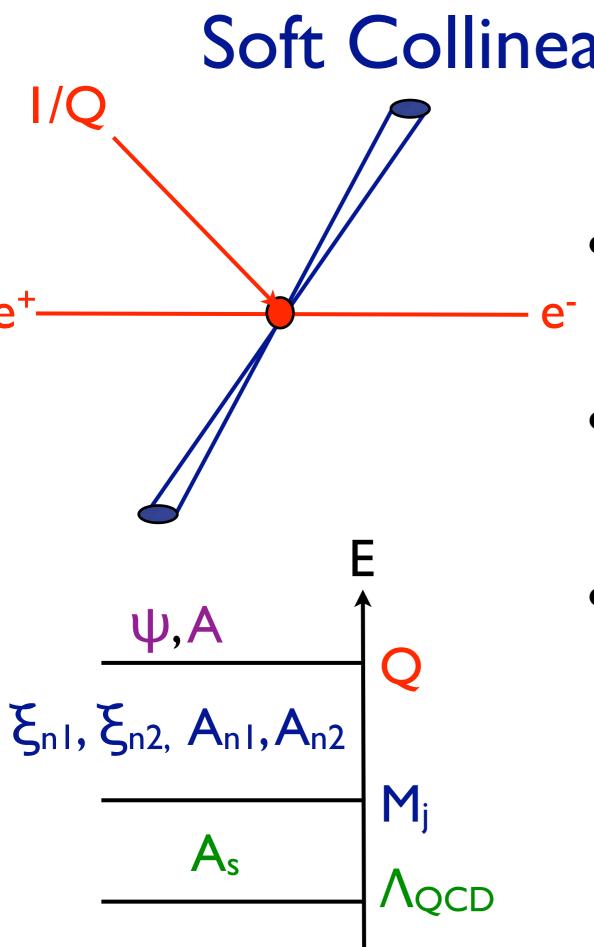
Effective Theory for Jets in the medium

Expectations from the effective theory



- Our goal is to construct an effective theory for highly energetic quarks and gluons in the medium
- Soft Collinear Effective Theory(SCET) is a good start
- Need to add the transverse(Glauber) gluons to the SCET lagrangian: SCET_G
- We want to go beyond the static source approximation
- Check gauge invariance of the broadening and bremsstrahlung

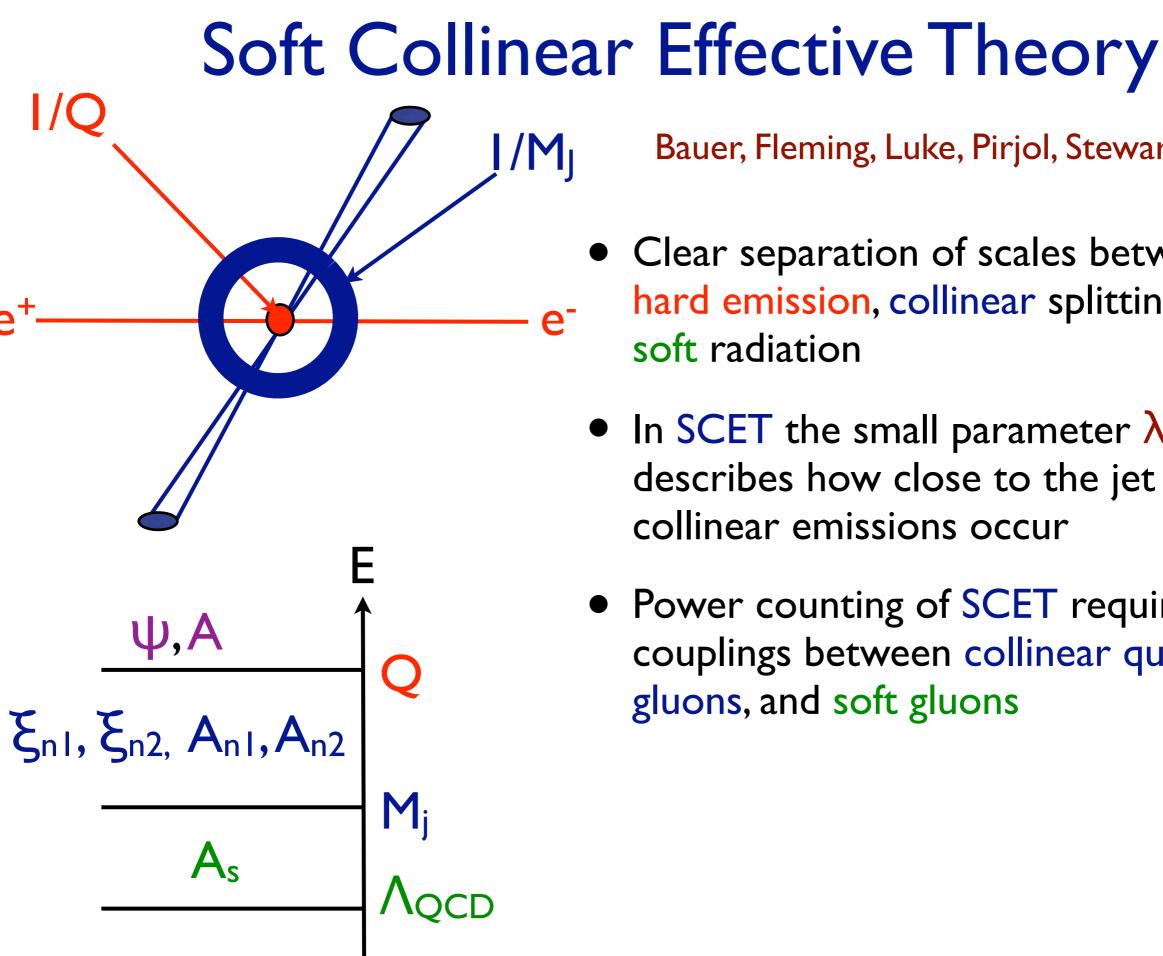




Soft Collinear Effective Theory

Bauer, Fleming, Luke, Pirjol, Stewart, (00-01)

