

# Boosted Massive Jets @ CDF & Implications

Gilad Perez

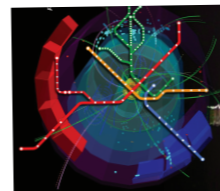
Weizmann Institute

*R, Alon, E. Duchovni, GP & P. Sinervo, for the CDF collaboration; blessed preliminary data (phase II);*

*R, Alon, E. Duchovni, GP, S. Pronko & P. Sinervo, arXiv:1101.xxxx;*

*G. Gur-Ari, M. Papucci & GP, arXiv:1101.xxxx;*




More to come ...



Boston Jet Physics Workshop

# Outline

---

- ◆ jet substructure, inner-jet energy flow:
  - (i) jet mass  $\Rightarrow$  perturbative @ high mass  $\Rightarrow$    
*theorist*
  - (ii) angularity  $\leftrightarrow$  2-body (iii) planar flow  $\leftrightarrow$  3 body.
- ◆ First measurements: CDF preliminary (phase II).   
*experimentalist*
- ◆ Data-driven method for pile up subtraction.
- ◆ Generic classification of jet shapes.   
*theorist*
- ◆ Some implications of CDF's data.

# Jet Mass-Overview

---

- ◆ Jet mass-sum of “massless” momenta in h-cal inside the cone:  $m_J^2 = \left( \sum_{i \in R} P_i \right)^2$ ,  $P_i^2 = 0$
- ◆ Jet mass is non-trivial both for S & B for concreteness mostly focus on top-jets.

# Non trivial top-jet mass distribution

---

- ◆ Naively the signal is  $J \propto \delta(m_J - m_t)$
- ◆ In practice  $m_J^t \sim m_t + \delta m_{QCD} + \delta m_{EW}$



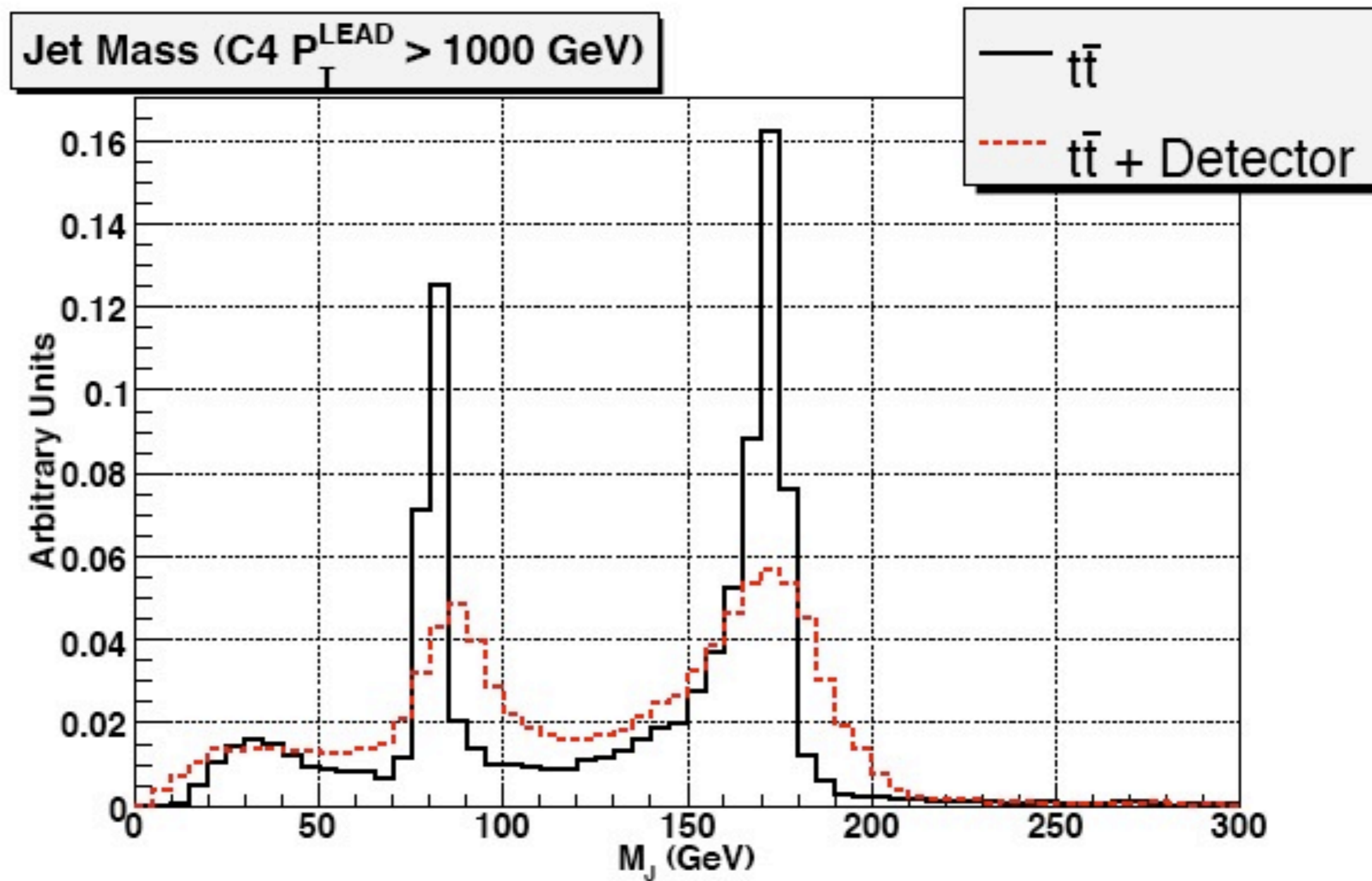
# Non trivial top-jet mass distribution

---

- ◆ Naively the signal is  $J \propto \delta(m_J - m_t)$
- ◆ In practice  $m_J^t \sim m_t + \delta m_{QCD} + \delta m_{EW}$   
+ detector smearing.

Almeida, Lee, Perez, Sung, & Virzi (08), see also Fleming, Hoang, Mantry, Stewart (07,08).

Sherpa => Transfer functions,  
(CKKW)



# QCD jet mass distribution

Ellis, Huston, Hatakeyama, Loch and Tonnesmann, (07); Almeida, Lee, Perez, Sung, & Virzi (08).

◆ Boosted QCD Jet via factorization:

$$\frac{d\sigma^i}{dm_J} = J^i(m_J, p_T^{\min}, R^2) \sigma^i(p_T^{\min})$$
$$\int dm_J J^i = 1 \quad i = Q, G$$

- can interpret the jet function as a probability density functions for a jet with a given  $p_T$  to acquire a mass between  $m_J$  and  $m_J + \delta m_J$

Full expression:

$$\frac{d\sigma_{H_A H_B \rightarrow J_1 J_2}}{dm_{J_1}^2 dm_{J_2}^2 d\eta} = \sum_{abcd} \int dx_a dx_b \phi_a(x_a, p_T) \phi_b(x_b, p_T) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{dp_T d\eta}(x_a, x_b, \eta, p_T)$$
$$S(m_{J_1}^2, m_{J_2}^2, \eta, p_T, R^2) J_1^{(c)}(m_{J_1}^2, \eta, p_T, R^2) J_2^{(d)}(m_{J_2}^2, \eta, p_T, R^2)$$

# QCD jet mass distribution

Ellis, Huston, Hatakeyama, Loch and Tonnesmann, (07); Almeida, Lee, Perez, Sung, & Virzi (08).

◆ Boosted QCD Jet via factorization:

$$\frac{d\sigma^i}{dm_J} = J^i(m_J, p_T^{min}, R^2) \sigma^i(p_T^{min})$$

$i = Q, G$

- can interpret the jet function as the probability for a parton to acquire a mass between  $m_J$  and  $m_J + dm_J$

For large jet mass & small R,  
no big corrections =>  
leading log can be captured via  
perturbative QCD.

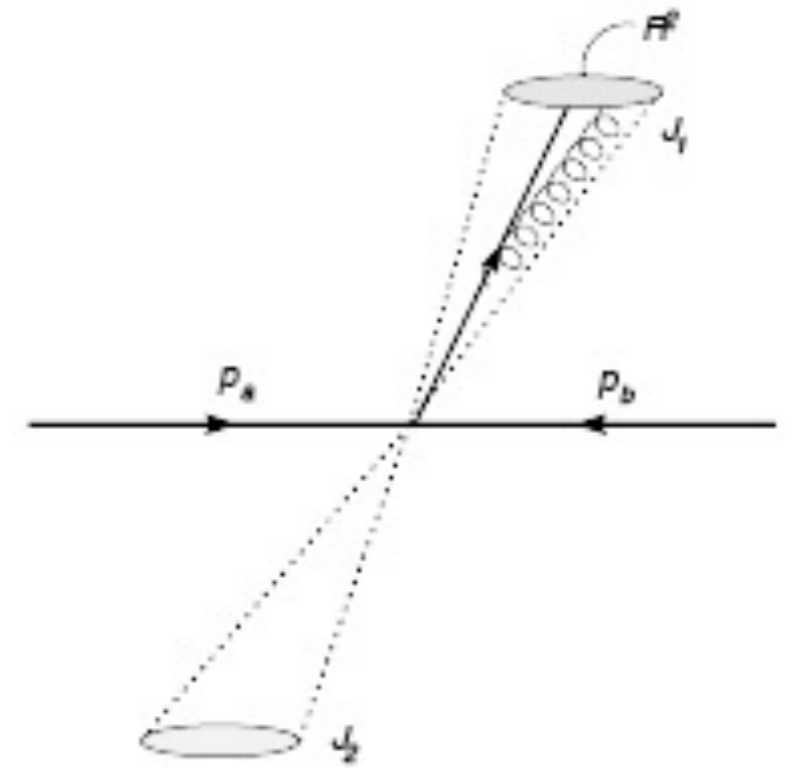
Full expression:

$$\frac{d\sigma_{HAHB \rightarrow J_1 J_2}}{dm_{J_1}^2 dm_{J_2}^2 d\eta} = \sum_{abcd} \int dx_a dx_b \phi_a(x_a, p_T) \phi_b(x_b, p_T) \frac{d\sigma_{ab \rightarrow cd}}{dp_T d\eta}(x_a, x_b, \eta, p_T) S(m_{J_1}^2, m_{J_2}^2, \eta, p_T, R^2) J_1^{(c)}(m_{J_1}^2, \eta, p_T, R^2) J_2^{(d)}(m_{J_2}^2, \eta, p_T, R^2)$$

# QCD jet mass distribution, Q+G

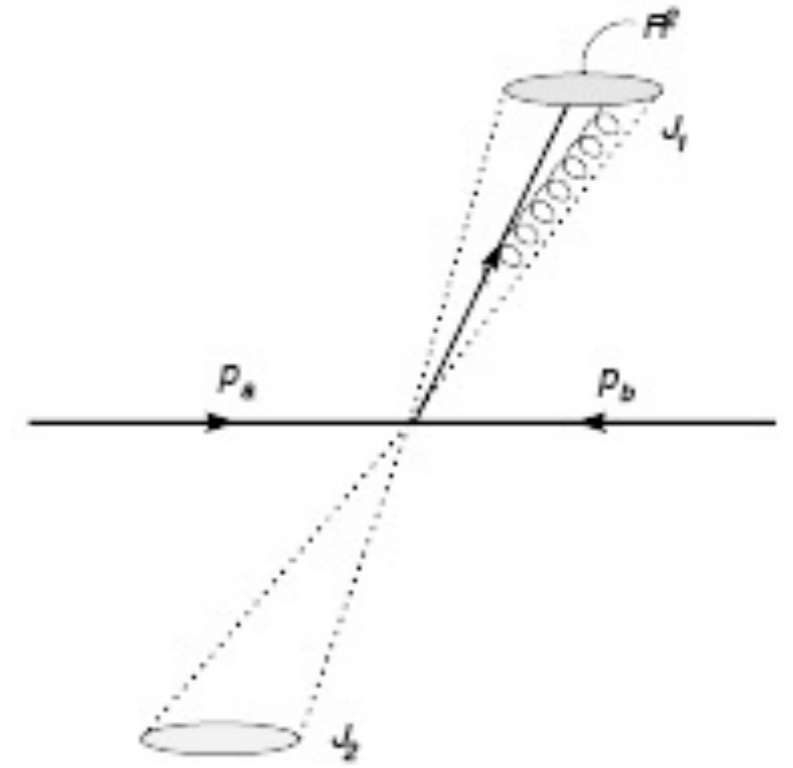
---

Main idea: calculating mass due to two-body QCD bremsstrahlung:



# QCD jet mass distribution, Q+G

Main idea: calculating mass due to two-body QCD bremsstrahlung:



$$J^{(eik),c}(m_J, p_T, R) \simeq \alpha_S(p_T) \frac{4C_c}{\pi m_J} \log \left( \frac{R p_T}{m_J} \right)$$

$C_F = 4/3$  for quarks,  $C_A = 3$  for gluons.



# QCD jet mass distribution, Q+G

---

$$J^{(eik),c}(m_J, p_T, R) \simeq \alpha_S(p_T) \frac{4C_c}{\pi m_J} \log\left(\frac{R p_T}{m_J}\right)$$

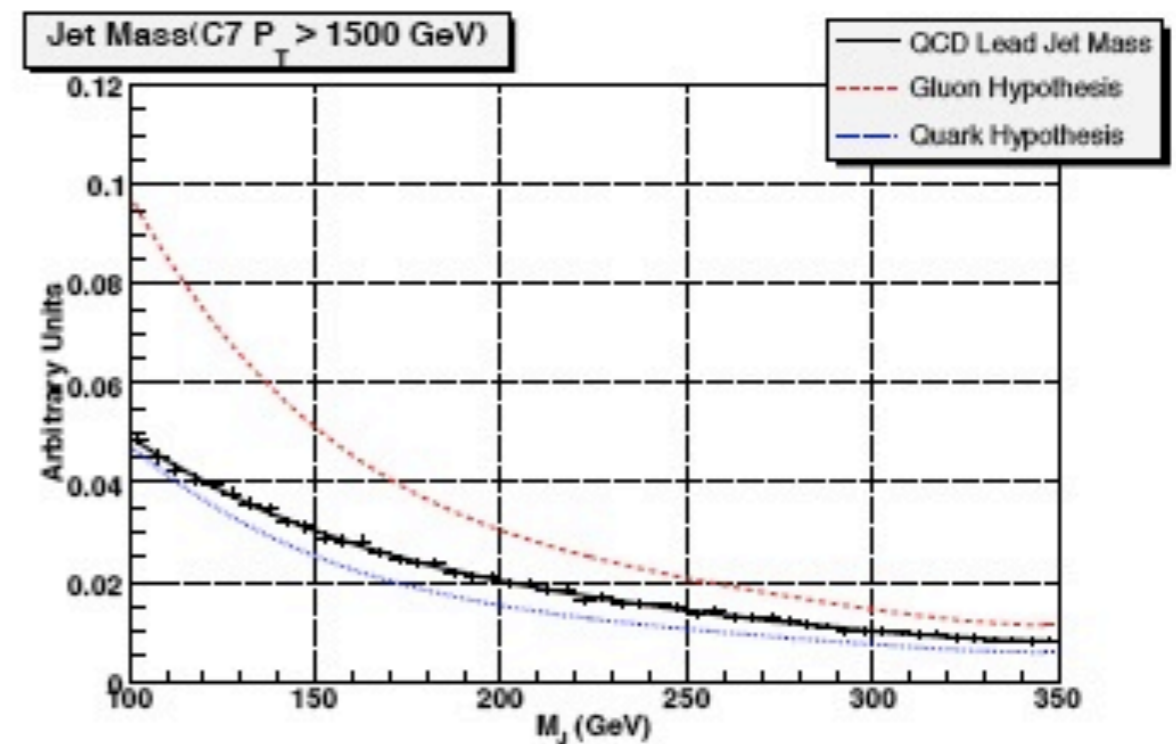
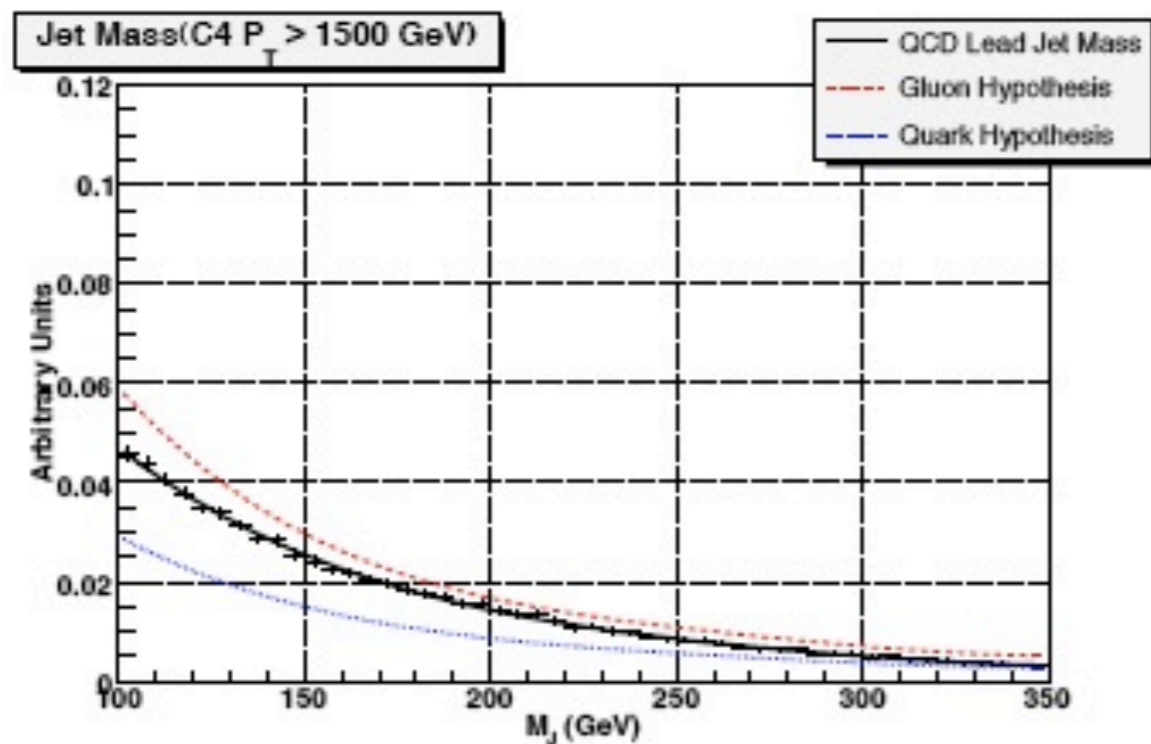
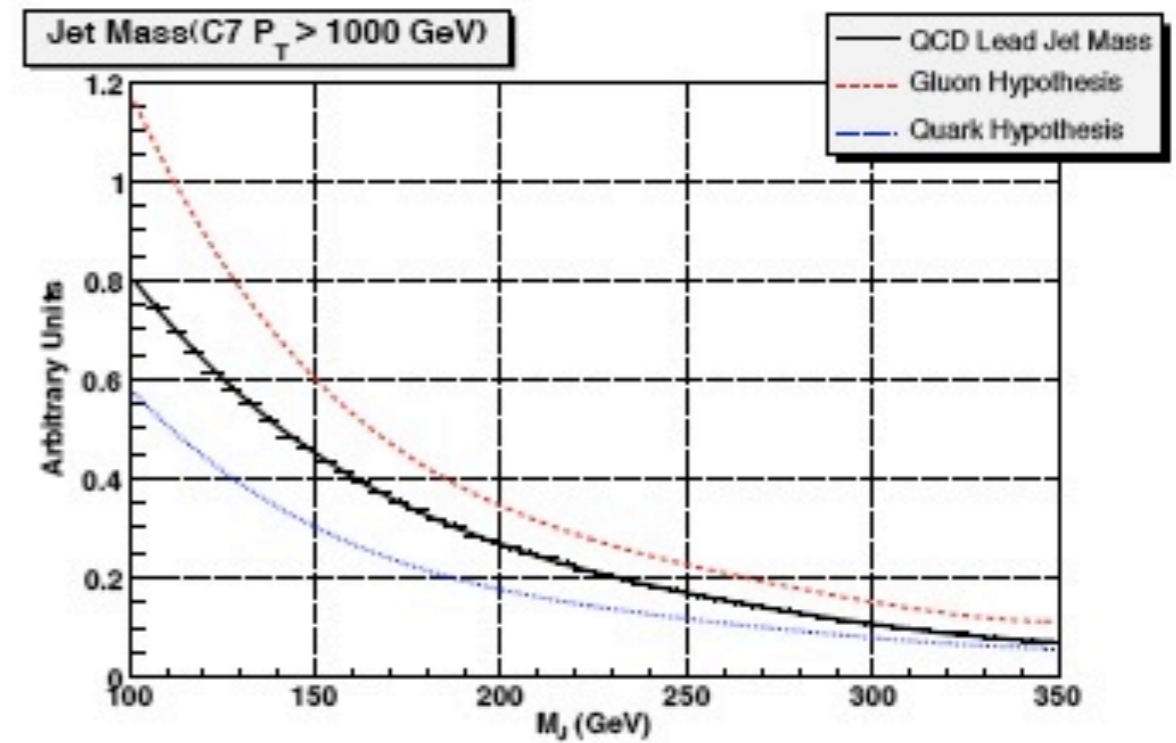
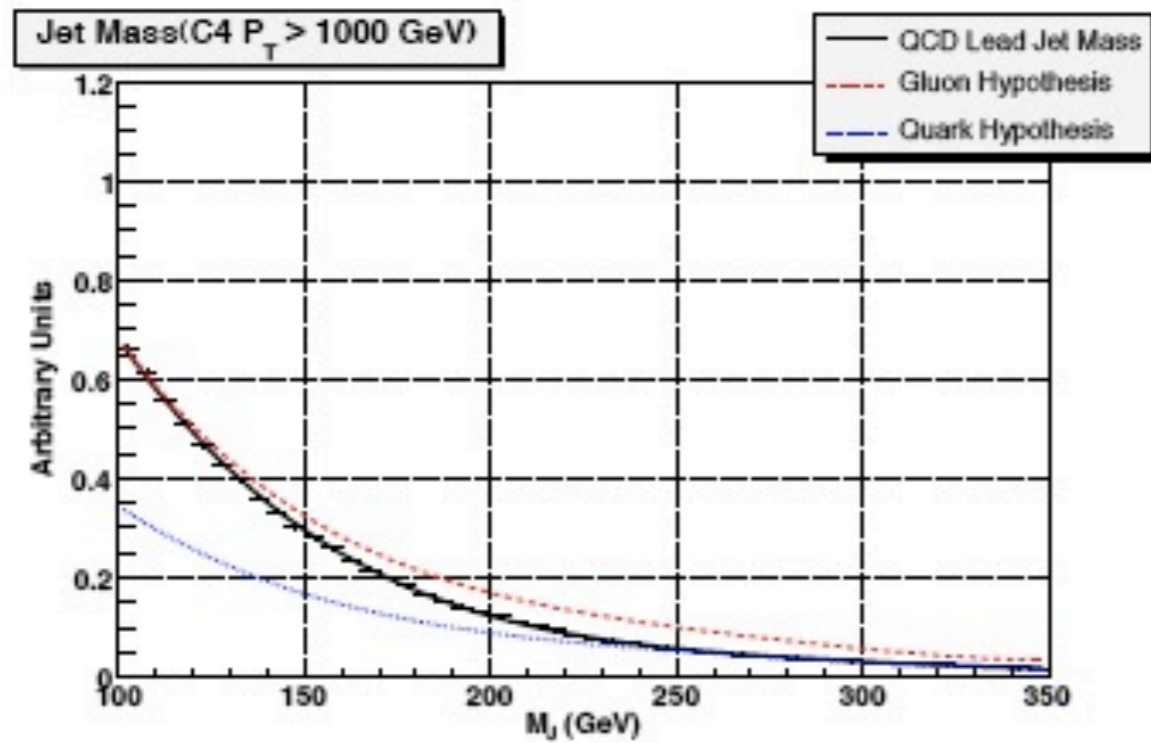
$C_F = 4/3$  for quarks,  $C_A = 3$  for gluons.

