Possibilities and precision goals for QCD measurements

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Precision calculations for future e^+e^- colliders: targets and tools
07.06.22, CERN

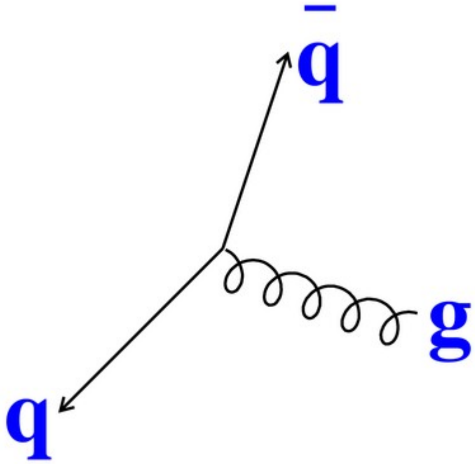
Event shapes (jets)
Colour factors
Particle spectra
Heavy Quark masses
Z, W EWPO
Hadronic τ decays
“Shopping list”

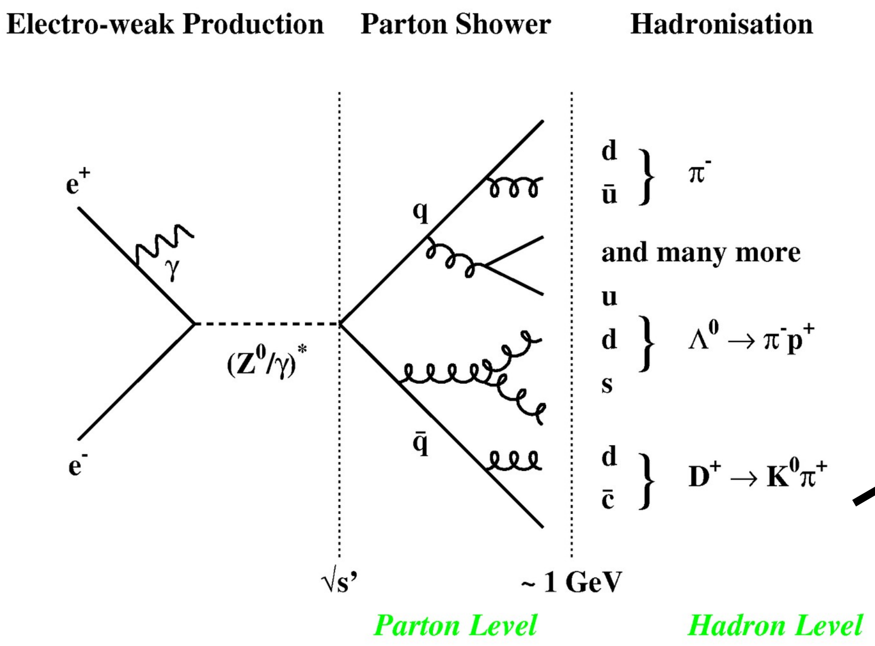
See also: D. d'Enterria, SK, G. Zanderighi (eds.), The strong coupling constant: State of the art and the decade ahead, 2203.08271

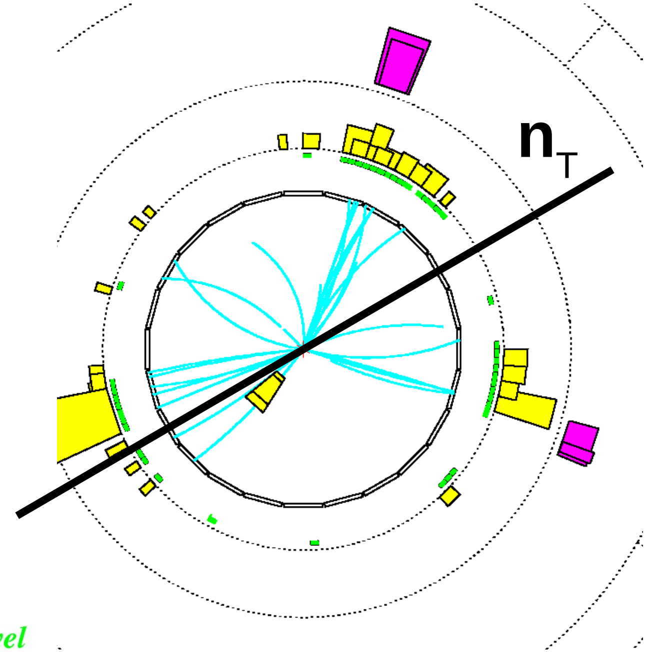
e^+e^- annihilation to hadrons

Interpretation: $e^+e^- \to q\bar{q}g$

OPAL $\sqrt{s}$ = 91.2 GeV







$$T = \max_{\mathbf{n}_T}[\, \sum_i |\mathbf{p}_i \cdot \mathbf{n}_T| \,/\, \sum_i |\mathbf{p}_i| \,]$$

Event shapes: predictions

Fixed order massless, $\bar{\alpha}_S = \alpha_S/(2\pi)$, e.g. $y = 1-T$

$d\sigma/dy^{\text{massless}}(Z \to 3 \text{ jets})$
$= \bar{\alpha}_S dA/dy + \bar{\alpha}_S^2 dB/dy + \bar{\alpha}_S^3 dC/dy$
+ ren. sc. + "$\sigma_0 \to \sigma_{had}$"

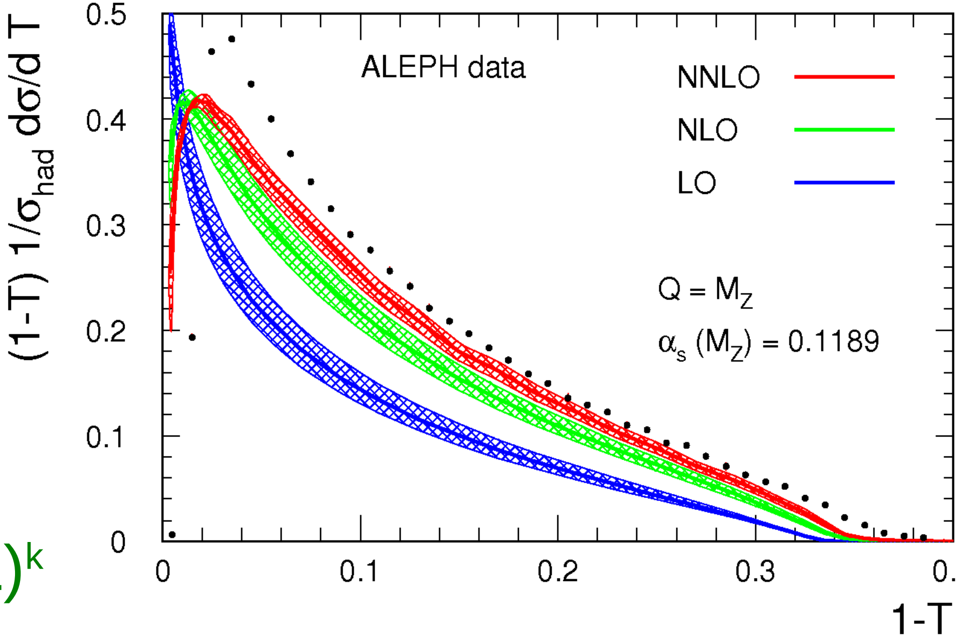
Resummation ($L=\log(1/y)$, $R=\int_0^y d\sigma/dy' dy'$)

