Soft Collinear Effective Theory & Jets

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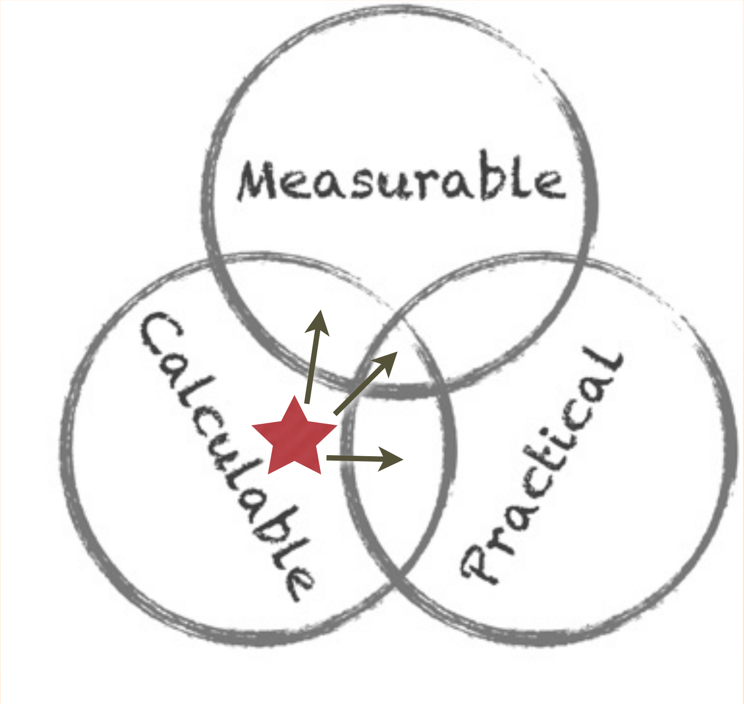
MIT & Harvard

Boston Jet Physics workshop, Harvard
January 2011

Outline:

- Introduction to Soft Collinear Effective Theory

  * scales & fields (organize the physics of jets)
  * systematic expansion (estimate theory errors)
  * simplify calculations, sum logs (higher precision)
  * factorization & universality

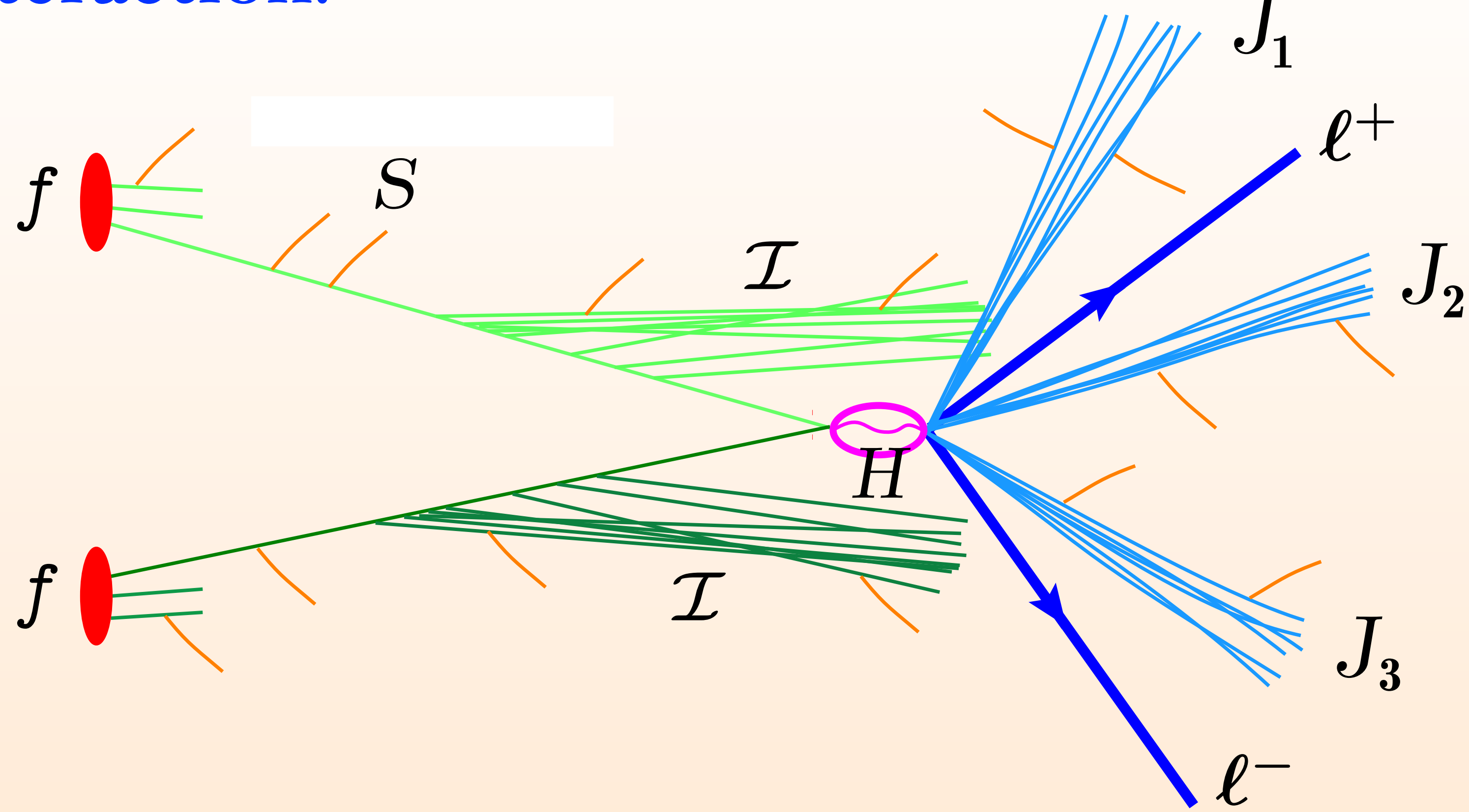


70's $\longrightarrow$ 2011

- Cross Sections with Jets
  - e^+e^- event shapes
  - jet algorithms
  - pp event shapes

- Jet Substructure
  - $\mathrm{SCET}_1 \rightarrow \mathrm{SCET}_2$ (collinear jet substructure)
  - Jet Shapes (event shapes in a jet)
  - Parton Shower

Typical Event with Hard Interaction:



Factorization:

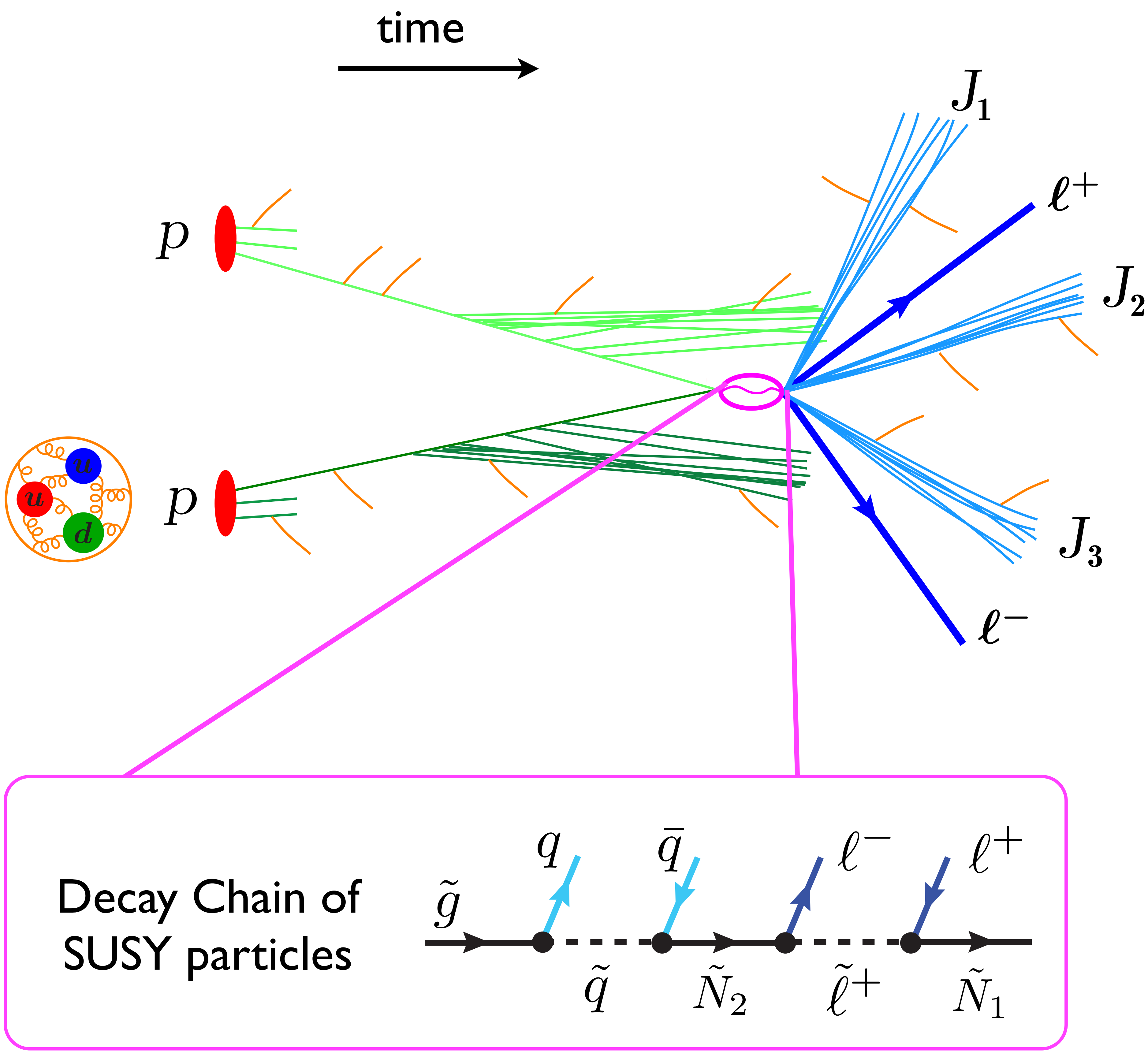
"cross section can be computed as product of independent pieces"

Shower MC programs assume factorization:

$d\sigma =$ initial state parton shower $\otimes$ hard scattering fixed order perturbative computation $\otimes$ final state parton showers $\otimes$ hadronization model, underlying event, ...

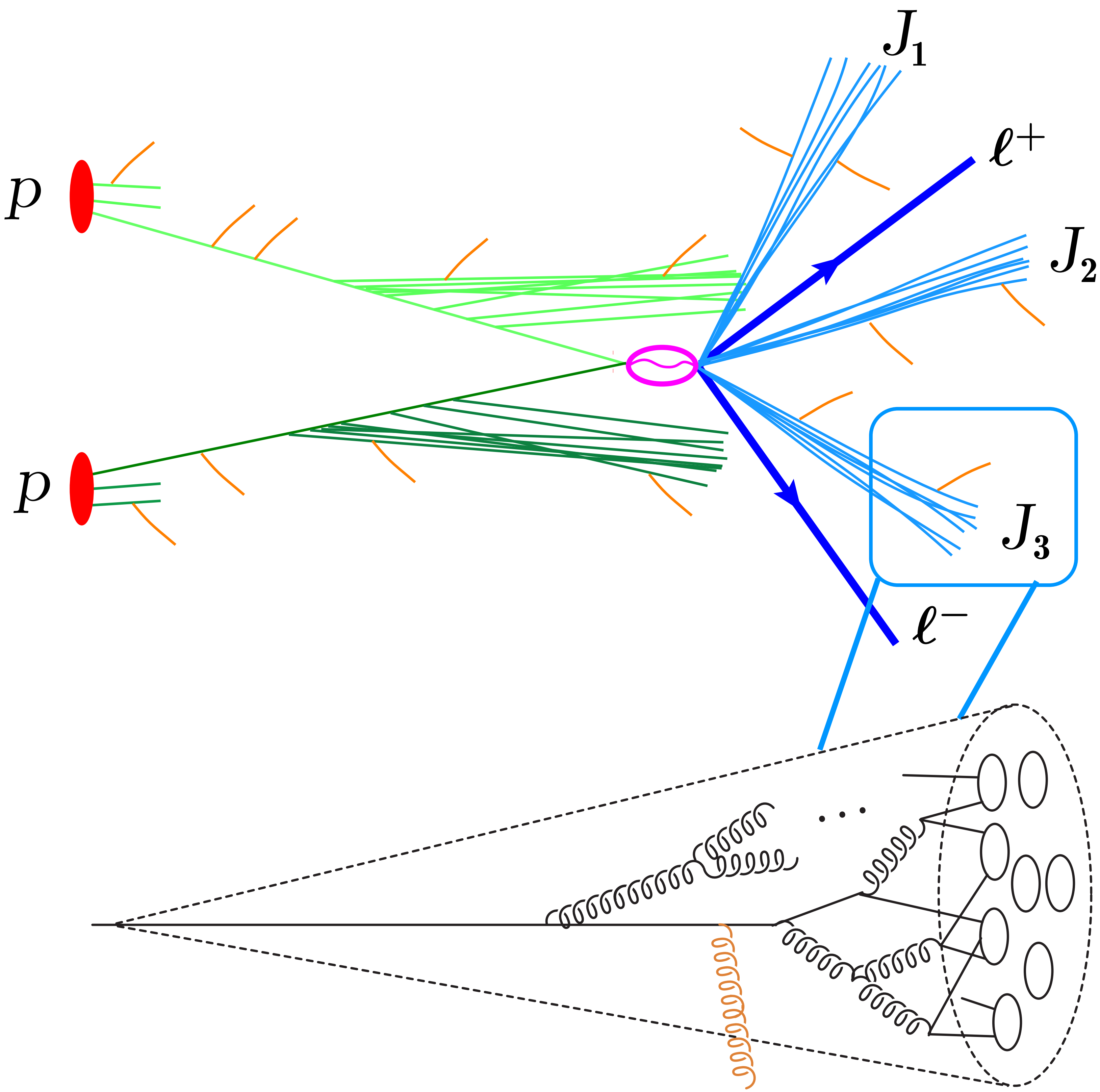
(with parton distributions)

Events with a Hard Interaction:



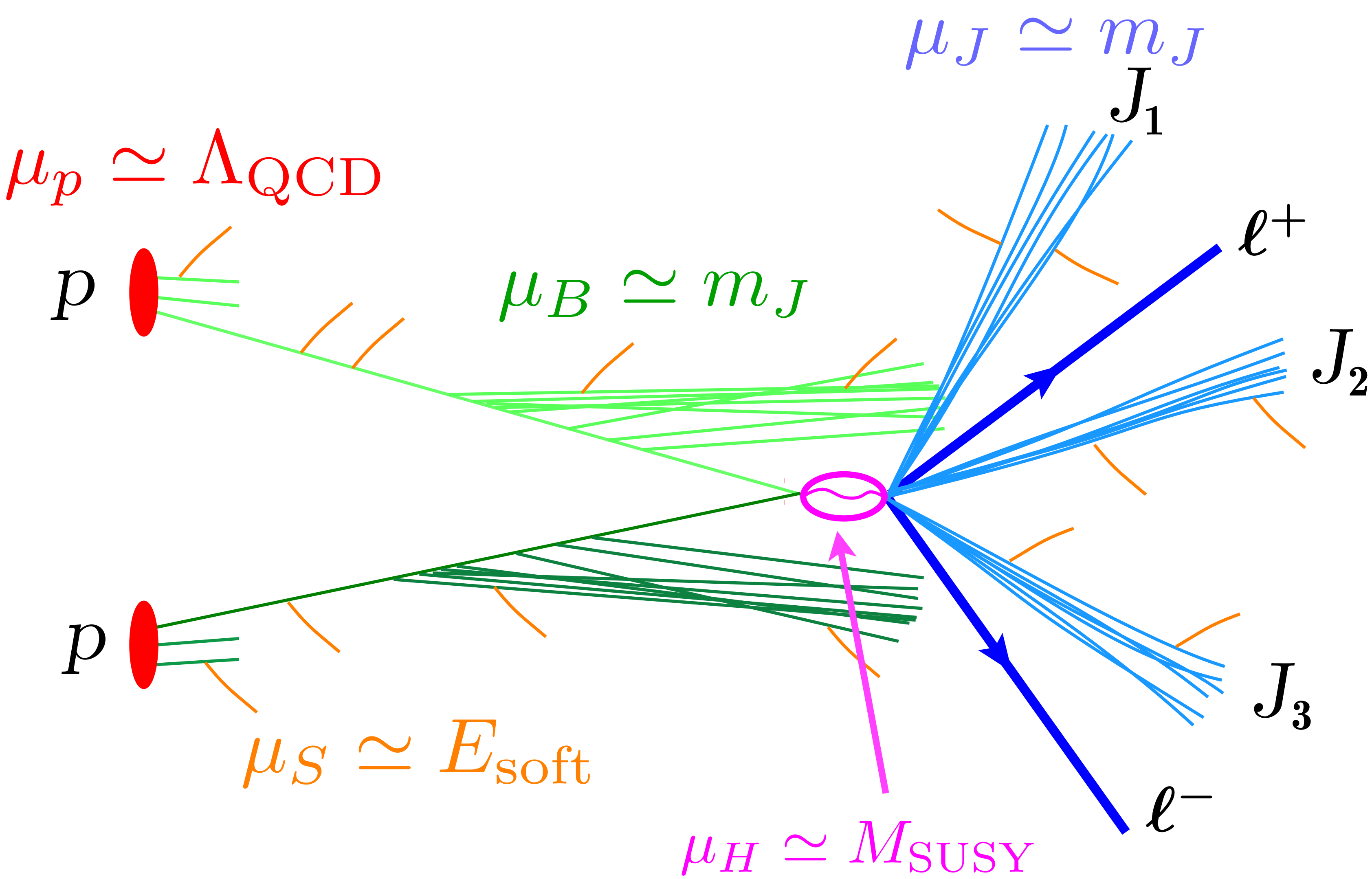
Search for New Heavy Particles at short distances

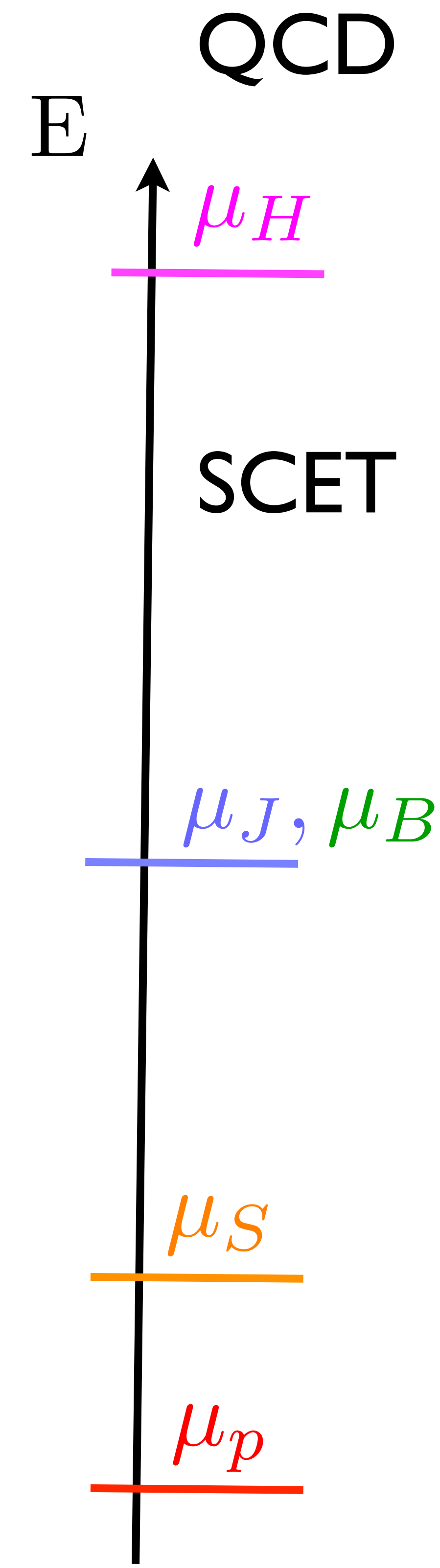
Events with a Hard Interaction:



Quarks and Gluons
Form Jets

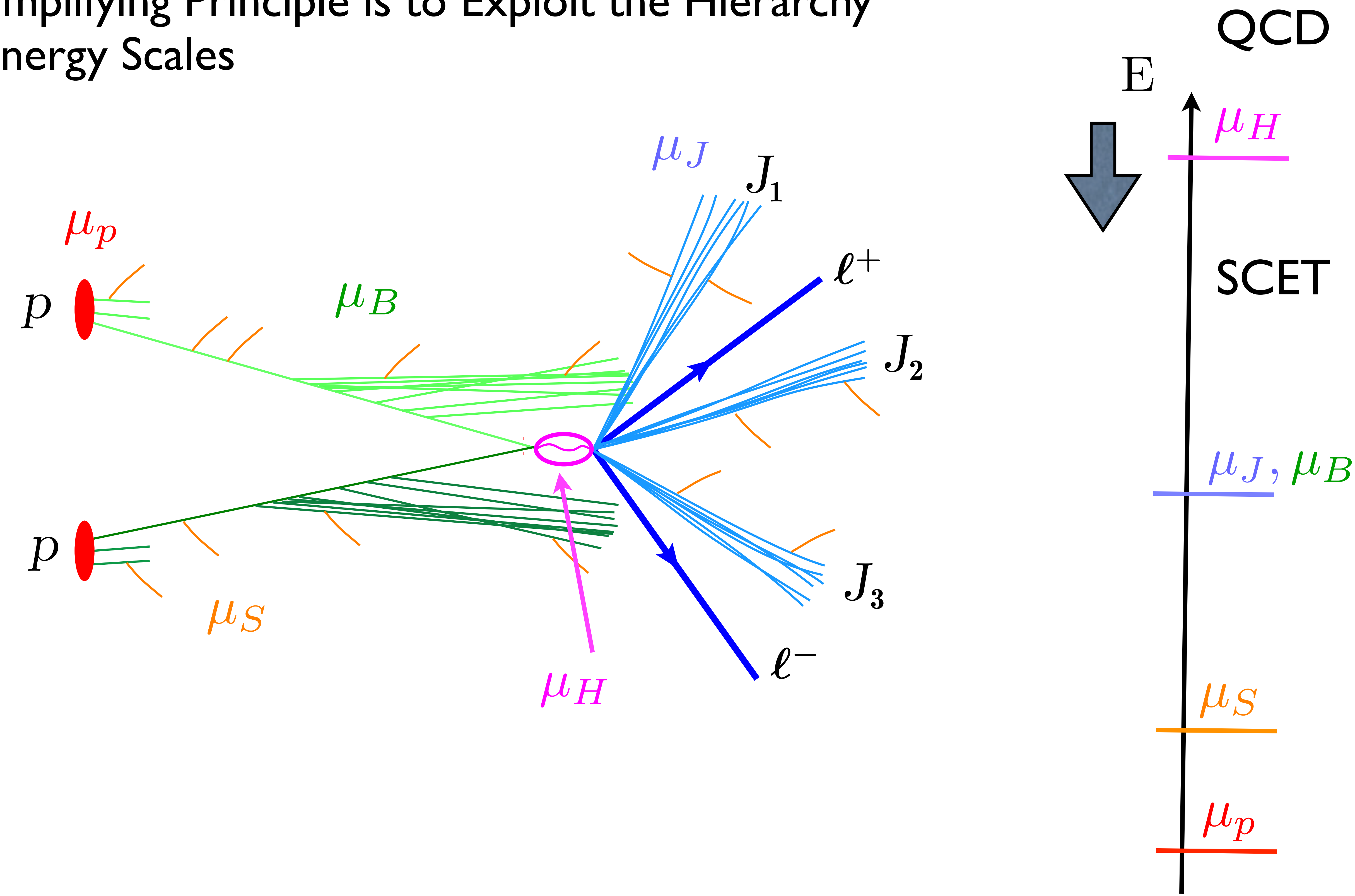
Key Simplifying Principle is to Exploit the Hierarchy of Energy Scales