Data is admixture of the two, should be bounded by them:

$$\frac{d\sigma_{pred}(R)}{dp_T dm_J} \text{ upper bound} = J^g(m_J, p_T, R) \sum_c \left( \frac{d\sigma^c(R)}{dp_T} \right),$$
$$\frac{d\sigma_{pred}(R)}{dp_T dm_J} \text{ lower bound} = J^q(m_J, p_T, R) \sum_c \left( \frac{d\sigma^c(R)}{dp_T} \right),$$

# Jet mass distribution theory vs. MC

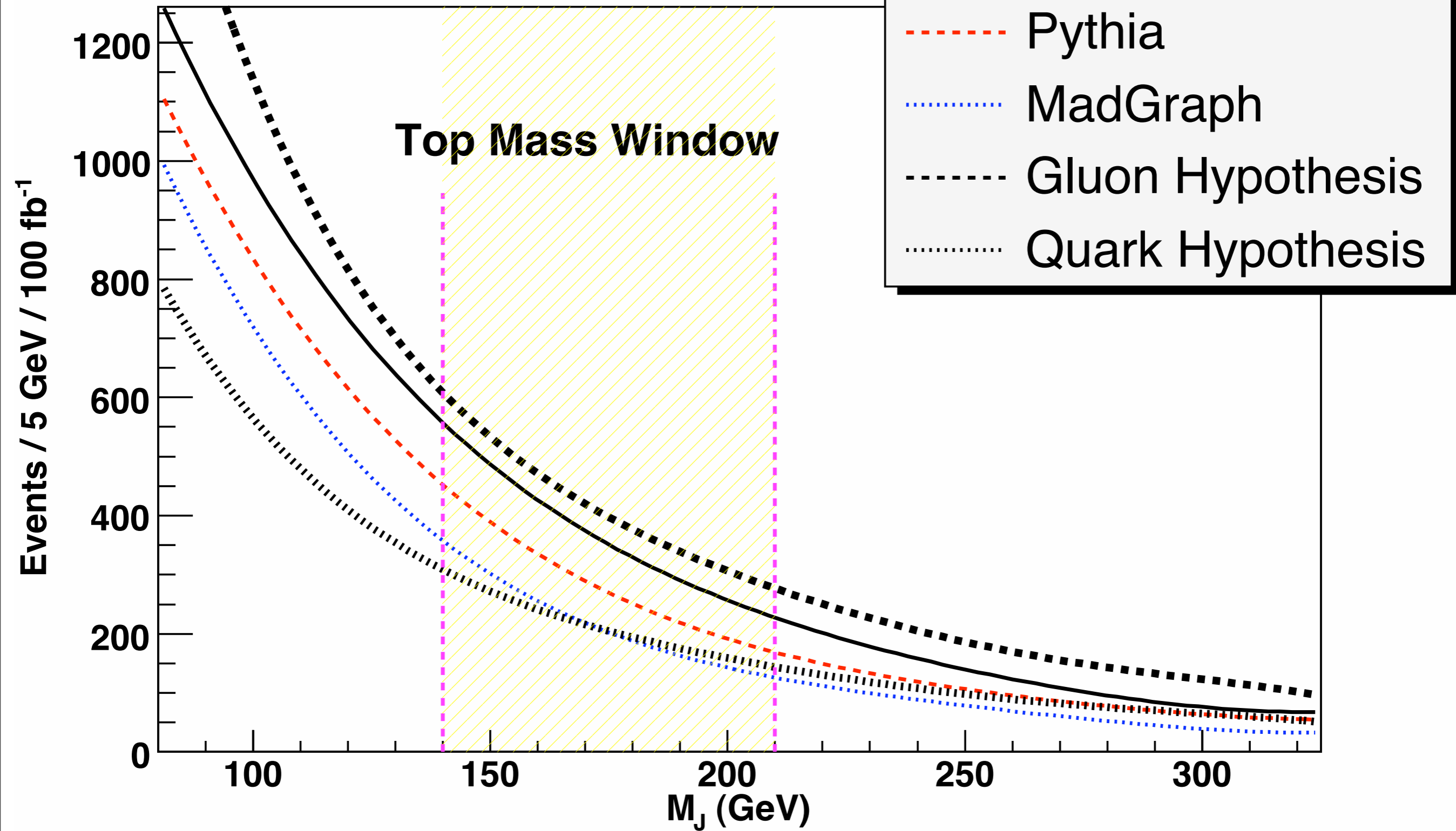
Sherpa, jet function convolved above  $p_T^{\min}$





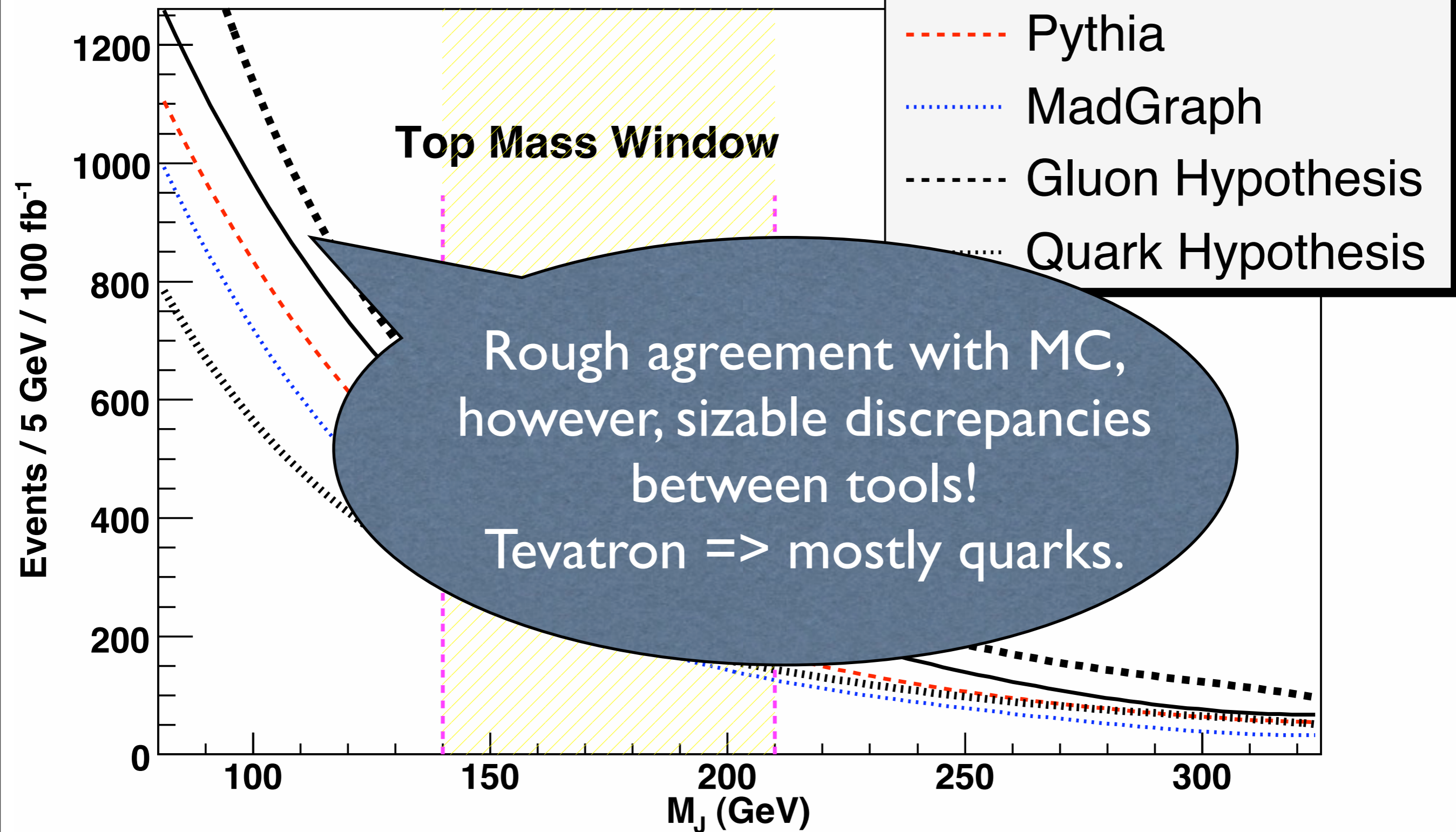
# Jet mass distribution theory vs. MC

## C4 Jet Mass ( $P_T = 1500$ GeV)

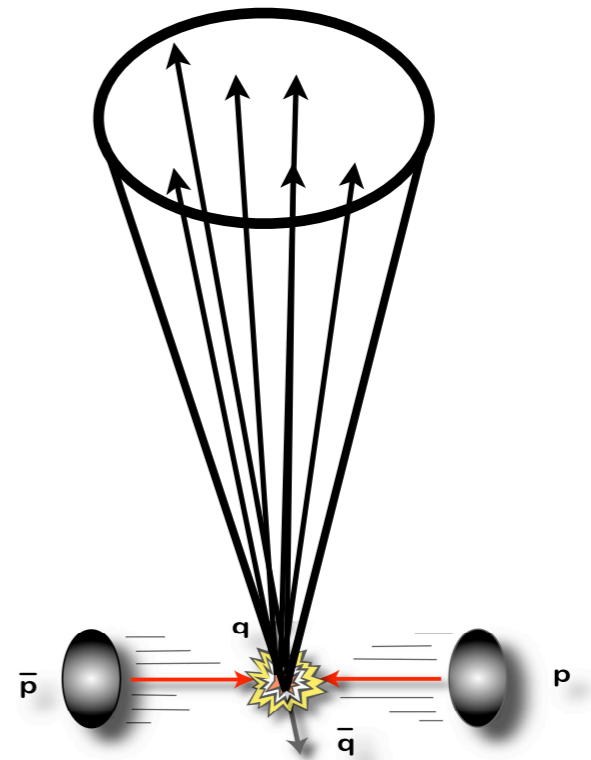
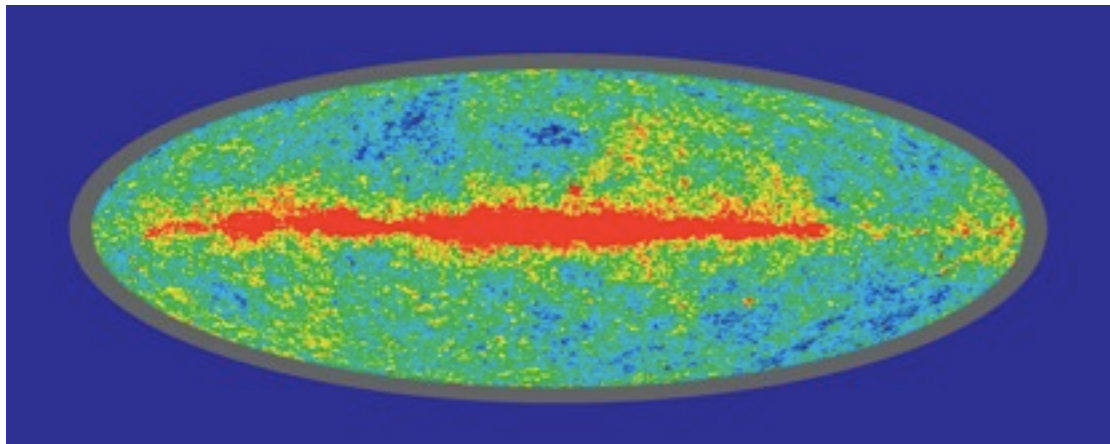


# Jet mass distribution theory vs. MC

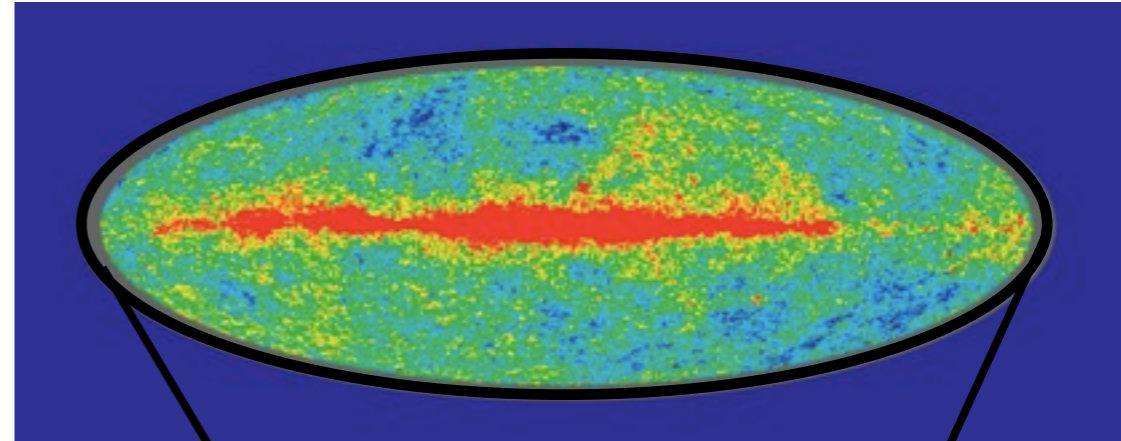
## C4 Jet Mass ( $P_T = 1500$ GeV)



# Jet sub-structure



# Jet sub-structure



Fixing mass  $\Rightarrow$  more control  
(looking @ set of moments):

- (i) Angularity.
- (ii) Planar flow.



(no manipulation of jet energy deposition)

IR-safe jet-shapes which distinguish  
between massive & QCD jets?

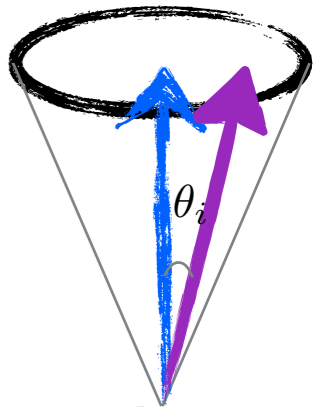
---

# IR-safe jet-shapes which distinguish between massive & QCD jets?

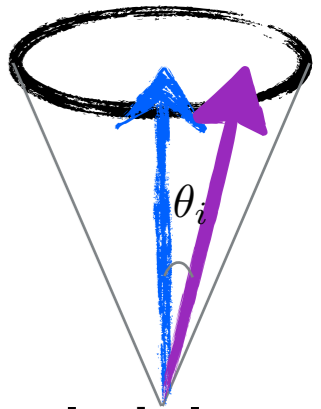
---

◆ Once jet mass fixed @ high scale

→ Large class of jet-shapes become perturbatively calculable



# IR-safe jet-shapes which distinguish between massive & QCD jets?



◆ Once jet mass fixed @ high scale

➔ Large class of jet-shapes become perturbatively calculable

◆ Angularity (2-body final state):

Berger, Kucs and Sterman (03)

$$\tau_a(R, p_T) = \frac{1}{m_J} \sum_{i \in \text{jet}} \omega_i \sin^a \theta_i [1 - \cos \theta_i]^{1-a} \sim \frac{2^{a-1}}{m_J} \sum_{i \in \text{jet}} \omega_i \theta_i^{2-a} \propto_{a=-2} \sum_i \omega_i \theta_i^4$$

emphasize cone-edge radiation

Almeida, Lee, GP, Sterman, Sung, & Virzi (08)

# Higher moments, angularity (2 body)

---

- Given jet mass & momenta, only one additional independent, variable to describe energy flow:

$$\tau_{-2} \sim \frac{1}{m} \sum_{i \in J} E_i \theta_i^4$$

- If mass is due to 2-body  $\Rightarrow$  sharp prediction (kinematics):

$$\theta_{\min} \sim \frac{m_J}{p_J} \Rightarrow \tau_{-2}^{\min} \approx \left( \frac{m_J}{p_J} \right)^3$$

$$\theta_{\max} \sim R \Rightarrow \tau_{-2}^{\max} \approx R^2 \frac{m_J}{p_J}$$



# 2-body jet's kinematics, $Z/W/h$

---

◆ Angularities “distinguish” between Higgs & QCD jets (2-body only one variable  $\Leftrightarrow$  asymmetry):

$$\frac{dJ^h}{d\bar{\tau}_a} \propto \frac{1}{|a| (\tau_a)^{1-\frac{2}{a}}} \quad \text{vs.} \quad \frac{dJ^{\text{QCD}}}{d\bar{\tau}_a} \propto \frac{1}{|a| \bar{\tau}_a}$$

# 2-body jet's kinematics, $Z/W/h$

---

◆ Angularities “distinguish” between Higgs & QCD jets (2-body only one variable  $\Leftrightarrow$  asymmetry):

$$\frac{dJ^h}{d\bar{\tau}_a} \propto \frac{1}{|a| (\tau_a)^{1-\frac{2}{a}}} \quad \text{vs.} \quad \frac{dJ^{\text{QCD}}}{d\bar{\tau}_a} \propto \frac{1}{|a| \bar{\tau}_a}$$

$$\tau_{-2} \propto \frac{1}{z}$$

$$z = \min(p_{T1}, p_{T2})/p_T$$

$$\frac{dJ^h}{dz} \propto z^4$$

vs.

$$\frac{dJ^{\text{QCD}}}{dz} \propto z^3$$

# 2-body jet's kinematics, $Z/W/h$

◆ Angularities “distinguish” between Higgs & QCD jets (2-body only one variable  $\Leftrightarrow$  asymmetry):

$$\frac{dJ^h}{d\bar{\tau}_a} \propto \frac{1}{|a| (\tau_a)^{1-\frac{2}{a}}}$$

vs.

$$\frac{dJ^{\text{QCD}}}{d\bar{\tau}_a} \propto \frac{1}{|a| \bar{\tau}_a}$$

Difference is  
not so big!!

$\tau_{-z}$

$= \min(p_{T1}, p_{T2}) / p_T$

$$\frac{dJ^h}{dz} \propto z^4$$

vs.

$$\frac{dJ^{\text{QCD}}}{dz} \propto z^3$$

# 2-body jet's kinematics, $Z/W/h$

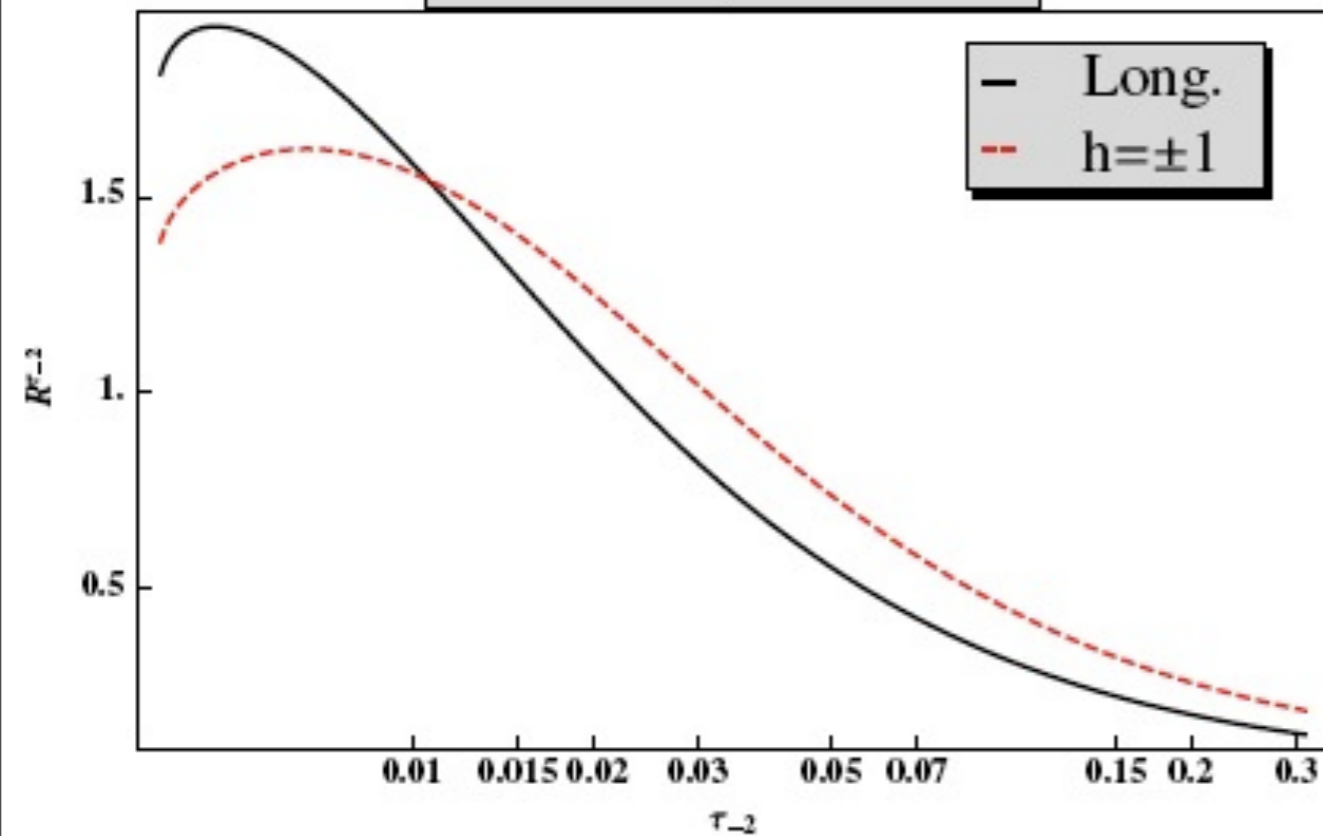
---

$$P^x(\theta_s) = (dJ^x/d\theta_s)/J^x \Rightarrow P^x(\tilde{\tau}_a); \quad R(\tilde{\tau}_a) = \frac{P^{\text{sig}}(\tilde{\tau}_a)}{P^{\text{QCD}}(\tilde{\tau}_a)}$$

# 2-body jet's kinematics, $Z/W/h$

$$P^x(\theta_s) = (dJ^x/d\theta_s)/J^x \Rightarrow P^x(\tilde{\tau}_a); \quad R(\tilde{\tau}_a) = \frac{P^{\text{sig}}(\tilde{\tau}_a)}{P^{\text{QCD}}(\tilde{\tau}_a)}$$

$R^{\tau_{-2}}$  vs.  $\tau_{-2}$  for  $z=0.05$



Angularity,  $\tau_a$  ( $a = -2, z = 0.05, R = 0.4$ )

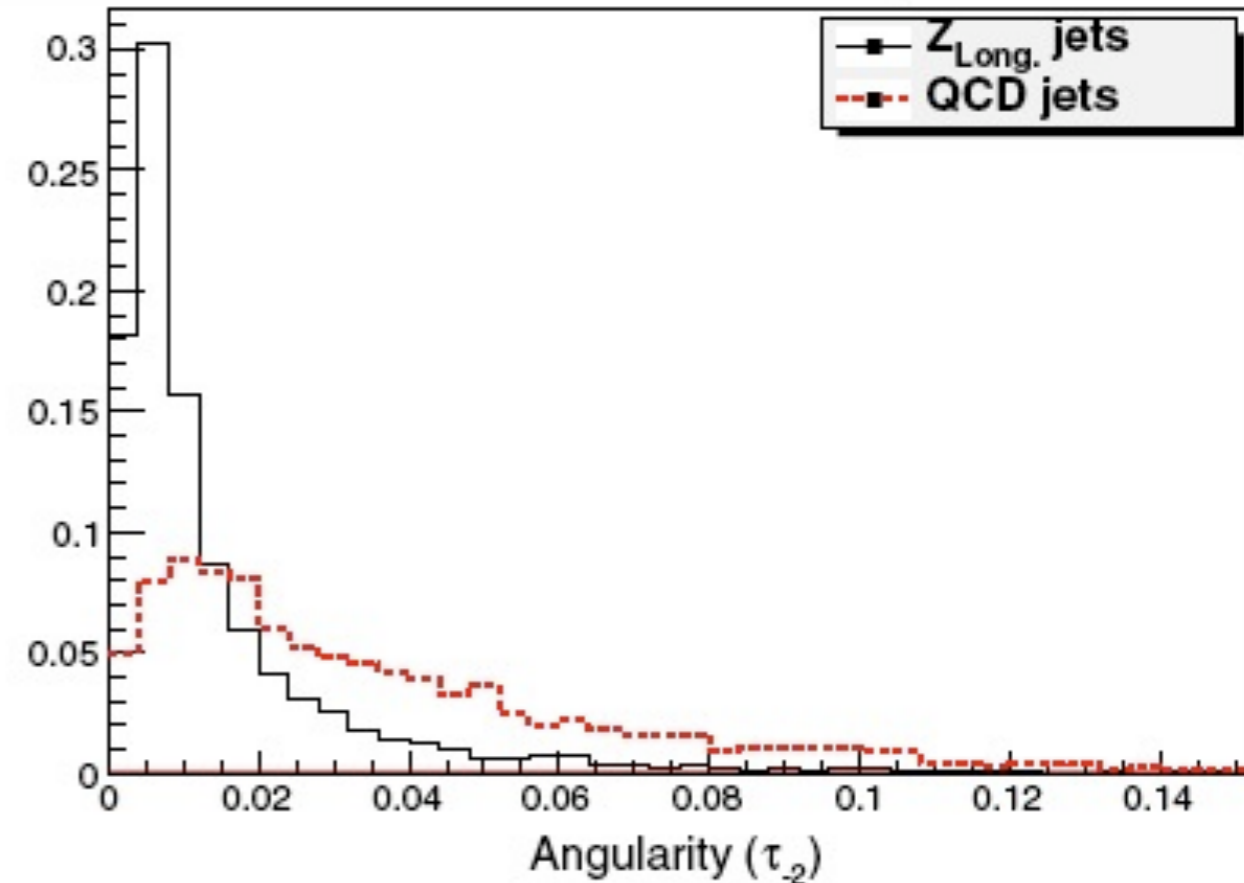


FIG. 3 (color online). The ratio between the signal and background probabilities to have jet angularity  $\tilde{\tau}_{-2}$ ,  $R^{\tilde{\tau}_{-2}}$ .

FIG. 4 (color online). The angularity distribution for QCD (red-dashed curve) and longitudinal Z (black-solid curve) jets obtained from MADGRAPH. Both distributions are normalized to the same area.

$$(z = m_J/p_T)$$

# 2-body jet's kinematics, $Z/W/h$

$$P^x(\theta_s) = (dJ^x/d\theta_s)/J^x \Rightarrow P^x(\tilde{\tau}_a); \quad R(\tilde{\tau}_a) = \frac{P^{\text{sig}}(\tilde{\tau}_a)}{P^{\text{QCD}}(\tilde{\tau}_a)}$$

Peak  $\Rightarrow$  special "democratic" configuration where the two particles have same energy & min' distance from

jet axis  $\theta_m \approx z$ .

$$(z = m_J/p_T)$$

Angularity,  $\tau_a$  ( $a = -2, z = 0.05, R = 0.4$ )

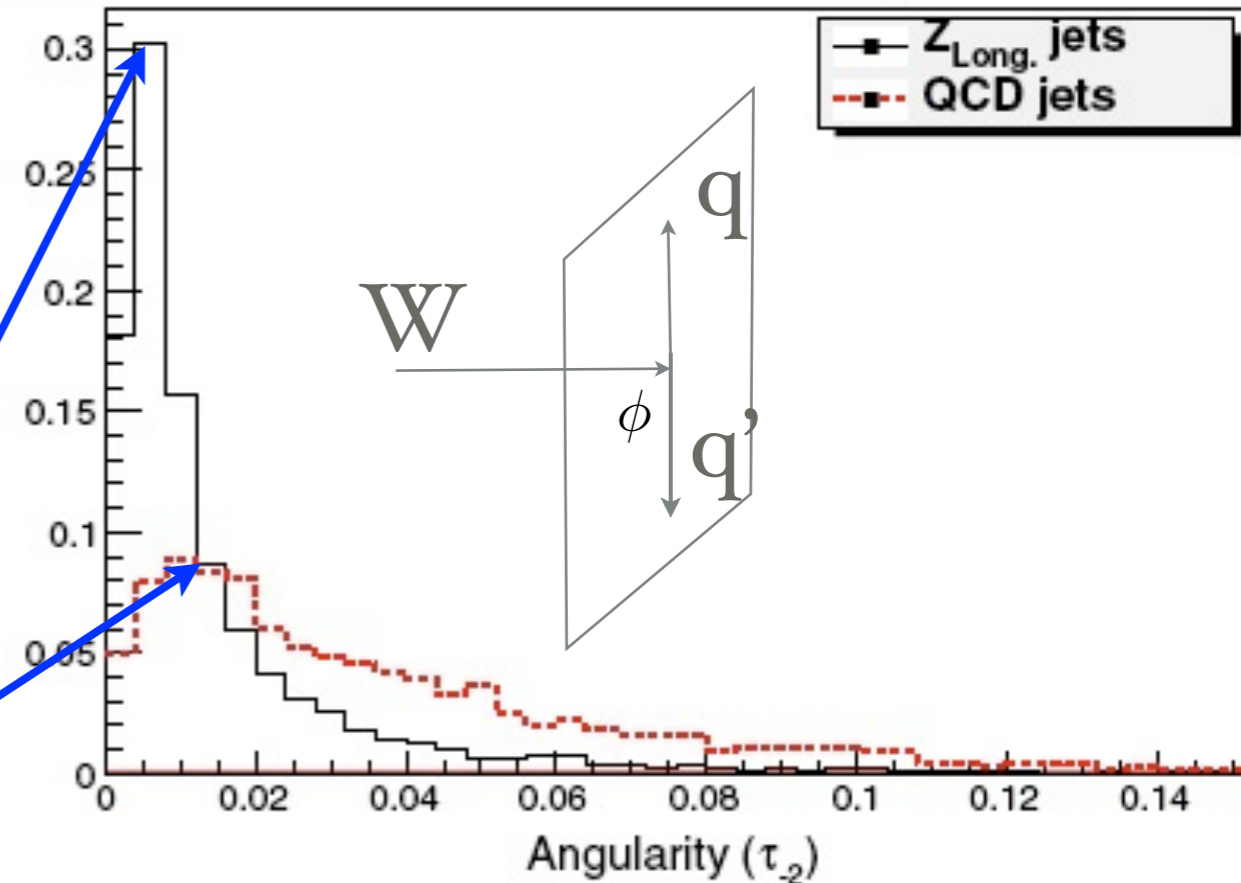


FIG. 4 (color online). The angularity distribution for QCD (red-dashed curve) and longitudinal Z (black-solid curve) jets obtained from MADGRAPH. Both distributions are normalized to the same area.