LL NLL

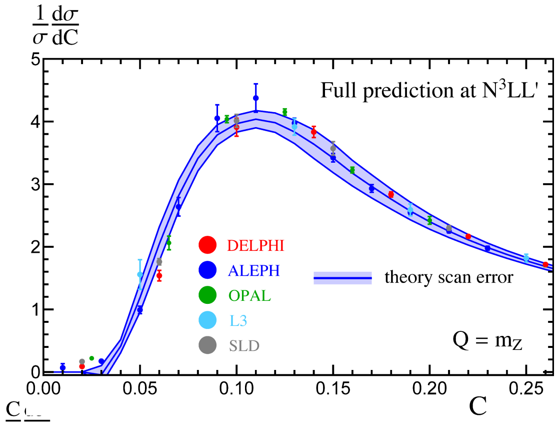
$\text{Log}(R) \sim L\sum_k(\alpha_S L)^k + \sum_k(\alpha_S L)^k$
$+ \alpha_S\sum_k(\alpha_S L)^k + \alpha_S^2\sum_k(\alpha_S L)^k$

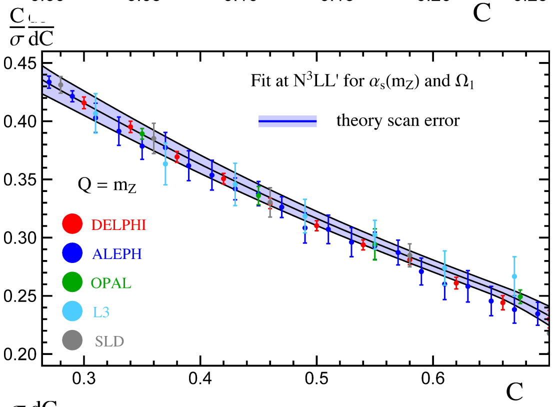
NNLL N3LL

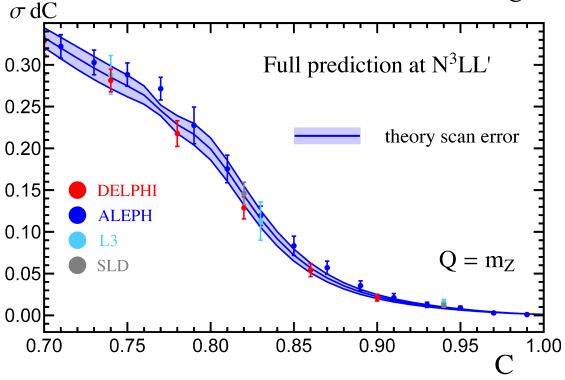
[GGGH, Phys. Rev. Lett. 99 (2007) 132002]



Event shapes: T and C





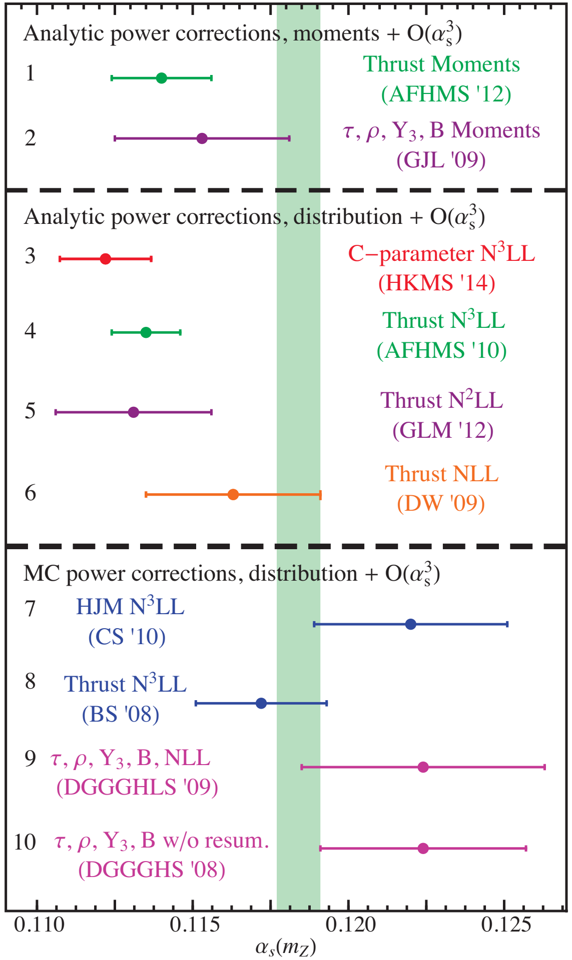


[Hoang, Kolodrubetz, Mateu, Stewart, Phys. Rev. D91 (2015) 094018]

C/6 ≈ 1-T; 1-T, C ≪ 1

Had. model links pQCD and (particle-level) data, e.g. dσ(C) → dσ(C – Ω_1^C/Q).

Consistency with MC based corrections?



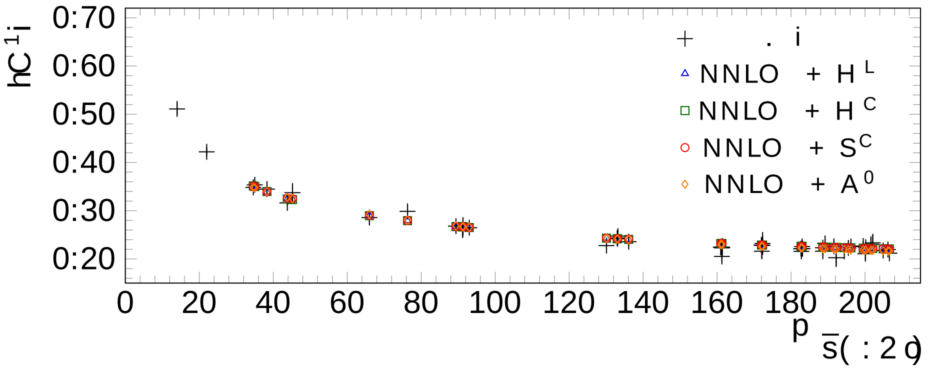
Event shape moments N3LO'

$$\langle y\rangle = \bar{\alpha}_S A_{\langle y\rangle} + \bar{\alpha}_S^{\,2} B_{\langle y\rangle} + \bar{\alpha}_S^{\,3} C_{\langle y\rangle} + \bar{\alpha}_S^{\,4} D_{\langle y\rangle}$$

