Precision targets at the Z pole

The mission

> The talk will review the physics potential of future lepton (e+e-) colliders at the Z pole, highlighting in particular the electroweak measurements that rely heavily on theory inputs, such as precision calculations. Primary examples are Z-pole measurements of EW parameters, as well as EW precision observables.

- In addition: this is the opening presentation of the workshop
  - Start with basic introduction
  - Well known by most of you, but may be useful anyway to set the scene
    - ➔ Motivation
    - ➔ Present landscape
    - ➔ Experimental and theoretical tasks
    - ➔ …

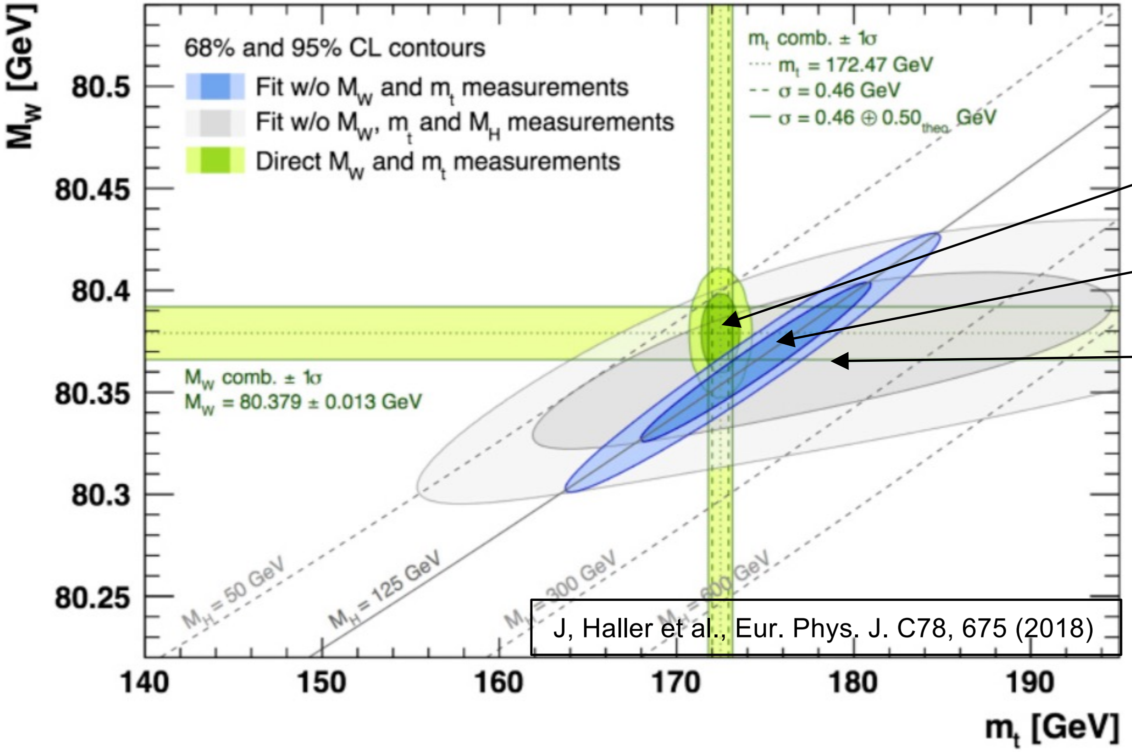
Motivation

- What do we need precision measurements (and related precision calculations) for?
  - With the Higgs boson discovery, the Standard Model was completed
    - The predictivity of the underlying theory was demonstrated (at the 10^{-3} level)
      - ➔ For example, LEP and SLC predicted the top quark and the Higgs boson masses

        … and the top quark (Tevatron) and the Higgs boson (LHC) were found at the right masses !
  - Precision measurements must be matched with SM predictions with the same accuracy (or better)
    - To make optimal use of the experimental data (and money!)
    - To provide sensitivity to "new-physics" phenomena such as
      - ➔ The origin of dark matter
      - ➔ The origin of the baryon asymmetry of the universe
      - ➔ The origin of the neutrino masses (and whoever comes with it, e.g., heavy neutral leptons)
    - Allowing the validity of a future theory (that would explain these new phenomena) to be tested
      - ➔ Of course, the accuracy of the future theory predictions must also match the measurement precision
  - The precision expected at future e^+e^- colliders will reject a multitude of new-physics models
    - Whether the precision measurements agree or deviate from the Standard Model predictions
      - ➔ And will provide a clear vision of what to look for , at high energy and/or feeble couplings

The current landscape

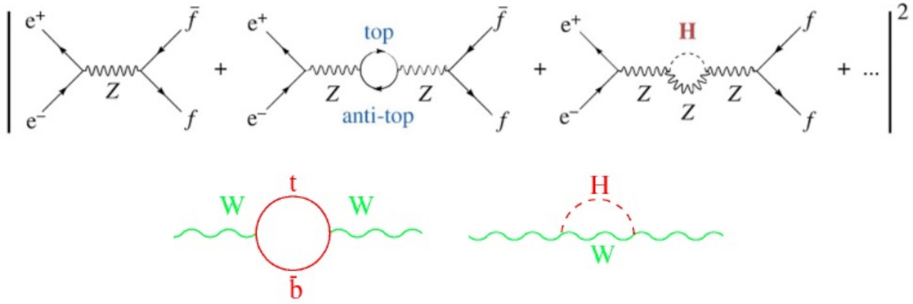
- Without the recent CDF m_W measurement



Direct m_W and m_{top} measurements

SM fit to EW + m_H precision measurements

SM fit to EW precision measurements only



- Precision measurements at the Z pole start to look like the poor relation in this plot!
  - One of the missions of future e^+e^- colliders is to very substantially improve on this front
    - Probably for the last time – the collider must therefore be chosen wisely

The current landscape

- **W mass in numbers (after top and Higgs observation) and related remarks**

Direct measurement

$$m_W = 80.379 \pm 0.012 \text{ GeV}$$

Fit of EWPO at the Z pole + m_H within the SM (and nothing else)

$$\begin{aligned} m_W &= 80.3584 \pm 0.0055_{m_{top}} \pm 0.0025_{m_Z} \pm 0.0018_{\alpha_{QED}} \\ &\pm 0.0020_{\alpha_S} \pm 0.0001_{m_H} \pm 0.0040_{theory} \text{ GeV} \\ &= 80.358 \pm 0.008_{total} \text{ GeV}, \end{aligned}$$

Estimates from S. Heinemeyer

- The theory accuracy (8 MeV) is at the same level as the measurement precision (12 MeV)
  - Note: The CDF precision on m_W reached 9 MeV
- The precision of the W mass direct measurement will improve to less than 0.5 MeV
  - EWPO measurements will have to improve accordingly at future e^+e^- colliders

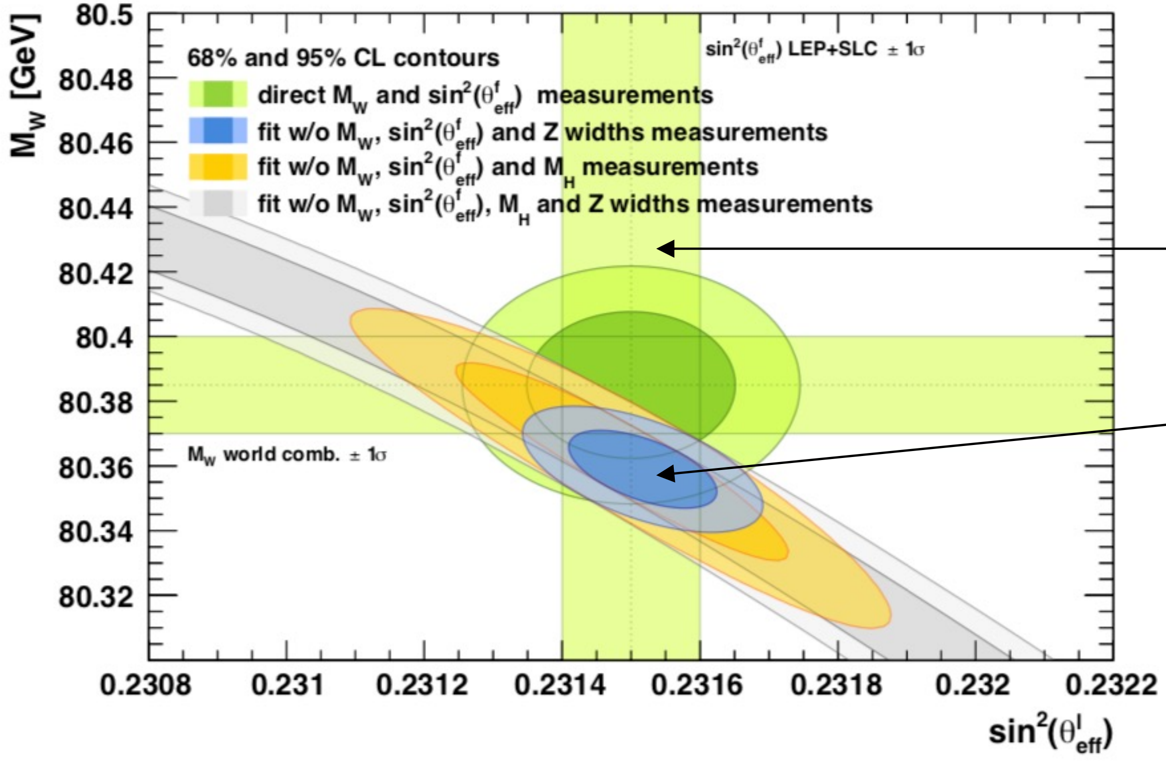
P. Azzurri
G. Wilson

- The theory accuracy is made of two components
  - Parametric uncertainties, which can be improved by better measurements of these parameters
    - ➔ m_Z, m_{top}, $\alpha_{QED}(m_Z)$, $\alpha_S(m_Z)$, m_H : ancillary measurements to be addressed by future e^+e^- colliders as well

      Z + WW + top required!
  - Intrinsic uncertainties, which can be improved by higher-order calculations

The current landscape

- **A (maybe) more useful presentation: the W mass and the weak mixing angle $\sin^2\theta^\ell_{eff}$**



Direct measurement from asymmetries (LEP/SLC)

EWPO fits to SM: $\sin^2\theta^\ell_{eff} = (1-m_W^2/m_Z^2)(1+\Delta\kappa)$

Similar remarks as for m_W

Direct measurement

$$\sin^2\theta_W^{eff} = 0.23153 \pm 0.00016$$

Fit of Z lineshape + m_H within the SM (and nothing else)

$$\begin{aligned}\sin^2\theta_W^{eff} &= 0.231488 \pm 0.000029_{m_{top}} \pm 0.000015_{m_Z} \pm 0.000035_{\alpha_{QED}} \\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000001_{m_H} \pm 0.000047_{theory} \\ &= 0.23149 \pm 0.00007_{total},\end{aligned}$$