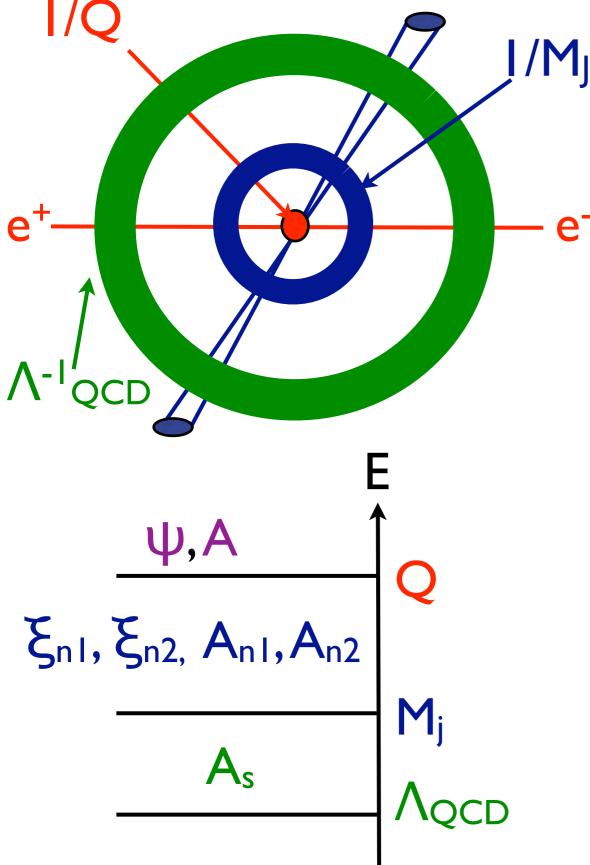
- Clear separation of scales between hard emission, collinear splittings and soft radiation
- In SCET the small parameter λ describes how close to the jet axis the collinear emissions occur
- Power counting of SCET requires couplings between collinear quarks, gluons, and soft gluons



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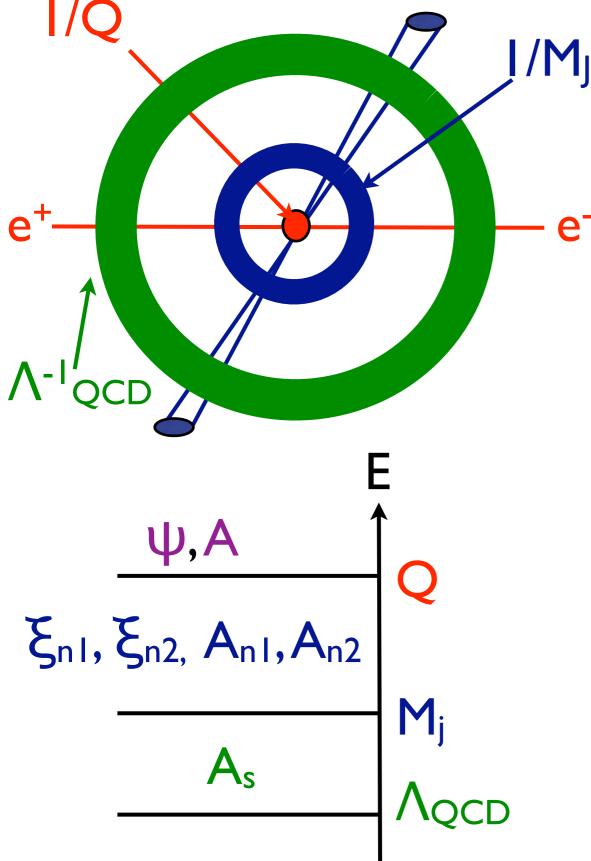
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Soft Collinear Effective Theory

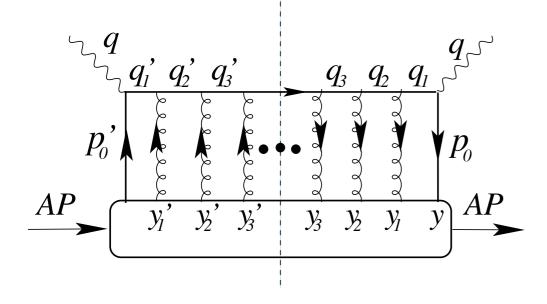


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Need to include Glauber gluons to SCET

SCET_G=SCET+Glauber gluons $q(\lambda^2, \lambda^2, \lambda)$



Idilbi, Majumder(08)

 $L_G(\xi_{\overline{n}}, A_G)$ SIDIS

- n-collinear source
- covariant, light-cone gauge

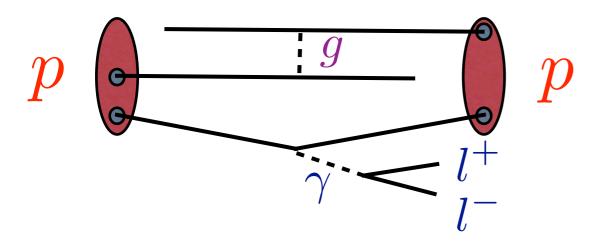
D'Eramo, Liu, Rajagopal(10)

$$P(k_{\perp}) = \int d^2 x_{\perp} e^{-ik_{\perp} \cdot x_{\perp}} \mathcal{W}_{\mathcal{R}}(x_{\perp})$$
$$\mathcal{W}_{\mathcal{R}}(x_{\perp}) = \frac{1}{d(\mathcal{R})} \left\langle \operatorname{Tr} \left[W_{\mathcal{R}}^{\dagger}[0, x_{\perp}] W_{\mathcal{R}}[0, 0] \right] \right\rangle$$
$$W_{F} \left[y^{+}, y_{\perp} \right] \equiv P \left\{ \exp \left[ig \int_{0}^{L^{-}} dy^{-} A^{+}(y^{+}, y^{-}, y_{\perp}) \right]$$

- Probability density of the scattered jet is equal to exp.value of two Wilson Lines
- Derived in the covariant gauge

SCET_G: effective theory for Drell-Yan





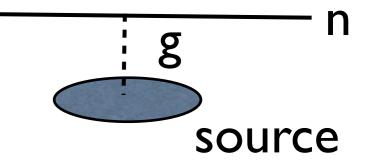
- An explicit calculation shows that for consistency of effective theory SCET should be expanded with Glauber modes to describe Drell-Yan process
- Factorization of Drell-Yan should be reconsidered using SCET_G

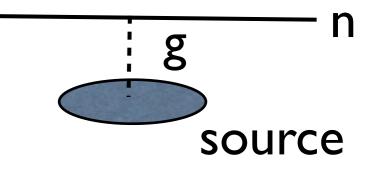
SCET_G: effective theory for Drell-Yan

Bowdin, Brodsky, Lepage, (81) Collins, Soper, Sterman, (82) Bauer, Lange, GO(10)

