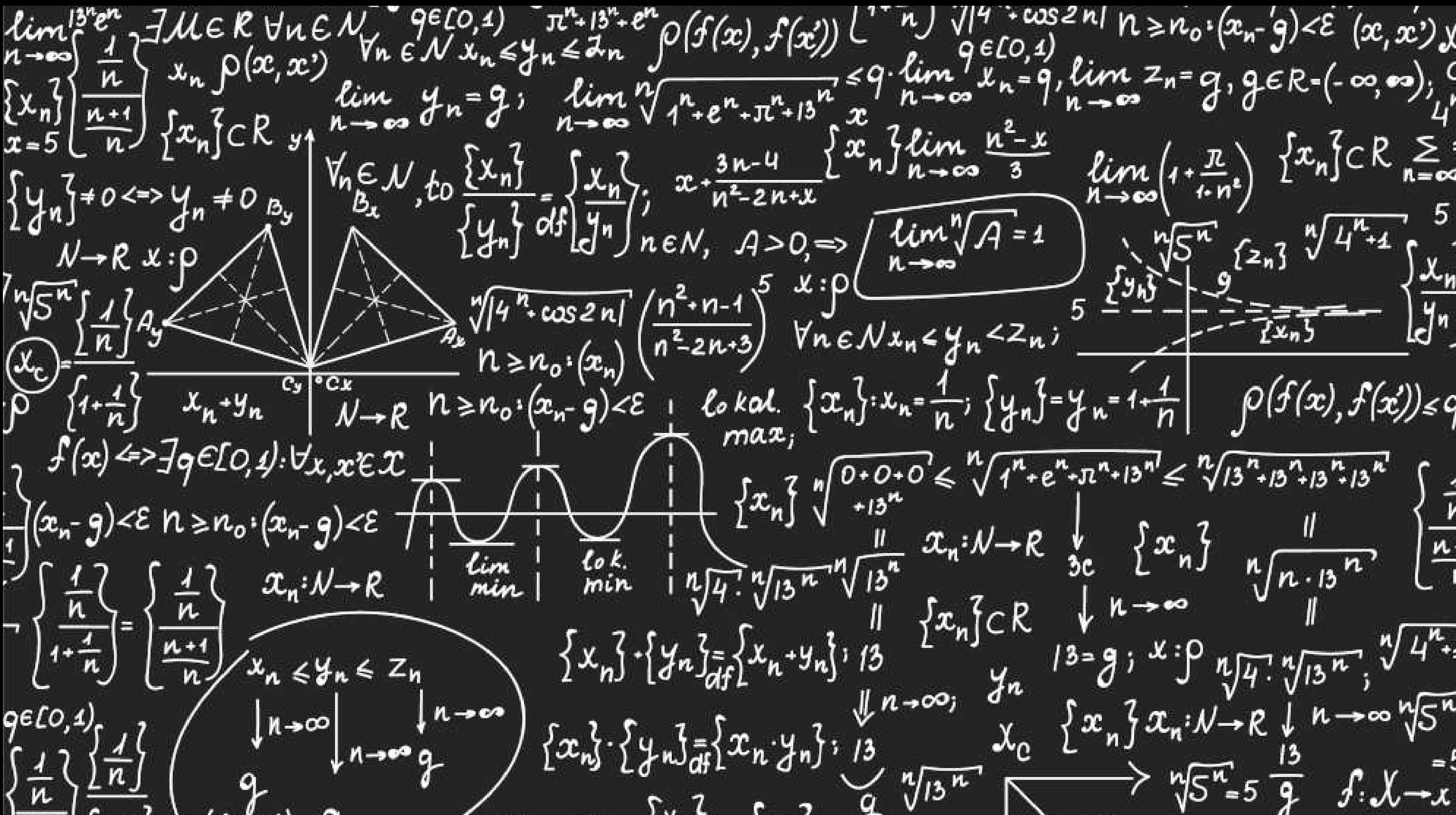
Status of theory calculations for Z-pole observables

A. Freitas

University of Pittsburgh

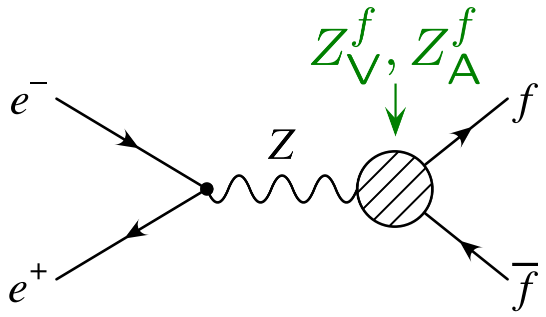


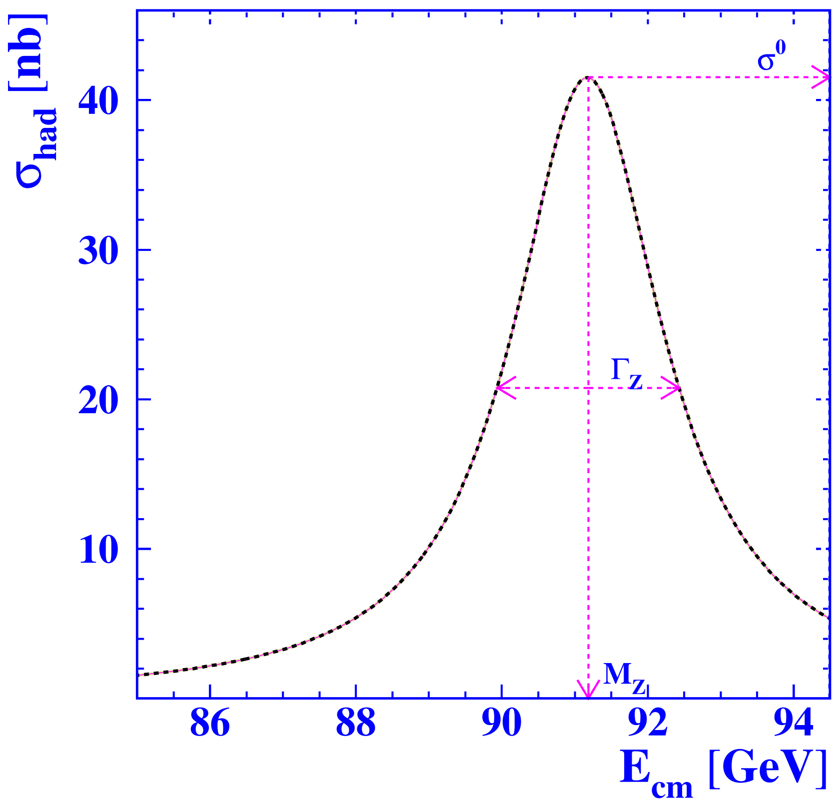
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2} R_e R_f$

$$\Gamma_{ff} = C\left[(Z_{\text{V}}^f)^2 + (Z_{\text{A}}^f)^2\right]$$

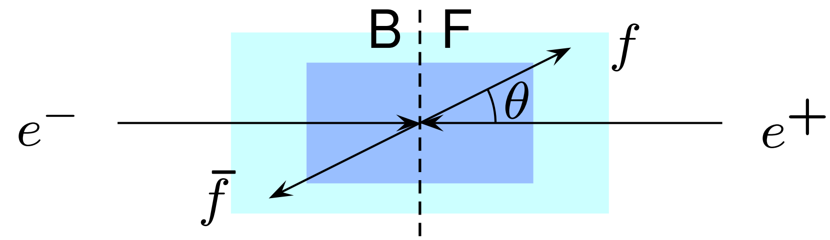




Z-pole asymmetries

Forward-backward asymmetry:

$$A_{\text{FB}} \equiv \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$



$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\text{eff}})}{1 + (1 - 4\sin^2\theta^f_{\text{eff}})^2} \qquad \sin^2\theta^f_{\text{eff}} = \frac{1}{4|Q_f|}\left[1 - \frac{Z^f_{\text{V}}}{Z^f_{\text{A}}}\right]$$

Left-right asymmetry:

With polarized e^- beam: $$A_{\text{LR}} \equiv \frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} = \mathcal{A}_e$$

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$: $$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