Estimates from S. Heinemeyer

Theory and experiment at the Z pole

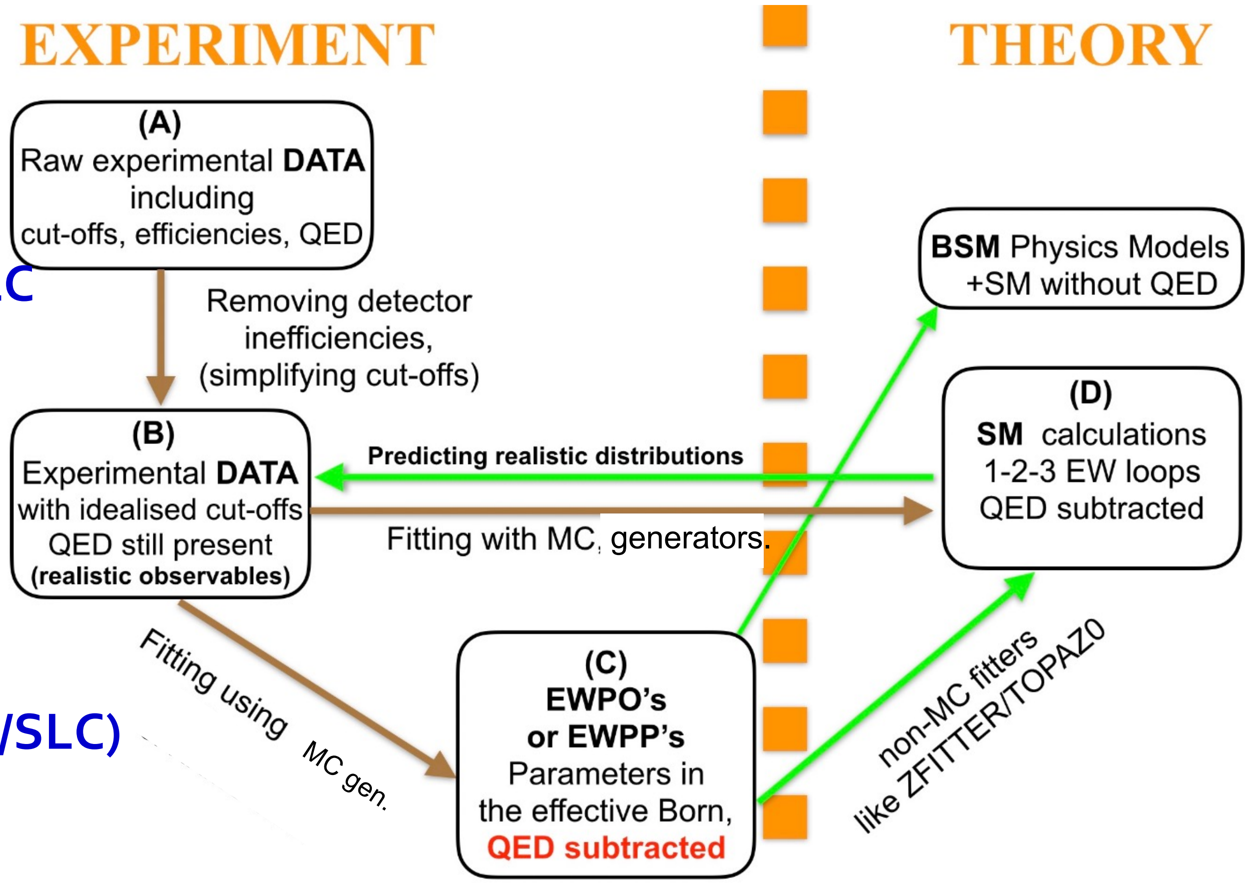
- At LEP/SLC, theory and experiment communicated by way of (pseudo)observables
  - Defined from exp'tal measurements from minimally model-dependent prescriptions
    - Experimental measurements
      - → Centre-of-mass energy, centre-of-mass energy spread
      - → Integrated luminosity, cross sections, angular distributions
    - Pseudo-observables
      - → Z mass, Z width, peak cross section (Z lineshape)
      - → Z partial widths or branching fractions
      - → Polarisation or forward-backward cross-section asymmetries
    - Assumptions (model dependence)
      - → QED is correct (ISR, FSR) ; Weak interaction is v-a ; Effective Born approximation.
      - → Z decays into SM fermion pairs (other decays were searched for exclusively)

- This scheme was well adapted to the situation (and the luminosity) at the time
  - What are the masses of the top quark and the Higgs boson?
  - Is there evidence of new physics in loops?

Theory and experiment at the Z pole

- This may be no longer possible at future e^+e^- colliders (10^3-10^5 larger luminosity)
  - Sophisticated MC event generators will have to be developed, with
    - Multi-loop EW and QCD corrections
    - Soft-photon resummation
    - Multi-body final states
  - QED (approx.) analytic formula @ LEP/SLC
    - May need to be replaced by MC fitting
  - Effective Born approximation
    - Might require re-defined EWPO (EWPP)
    - Might also be no longer valid
  - May have to replace B → C → D → B (LEP/SLC)
    - By direct MC fitting : B → D

https://arxiv.org/abs/1903.09895



- It is assumed in the following that EWPO (EWPP) are available and sound (tbc!)

Theory and experiment at the Z pole

- **Tasks for theory**
  - Identify observables/parameters that contain sensitivity to new phenomena
    - Via loops in γ, Z, W propagators (flavour universal), e.g., S, T, U @LEP/SLC
    - Via boxes and vertices (flavour dependent), e.g., δ_b @ LEP/SLC
    - Via direct long distance propagator effects (universality violation): e.g., new Z′
    - Via mixing with known particles, e.g., Z′/Z mixing, ν/N mixing, …
  - Develop high-precision SM procedures to extract these parameters from measurements
    - Precise (maybe not universal?) QED/QCD Monte Carlo / radiator for ISR/FSR/IFI, …
  - Perform high-precision calculations of these observables/parameters in the SM
    - Precise multi-loop calculations with, e.g., m_Z, G_F, $\alpha_{QED}(0)$ as basic inputs
      - ➔ Also requires high-precision theory to extract ancillary quantities from experimental measurements
        $\alpha_{QED}(m_Z)$, $\alpha_S(m_Z)$, m_{top}, m_b, m_H, etc. to reduce parametric uncertainties
  - Develop sophisticated MC event generators, for direct tests of the theoretical prediction
    - Also needed to remove detector acceptance and selection inefficiencies

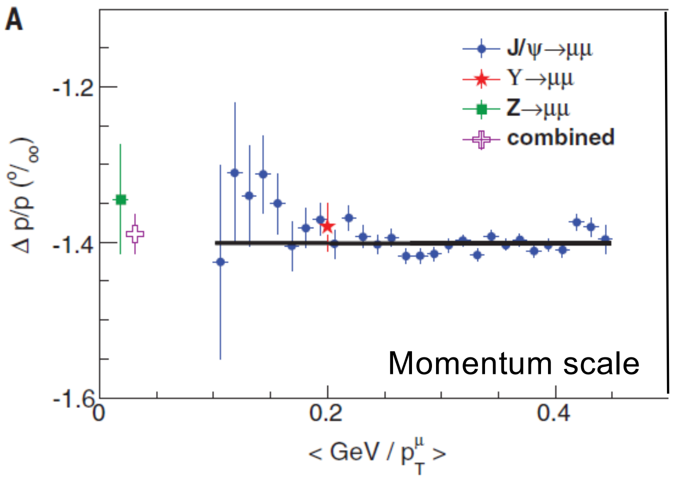
Theory and experiment at the Z pole

- **Tasks for experiment and collider**
  - Maximize the luminosity produced by the collider at the Z pole, w/ clean exp'tal conditions
  - Tune the operation model ($L_{uminosity}$, E_{nergy}, $P_{olarisation}$) for optimal EWPO statistical precision
  - Design ways to accurately measure the centre-of-mass energy and its spread
  - Operate several detectors simultaneously to increase statistics
  - Design the detectors to match the systematic uncertainties with the statistical precision
    - Often requires ancillary measurements to be performed and subtle tricks to be developed

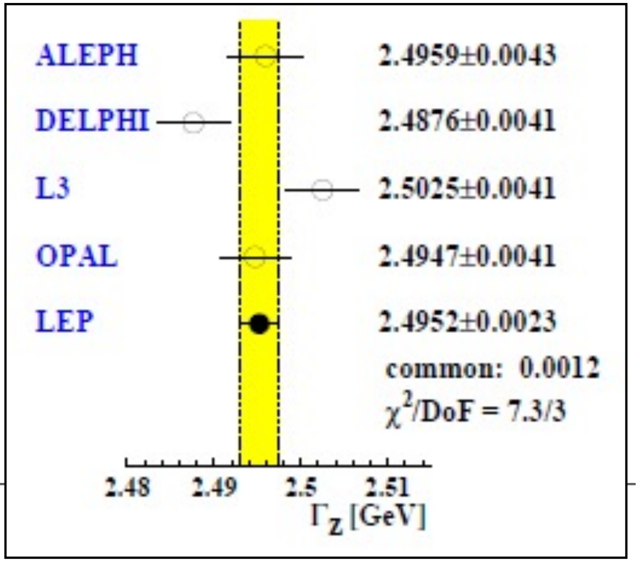
- **Past experience proves that "statistics is the limit" (and that this limit is reached)**
  - Experimental systematic uncertainties are often of statistical nature
    - The analysis of real data provides the needed additional motivation boost for hard work
  - Parametric uncertainties are often of statistical nature
    - If the parameters can be measured independently
  - The plan must be to match intrinsic theoretical uncertainties to the statistical precision
    - Nobody wants to be in the way of a discovery by being the dominant source of uncertainty

Statistics is the limit

- **Recent CDF measurement with full Run2 stat :** m_W = 80433.5 ± 6.4 (stat.) ± 6.9 (syst.) MeV
  - Systematic uncertainty similar to statistical precision !
    - Required 10 years of work and motivation
  - Relies on the precise measurements of J/ψ, Υ, Z masses
    - All measured in e^+e^- colliders (using resonant depolarisation)
  - Measured value inconsistent with previous measurements …
    - Raises questions that will require more work
      - ➔ Or just wait for FCC-ee that will measure m_W 40 times better



- **Z width measurement at LEP:** Γ_Z = 2495.2 ± 1.8 (stat.) ± 1.2 (syst.) MeV
  - Original systematic uncertainty estimate was 20 MeV (1986)
    - Requires hard work and ingenuity from LEP energy WG for 5 years
      - ➔ Until the systematic error was smaller than the statistical precision
        - √s calibration with resonant depolarization (not during physics runs, e^- only)
        - Systematic uncertainties due to tides, rain, train effects in extrapolation



Operation models at the Z pole

- **Two generic configurations**
  - In the core programme of FCC-ee with two interaction points (4 years) : TeraZ
    - 150 ab^{-1} at and around the Z pole – up to 5×10^{12} Z produced, 2×10^{5} times LEP statistics
      - ➔ Instantaneous luminosity ~$4\times10^{36}cm^{-2}s^{-1}$

To be multiplied by 1.7 with 4 interaction points

    - Scan of the Z resonance with 3 energy points – 87.69 GeV, 91.21 GeV, 93.85 GeV
      - ➔ Beam energies corresponding to half-integer spin tunes: precise calibration with resonant depolarization
    - Transverse polarization for ~250 e^+ and e^- non-colliding bunches (out of ~10,000)
      - ➔ Continuous in-situ beam energy calibration for electrons and positrons, much reduced systematic errors
  - Not in the core programme of ILC – layout still in the work : GigaZ
    - About 0.1 ab^{-1} at and around the Z pole – a few 10^{9} Z produced, about 10^{4} times SLC statistics
      - ➔ Instantaneous luminosity ~$2\times10^{33}cm^{-2}s^{-1}$
    - Scan of the resonance with 7 energy points, typically 91.2 GeV, ±1.05 GeV, ±2.1 GeV, ±3.15 GeV
    - Longitudinal polarization: 80% for electrons, (possibly) 30% for positrons
      - ➔ Gives access to A_{LR}, the observable most sensitive to the effective weak mixing angle, $\sin^2\theta^{\ell}_{eff}$

