Shape Variables and Power Corrections

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Shape variables and QCD

Shape variables in e^+e^- annihilation are the simplest contest where we can study perturbative QCD.

For example, thrust:

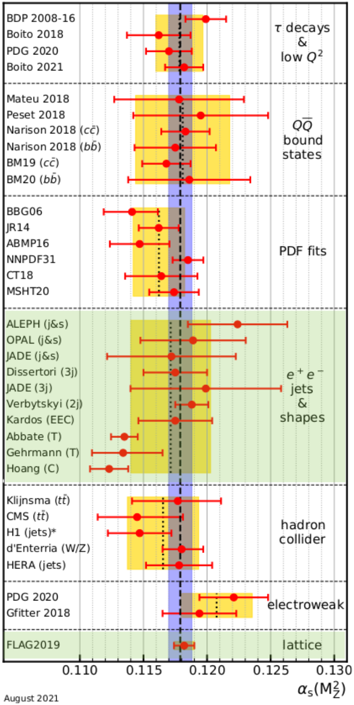
$$T = \max_{\vec{t}} \frac{\sum |\vec{p}_i \cdot \vec{t}|}{\sum |\vec{p}_i|}$$

equals 1 for two narrow back-to-back jets, and $2/3 < T < 1$ for three narrow jet.
Thus in the region $2/3 < T < 1$ the thrust distribution is proportional to α_s, and can be used for its determination.

On the other hand, the thrust distribution is sensitive to non-perturbative hadronization effects.

For example, the emission of a soft hadron with momentum 500 MeV, perpendicular to the thrust direction, affects the thrust by an amount $0.5/91 \approx 0.005$ on the Z peak. This shift in T can affect the thrust distribution by an amount of the order of 5%.

In practice non-perturbative corrections can reach the 10% level, and can affect at the same level the extracted value of α_S.



α_s determinations (PDG)

Determination of α_s from the first seven rows of the jets & shapes cathegory (highlighted in green) use Monte Carlo model to correct for non-perturbative effects.

The following three lines (Abbate, Gehrmann, Hoang) are based upon analytic modeling of non-perturbative effects.

Status

- The use of Monte Carlo modeling for hadronization corrections is not totally satisfying, since it lacks a sound theoretical basis.
- Analytic models seem to favour a too low value of α_s as compared to the precise lattice determination.
- No bridge between MC and analytic models
- It is disturbing that we do not fully understand the role of non-perturbative effects at least in the simplest context where they can be studied.
- Understanding non-perturbative effects can have important consequences also for precision physics at hadron colliders, where linear power corrections can play an important role.

Recent progress

There have been recently new findings regarding the structure of linear power corrections in collider observables:

- In ref. Eur.Phys.J.C 81 (2021), (Luisoni, Monni, Salam) it was shown that linear power corrections to the C parameter in the 3-jet symmetric limit are about $1/2$ of those in the two jet limit.
- In ref. JHEP 01 (2022) 093, (Caola, Ferrario-Ravasio, Limatola, Melnikov, P.N.) it was demonstrated that linear power corrections are absent in sufficiently inclusive observables, in a variety of processes, in a model theory (large n_f QCD) that shares some properties with the full theory. These findings confirmed previous results obtained at the numerical level JHEP 06 (2021) 018, (Ferrario-Ravasio,Limatola,P.N.).
- The same findings opened the possibility to compute linear power corrections to shape variables in the 3-jet configuration arXiv:2204.02247, (Caola,Ferrario-Ravasio,Limatola,Melnikov, Ozcelik,P.N.)

The results of Luisoni, Monni, Salam are based upon the so called "dispersive approach", where one assumes that the strong coupling at low energy can be given by an effective coupling

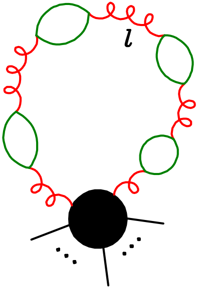
The results of Caola et al. and Ferrario-Ravasio et al. are obtained from the study if IR renormalons.

The two approaches bear some relation among each other.

ABC of I.R. Renormalons

All-orders contributions to QCD amplitudes of the form

$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p\, \frac{1}{b_0 \log(k^2/\Lambda^2)}$$
$$= \int_0^m \mathrm{d}k^p \frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2) \log \frac{k^2}{m^2}}$$
$$= \alpha_s(m^2) \sum_{n=0}^{\infty} (2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!}.$$



Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p \alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p$.
- Typical scale dominating at order α_s^{n+1}: $m \exp(-np)$.

ABC of I.R. Renormalons

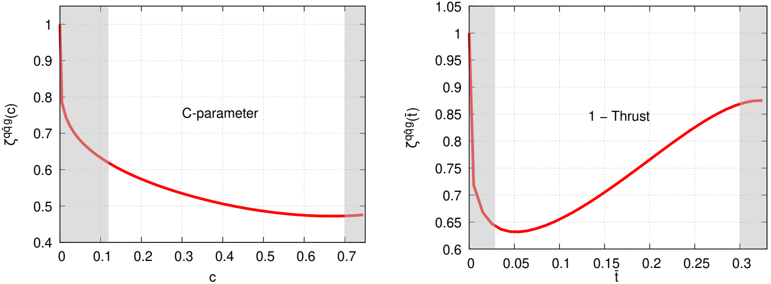
- Renormalon arise due to all radiative corrections that build up the running of α_s.
- When $\alpha_s \approx 1$ perturbation theory breaks down. This happens when $k \approx \Lambda$. The volume of this region is

$$\int_0^\Lambda \mathrm{d}k^p \approx \Lambda^p. \tag{1}$$

  This corresponds to the size of the minimal term in the asymptotic expansion. For shape variables $p = 1$.
- The running of α_s in QCD is not only due to fermionic bubble insertion in the gluon propagator. However:
  In the limit of large n_f these are the dominant corrections.
- The resummation of the bubble graphs leads to some sort of effective coupling, and from here the relation to the dispersive approach.