FIG. 3 (color online). Ground probability and background probability for the angularity distribution.

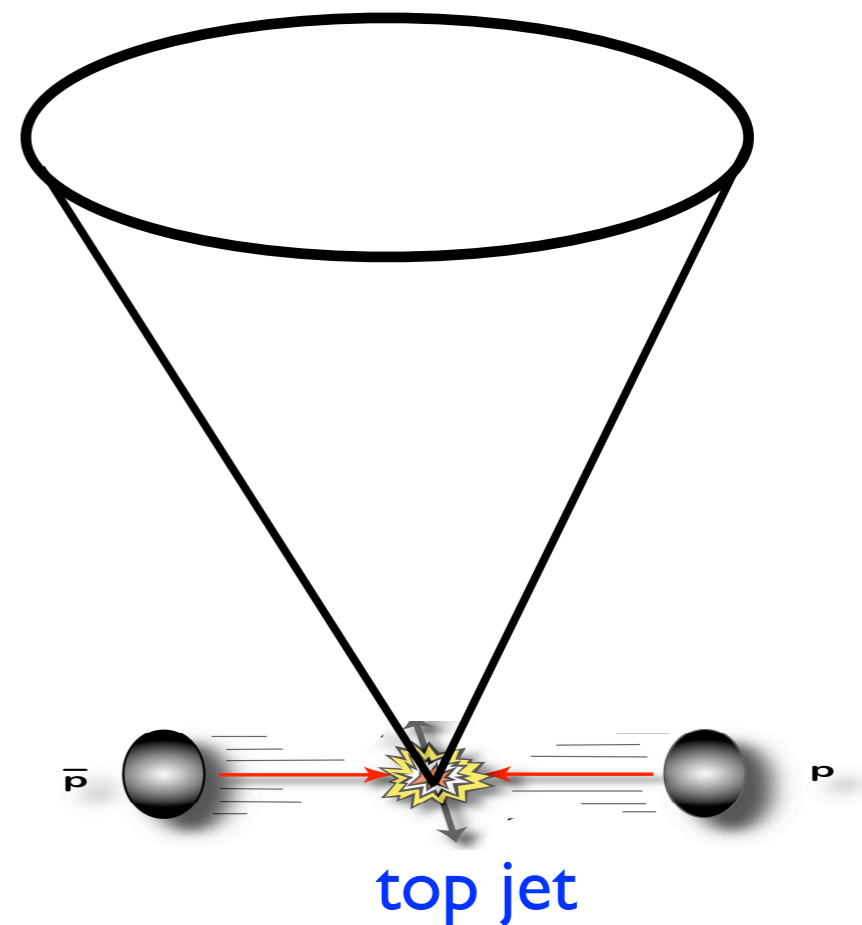
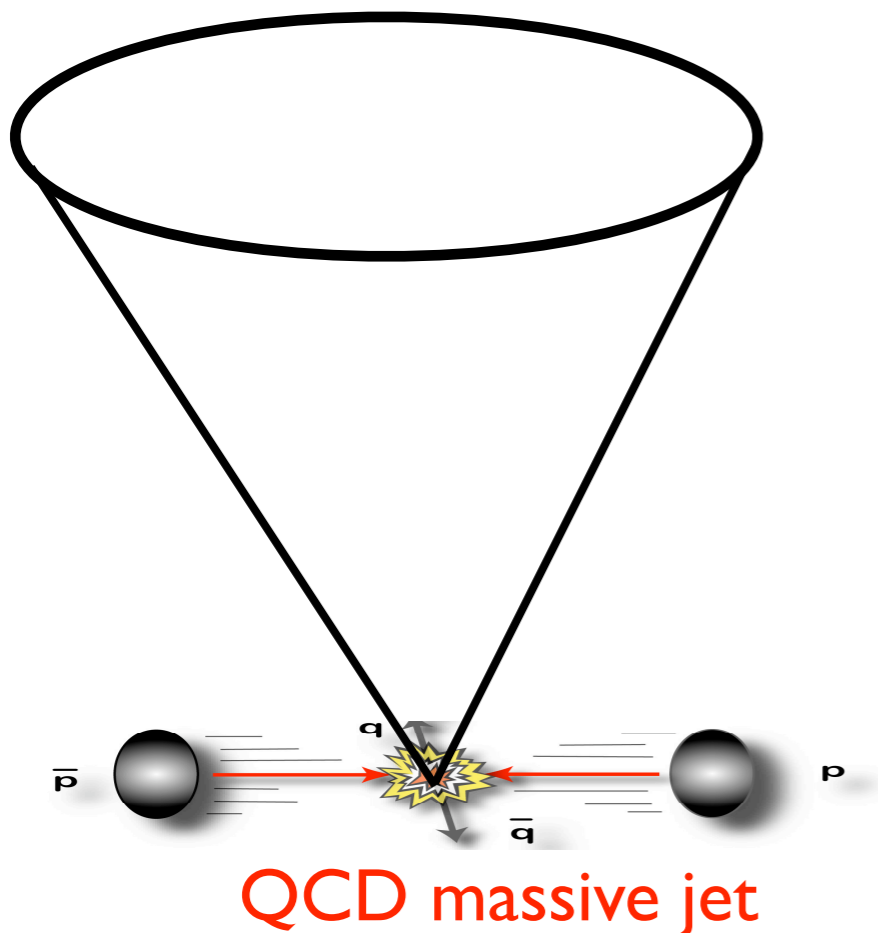


# Planar flow

- Top-jet is 3 body vs. massive QCD jet  $\Leftrightarrow$  2-body (previous result)

Thaler & Wang, JHEP (08);

Almeida, Lee, GP, Sterman, Sung & Virzi, PRD (09).

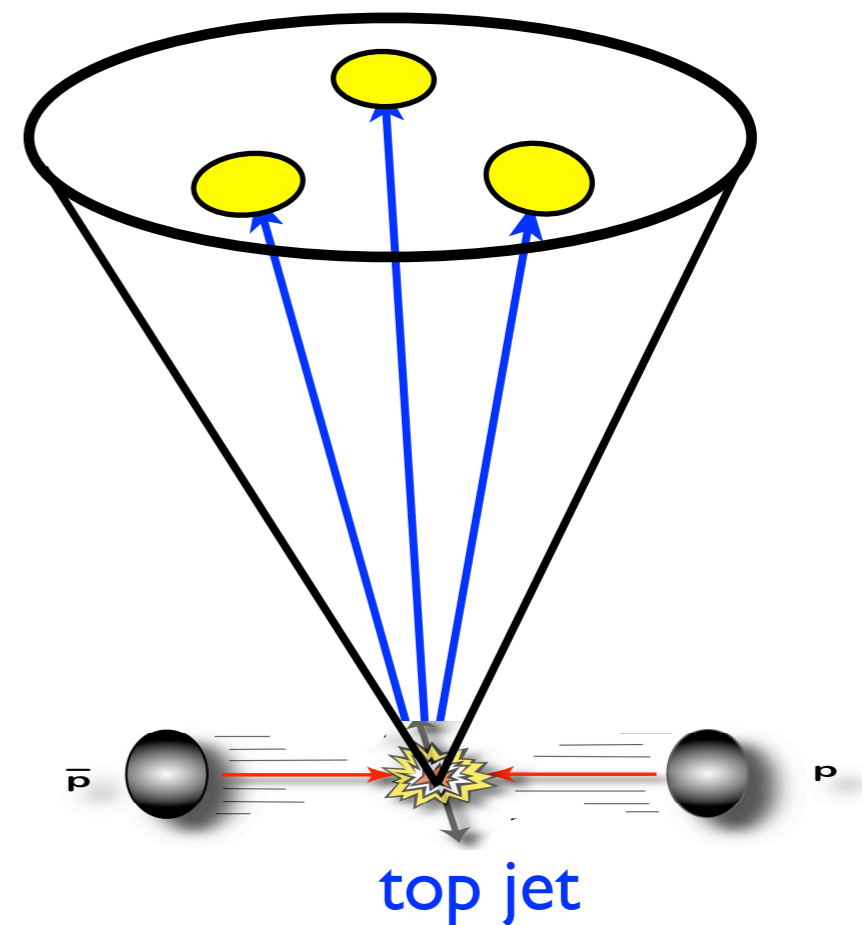
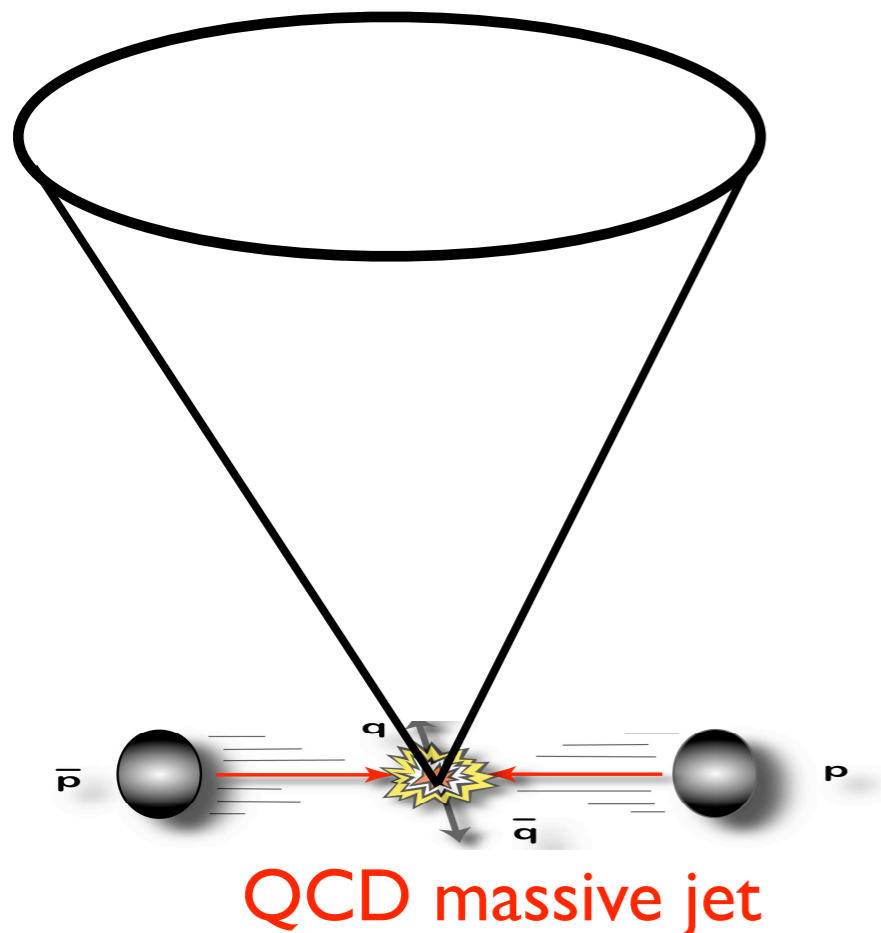


# Planar flow

- Top-jet is 3 body vs. massive QCD jet  $\Leftrightarrow$  2-body (previous result)

Thaler & Wang, JHEP (08);

Almeida, Lee, GP, Sterman, Sung & Virzi, PRD (09).



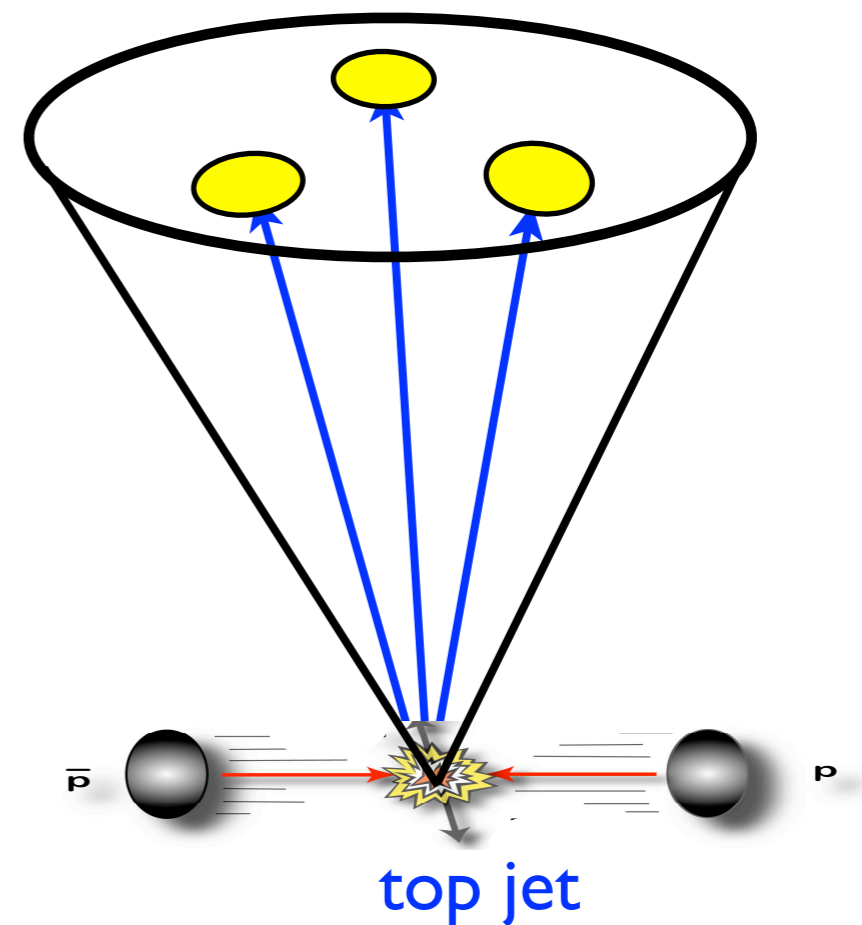
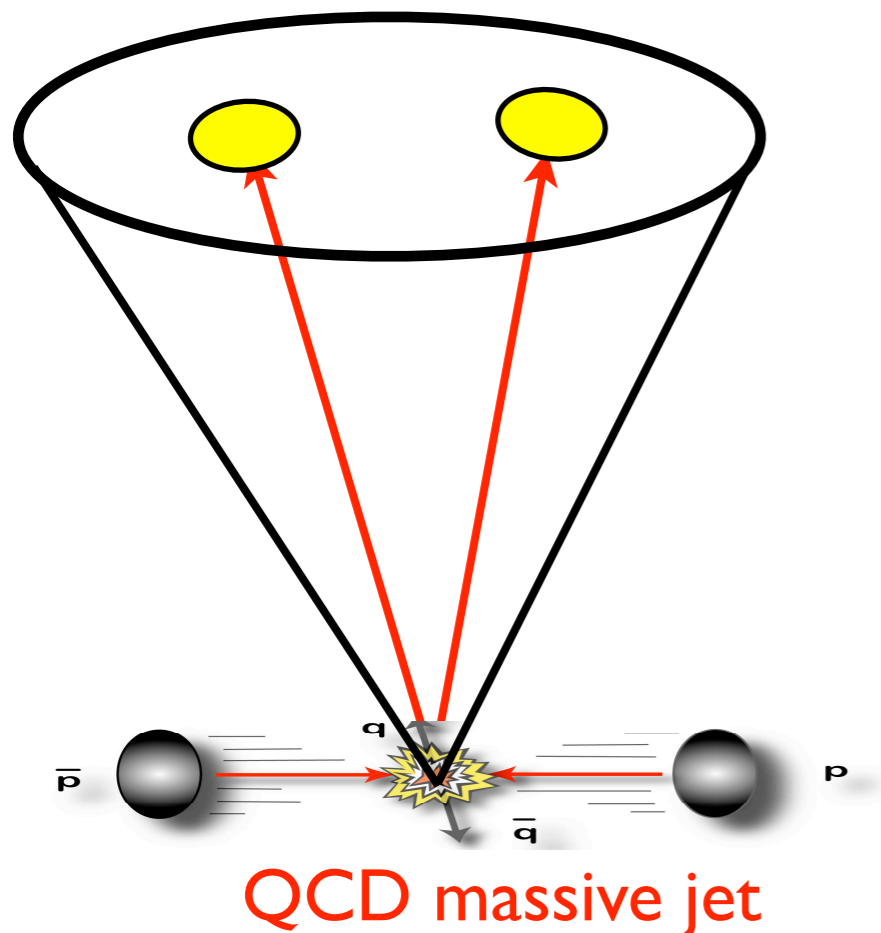


# Planar flow

- Top-jet is 3 body vs. massive QCD jet  $\Leftrightarrow$  2-body (previous result)

Thaler & Wang, JHEP (08);

Almeida, Lee, GP, Sterman, Sung & Virzi, PRD (09).



# Planar flow

- Top-jet is 3 body vs. massive QCD jet  $\Leftrightarrow$  2-body

Thaler & Wang, JHEP (08);

Almeida, Lee, GP, Sterman, Sung & Virzi, PRD (09).

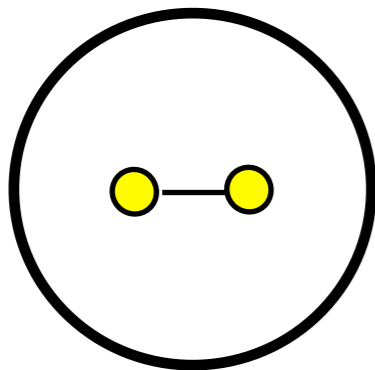
- Planar flow,  $Pf$ , measures the energy ratio between two primary axes of cone surface:

(i) “moment of inertia “:

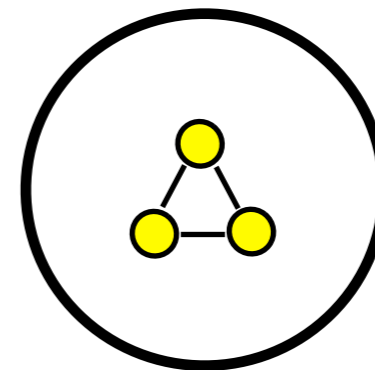
$$I_E^{kl} = \frac{1}{m_J} \sum_{i \in R} E_i \frac{p_{i,k}}{E_i} \frac{p_{i,l}}{E_i},$$

(ii) Planar flow:

$$Pf = 4 \frac{\det(\mathbf{I}_E)}{\text{tr}(\mathbf{I}_E)^2} = \frac{4\lambda_1\lambda_2}{(\lambda_1 + \lambda_2)^2}$$

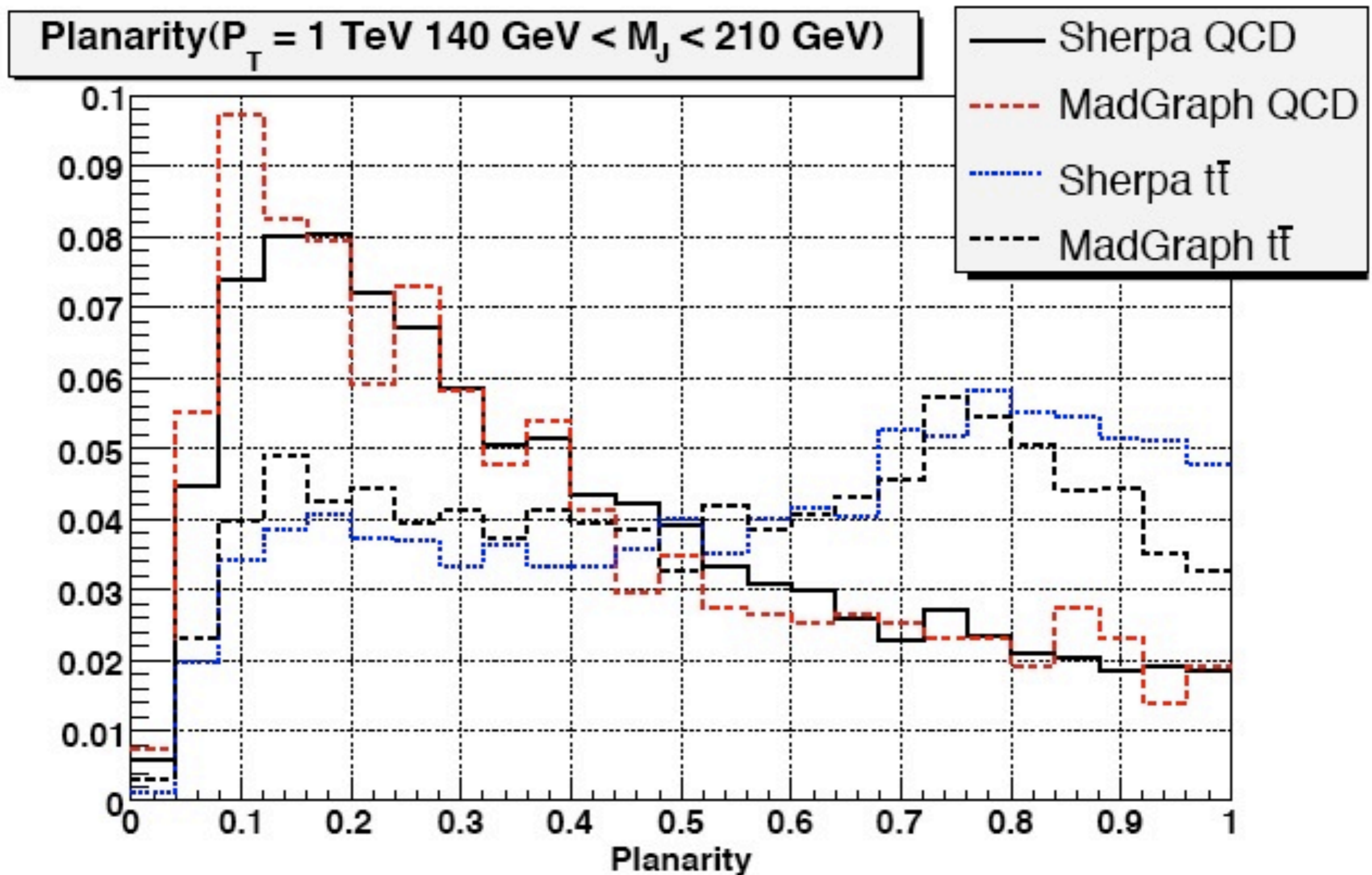


leading order QCD,  $Pf=0$



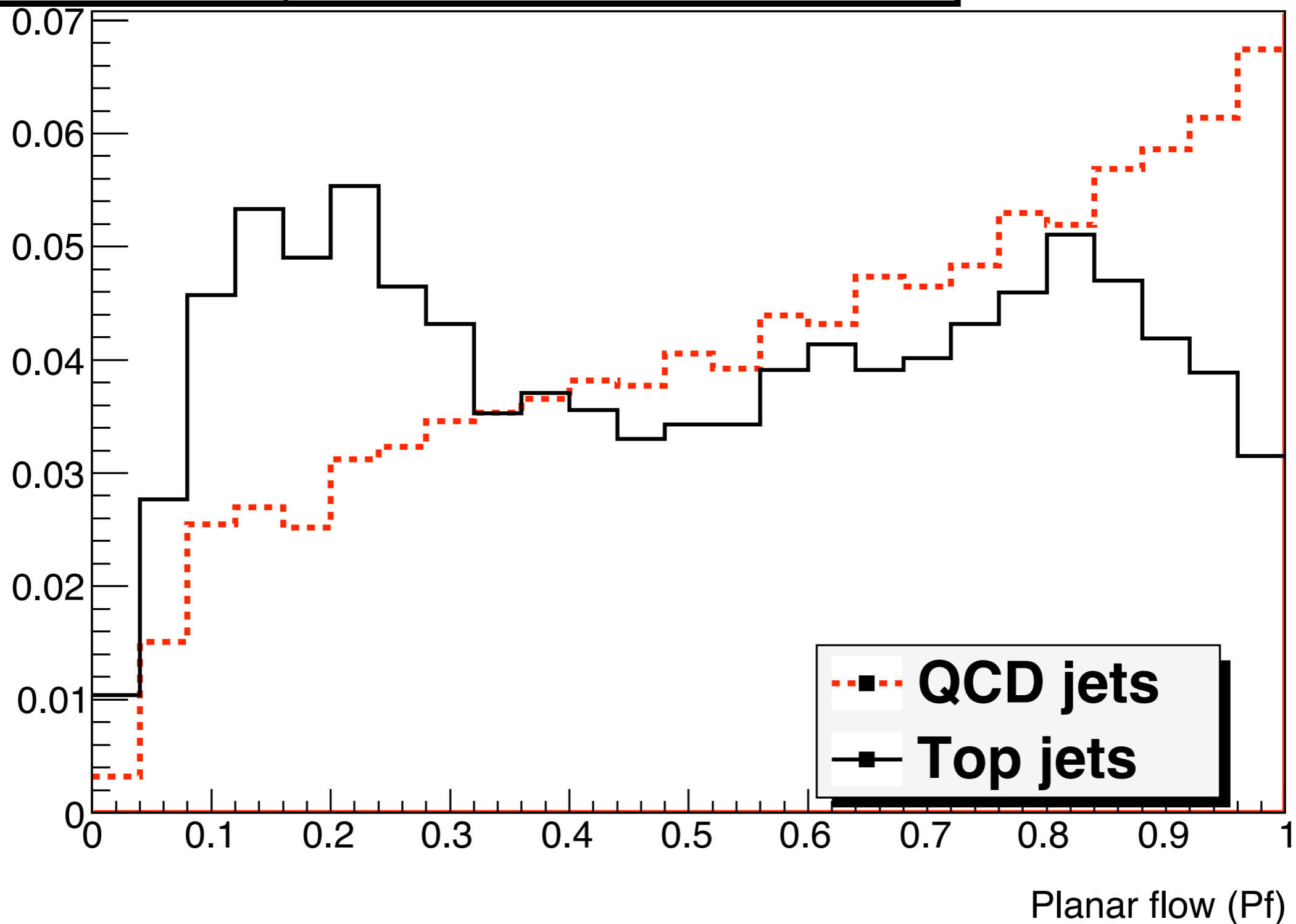
top jet,  $Pf=1$

# Planar flow, QCD vs top jets

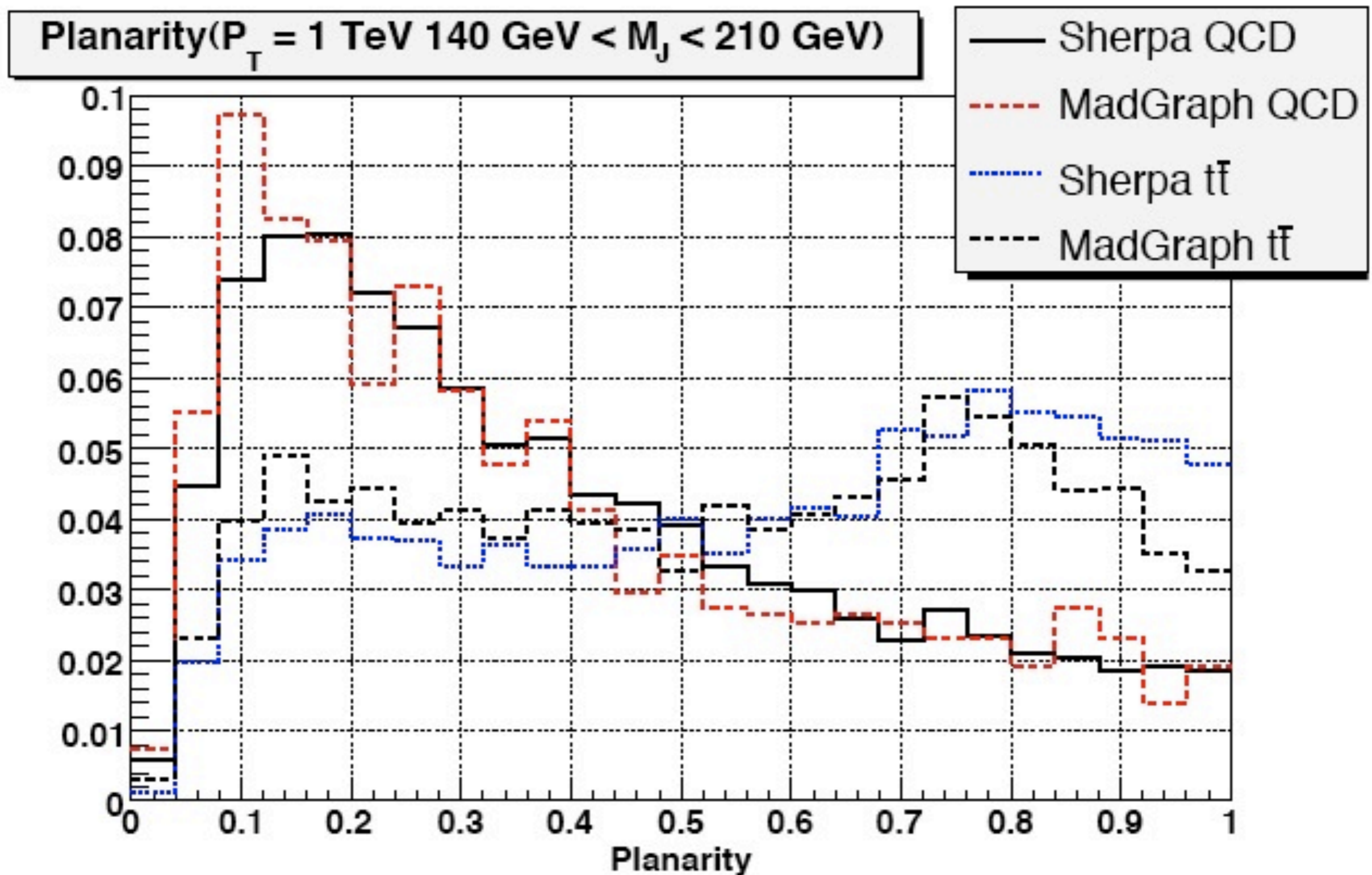


# Planar flow, QCD vs top jets

Planar flow, Pf ( $P_T = 1$  TeV,  $R = 0.4$ , "no mass cuts")