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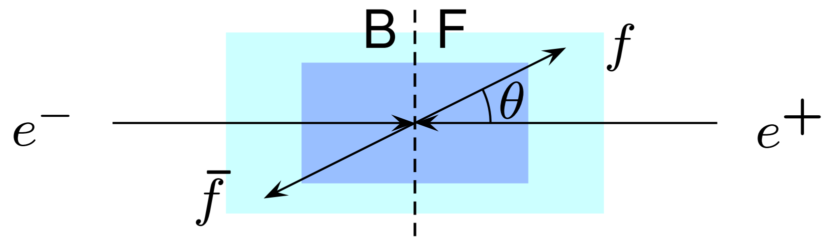

$$\mathcal{A}_f = \frac{2(1-4\sin^2\theta^f_{\text{eff}})}{1+(1-4\sin^2\theta^f_{\text{eff}})^2} \qquad \sin^2\theta^f_{\text{eff}} = \frac{1}{4|Q_f|}\left[1 - \frac{Z^f_{\text{V}}}{Z^f_{\text{A}}}\right]$$

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Decay widths in terms of $\sin^2\theta^f_{\text{eff}}$:

$$\Gamma_{ff} = C\left[F^f_V + F^f_A\right] = C\left[F^f_A(1-4|Q_f|\sin^2\theta^f_{\text{eff}})^2 + F^f_A\right]$$

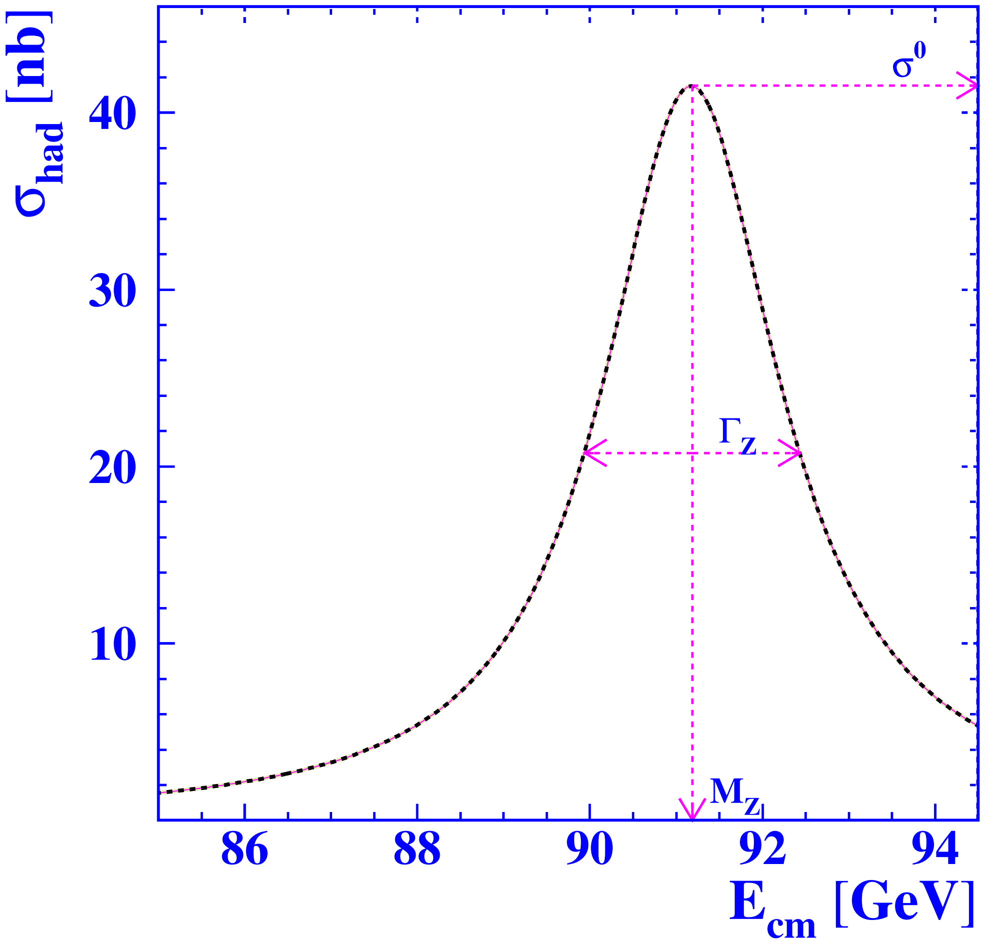
Need for theory input

- Comparison of EWPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

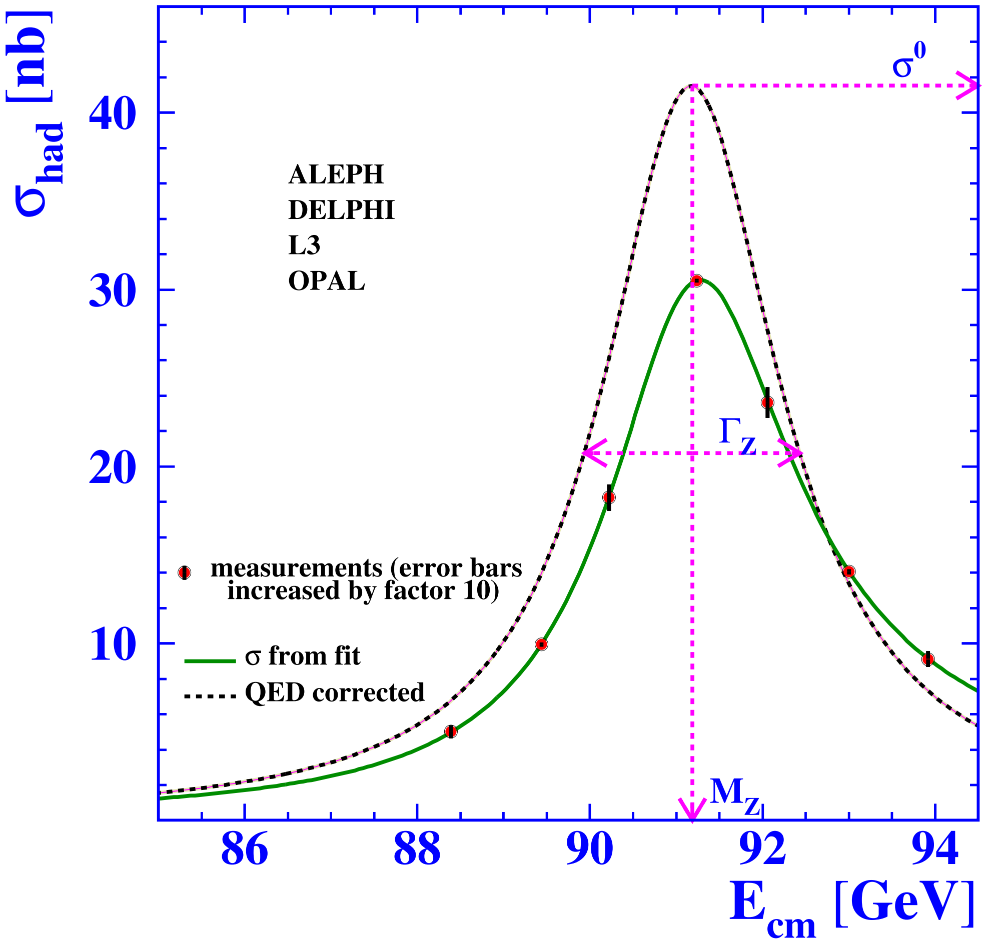
- Extraction of EWPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_{t}, α_{s}, etc. extracted from other processes

Real observables vs. pseudo-observables



Real observables vs. pseudo-observables



LEP EWWG '05

→ Large effects from initial-state QED radiation

→ Theory input necessary to extract relevant EWPOs ("pseudo-observables")