SCET = Soft-Collinear Effective Theory

Key Simplifying Principle is to Exploit the Hierarchy of Energy Scales



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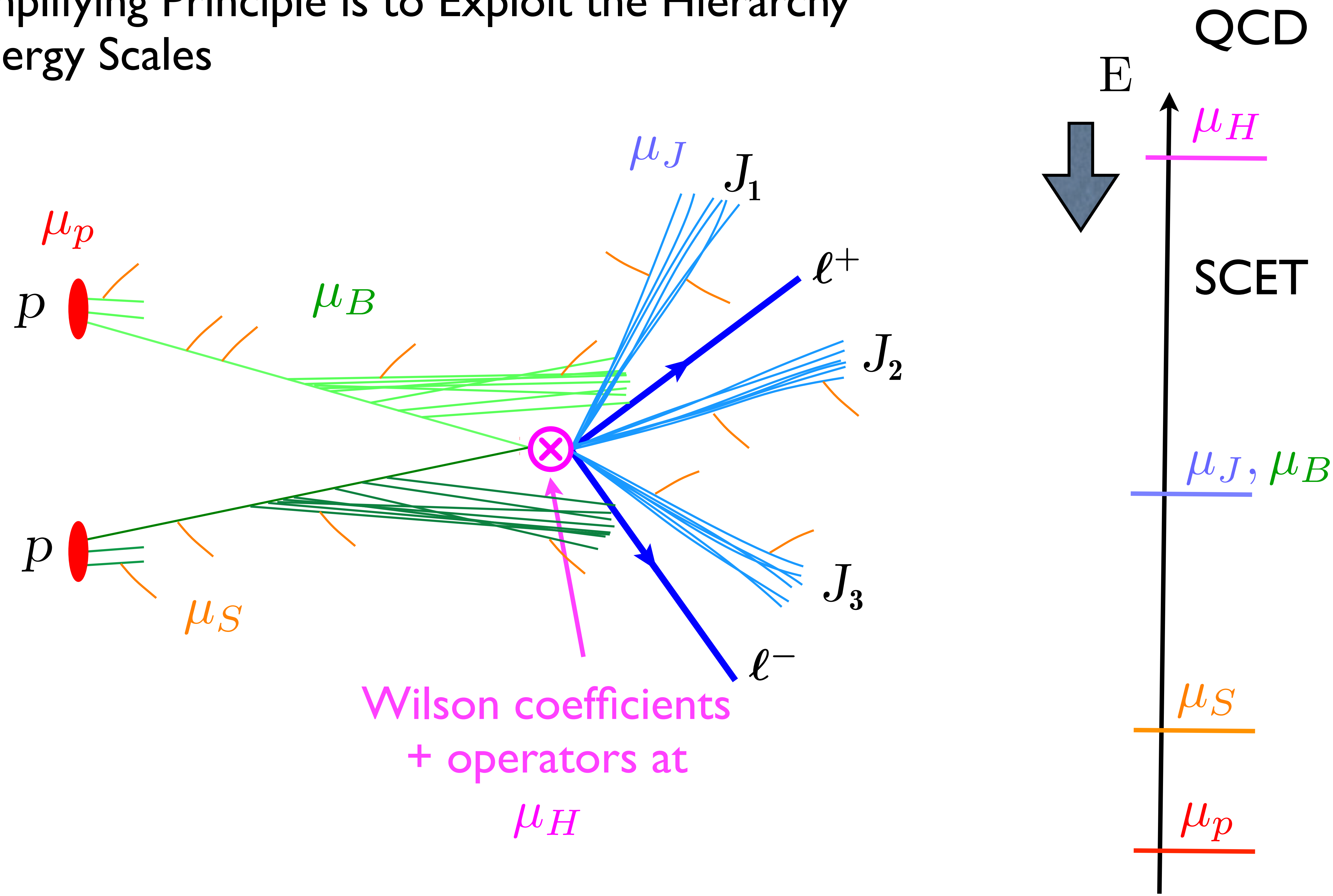

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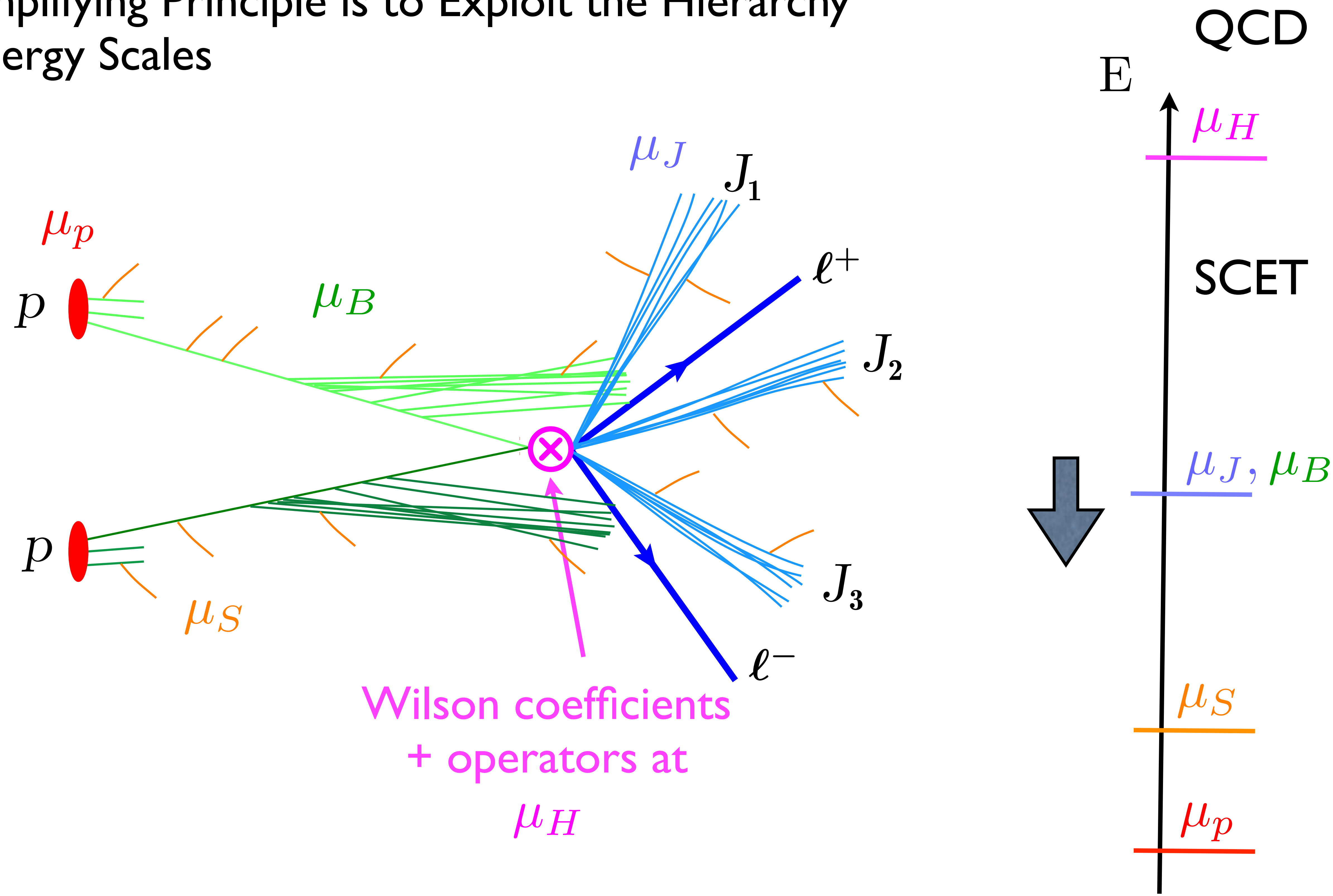
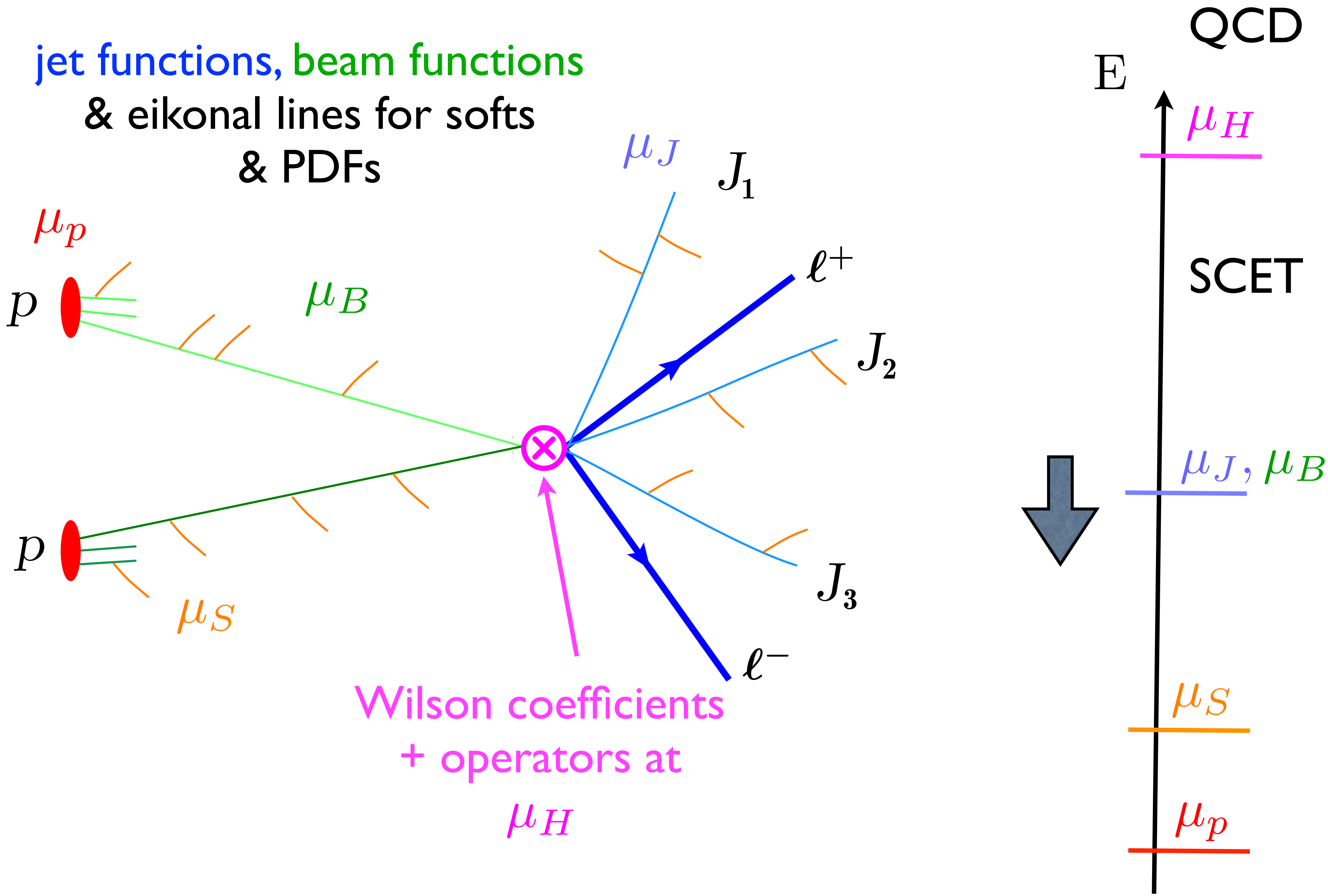




Factorization: $\mathrm{d}\sigma = f_{a,b} \otimes \mathcal{I}_{a,b} \otimes H \otimes \prod_i J_i \otimes S$

Λ_{QCD} $\quad \mu_B \quad \mu_H \quad \mu_J \quad \mu_S$

SCET can be used for:

- Factorization $d\sigma^{\rm had} = f \otimes f \otimes d\sigma^{\rm part}$
- Sum Large logs $\alpha_s \ln^2 z,\ \alpha_s^2 \ln^4 z,\ \ldots$ QCD Sudakov's, EW Sudakov's
- Analytic calculations of perturbative corrections
  NLO, NNLO in α_s
- Nonperturbative corrections (hadronization $\to$ matrix elements)
- Parton Shower: ISR, High multiplicity final states
- Softer physics (underlying event?)
- Precision Measurements:

Tevatron $m_t = 173.3 \pm 0.6_{\rm stat} \pm 0.9_{\rm syst}\ {\rm GeV}$ theory error? what mass is it?

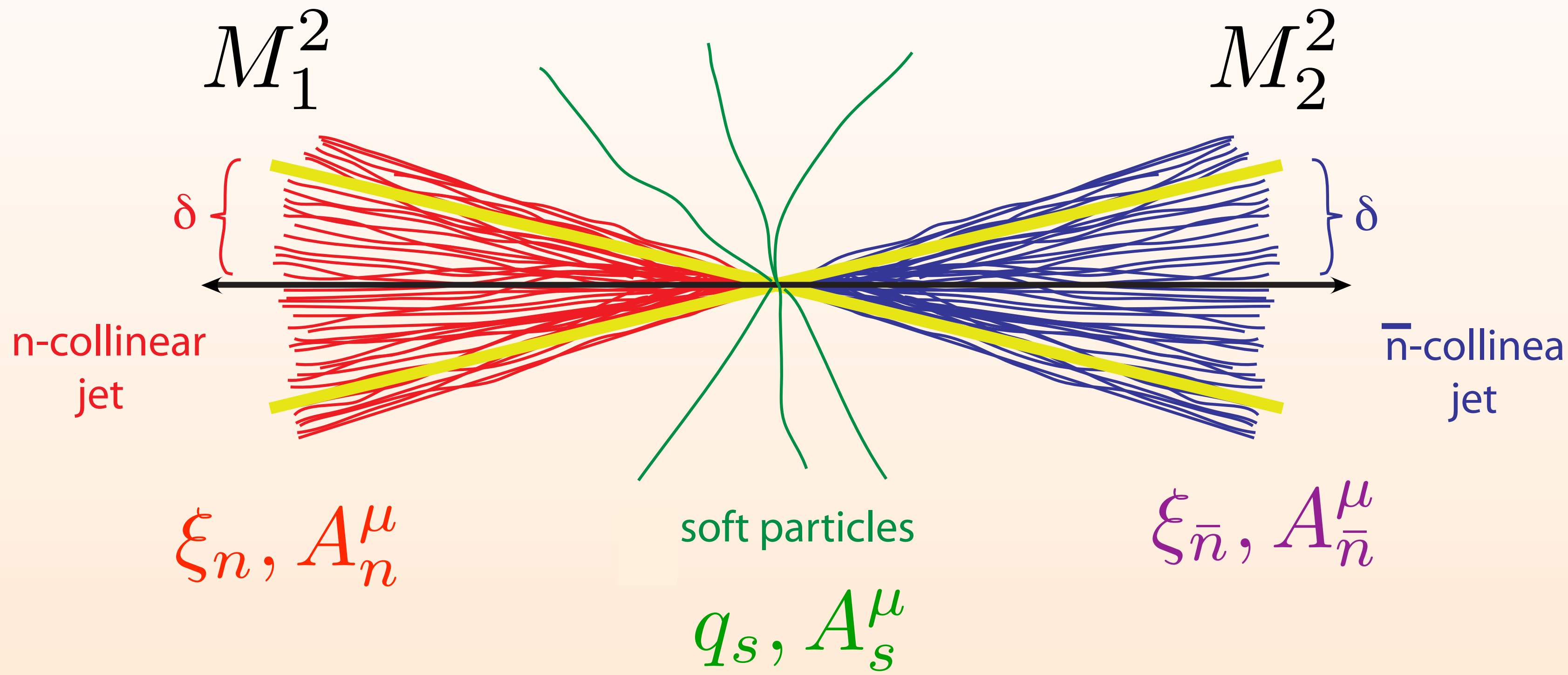
(in this talk I'll avoid Glaubers & pT dependent functions)

SCET energetic jets

eg. $e^+e^- \to 2$ jets

$M_i^2 \sim \Delta^2$

$\Lambda^2 \ll \Delta^2 \ll Q^2$



$\lambda \sim \frac{\Delta}{Q}$ is the expansion parameter

$\delta \sim \lambda$

Jet constituents : $p^\mu \sim \overset{+\quad -\quad \perp}{\Big(\frac{\Delta^2}{Q}, Q, \Delta\Big)} \sim Q(\lambda^2, 1, \lambda)$

$$p^2 = p^+p^- + p_\perp^2 \quad , \quad p^2 \sim \Delta^2$$

Soft particles:

$$p^\mu \sim \Big(\frac{\Delta^2}{Q}, \frac{\Delta^2}{Q}, \frac{\Delta^2}{Q}\Big) \sim Q(\lambda^2, \lambda^2, \lambda^2)$$

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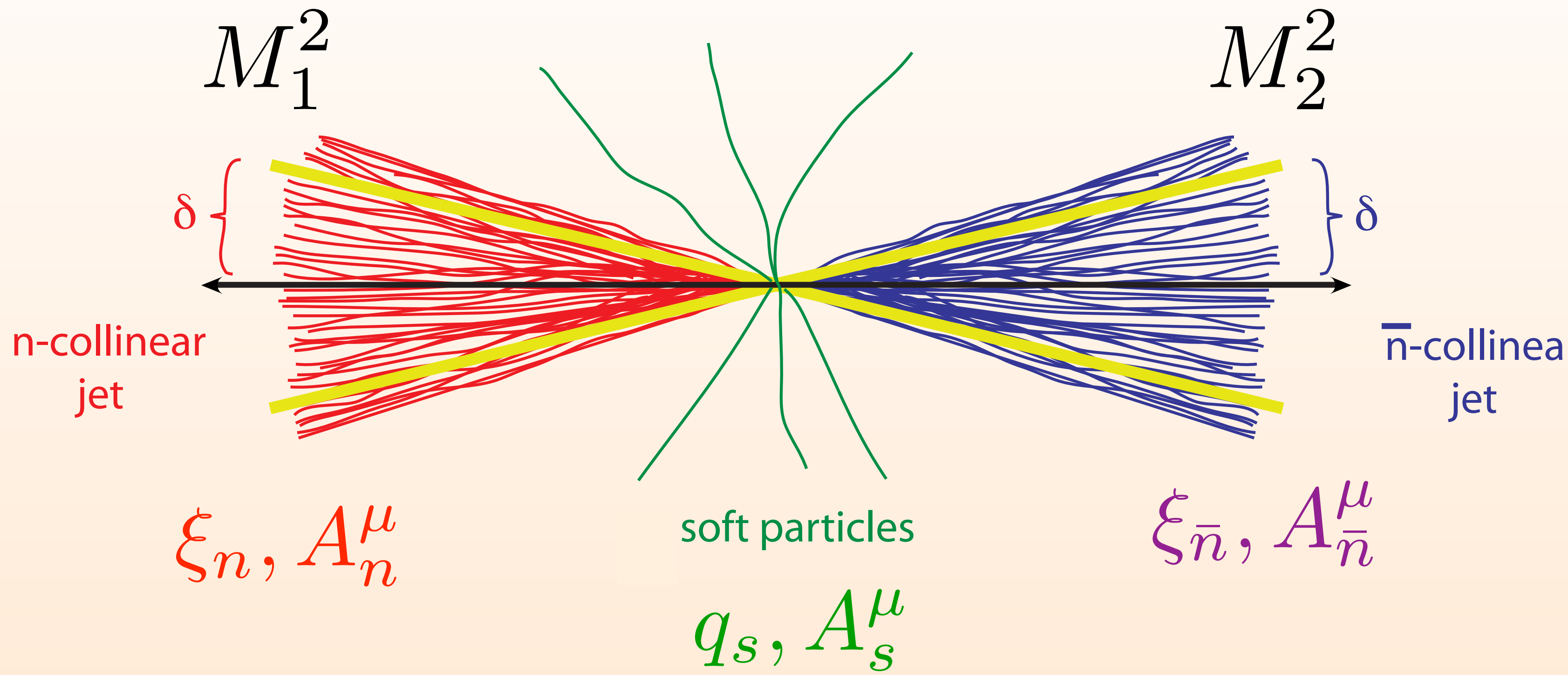
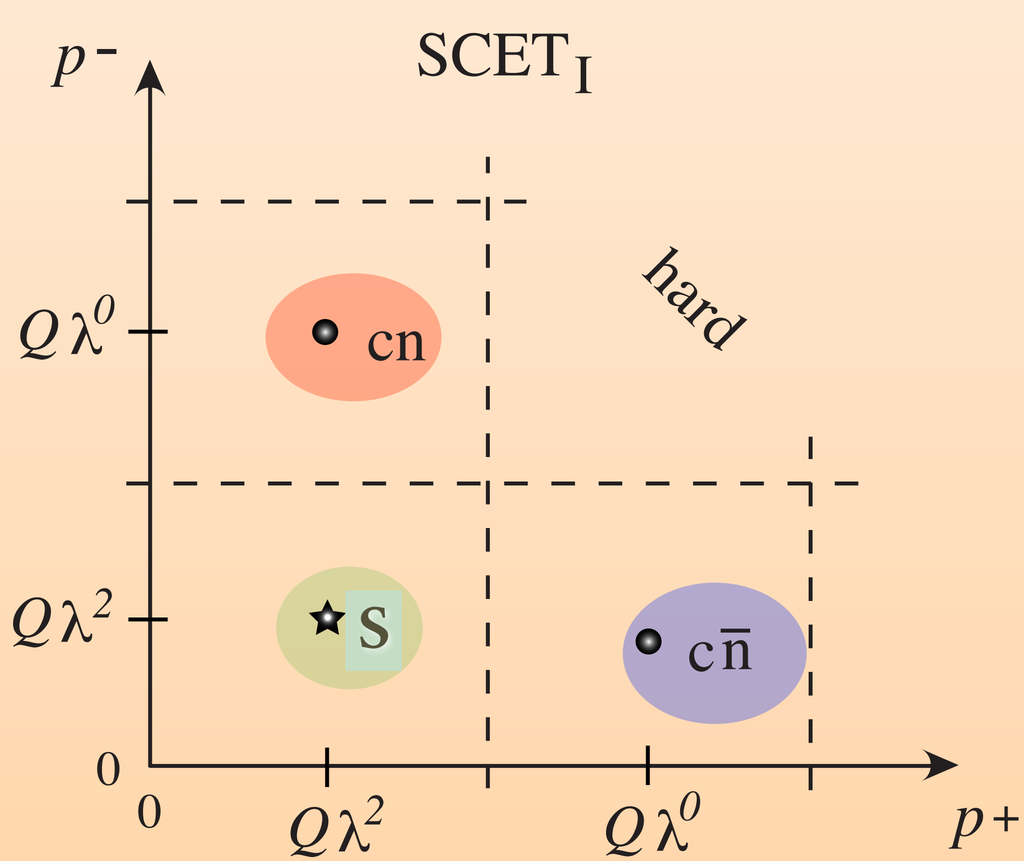
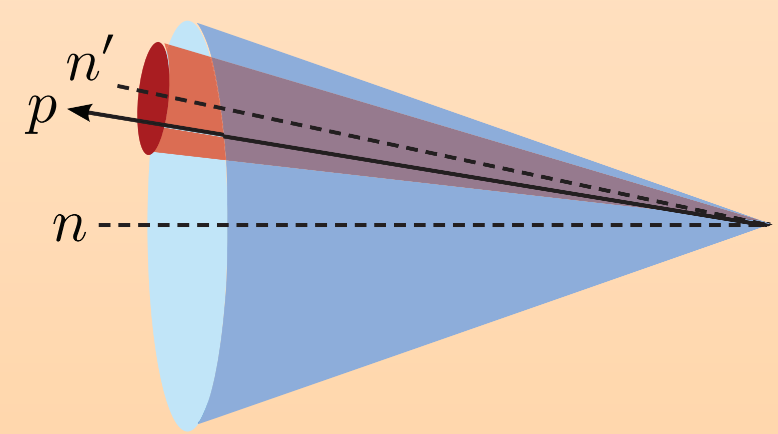