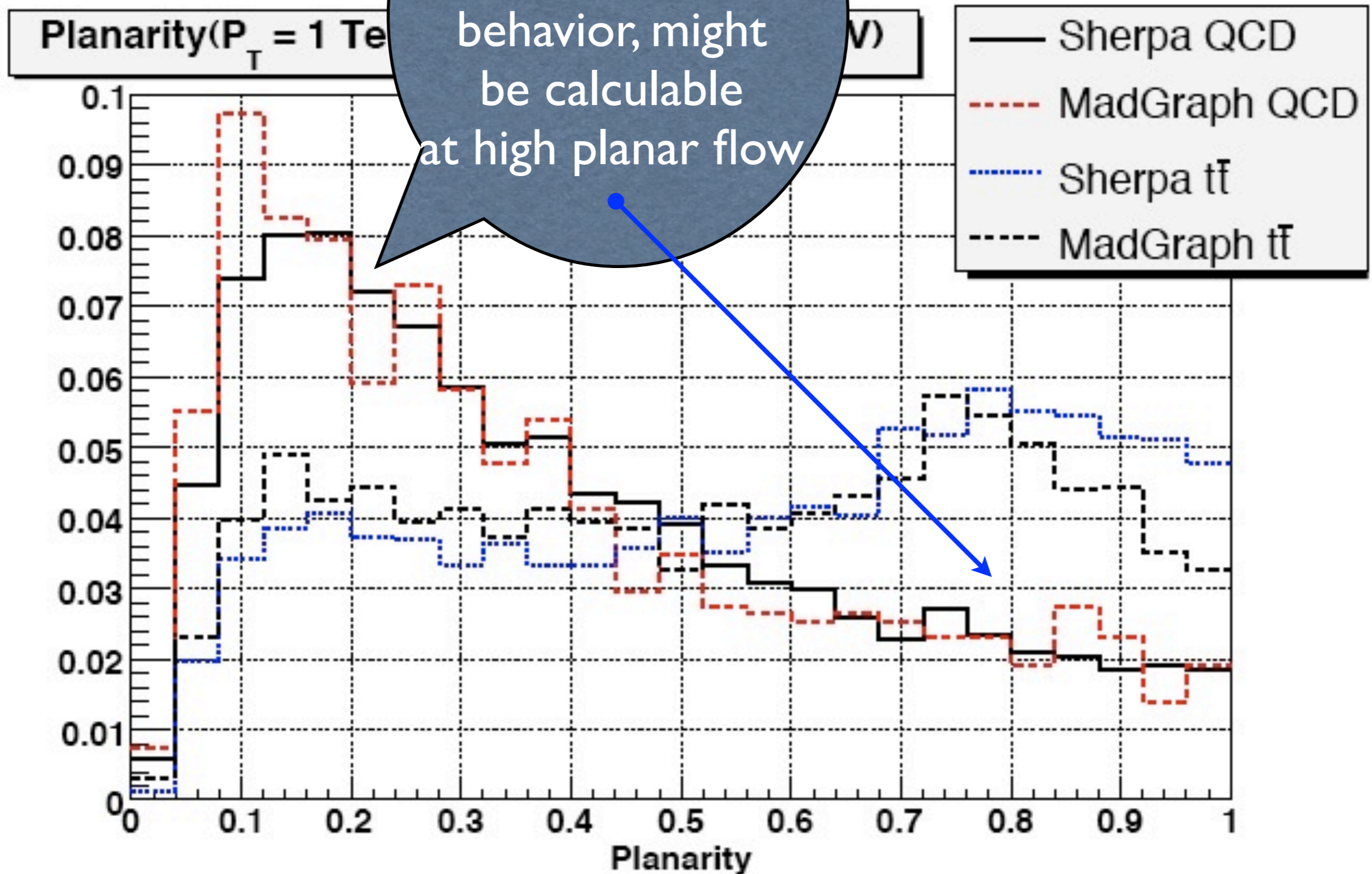
# Planar flow, QCD vs top jets



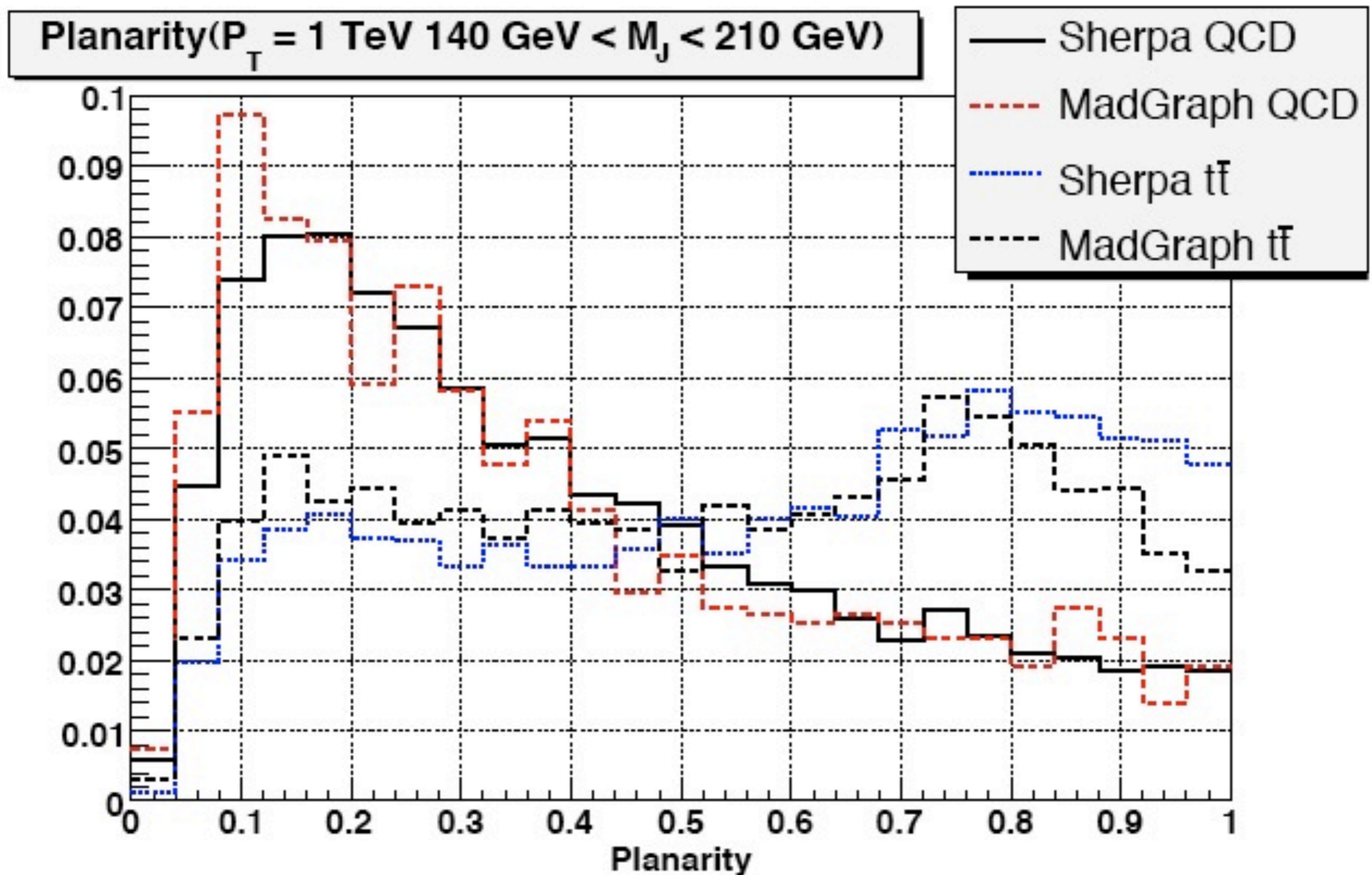


# Planar flow in QCD vs top jets

Guess: QCD  
Planar flow shows  
a "typical" QCD  
behavior, might  
be calculable  
at high planar flow

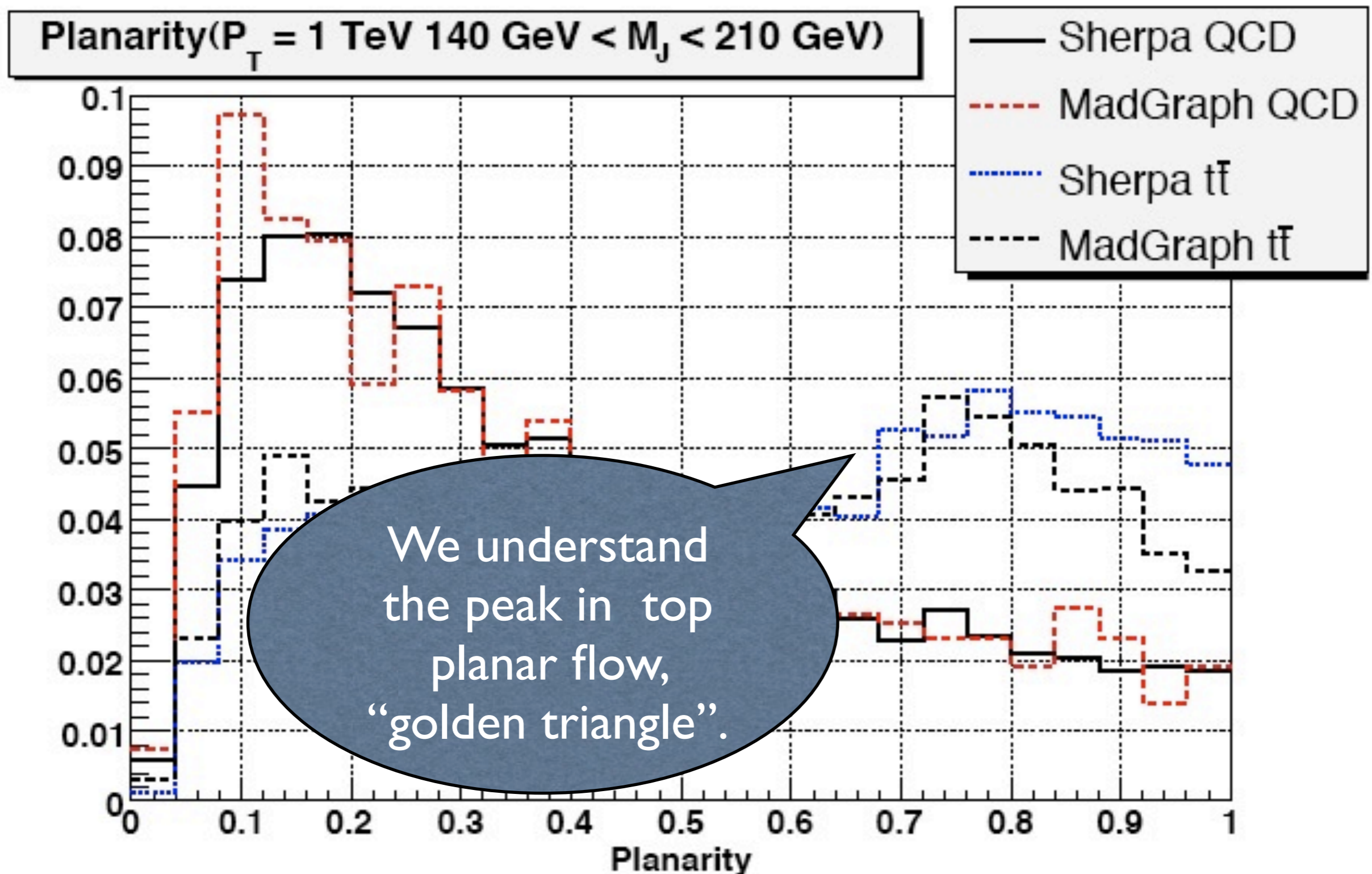


# Planar flow, QCD vs top jets





# Planar flow, QCD vs top jets





# Boosted massive jets @ CDF (phase II)



*R, Alon, E. Duchovni, GP & P. Sinervo, for the CDF; blessed **preliminary** data;*



*experimentalist*

# The preliminary data to be looked at

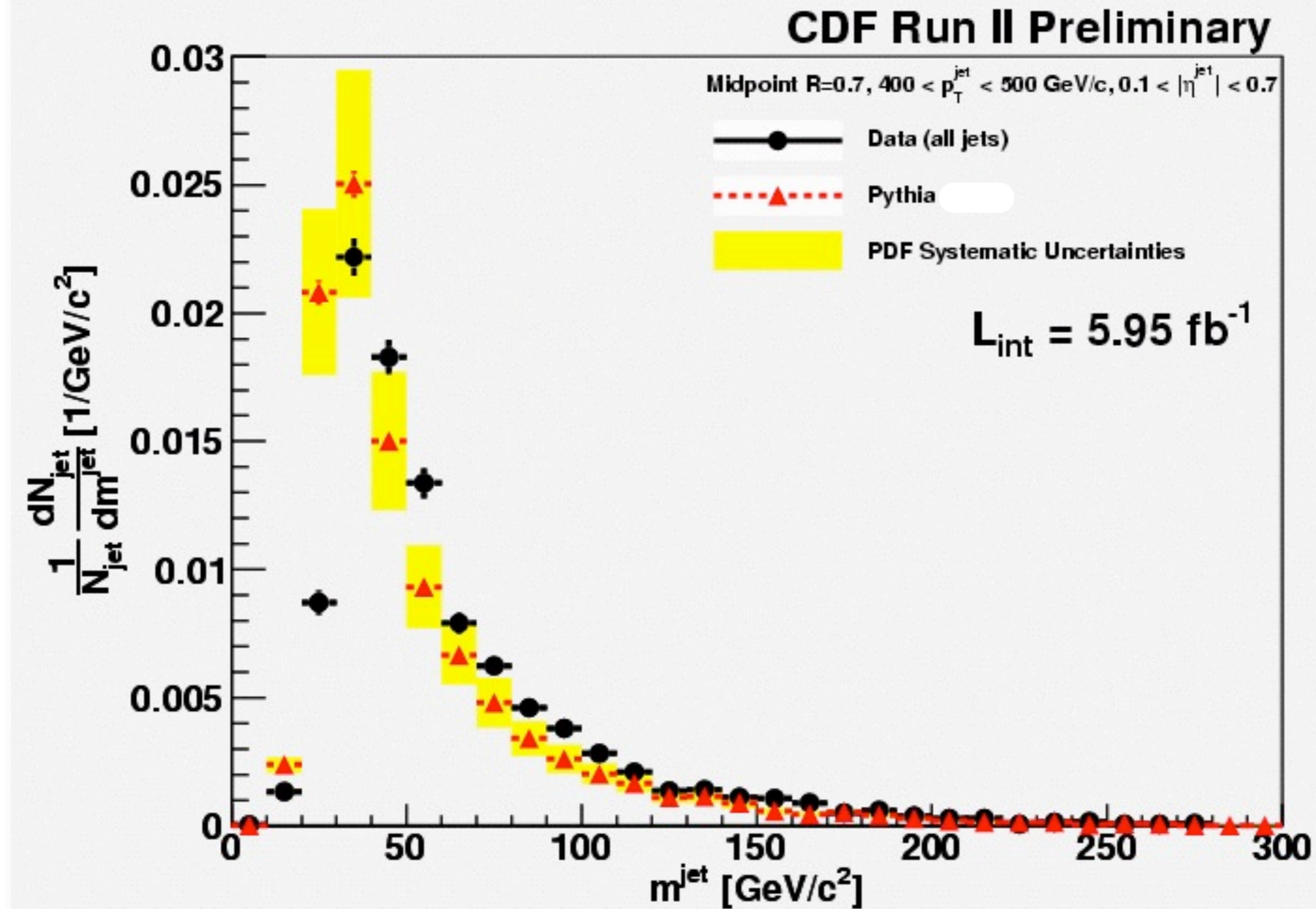
---

Cut Flow		
	R = 0.4	R = 0.7
All Data, $5.95 \text{ fb}^{-1}$	75,764,270 events	
At least one jet with $p_T > 400 \text{ GeV}/c$ , $0.1 <  \eta  < 0.7$ , and event quality cuts	2,153	2,700
$m^{\text{jet}2} < 100 \text{ GeV}/c^2$ and $S_{\text{MET}} < 4$ (with $p_T^{\text{jet}2} > 100 \text{ GeV}/c$ and MI corrections)	1,837	2,108

Top rejection cut.

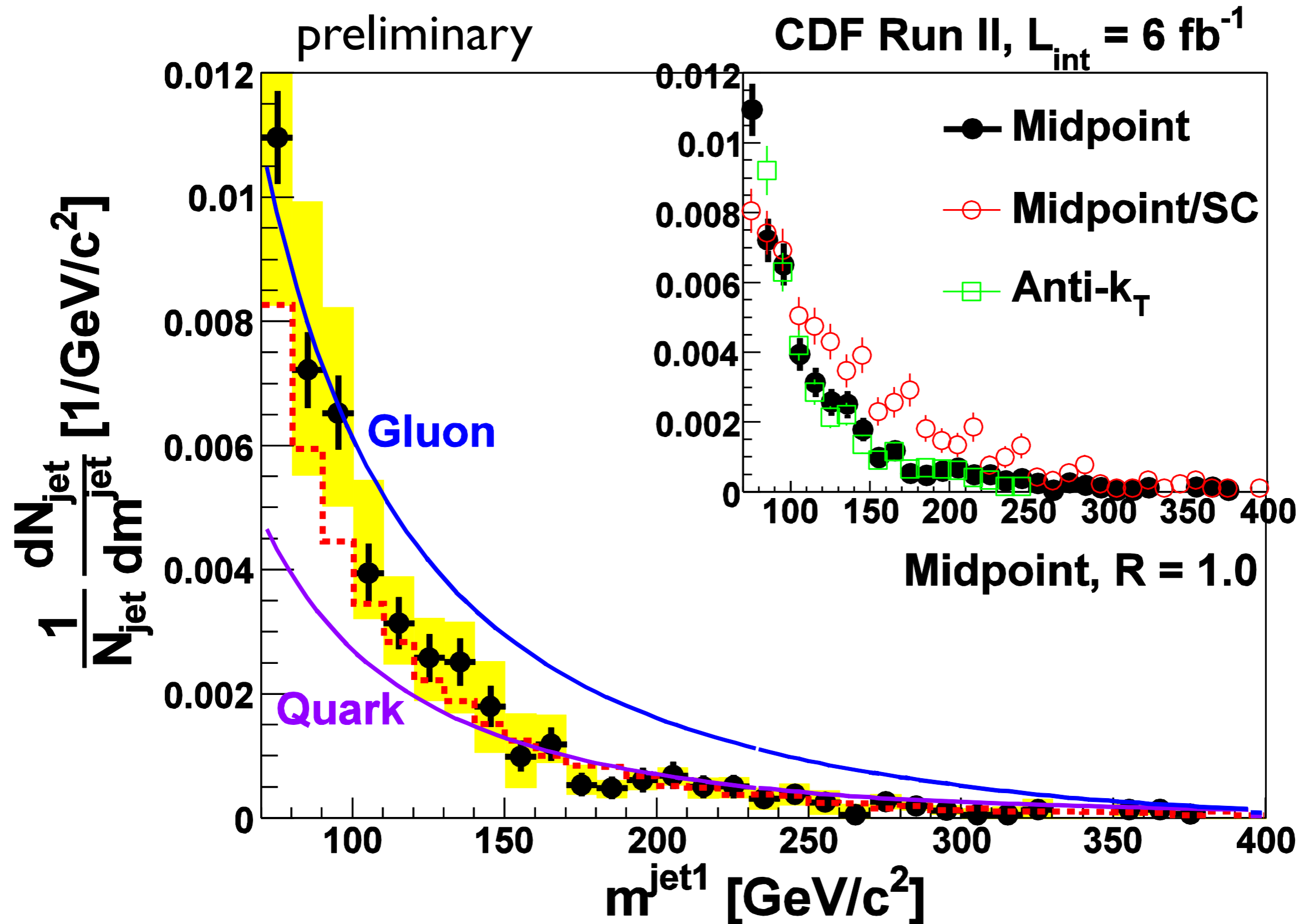


# Jet mass distribution

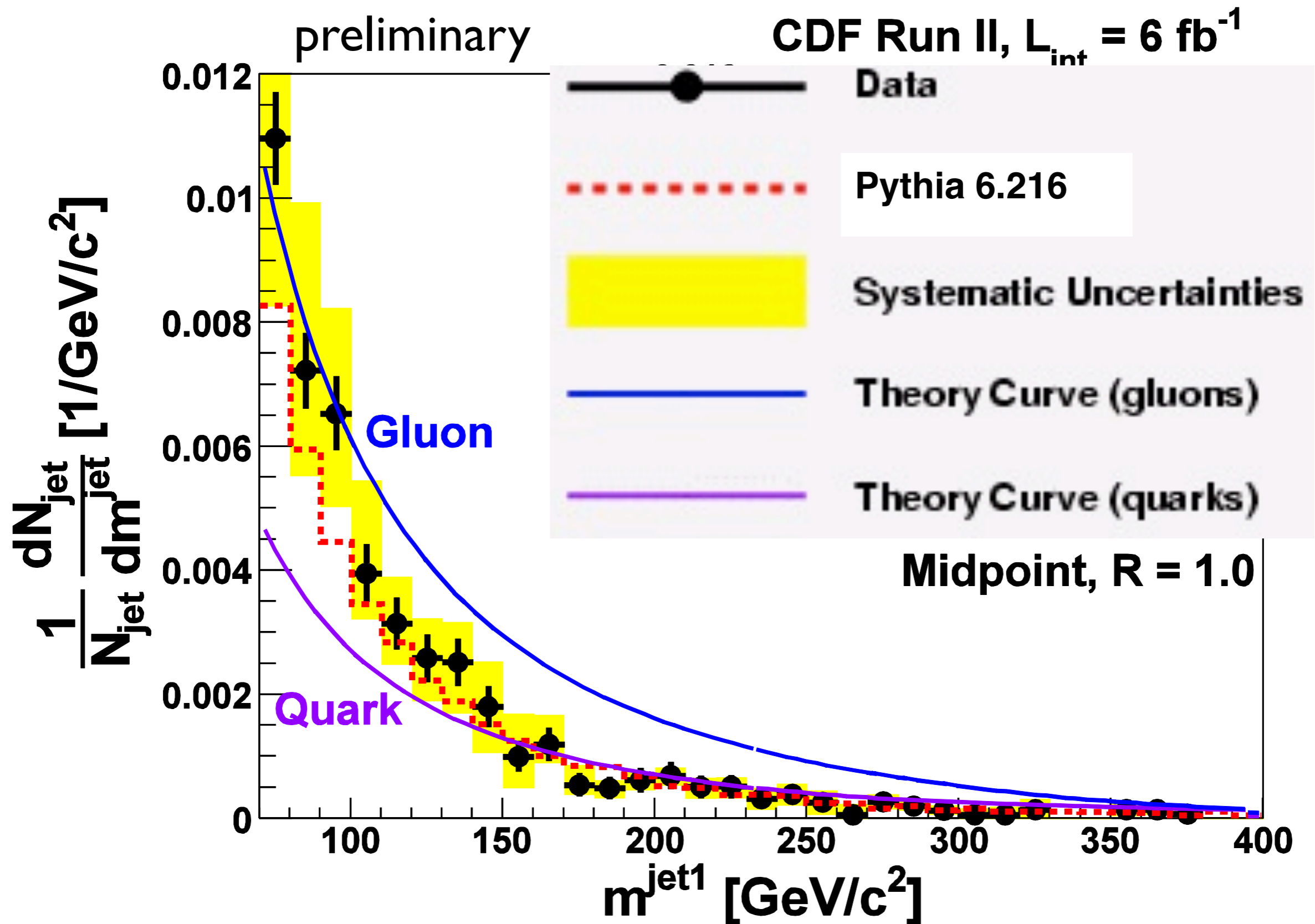


Distribution of jet mass after MI correction for jets with  $400 < p_T < 500$  GeV/c, cone  $R=0.7$ , data and QCD MC