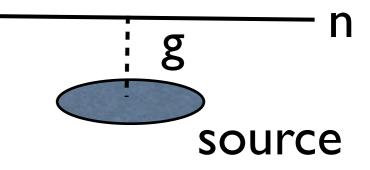
Solution Glauber (Coulomb) gluon, transverse to the beam axis: $q(\lambda^2, \lambda^2, \lambda)$

- An explicit calculation shows that for consistency of effective theory SCET should be expanded with Glauber modes to describe Drell-Yan process
- Factorization of Drell-Yan should be reconsidered using SCET_G



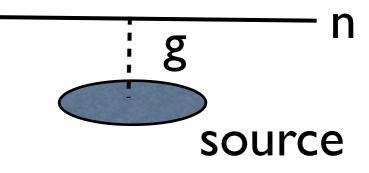


- Use full QCD fields to derive the scaling of the vector potential created by the source of the Glauber gluons
- Consider collinear, static and soft quark sources
- Use background field method to deduce the Feynman rules of interaction of the target jet with the Glauber vector potential



- Use full QCD fields to derive the scaling of the vector potential created by the source of the Glauber gluons
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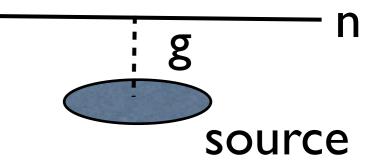
$$\mathcal{L}_{\mathcal{G}}\left(\xi_{n},A_{n},\eta\right) = \sum_{p,p',q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'}\Gamma^{\mu,a}_{qqA_{\mathcal{G}}}\frac{\vec{\eta}}{2}\xi_{n,p} - i\Gamma^{\mu\nu\lambda,abc}_{ggA_{\mathcal{G}}}\left(A^{b}_{n,p'}\right)_{\nu} \left(A^{c}_{n,p}\right)_{\lambda}\right) \,\bar{\eta}\,\Gamma^{\nu,a}_{s}\,\eta\,\Delta_{\mu\nu}(q)$$



- Use full QCD fields to derive the scaling of the vector potential created by the source of the Glauber gluons
- Consider collinear, static and soft quark sources
- Use background field method to deduce the Feynman rules of interaction of the target jet with the Glauber vector potential

Gauge	Object	Collinear source	Static source	Soft source
	p	$\left[\lambda^2, 1, oldsymbol{\lambda} ight]$	$[1,1,\boldsymbol{\lambda}]$	$[\lambda, \lambda, oldsymbol{\lambda}]$
	$a_{oldsymbol{p}},a_{oldsymbol{p}}^{\dagger}$	λ^{-1}	$\lambda^{-3/2}$	$\lambda^{-3/2}$
	u(p)	1	1	$\lambda^{1/2}$
	$\bar{u}(p_2)\gamma_{\nu}u(p_1)$	$\left[\lambda^2,1,oldsymbol{\lambda} ight]$	$[1,1,\boldsymbol{\lambda}]$	$[\lambda,\lambda,oldsymbol{\lambda}]$
R_{ξ}	$A^{\mu}(x)$	$\left[\lambda^4,\lambda^2,oldsymbol{\lambda}^3 ight]$	$\left[\lambda^2,\lambda^2,oldsymbol{\lambda}^3 ight]$	$\left[\lambda^2,\lambda^2,oldsymbol{\lambda}^2 ight]$
	Γ_{qqA_G}	$ \begin{array}{c} \Gamma_1^{\mu} \\ \Sigma_1^{\mu\nu\lambda} \end{array} $	$\begin{array}{c} \Gamma_1^{\mu} \\ \Sigma_1^{\mu\nu\lambda} \end{array}$	$ \begin{array}{c} \Gamma_1^{\mu} \\ \Sigma_1^{\mu\nu\lambda} \end{array} $
	$\Gamma_{\rm ggA_G}$	$\Sigma_1^{\mu\nu\lambda}$	$\Sigma_1^{\mu\nu\lambda}$	$\Sigma_1^{\mu\nu\lambda}$
	$\Gamma_{ m s}$	$\Gamma_1^{\mu} \left(n \leftrightarrow \bar{n} \right)$	Γ^{μ}_{3}	Γ_4^{μ}
$A^+ = 0$	$A^{\mu}(x)$	$\left[0,\lambda^2,oldsymbol{\lambda}^3 ight]$	$\left[0,\lambda^2,oldsymbol{\lambda} ight]$	$\left[0,\lambda^2,oldsymbol{\lambda} ight]$
	Γ_{qqA_G}	Γ^{μ}_{1}	$\Gamma_1^{\mu} + \Gamma_2^{\mu}$	$\Gamma_1^{\mu} + \Gamma_2^{\mu}$
	$\Gamma_{\rm ggA_G}$	$\Sigma_2^{\mu u\lambda}$	$\Sigma_2^{\mu\nu\lambda}$	$\Sigma_2^{\mu u\lambda}$
	$\Gamma_{ m s}$	$\Gamma_2^{\mu} \left(n \leftrightarrow \bar{n} \right)$	Γ^{μ}_{3}	Γ^{μ}_{4}
$A^- = 0$	$A^{\mu}(x)$	$\left[\lambda^2,0,oldsymbol{\lambda} ight]$	$\left[\lambda^2,0,oldsymbol{\lambda} ight]$	$\left[\lambda^2,0,oldsymbol{\lambda} ight]$
	Γ_{qqA_G}	$ \begin{array}{c} \Gamma_2^\mu \\ \Sigma_3^{\mu\nu\lambda} \end{array} $	Γ_2^{μ}	Γ_2^{μ}
	$\Gamma_{ m ggA_G}$	$\Sigma_3^{\mu\nu\lambda}$	$ \begin{array}{c} \Gamma^{\mu}_{2} \\ \Sigma^{\mu\nu\lambda}_{3} \end{array} $	$ \begin{array}{c} \Gamma_2^\mu \\ \Sigma_3^{\mu\nu\lambda} \end{array} $
	$\Gamma_{ m s}$	$\Gamma_1^{\mu} \left(n \leftrightarrow \bar{n} \right)$	Γ^{μ}_{3}	Γ^{μ}_{4}

$$\mathcal{L}_{\mathrm{G}}\left(\xi_{n},A_{n},\eta\right) = \sum_{p,p',q} \mathrm{e}^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'}\Gamma^{\mu,a}_{\mathrm{qqA_{G}}}\frac{\vec{\eta}}{2}\xi_{n,p} - i\Gamma^{\mu\nu\lambda,abc}_{\mathrm{ggA_{G}}}\left(A^{b}_{n,p'}\right)_{\nu} \left(A^{c}_{n,p}\right)_{\lambda}\right) \,\bar{\eta}\,\Gamma^{\nu,a}_{\mathrm{s}}\,\eta\,\Delta_{\mu\nu}(q)$$