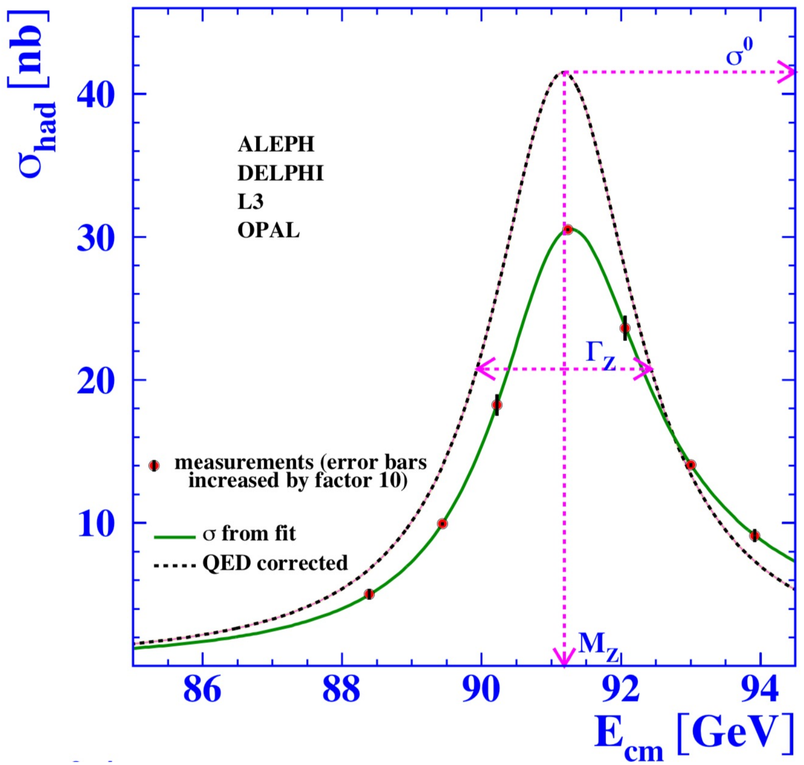
        Partially compensates for the smaller luminosity (for this parameter)

Scan of the Z lineshape: m_Z, Γ_Z, σ^o_{had}

- **Statistical precision sets the scene**

	m_Z	Γ_Z	σ_{had}
FCC-ee	4 keV	4 keV	$< 10^{-6}$
ILC	120 keV	120 keV	$< 10^{-4}$

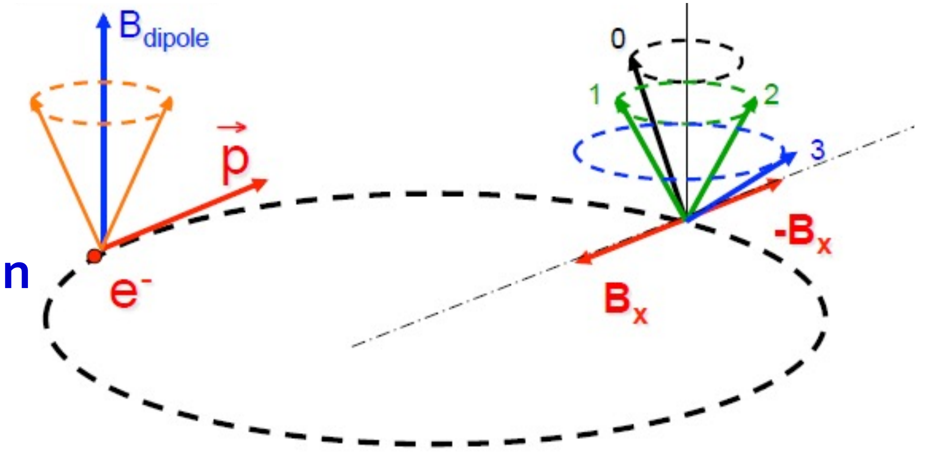
- **Experimentally**
  - m_Z requires absolute determination of √s
  - Γ_Z requires relative (pt-to-pt) determination of √s
    - Also: absolute determination of √s spectrum (spread)
  - σ_{had} requires absolute determination of luminosity



- **Theoretically**
  - High-precision QED prodecures to go from the exp'tal green curve to the pink curve
  - High-precision SM calculations to go from the pink curve to the Z parameters
    - With the statistical precision as a target

Absolute determination of √s at FCC-ee

- **Continuous resonant depolarization to determine the beam energies**
  - Transverse polarization (with wigglers)
  - Spin precession frequency $\nu_0 = E_{beam}/0.4406486$
    - $\nu_0 = 103.5$ at the Z peak (called "spin tune")
  - Kicker with frequency ν provokes sharp depolarization
    - Simulation with CDR FCC-ee layout



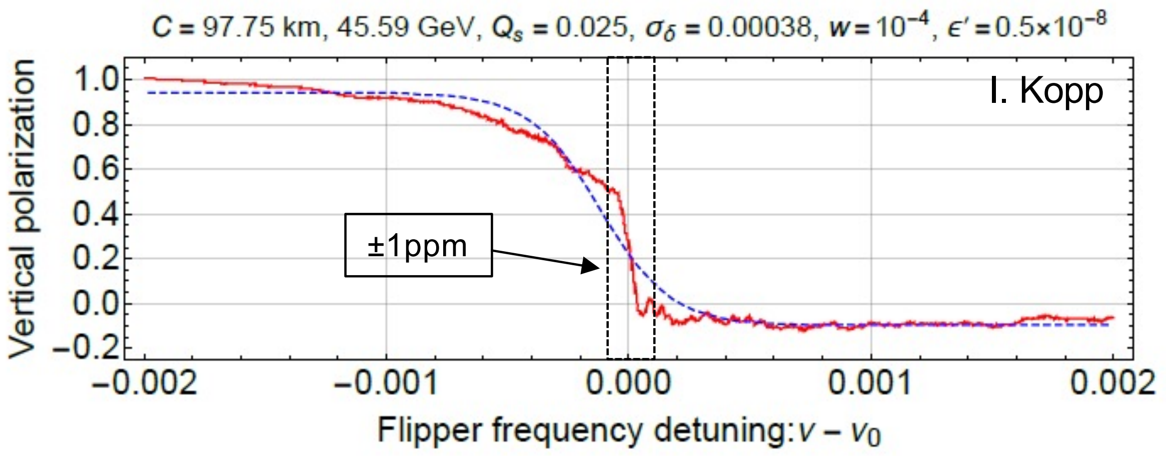


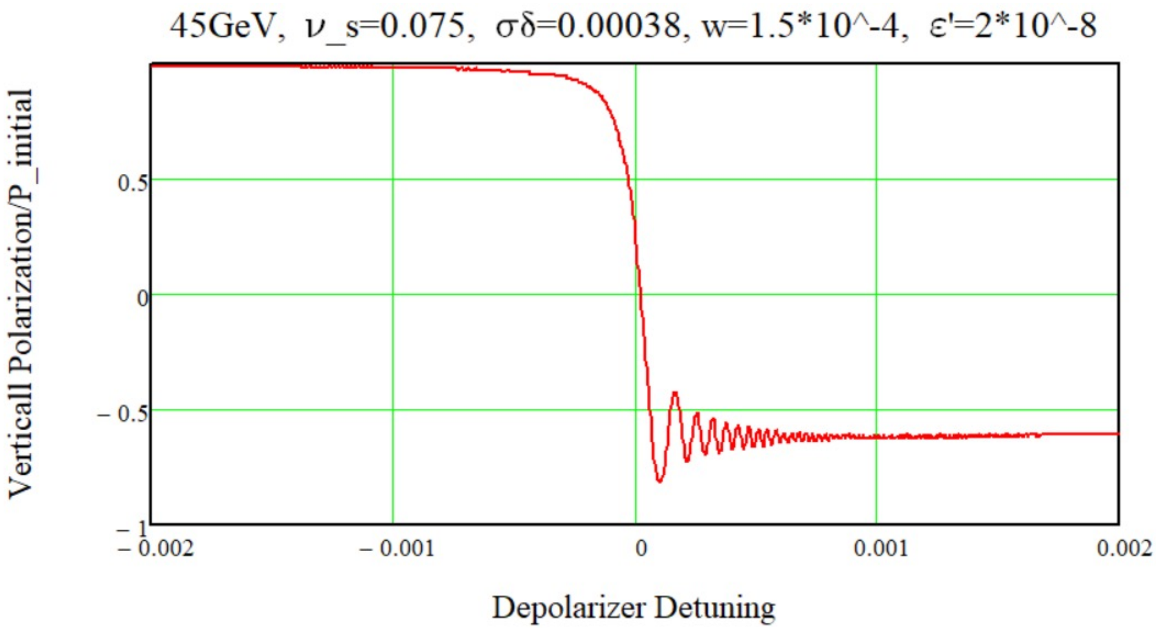
Figure 39. Simulation of a frequency sweep with the depolarizer on the Z pole showing a very sharp depolarization at the exact spin tune value.

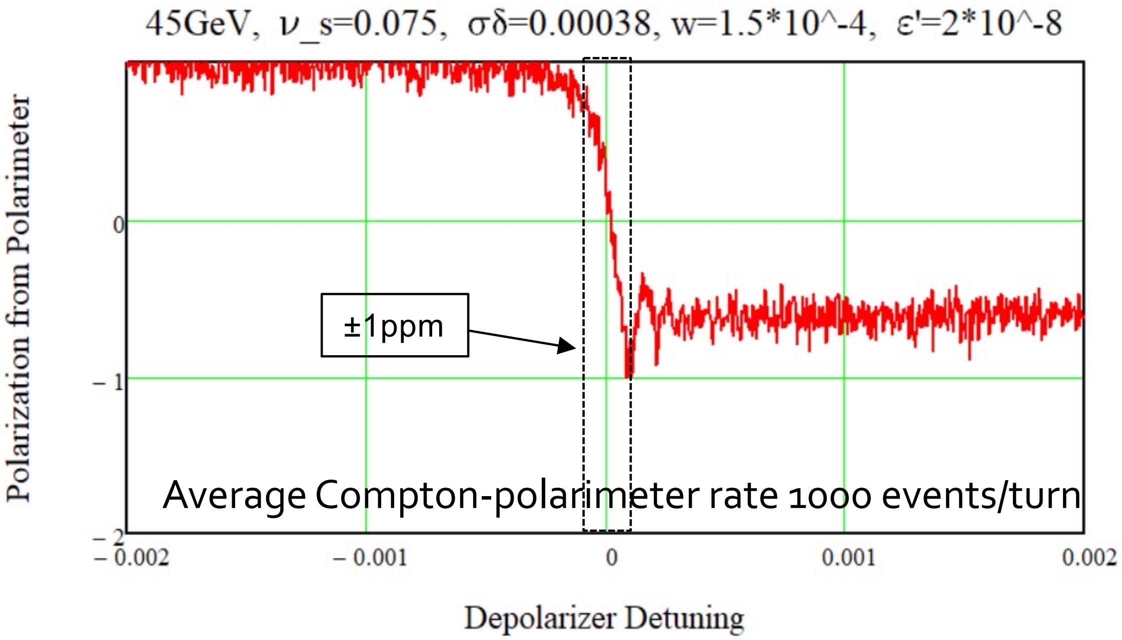
260 seconds sweep of the kicker frequency

- **Reach ppm precision or better on √s**
  - Realistic assumption: < 100 keV
    - Ultimate reach: 10 keV or better ?
- **Crossing angle α: $\sqrt{s} = 2\,E_+E_-\cos\alpha/2$**
  - α (30 mrad) can be measured in situ
    - With $e^+e^- \to \mu^+\mu^-(\gamma)$ events

Absolute determination of $\sqrt{s}$ at FCC-ee

- More recent work presented at the FCC Week 2022 in Paris (I. Koop)
  - Expected polarization = f $(\nu-\nu_0)$ — Measured polarization (simulation)