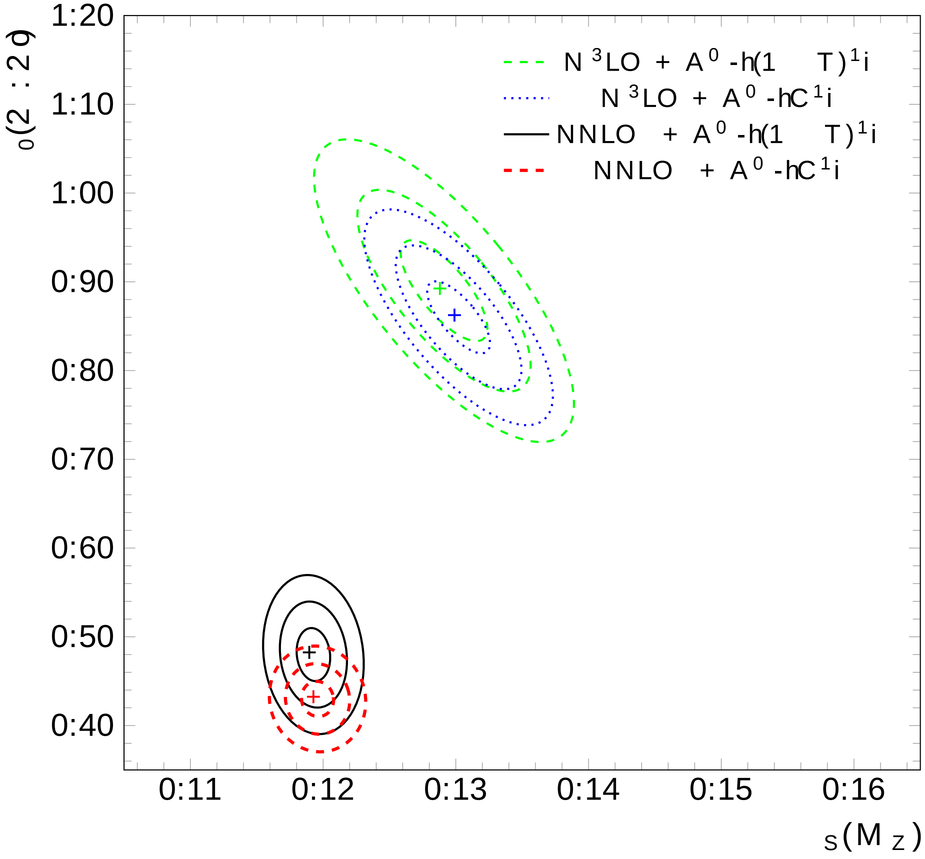
D not known, free in fit

DMW power corrections, or MC

"Consequently, the improvement of the hadronization modeling and a better understanding of hadronization itself is more important for increasing the precision of $\alpha_s(m_z)$ extractions than the calculation of perturbative corrections beyond NNLO"



[Kardos, Somogy, Verbytskyi,
Eur. Phys. J. C81 (2021) 292]

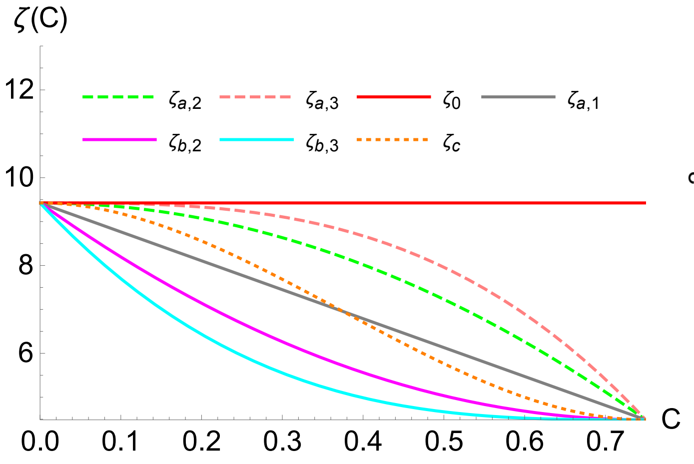


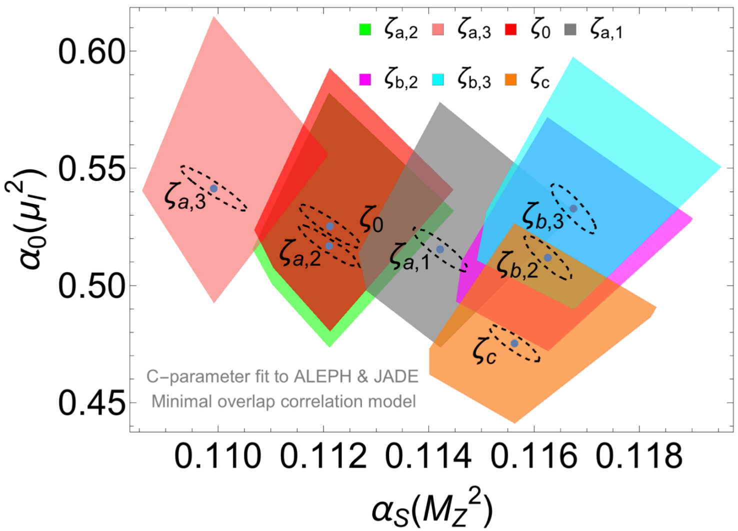
Power corrections in 3-jet limit

DMW power corrections in the 2- and 3-jet limit for C-parameter (C=0 or C=3/4) with C-dependence in ζ(C):

$\langle \delta C_{pc}(C) \rangle = \zeta(C)\ M\ \mu_I/Q\ 4C_F/\pi^2\ (\ \alpha_0(\mu_I^2) - \alpha_S(\mu_R^2) + \dots\ )$

$\zeta(C=0) = 3\pi = 9.42 \neq \zeta(C=3/4) = 4.49$





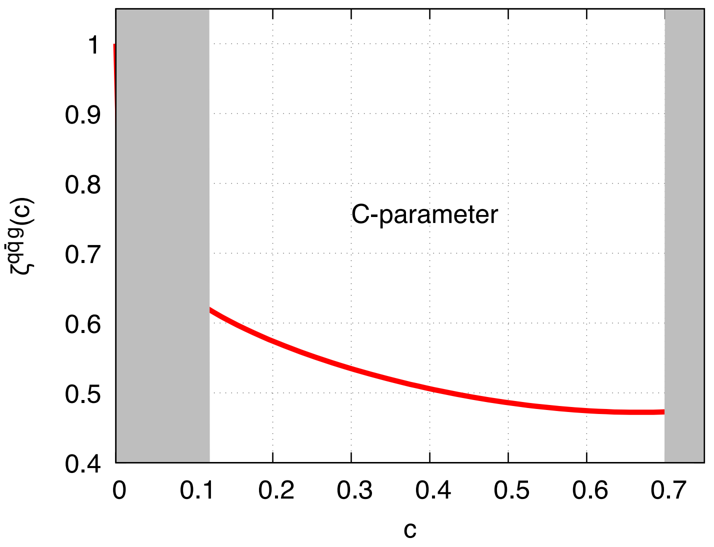
"Overall, the results suggest that one should allow for a 3−4% uncertainty in α_s extractions from e^+e^- C-parameter data, associated with limitations in our current ability to estimate hadronisation corrections"

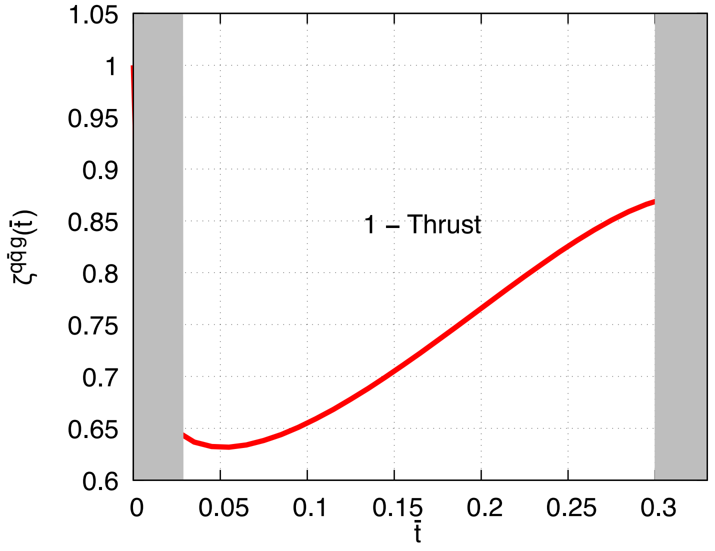
[Luisoni, Monni, Salam, Eur. Phys. J. C81 (2021) 2]

Power corrections in 3-jet limit

[Caola, Ravasio, Limatola, Melnikov, Nason, Ozcelik, 2204.02247]

Calculate power corrections in abelian large n_f approximation for 2, 3 and n-jets $\gamma^* \to q\bar{q}\gamma + (g^* \to q\bar{q})$ for C and 1-T





Reproduce LMS for C=0,3/4, “non-trivial shape” in fit regions

Radiative Z decays

Select “radiative” hadronic Z decays with isolated high energy photon ($E_\gamma > 10$ GeV, $|\cos\theta_\gamma| < 0.72$, $\alpha_{\gamma\text{-jet}} > 25^o$) → FSR dominated