Defining concepts:

- hard scale Q
- collinear sectors $\{[n_i]\}$
- power counting parameter λ



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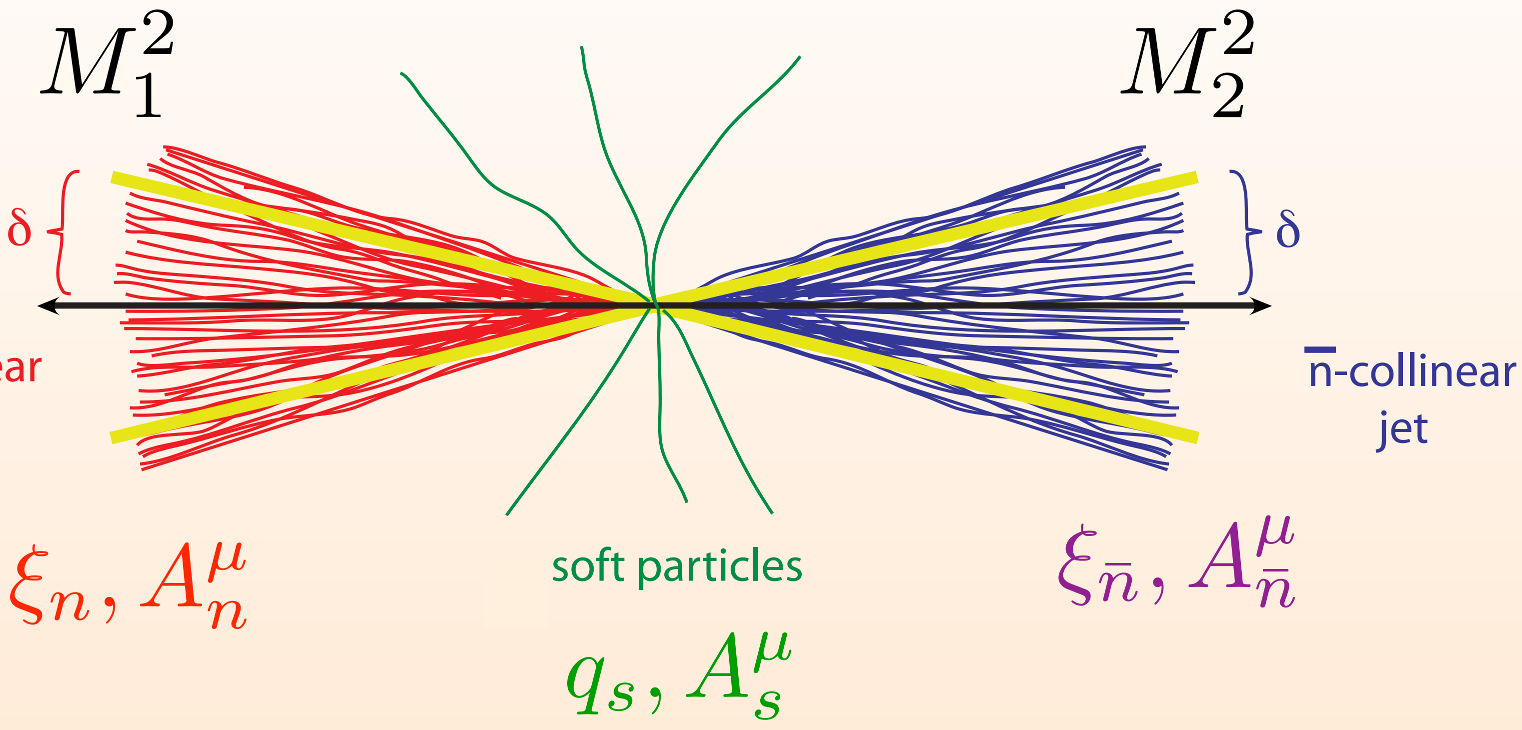
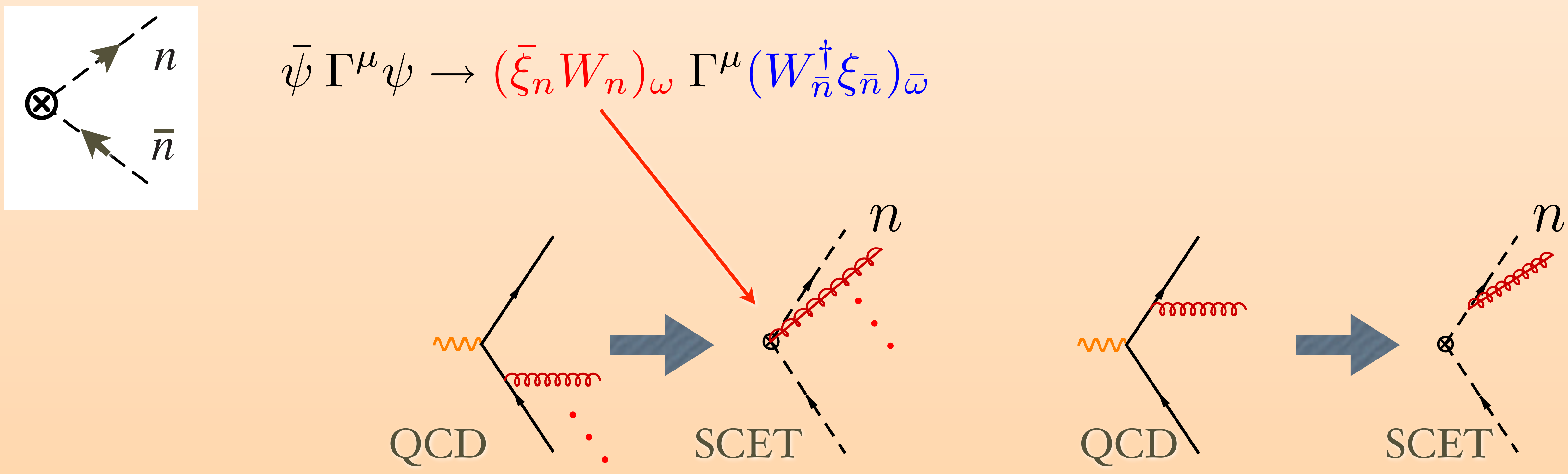


Production Current: $Q \gg \Delta$

$$\bar\psi\, \Gamma^\mu \psi \to (\bar\xi_n W_n)_\omega\, \Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$



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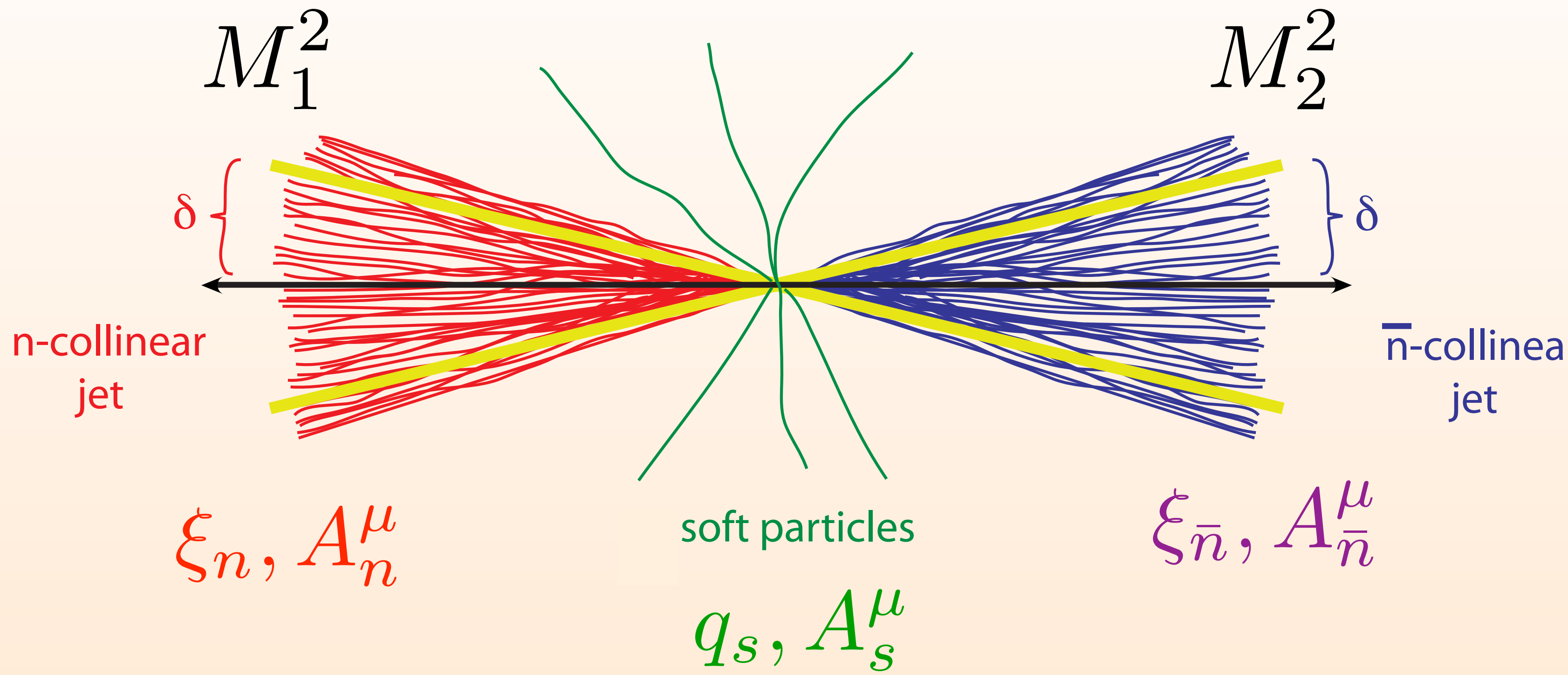
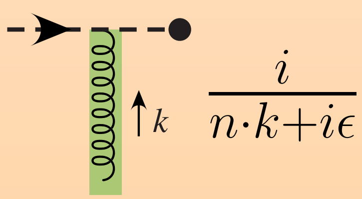


SCET Lagrangian:

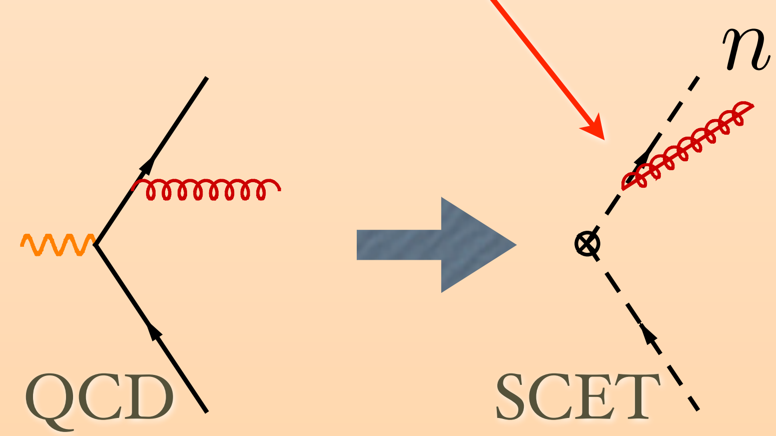
$$\mathcal{L}_n^{(0)} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i\not{D}_\perp^n \frac{1}{i\bar{n}\cdot D_n} i\not{D}_\perp^n \Big\} \frac{\bar{\not{n}}}{2} \xi_n$$

propagator: $$\frac{i\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon} = \frac{i\not{n}}{2}\,\frac{1}{n\cdot p - \frac{\vec{p}_\perp^{\,2}}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)}$$

eikonal softs:



$$\xi_n \to Y\xi_n$$
$$A_n \to Y A_n Y^\dagger$$
$$Y(x) = P\exp\Big(ig\int_{-\infty}^0 ds\, n\cdot A_{us}(x+ns)\Big)$$



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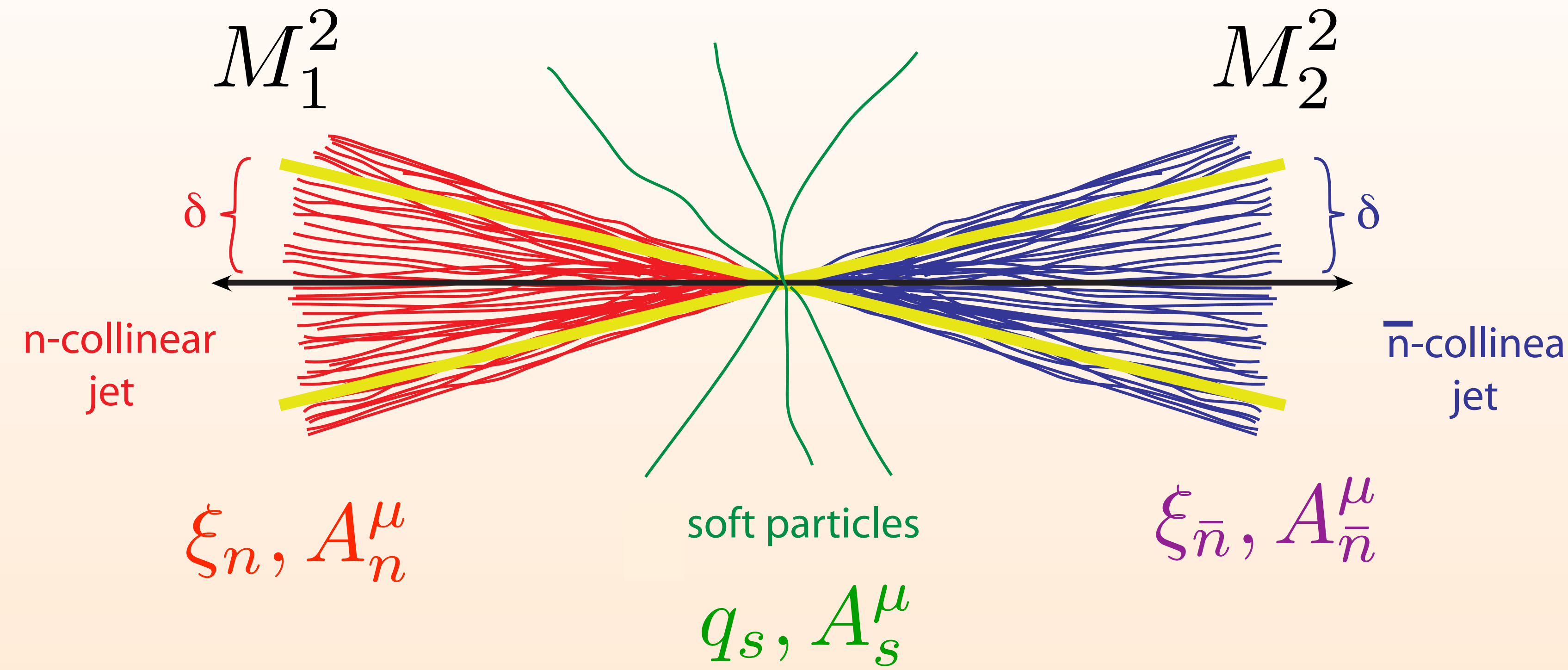

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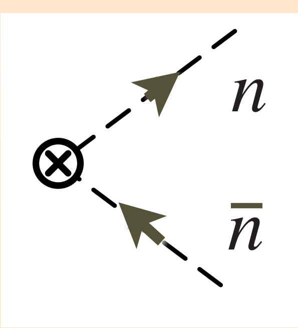
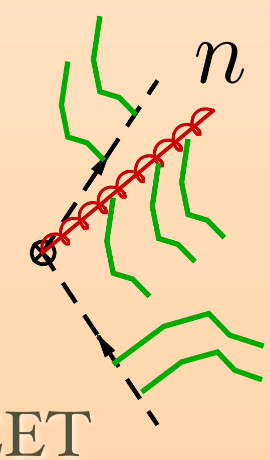


$$\bar\psi\,\Gamma^\mu\psi \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} \Rightarrow (\bar\xi_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar n} \underbrace{(W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}}_{\chi_{\bar n,\bar\omega}}$$



c. gauge invariant "parton" field

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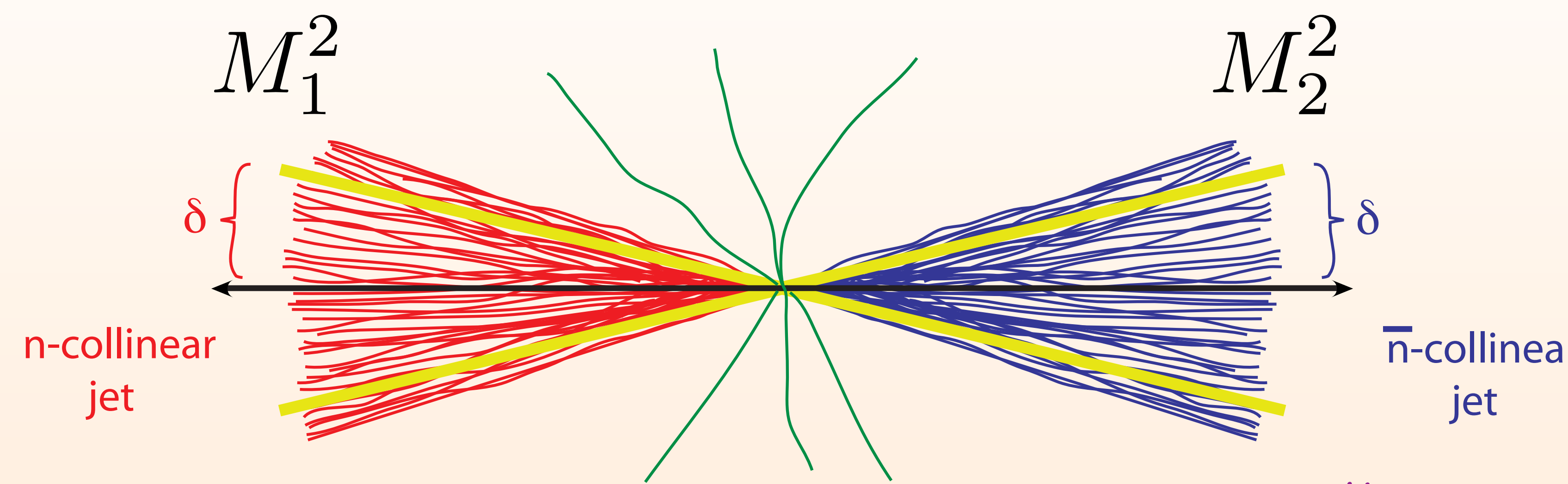
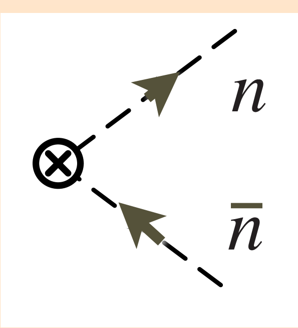


ξ_n, A_n^μ q_s, A_s^μ $\xi_{\bar n}, A_{\bar n}^\mu$

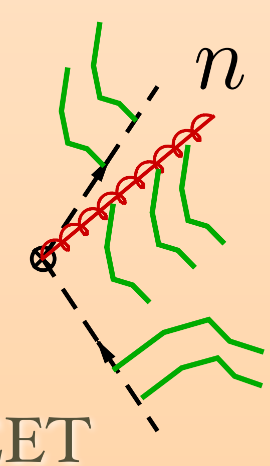
Production Current: $Q \gg \Delta$



$\mathcal{L}_n^{(0)}$ $\mathcal{L}_s^{(0)}$ $\mathcal{L}_{\bar n}^{(0)}$

$$\bar\psi\,\Gamma^\mu\psi \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} \Rightarrow (\bar\xi_n W_n)_\omega\; Y_n^\dagger \Gamma^\mu Y_{\bar n}\,(W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$

$\underbrace{(W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}}_{\chi_{\bar n,\bar\omega}}$