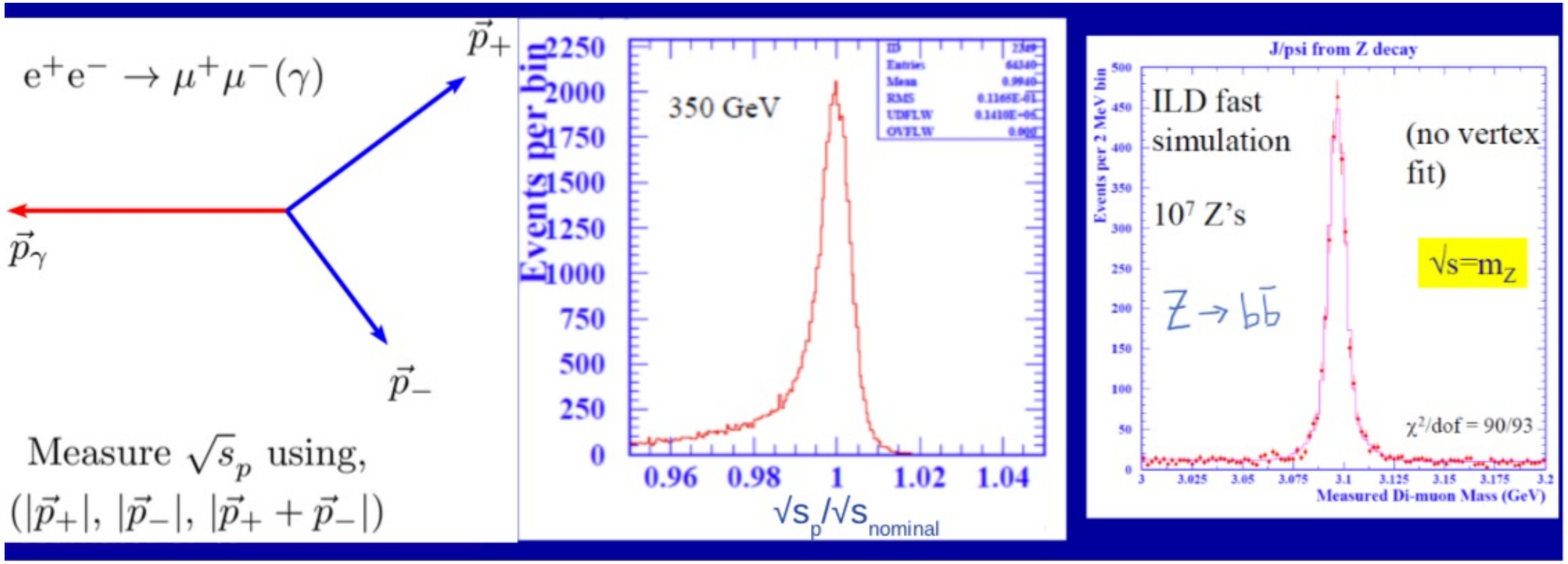


- Precision of 0.1 ppm (0.00001) on $\nu-\nu_0$ does not seem out of reach
  - ➔ Would corresponds to about 10 keV on m_Z

Absolute determination of √s at ILC

G. Wilson

- Use “calibrated” dimuon events $e^+e^- \to \mu^+\mu^- (\gamma)$
  - Use $E_+ + E_- + p_{miss}$ as an estimator for √s – requires excellent momentum resolution



  - Tie detector momentum scale to known masses (a la CDF): J/ψ, K^0, Λ – known to ~2 ppm
    - Expect ~ppm statistical uncertainty on p-scale with 1.2M $J/\psi \to \mu^+\mu^-$ events (full statistics)
  - Ultimate (systematic) target for √s determination at the Z peak : 200 keV
    - Requires complete systematic study to demonstrate the feasibility of the method

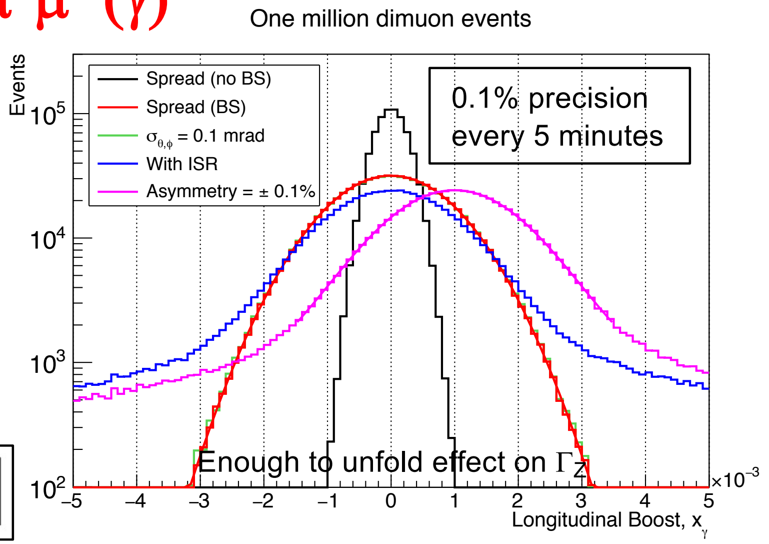
√s spread and point-to-point determination at FCC-ee

- In situ measurement with the same dimuon events $e^+e^- \rightarrow \mu^+\mu^- (\gamma)$
  - Energy spread = relative longitudinal boost $x_\gamma = p_z^{miss} / \sqrt{s}$
    - Full spectrum obtained from μ directions and E,p conservation
      - ➔ Method also provides absolute directions wrt the beams
      - ➔ Requires ~0.1 mrad angular resolution or less
      - ➔ Good ISR description needed

$$x_\pm = \frac{\mp \sin\theta^\mp \sin\varphi^\mp}{\sin\theta^+ \sin\varphi^+ - \sin\theta^- \sin\varphi^-}$$

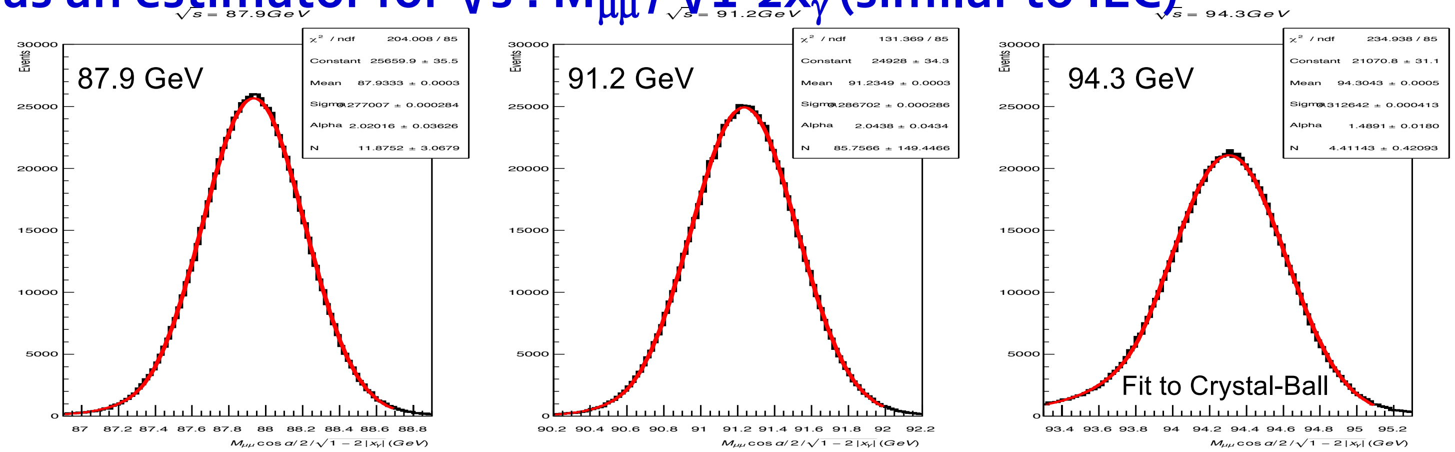
$$x_\gamma = -\frac{x_+ \cos\theta^+ + x_- \cos\theta^-}{\cos(\alpha/2) + |x_+ \cos\theta^+ + x_- \cos\theta^-|}$$

$$\alpha = 2 \arcsin\left[\frac{\sin(\varphi^- - \varphi^+) \sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+}\right]$$



  - Use ISR-corrected dimuon mass as an estimator for √s : $M_{\mu\mu} / \sqrt{1-2x_\gamma}$ (similar to ILC)

- Target for pt-to-pt uncertainty: < 10 keV
  - ➔ Would translate to ~5 keV error on Γ_Z
  - ➔ Present estimate 40 keV (25 keV on Γ_Z)
- Systematic uncertainty: ISR description
  - ➔ Shift of the peak by ~30 MeV [*]
  - ➔ Multi-photons, angular distribution, …
- Complete case study required



[*] It is therefore not clear that this method can be used for an absolute determination of √s

https://arxiv.org/abs/1909.12245

Absolute luminosity determination

- Measured with low angle Bhabha scattering $e^+e^- \to e^+e^-$
  - Statistical uncertainty (10^{-6} at FCC-ee, 3×10^{-5} at ILC) seems impossible to reach
  - Theoretical uncertainty at LEP: 0.061%, recently reduced to 0.037% https://arxiv.org/abs/1912.02067
    - 0.061% deemed adequate for ILC – no additional work required ☺
    - Achievable target for FCC-ee is 0.01% (10^{-4}) – actual calculation needed https://arxiv.org/abs/1812.01004
  - Measuring the Bhabha rate at the 10^{-4} level is experimentally challenging
    - Construction of luminometer inner radius at the μm level https://arxiv.org/abs/2107.12837

- The point-to-point luminosity uncertainty is at least one order of magnitude smaller
  - σ^0_{had} is the only observable affected by this 10^{-4} limitation
    - And therefore, the number of light neutrino species N_ν [*]

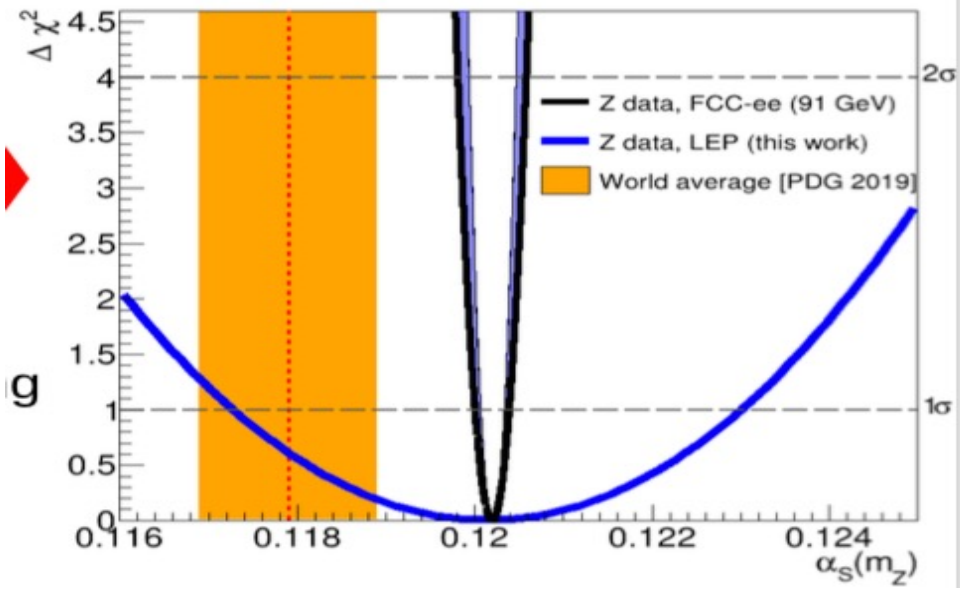
$$N_\nu \left(\frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}\right)_{SM} = \left(\frac{12\pi}{m_Z^2}\frac{R^0_\ell}{\sigma^0_{had}}\right)^{\frac{1}{2}} - R^0_\ell - 3 - \delta_\tau$$