# Jet mass distribution, high mass region

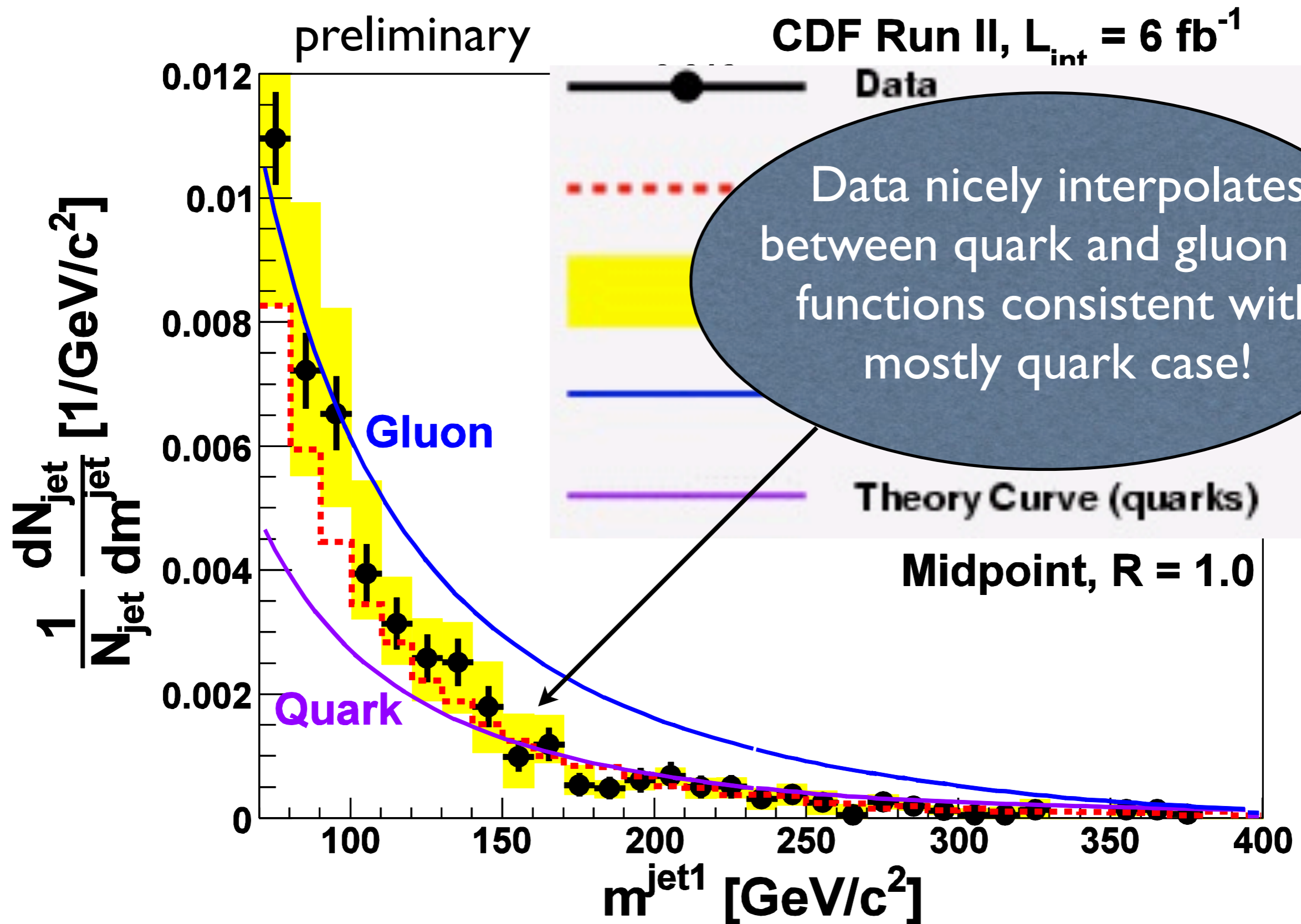


# Jet mass distribution, high mass region



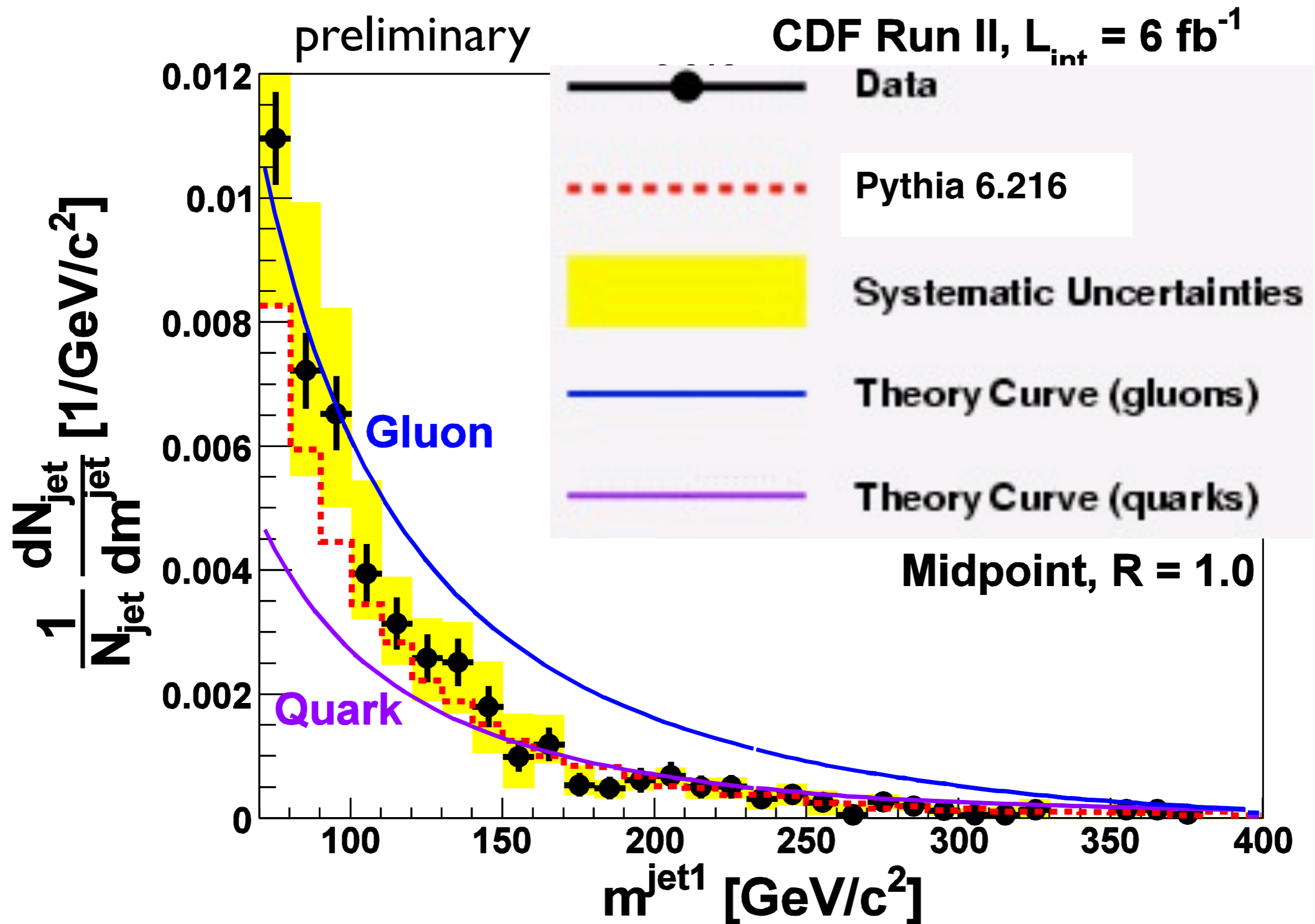


# Jet mass distribution, high mass region

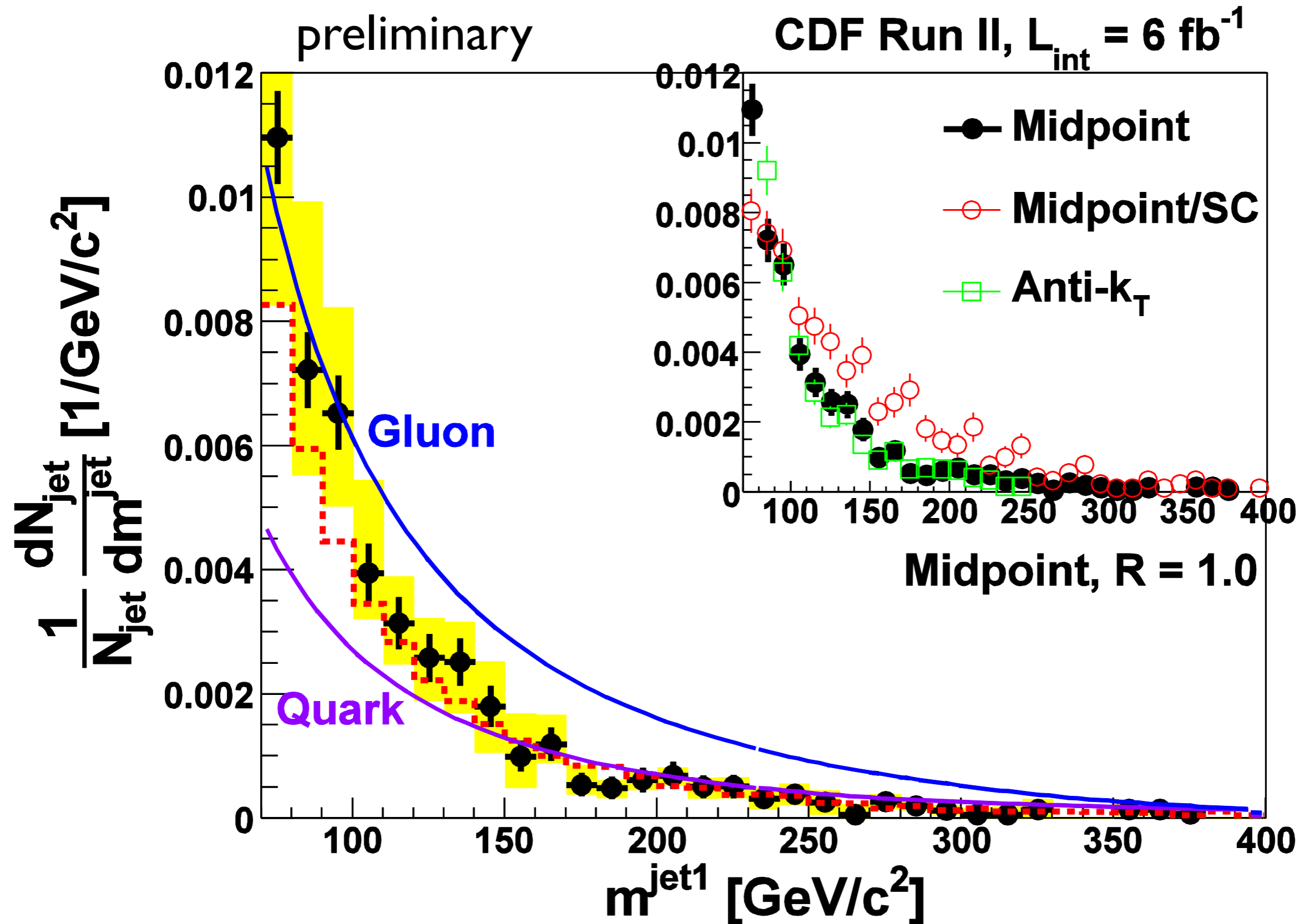




# Jet mass distribution, high mass region

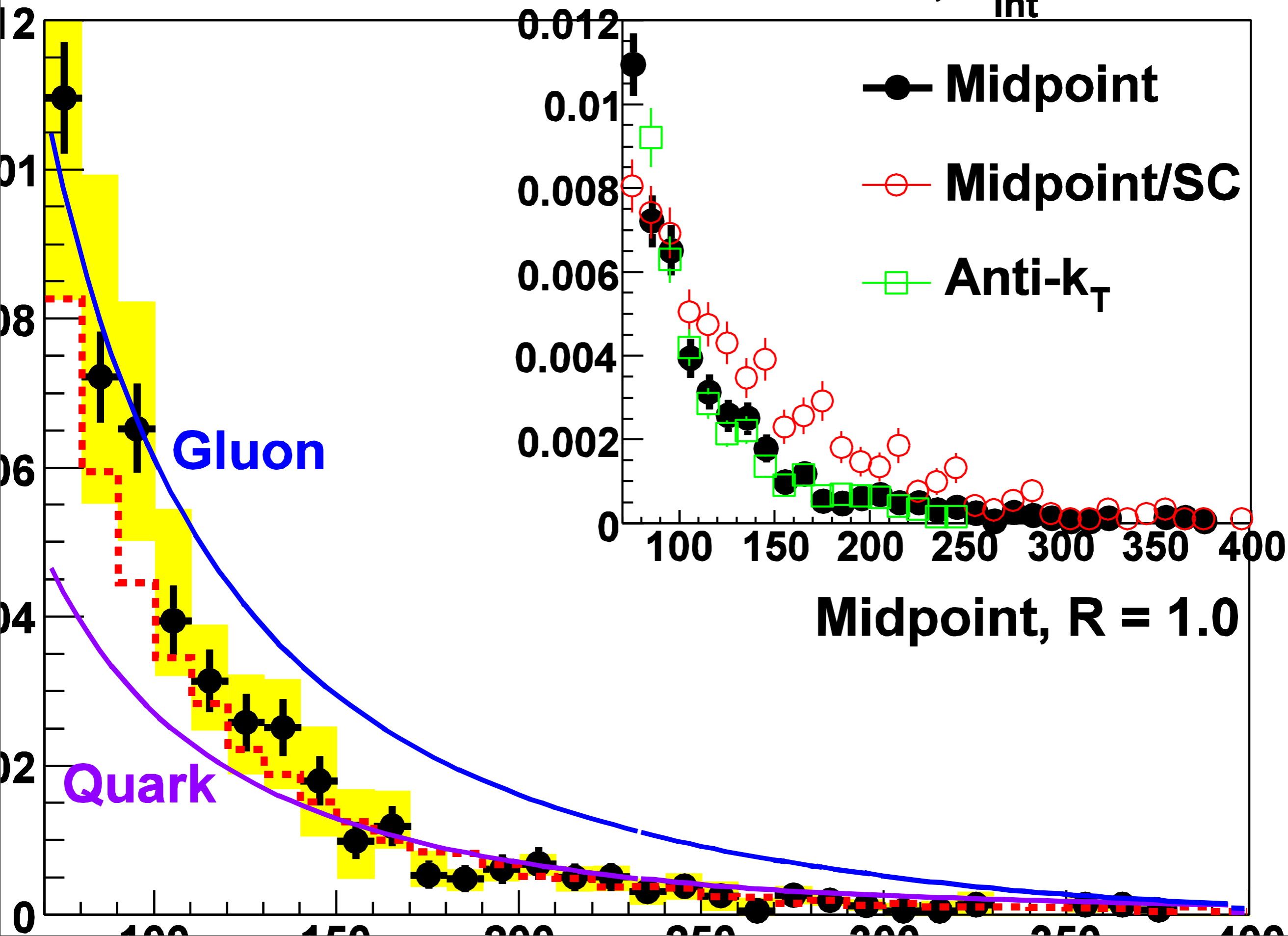


# Jet mass distribution, high mass region



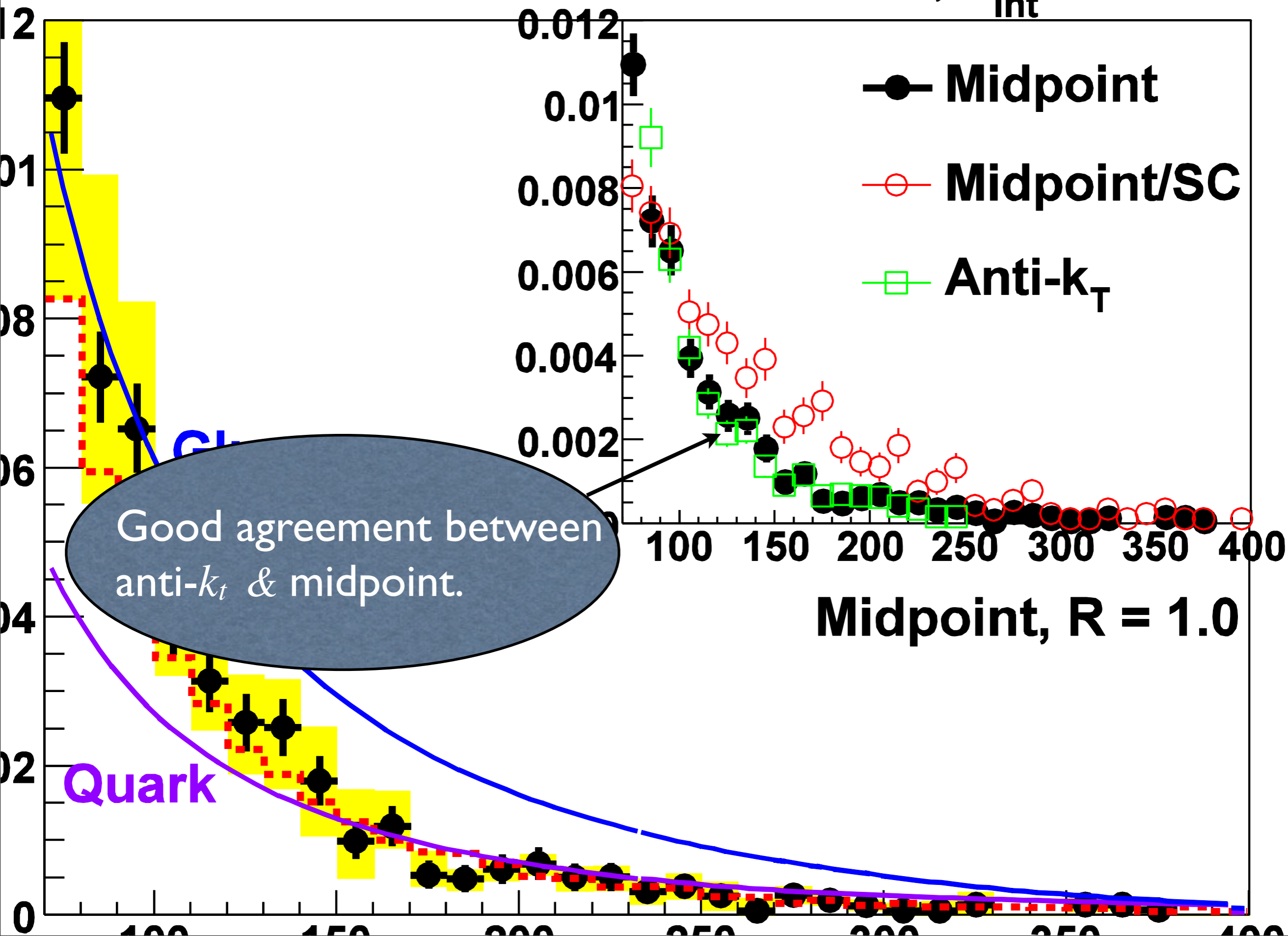
preliminary

CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$



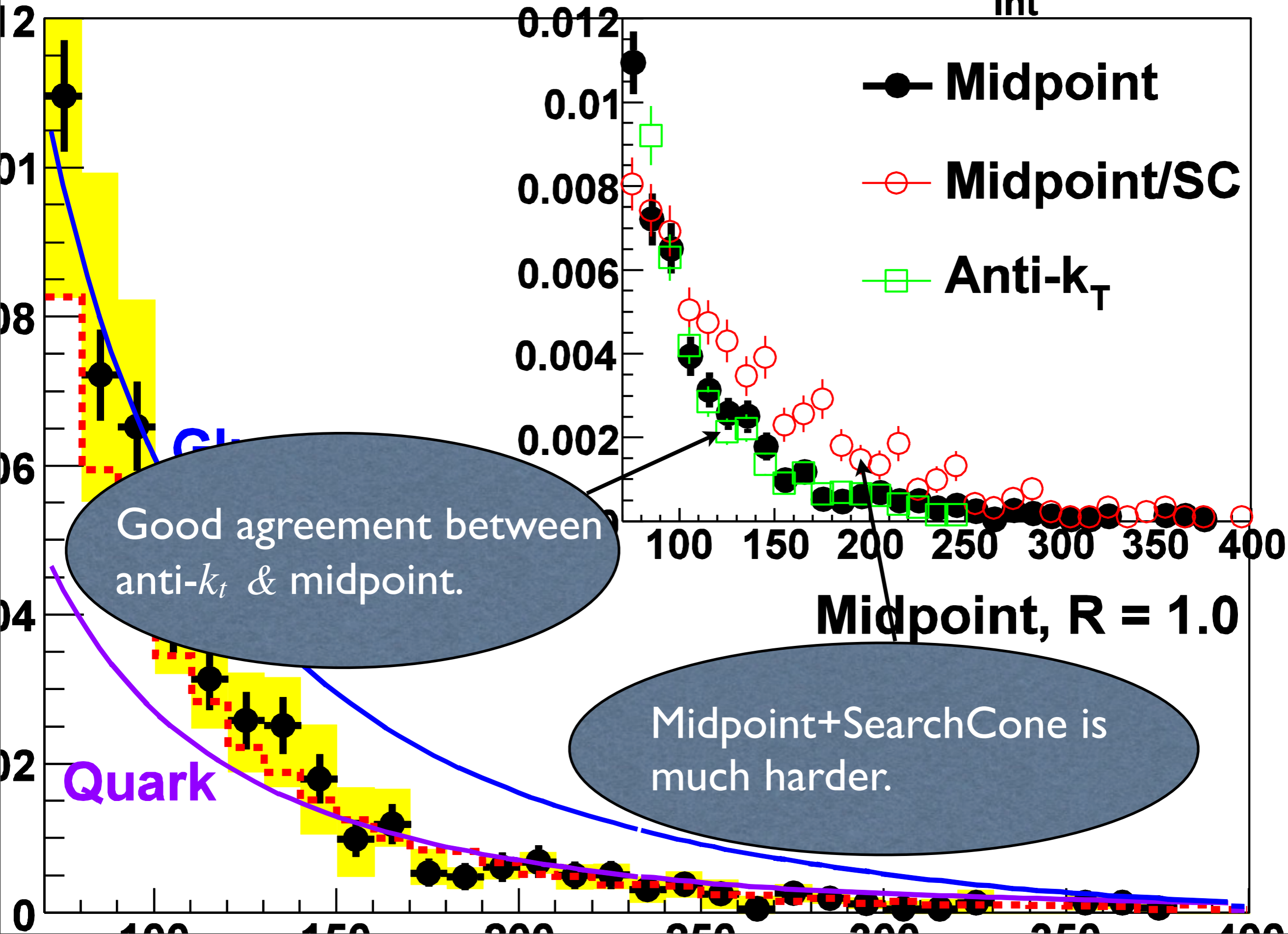
preliminary

CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$



preliminary

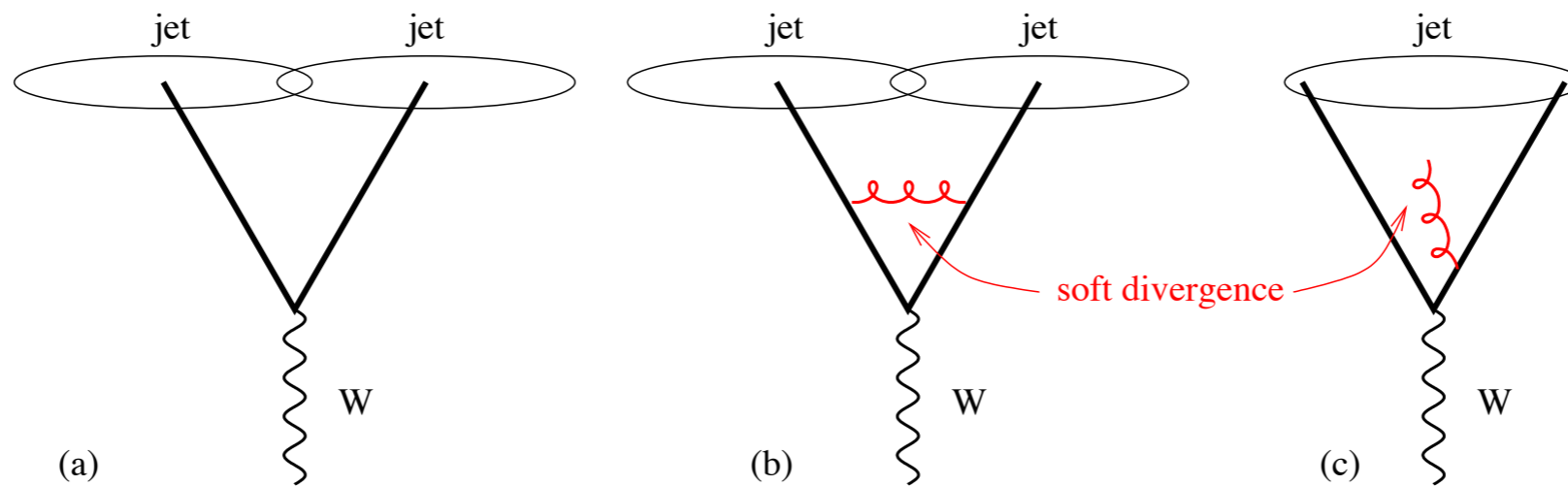
CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$



# IR-collinear sensitivity & jet mass

MidPoint searchcone  $IR_{2+1} \Rightarrow$  harder jets.

Salam, Eur. Phys. J. (2010)



2 perturbative  
massless jets

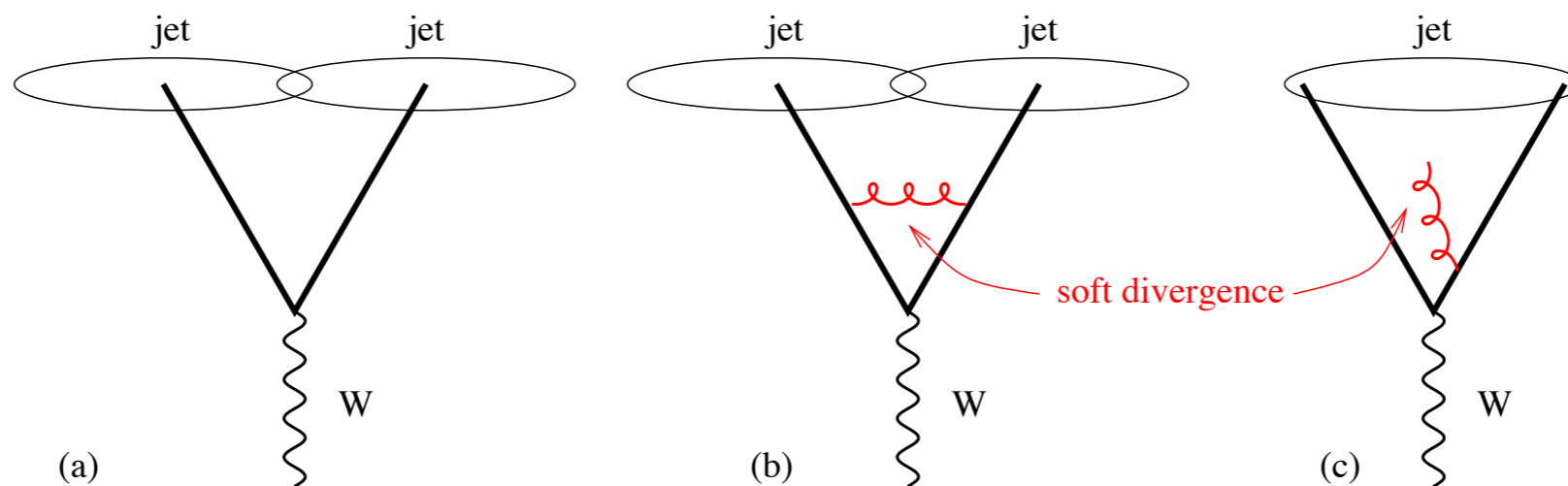
massive jet



# IR-collinear sensitivity & jet mass

MidPoint searchcone  $IR_{2+1} \Rightarrow$  harder jets.