SCET

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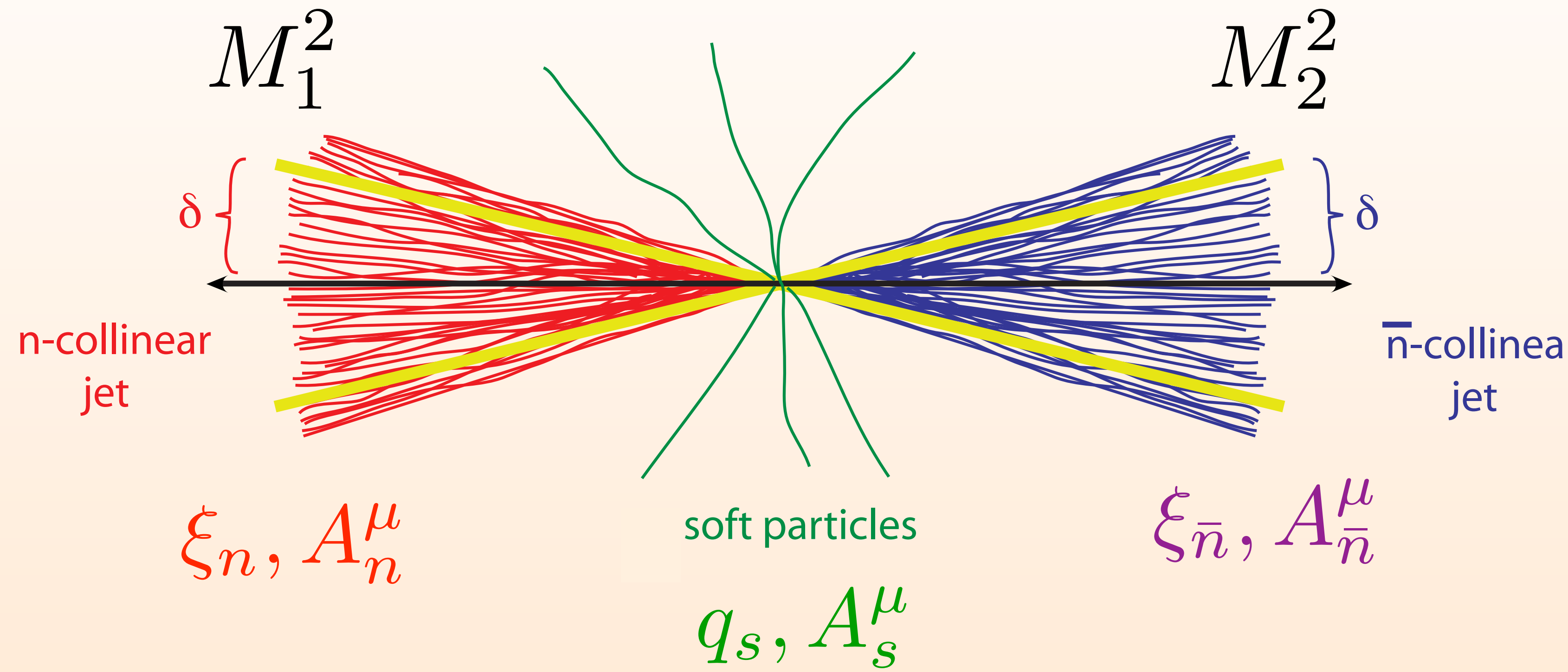

Factorization:

$$|X\rangle = |X_n X_{\bar n} X_s\rangle$$

$$\sigma = K_0 \sum_{\vec n} \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \langle 0|\overline{Y}_{\bar n}\, Y_n|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger|0\rangle$$

$$\times\ |C(Q,\mu)|^2\, \langle 0|\bar{\not{n}}\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle \langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\not{n}\chi_{\bar n,\bar\omega}|0\rangle$$

all-orders in α_s

SCET energetic jets

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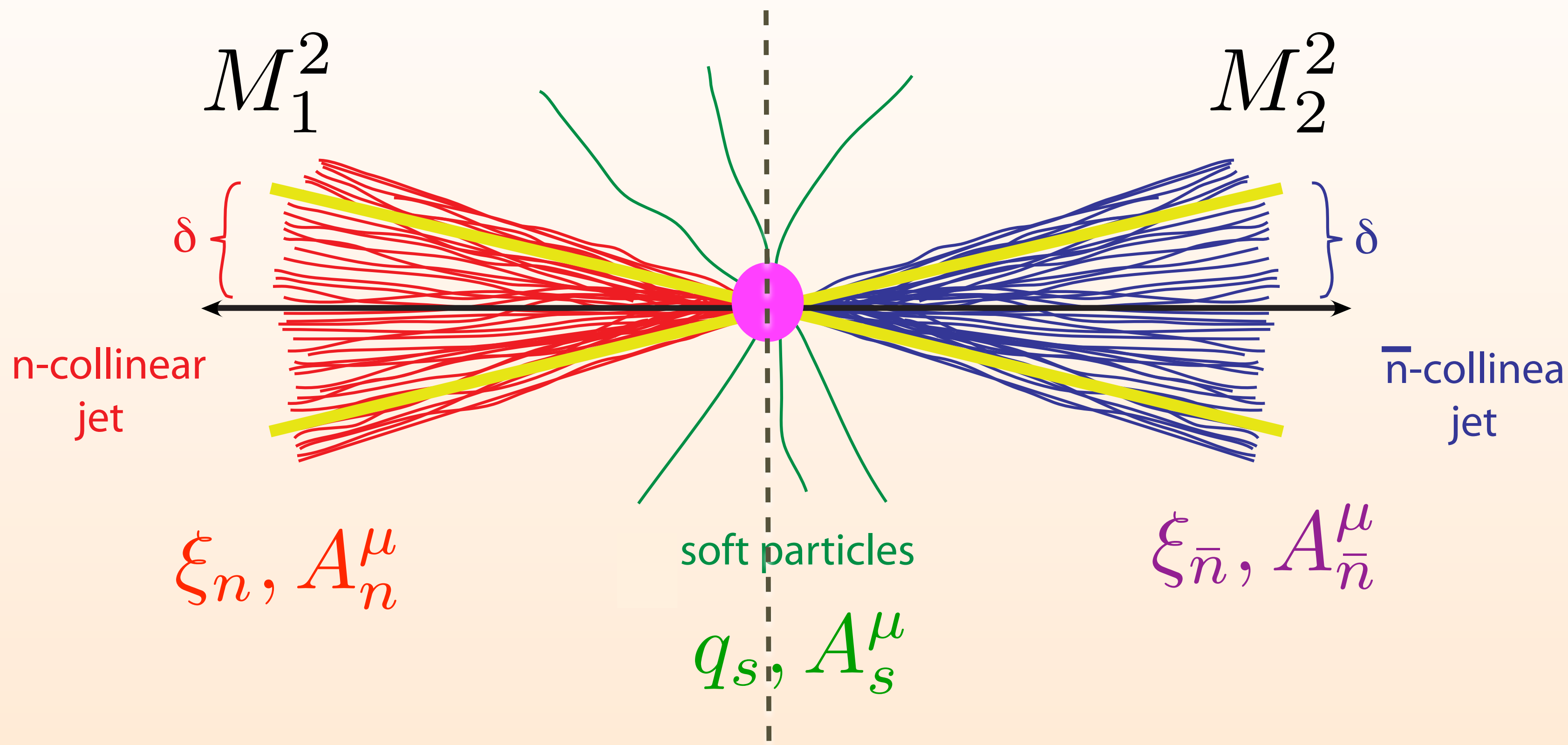

Factorization:

$\mu_H \sim Q$ $\qquad\qquad$ $\mu_J \sim M_i$ $\qquad\qquad$ $\mu_S \sim \ell^\pm$

$$\frac{d^2\sigma}{dM_1^2 dM_2^2} = \sigma_0 H(Q,\mu) \int d\ell^+ \, d\ell^- \, J_n\big(M_1^2 - Q\ell^+, \mu\big) J_{\bar n}\big(M_2^2 - Q\ell^-, \mu\big) S(\ell^+, \ell^-, \mu)$$

Hard Function $\qquad$ Jet Functions $\qquad$ Soft Function

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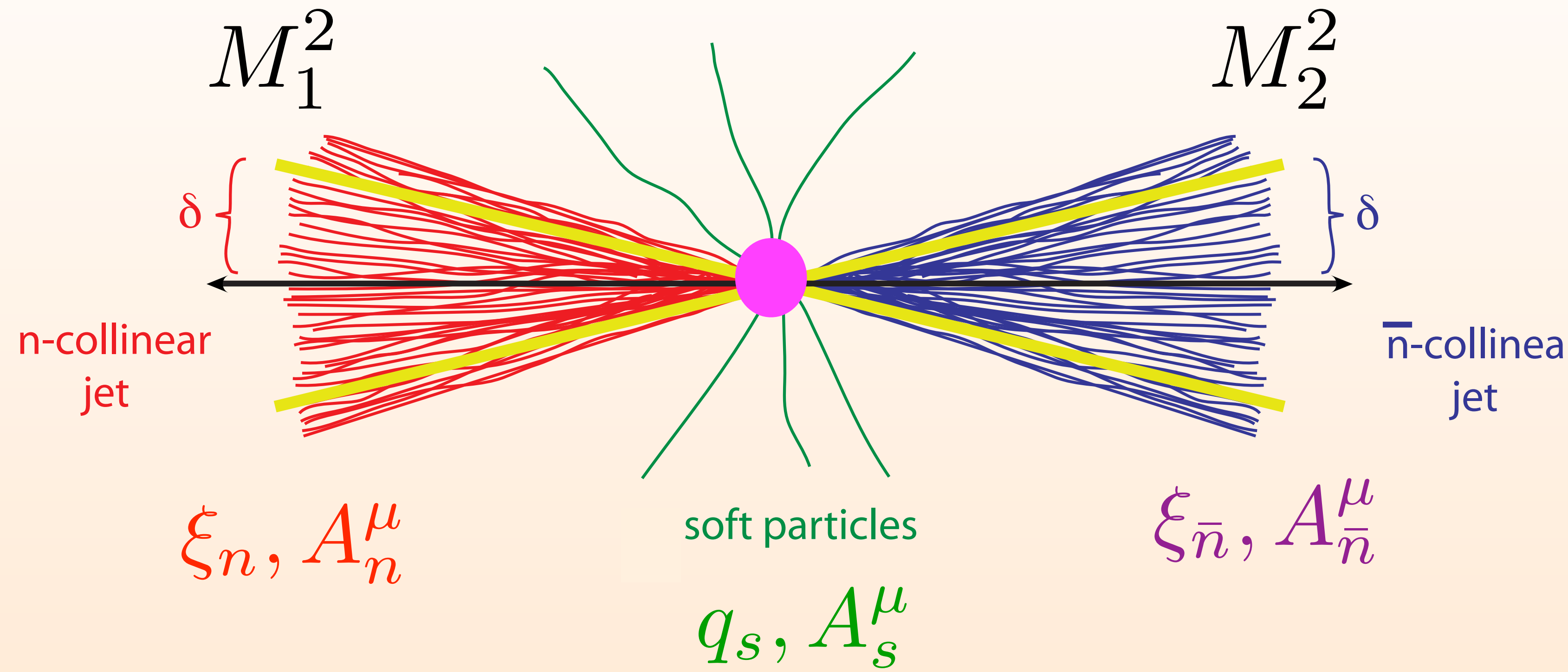
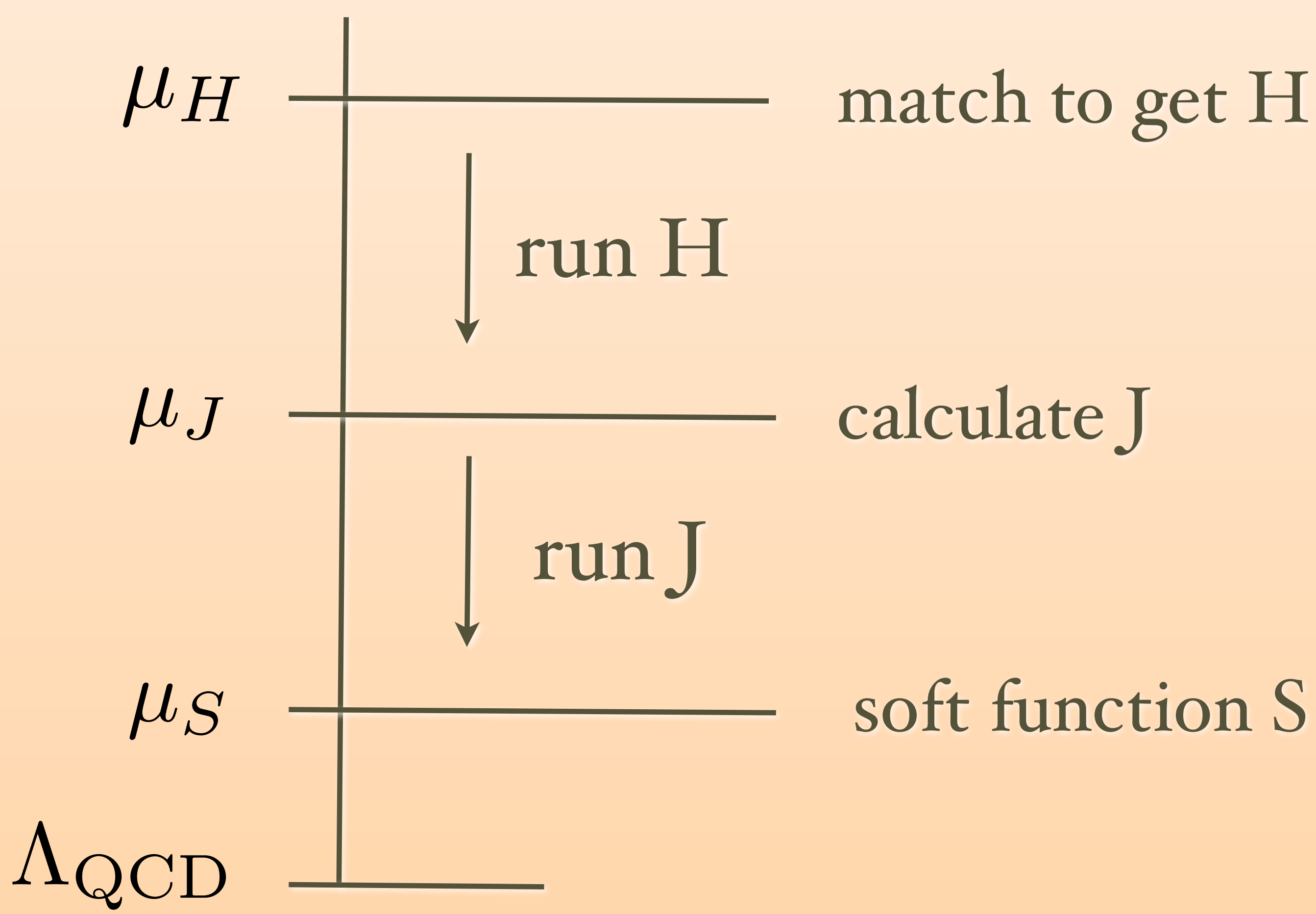


Sum Large Logs

run between scales & not below $\Lambda_{\rm QCD}$



Large Logs

log summation

$\alpha_s L \sim 1$

$\alpha_s \ll 1$

$$L = \ln(\mu_H/\mu_J) = \ln(\mu_J/\mu_S) = \ln(Q^2/\Delta^2)$$

	LO	NLO	NNLO	N^{3}LO		
$\sigma(\widetilde{\Delta}) =$	1	$+\,\alpha_s L^2$	$+\,\alpha_s^2 L^4$	$+\,\alpha_s^3 L^6$	$+\ldots$	LL
		$+\,\alpha_s L$	$+\,\alpha_s^2 L^3$	$+\,\alpha_s^3 L^5$	$+\ldots$	NLL
		$+\,\alpha_s$	$+\,\alpha_s^2 L^2$	$+\,\alpha_s^3 L^4$	$+\ldots$	NNLL
			$+\,\alpha_s^2 L$	$+\,\alpha_s^3 L^3$	$+\ldots$	
			$+\,\alpha_s^2$	$+\,\alpha_s^3 L^2$	$+\ldots$	N^{3}LL
				$+\,\alpha_s^3 L$	$+\ldots$	
				$+\,\alpha_s^3$	$+\ldots$	
				$\ddots$		

small print:

here $\sigma(\widetilde{\Delta}) = \int_0^{\widetilde{\Delta}} d\tau \frac{d\sigma}{d\tau}$; sum's are actually in exponent

Large Logs

$$\text{NLL}' \longrightarrow \text{NNLL} \longrightarrow \text{NNLL}' \longrightarrow \text{N}^3\text{LL} \longrightarrow \text{N}^3\text{LL}'$$

sum logs	add $\mathcal{O}(\alpha_s^2)$	sum logs	add $\mathcal{O}(\alpha_s^3)$

	LO	NLO	NNLO	N^{3}LO		
$\sigma(\Delta) =$	1	$+\,\alpha_s L^2$	$+\,\alpha_s^2 L^4$	$+\,\alpha_s^3 L^6$	$+\ldots$	LL
		$+\,\alpha_s L$	$+\,\alpha_s^2 L^3$	$+\,\alpha_s^3 L^5$	$+\ldots$	NLL$'$
		$+\,\alpha_s$	$+\,\alpha_s^2 L^2$	$+\,\alpha_s^3 L^4$	$+\ldots$	NNLL$'$
			$+\,\alpha_s^2 L$	$+\,\alpha_s^3 L^3$	$+\ldots$	
		NLO	$+\,\alpha_s^2$	$+\,\alpha_s^3 L^2$	$+\ldots$	N^{3}LL$'$
				$+\,\alpha_s^3 L$	$+\ldots$	
			NNLO	$+\,\alpha_s^3$	$+\ldots$	
				N^{3}LO	$\ddots$	

Operators

- built from $\{\ \chi_n,\ \mathcal{B}^\mu_{n\perp},\ i\partial^\mu_{n\perp}\}$, + usoft terms

  $\qquad\qquad\mathcal{O}(\lambda)\quad\mathcal{O}(\lambda)\quad\mathcal{O}(\lambda)$

N-jet amplitude $\quad O_N$

$$O_2 = \bar\chi_{n_1}\Gamma\chi_{n_2}$$

$$O_3 = \bar\chi_{n_1}\Gamma\mathcal{B}^\mu_{n_3\perp}\chi_{n_2}$$

$$O_4 = \bar\chi_{n_1}\Gamma\mathcal{B}^\mu_{n_3\perp}\mathcal{B}^\nu_{n_4\perp}\chi_{n_2}$$

$$O_2' = \bar\chi_{n_1}\Gamma\mathcal{B}^\mu_{n_2\perp}\chi_{n_2}$$

$$O_2'' = \bar\chi_{n_1}\Gamma\mathcal{B}^\mu_{n_2\perp}i\partial^\nu_{n_2\perp}\chi_{n_2}$$

$$O_3' = \bar\chi_{n_1}\Gamma\mathcal{B}^\mu_{n_3\perp}\mathcal{B}^\nu_{n_3\perp}\chi_{n_2}$$

$$O_{\rm pdf} = \bar\chi_{n_1}\Gamma\chi_{n_1,\omega}$$

$$O_{\text{jet fn.}} = \bar\chi_{n_1}(x^-)\Gamma\chi_{n_1,\omega}(0)$$

For more introduction see the lecture notes:

http://www2.lns.mit.edu/~iains/talks/SCET_Lectures-Stewart-2009.pdf

Event shapes

$$e^+e^- \to \text{jets}$$

$\alpha_s(m_Z)$ from Thrust

$e^+e^- \to$ jets

Aim at 1% precision

Becher, Schwartz;

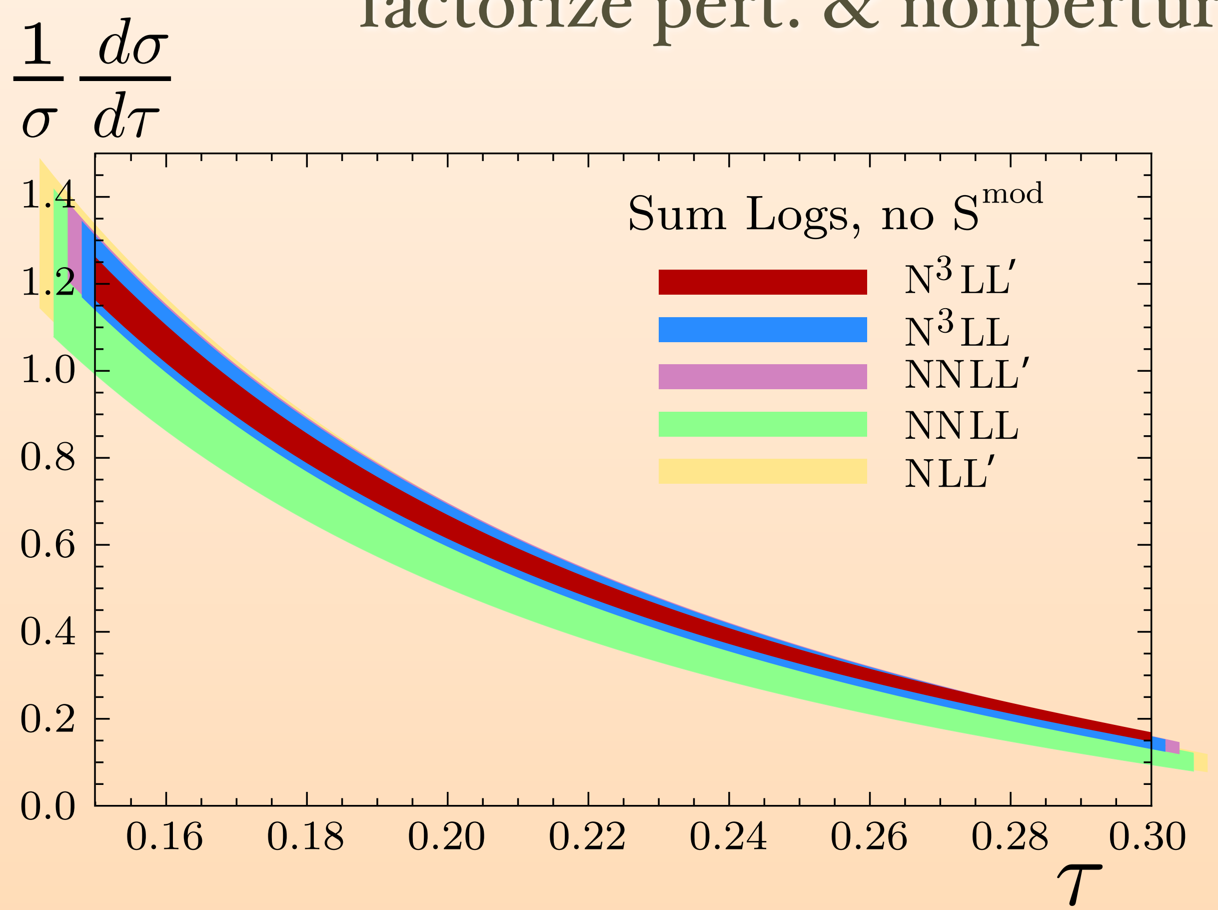
Abbate, Fickinger, Hoang, Mateu, I.S.