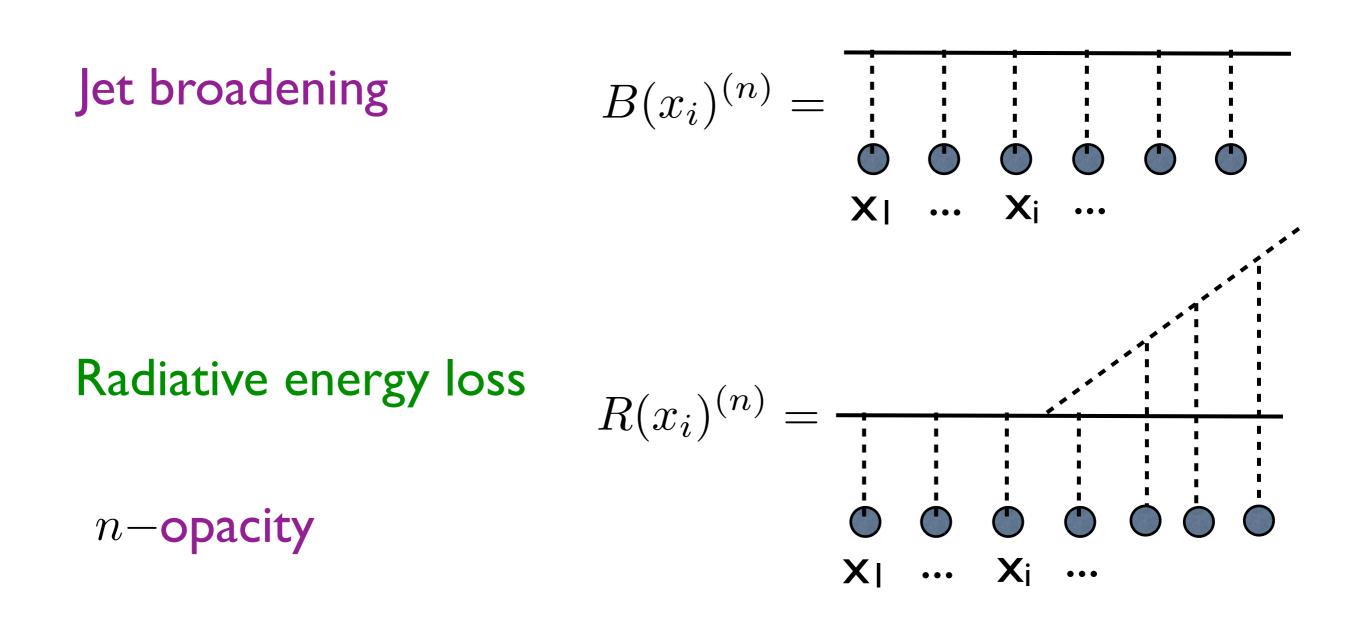
Elements in the table:

$$\begin{split} \Gamma_{1}^{\mu} &= igT^{a} \, n^{\mu} \frac{\vec{n}}{2}, \\ \Gamma_{2}^{\mu} &= igT^{a} \, \frac{\gamma_{\perp}^{\mu} \not{p}_{\perp} + \not{p}_{\perp}^{\prime} \gamma_{\perp}^{\mu} \vec{n}}{\bar{n} \cdot p} \frac{\gamma_{\perp}^{\mu} \varphi_{\perp}^{\mu}}{2}, \\ \Gamma_{4}^{\mu} &= igT^{a} \, \gamma^{\mu}, \\ \Gamma_{4}^{\mu} &= igT^{a} \, \gamma^{\mu}, \\ \Sigma_{3}^{\mu,\nu,\lambda} &= gf^{abc} \left[g_{\perp}^{\mu\lambda} \left(\frac{\bar{n}^{\nu}}{2} (q_{1}^{-} - 2q_{2}^{-}) + q_{\perp}^{\nu} - 2q_{2\perp}^{\nu} \right) + g_{\perp}^{\mu\nu} \left(\frac{\bar{n}^{\lambda}}{2} (q_{2}^{-} - 2q_{1\perp}^{-}) + q_{\perp}^{\lambda} - 2q_{\perp}^{\lambda} \right) + g_{\perp}^{\nu\lambda} \left(q_{1\perp}^{\mu} + q_{\perp}^{\mu} \right) \right]$$

- Our Glauber Lagrangian is invariant under the gauge symmetries of SCET $\mathcal{L}_{G}(\xi_{n}, A_{n}, \eta) \rightarrow \mathcal{L}_{G}(W_{n}^{\dagger}\xi_{n}, \mathcal{B}_{n}(A_{n}), \eta) \equiv \mathcal{L}_{G}(\chi_{n}, \mathcal{B}_{n}, \eta)$
- We derived the Feynman rules for three types of sources: collinear, static, and soft quarks and three gauges: covariant, A⁺=0 and A⁻=0 gauges.