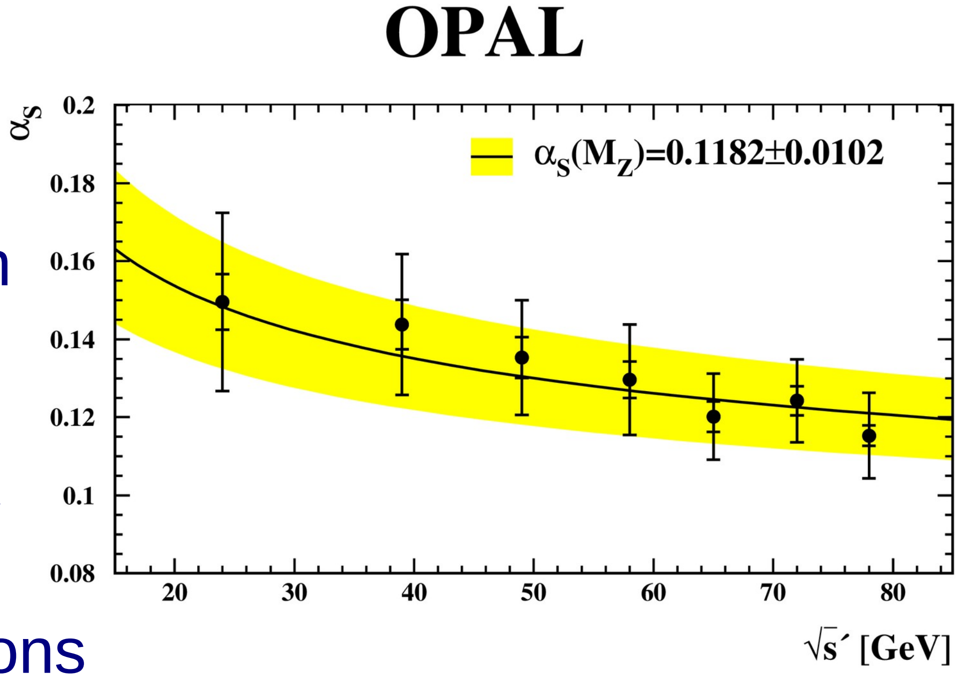
$\sqrt{s'} = 2E_{beam}\sqrt{(1-E_\gamma/_{Ebeam})}$

78.1 GeV < $\sqrt{s'}$ < 24.4 GeV,

7 $\sqrt{s'}$ bins, ~300-1500 events/bin

1–T, M_H, C, B_T, B_W, $y_{23,D}$

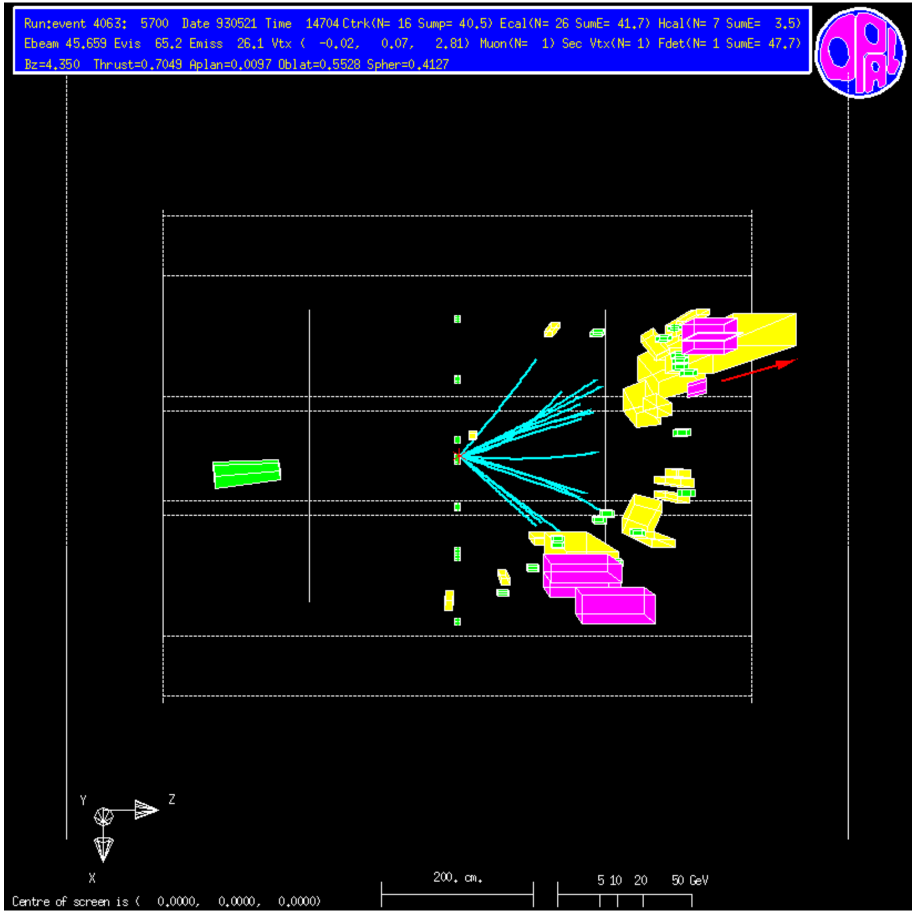
Problem: α_S extractions with MC based correction for γ ISR/FSR, QCD analysis w/o QED corrections



[OPAL coll., Eur. Phys. J. C53 (2008) 21]

Radiative events: future colliders

[opal.web.cern.ch/Opal/events/opalpics.html]



Future e^+e^- colliders expect HUGE Z statistics: 10^3 to $10^6 \cdot$ LEP

ISR dominated event selection: high energy γ in luminosity detector, invisible beam-collinear γ from boosted hadronic event kinematics, ~O(10^{-4}) (boe!) selection fraction

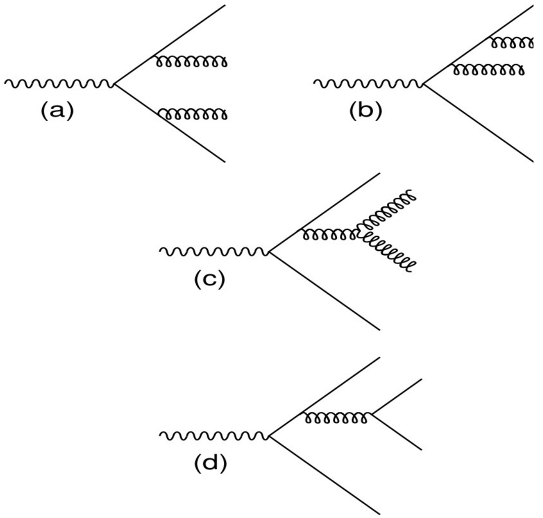
Need QED/EW corrections for $e^+e^- \rightarrow \gamma + n$ jets at $\sqrt{s} \approx m_Z$

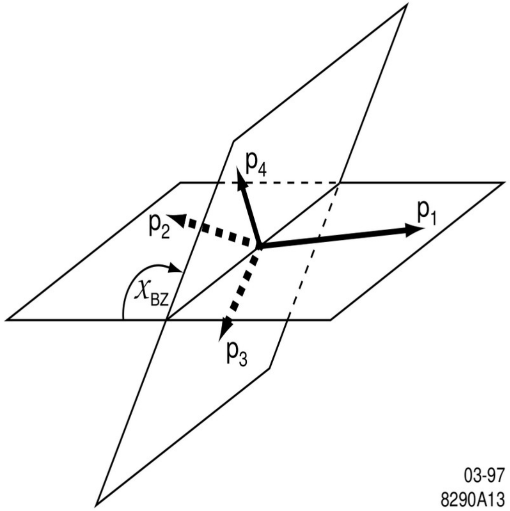
[SNOWMASS21-EF5_EF4_Andrii_Verbytskyi-208.pdf]