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

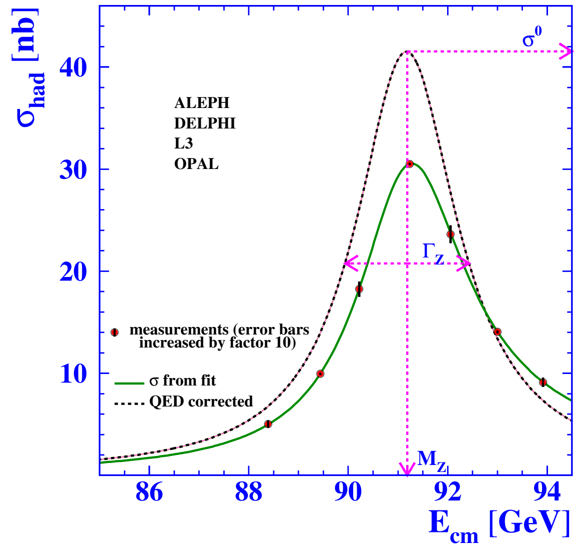
Universal ($m{=}n$) logs known to $n = 6$,
also some sub-leading terms

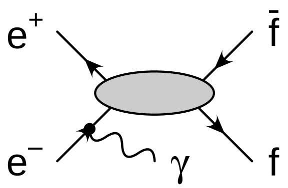
Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools

→ talk by S. Frixione

LEP EWWG '05





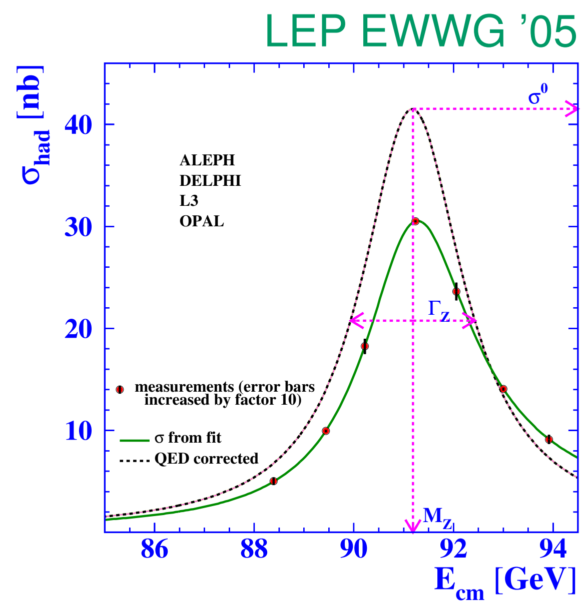
Z lineshape

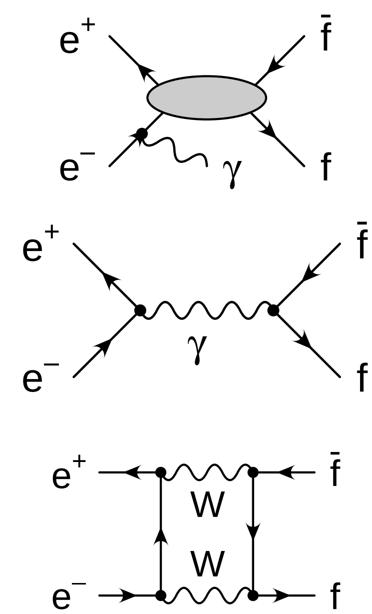
- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{Z} + \sigma_{\gamma} + \sigma_{\gamma Z} + \sigma_{\text{box}}$$





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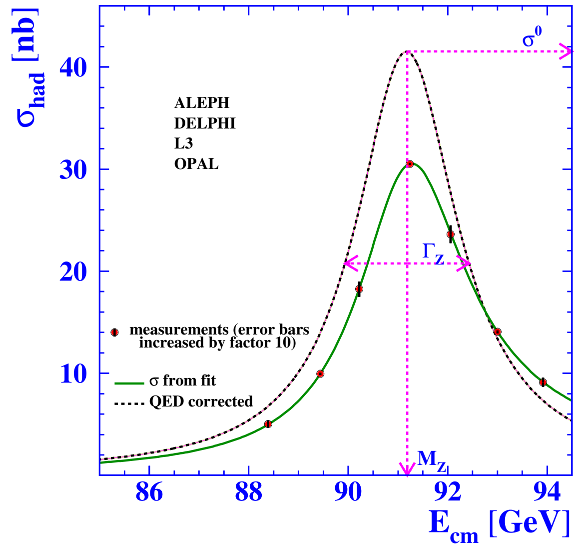
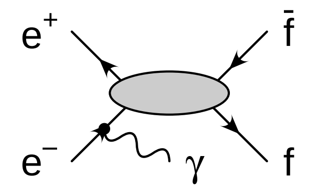
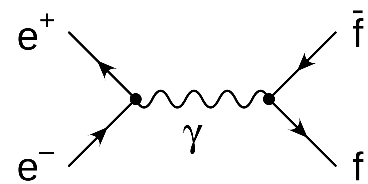
- Z-pole contribution:

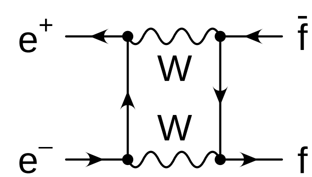
$$\sigma_{\text{Z}} = \frac{R}{(s-\overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

LEP EWWG '05









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computed in SM

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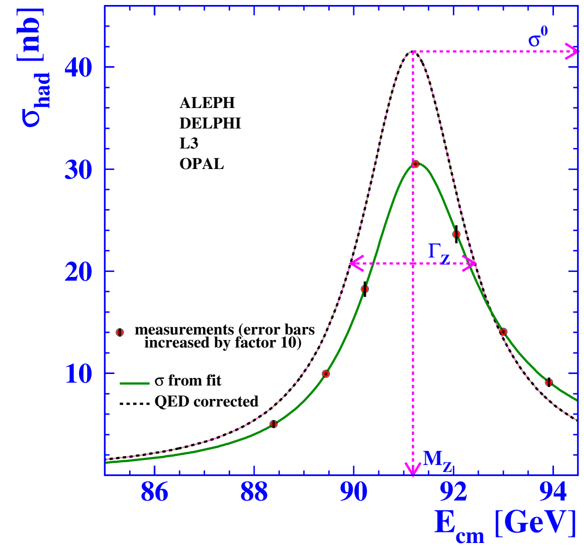
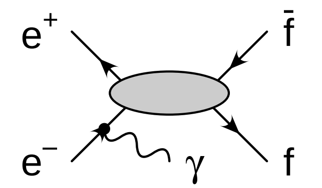
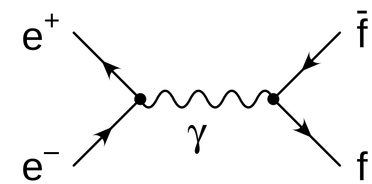
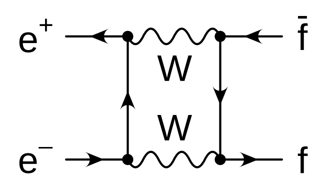
LEP EWWG '05









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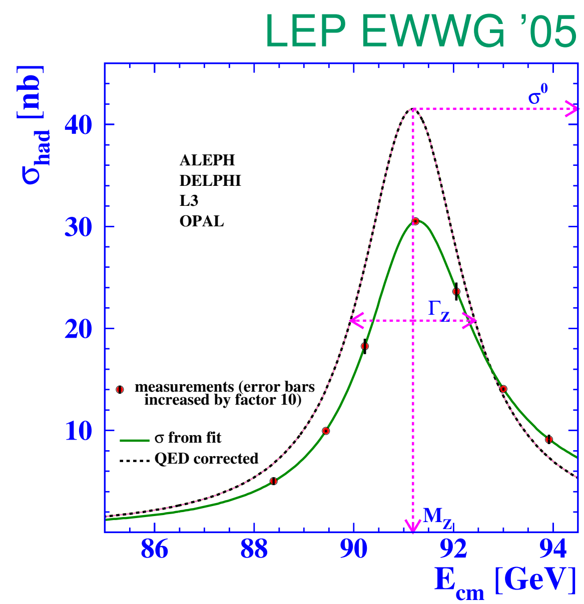
$$\sigma_{\text{Z}} = \frac{R}{(s-\overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

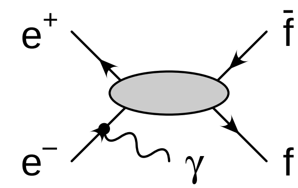
- In experimental analyses:

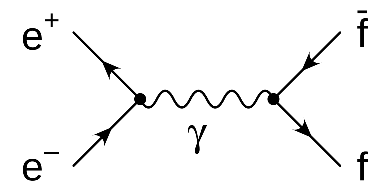
$$\sigma \sim \frac{1}{(s-M_{\text{Z}}^2)^2 + s^2\Gamma_{\text{Z}}^2/M_{\text{Z}}^2}$$

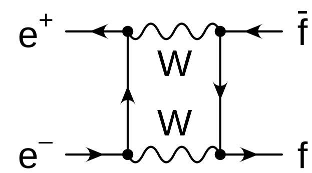
$$\overline{M}_{\text{Z}} = M_{\text{Z}}\Big/\sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx M_{\text{Z}} - 34\ \text{MeV}$$

$$\overline{\Gamma}_{\text{Z}} = \Gamma_{\text{Z}}\Big/\sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx \Gamma_{\text{Z}} - 0.9\ \text{MeV}$$







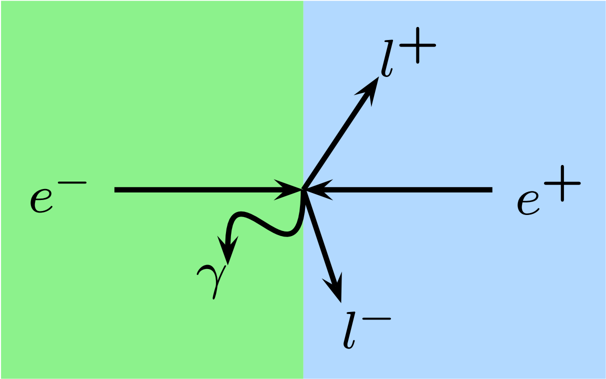


QED radiation in differential observables

QED radiation in principle cancels in asymmetries, e.g. $A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$

Some effects from detector acceptance and cuts

Typical influence $< 10^{-3}$



Implementation of QED effects:

a) Analytical formulae, e.g. ZFITTER

Arbuzov, Bardin, Christova, Kalinovskaya, Riemann, Riemann, ...

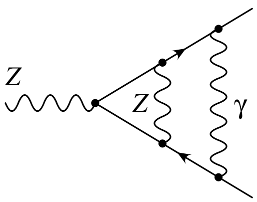
b) Monte Carlo event generator, e.g. KORALZ, KKMC

Jadach, Ward, ...

QED radiation in differential observables

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\mathcal{R}_V^f F_V^f + \mathcal{R}_A^f F_A^f \right]_{s=\overline{M}_Z^2}$$



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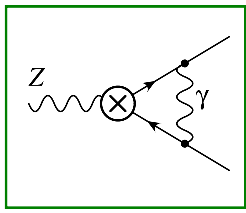
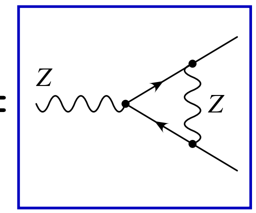


+ finite, with $\otimes$ =


$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation; → talk by S. Kluth

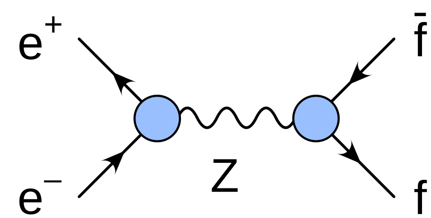
known inclusively to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

Baikov, Chetyrkin, Kühn, Rittinger '12

or compute exclusively using MC methods

F_V^f, F_A^f: Electroweak corrections



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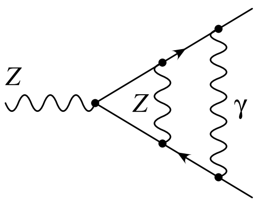

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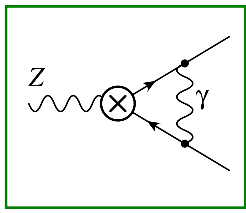

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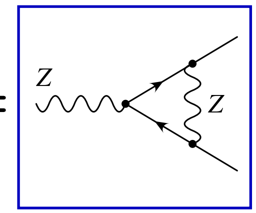
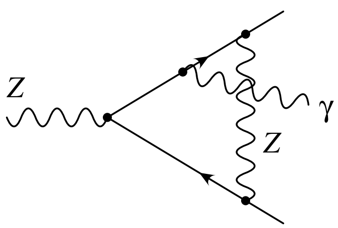


Additional non-factorizable contributions, e.g.



→ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

→ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

→ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

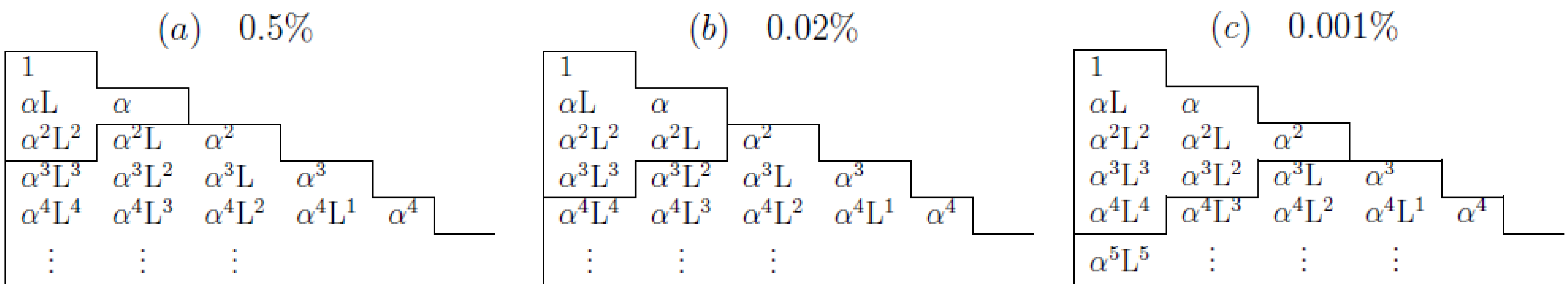
→ How to account for in MC simulations?

Monte-Carlo methods for QED effects

- Implementation in MC program to evaluate exp. efficiency and particle ID
- Current state of art: e.g. `KORALZ, KKMC` — Jadach, Ward, ...
  $\rightarrow \mathcal{O}(\alpha^2 L)$ accuracy [$L = \ln(s/m_e^2)$]
- One to two orders improvement needed:

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z \pm 3.5\rm GeV}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

Jadach, Skrzypek '19



$\rightarrow$ Need matching of h.o. matrix elements with QED parton shower (exclusive in all fs particles)

Pole expansion

Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$:

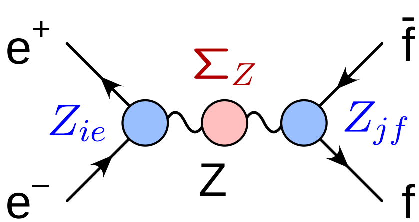
$\rightarrow$ All terms are individually gauge-invariant