Salam, Eur. Phys. J. (2010)



(a)  
2 perturbative  
massless jets

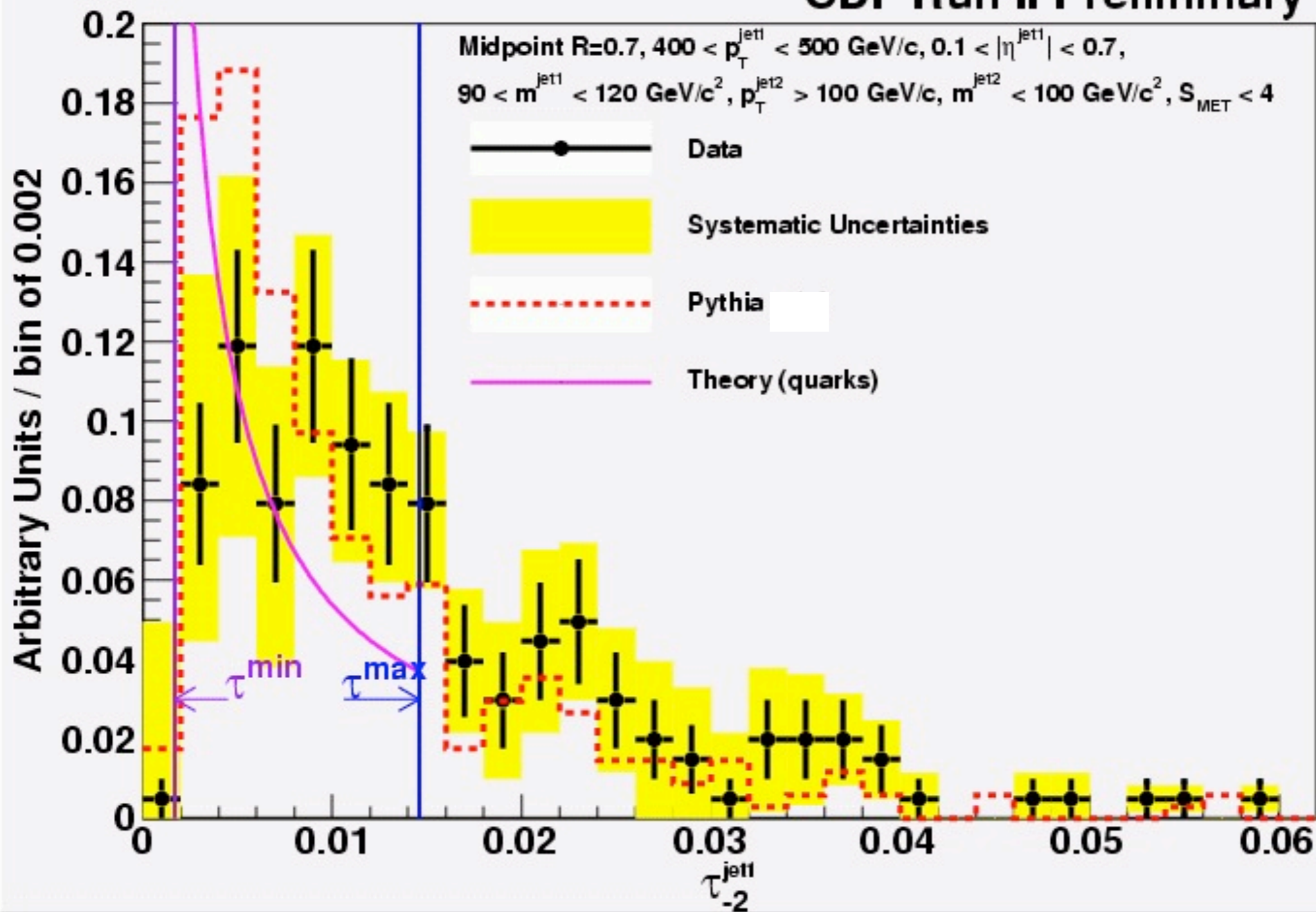
(b) (c)  
massive jet

MidPoint  $IR_{3+1} \Rightarrow$  problem postponed to NLO.

# Angularity

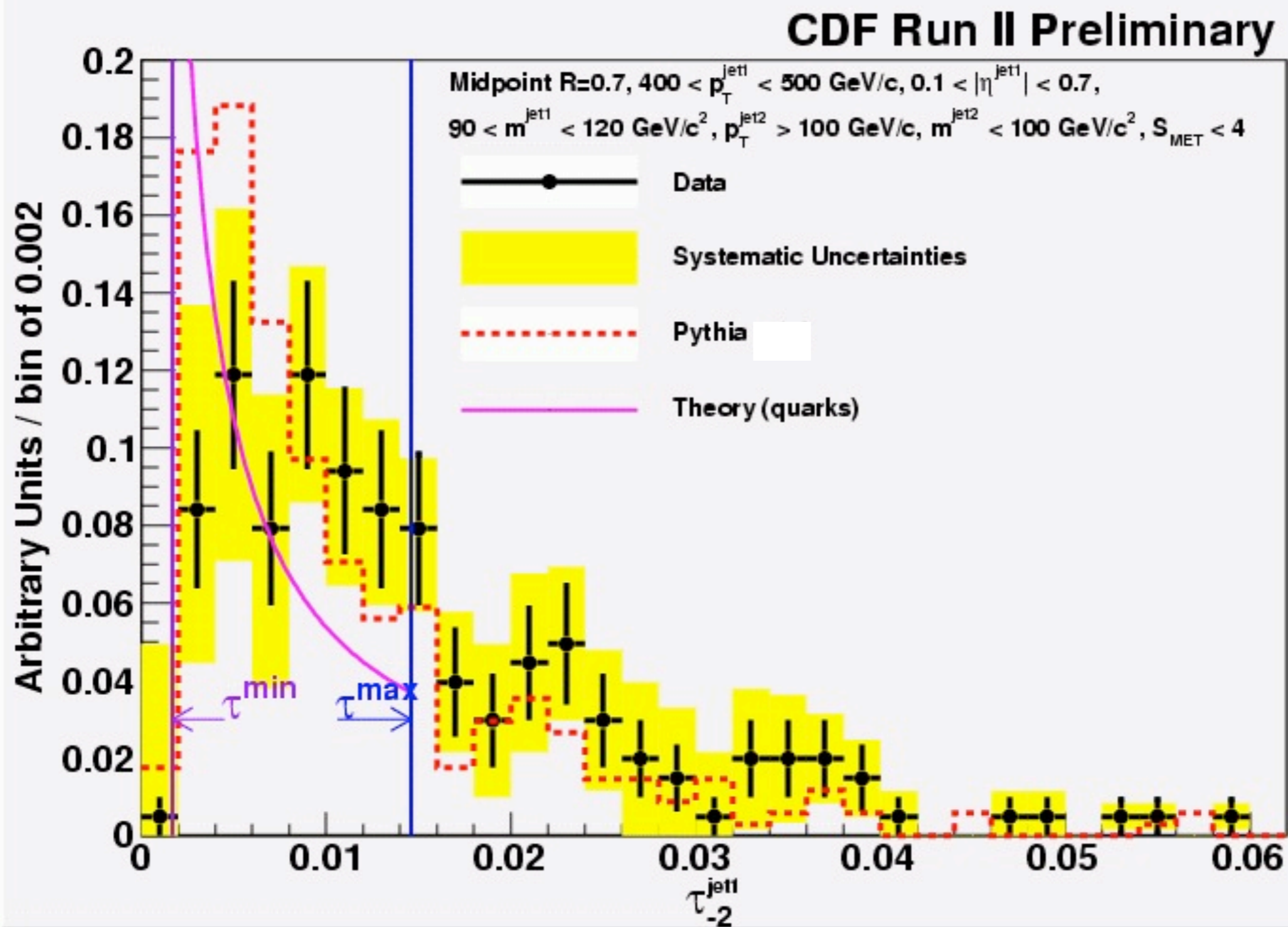
$$\left( \propto_{a=-2} \sum_i \omega_i \theta_i^4 \right)$$

## CDF Run II Preliminary



# Angularity

$$\left( \propto_{a=-2} \sum_i \omega_i \theta_i^4 \right)$$

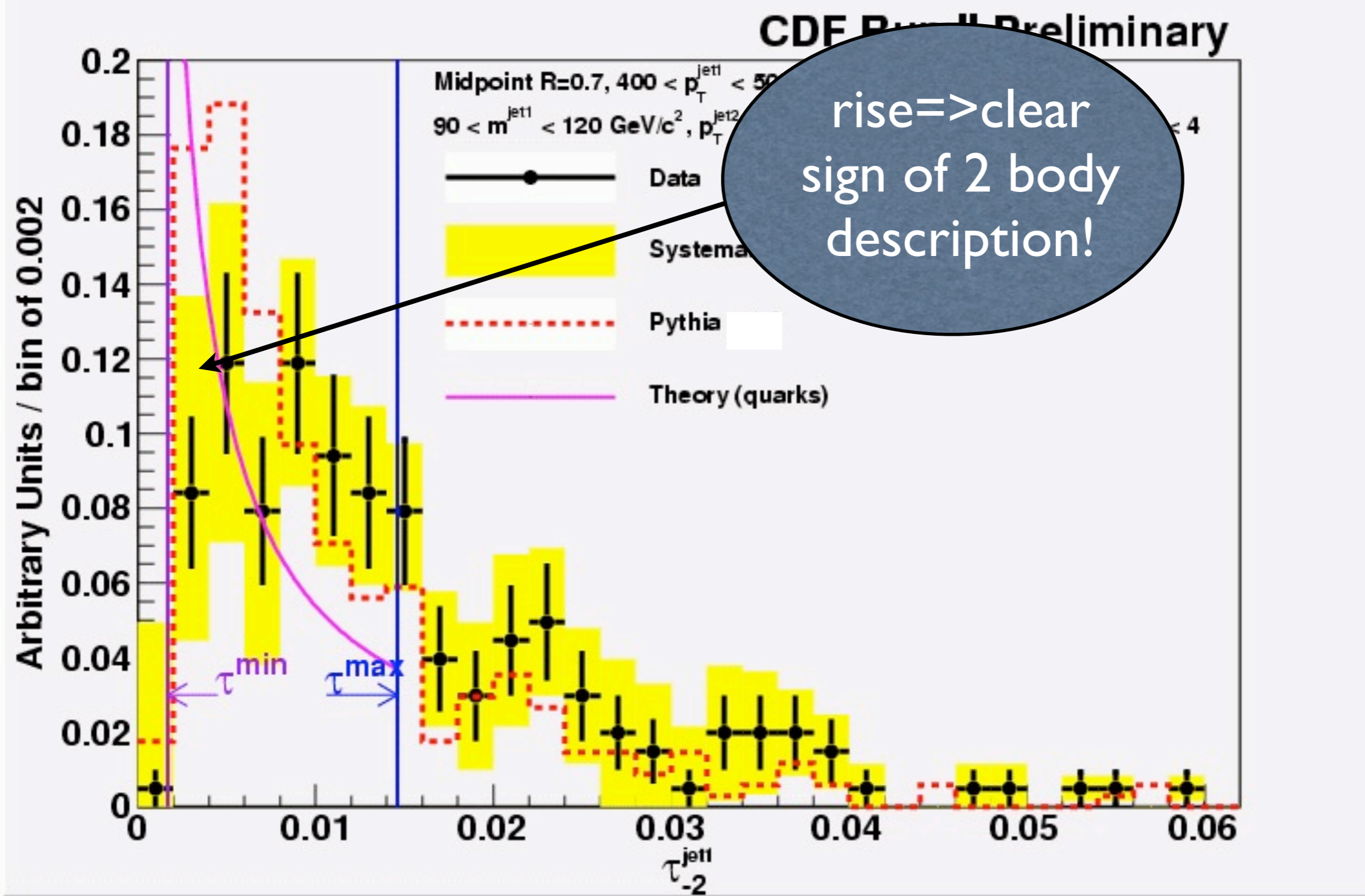


$$\tau_a^{\text{min}}(z) \sim \left(\frac{z}{2}\right)^{1-a}, \quad \tau_a^{\text{max}}(R, p_T) \sim 2^{a-1} R^{-a} z$$



# Angularity

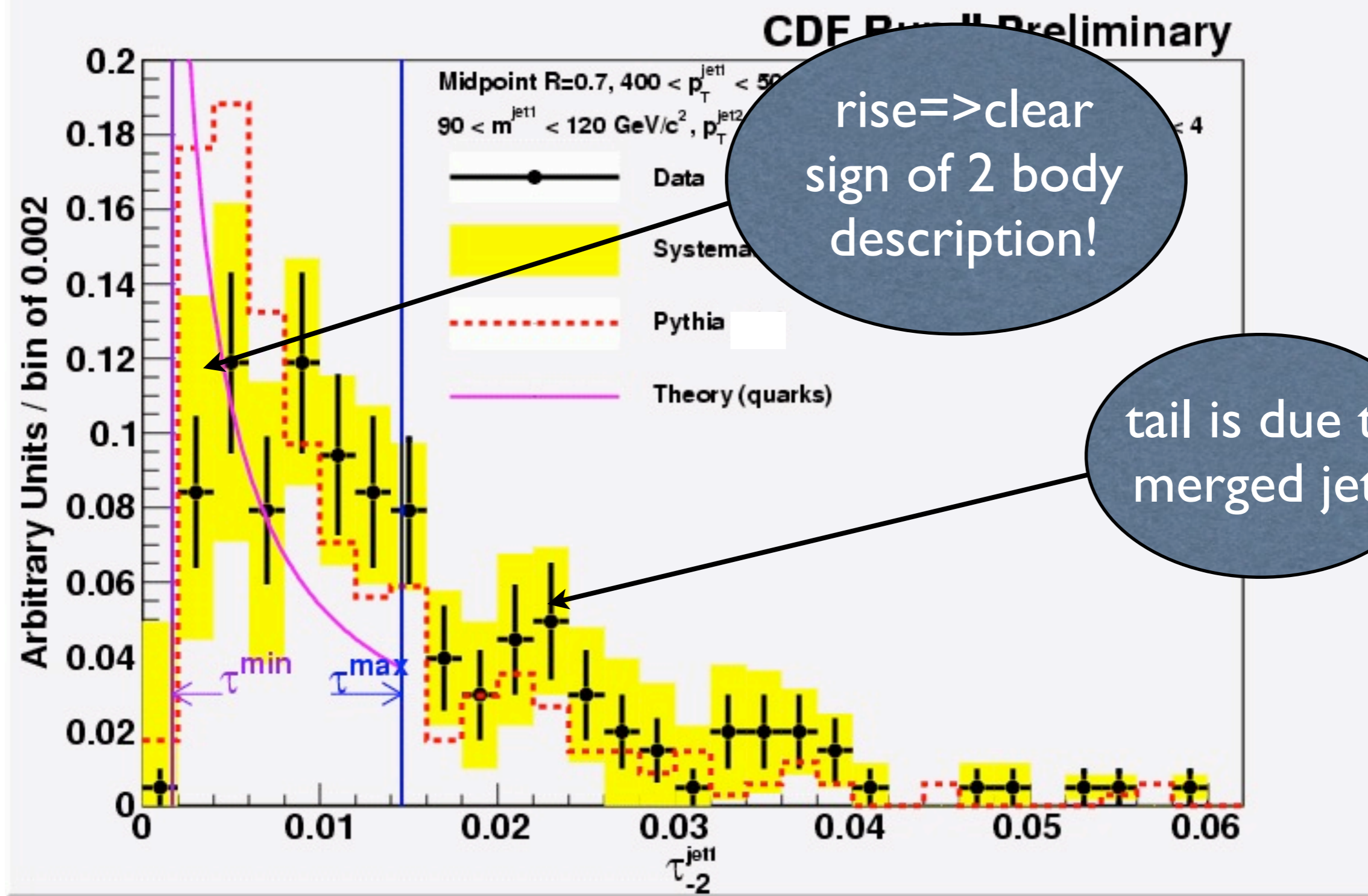
$$\left( \propto_{a=-2} \sum_i \omega_i \theta_i^4 \right)$$



$$\tau_a^{\text{min}}(z) \sim \left(\frac{z}{2}\right)^{1-a}, \quad \tau_a^{\text{max}}(R, p_T) \sim 2^{a-1} R^{-a} z$$

# Angularity

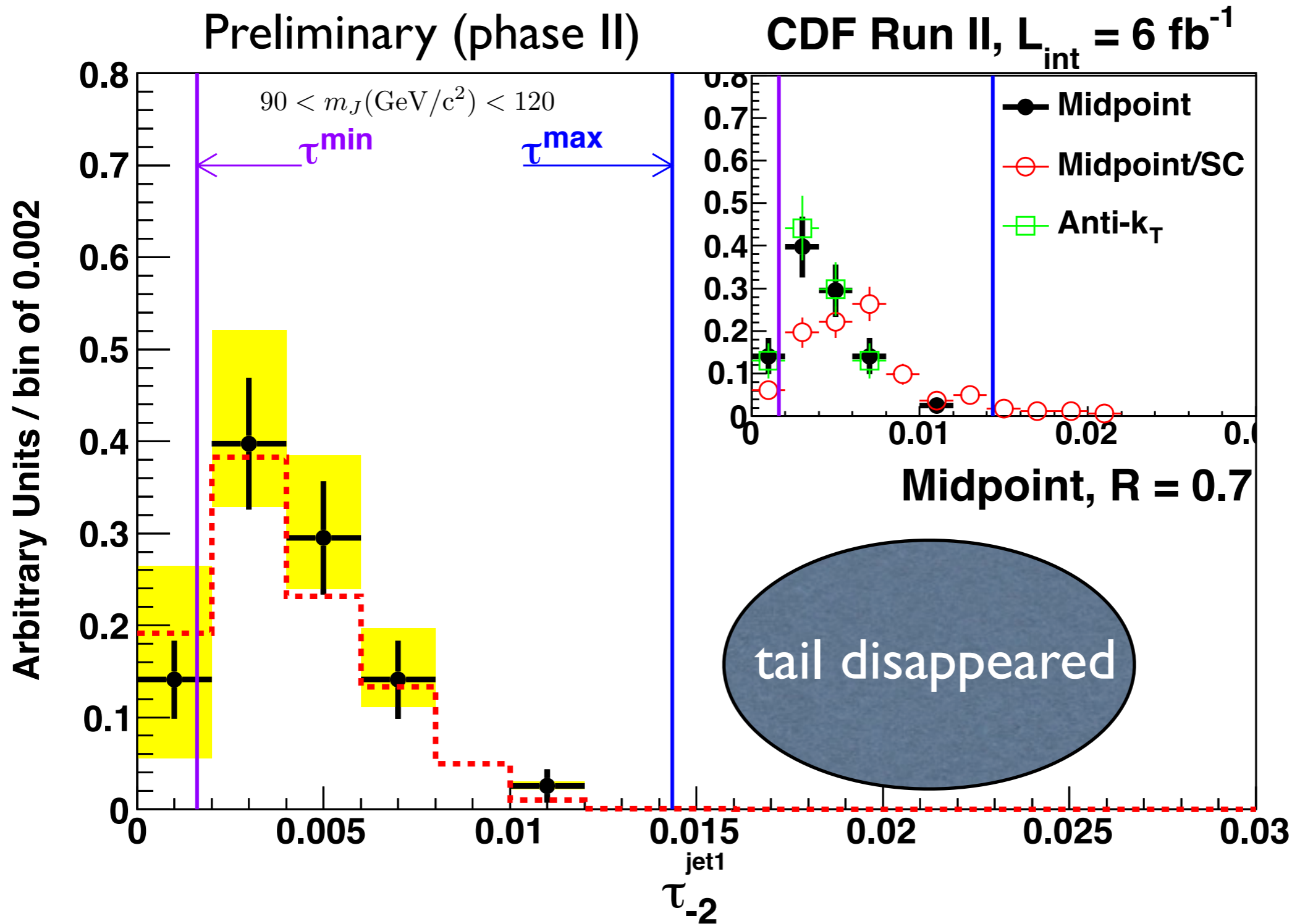
$$\left( \propto_{a=-2} \sum_i \omega_i \theta_i^4 \right)$$



$$\tau_a^{\text{min}}(z) \sim \left(\frac{z}{2}\right)^{1-a}, \quad \tau_a^{\text{max}}(R, p_T) \sim 2^{a-1} R^{-a} z$$

# Angularity

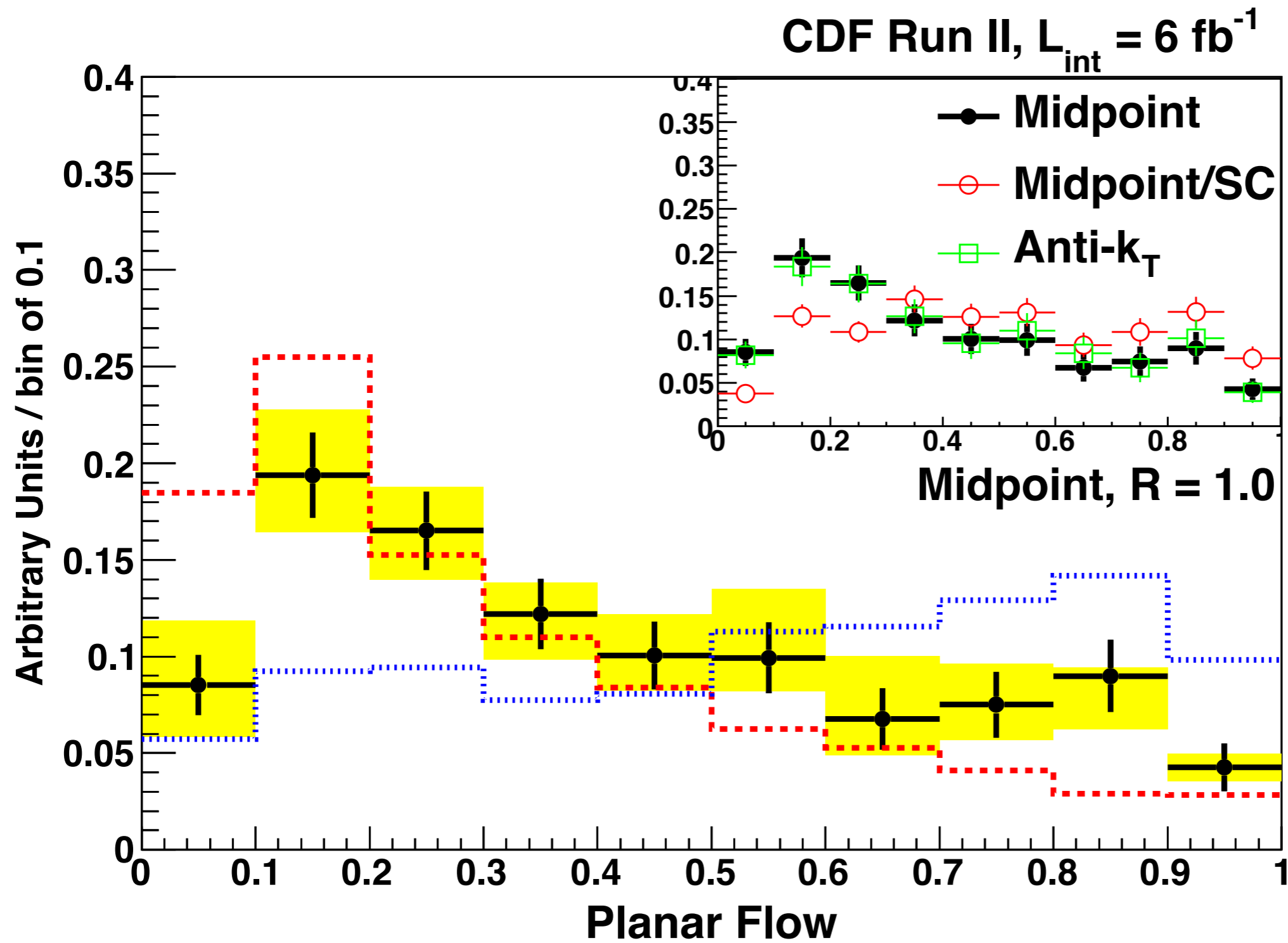
$$\left( \propto_{a=-2} \sum_i \omega_i \theta_i^4 \right)$$



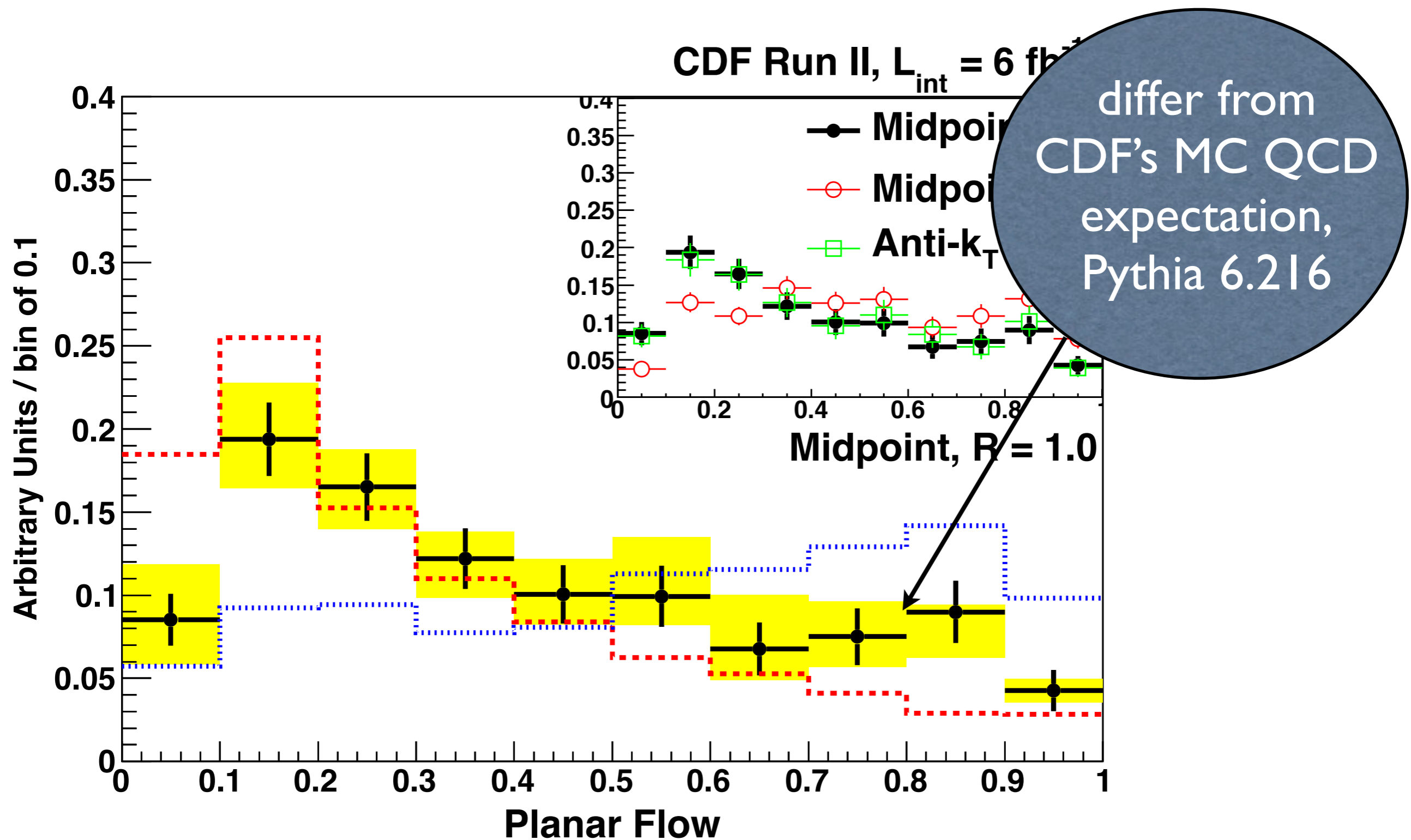
$$\tau_a^{\min}(z) \sim \left(\frac{z}{2}\right)^{1-a}, \quad \tau_a^{\max}(R, p_T) \sim 2^{a-1} R^{-a} z$$