- Alternative absolute luminosity measurement with large angle $e^+e^- \to \gamma\gamma$ events
  - Statistical uncertainty of 2×10^{-5} at FCC-ee – Feasibility study synergistic with R_ℓ (next slide)
  - Potential theory uncertainty: 10^{-5} – NNLO calculation required https://arxiv.org/abs/1906.08056

[*] N_ν can be also measured above the Z pole with the ratio $\sigma(\nu\nu\gamma)/\sigma(\mu\mu\gamma)$

Measurement of $R_\ell=\Gamma_{had}/\Gamma_\ell$ and $\alpha_S(m_Z)$ determination

- **Relative measurement independent of luminosity determination**
  - At FCC-ee, relative statistical precision of 3×10^{-6} for each of R_e, R_μ and R_τ
    - Sensitive to new physics (test of lepton universality and quark-lepton universality)
    - In the SM, leads to a determination of $\alpha_S(m_Z)$ through Γ_{had}
  - At LEP, $R_\ell = 20.767 \pm 0.025$ yielded $\alpha_S(m_Z) = 0.1196 \pm 0.0028$ (exp.) ± 0.0009 (th.)
    - Main experimental systematic uncertainty came for lepton acceptance ($\cos\theta_{cut} < 0.95$, $\varepsilon \sim 90\%$)
  - At FCC-ee, the lepton acceptance must be better controlled
    - Acceptance down to 100 mrad ($\cos\theta_{cut} < 0.995$) ?
    - Clean design of the low angle detector fiducial
      - ➔ Target precision of 0.001 for R_ℓ
  - Calls for a reduction of theory error by a factor > 4
    - Computing missing α_S^5, α^3, $\alpha\alpha_S^2$, $\alpha^2\alpha_S$ terms
      - ➔ $\alpha_S(m_Z) = 0.11960 \pm 0.00014$ (exp.) ± 0.00022 (th.) [*]
  - Level of details in the dilepton generator
    - To improve accordingly



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[*] $\alpha_S(m_W)$ can be also measured from leptonic and hadronic W decays

ers

https://arxiv.org/abs/2005.04545

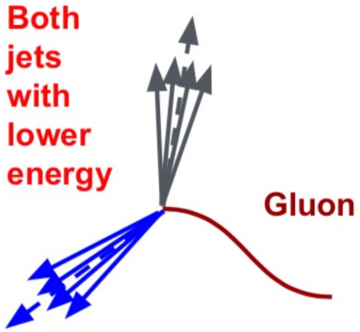
Measurement of $R_{b\,(c,s)} = \Gamma_{b\,(c,s)} / \Gamma_{had}$

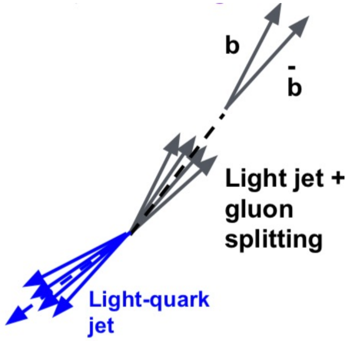
- **Largest expected improvement from FCC-ee with respect to LEP (> 2000)**
  - Factor 500 in statistical precision (+ no R_s measurement at LEP)
  - Factor 5 in beam pipe radius (10 mm for FCC-ee, 15 mm for ILC)
  - Much developments in flavour tagging algorithms from LHC
    - Relative stat. precision on R_b of 1.5×10^{-6} with 7×10^{11} $Z\rightarrow bb$ events !
  - R_b sensitive to new physics via a specific top/W vertex correction
- **Largest improvement of (theoretical) uncertainties needed**
  - Gluon radiation, gluon splitting, decay models, b,c fragmentation …
    - Huge available statistics to study such effects: define strategies
      - ➔ Improve the QCD calculations and the MC generators accordingly

LEP uncertainties

Source	R_b^0 $[10^{-3}]$	R_c^0 $[10^{-3}]$
statistics	0.44	2.4
internal systematics	0.28	1.2
QCD effects	0.18	0
$B(D \rightarrow \text{neut.})$	0.14	0.3
D decay multiplicity	0.13	0.6
B decay multiplicity	0.11	0.1
$B(D^+ \rightarrow K^-\pi^+\pi^+)$	0.09	0.2
$B(D_s \rightarrow \phi\pi^+)$	0.02	0.5
$B(\Lambda_c \rightarrow p K^-\pi^+)$	0.05	0.5
D lifetimes	0.07	0.6
B decays	0	0
decay models	0	0.1
non incl. mixing	0	0.1
gluon splitting	0.23	0.9
c fragmentation	0.11	0.3
light quarks	0.07	0.1
beam polarisation	0	0
total correlated	0.42	1.5
total error	0.66	3.0

https://arxiv.org/abs/hep-ex/0509008





Summary: Theory inputs for Z lineshape observables

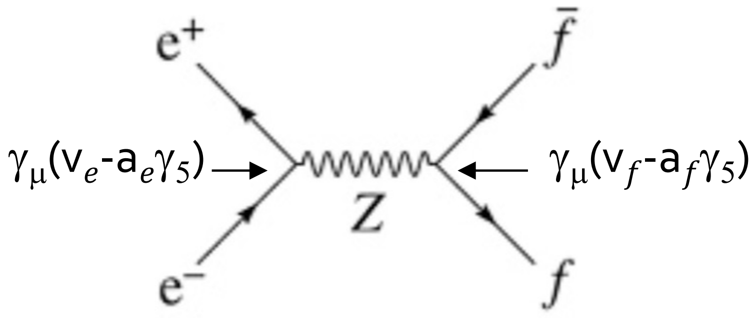
- Numbers are given here for FCC-ee (best prospects)

Observables	Present value	FCC-ee stat.	FCC-ee current syst.	FCC-ee ultimate syst.	Theory input (not exhaustive)
m_Z (keV)	**91187500 ± 2100**	4	100	10 ?	Lineshape QED unfolding Relation to measured quantities
Γ_Z (keV)	**2495500 ± 2300 [*]**	4	25	5 ?	Lineshape QED unfolding Relation to measured quantities
σ^0_{had} (pb)	**41480.2 ± 32.5 [*]**	0.04	4	0.8	Bhabha cross section to 0.01% $e^+e^- \to \gamma\gamma$ cross section to 0.002%
N_ν ($\times 10^3$) from σ_{had}	**2996.3 ± 7.4**	0.007	1	0.2	Lineshape QED unfolding $(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{SM}$
R_ℓ ($\times 10^3$)	**20766.6 ± 24.7**	0.04	1	0.2 ?	Lepton angular distribution (QED ISR/FSR/IFI, EW corrections)
$\alpha_s(m_Z)$ ($\times 10^4$) from R_ℓ	**1196 ± 30**	0.1	1.5	0.4 ?	Higher order QCD corrections for Γ_{had}
R_b ($\times 10^6$)	**216290 ± 660**	0.3	?	< 60 ?	QCD (gluon radiation, gluon splitting, fragmentation, decays, …)

- **And also sophisticated and state of the art MC generators (signal and backgrounds)**
  - **Plus, maybe, redefined EW Precision Parameters (EWPP) and extraction procedures ?**

Asymmetries and $\sin^2\theta^{\ell}_{\mathrm{eff}}$

- **Parity-violating (L ≠ R) weak couplings at the Z pole**



$a_f = \pm\, 1/2$

$v_f = a_f(1 - 4|Q_f|\,\sin^2\theta^{\ell}_{\mathrm{eff}})$

Asymmetry parameter

$$\mathcal{A}_f = \frac{2 a_f v_f}{v_f^2 + a_f^2}$$

Polarisation parameter

$$P = \frac{P_{e^-} - P_{e^+}}{(1 - P_{e^-} P_{e^+})}$$

- **Longitudinally polarized incoming beams**

$$A_{LR} = \frac{\sigma_{tot}(P) - \sigma_{tot}(-P)}{\sigma_{tot}(P) + \sigma_{tot}(-P)} = P\mathcal{A}_e$$

$$A_{FB}^{pol\,f} = \frac{\sigma_{Ff}(P) - \sigma_{Ff}(-P) - [\sigma_{Bf}(P) - \sigma_{Bf}(-P)]}{\sigma_{tot\,f}(P) + \sigma_{tot\,f}(-P)} = \frac{3}{4} P\mathcal{A}_f$$

- **Longitudinally unpolarized beams produce longitudinally polarized fermions (Z couplings)**
  - Longitudinal polarization of the τ's obtained from the decay particle spectrum (π, ρ, etc.)

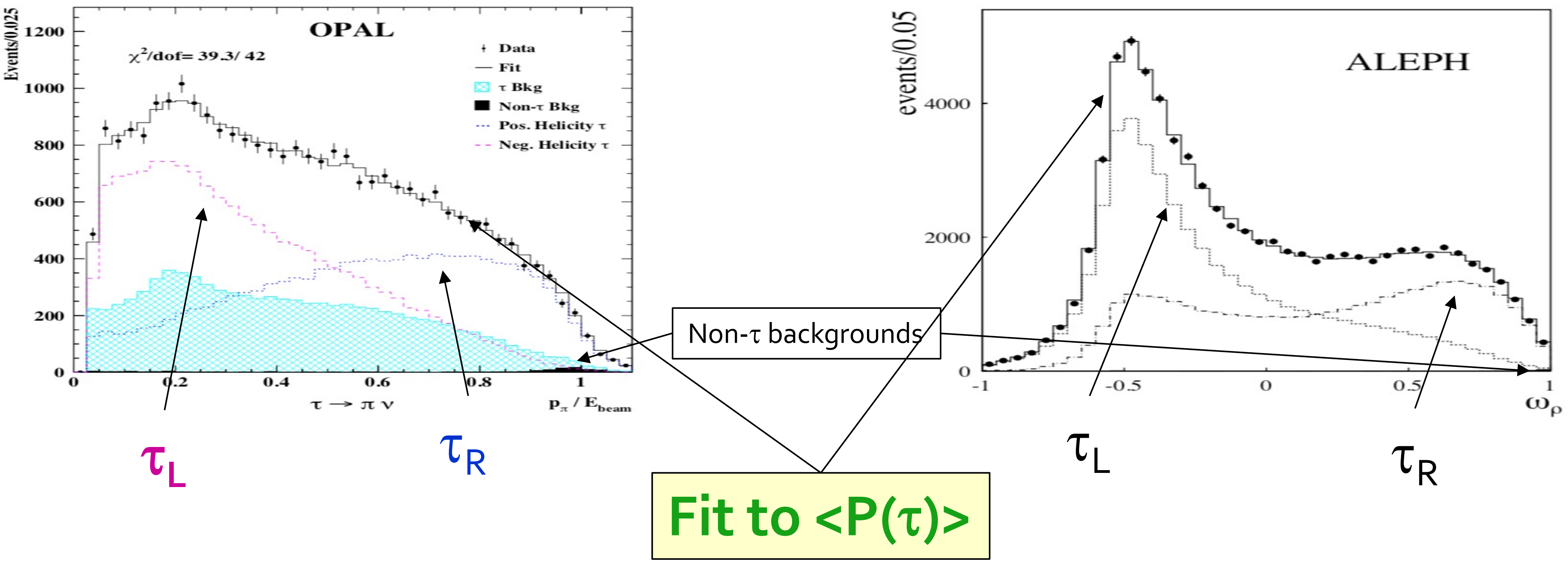
$$\langle P_\tau \rangle = \frac{\sigma_{R\tau} - \sigma_{L\tau}}{\sigma_{R\tau} + \sigma_{L\tau}} = -\mathcal{A}_\tau$$

$$A_{FB}^{pol\,\tau} = \frac{\sigma_{RF\tau} - \sigma_{LF\tau} - [\sigma_{RB\tau} - \sigma_{LB\tau}]}{\sigma_{R\tau} + \sigma_{L\tau}} = -\frac{3}{4}\mathcal{A}_e$$

$$A_{FB}^{f} = \frac{\sigma_{Ff} - \sigma_{Bf}}{\sigma_{Ff} + \sigma_{Bf}} = \frac{3}{4}\mathcal{A}_e\,\mathcal{A}_f$$