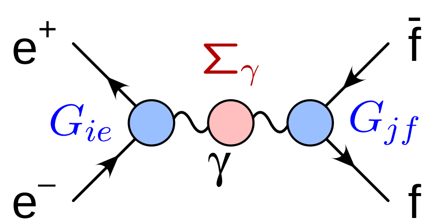
$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i,j = V,A)$$

$$R_{ij} = \left.\frac{Z_{ie}Z_{jf}}{1+\Sigma'_Z}\right|_{s=s_0}$$

$$S_{ij} = \left[\frac{Z_{ie}Z'_{jf} + Z'_{ie}Z_{jf}}{1+\Sigma'_Z} - \frac{Z_{ie}Z_{jf}\Sigma''_Z}{2(1+\Sigma'_Z)^2} + \frac{G_{ie}G_{jf}}{s+\Sigma_\gamma} + B_{ij}\right]_{s=s_0}$$

$$S'_{ij} = \ldots$$





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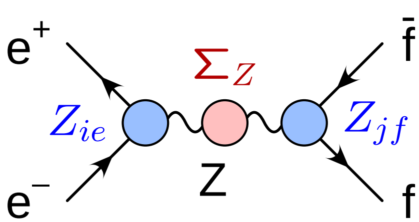
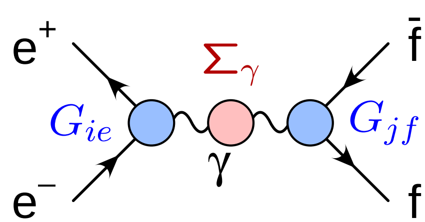
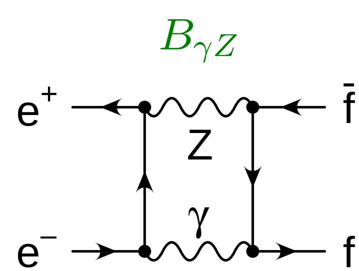
$$R_{ij} = \left.\frac{Z_{ie}Z_{jf}}{1+\Sigma'_Z}\right|_{s=s_0} + B^R_{\gamma Z,ij} + B^{RL}_{\gamma Z,ij}\ln(1-\tfrac{s}{s_0})$$

$$S_{ij} = \left[\frac{Z_{ie}Z'_{jf} + Z'_{ie}Z_{jf}}{1+\Sigma'_Z} - \frac{Z_{ie}Z_{jf}\Sigma''_Z}{2(1+\Sigma'_Z)^2} + \frac{G_{ie}G_{jf}}{s+\Sigma_\gamma} + B_{ij}\right]_{s=s_0}$$
$$+B^S_{\gamma Z,ij} + B^{SL}_{\gamma Z,ij}\ln(1-\tfrac{s}{s_0})$$

$$S'_{ij} = \ldots$$







Pole expansion

Express R_{ij} in terms of $\sin^2\theta^f_{\rm eff}$ and F^f_A (with NNLO corrections):

$$R_{ij} = 4 I^3_e I^3_f \sqrt{F^e_A F^f_A}\Big[Q^e_i Q^f_j\big(1 + i\, r^I_{AA} - \tfrac{1}{2}(r^I_{AA})^2 + \tfrac{1}{2}\delta\overline{X}_{(2)}\big) + (Q^e_i I_{j,f} + Q^f_j I_{i,e})(i - r^I_{AA}) - I_{i,e} I_{j,f}\Big] + M_Z\Gamma_Z\, Z_{ie(0)} Z_{jf(0)}\, x^I_{ij}\,,$$

$$Q^f_V = 1 - 4|Q_f|\sin^2\theta^f_{\rm eff}, \qquad Q^f_A = 1$$

$$I_{V,f} = \frac{1}{(a^Z_{f(0)})^2}\Big[a^Z_{f(0)}\,\mathrm{Im}\,Z_{Vf(1)} - v^Z_{f(0)}\,\mathrm{Im}\,Z_{Af(1)}\Big], \qquad I_{A,f} = 0$$

$$\delta\overline{X}_{(2)} = -(\mathrm{Im}\,\Sigma'_{Z(1)})^2 + 2\,\overline{b}^R_{\gamma Z(1)},$$

$$r^I_{ij} = \frac{\mathrm{Im}\,Z_{ie(1)}}{Z_{ie(0)}} + \frac{\mathrm{Im}\,Z_{jf(1)}}{Z_{jf(0)}} - \mathrm{Im}\,\Sigma'_{Z(1)},$$

$$x^I_{ij} = \frac{\mathrm{Im}\,Z'_{ie(1)}}{Z_{ie(0)}} + \frac{\mathrm{Im}\,Z'_{jf(1)}}{Z_{jf(0)}} - \frac{1}{2}\,\mathrm{Im}\,\Sigma''_{Z(1)},$$

C++ library GRIFFIN (in preparation)

Chen, Freitas '22

Pole expansion

Expand amplitude for $e^+e^- \to f\bar{f}$ about **complex pole** $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\Gamma_Z$:

$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad (i, j = V, A)$$

Current state of art: R @ NNLO + leading higher orders
S @ NLO
S' @ (N)LO

For future ee colliders: (at least) one order more!

$\rightarrow$ also matching to Monte-Carlo for QED/QCD ISR/FSR/IFI

“Input” parameters

- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape; δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV at FCC-ee
  $\rightarrow$ Main theory uncertainties: QED ISR

- m_t: Current status $\delta m_t \sim 0.3$ GeV at LHC PDG '20
  $\rightarrow$ Theoretical ambiguity in mass def. of $\mathcal{O}(0.5\text{ GeV})$
  Hoang, Plätzer, Samitz '18; Ferrario Ravasio, Nason, Oleari '18

  $\rightarrow$ Robust det. from $e^+e^- \rightarrow t\bar{t}$ at $\sqrt{s} \sim 350$ GeV $\rightarrow$ talk by M. Beneke

- m_b, m_c: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\text{MS}}} \sim 30$ MeV, $\delta m_c^{\overline{\text{MS}}} \sim 25$ MeV LHC HXSWG '16

  $\rightarrow$ estimated improvements $\delta m_b^{\overline{\text{MS}}} \sim 13$ MeV, $\delta m_c^{\overline{\text{MS}}} \sim 7$ MeV
  Lepage, Mackenzie, Peskin '14

- M_H: from kinematic constraint fits $HZ(\ell\ell)$, $H(b\bar{b})Z$
  $\rightarrow$ $\delta M_H \sim 10...20$ MeV
  $\rightarrow$ theory errors subdominant

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

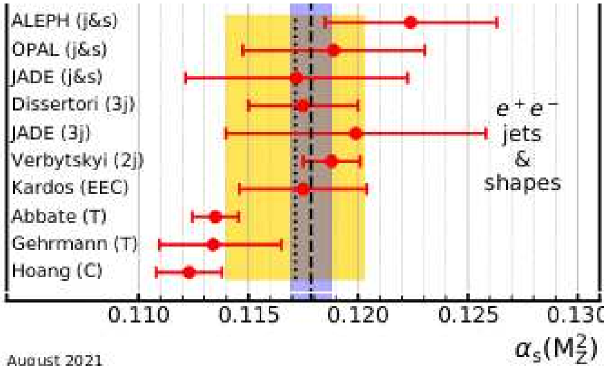
    $\rightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes: $\alpha_s \sim 0.113...0.119$

    $\rightarrow$ Large non-pertubative power corrections

    $\rightarrow$ Systematic uncertainties?

PDG '21



  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

    $\rightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

- α_s: → talk by S. Kluth
  - Electroweak precision ($R_\ell = \Gamma_Z^{had}/\Gamma_Z^{\ell}$):

    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    → No (negligible) non-perturbative QCD effects

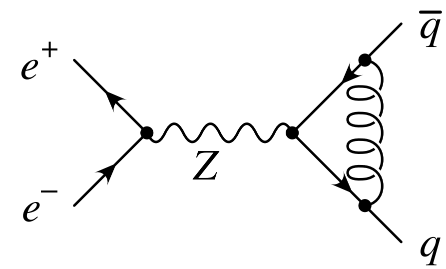
    FCC-ee: $\delta R_\ell \sim 0.001$

    $\Rightarrow \quad \delta\alpha_s < 0.0001$

    Theory input: N^{3}LO EW corr. + leading N^{4}LO

    to keep $\delta_{th} R_\ell \lesssim \delta_{exp} R_\ell$

    Caviat: R_ℓ could be affected by new physics



Strong coupling

- α_s: → talk by S. Kluth
  - Electroweak precision ($R_\ell = \Gamma_Z^{\rm had}/\Gamma_Z^\ell$):
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    → No (negligible) non-perturbative QCD effects

    FCC-ee: $\delta R_\ell \sim 0.001$
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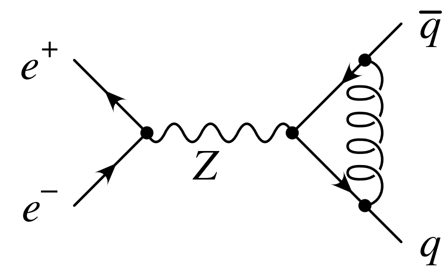
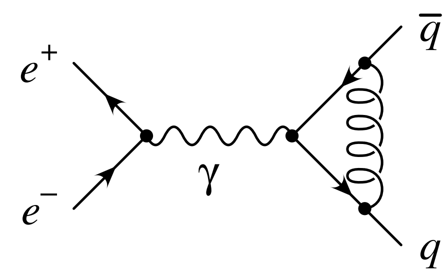