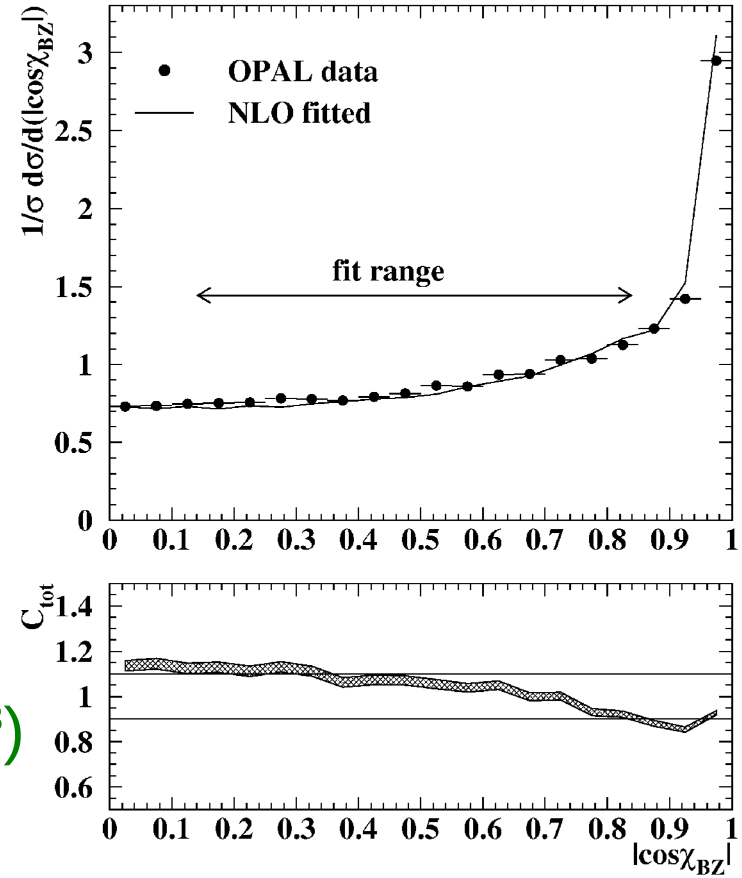
Or: dedicated runs $\sqrt{s} < m_Z$!

QCD gauge structure: 3-g vertex

[PN Burrows, SLAC-PUB-7434, hep-ex/9705013]







$$d\sigma/dy(Z \to 4\ \text{jets}) = \bar{\alpha}_S^{\,2} \left(C_F^{\,2} dB_{CF}/dy + C_F T_F dB_{TF}/dy + C_F C_A dB_{CA}/dy \right) + O(\bar{\alpha}_S^{\,3})$$

Limited by th. and had. uncertainties

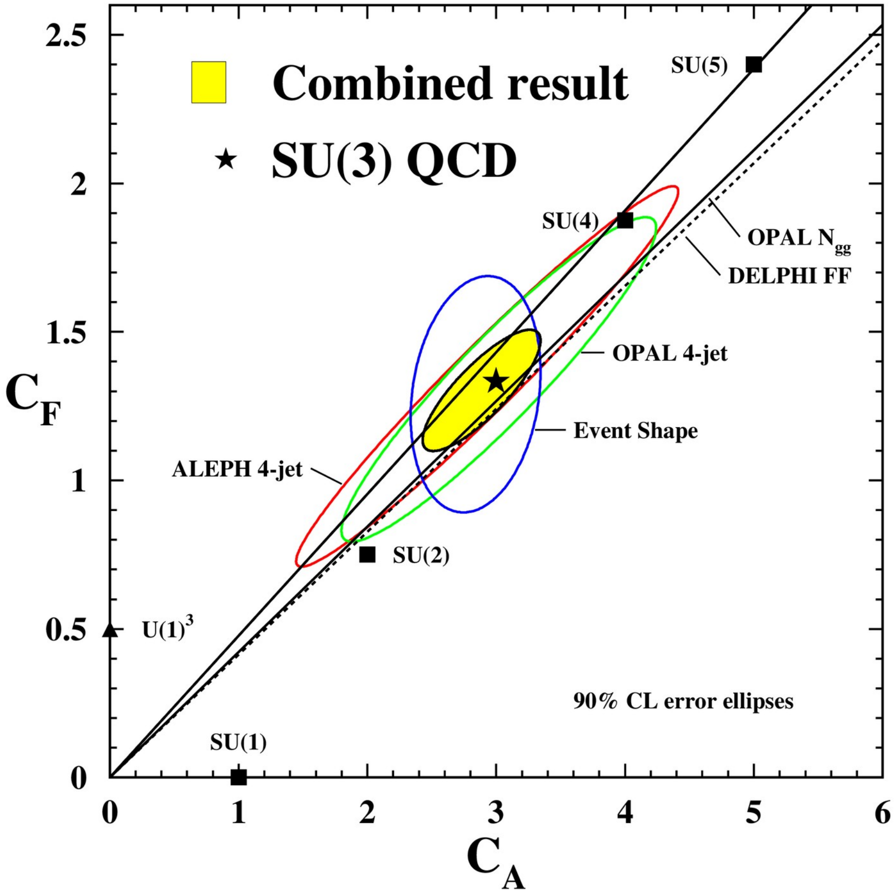
QCD gauge structure: 3-g vertex

Event shapes: fits to 1-T, C et al.
$\sqrt{s}$ = 14-189 GeV
$O(\alpha_S^2)$ + NLL + DMW pc

Today: $O(\alpha_S^3)$ + N3LL + pc, but still had. systematics

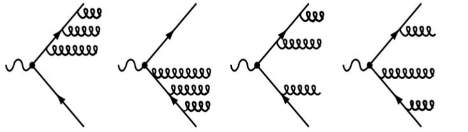
Improved had. models?

Impact of colour factors on MC had. models?

