Near subsurface density reconstruction by full waveform inversion in the frequency domain

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Context and position of the problem

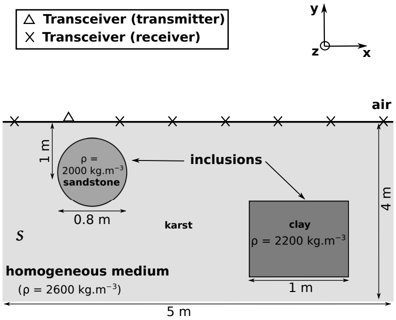


Figure – Seismic reflection configuration

- Goal $\Rightarrow$ subsurface density imaging
- Hypothesis :
  - 2D
  - plane deformation
  - homogeneous Lamé parameters
  - harmonic time dependence
  - Common Mid-Point and Common Off-set
- Forward and inverse problems

- Equation of motion : $(\lambda + \mu)\nabla\vec{\nabla} \cdot \mathbf{u} + \mu\Delta\mathbf{u} + \rho(2\pi f)^2\mathbf{u} = \mathbf{f}\delta(\mathbf{r} - \mathbf{r}_m)$
- Forward problem $\Rightarrow$ $\mathbf{u} = O(\rho, \lambda, \mu)$, non linear (Solved by Finite Element Method)
- Inverse problem $\Rightarrow$ $(\rho, \lambda, \mu) = O^{-1}(\mathbf{u})$, non linear and ill-posed problem
- Solved by a Gauss-Newton optimisation algorithm (robust, easy to use)

Forward and inverse problems

Gauss-Newton Scheme

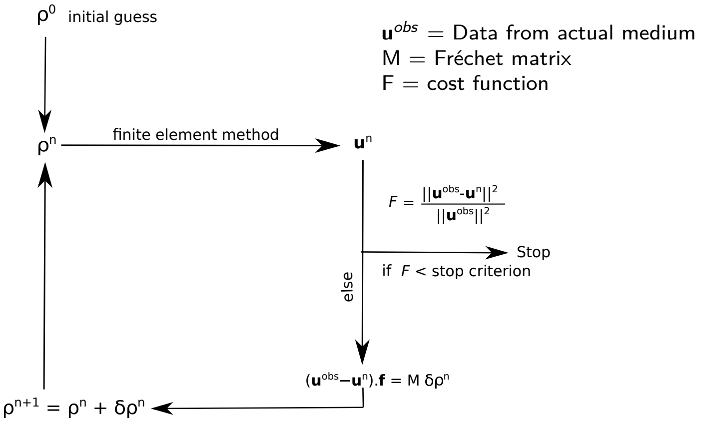


Figure – Gauss-Newton scheme

- At the first order approximation :

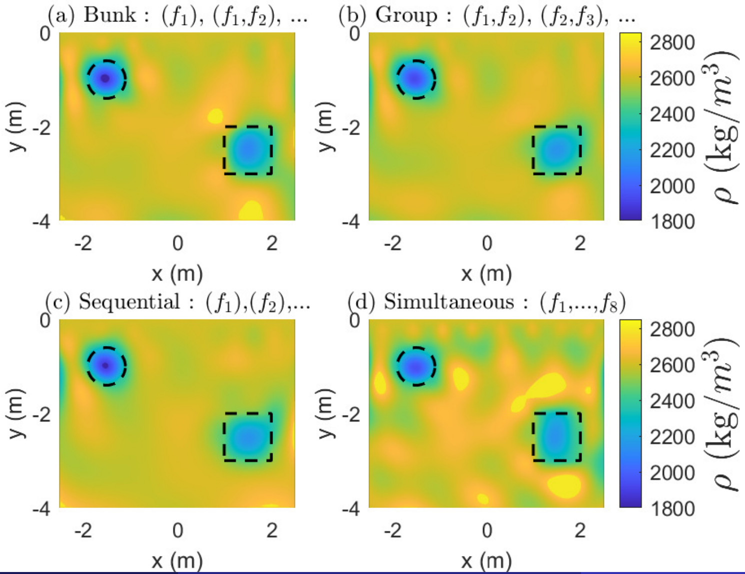
$$(\mathbf{u}^{obs}(\mathbf{r}_l, \mathbf{r}_m, f) - \mathbf{u}^n(\mathbf{r}_l, \mathbf{r}_m, f)) \cdot \mathbf{f}_m \approx 4\pi^2 f^2 \times \int_{\mathcal{S}} \mathbf{u}^n(\mathbf{r}, \mathbf{r}_m, f) \cdot \mathbf{u}^n(\mathbf{r}, \mathbf{r}_l, f)\, \delta\rho^n \,\mathrm{d}\mathcal{S}$$

- 8 frequencies are involved in the inversion process from 400 to 900 Hz [1]
- 4 frequency strategies :
  - Bunk [2] : using (f_1), (f_1,f_2), ..., (f_1, .. ,f_8)
  - Group : using (f_1,f_2), (f_2,f_3), ..., (f_7,f_8)
  - Sequential : using frequencies, one at a time in increasing order
  - Simultaneous : all available frequencies at once

[1] L. Sirgue and R. Pratt, “Efficient waveform inversion and imaging : A strategy for selecting temporal frequencies", in **GEOPHYSICS**

[2] D. Feng, X. Wang and B. Zhang, “A Frequency-Domain Quasi-Newton-Based Biparameter Synchronous Imaging Scheme for Ground-Penetrating Radar With Applications in Full Waveform Inversion", in **IEEE Transactions on Geoscience and Remote Sensing**

Numerical results



Numerical results

Quantitative comparison

- 200000 unknowns and 66 separate data
- $CC(\rho) = \frac{1}{M \times N - 1} \sum_{i=1}^{N} \sum_{j=1}^{M} \left(\frac{\rho_{ij}^{end} - \bar{\rho}^{end}}{\sigma_{\rho^{end}}} \right) \left(\frac{\rho_{ij}^{true} - \bar{\rho}^{true}}{\sigma_{\rho^{true}}} \right)$

	Bunk $(f_1),(f_1,f_2),..$	Group $(f_1,f_2),(f_2,f_3),..$	Sequential $(f_1),(f_2),..$	Simultaneous $(f_1,..,f_2)$
Time (min)	95	47	30	45
Iteration number	28	18	22	6
CC (ρ)	81.15%	83.5%	82.8%	74.5%

Table – Summary table of the results

Conclusion and perspectives

Conclusion

- Group strategy gives the best result and isn't time consuming
- Simultaneous is not satisfying qualitatively and quantitatively

Perspectives

- Multiparameter inversion
- Inversion with real data at laboratory (50 cm & 50 kHz) and geophysical (100 m & 200 Hz) scales
- 3D inversion

Thank you for your attention
Questions ?

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