  - $R = \frac{\sigma[ee \to \text{had.}]}{\sigma[ee \to \mu\mu]}$ at lower $\sqrt{s}$
    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
    Kühn, Steinhauser, Teubner '07



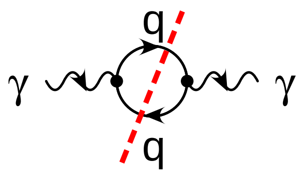
    → dominated by s-channel photon, less room for new physics
    → QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

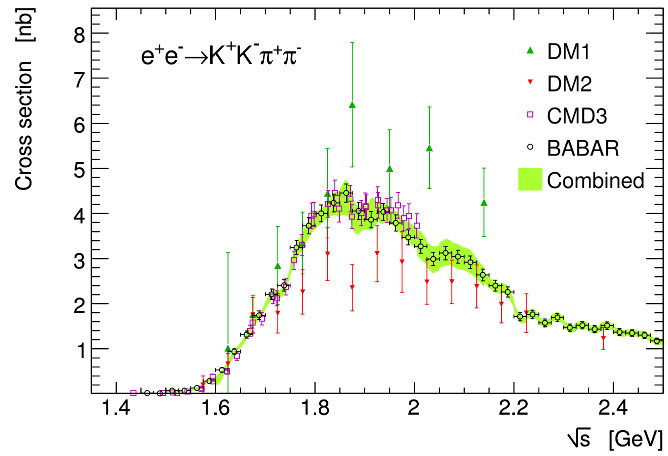
Electromagnetic coupling

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \to$ had.
using dispersion relation



Davier et al. '17

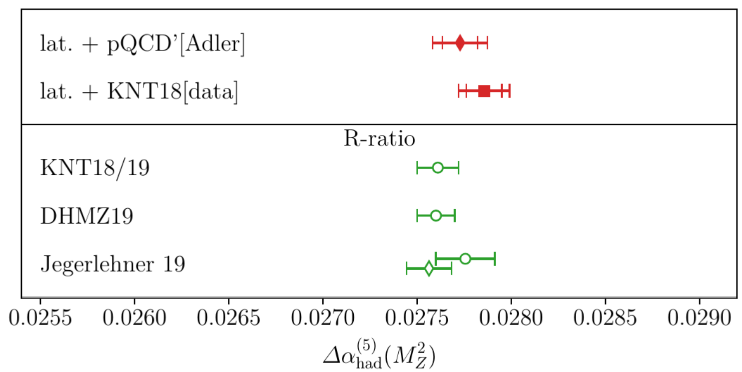


New data from BaBar, VEPP, BES, KLOE
$\rightarrow$ Robust precision $\sim 10^{-4}$
Davier et al. '19; Jegerlehner '19; Keshavarzi, Nomura, Teubner '19

Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely — Jegerlehner '19

b) $\Delta\alpha_{\text{had}}$ from Lattice QCD
(challenging but much progress)

Burger et al. '15
Cè et al. '22

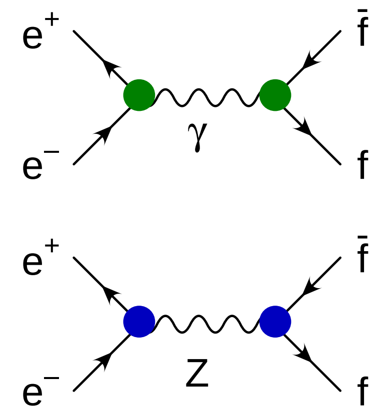


- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$

  c) Direct det. of $\Delta\alpha_{\text{had}}$ from $e^+e^- \to \mu^+\mu^-$ off the Z peak   Janot '15

$$|\mathcal{M}_{ij}|^2 \propto |g_i^\ell|^2 |g_j^\ell|^2 + (s - M_Z^2)\,\alpha(M_Z)|g_{i,j}^\ell|^2 + \ldots$$

↑ determined from Z pole



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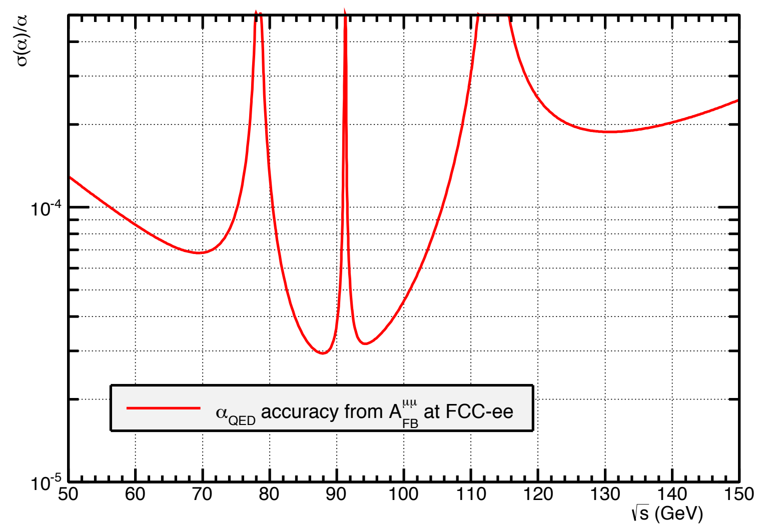
  → Use $A_{\text{FB}}^{\mu\mu}$ and two cms energies to reduce systematics

  → Sensitivity maximized for $\sqrt{s_1} \sim 88$ GeV, $\sqrt{s_2} \sim 95$ GeV

  → $\delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ for $\mathcal{L}_{\text{int}} = 85$ ab^{-1}



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  $\to$ $\delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ for $\mathcal{L}_{\text{int}} = 85\ \text{ab}^{-1}$

Requires theory input:
2-/3-loop corrections for
$e^+e^- \to \mu^+\mu^-$

Comparison of EWPOs with theory

- To probe new physics, compare EWPOs with SM theory predictions

	Current exp.	Current th.	CEPC exp.	FCC-ee exp.
M_W* [MeV]	15	4	0.5	0.4
Γ_Z [MeV]	2.3	0.4	0.025	0.025
$R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^\ell$ $[10^{-3}]$	25	5	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\text{had}}$ $[10^{-5}]$	66	10	4.3	6
$\sin^2\theta^\ell_{\text{eff}}$ $[10^{-5}]$	16	4.5	<1	0.5

* computed from G_μ

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_{W}, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '14
  Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Approximate 3- and 4-loop results (enhanced by Y_{t} and/or N_f)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06
  Chen, Freitas '20

Future theory and parametric uncertainties

Freitas, Heinemeyer, et al. '19

	CEPC	FCC-ee	perturb. error with 3-loop†	Param. error scen. 1*	Param. error scen. 2*
$M_{\rm W}$ [MeV]	0.5	0.4	1	2.1	0.6
Γ_Z [MeV]	0.025	0.025	0.15	0.15	0.1
R_b [10^{-5}]	4.3	6	5	< 1	< 1
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	<1	0.5	1.5	2	1

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\rm s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\rm s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\rm s})$, leading 4-loop

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***Scenario 1:** $\delta m_t = 600$ MeV, $\delta\alpha_{\rm s} = 0.0002$, $\delta M_{\rm Z} = 0.5$ MeV,
$\delta(\Delta\alpha) = 5\times10^{-5}$

***Scenario 2:** $\delta m_t = 50$ MeV, $\delta\alpha_{\rm s} = 0.0002$, $\delta M_{\rm Z} = 0.5$ MeV,
$\delta(\Delta\alpha) = 3\times10^{-5}$

Calculational techniques

Analytical techniques:

- Computational intensive reduction to master integrals (MIs)
- Not fully understood function space of MIs
- Works best for problems with few (no) masses

Numerical techniques:

- Large computing time
- Numerical instabilites, in particule for diagrams with physical cuts
- Works best for problems with many masses

New techniques, e.g.:

- Numerical reduction to MIs, numerical MIs via differential equations (DEs)
  Mandal, Zhao '18, Czakon, Niggetiedt '20
- DEs with respect to auxialiary parameter, $\frac{1}{k_i^2 - m_i^2 + i\epsilon}$ Liu, Ma, Wang '17
  Liu, Ma '18,21,22
- Series solutions of DEs Moriello '19, Hidding '20
- Dispersion relations + Feynman parameters Song, Freitas '21

$\rightarrow$ next week's talks

“SM Effective Field Theory” (SMEFT)

Extension of SM by **higher-dimensional operators**:

Wilson ’69
Weinberg ’79

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_i c_i \, \mathcal{O}_i^{(d)}$$

- $\mathcal{O}_i^{(d)}$ depend on all SM fields (including Higgs doublet)
- Operators must satisfy SM gauge invariance
- Valid description for energies $E \ll \Lambda$ ($\Lambda \sim$ mass of heavy particles)
- Operators ranked by suppression power Λ^{4-d}

"Higgs Effective Field Theory" (HEFT)

Review: 1706.08945

HEFT: similar to SMEFT, but Higgs and Goldstone bosons treated independently

Higgs ↑ gauge singlet; Goldstone ↑ SU(2) triplet

Example:

HEFT: $\frac{c_1}{\Lambda} h(\overline{\psi}\not{D}\psi), \quad \frac{c_2}{\Lambda^2} h^2(\overline{\psi}\not{D}\psi), \quad \ldots$

SMEFT: $\frac{c_{\phi\psi}}{\Lambda^2}(\phi^\dagger\phi)(\overline{\psi}\not{D}\psi) \quad \to \quad \frac{c_{\phi\psi}}{2\Lambda^2}(v+h)^2(\overline{\psi}\not{D}\psi)$

$$\Rightarrow \quad c_1 = c_{\phi\psi} v, \quad c_2 = \tfrac{1}{2} c_{\phi\psi}$$

→ Fewer operators, less freedom in SMEFT

→ HEFT needed to describe some composite-Higgs theories

SMEFT operators for electroweak precision

Leading dim-6 contribution: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = \frac{1}{4}|\Phi^\dagger \overset{\leftrightarrow}{D}_\mu \Phi|^2 \qquad \alpha\Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{\mathrm{BW}} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu}\Phi \qquad \alpha\Delta S = -e^2 v^2 \frac{c_{\mathrm{BW}}}{\Lambda^2}$$

$$O_{\mathrm{LL}}^{(3)\mu e} = (\bar{L}_{\mathrm{L}}^{\mu}\sigma^a\gamma_\mu L_{\mathrm{L}}^{\mu})(\bar{L}_{\mathrm{L}}^{e}\sigma^a\gamma^\mu L_{\mathrm{L}}^{e}) \qquad \Delta G_F = -\sqrt{2}\frac{c_{\mathrm{LL}}^{(3)\mu e}}{\Lambda^2}$$

$$O_{\mathrm{R}}^{f} = i(\Phi^\dagger \overset{\leftrightarrow}{D}_\mu \Phi)(\bar{f}_{\mathrm{R}}\gamma^\mu f_{\mathrm{R}}) \qquad f = e, \mu\, \tau, b, lq$$

$$O_{\mathrm{L}}^{F} = i(\Phi^\dagger \overset{\leftrightarrow}{D}_\mu \Phi)(\bar{F}_{\mathrm{L}}\gamma^\mu F_{\mathrm{L}}) \qquad F = \begin{pmatrix}\nu_e\\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}, \begin{pmatrix}u, c\\ d, s\end{pmatrix}, \begin{pmatrix}t\\ b\end{pmatrix}$$

$$O_{\mathrm{L}}^{(3)F} = i(\Phi^\dagger \overset{\leftrightarrow}{D}{}^a_\mu \Phi)(\bar{F}_{\mathrm{L}}\sigma_a\gamma^\mu F_{\mathrm{L}})$$

More operators than EWPOs $\longrightarrow$ need assumptions:

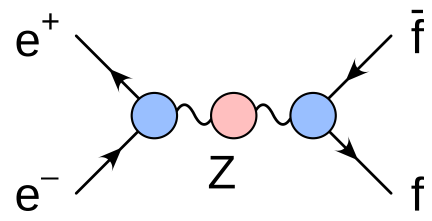
- Family universality, e.g. $c_{\mathrm{R}}^{e} = c_{\mathrm{R}}^{\mu} = c_{\mathrm{R}}^{\tau}$
- U(2)×U(1) flavor symmetry, e.g. $c_{\mathrm{R}}^{e} = c_{\mathrm{R}}^{\mu} \neq c_{\mathrm{R}}^{\tau}$

Translation SMEFT ↔ EWPOs

Leading Z pole contribution:

Modified propagators:

e.g. $\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi \, \Phi^\dagger (D^\mu \Phi)$
$\mathcal{O}_{\rm BW} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$



Modified Z-fermion couplings:

e.g. $O^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar f \gamma^\mu f)$ $\quad f = e, \mu \, \tau, b, \ldots$

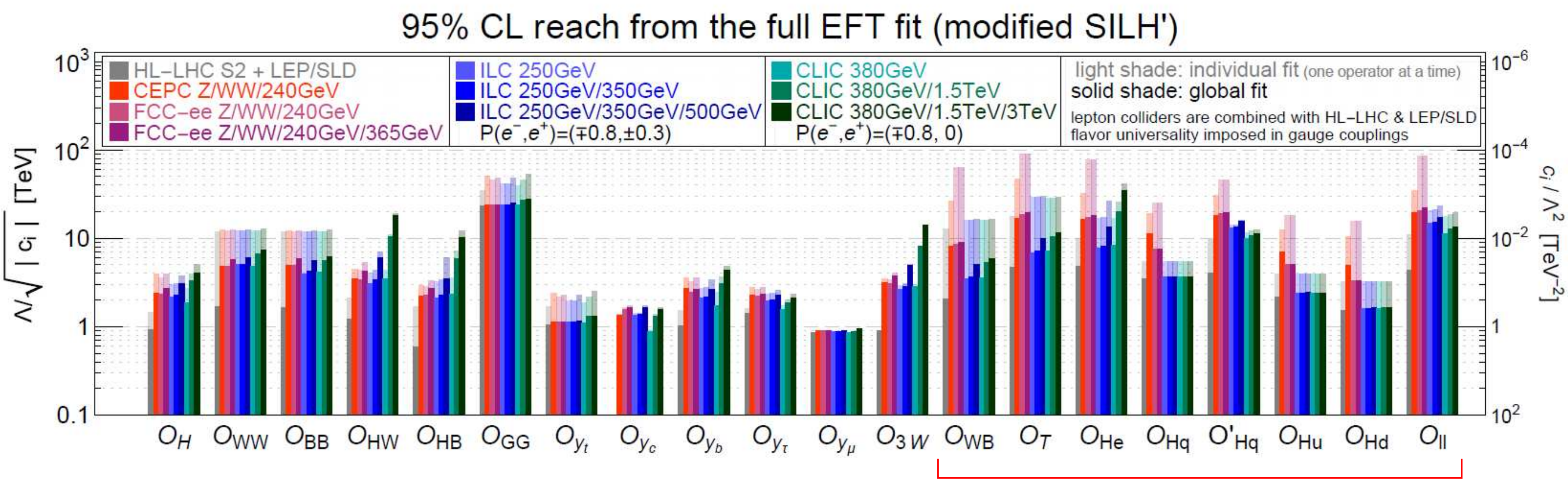
Direct correspondence between pseudo-observables and SMEFT ops.:

$$\frac{\delta \Gamma_{ff}}{\Gamma_{ff}} = 2\left[\frac{\delta Z^f_L}{Z^f_L} + \frac{\delta Z^f_R}{Z^f_R}\right] \qquad \frac{\delta \sin^2\theta^f_{\rm eff}}{\sin^2\theta^f_{\rm eff}} = \frac{1}{1 - Z^f_L/Z^f_R}\left[\frac{\delta Z^f_L}{Z^f_L} - \frac{\delta Z^f_R}{Z^f_R}\right]$$

$$\delta Z^f_X = \frac{1}{\Lambda^2}\left[I_3^{fX} c_L^{(3)f} - \frac{1}{2} c^f_X - Q_f \frac{sc}{c^2 - s^2} c_{WB} - \left(\tfrac{1}{16} c_{\phi 1} - \tfrac{1}{4} c^{(3)}_{LL} + c^\ell_L\right)\left(I_3^{fX} + Q_f \frac{s^2}{c^2 - s^2}\right)\right]$$

$$\frac{\delta M_{\rm W}}{M_{\rm W}} = \delta Z^\nu_L - \delta Z^e_L - \tfrac{1}{4} c^{(3)}_{LL}$$