τ Longitudinal Polarisation: A_e and A_τ

- **Longitudinal polarisation measurement (all decay channels are used)**
  - $\tau \to \pi\nu_\tau$: pion energy
  - $\tau \to \rho\nu_\tau$: optimal observable ω_ρ



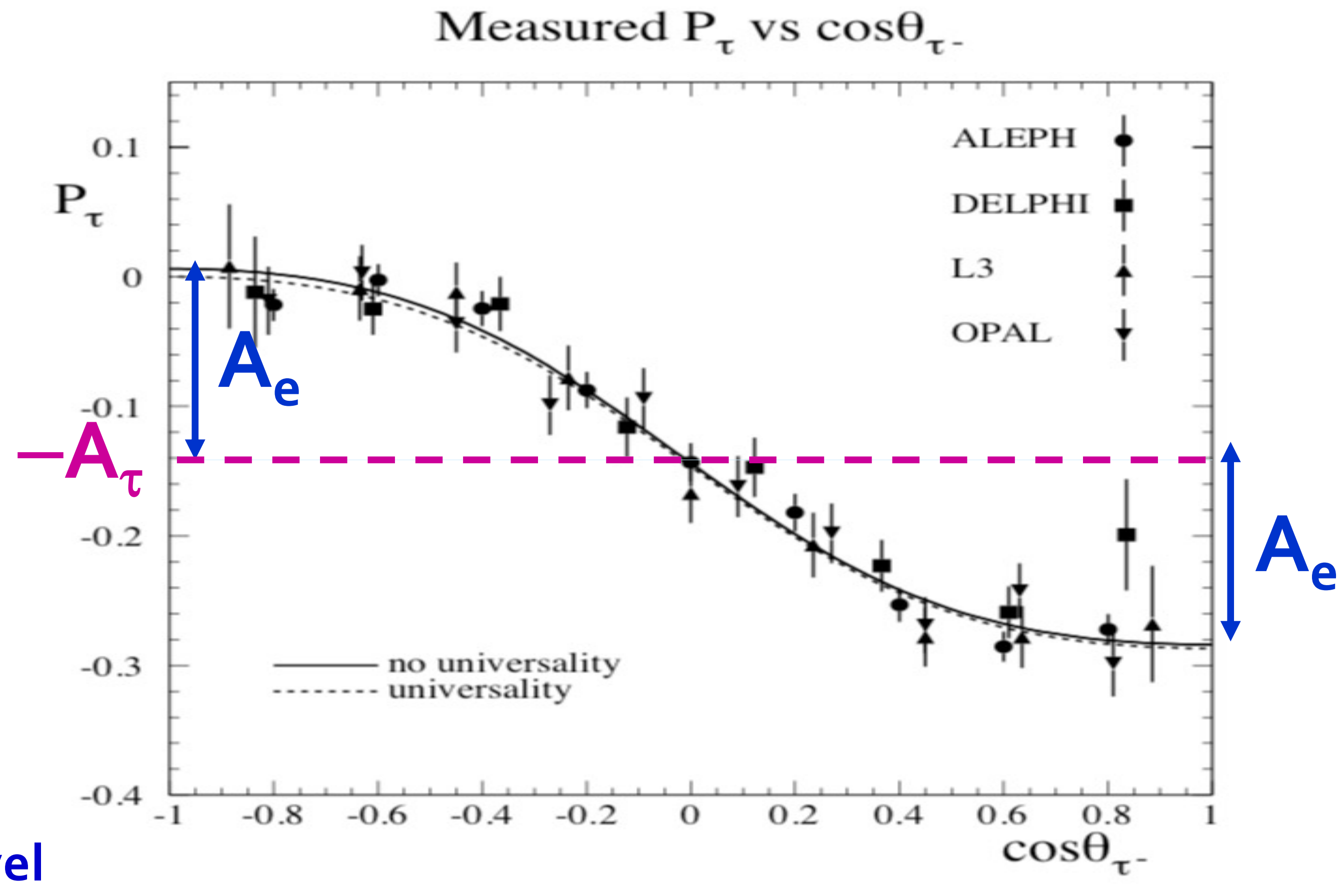
  - Important: perform this fit in each bin of the τ polar angle, $\cos\theta_\tau$

τ Longitudinal Polarisation: A_e and A_τ

- **Angular distribution of P_τ**

$$P(\cos\theta) = -\frac{\mathcal{A}_\tau(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + 2\mathcal{A}_e\mathcal{A}_\tau\cos\theta}$$

  - Average $\langle P_\tau \rangle$ gives $-A_\tau$
  - P_τ FB Asymmetry $A_{FB}^{pol\tau}$ gives A_e
  - Very high FCC-ee statistics !
    - Use best channel(s) only ($\pi\nu_\tau$, $\rho\nu_\tau$)



- **Theory inputs**
  - Above formula at improved Born level
    - Higher order calculations needed – also for optimal observable definition
  - Non-τ ($\gamma\gamma$) backgrounds will need a refined prediction and MC generators
    - FCC-ee control samples might help too (also for τ decay modelling and branching fractions)

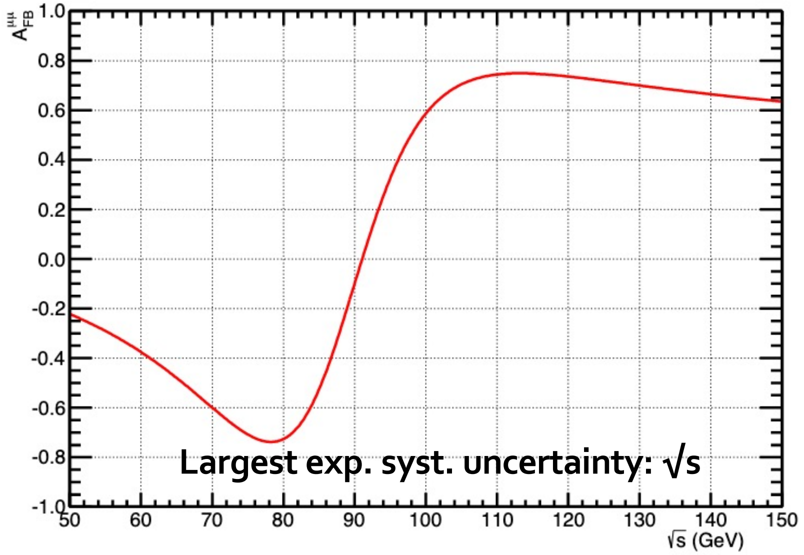
Muon Forward-Backward asymmetry: A_μ

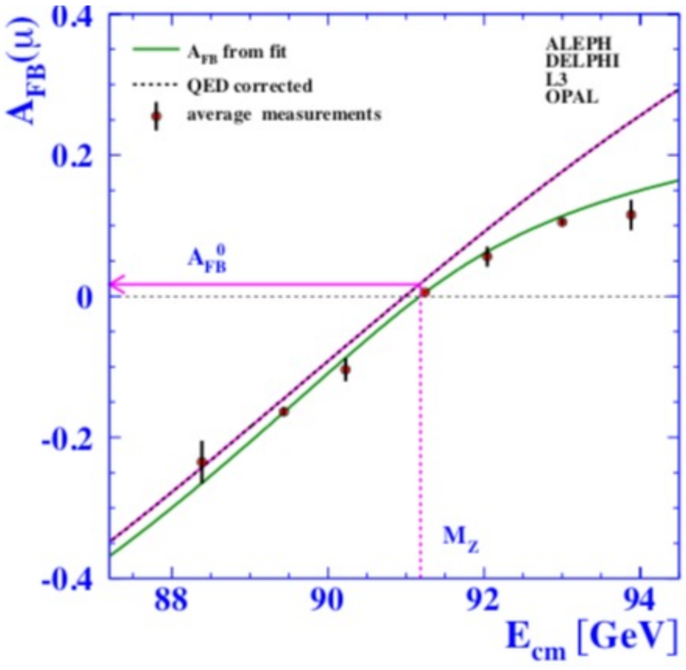
- Obtained from the charged muon angular distribution

$$\frac{dN^\pm}{dc} \propto \left\{ \frac{3}{8}\left(1+c^2\right) \pm A(s)c \right\} \times \varepsilon(c)$$

$$A(s) = \frac{3}{8}\frac{1+c^2}{c}\frac{N^+(c)-N^-(c)}{N^+(c)+N^-(c)}$$

$$A(m_Z) = \frac{3}{4}A_e\, A_\mu$$





- Theory inputs
  - High-precision QED prodecures to go from the exp'tal green curve to the pink curve
    - Accurate ISR, IFI, FSR Monte Carlo generators are also needed
      - ➔ Initial State radiation higher orders (several photons, emission angular distribution, etc.)
      - ➔ Initial-Final State interference adds a pure QED asymmetry which needs to be simulated/predicted
  - High-precision SM calculations to go from the pink curve to the SM parameters
    - E.g., higher-order calculations for $A_{FB}(s)$

Left-Right Asymmetry: A_e

- Obtained from the total cross sections measured in four beam helicity configurations
  - To reduce P dependence, a.k.a. "Blondel scheme"

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

	$\mathrm{sgn}(P(e^-), P(e^+)) =$ (-,+)	(+,-)	(-,-)	(+,+)	sum
luminosity [fb^{-1}]	40	40	10	10	
$\sigma(P_{e^-}, P_{e^+})$ [nb]	83.5	63.7	50.0	40.6	
Z events [10^9]	2.4	1.8	0.36	0.29	4.9
hadronic Z events [10^9]	1.7	1.3	0.25	0.21	3.4

L (fb^{-1})	N_Z^{had}	$\lvert P(e^-)\rvert$ (%)	$\lvert P(e^-)\rvert$ (%)	ΔA_{LR} (stat.)	ΔA_{LR} (syst).
100	3.3×10^9	80	30	4.3×10^{-5}	1.3×10^{-5}
100	4.2×10^9	80	60	2.4×10^{-5}	1.3×10^{-5}
250	8.4×10^9	80	30	2.7×10^{-5}	1.3×10^{-5}
250	1.1×10^{10}	80	60	1.5×10^{-5}	1.3×10^{-5}