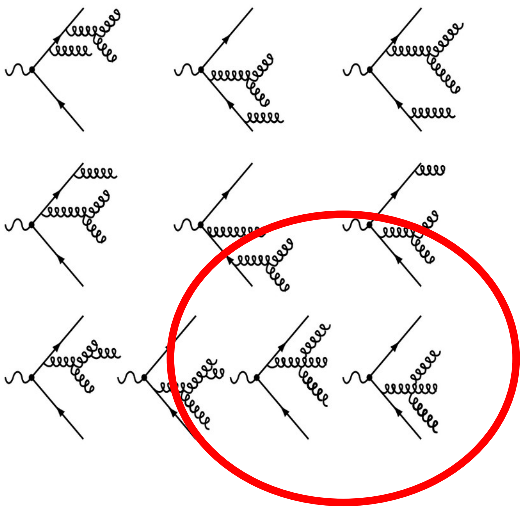


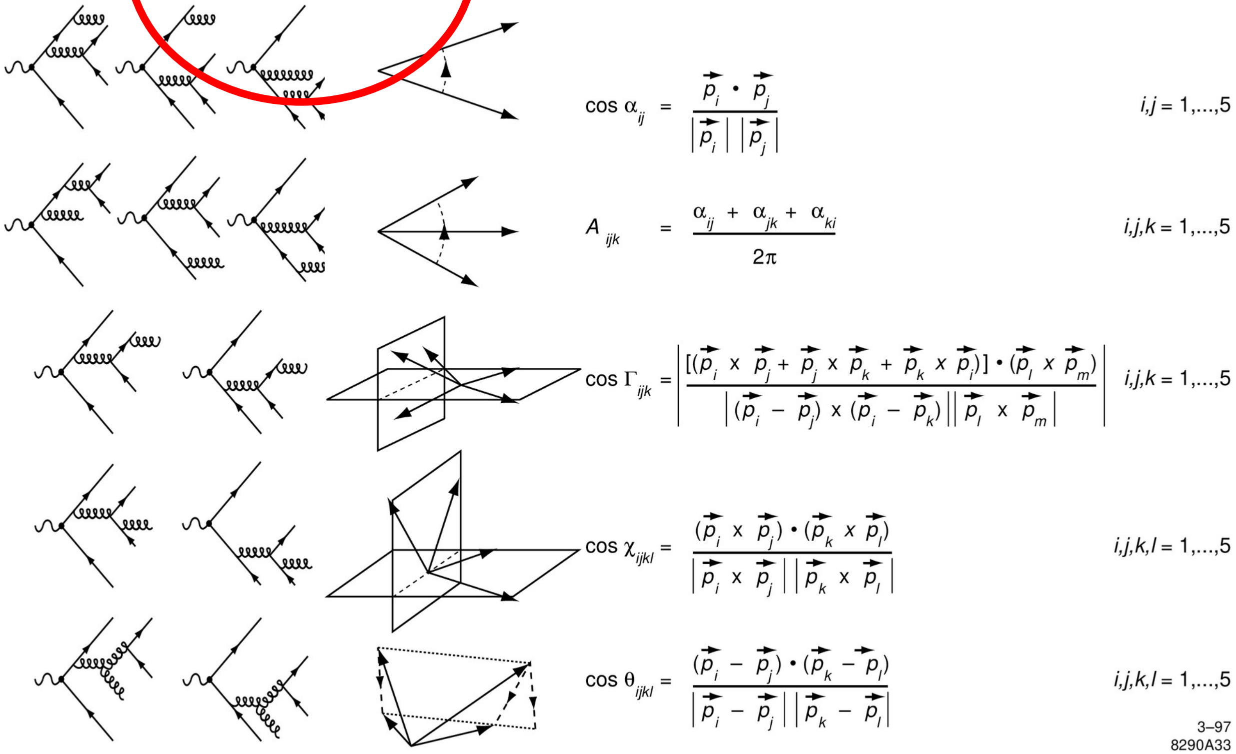
QCD gauge structure: 4-g vertex

$$d\sigma/dy(Z \to 5\text{ jets}) = \bar{\alpha}_S^{\,3}\,(\,C_F^{\,3}\,dC_{CF}/dy + C_F^{\,2}C_{TF}\,dC_{TF}/dy + C_F^{\,2}C_A\,dC_{CA}/dy + C_F C_A^{\,2}\,dC_{CA2}/dy\,) + O(\bar{\alpha}_S^{\,4})$$

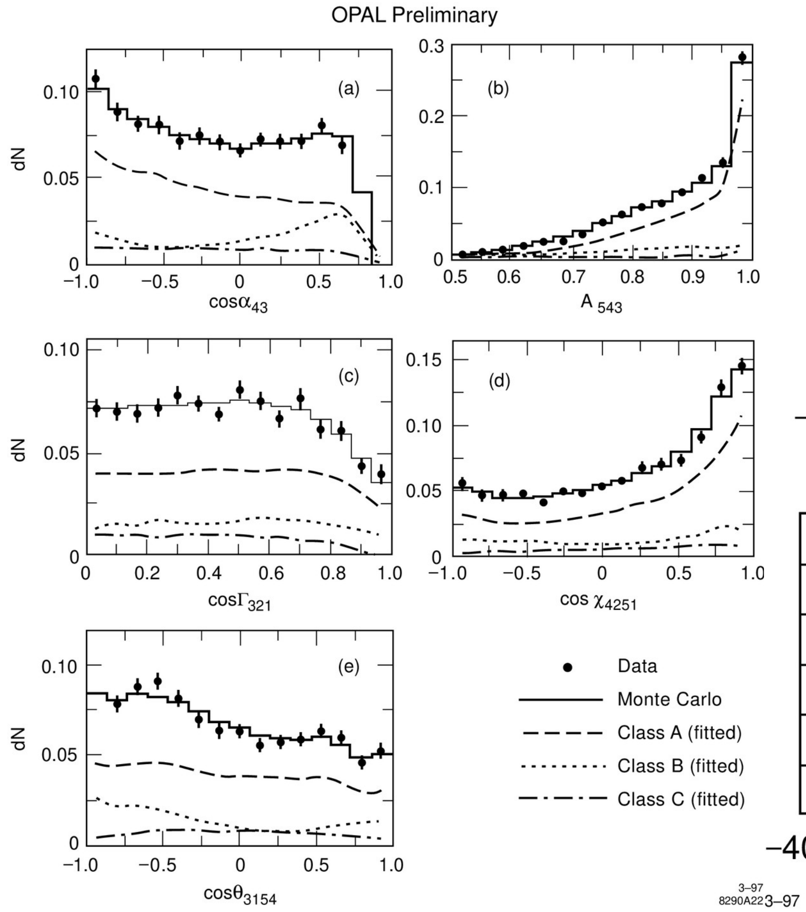


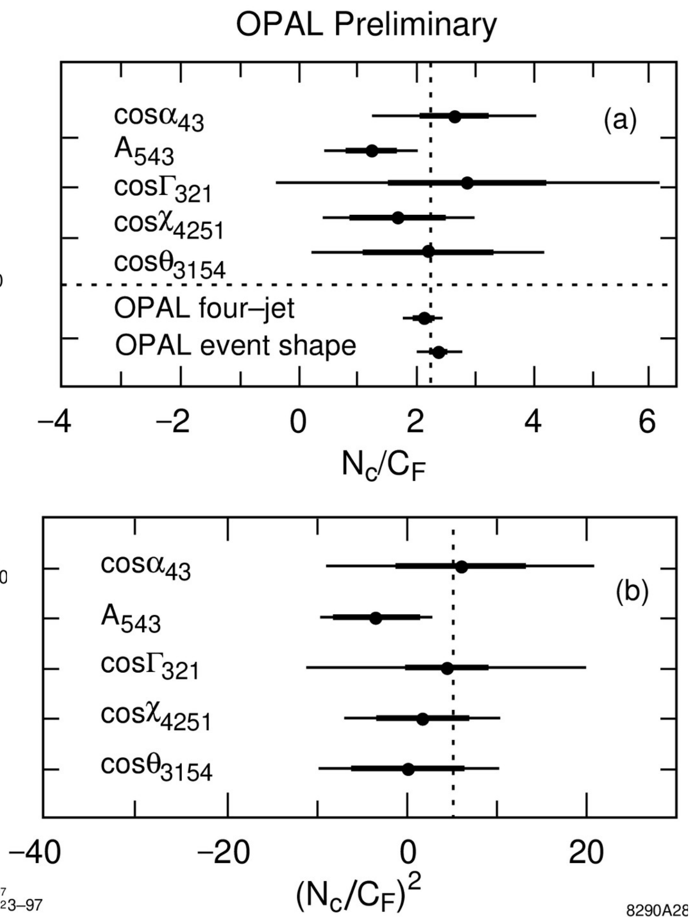
[PN Burrows, SLAC-PUB-7434, hep-ex/9705013]