(using work by Gehrmann et al. & Weinzierl)

- $\mathcal{O}(\alpha_s^3)$ + N^{3}LL + $\frac{\Omega_1}{Q\tau}$ power correction + renormalon subtractions, R-RGE

+ full treatment of {peak, tail, multijet} + QED effects + b-mass effects + global fit, various Q's

factorize pert. & nonperturbative soft effects: $S = S^{\text{pert}} \otimes S^{\text{mod}}$



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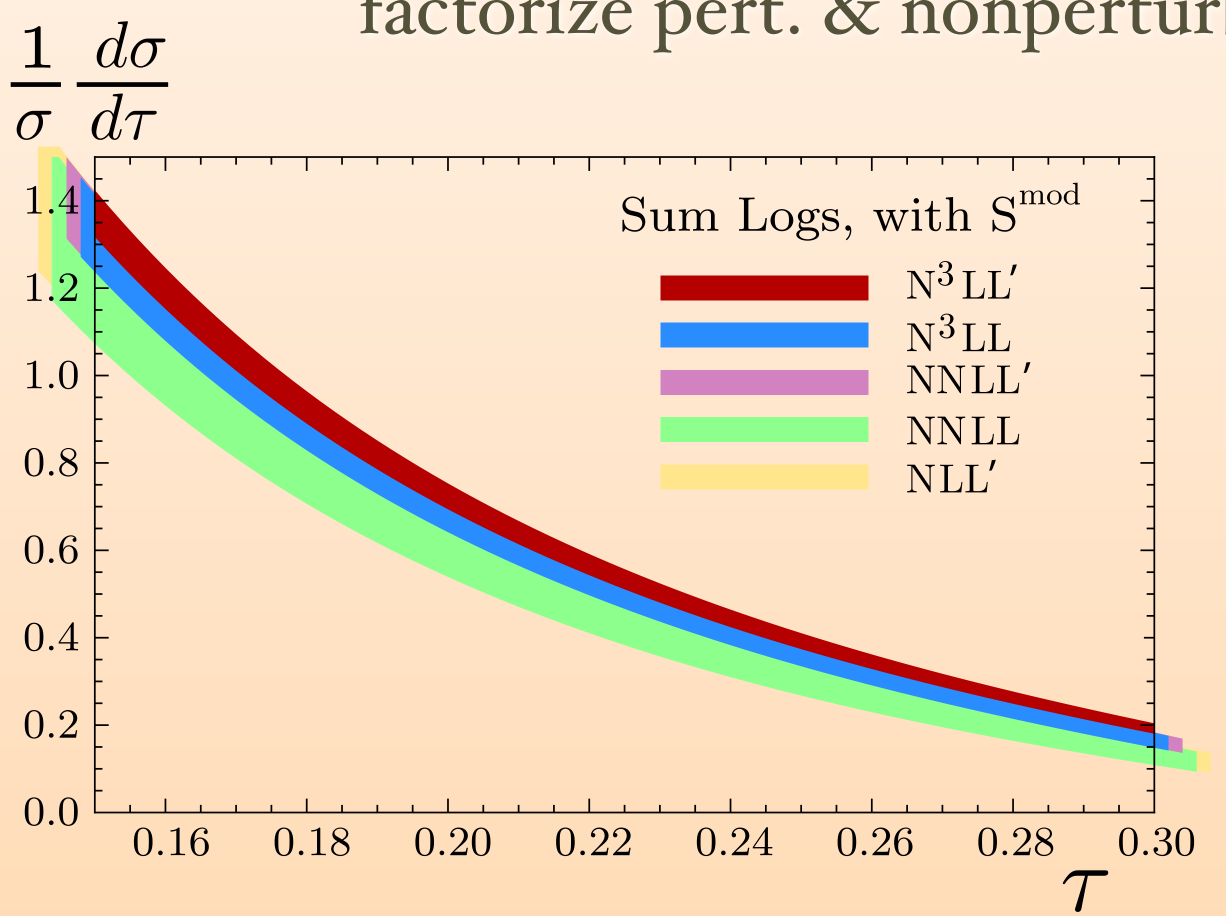

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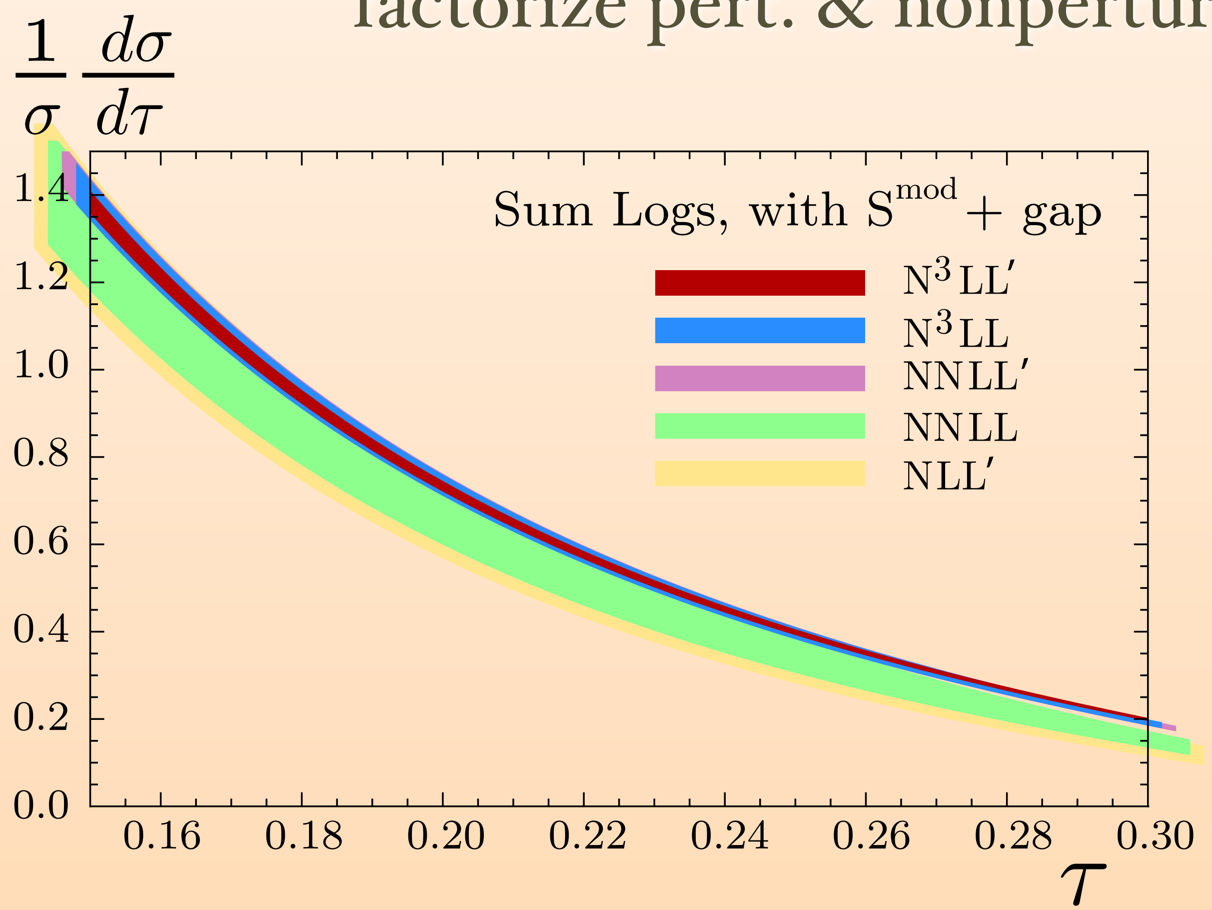

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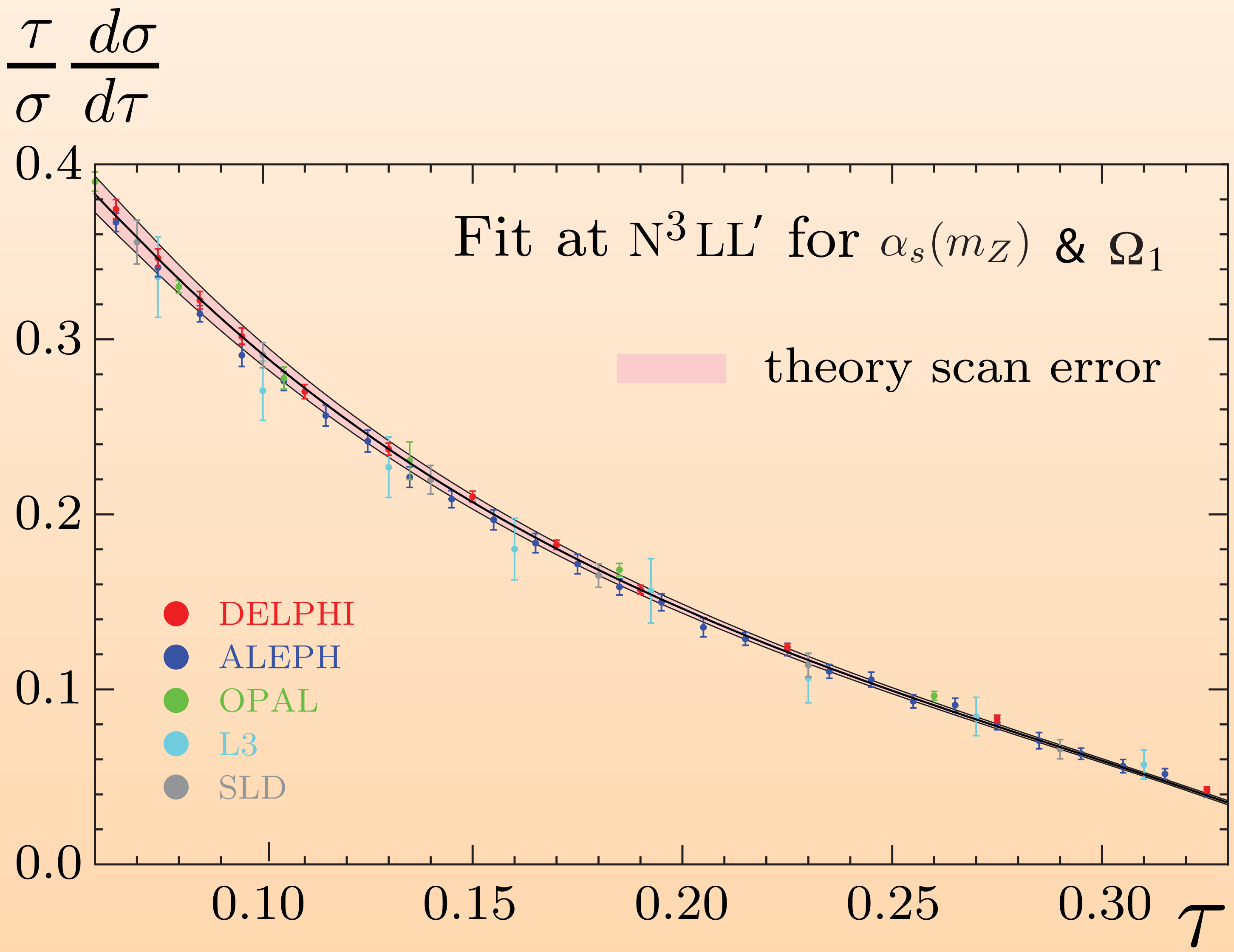

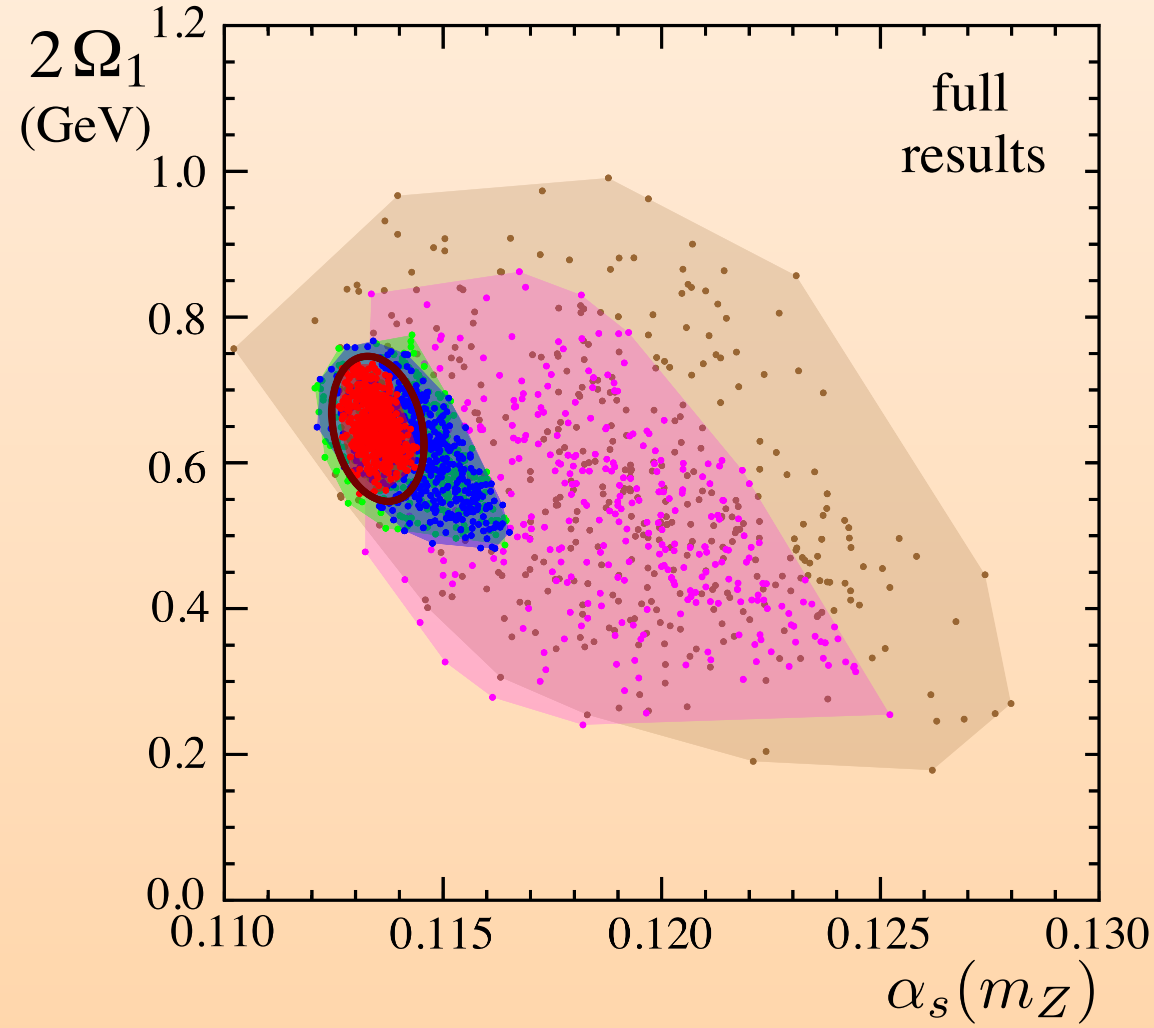

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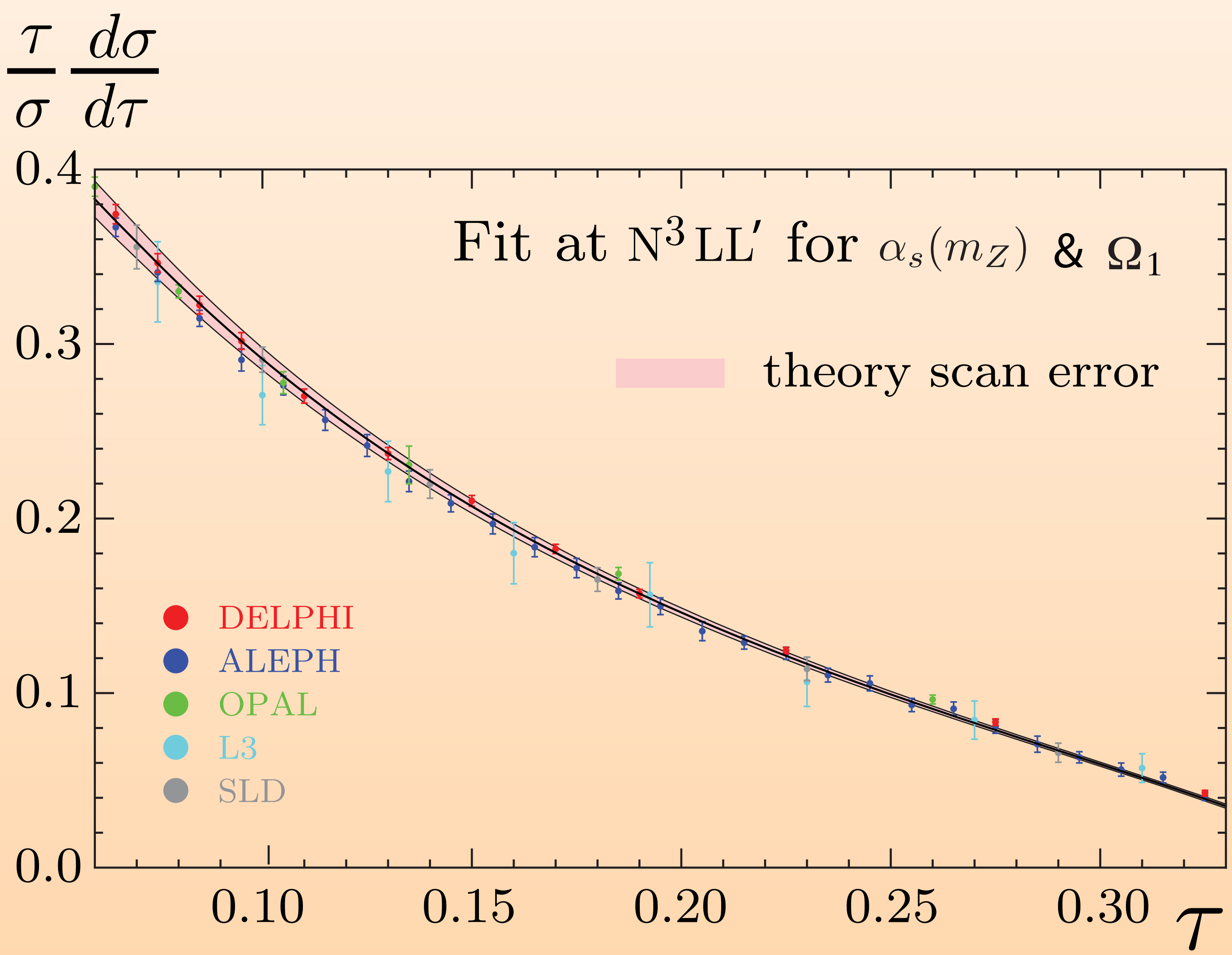

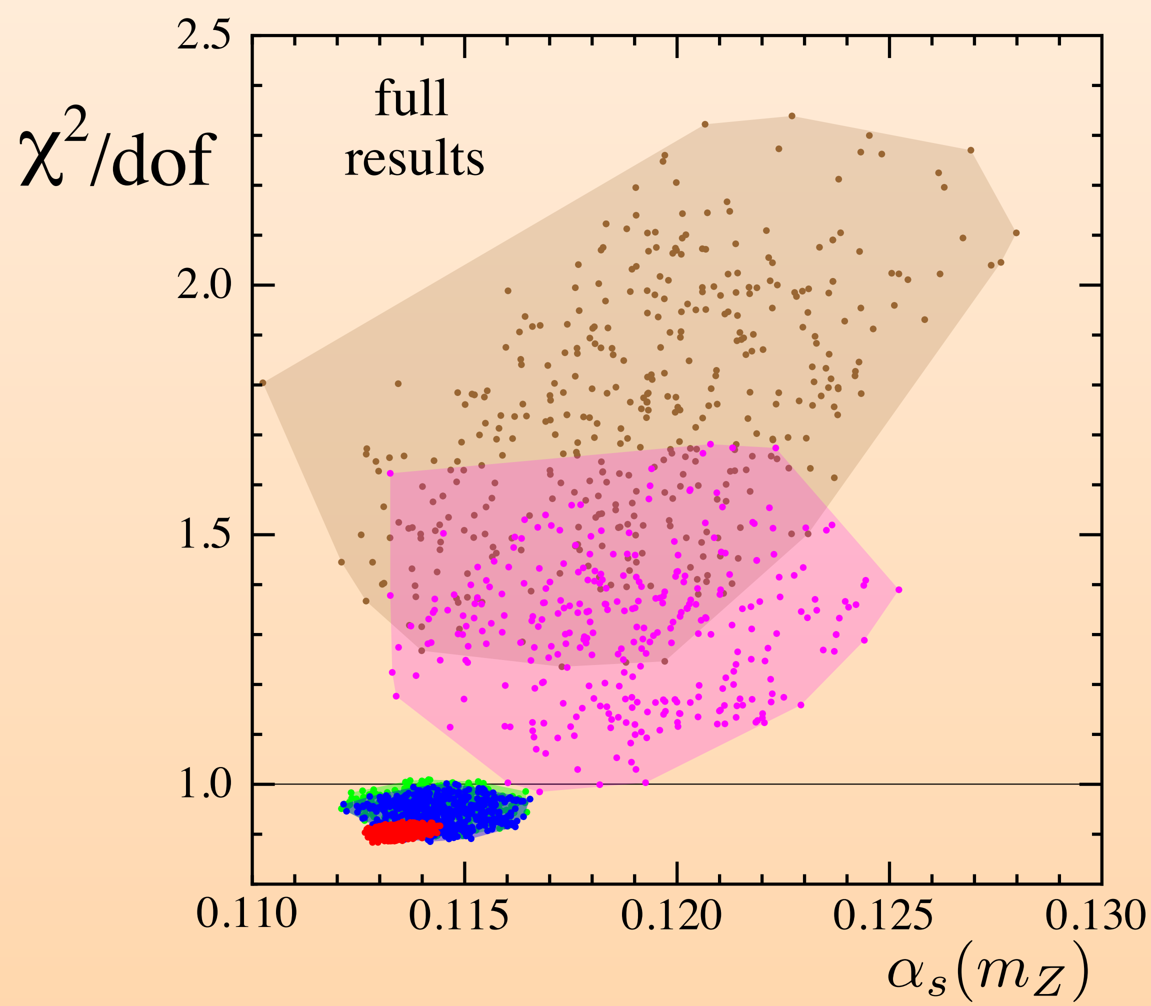

$\alpha_s(m_Z)$ from Thrust

Aim at 1% precision

Becher, Schwartz;

Abbate, Fickinger, Hoang, Mateu, I.S.

(using work by Gehrmann et al. & Weinzierl)

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$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm expt} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

χ^2 fit (expt, hadr; Ω_1: expt, $\alpha_s(m_Z)$)

Parameter Scan (pert; Ω_1: Ω_2, pert)

$$\Omega_1 = 0.324 \pm (0.009)_{\rm expt} \pm (0.013)_{\Omega_2} \pm (0.030)_{\alpha_s(m_Z)} \pm (0.045)_{\rm pert}\ {\rm GeV}$$

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- power correction shifts result for $\alpha_s(m_Z)$ by -9%

(in tuned MC effect is small at m_Z, similar at other scales)

$\alpha_s(m_Z)$ from Thrust

Aim at 1% precision

Becher, Schwartz;

Abbate, Fickinger, Hoang, Mateu, I.S.