# Planar flow

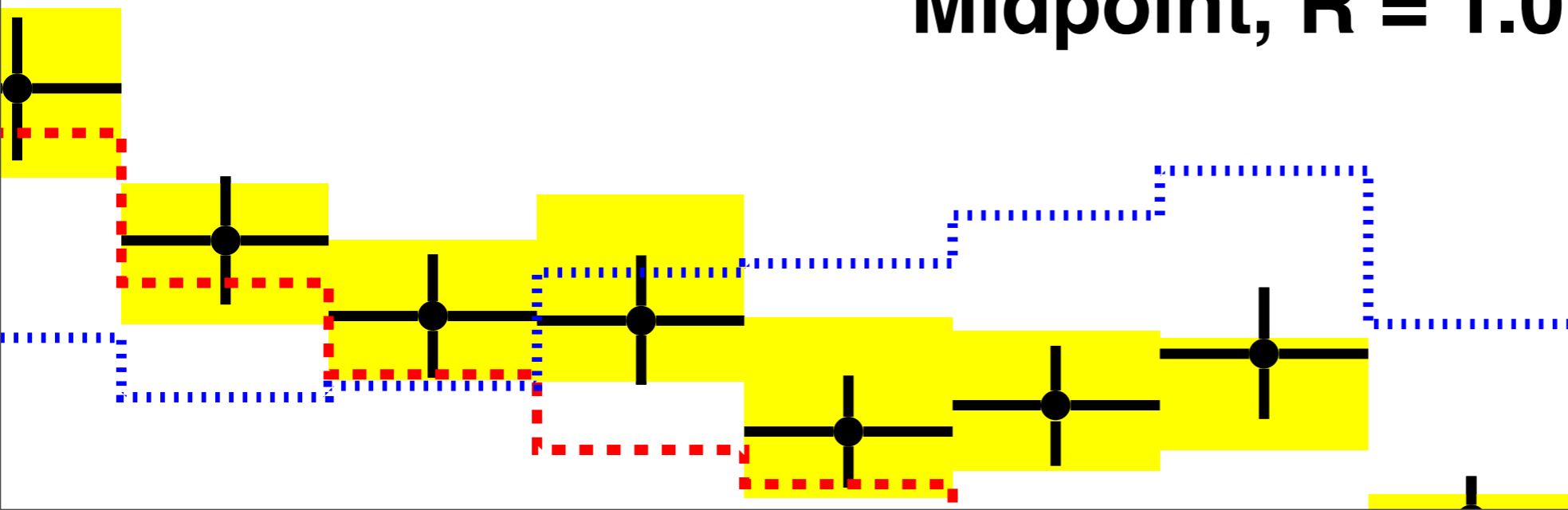
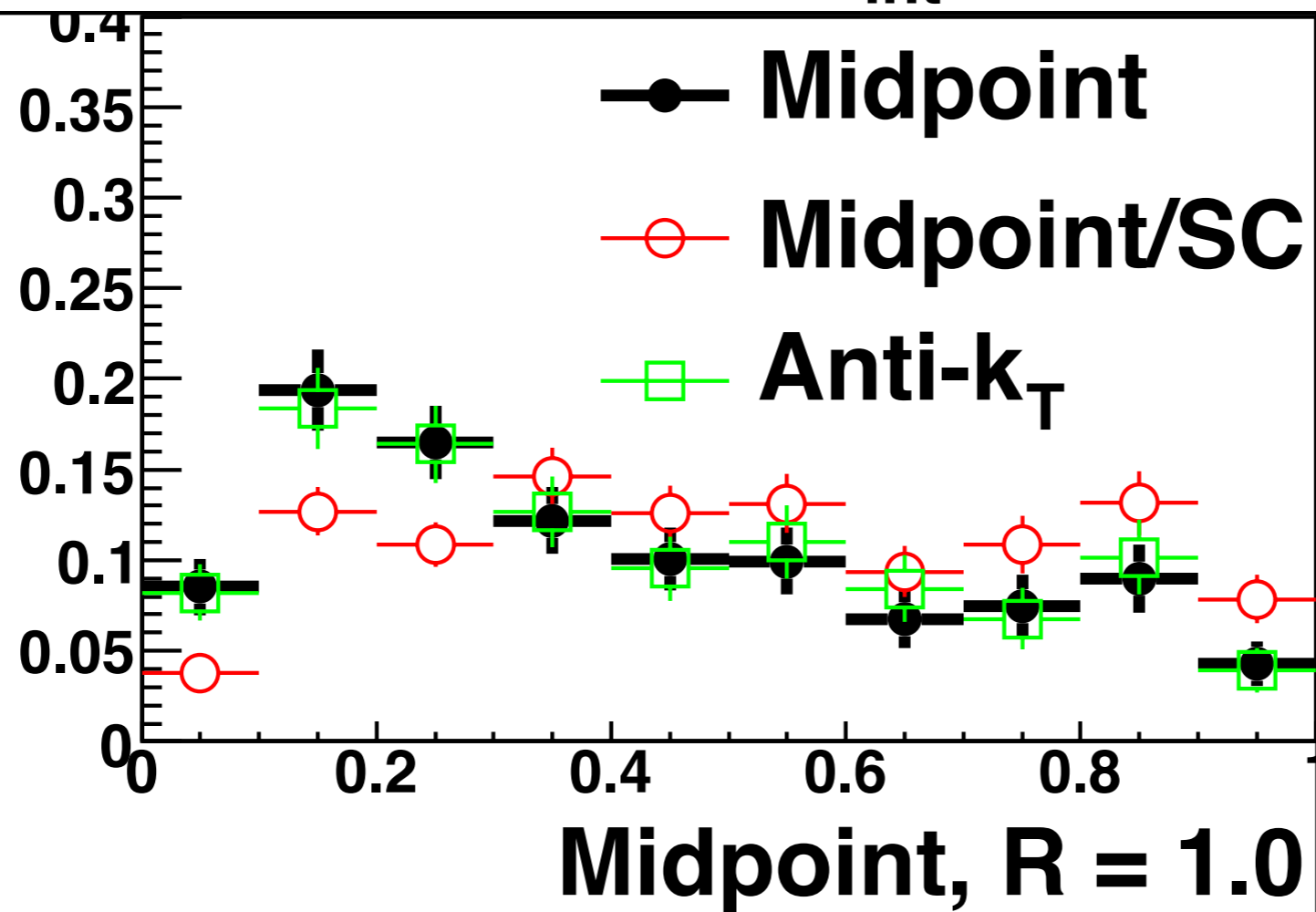


# Planar flow



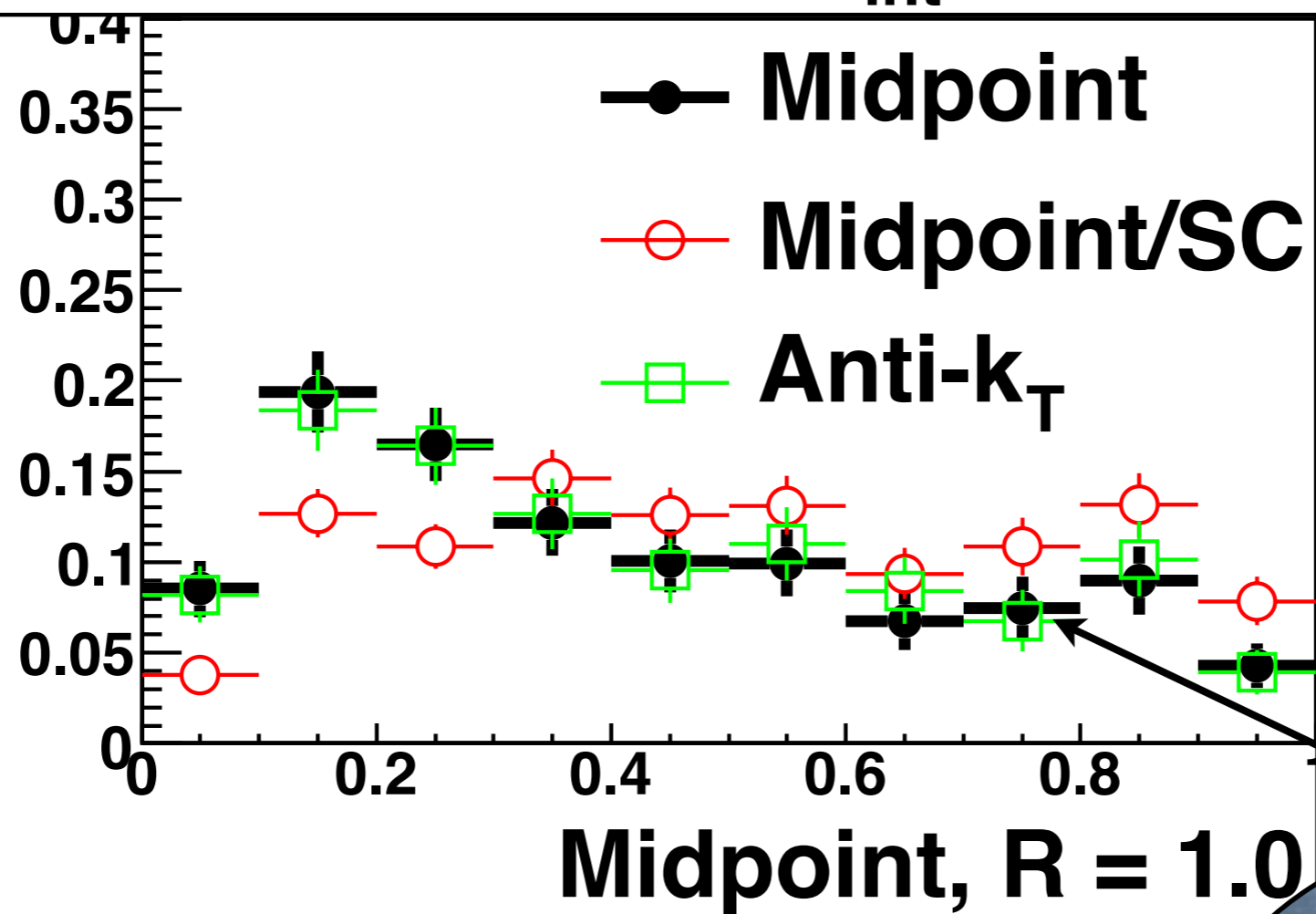
# Planar flow

CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$



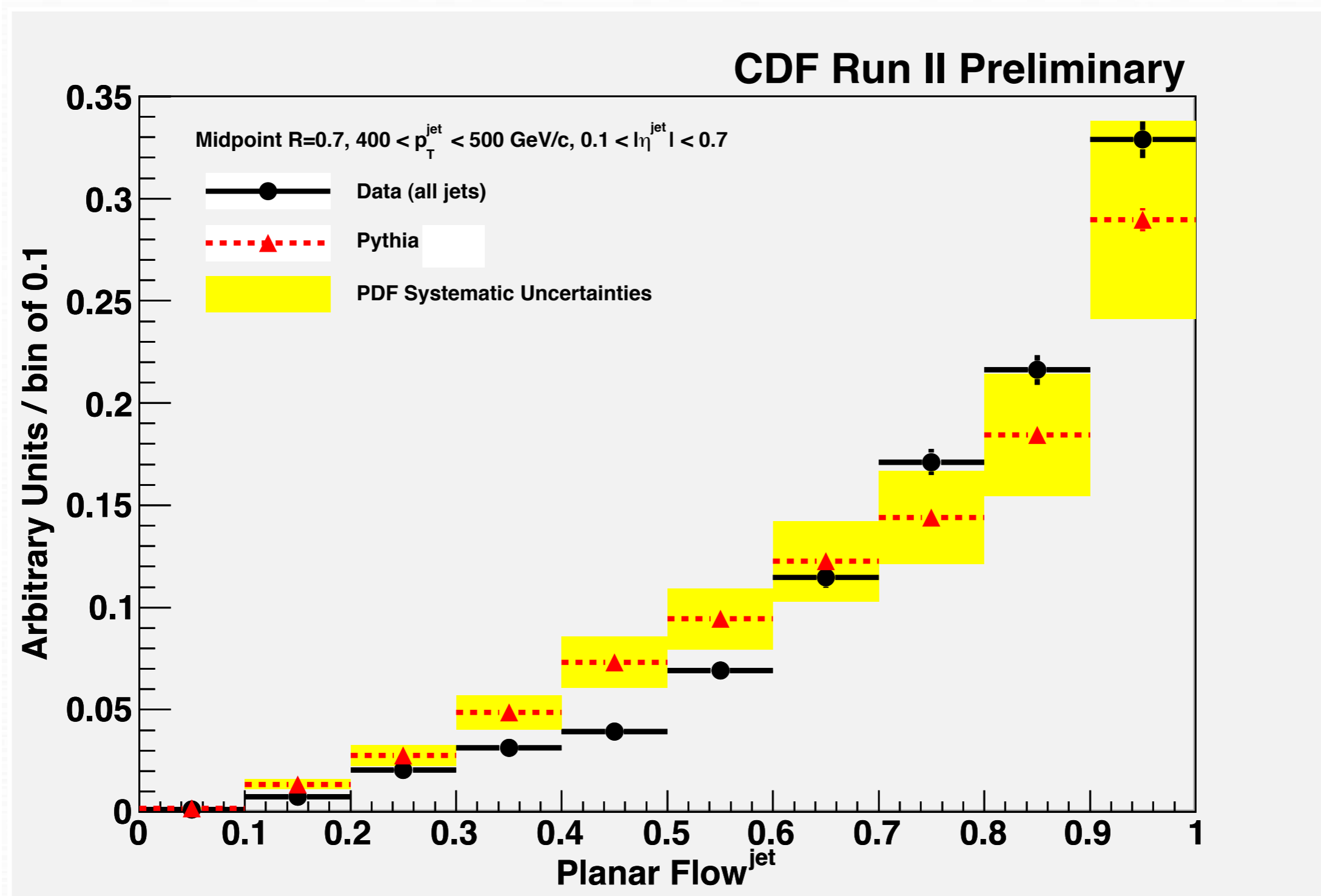
# Planar flow

CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$

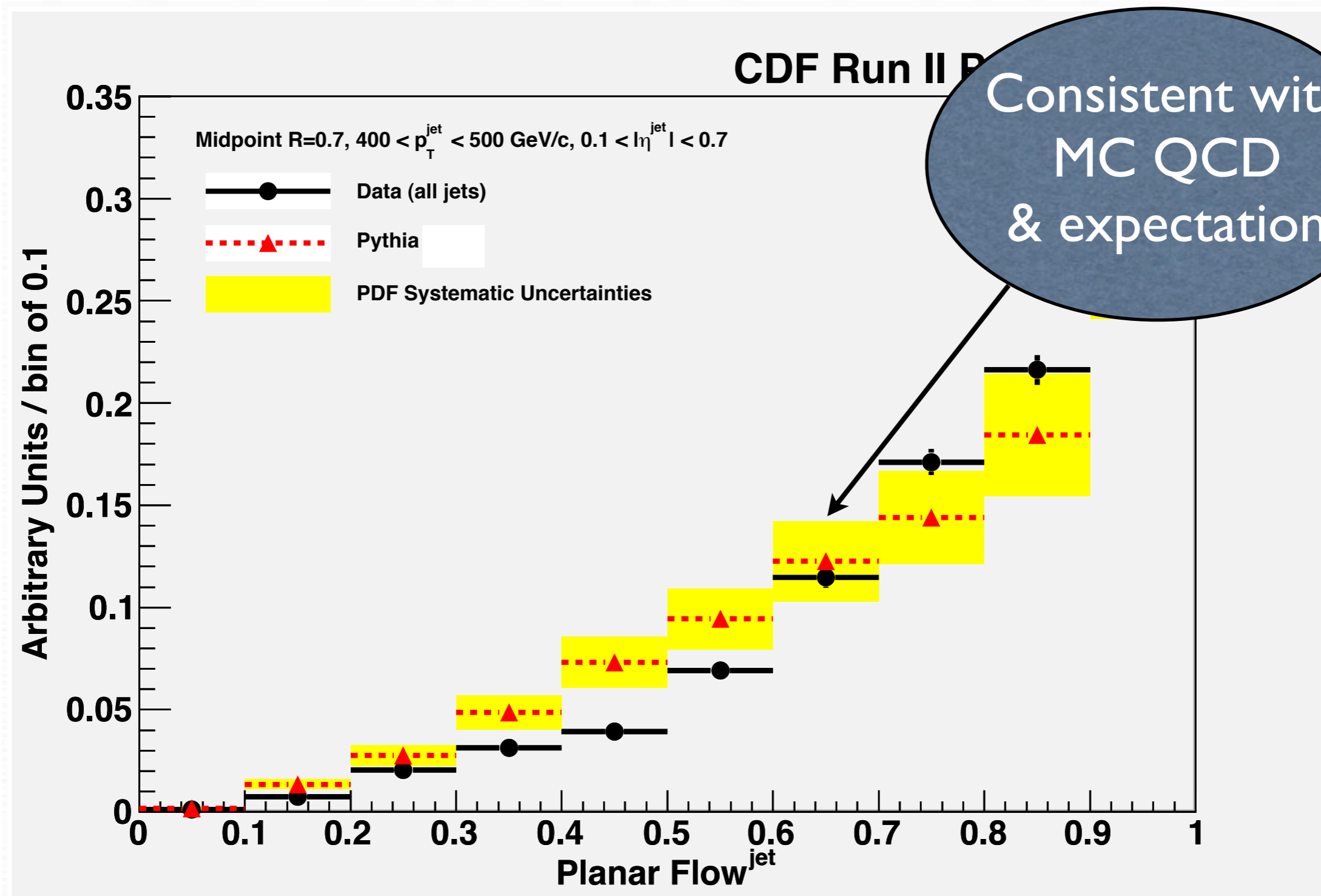


No difference  
between Midpoint  
& anti- $k_t$

# Planar flow, no mass cut



# Planar flow, no mass cut





# Excess in di-massive jets

---

Let us look at the “SL” & ‘hadronic’ data samples separately  
(including 30% sys’ uncertainties from JES & mass measurements):

# Excess in di-massive jets

---

Let us look at the “SL” & ‘hadronic’ data samples separately  
(including 30% sys’ uncertainties from JES & mass measurements):

**1 massive jets+MET:**      QCD<sub>data driven</sub> :     $31 \pm 8.1$  (stat.)  $\pm 9.3$  (syst.),  
[ $130 < m_j < 210$  (GeV),  $4 < s_{MET} < 10$ ]       $t\bar{t}$  :                       $1.9 \pm 0.5$ .

# Excess in di-massive jets

---

Let us look at the “SL” & ‘hadronic’ data samples separately  
(including 30% sys’ uncertainties from JES & mass measurements):

**1 massive jets+MET:**       $\text{QCD}_{\text{data driven}} : 31 \pm 8.1 \text{ (stat.)} \pm 9.3 \text{ (syst.)}$ ,  
[ $130 < m_j < 210 \text{ (GeV)}, 4 < s_{\text{MET}} < 10$ ]       $t\bar{t} : 1.9 \pm 0.5$ .

**26 observed events  $\Rightarrow \sim -0.6$  standard deviations**

# Excess in di-massive jets

---

Let us look at the “SL” & ‘hadronic’ data samples separately  
(including 30% sys’ uncertainties from JES & mass measurements):

**1 massive jets+MET:**       $\text{QCD}_{\text{data driven}} : 31 \pm 8.1 \text{ (stat.)} \pm 9.3 \text{ (syst.)}$ ,  
[ $130 < m_j < 210 \text{ (GeV)}, 4 < s_{\text{MET}} < 10$ ]       $t\bar{t} : 1.9 \pm 0.5$ .

**26 observed events  $\Rightarrow \sim -0.6$  standard deviations**

**2 massive jets:**       $\text{QCD}_{\text{data driven}} : 13 \pm 2.4 \text{ (stat.)} \pm 3.9 \text{ (syst.)}$ ,  
[ $130 < m_j < 210 \text{ (GeV)}$ ]       $t\bar{t} : 3.0 \pm 0.8$ .

# Excess in di-massive jets

---

Let us look at the “SL” & ‘hadronic’ data samples separately  
(including 30% sys’ uncertainties from JES & mass measurements):

**1 massive jets+MET:**       $\text{QCD}_{\text{data driven}} : 31 \pm 8.1 \text{ (stat.)} \pm 9.3 \text{ (syst.)}$ ,  
[ $130 < m_j < 210 \text{ (GeV)}, 4 < s_{\text{MET}} < 10$ ]       $t\bar{t} : 1.9 \pm 0.5$ .

**26 observed events => ~ -0.6 standard deviations**

**2 massive jets:**       $\text{QCD}_{\text{data driven}} : 13 \pm 2.4 \text{ (stat.)} \pm 3.9 \text{ (syst.)}$ ,  
[ $130 < m_j < 210 \text{ (GeV)}$ ]       $t\bar{t} : 3.0 \pm 0.8$ .

**32 observed events => ~ 3.4 standard deviations**



# Back to Theory



(i) Method for pile up subtraction for massive jets.

*R, Alon, E. Duchovni, GP, S. Pronko & P. Sinervo, arXiv:1101.xxxx.*

(ii) Characterization of massive jets.

*G. Gur-Ari, M. Papucci & GP, arXiv:1101.xxxx;*

(iii) Some trivial implications of the recent data.

# Data-driven method of pile-up correction for substructure of massive jets (brief)

---

- Soft semi-coherent contributions smear E-flow distributions.

Dokshitzer, Lucenti, Marchesini and Salam, JHEP (98); Webber, PLB (94).

- Global corrections elegantly dealt with the concept of jet area.

Cacciari and Salam, PLB (08); Cacciari, Salam and Soyez, JHEP (08).

- What about jet shape specific correction (differential correction)?

- Can be addressed by generalization of the jet area concept.

Cacciari and Salam, PLB (08); Cacciari, Salam and Soyez, JHEP (08); Sapeta and Q. C. Zhang, 1009.1143.

$$A_X = [ X(\{p_i, g_i\}) - X(\{p_i\}) ] / (\nu_g \langle g_t \rangle)$$

$$X_{\text{pileup subtracted}} = X - A_X * \rho$$

(where  $X(\{p_i, g_i\})$  is the value of  $X$  in the presence of ghosts and genuine jet particles  $p_i$  and  $X(\{p_i\})$  is its value given just the particles  $p_i$ ,  $\nu_g$  is the ghost density and  $\langle g_t \rangle$  average ghost momentum.)

# Data-driven method of pile-up correction for massive jets

- An analytical close form can be obtained for narrow massive jets, mass, angularity & Pf (qualitatively verified by recent data).

*R, Alon, E. Duchovni, GP, S. Pronko & P. Sinervo, arXiv:1101.xxxx.*

$$\Delta X|_{p_J, m_J} = \frac{\partial X}{\partial m_J}|_{p_J, m_J} \delta m_J + \sum_{i \in R^{90^\circ}} \frac{\partial X}{\partial E_i}|_{p_J, m_J} \delta E_i,$$

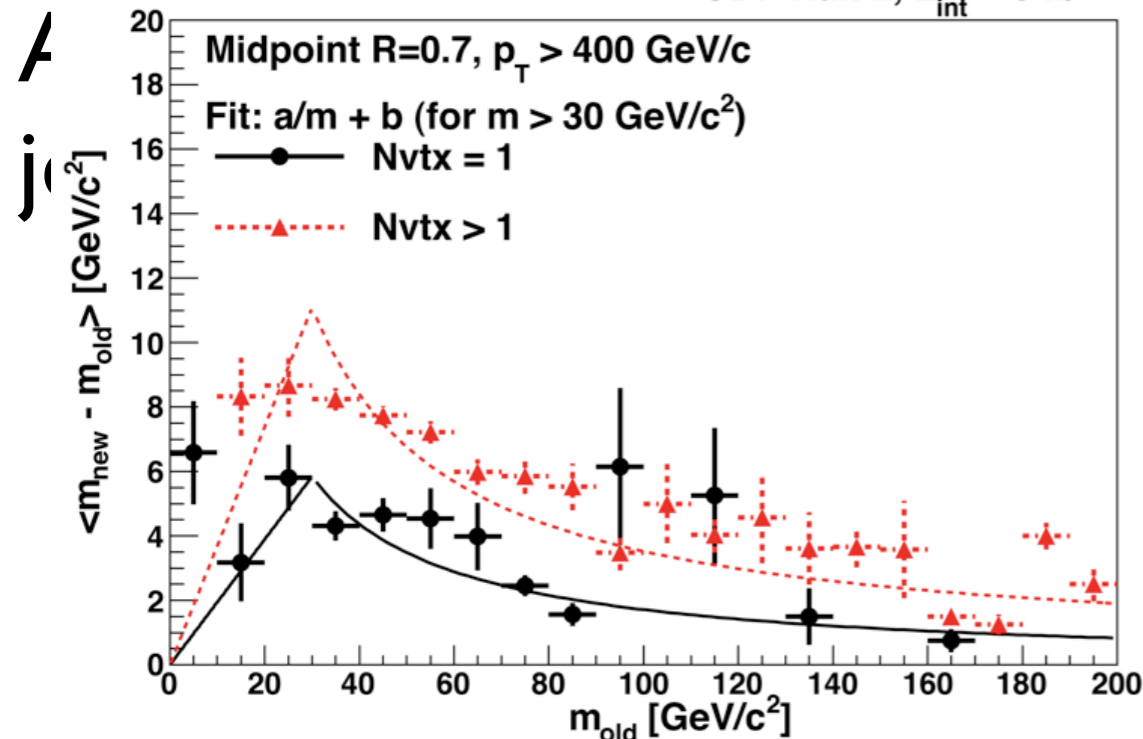
$$\Delta X(p_J, m_J) = f(X, p_J, m_J) \delta m_J^2 \oplus g(X, p_J, m_J) \delta E,$$

- Trivial ex., jet mass:  $\Delta m_J|_{p_J, m_J} = \sum_{i \in R^{90^\circ}} \frac{\partial m}{\partial E_i}|_{p_T, m_J} \delta E_i$ .  $\Delta m_J^2 \sim p_J \sum_{i \in R^{90^\circ}} \delta E_i \theta_i^2 \equiv \sum_{i \in R^{90^\circ}} \delta m_i^2$ .