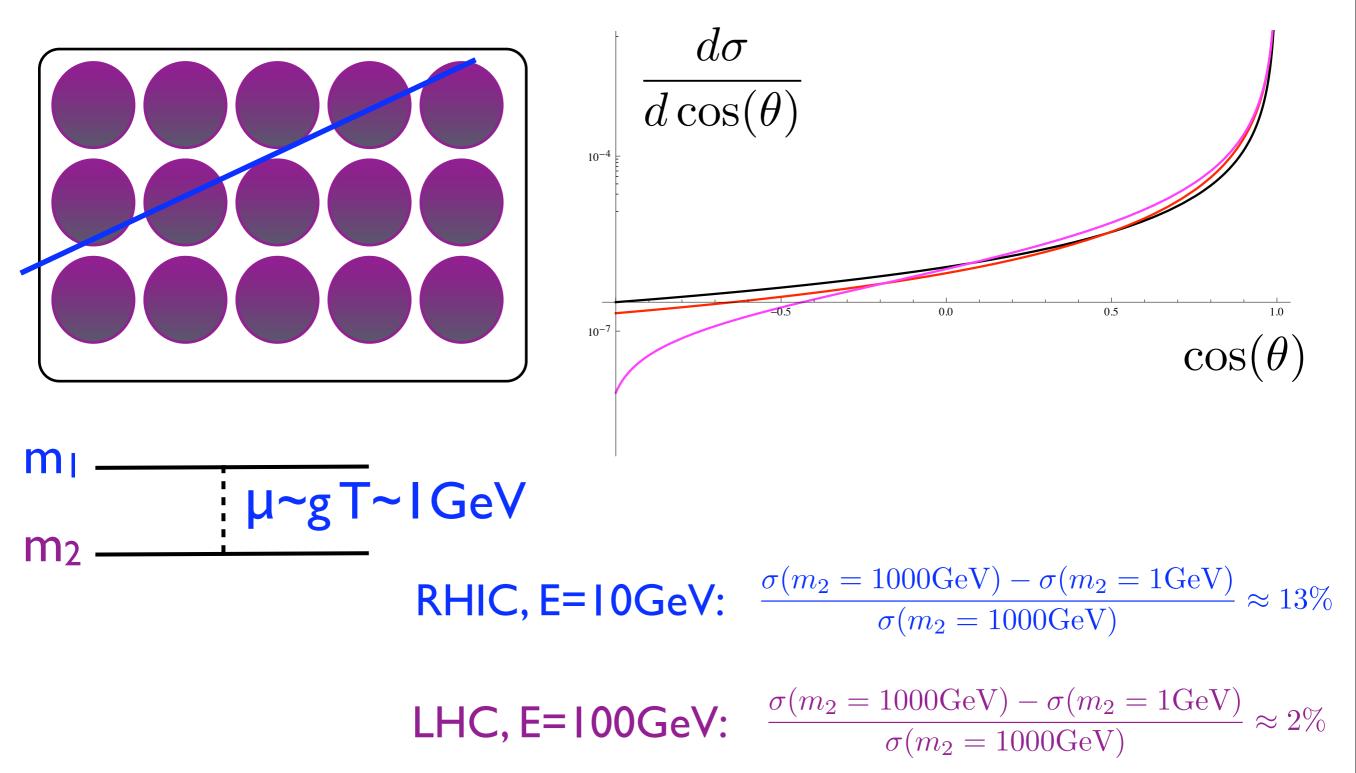
Results for jet broadening and jet energy loss

Diagrams to be evaluated

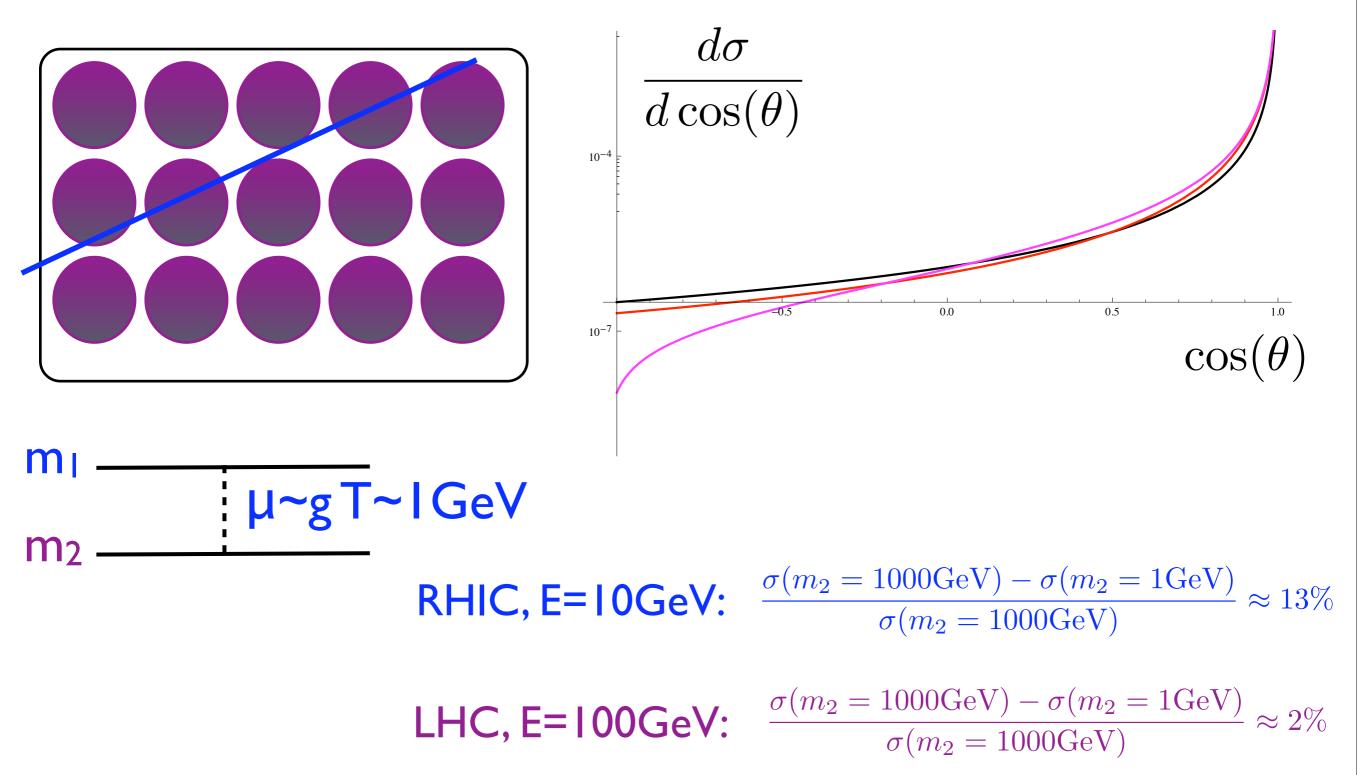


What kinematics should we assume for the source term?