QCD gauge structure: 4-g vertex





LO only, large had. systematics

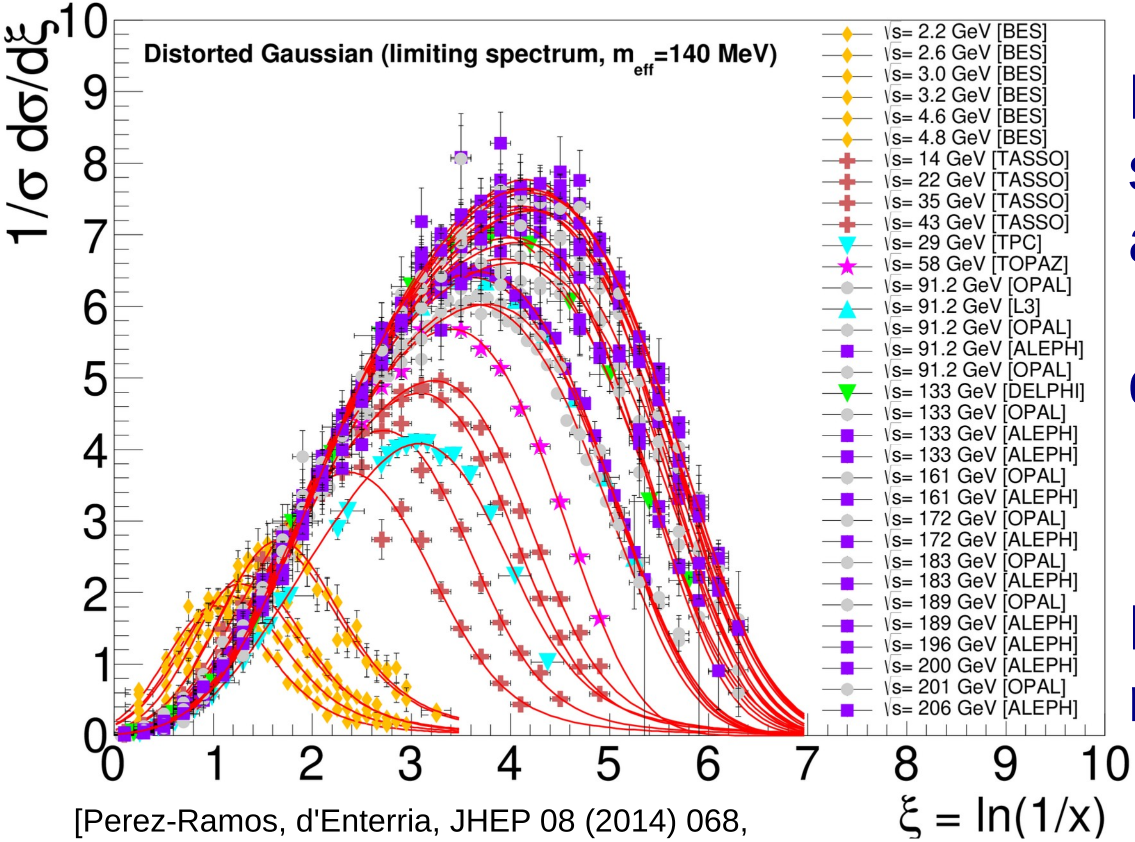
Possible: $d\sigma/dy(Z \to 5 \text{ jets})$ in NLO with colour decomposition

$d\sigma/dy(Z \to 5 \text{ jets})$ in NNLO?

Improved had. models?

Fragmentation functions

Parton-hadron fragmentation $x = 2p/\sqrt{s}$



[Perez-Ramos, d'Enterria, JHEP 08 (2014) 068, 2203.08271]

NMLLA + NLO* for limiting spectra fits and evolution, also include DIS data

$\alpha_S(m_Z) = 0.1205 \pm 0.0010 \text{ (fit)} {}^{+0.0022}_{-0.0000} \text{ (evolution)}$

Extensions: include NLO + resummed splitting functions

Heavy quark effects

$\Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons}) = R_b^0 = 0.21629 \pm 0.00066$

$\Rightarrow$ 21.6% of hadronic events at Z peak are $b\bar{b}$ (and 17% $c\bar{c}$)

$$d\sigma/dy^{udscb}(Z \to 3\text{ jets}) = R_b^0(\,\bar{\alpha}_S dA^b/dy + \bar{\alpha}_S^2 dB^b/dy\,)$$
$$+ (1 - R_b^0)(\,d\sigma/dy^{massless}(Z \to 3\text{ jets})\,)$$

Dilutes precision: ~1/5 of NLO scale unc. (N3LO?)

Mass effects in resummation for jets up to NLL

[KR, Phys. Lett. B576 (2003) 135]

Limits flavour dependent analyses: α_S universality, $m_b(m_Z)$ from jets/event shapes for $m_b(Q)$ evolution studies, ...

$m_b(Q)$ evolution

$\Gamma(H \to b\bar{b})/\Gamma(H \to ZZ)$ (ATLAS, CMS)

$m_b(m_H) = (2.60^{+0.36}_{-0.31})$ GeV

$\Delta m_b(m_H) \approx 0.01$ GeV (future coll.)

$m_b(\mu) = y(\mu)v(\mu)/\sqrt{2}$,

$y(\mu)$ or $m_b(\mu)$ but not both

3-jet rates in b-tagged e^+e^- hadronic final states at $\sqrt{s} = m_Z$ (LEP, SLD)

$m_b(m_Z) = (2.90 \pm 0.31)$ GeV

$\Delta m_b(m_Z) \approx 0.12$ GeV (FCC-ee, NNLO, Improved had.)

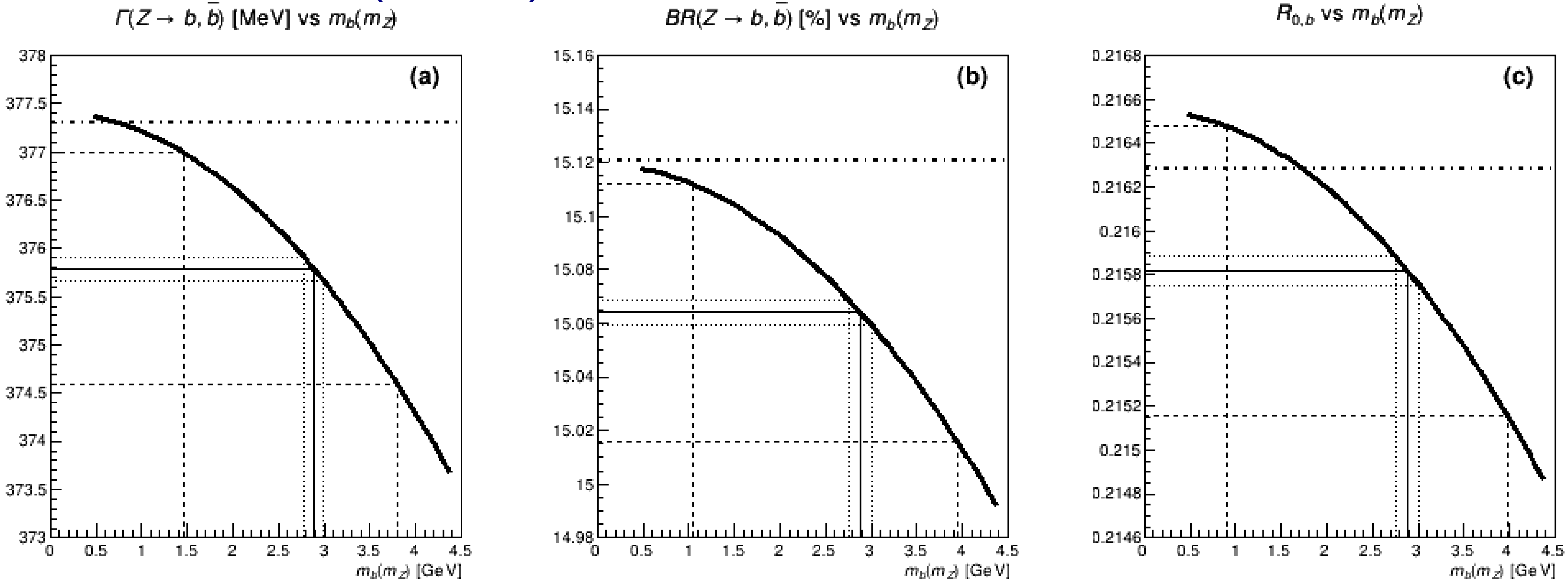
[J. Aparisi et al., Phys. Rev. Lett. 128 (2022) 122001,
SK, Rept. Prog. Phys. 69 (2006) 1771,
J. Fuster et al., ILD-PHYS-PUB–2021-001]