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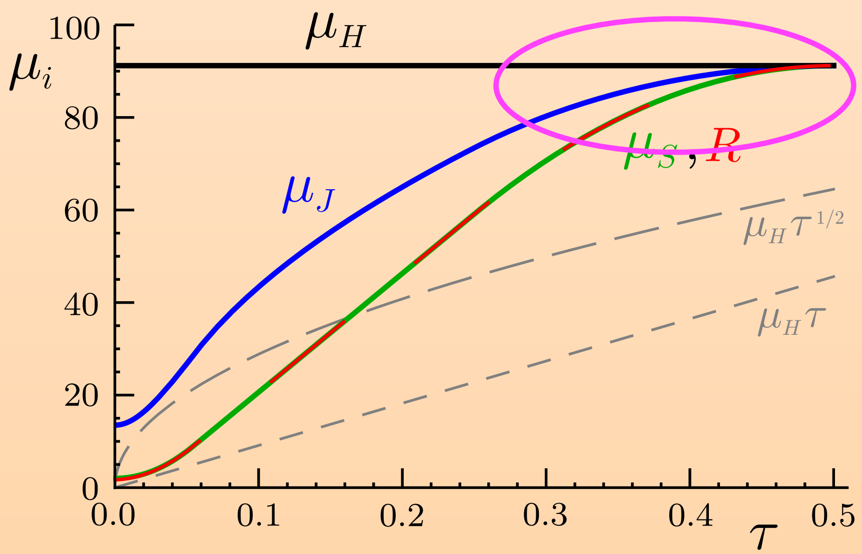
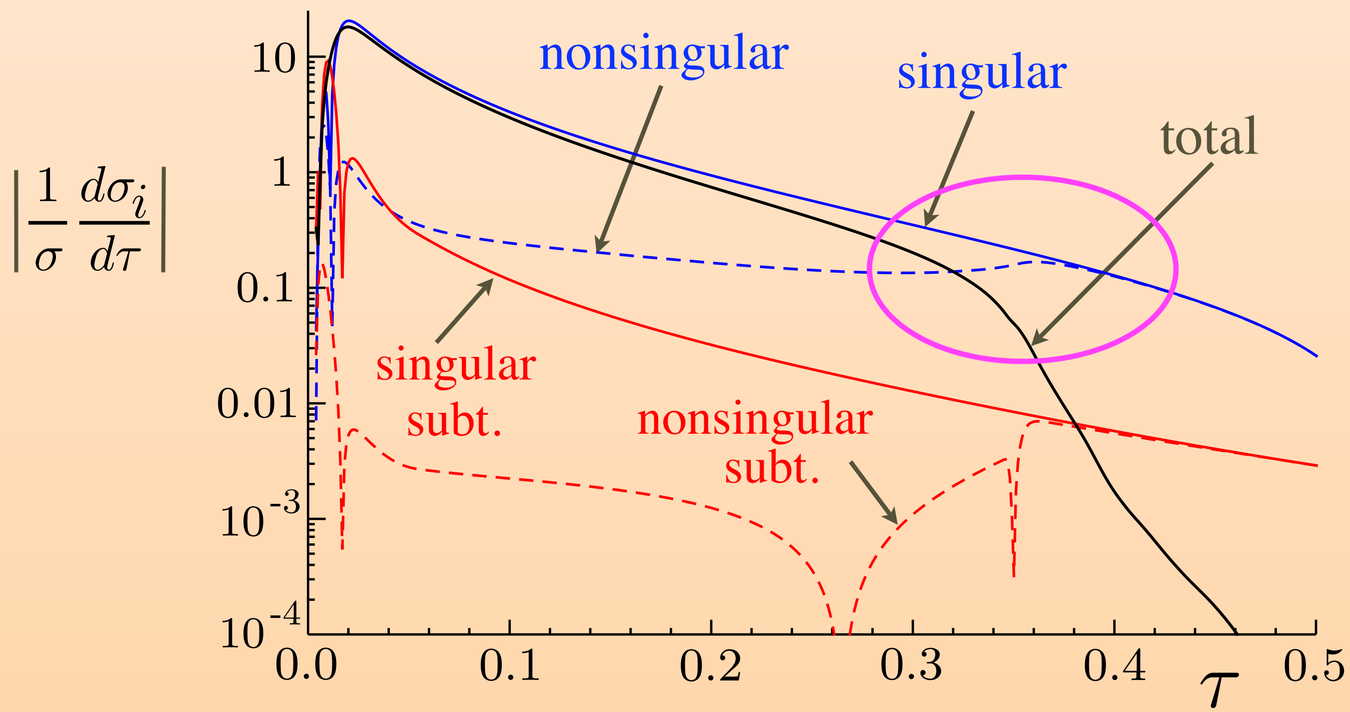
- profile functions $(+4\%)$





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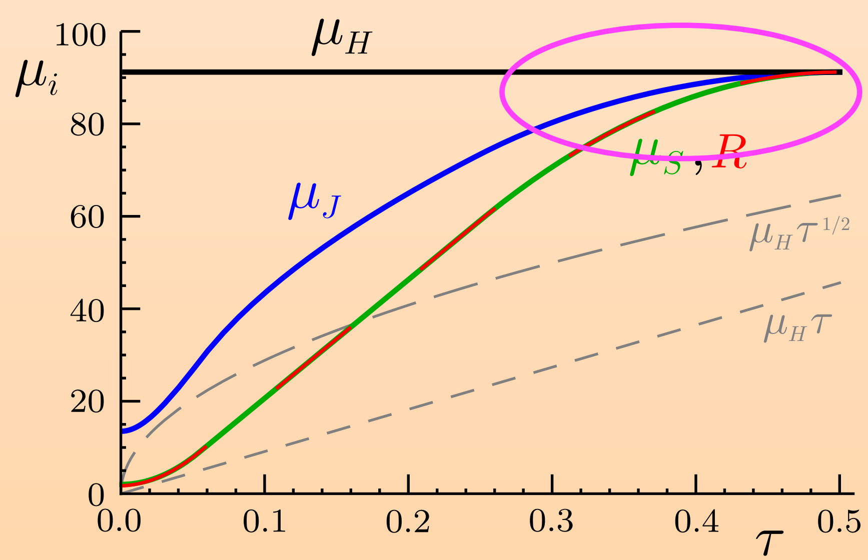
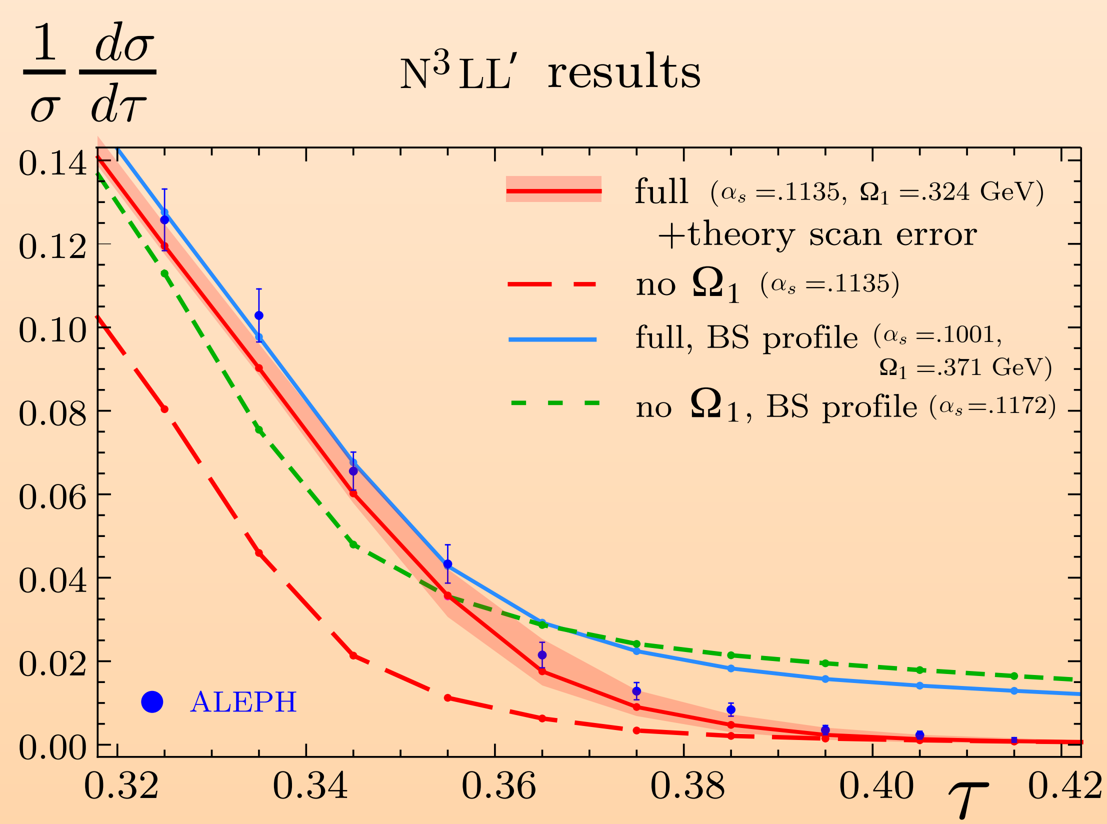


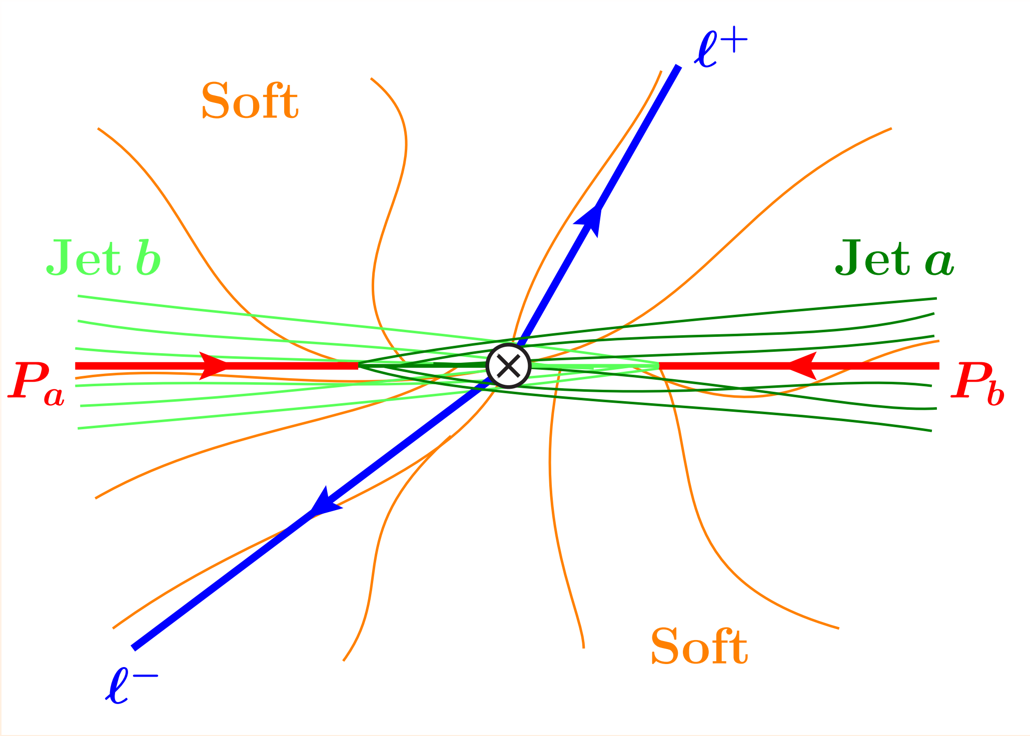


Event shapes

$$pp \to \text{jets} + \text{leptons}$$

Isolated Drell-Yan

IS, Tackmann, Waalewijn



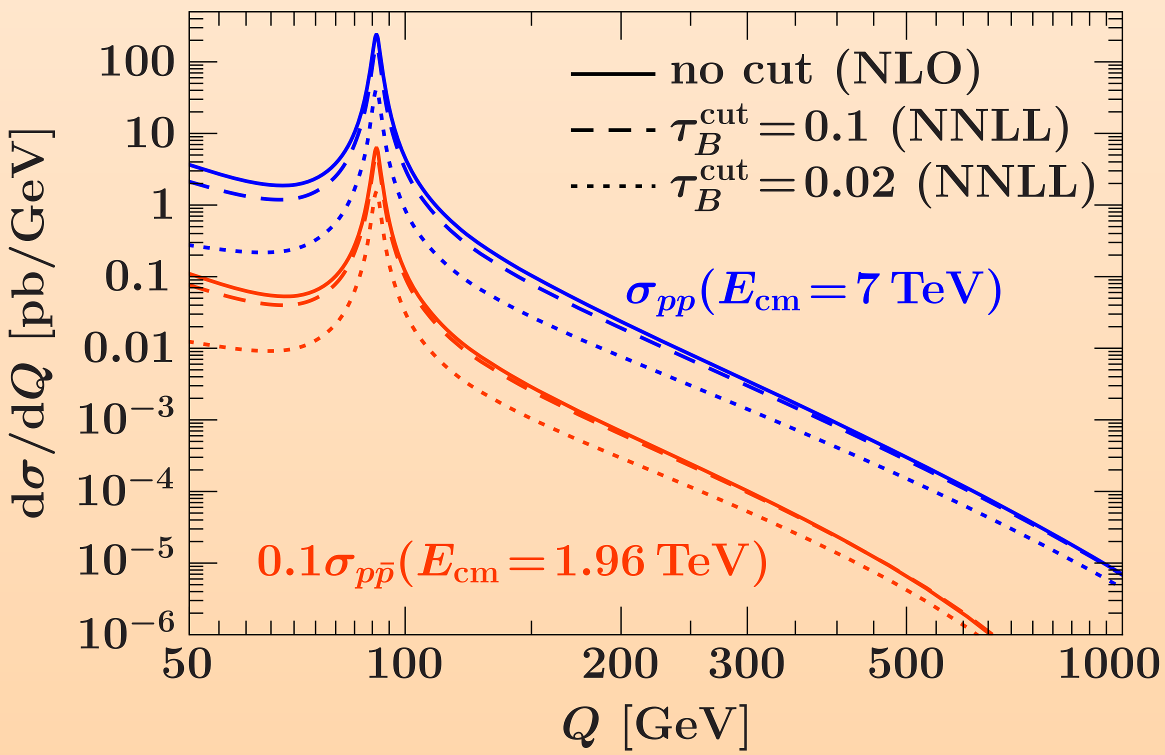
- measure Beam Thrust

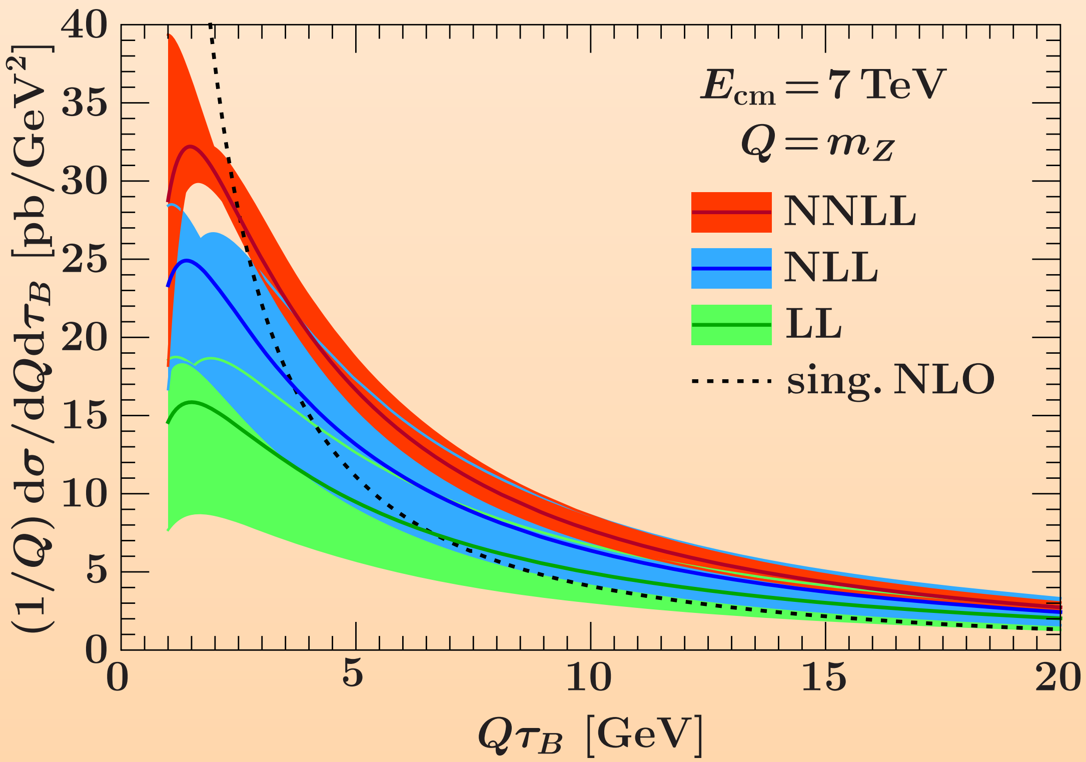
$$\mathcal{T}_B = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

$$\tau_B = \mathcal{T}_B / Q$$

- $\tau_B \ll 1$ is veto for central jets
- sensitive probe of Initial State Parton shower (test & tune MC)

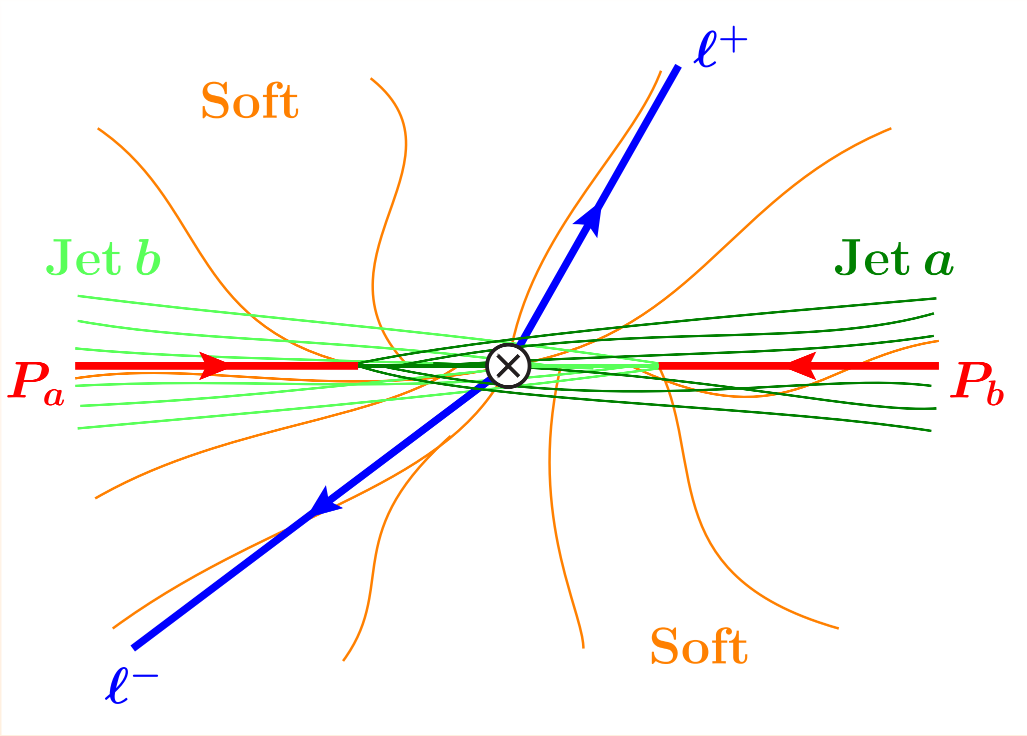
Captures a large part of cross section even for: $0 \leq \tau_B \leq \tau_B^{\rm cut} = 0.1$





Isolated Drell-Yan

IS, Tackmann, Waalewijn



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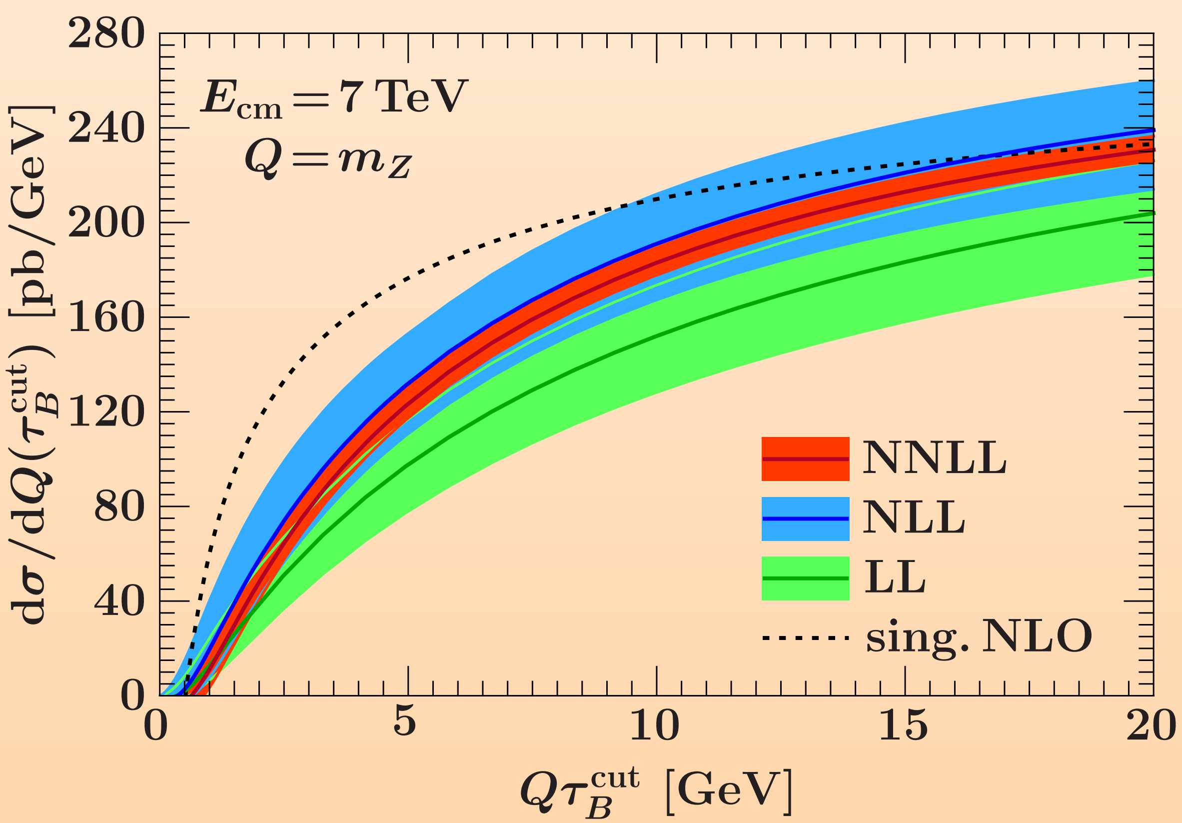
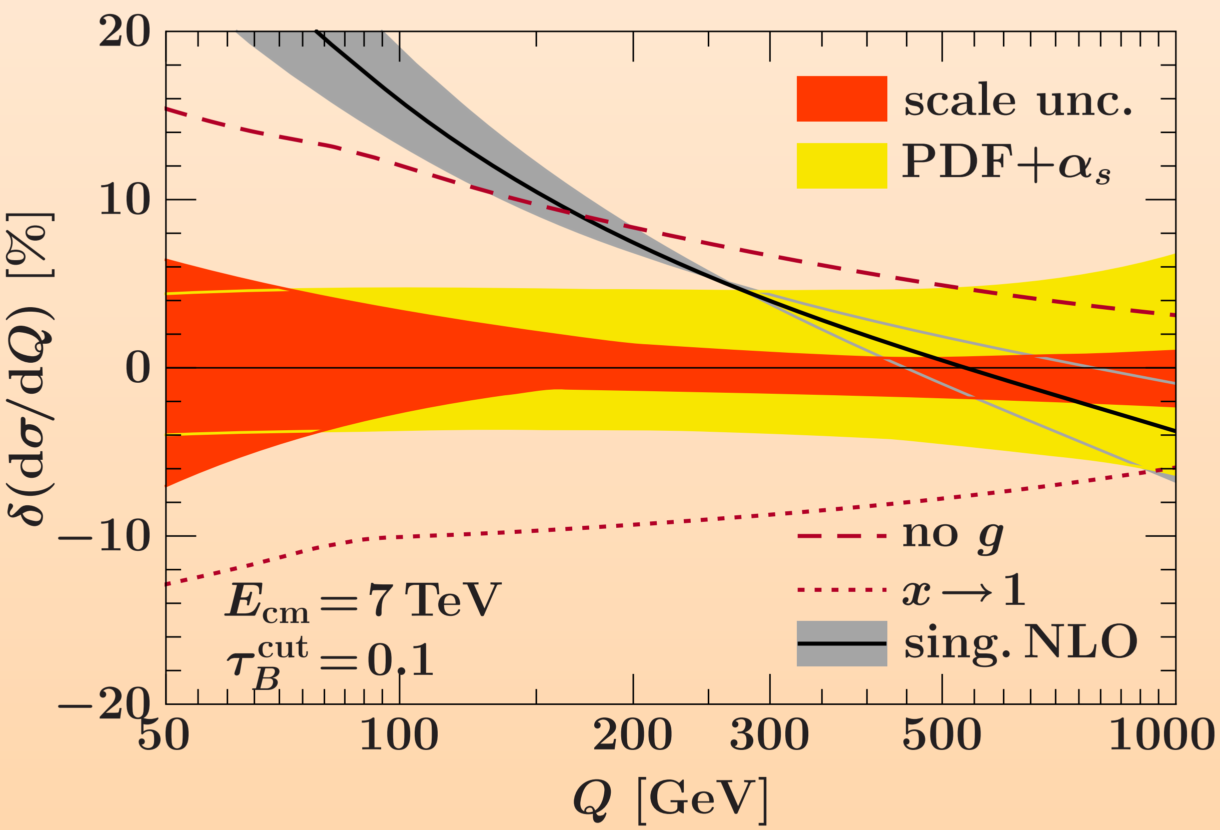
- $\tau_B \ll 1$ is veto for central jets
- sensitive probe of Initial State Parton shower (test & tune MC)