Kinematics of the source



Kinematics of the source



Static source is a realistic assumption

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Gauge invariance

Broadening $\mathcal{L}_{G}\left(\xi_{n}, A_{n}, \eta\right) = \sum_{p, p', q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n, p'} \Gamma^{\mu, a}_{qqA_{G}} \frac{\vec{\eta}}{2} \xi_{n, p} - i \Gamma^{\mu\nu\lambda, abc}_{ggA_{G}} \left(A^{b}_{n, p'}\right)_{\nu} \left(A^{c}_{n, p}\right)_{\lambda}\right) \bar{\eta} \Gamma^{\nu, a}_{s} \eta \Delta_{\mu\nu}(q)$ $\Delta_{\mu\nu}(q) = \frac{(-d_{\mu\nu})}{q^{2} - \mu^{2}}$

$$M_{1}^{(q)} = \underbrace{\overset{x_{0}}{\overbrace{p-q}}_{x,0,b}}_{p-q} \underbrace{\underset{x,0,b}{\uparrow q}}_{x,0,b} M_{1}^{(p)} \underbrace{M_{1}^{(q)}}_{p} = -e^{ipx_{0}} T_{r}^{a} T_{r'}^{a} J(p) \int \frac{d^{2}q_{\perp}}{(2\pi)^{2}} \tilde{v}(q_{\perp}) e^{-iq_{\perp}(\boldsymbol{x}-\boldsymbol{x}_{0})_{\perp}} \left[e^{i\omega(z-z_{0})} + \Delta_{(A^{+},A^{-})} \right]$$

$$M_{1}^{(g)} = \underbrace{\overset{x_{0}}{\underset{p \to q}{\longrightarrow}}}_{p \to q} \underbrace{(M_{1}^{(g)})}_{(R_{\xi},A^{+},A^{-})} = -i e^{ipx_{0}} f^{abc} T_{r'}^{b} J(p)_{\nu_{1}} \int \frac{d^{2}q_{\perp}}{(2\pi)^{2}} \tilde{v}(q_{\perp}) \varepsilon_{\lambda}(q_{2}) \left(g_{\perp}^{\nu_{1}\lambda}\right) e^{-iq_{\perp}(\boldsymbol{x}-\boldsymbol{x}_{0})_{\perp}} \left[e^{i\omega(\boldsymbol{z}-\boldsymbol{z}_{0})} + \Delta_{(R_{\xi},A^{+},A^{-})}\right]$$

Prescription	$\frac{1}{[k^+]}$	Δ_{A^+}	Δ_{A^-}
+i0	$\frac{1}{k^++i0}$	1	0
-i0	$\frac{1}{k^+-i0}$	0	-1
PV	$\left \frac{1}{2} \left(\frac{1}{k^+ + i0} + \frac{1}{k^+ - i0} \right) \right $	$\frac{1}{2}$	$-\frac{1}{2}$
ML	$\frac{1}{k^+ + i0 \text{sign}(k^-)}$	$\frac{1}{2}$	$-\frac{1}{2}$

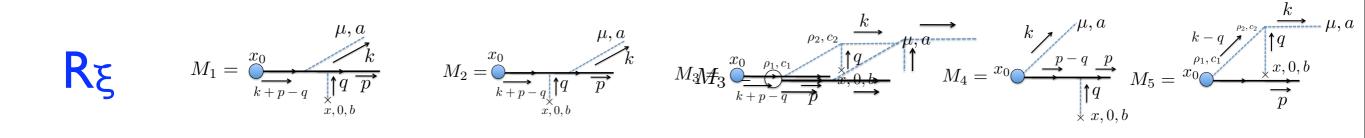
The light-cone prescription dependence suggest that a new Wilson line be introduced

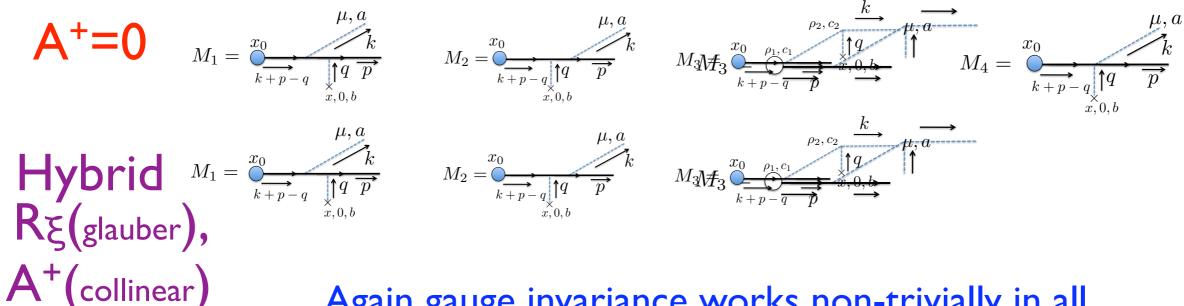
Idilbi, Scimemi, 10

Gauge invariance

Radiative Energy Loss

$$\mathcal{L}_{G}\left(\xi_{n},A_{n},\eta\right) = \sum_{p,p',q} e^{-i(p-p'+q)x} \left(\bar{\xi}_{n,p'}\Gamma^{\mu,a}_{qqA_{G}}\frac{\vec{\eta}}{2}\xi_{n,p} - i\Gamma^{\mu\nu\lambda,abc}_{ggA_{G}}\left(A^{b}_{n,p'}\right)_{\nu}\left(A^{c}_{n,p}\right)_{\lambda}\right) \bar{\eta}\,\Gamma^{\nu,a}_{s}\,\eta\,\Delta_{\mu\nu}(q)$$
$$\Delta_{\mu\nu}(q) = \frac{(-d_{\mu\nu})}{q^{2}-\mu^{2}}$$



