Full-wave Radar Tomography for Astrophysics: Interior structure of small body vs. carrier effects

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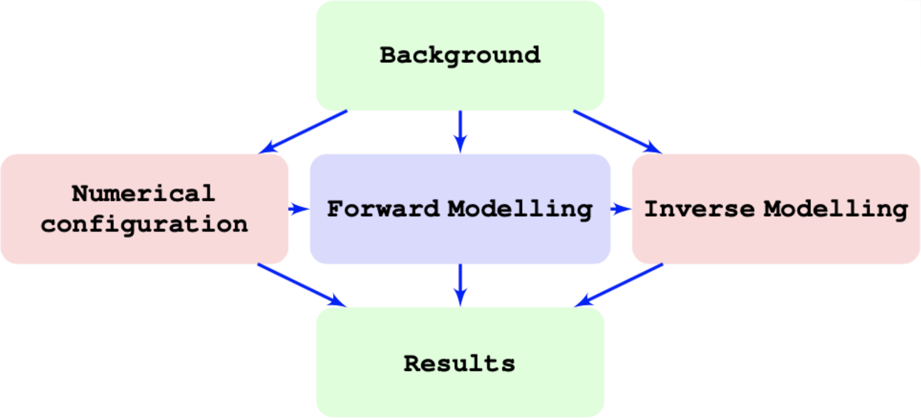
Outline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Background

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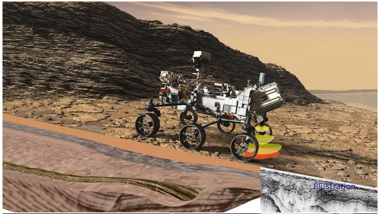
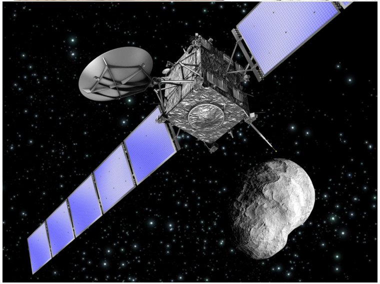
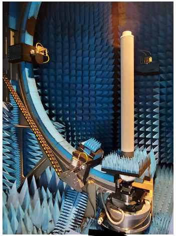


A



B



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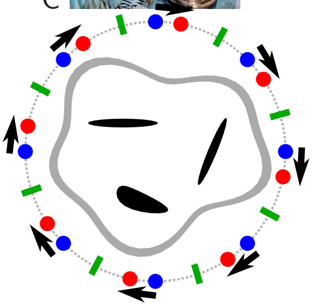


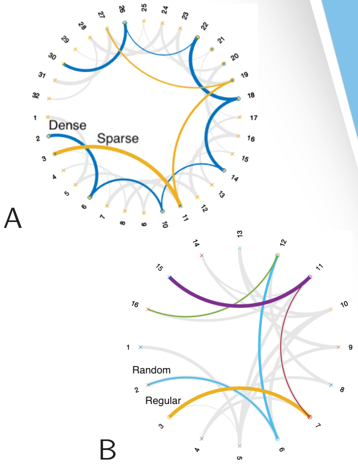
D

[1](A) & (B) credits: NASA.

Numerical configuration

Numerical considerations and Data processing schemes :

- Constant background model $(\tilde{\varepsilon}'_r = 4)$ vs. Detailed model $(\varepsilon'_r = 3, 4, 1)$
- Low frequency (20 MHz) vs. High frequency (60 MHz)
- Low noise (20 dB SNR) vs. High noise (12 dB SNR)
- Spatial point density selection (Sparse vs. Dense)
- Randomised inverse solution averaging vs Regular inverse solution averaging.
- Data Filtering with Truncated Singular Value Decomposition (TSVD) vs. nonfiltered data.



Forward model

Given the total electric field u, we consider a TE-field that satisfies

$$\varepsilon_r' \frac{\partial^2 u}{\partial t^2} + \sigma \frac{\partial u}{\partial t} - \Delta u = \frac{\partial \mathfrak{V}}{\partial t}.$$

The domain is assumed to be decomposed by a triangular mesh $\mathcal{T}$, whose j-th triangle $\mathcal{T}_j$ corresponds to a set indicator function χ_j and a perturbation vector s_j.

$$\varepsilon_{r(j)}' = \tilde{\varepsilon}_r' + \sum_{j=1}^{M} s_j \chi_j,$$

- FETD, FDTD, Spectral method

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[1] L.-I. Sorsa, et al., IEEE TCI, 2020