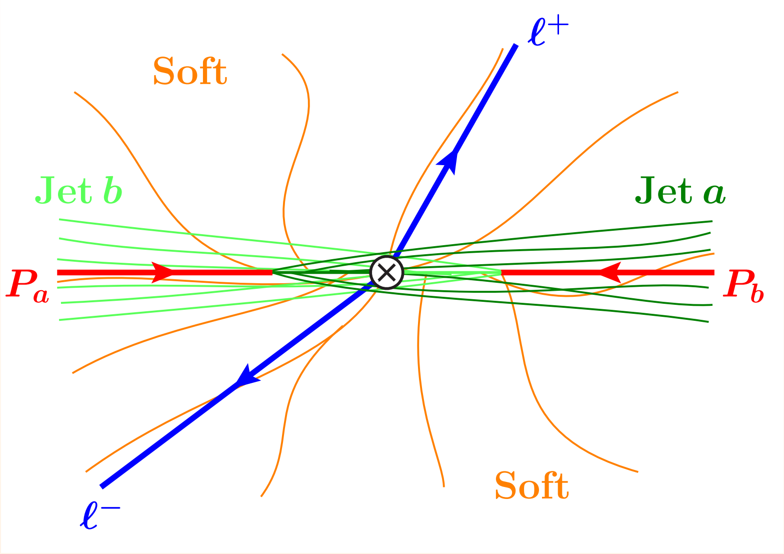
$0 \leq \tau_B \leq \tau_B^{\text{cut}}$ nice convergence



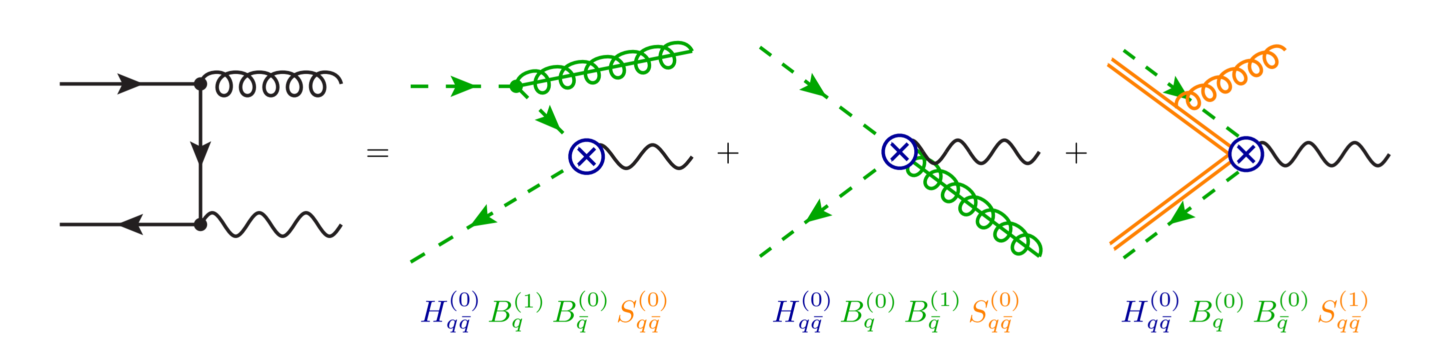


Calculation here involves:

- ISR from proton

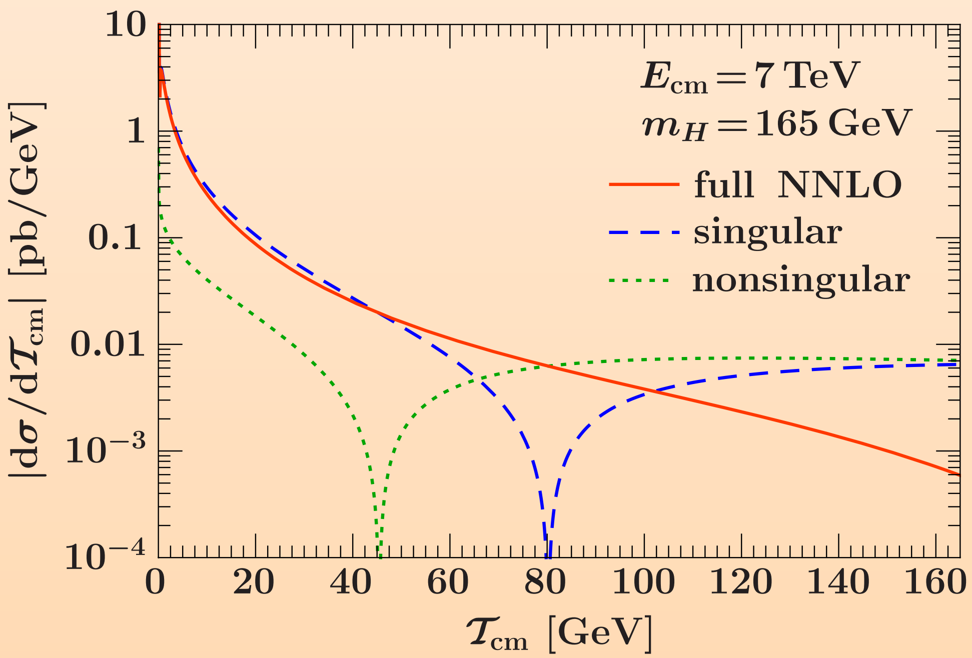


- summing large logs from t - channel singularities



$$\frac{\alpha_s^n \ln^m(t/Q^2)}{t}$$

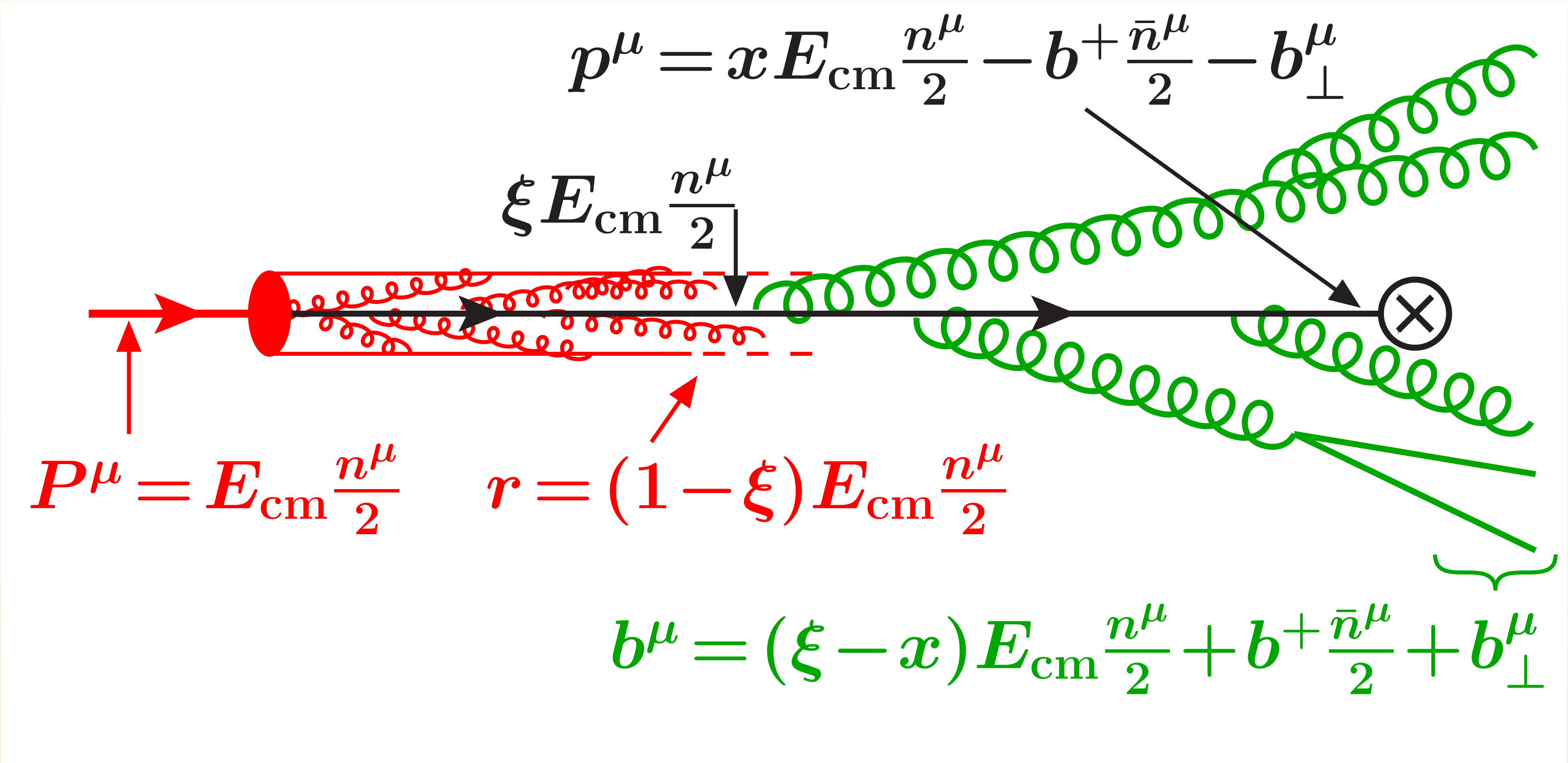
- Singular terms dominate numerically



- Factorization involves Beam Functions

$$d\sigma = \sum_{ij} H_{ij} \int B_i(t_a, x_a) B_j(t_b, x_b) \otimes S_B$$

- Measurement probes proton PRIOR to hard collision



$$t = x E_{\rm CM} b^+$$

$$t \gg \Lambda_{\rm QCD}^2$$

Beam function factorization:

$$B_i(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mu\Big) f_j(\xi,\mu)$$

perturbative & calculable

Fleming, Leibovich, Mehen
IS, Tackmann, Waalewijn

As proton matrix element:

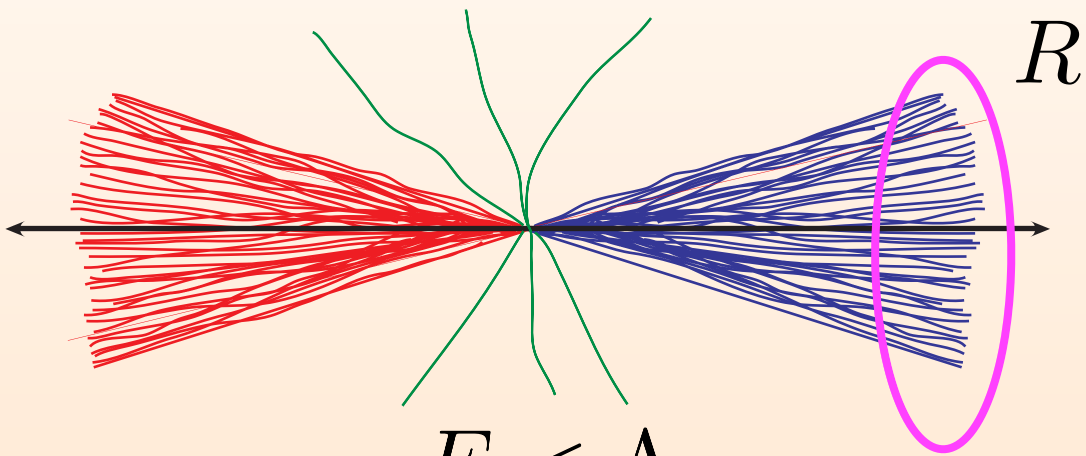
$$B_q(\omega b^+, \omega/P^-, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{{\rm d}y^-}{4\pi} e^{ib^+ y^-/2} \Big\langle p_n(P^-) \Big| \bar\chi_n\Big(y^- \frac{n}{2}\Big) \delta(\omega - \overline{\mathcal{P}}_n) \frac{\bar{n}\!\!\!/}{2} \chi_n(0) \Big| p_n(P^-) \Big\rangle$$

Factorization with Jet Algorithms

Berger, Kucs, Sterman;
Almeida, Lee, Perez, Sterman, Sung, Virzi; Jouttenus; Cheung et al

Jet & Soft functions depend on the algorithm

Ellis, Hornig, Lee, Vermilion, Walsh



$$\mu_H = Q$$

$$\mu_J = Q \tan \frac{R}{2}$$

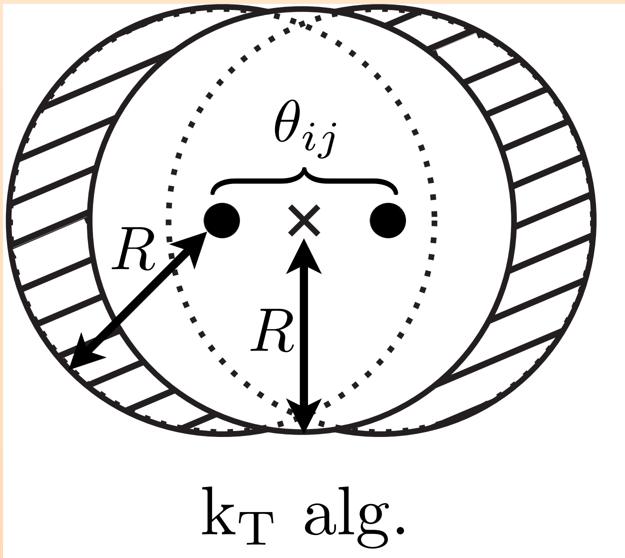
soft

$$\mu_S^I = \Lambda$$

$$\mu_S^{II} = 2\Lambda \tan \frac{R}{2}$$

measurements induce more scales

Grouping of Soft Radiation



kT: $R \gg \theta_{ij} \sim \lambda$

anti-kT, cone: $R \gtrsim \lambda$

non global logs: $\alpha_s^2 \ln^2 + \ldots$

N - Jettiness

IS, Tackmann, Waalewijn

Define (massless) reference momenta for each jet and beam

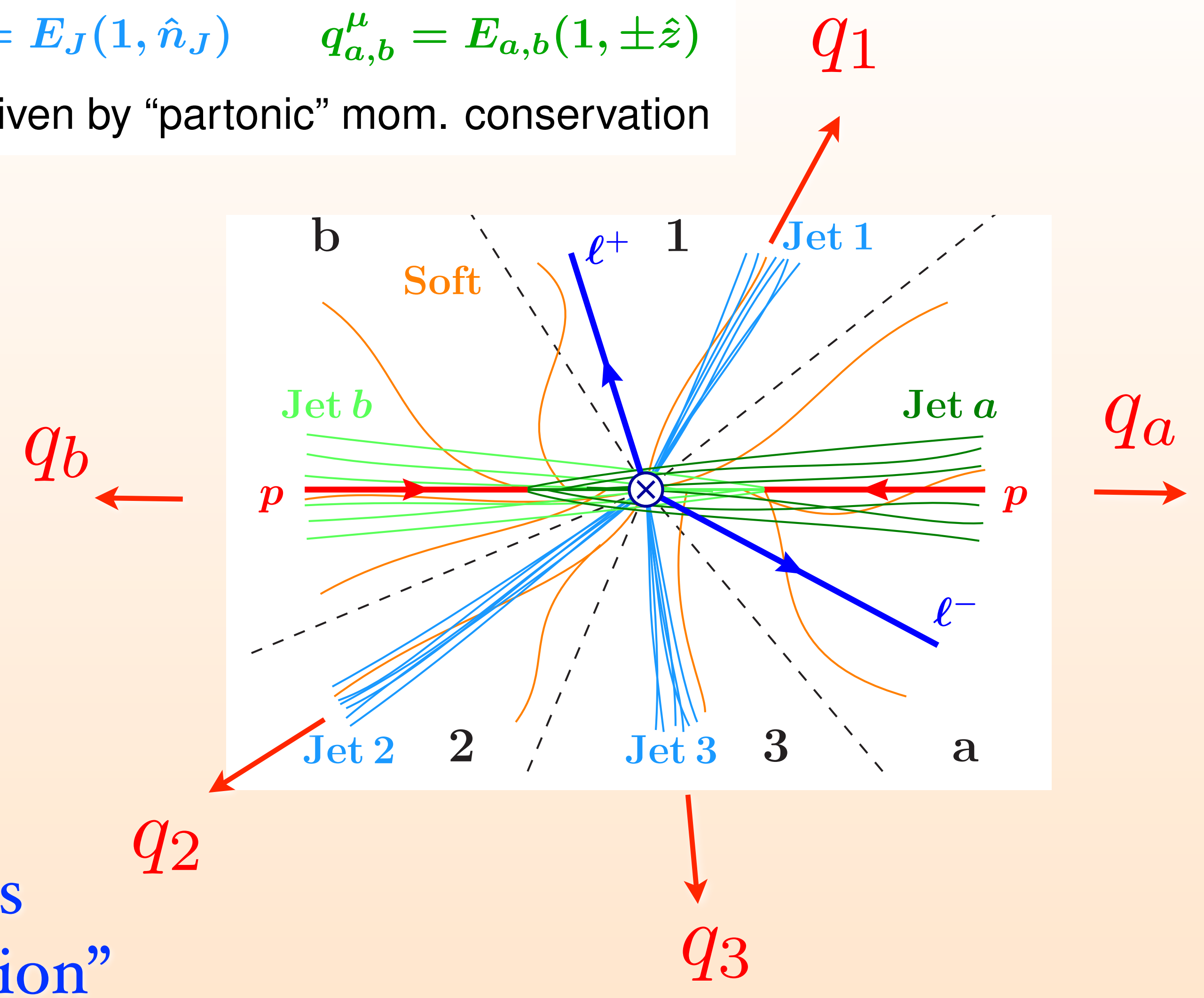
$q_J^\mu = E_J(1, \hat{n}_J) \qquad q_{a,b}^\mu = E_{a,b}(1, \pm\hat{z})$

$E_{a,b}$ given by "partonic" mom. conservation

$$\mathcal{T}_N = \sum_k \min\left\{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k \right\}$$

normalize it if desired

can pick another distance measure if desired



Practical: use jet algorithm to determine $\{E_J, \hat{n}_J\}$ and hence q_i

$\mathcal{T}_N \ll Q^2$ ensures there are N jets "exclusive N-jet cross section"

nice scales: $\mu_H = Q \qquad \mu_{B,J}^2 = \mathcal{T}_N \qquad \mu_S = \mathcal{T}_N / Q$

Inclusive Jet and Beam functions, N-jettiness soft function

$$\frac{d\sigma}{d\tau_N \, d(q_m)} = H \, B_i \otimes B_j \otimes \prod J_k \otimes S_{\tau_N}^{(q_m)}$$

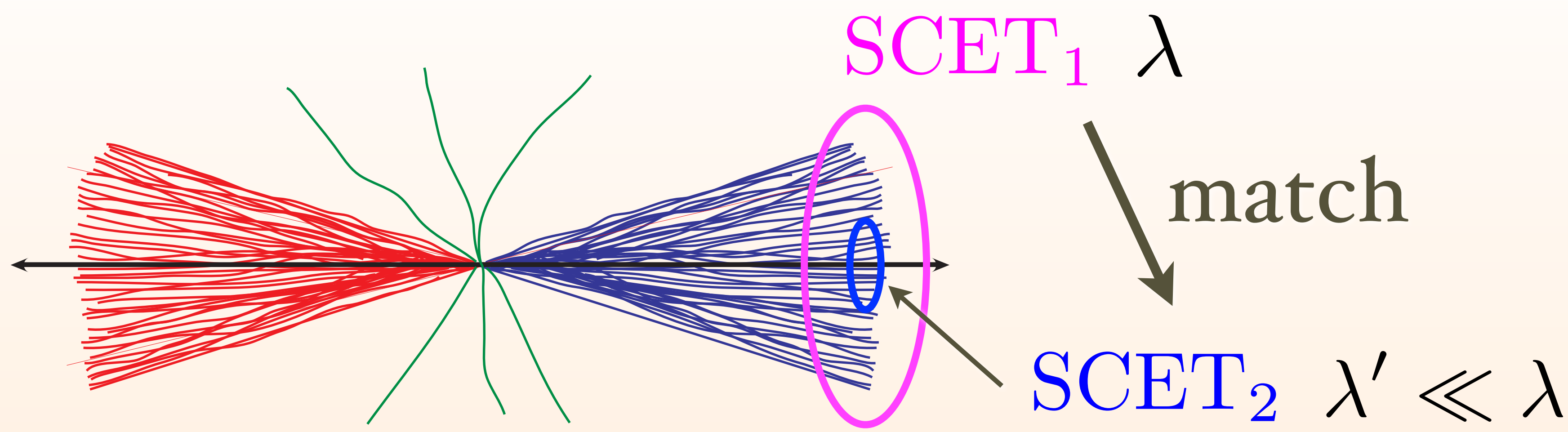
$$\sum_{k \in \mathrm{coll}_J} \min_m \{2 q_m \cdot p_k\} = \sum_{k \in \mathrm{coll}_J} 2 q_J \cdot p_k = s_J$$