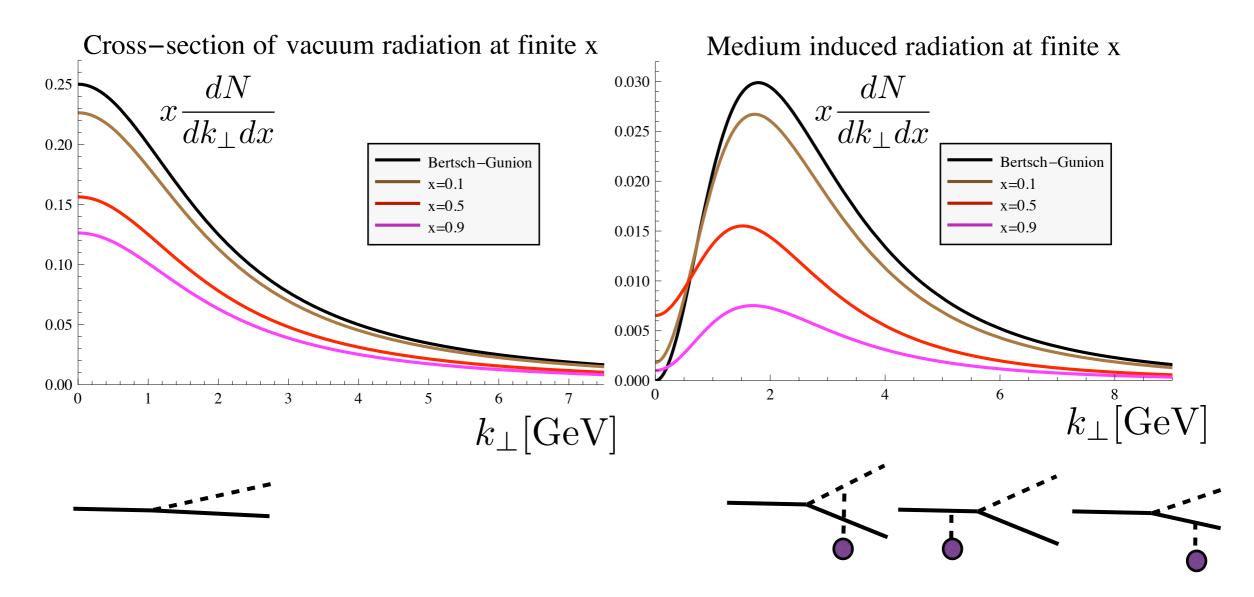


Again gauge invariance works non-trivially in all cases, up to T-Wilson line necessity in $A^+=0$ gauge

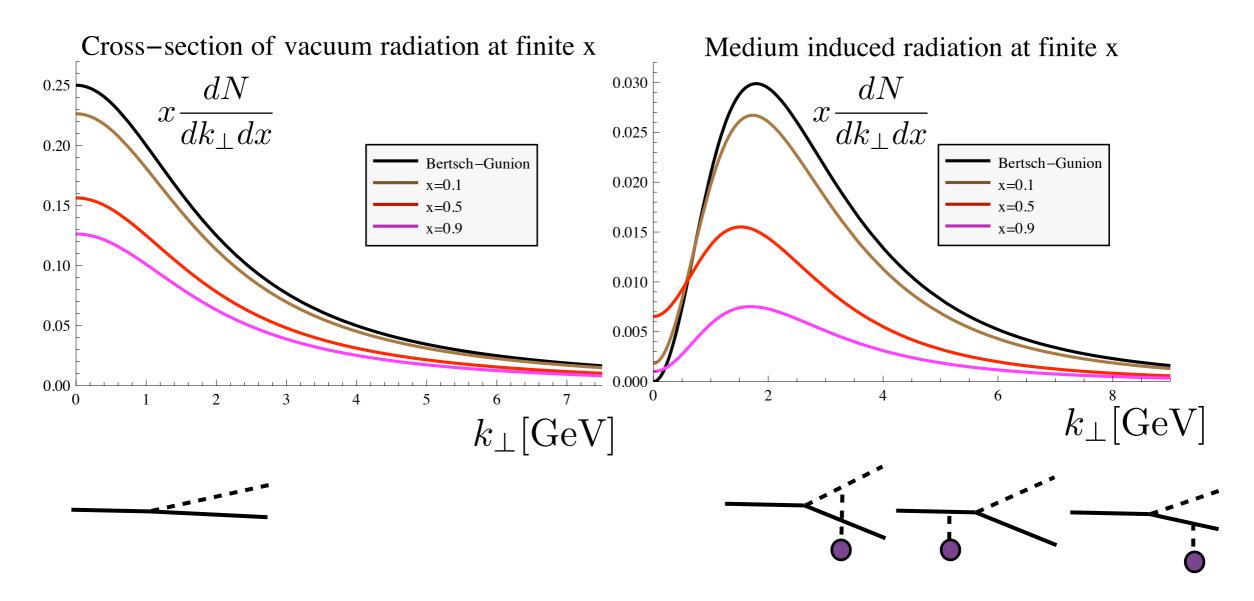
Comparison with previous results Gyulassy, Levai, Vitev, 00

- For Jet broadening and Radiative energy loss we have verified that the scattering kernels are gauge invariant
- For the Jet broadening our results agree with previously derived results in GLV approach
- For the Radiative energy loss our results agree with GLV approach in the soft gluon approximation