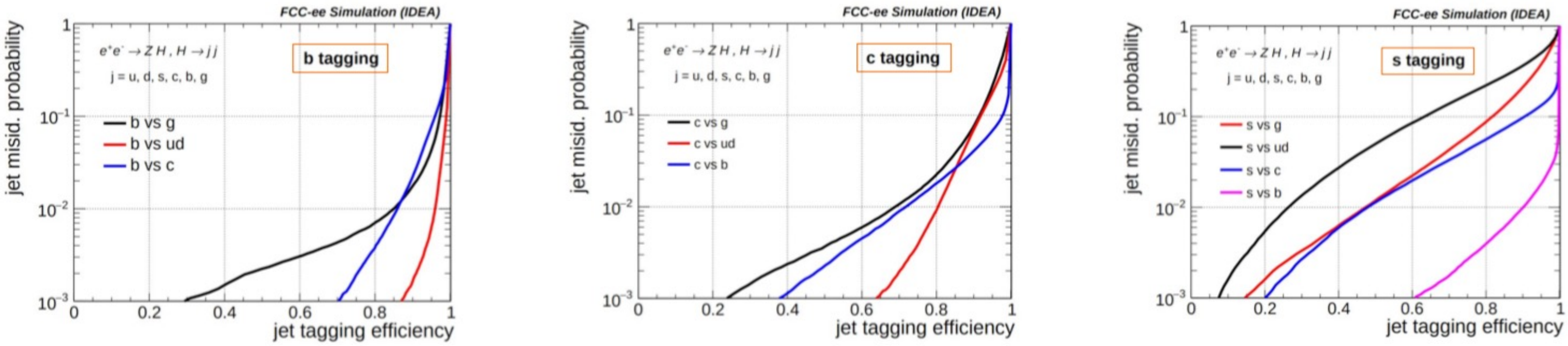
Assumes 500 keV precision on $\sqrt{s}$

- Bottom line: A_{LR} precision of 10^{-4} is a very realistic assumption with GigaZ

- Theory inputs
  - Almost none, besides high-precision SM calculations to go from A_{LR} to SM parameters

Other fermion asymmetries: A_b, A_c, A_s

- From forward-backward asymmetries (polarized or not) of $e^+e^- \to b\bar{b}$, $c\bar{c}$, $s\bar{s}$
  - Rely on efficient and pure flavour tagging algorithm (as for R_b, R_c, R_s)
    - Example of performance with IDEA detector at FCC-ee (Latest update at FCC Week 2022)
      - ➔ PID: cluster counting + TOF – 30 ps. Displacement: Beam pipe 10mm, VDet 3 layers



➔ Can tag $Z \to s\bar{s}$ with 40% efficiency, with 4% contamination from $Z \to u\bar{u} + d\bar{d}$

Open the way to several additional EW measurements in the strange sector

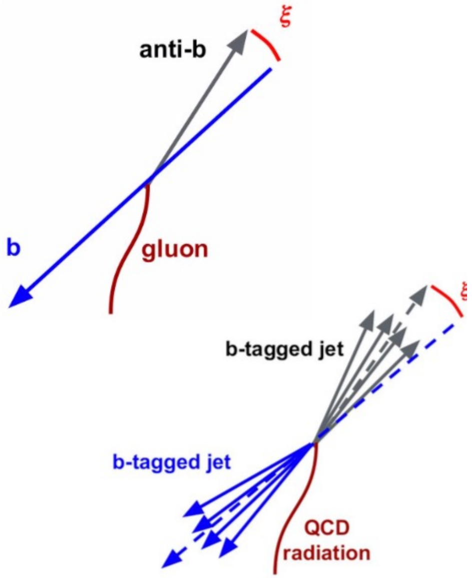
- Use double tagging technique to remove dependence on the tagging efficiency
  - Except with correlations between hemispheres (primary vertex, gluon radiation/splitting, bkgds)

Other fermion asymmetries: Theory inputs

- **Dominant systematic uncertainties (from LEP experience)**
  - Polarisation measurement for polarised asymmetries
  - QCD effects for all measurements (100% correlated)
  - New developments in arXiv:2010.08604 (J. Alcaraz)

QCD corrections and uncertainties can be reduced significantly using acollinearity (ξ) cut, which rejects events with (hard) gluon radiation. Assume a factor 5 for now.

Full systematic study required
QCD higher-order corrections welcome



https://arxiv.org/abs/hep-ex/0509008

Source	$A_{FB}^{0,b}$ $[10^{-3}]$	$A_{FB}^{0,c}$ $[10^{-3}]$	$\mathcal{A}_b$ $[10^{-2}]$	$\mathcal{A}_c$ $[10^{-2}]$
statistics	1.5	3.0	1.5	2.2
internal systematics	0.6	1.4	1.2	1.5
QCD effects	0.4	0.1	0.3	0.2
$B(D \to \text{neut.})$	0	0	0	0
D decay multiplicity	0	0.2	0	0
B decay multiplicity	0	0.2	0	0
$B(D^+ \to K^-\pi^+\pi^+)$	0	0.1	0	0
$B(D_s \to \phi\pi^+)$	0	0.1	0	0
$B(\Lambda_c \to p K^-\pi^+)$	0	0.1	0	0
D lifetimes	0	0.2	0	0
B decays	0.1	0.4	0	0.1
decay models	0.1	0.5	0.1	0.1
non incl. mixing	0.1	0.4	0	0
gluon splitting	0.1	0.2	0.1	0.1
c fragmentation	0.1	0.1	0.1	0.1
light quarks	0	0	0	0
beam polarisation	0	0	0.5	0.3
total correlated	0.4	0.9	0.6	0.4
total error	1.6	3.5	2.0	2.7

- Exclusive decays can also be used
  - To improve the b, c, s purity (or calibrate other hemisphere efficiency)

Summary: Theory inputs for asymmetries

Observables	Present value (×10⁴)	TeraZ / GigaZ stat.	TeraZ / GigaZ current syst.	Theory input (not exhaustive)
A_e from P_τ (FCC-ee)	1514 ± 19	0.07	0.20	SM relation to measured quantities
A_e from A_{LR} (ILC)		0.15	0.80	
A_μ from A_{FB} (FCC-ee)	1456 ± 91	0.23	0.22	Accurate QED (ISR, IFI, FSR)
A_μ from A_{FB}^{pol} (ILC)		0.30	0.80	
A_τ from P_τ (FCC-ee)	1449 ± 40	0.05	2.00	Prediction for non-τ backgrounds
A_τ from A_{FB} (FCC-ee)		0.23	1.30	
A_τ from A_{FB}^{pol} (ILC)		0.30	0.80	
A_b from A_{FB} (FCC-ee)	8990 ± 130	0.24	2.10	QCD calculations
A_b from A_{FB}^{pol} (ILC)		0.90	5.00	
A_c from A_{FB} (FCC-ee)	65400 ± 210	2.00	1.50	
A_c from A_{FB}^{pol} (ILC)		2.00	3.70	

Projections from https://www.overleaf.com/read/xnfpvbdgfqpr

- **And also sophisticated and state of the art MC generators (signal and backgrounds)**
  - **Plus, maybe, redefined EW Precision Parameters (EWPP) and extraction procedures ?**

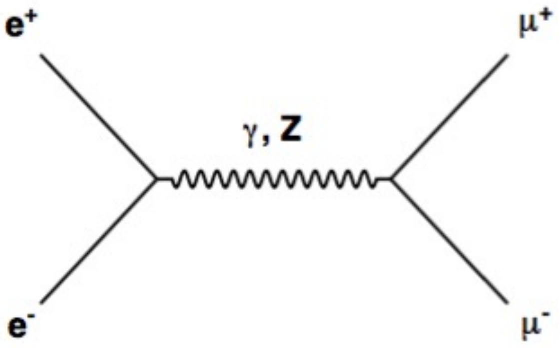
Electromagnetic coupling constant (FCC-ee)

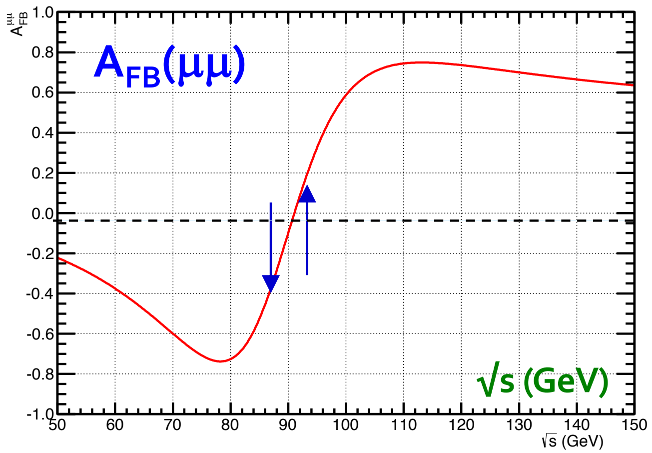
❑ Muon forward-backward asymmetry off-peak measurement

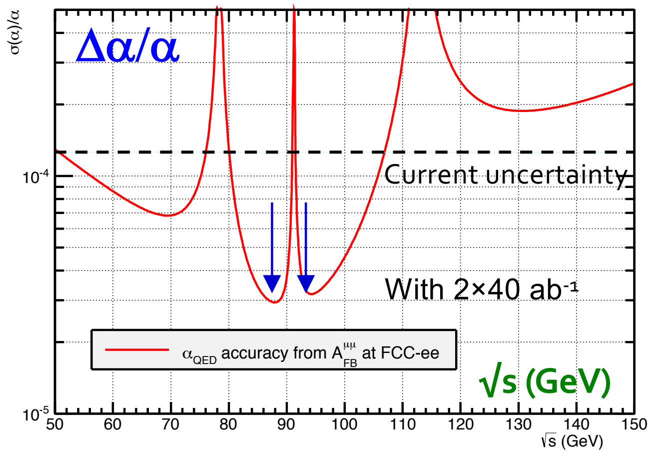
https://arxiv.org/abs/1512.05544

From γ-Z interference:

$$A_{FB}^{\mu\mu} = \frac{N_F^{\mu+} - N_B^{\mu+}}{N_F^{\mu+} + N_B^{\mu+}} \approx f(\sin^2 \vartheta_W^{eff}) + \alpha_{QED}(s) \frac{s - m_Z^2}{2s} g(\sin^2 \vartheta_W^{eff})$$







- ◆ **Statistical optimum is a compromise**
  - **The number of events (be as close as m_Z as possible)**
  - **The absolute asymmetry (be as close as 78 and 115 GeV as possible)**
  - **The ability to measure the beam energy (half-integer spin tune)**
    - ➔ **Two optimal centre-of-mass energies : 87.69 GeV and 94.71 GeV (or 93.83 GeV)**

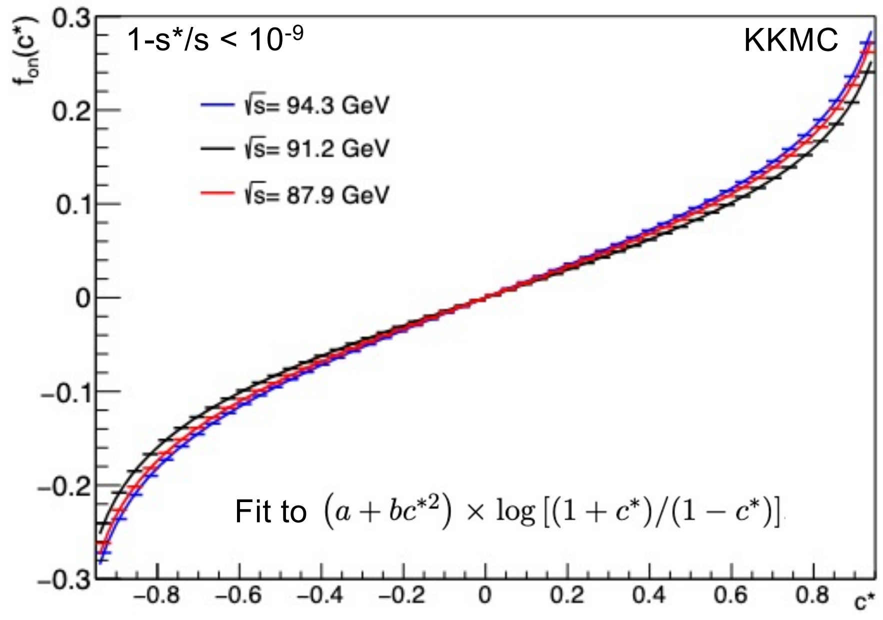
Used primarily for Γ_Z measurement

Electromagnetic coupling constant (FCC-ee)

https://arxiv.org/abs/1512.05544

- $\alpha_{QED}(m_Z^2)$ obtained from the difference of the two asymmetries
  - Lots of parametric and theoretical uncertainties cancel in the difference
    - Perfect cancellation for A_{FB}^0, m_Z, ISR, FSR…
    - Only approximate cancellation for IFI asymmetry

$$\frac{dN^{\pm}}{ds^*dc^*}(s,s^*,c^*) \propto \left\{\frac{3}{8}\left(1+c^{*2}\right) \pm A(s^*)c^*\right\} \times \left[1 \pm f(s,s^*,c^*)\right] \times \varepsilon(c^*).$$



Statistics limited !