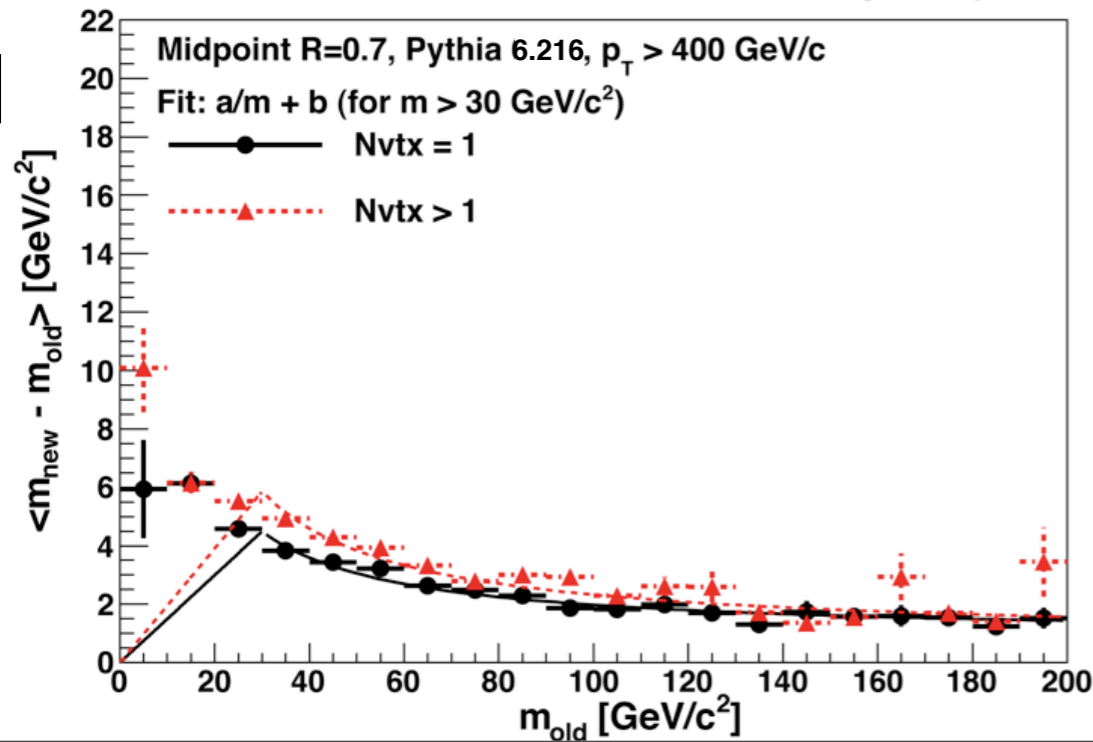
$$\Delta m_J^2 = 2m_J \delta m_J \quad \longrightarrow \quad \delta m_J \sim \sum_{i \in R^{90^\circ}} \frac{\delta m_i^2}{2m_J}.$$

# Data-driven method of pile-up correction for massive jets

Preliminary CDF Run II,  $L_{\text{int}} = 6 \text{ fb}^{-1}$



Preliminary CDF Run II



be obtained for narrow massive jets (qualitatively verified by recent data).

R, Alon, E. Duchovni, GP, S. Pronko & P. Sinervo, arXiv:1101.xxxx.

$$\delta m_J + \sum_{i \in R^{90^\circ}} \frac{\partial X}{\partial E_i} \Big|_{p_J, m_J} \delta E_i,$$

$$, m_J) \delta m_J^2 \oplus g(X, p_J, m_J) \delta E,$$

$$= \sum_{i \in R^{90^\circ}} \frac{\partial m}{\partial E_i} \Big|_{p_T, m_J} \delta E_i. \quad \Delta m_J^2 \sim p_J \sum_{i \in R^{90^\circ}} \delta E_i \theta_i^2 \equiv \sum_{i \in R^{90^\circ}} \delta m_i^2.$$

$$\delta m_J \sim \sum_{i \in R^{90^\circ}} \frac{\delta m_i^2}{2m_J}.$$

# Data-driven method of pile-up correction for angularity

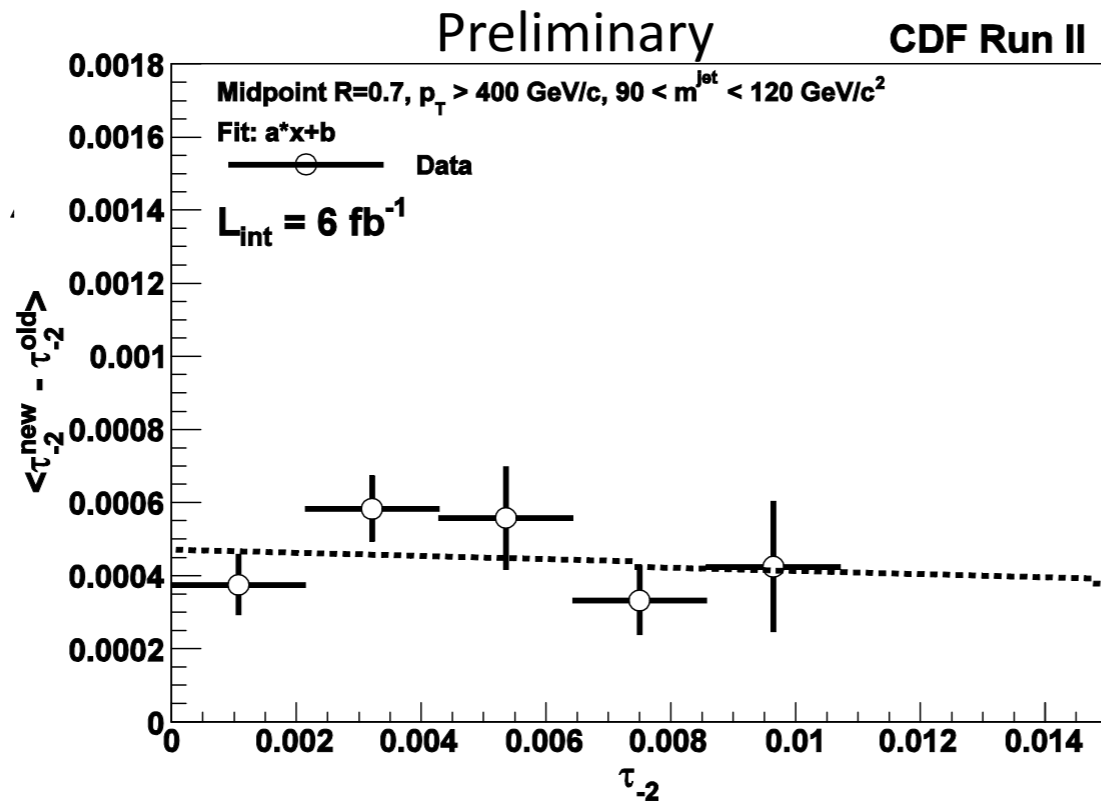
---

● **Angularity:**  $\Delta\tau_a = \frac{\partial\tau_a}{\partial m_J}\delta m_J + \sum_{i \in R^{90^\circ}} \frac{\partial\tau_a}{\partial E_i}\delta E_i \longrightarrow \sum_{i \in R^{90^\circ}} \frac{\delta m_i^2}{2m_J^2} \left( \frac{2^a m_J}{p_J} \theta_i^{-a} \oplus \tau_a^J \right)$

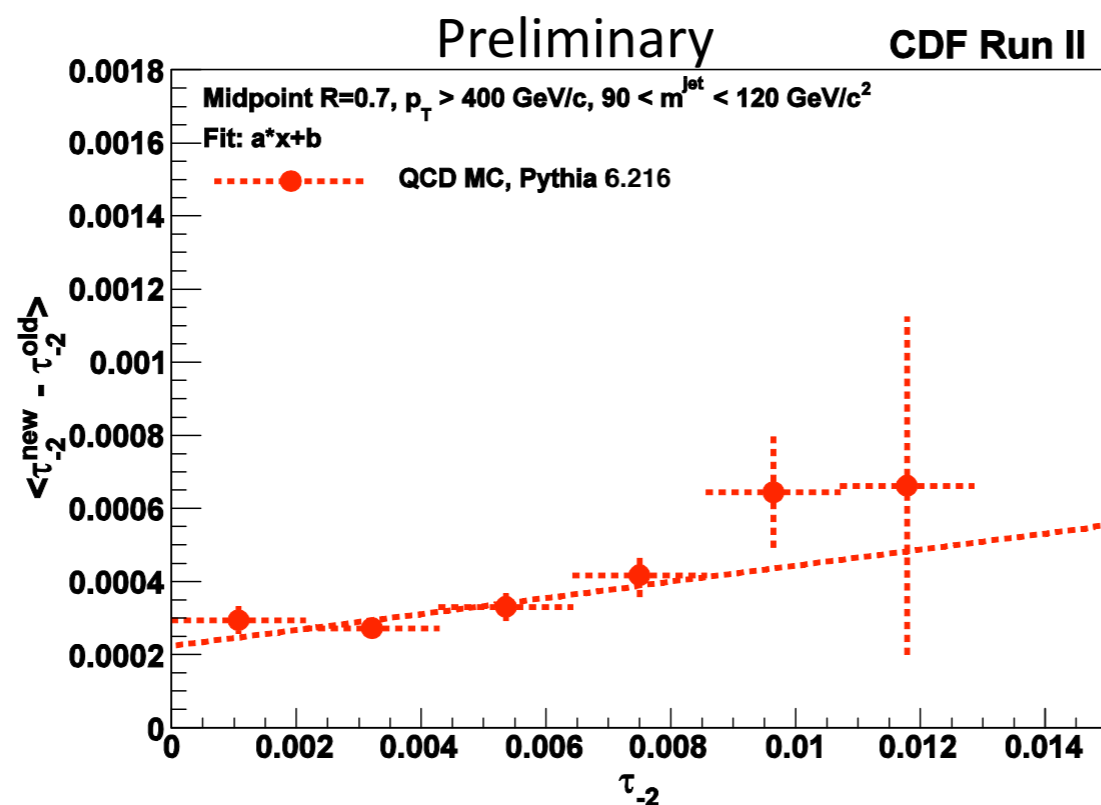


# Data-driven method of pile-up correction for angularity

- Angularity:



$$\sum_{i \in R^{90^\circ}} \frac{\delta m_i^2}{2m_J^2} \left( \frac{2^a m_J}{p_J} \theta_i^{-a} \oplus \tau_a^J \right)$$



# Data-driven method of pile-up correction for planar flow

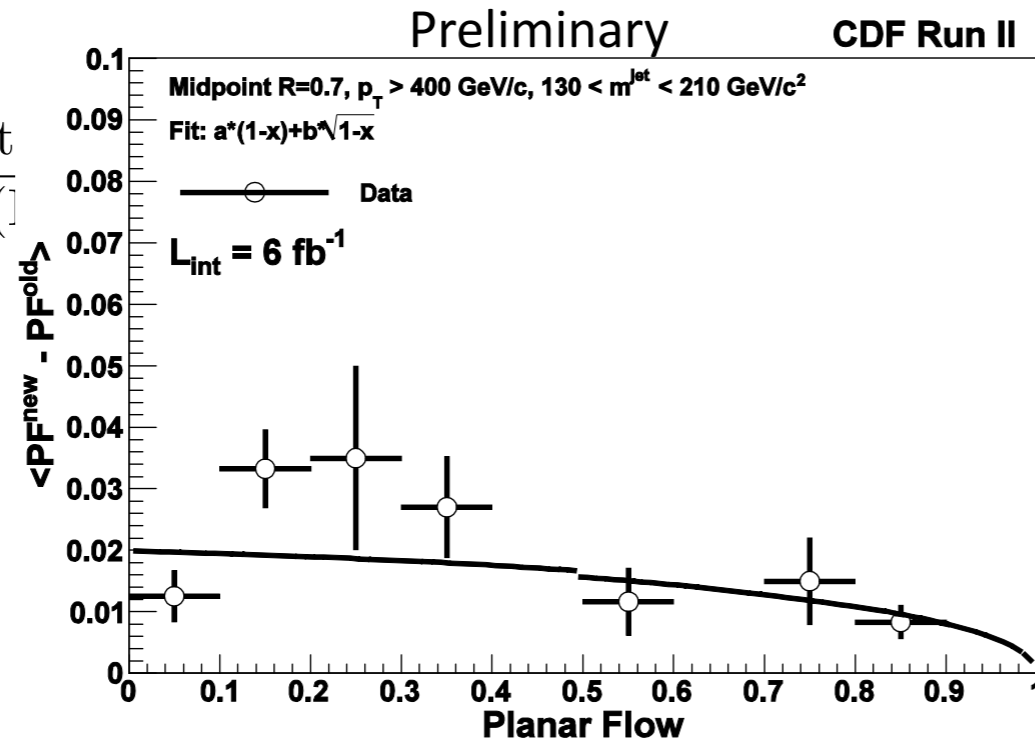
---

● **PF:**  $Pf = 4 \frac{\det(\mathbf{I}_E)}{\text{tr}(\mathbf{I}_E)^2} = \frac{4\lambda_1\lambda_2}{(\lambda_1 + \lambda_2)^2}, \quad \mathbf{I}_E = p_0 \sigma_0 + p_x \sigma_x + p_z \sigma_z, \quad p_0 \simeq \frac{m_J}{\sqrt{2} P_J}$

$$\Delta Pf = \frac{\sqrt{2} P_J}{m_J} \left[ (1 - Pf) \delta p_0 \oplus \sqrt{1 - Pf} \delta p_i \right].$$

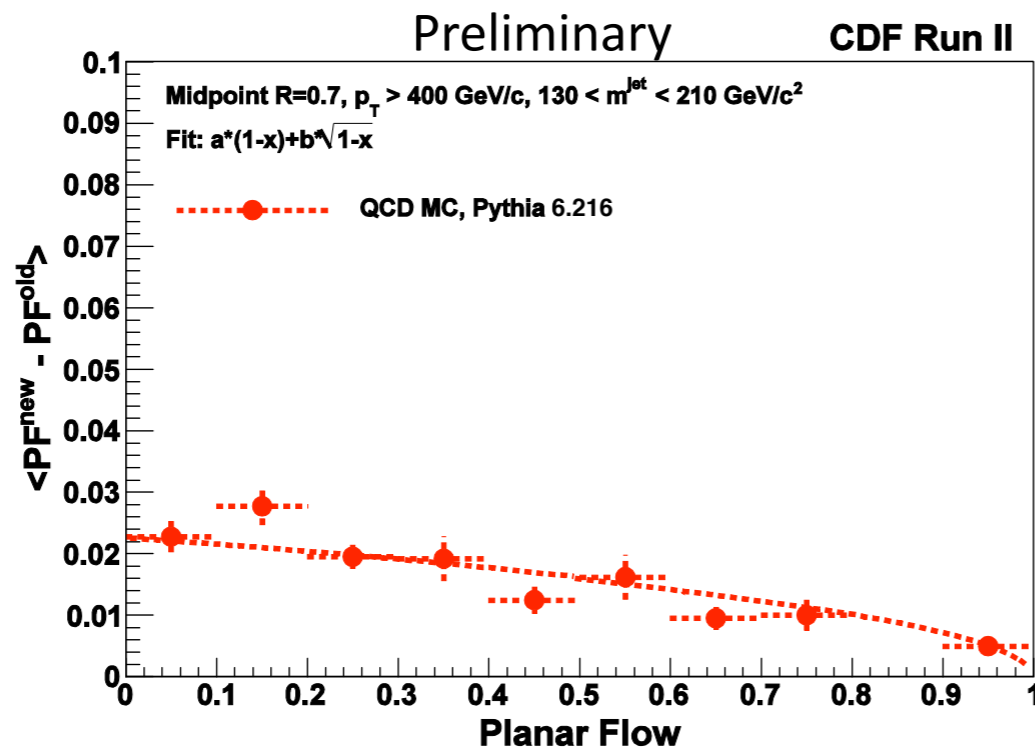
# Data-driven method of pile-up correction for planar flow

● PF:  $Pf = 4 \frac{\det}{\text{tr}(\dots)}$



$$p_0 \simeq \frac{m_J}{\sqrt{2} P_J}$$

$$\left[ \delta p_i \right]$$



# Classification of LO jet shapes (brief)

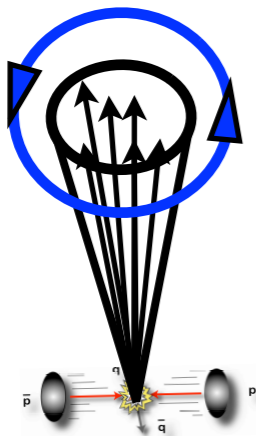
*G. Gur-Ari, M. Papucci & GP, arXiv:1101.xxxx;*

$$I_{i_1 \dots i_n} = \int d^2 x \varepsilon(x) x_{i_1} \cdots x_{i_n} .$$

$$I_w^{kl} = \sum_{i \in \text{particles}} E_i \frac{p_{i,k}^\perp}{E_i} \frac{p_{i,l}^\perp}{E_i} \approx \sum_{i \in \text{particles}} E_i \theta_i f_k(\phi_i) \theta_i f_l(\phi_i) ,$$

$\phi$  is the azimuthal angle, and  $f_1(\phi) = \cos(\phi)$ ,  $f_2(\phi) = \sin(\phi)$ .

$$I_{k_1, \dots, k_n} \equiv \int d^2 x \varepsilon(x) x_{k_1} \cdots x_{k_n} = \frac{1}{E_J} \sum_{i \in \text{particles}} E_i x_{k_1}^{(i)} \cdots x_{k_n}^{(i)} .$$



invariance under the little group  $SO(2)$  (same w splitting function of QCD)

$$I_0 = 1, \quad I_1 = 0, \quad I_{ii} \approx \frac{m_J^2}{E_J^2} .$$

Next, consider a tensor product  $I_2 \otimes I_2$ . There are three nontrivial scalars one may construct,

$$I_{ii} I_{jj}, \quad I_{ij} I_{ij}, \quad \epsilon_{ij} \epsilon_{kl} I_{ik} I_{jl} .$$

Of these, only two are independent, since

$$\epsilon_{ij} \epsilon_{kl} I_{ik} I_{jl} = 2(I_{ii} I_{jj} - I_{ij}^2) = 2 \det I \propto \text{Pf}$$

# Classification of LO jet shapes (brief)

$$I_{iijj} = \frac{1}{E_J} \sum_{i \in \text{particles}} E_i \theta_i^4 \propto \tau_{-2} \quad I_{iijjkk} = \frac{1}{E_J} \sum_{i \in \text{particles}} E_i \theta_i^6 \propto \tau_{-4}$$

At the next order we find  $I_2 I_4$ ,  $(I_3)^2$ , and  $I_8$ , with the following independent contractions:

$$I_2 I_4 : \epsilon_{ij} \epsilon_{kl} I_{ik} I_{jlm} , \epsilon_{ij} I_{ik} I_{jkl}$$

$$(I_3)^2 : \epsilon_{ij} \epsilon_{kl} I_{ikm} I_{jlm} , I_{ijk} I_{ijk}$$

$$I_8 : I_{iijjkkll}$$

$R \backslash \Delta\epsilon$	1	2	3	4
2	$I_2$	-	-	-
4	$I_4$	$(I_2)^2$	-	-
6	$I_6$	$I_2 I_4, (I_3)^2$	$(I_2)^3$	-
8	$I_8$	$I_2 I_6, I_3 I_5, (I_4)^2$	$I_2 (I_3)^2, (I_2)^2 I_4$	$(I_2)^4$

# Zernike polynomials

$$\varepsilon(r, \phi) = \frac{a_{0,0}}{R^2} + \frac{1}{R^2} \sum_{n=1}^{\infty} \sum_{\substack{0 \leq m \leq n, \\ n-m \text{ even}}} \left[ a_{n,m} R_n^m \left( \frac{r}{R} \right) \cos(m\phi) + a_{n,-m} R_n^m \left( \frac{r}{R} \right) \sin(m\phi) \right],$$

where  $R_n^m(\rho)$  are a set of polynomials of degree  $n$  respecting the orthogonality condition

$$\int_0^1 d\rho \rho R_n^m(\rho) R_{n'}^m(\rho) = \frac{1}{2n+2} \delta_{n,n'}.$$

$$\frac{m_J^2}{E_J^2} = \frac{\pi}{6} R^2 (a_{2,0} + 3a_{0,0}),$$

$$\frac{8s}{E_J} \tau_{-2} = \frac{\pi}{30} R^4 (a_{4,0} + 5a_{2,0} + 10a_{0,0}),$$

$$\frac{32s}{E_J} \tau_{-4} = \frac{\pi}{140} R^6 (a_{6,0} + 7a_{4,0} + 21a_{2,0} + 35a_{0,0}),$$

$$(\text{Pf} - 1) \frac{m_J^4}{E_J^4} = \frac{\pi^2}{36} R^4 (a_{2,2}^2 - a_{2,-2}^2).$$

## Rotation Moment Invariants for Recognition of Symmetric Objects

Jan Flusser, *Senior Member, IEEE*, and Tomáš Suk



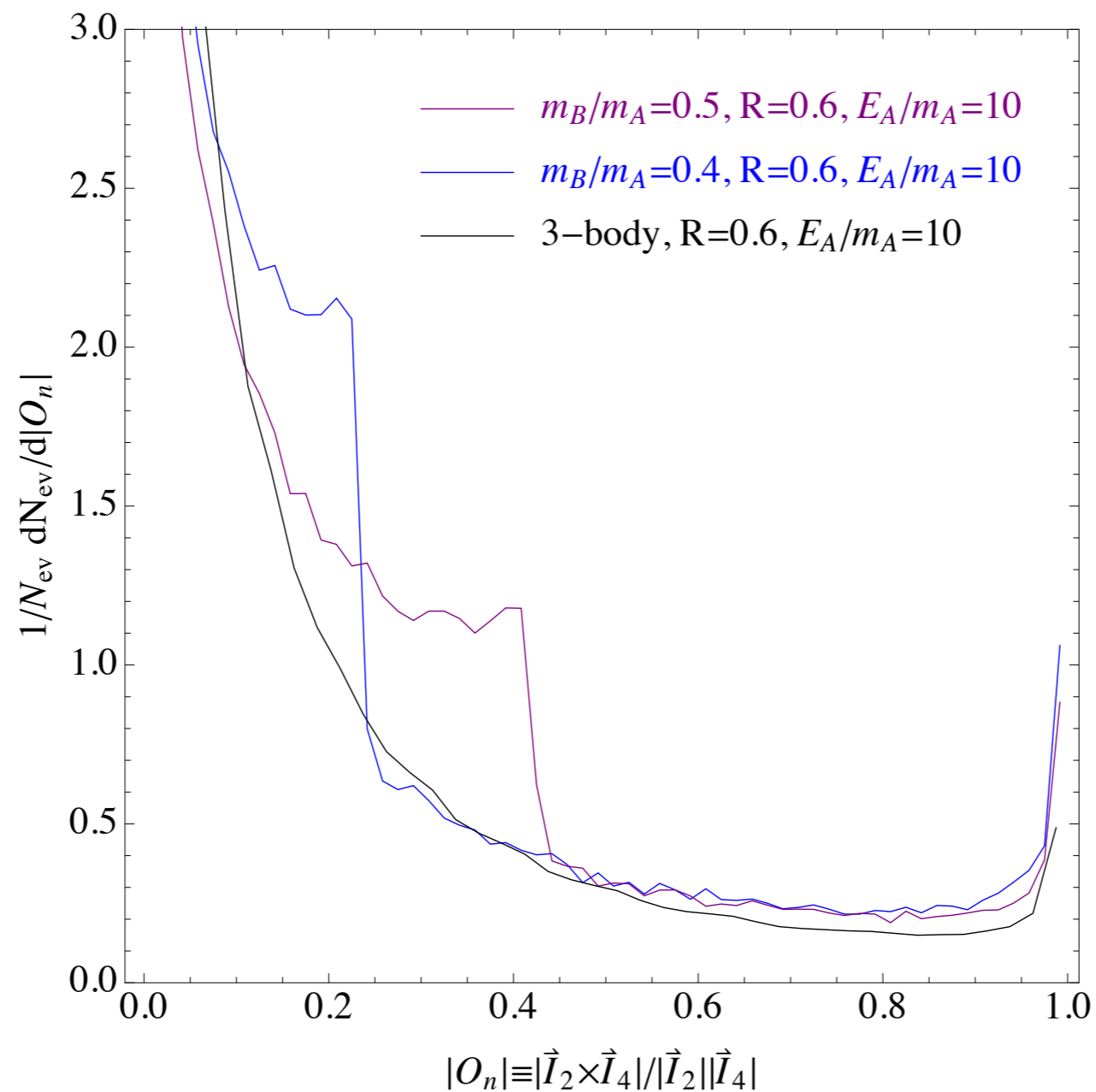
Fig. 1. Test trademarks (from left to right): Mercedes-Benz, Mitsubishi, Recycling, Fischer, and Woolen Stuff.

**Abstract**—In this paper, a new set of moment invariants with respect to rotation, translation, and scaling suitable for recognition of objects having  $N$ -fold rotation symmetry are presented. Moment invariants described earlier cannot be used for this purpose because most moments of symmetric objects vanish. The invariants proposed here are based on complex moments. Their independence and completeness are proven theoretically and their performance is demonstrated by experiments.



# The pseudo scalar jet shape variable?

$$\mathcal{O} = 2\epsilon_{ij}I_{ik}I_{jkm} = 2\text{Tr}(I_2\epsilon I_4') = \epsilon_{ij}I_{2,i}I_{4,j} = \vec{I}_2 \times \vec{I}_4,$$



# Some Interpretation of CDF's di-mass boosted jet excess

---

- Simplest explanation is QCD:  $R_{\text{mass}} \equiv \frac{n_B n_C}{n_A n_D} = 1$ ,  
not coming from PDF, since the ratio is close to unity.  
(thanks to S. Ellis for questioning)
- Requires 7-14 fb of hadronic top equivalence Xsec.
- Assuming new source of tops, tension with “SL” sample is  $\sim 1.4\sigma$
- Pf: Deviation from MC is reduced when looking at new Pythia,  
MG/ME+matching & Herwig (however none includes 1- $\rightarrow$ 3 SF).

# Summary

---

- ◆ LHC => new era, boosted massive jets important for studying QCD & NP discoveries.
- ◆ Jet function (gluon emission) gives correct qualitative description of **data** => 2 body physics; quark jets.
- ◆ Angularity distribution further confirmed this description, affected by jet algorithm (due to IR safety issues).
- ◆ Interesting excess of di-massive jet events (not in ones \w MET).
- ◆ Planar flow (3 body) shows larger deviation at large masses.
- ◆ Data driven pile up corrections works, jet-shape classification.