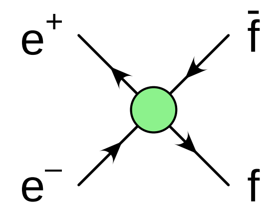
Project reach of future colliders to constrain SMEFT operators:



de Blas, Durieux, Grojean, Gu, Paul ’19

Beyond leading Z pole ("background"):

Additional 4-fermion operators

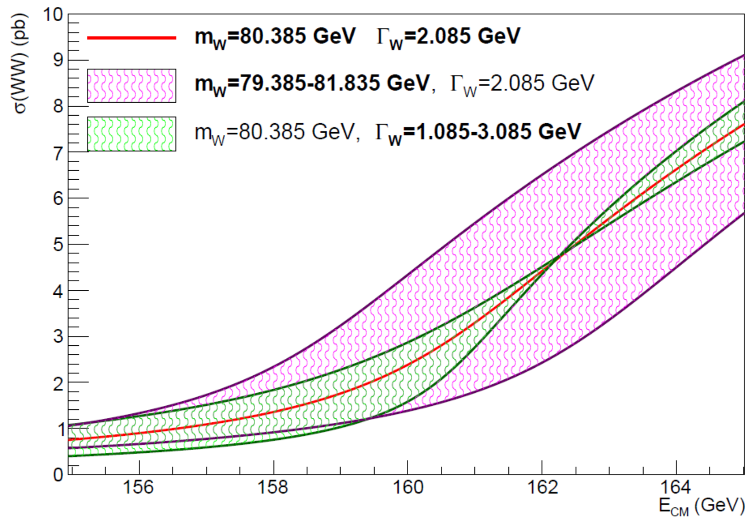


e.g. $O^{(1)}_{ff'} = (\bar{f}\gamma_\mu f)(\bar{f}'\gamma^\mu f')$ $\quad f, f' = e, \mu\, \tau, b, \ldots$

(related via e.o.m. to energy-dependent $Z f f/\gamma f f$ couplings $O^D_f \sim \bar{f}\gamma^\mu F_{\mu\nu} D^\nu f$)

$\longrightarrow$ Model-independent analysis requires additional parameters (or Wilson coeffs.) in non-resonant terms

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold



- a) Corrections near threshold enhanced by $1/\beta$ and $\ln \beta$

  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important

- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05

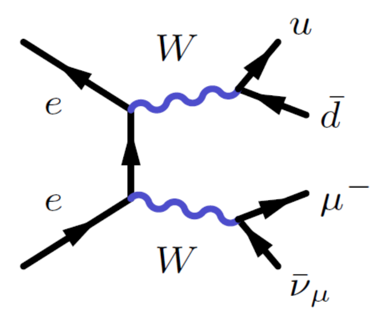
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

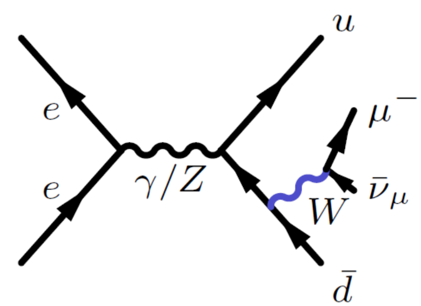
  Beneke, Falgari, Schwinn, Signer, Zanderighi '07

  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\rm th} M_W \sim 3$ MeV

    Actis, Beneke, Falgari, Schwinn '08

  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\rm th} M_W \lesssim 0.6$ MeV





WW threshold

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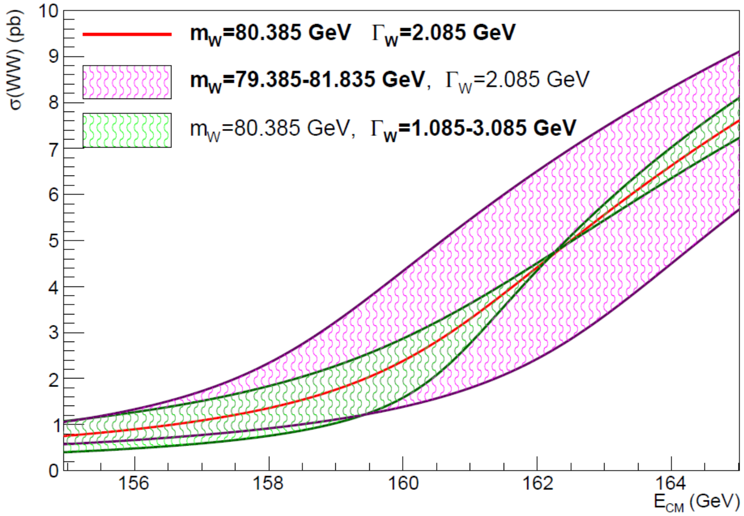

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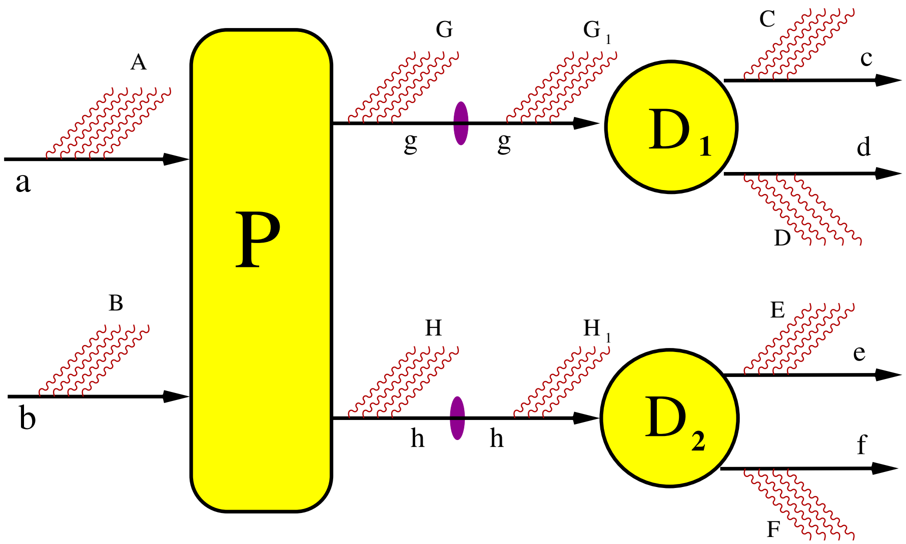
  b) Non-resonant contributions are important

- Resummation of soft photon radiation

Jadach, Płaczek, Skrzypek '19



Summary

- Electroweak precision tests require theory input for **measurements of pseudo-observables** (BRs, widths, masses, cross-sections, ...) and their **SM/BSM interpretation**

- **Future e^+e^- colliders** improve precision by 1–2 orders of magnitude

- Theory progress needed both for **fixed-order loop corrections** as well as **MC tools**

- Model-independent description of BSM effects through **EWPOs** (for leading Z-pole term) and **SMEFT operators**

- Accurate description of non-resonant terms (backgrounds) requires a **gauge-invariant framework**, **SM corrections**, and accounting of **BSM effects**

Backup slides

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_s$, $\alpha\alpha_s^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_s$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_s$
$\sin^2\theta^\ell_{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_s$

- Theory error estimate is not well defined, ideally $\Delta_{th} \ll \Delta_{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series $\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26\ \text{MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.30\ \text{MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23\ \text{MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035\ \text{MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1\ \text{MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z\alpha^2 \sim 0.1\ \text{MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29\ \text{MeV}$$

Total: $\delta\Gamma_Z \approx 0.5\ \text{MeV}$