Energy loss at finite x, using $SCET_G$

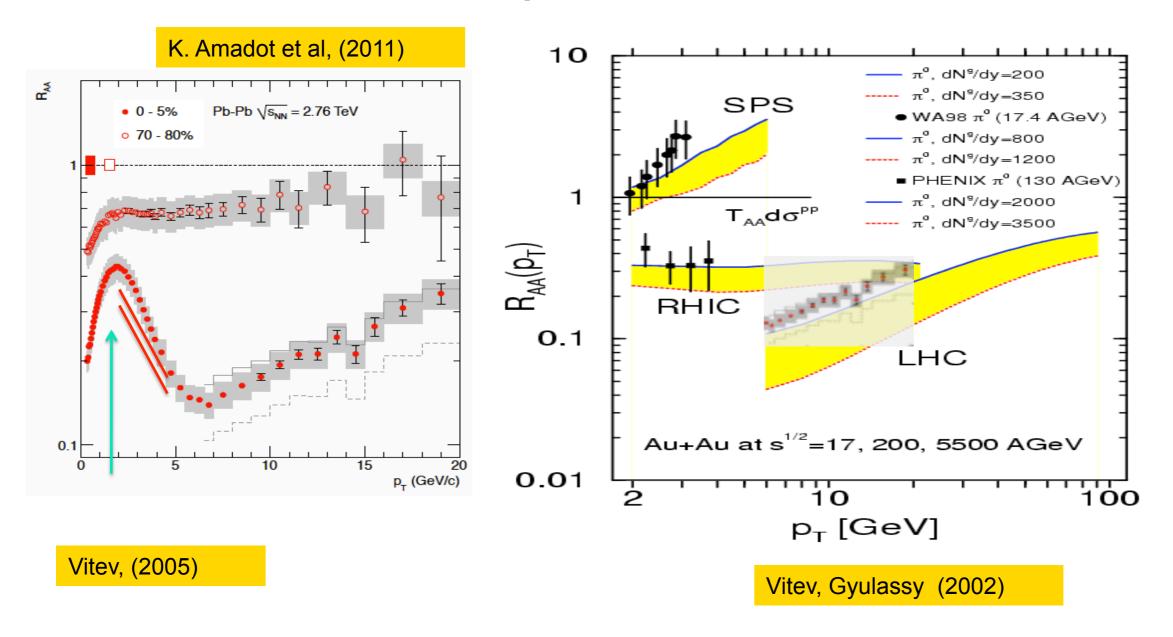


Energy loss at finite x, using SCET_G



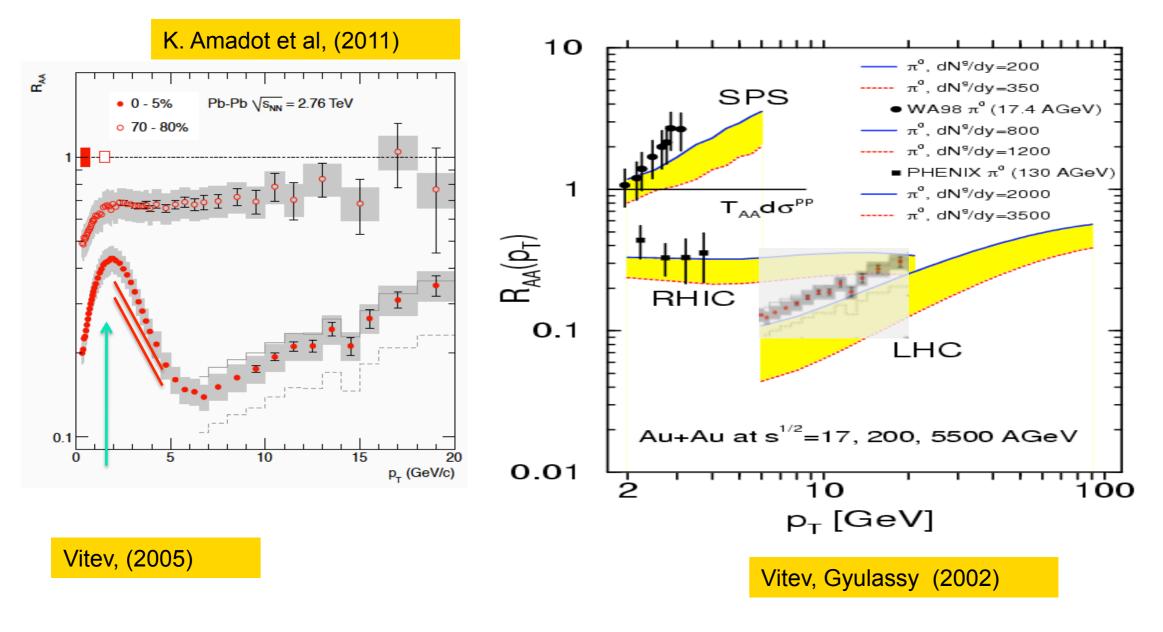
- Typical treatment of energy loss uses soft gluon approximation x<<I
- Our results show that the finite x effects push towards the smaller energy loss

First heavy ion data from LHC well explained by GLV 2002 prediction!



Larger R_{AA} can be explained by:

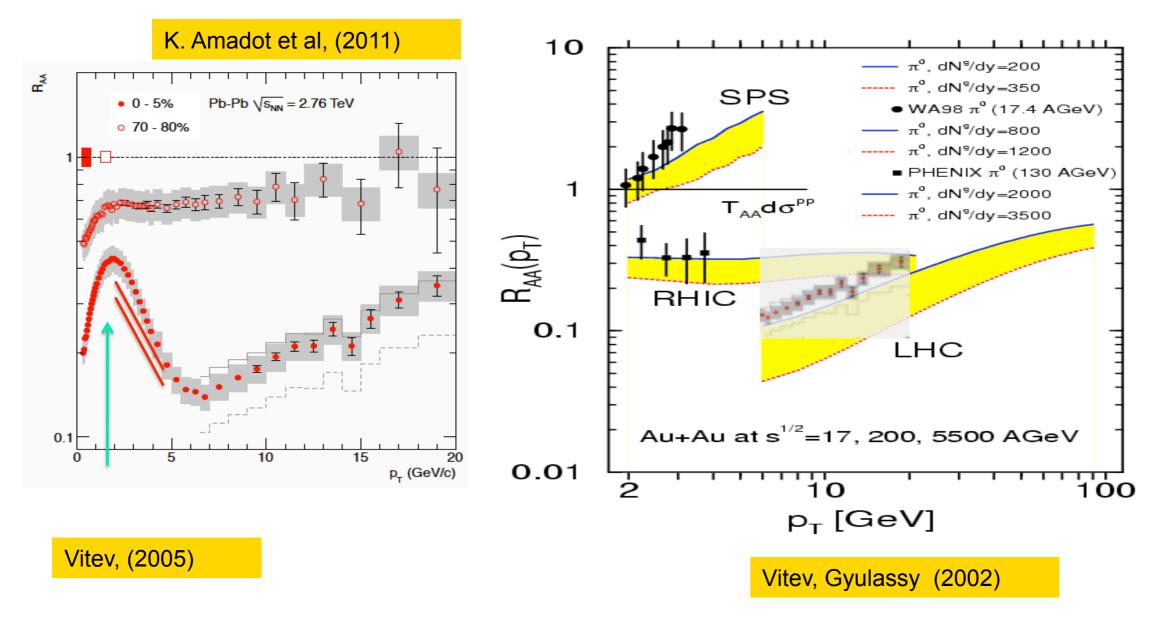
First heavy ion data from LHC well explained by GLV 2002 prediction!



Larger R_{AA} can be • Smaller coupling to medium at higher explained by: • energies

Smaller energy loss

First heavy ion data from LHC well explained by GLV 2002 prediction!



Larger R_{AA} can be • Smaller coupling to medium at higher explained by: • energies

• Smaller energy loss (finite x corrections?)

Conclusions

- We constructed effective theory for jet propagation in medium, including collinear quarks and gluons
- We formulated the effective theory in three different gauges and demonstrated the gauge invariance of the scattering kernels, derived previously in PQCD in soft gluon approximation
- Effective theory Feynman rules allowed us to go beyond the soft gluon approximation
- Constructed Lagrangian of SCET_G needs to be completed with soft gluons
- Such effective theory should be applied to factorization of the Drell-Yan process