Type	Source	Uncertainty
Experimental	$E_{\rm beam}$ calibration	1×10^{-5}
	$E_{\rm beam}$ spread	$< 10^{-5}$
	Acceptance and efficiency	negl.
	Charge inversion	negl.
	Backgrounds	negl.
Parametric	m_Z and Γ_Z	1×10^{-6}
	$\sin^2\theta_W$	5×10^{-6}
	G_F	5×10^{-7}
Total	Systematics	1.2×10^{-5}
	Statistics	3×10^{-5}
Theoretical	QED (ISR, FSR, IFI)	$< 10^{-6}$
	QED (IFI)	few 10^{-5}
	Missing EW higher orders	few 10^{-4}
	New physics in the running	0.0

Projected accuracies at FCC-ee

- **From a complete set of EWPO measurements at LEP + SLC (reminder)**

EWPO Fit to the SM (and nothing else)

Direct measurement

$$\sin^2\theta_{\rm W}^{\rm eff} = 0.23153 \pm 0.00016$$

$$\begin{aligned}\sin^2\theta_{\rm W}^{\rm eff} &= 0.231488 \pm 0.000029_{m_{\rm top}} \pm 0.000015_{m_{\rm Z}} \pm 0.000035_{\alpha_{\rm QED}} \\ &\pm 0.000010_{\alpha_{\rm S}} \pm 0.000001_{m_{\rm H}} \pm 0.000047_{\rm theory} \\ &= 0.23149 \pm 0.00007_{\rm total},\end{aligned}$$

EWPO Fit to the SM (and nothing else)

Direct measurement

$$m_{\rm W} = 80.379 \pm 0.012 \ \ {\rm GeV}$$

$$\begin{aligned}m_{\rm W} &= 80.3584 \pm 0.0055_{m_{\rm top}} \pm 0.0025_{m_{\rm Z}} \pm 0.0018_{\alpha_{\rm QED}} \\ &\pm 0.0020_{\alpha_{\rm S}} \pm 0.0001_{m_{\rm H}} \pm 0.0040_{\rm theory} \ \ {\rm GeV} \\ &= 80.358 \pm 0.008_{\rm total} \ \ {\rm GeV},\end{aligned}$$

Projected accuracies at FCC-ee

- From a complete set of EWPO measurements at FCC-ee (projections)

Direct measurement

$$\sin^2\theta_W^{\rm eff} = 0.23153 \pm 0.000002$$

$\approx A_\ell/16\ \Delta A_{FB}^{\mu\mu} / A_{FB}^{\mu\mu}$
(w/ lepton universality)

(ILC projection: $\approx \frac{\Delta A_{LR}}{8} = \pm\ 0.000010$)

Direct measurement

$$m_W = 80.379 \pm 0.0003\,\mathrm{GeV}$$

EWPO Fit to the SM (and nothing else)

$$\sin^2\theta_W^{\rm eff} = 0.231488 \pm 0.000001_{m_{top}} \pm 0.000001_{m_Z} \pm 0.000009_{\alpha_{QED}} \pm 0.000001_{\alpha_S} \pm 0.000000_{m_H} \pm 0.000047_{\rm theory}$$

EWPO Fit to the SM (and nothing else)

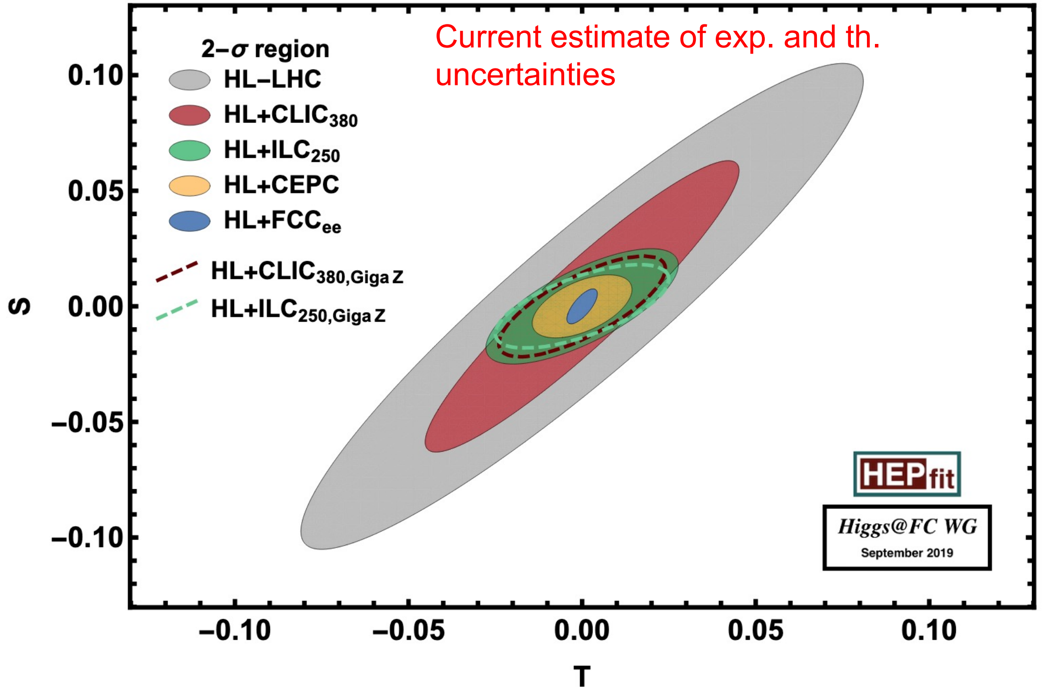
$$m_W = 80.3584 \pm 0.0001_{m_{top}} \pm 0.0001_{m_Z} \pm 0.0005_{\alpha_{QED}} \pm 0.0002_{\alpha_S} \pm 0.0000_{m_H} \pm 0.0040_{\rm theory}\,\mathrm{GeV}$$

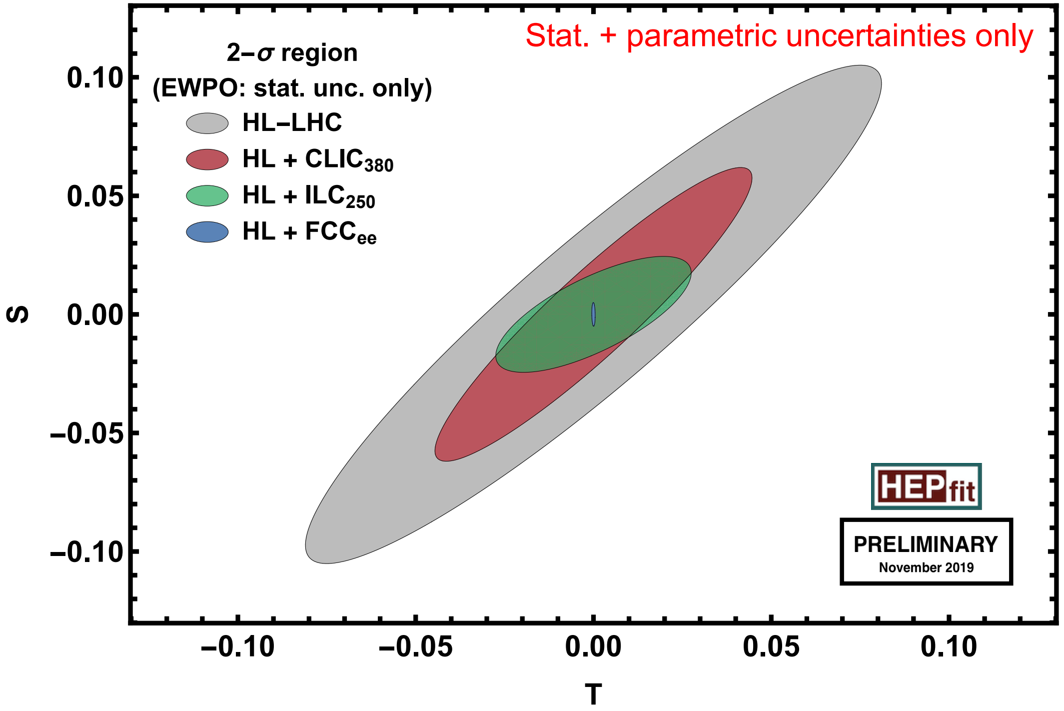
- Additional improvement for $\alpha_{QED}(m_Z^2)$ would be welcome (factor 2 to 4)
- A factor 10 to 20 improvement is required for intrinsic theoretical uncertainties

Estimates from S. Heinemeyer

Statistics is the limit

- Challenge is to match systematic uncertainty with the statistical precision
  - Precision = discovery potential
    - Example: New physics in W and Z propagators, parameterized here with S and T variables





➔ At FCC-ee, a lot of potential to exploit (e.g., with a good detector design)

➔ Theory work is critical

Conclusions (1)

- EWPO measurements at the Z pole have a considerable physics potential
  - Combined with W, top, and Higgs measurements, they probe the BSM origins of the SM
    - As a EFT of an underlying UV theory it originates from
- Statistics is the name of the game and polarisation is the cornerstone of the program
  - At FCC-ee, resonant depolarisation allow for EWPO improvements by factors 10 to 2000
    - e.g., W mass to ±250 keV, Z mass and width to ±4 keV, $\sin^2\theta_W^{eff}$ to 2×10^{-6}, α_{QED} to 3×10^{-5} etc.
  - At ILC, beam polarisation partially compensates for the 1000 times smaller statistics
    - For some of the EWPO's, e.g. , $\sin^2\theta_W^{eff}$ to 1×10^{-5}
      - ➔ (Note: It was checked that there is nothing that FCC-ee can do better with beam polarisation)
- Today, systematic uncertainties are the limiting factor in many of the measurements
  - The challenge arise from matching these uncertainties to the statistical precision
    - Optimized detector design, new analysis strategies, new control samples, detailed studies
    - Theory developments

Conclusions (2)

- History has shown that exp. systematic uncertainties are usually statistics limited
  - FCC-ee statistical precision is the target
    - Experimenters will do it!

- FCC-ee statistics allows control of parametric uncertainties to the desired level
  - e.g., direct determination of $\alpha_{QED}(m_Z^2)$
    - [Additional factor 2 improvement would still be welcome]

- The physis case of FCC-ee will therefore be made significantly stronger
  - With robust estimate of theoretical uncertainties
  - With a strategy towards matching them to the FCC-ee statistical precision
  - With theoretical work to explore sensitivity for specific new physics
    - In order to optimize strategies in an informed way

- Today it may look like a brick wall
  - But it may be a mine of gold in our quest for the BSM origins of the laws of our Universe