Linearised forward model

The difference between the actual real relative permittivity distribution ε'_r and its background estimate $\tilde{\varepsilon}'_r$ can be estimated as follows:

$$y_j(t) = \mathfrak{D}\mathfrak{M}[u(t,\vec{p}_i) - \tilde{u}(t,\vec{p}_i)] = \mathfrak{D}\mathfrak{M}\left[\sum_{j=1}^{M} s_j \frac{\partial}{\partial s_j} u(t,\vec{p}_i)\right].$$

The Jacobian matrices $\mathbf{J}_\ell$ is define as

$$(J_\ell)_{i,j} = \mathfrak{D}\mathfrak{M}\left[\frac{\partial}{\partial s_j} u(t_\ell,\vec{p}_i)\right].$$

Hence, one obtains a linearized forward model

$$\mathbf{y}_\ell = \mathbf{J}_\ell \mathbf{x} + \mathbf{n}_\ell \quad \ell = 1, 2, \ldots, N, \quad \text{i.e.,} \quad \mathbf{y} = \mathbf{L}\mathbf{x} + \mathbf{n}$$

[2] M. Takala, et al., IEEE TCI, 2018

Filtering and inversion model

The truncated SVD (TSVD) is used to filter spatio-temporal data and then backpropagated as follows:

$$\mathbf{x}^{\ddagger} = \mathbf{L}^{*}\mathbf{y} \approx \sum_{n=1}^{p} \sigma_n \mathbf{K}_n^{*} \mathbf{u}_n, \quad \text{where} \quad \mathbf{K}_n = \sum_{\ell=1}^{N} \overline{v}_{\ell,n} \mathbf{J}_{\ell}.$$

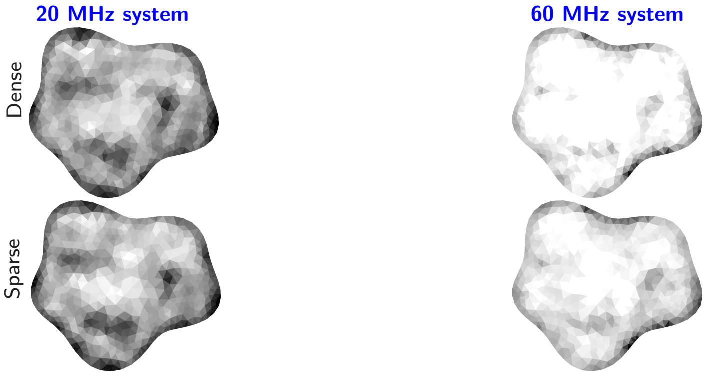
We obtain a Total Variation (TV) regularised solution $\mathbf{x}$ of the linearized forward model as:

$$\mathbf{x}^{\ddagger} = \mathbf{L}^{*}\mathbf{y},$$

$$\mathbf{x}_{l+1} = (\mathbf{L}^{*}\mathbf{L} + \alpha \mathbf{D}^{\mathrm{T}} \Gamma_l \mathbf{D})^{-1} \mathbf{x}^{\ddagger}.$$

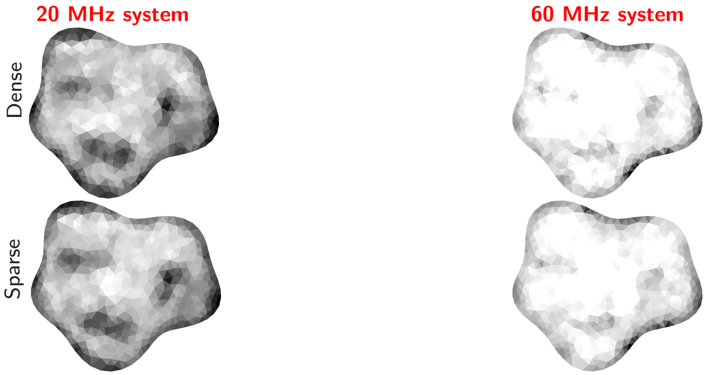
$$D_{i,j} = \beta I_{i,j} + \frac{(2I_{i,j} - 1) \int ds}{\max_{i,j} \int ds}.$$

Results



Reconstruction obtained with the total variation (TV) regularisation for the 20 and 60 MHz centre frequencies.

Results



Reconstruction obtained for the TSVD filtered data with total variation (TV) regularisation for the 20 and 60 MHz centre frequencies.

[3]Y.O. Yusuf, et al., ArXiv Preprint, 2021

Summary

- Numerical modelling of a 2-Dimensional asteroid model.
- 20 and 60 MHz systems with different configurations i.e sparse vs. dense, regular vs. random, low vs. high noise, filtered vs. nonfiltered.
- 53% structural similarity, 40% void overlap, and 60% surface overlap, for the 20 MHz system.
- 67% structural similarity, 30% void overlap, and 70% surface overlap, for the 60 MHz system.
- Lower MSE for both surface and void in the 20 MHz system compared to the 60 MHz system.

Thanks

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