$m_b(Q)$ evolution from $Z \to b\bar{b}$ EWPO

LEP/SLD data — dash-dotted
$m_b(m_Z)$ = 2.88 GeV — solid
LEP/SLD errors — dashed
LEP/SLD errors 1/10 (FCC-ee) — dotted

With LEP/SLD data impossible, study errors at reference $m_b(m_Z)$

$\Delta m_b(m_Z) \approx 0.14$ GeV FCC-ee



[SK, Eur. Phys. J. C82 (2022) 240]

Z EWPO

Full EW fit, or Γ_Z^{tot}, R_Z, and σ_Z^{had}, updated for LEP lumi correction and 2-loop (partial 3-loop) EW corrections

today FCC-ee

For FCC-ee: missing α_s^5, α^3, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

[d'Enterria, Jacobsen, 2005.04545]

W EWPO

Γ_W^{lep}, Γ_W^{had}, Γ_W^{tot}, w/ or w/o CKM unit., N3LO, α and $\alpha\alpha_s$ EW corrections

today

FCC-ee

For FCC-ee: missing α_s^5, α^2, α^3, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

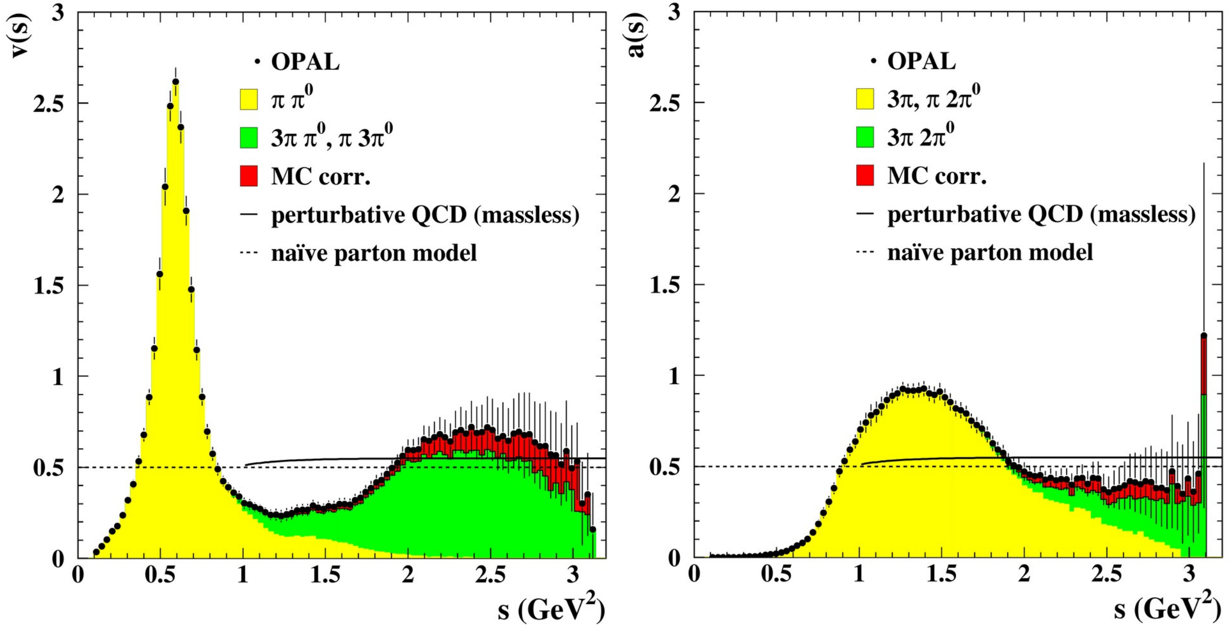
[d'Enterria, Jacobsen, 2005.04545]

Hadronic τ decays

Spectral functions: $R_{\tau,v/a}(s) = \Gamma(\tau \to \text{pions}\ \nu_\tau)/\Gamma(\tau \to e\nu_e\nu_\tau)$

Spectral moments: $R_{\tau,v/a}^{kl}(s_0) = \int_0^{s0} (1-s/s_0)^k (s/m_\tau^2)^l\, dR_{\tau,v/a}/ds\, ds$

$R_{\tau,v/a}^{kl}(s_0) = S_{EW} |V_{ud}|^2 (1 + \delta_{EW} + \delta_{pert} + \delta_{non-pert,v/a})$



PDG '22 (τ decays):
$\alpha_S(m_Z) = 0.1178 \pm 0.0019$

Uncertainty limited by:
δ_{pert} (pert. effects)
$\delta_{non-pert,v/a}$ (non-pert effects)

Hadronic τ decays

δ_{pert}: up to $(\alpha_S(m_\tau)/\pi)^4$, but FOPT vs CIPT?

[Benitez-Rathgeb, Boito, Jamin, Hoang, t.a.]

CIPT^{RS} (renormalon subtracted) seems consistent, depends on “gluon condensate norm” and fact. scale

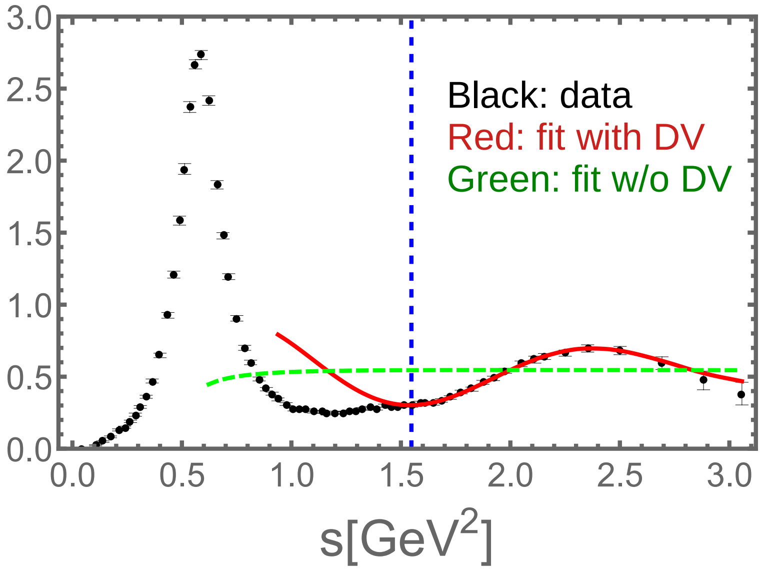
$\delta_{non-pert,v/a}$: truncated OPE vs duality violation (DV) modelling,

$\delta_{non-pert,OPE} \sim c_4/q^4 - c_6/q^6 + c_8/q^8 \pm \ldots$

choose spectral moments to suppress

$\delta_{non-pert,DV}$: include model for DV effects

[Boito et al., Phys. Rev. D103, 034028 (2021)]



Shopping list

	f.o.	resummation	soft corrections
$e^+e^- \rightarrow$ 3 jets	N3LO?	beyond N3LL?	y dependent
$e^+e^- \rightarrow$ 4 jets	NNLO?	MC N(N?)LL PS	possible? MC
$e^+e^- \rightarrow$ 5 jets	NNLO?	MC N(N?)LL PS	possible? MC
$e^+e^- \rightarrow$ hadrons		beyond NMMLA?	
$e^+e^- \rightarrow$ γ+n jets	NLO EW		
$e^+e^- \rightarrow Q\bar{Q}g$	NNLO	NNLL? N(N?)LL MC PS?	
Z/W EWPO	α_s^5, α^2, α^3, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$		
τ had. decays	α_s^5? FOPT/CIPT^{RS}		OPE vs DV models