Jet Substructure

$\mathrm{SCET}_1 \Rightarrow \mathrm{SCET}_2$

$\text{SCET}_1 \Rightarrow \text{SCET}_2$

$$J_1 = C \otimes J_2$$

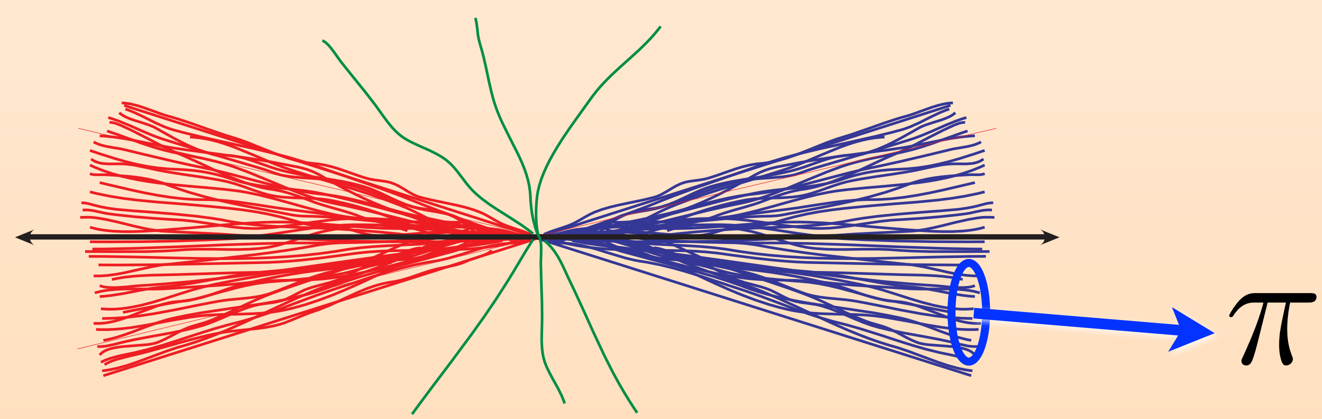


- $B \to \pi \ell \bar{\nu}$
- Beam functions are an example $B_i = \sum_j \mathcal{I}_{ij} \otimes f_j$
- Study fragmentation in a jet of measured invariant mass

IS, Procura

$$J(s) \to \frac{1}{16\pi^3} \mathcal{G}^\pi(s,z)dz$$

fragmenting jet function



$$\mathcal{G}_i^\pi(s,z) = \sum_j \int \frac{dx}{x} \mathcal{J}_{ij}\left(s, \frac{z}{x}\right) D_j^\pi(x)$$

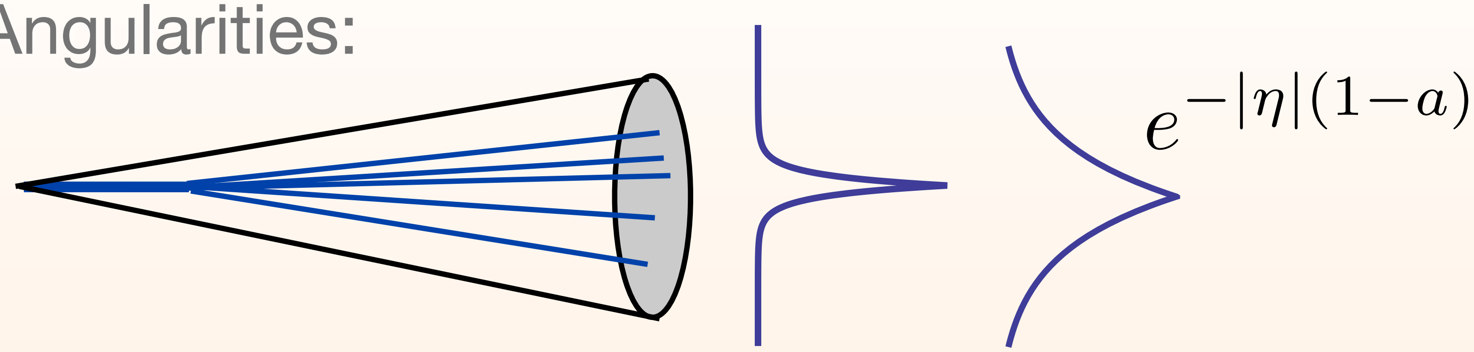
cute: $$\sum_j \int dz\, z\, \mathcal{J}_{ij}(s,z) = J_i^{\text{inclusive}}(s)$$

Jain, Procura, Waalewijn

- **Jet Shapes with Jet Algorithm** (event shapes in jets)

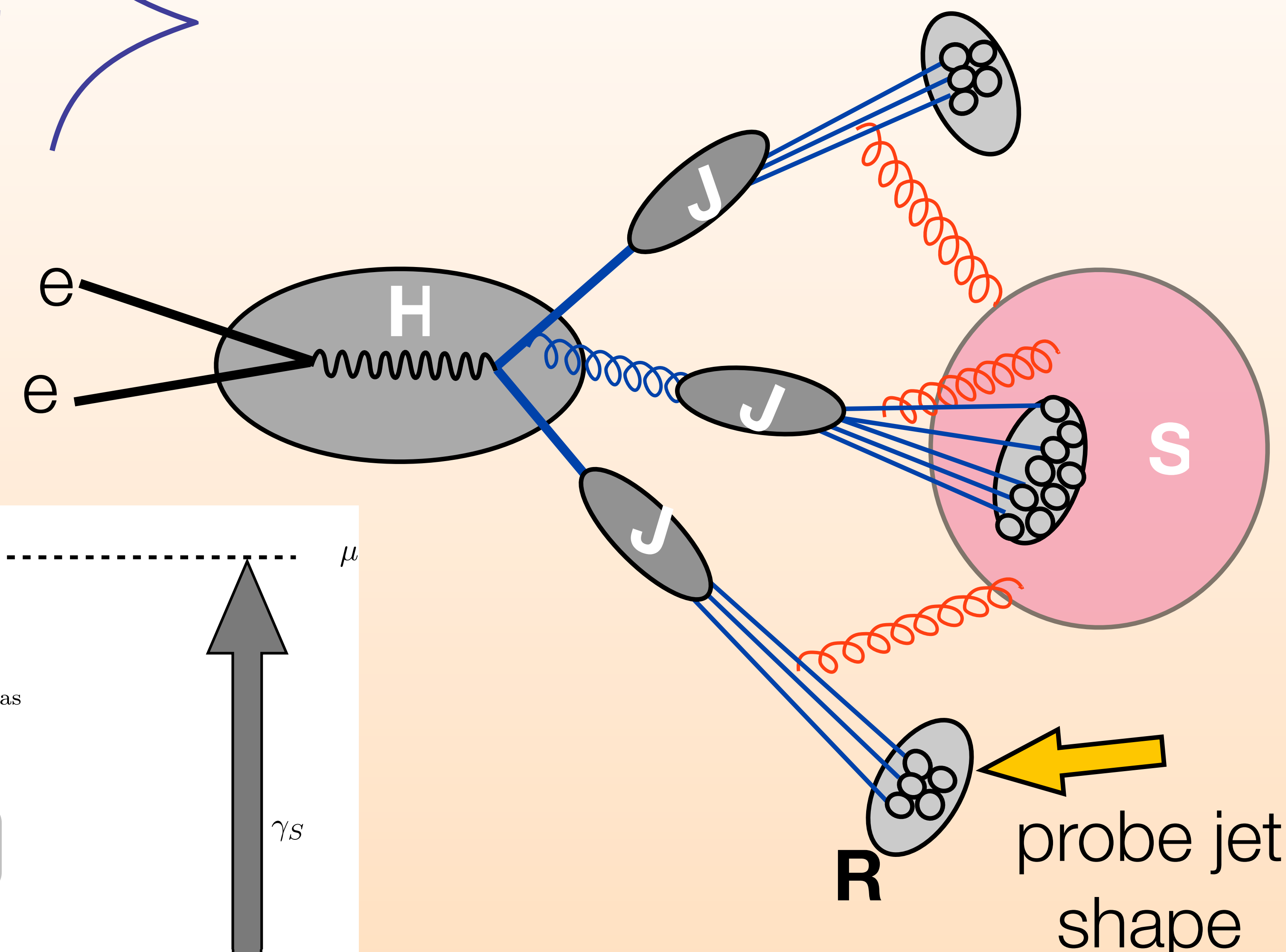
Berger, Kucs, Sterman; Almeida, Lee, Perez, Sterman, Sung, Virzi; Jouttenus; Cheung et al

Angularities:

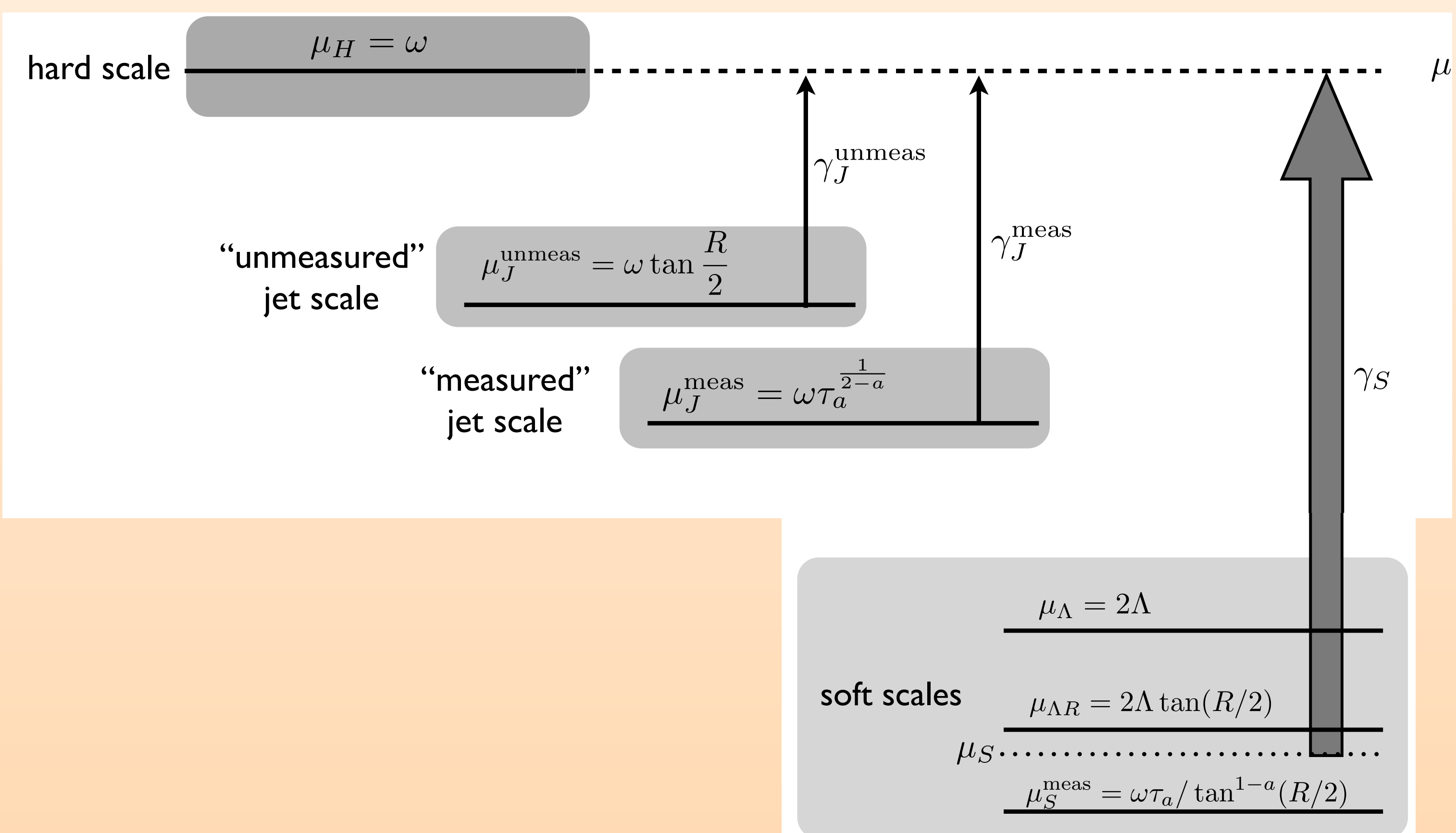


Ellis, Hornig, Lee, Vermilion, Walsh

$$\tau_a(\text{jet}) = \frac{1}{2E_{\text{jet}}} \sum_{i\in\text{jet}} |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$



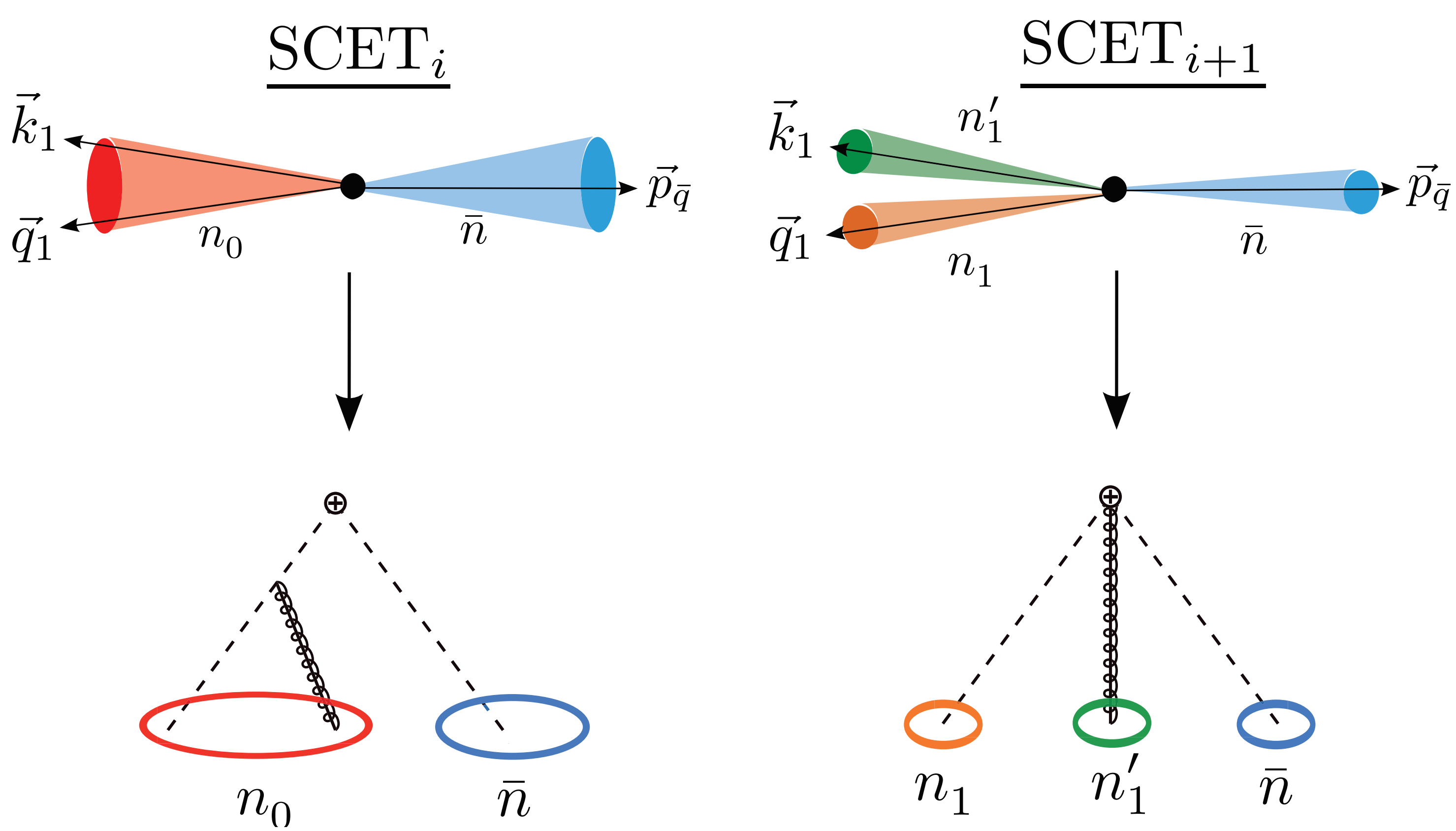
more scales:



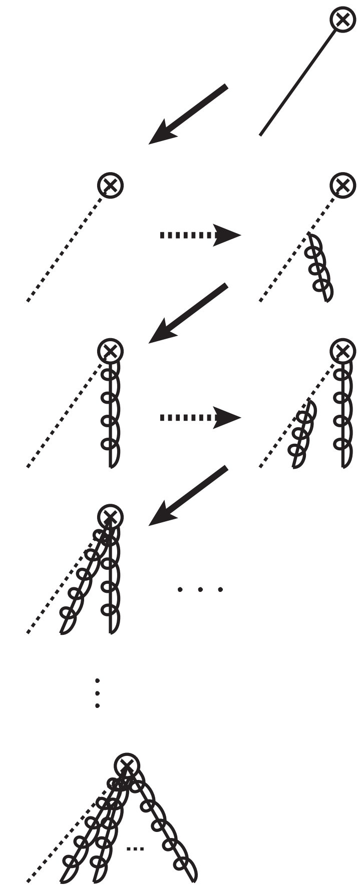
$$\frac{d\sigma}{d\tau_{a_1}\cdots d\tau_{a_N}} \sim \int d\Phi(q_J) F_N(q_J)\, \text{tr}\, \widehat{H}_\kappa(\mu) \Big[\prod_J J_{a_J}(\mu)\Big] \otimes \widehat{S}_{a_1\cdots a_N}(\mu)$$

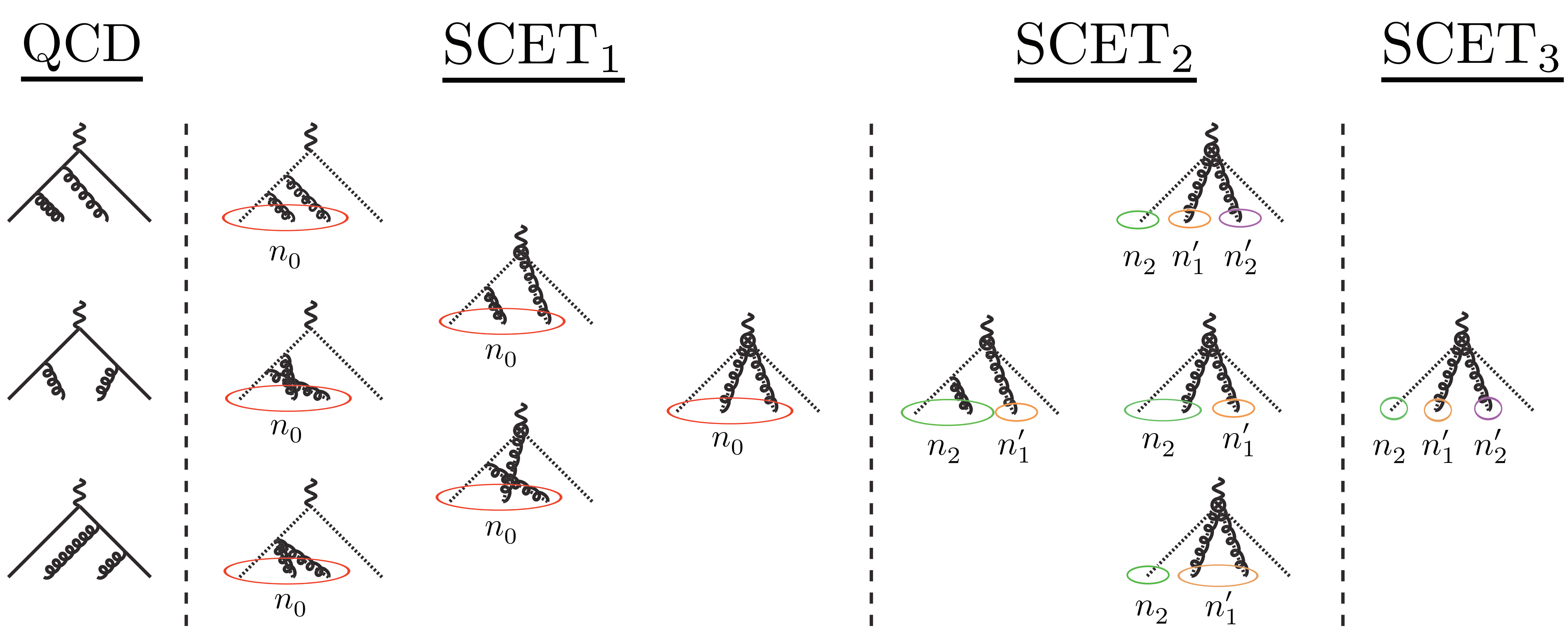
Parton Shower in SCET

Bauer & Schwartz; Baumgart et.al.

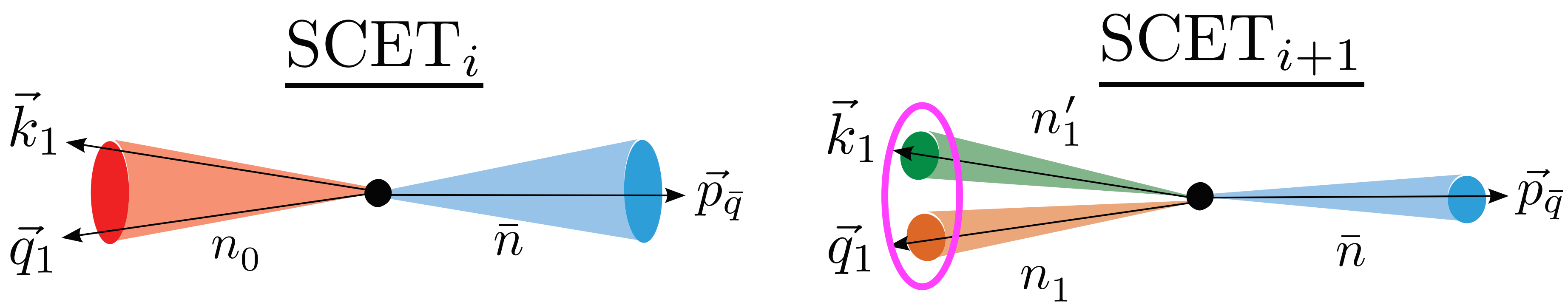


$p^2 \sim Q^2$	QCD
$p^2 \sim Q^2\lambda^2$	SCET$_1$
$p^2 \sim Q^2\lambda^4$	SCET$_2$
$p^2 \sim Q^2\lambda^6$	SCET$_3$
⋮	⋮
$p^2 \sim Q^2\lambda^{2i}$	SCET$_i$





Can also use these techniques to derive factorization theorems for identified subjets:



Summary

e^+e^- event shapes & $\alpha_s(m_Z)$

- SCET analysis provides high precision.
  Log summation and nonperturbative effects are important.

jet substructure & jet algorithms

- Sensitive probe of events. Calculations tractable with SCET

threshold factorization

- simple method to get an (often important) subset of higher order terms

hadron-hadron event shapes

- new methods to test MC, new methods to veto jets

Beam Functions

- universal function that describes ISR for broad class of processes
  (Exclusive Jet Production)