Non perturbative corrections are seen to arise from the emission of a very soft gluon (gluer).

- The calculability of the non-perturbative correction in the two-parton case is based upon the fact that all shape variables have a well defined value for two-partons final states
- The calculability of the non-perturbative correction to the C parameter in the three-jet symmetric limit (Luisoni et al.) is based upon the fact that the C parameter acquires a well-defined value near the 3-partons symmetric limit, up to quadratic effects in the deviation from the 3-partons symmetric configuration.
- The calculability of the non-perturbative correction in the generic case is based upon the findings of Caola et al., that in suitable recoil schemes recoil effects cannot generate linear power corrections



Non-perturbative corrections are usually parametrized as a shift in the perturbative cumulant distribution:

$$\delta_{\rm np}(s) = \Sigma(s + h\zeta(s)) - \Sigma(s) \approx \frac{d\sigma}{ds} h\zeta(s), \qquad \Sigma(s) = \int d\sigma(\Phi)\theta(s - s(\Phi))$$

and $h \approx \Lambda/Q$ is the shift in the two jet limit.

Notice the consistency with Luisoni et al.: for the C parameter at the symmetric limit (i.e. $c = 0.75$) the non-perturbative shift is about one half of the two-jet limit shift.

Implications

- There is a clear indication that the non-perturbative correction in the two jet limit cannot be safely extrapolated in the region where α_s is fitted.
- There is a hint that the small values of α_s found in fits using analytic models may be due to this assumption
- It is likely that this is not the whole story, and more needs to be understood before these findings can be safely used.

Mass corrections

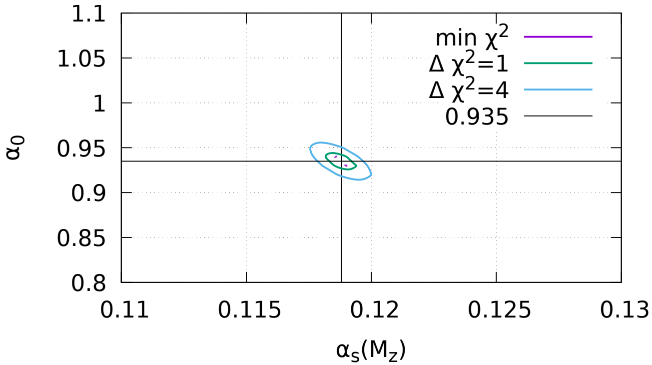
- Shape variables are defined for massless partons, and the analytic models refer to the "massless" definition.
- Final state hadrons are massive; so the definition of the shape variables must be extended to massive objects, in such a way that the factorization of non-perturbative corrections is not spoiled.
- This problem has been extensively studied in JHEP 05 (2001) 061, (Salam, Wicke). One scheme that should satisfy this requirement is the so called "Decay scheme", where massive hadrons are decayed isotropically into a pair of fictitious massless particles before the shape variable is computed.

In the following I will illustrate preliminary results (Zanderighi,P.N., in preparation) obtained by fitting ALEPH data.
The non-perturbative shift for C and t are available from arXiv:2204.02247, Caola et al..
In Zanderighi,P.N. we also computed it for the y_3 in the Durham scheme, the Heavy jet mass M_h^2 and the heavy-light mass difference $M_h^2 - M_l^2$.

- Theoretical errors were estimated with a 3-point scale variation $\mu_R/Q = 0.25, 0.5, 1$, and added in quadrature to the systematic experimental error.
- Diagonal terms of the covariant matrix was computed by summing in quadrature the systematic and statistical error. The off-diagonal terms were computed as $E_{ij} = \min(\delta\sigma^2_{\mathrm{syst},i}, \delta\sigma^2_{\mathrm{syst},j})$ (the so called minimal-overlap model).
- We adopted the decay scheme to account for hadron masses, and computed the associated bin migration matrix using Pythia8. Using Herwig7 we obtain compatible results with a slightly worse χ^2.

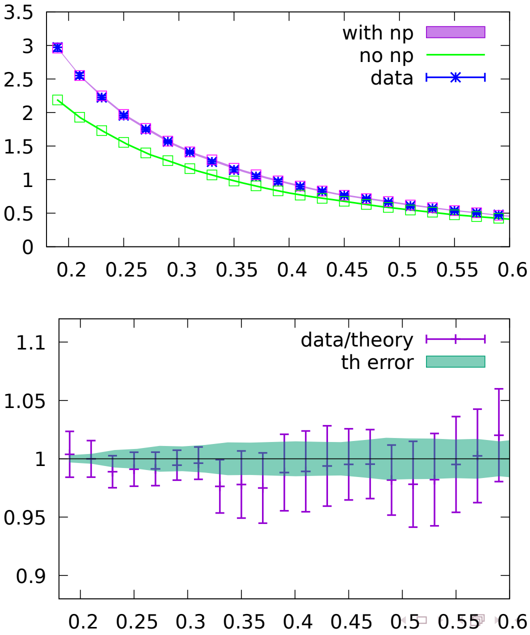
PRELIMINARY RESULTS

Simultaneous fit to C, t and y_3:



The minimum χ^2 is 46, with $\chi^2/n_{\mathrm{deg}} = 0.84$.
The central value is at $\alpha_s(M_Z) = 0.1188$, $\alpha_0 = 0.935$.

C



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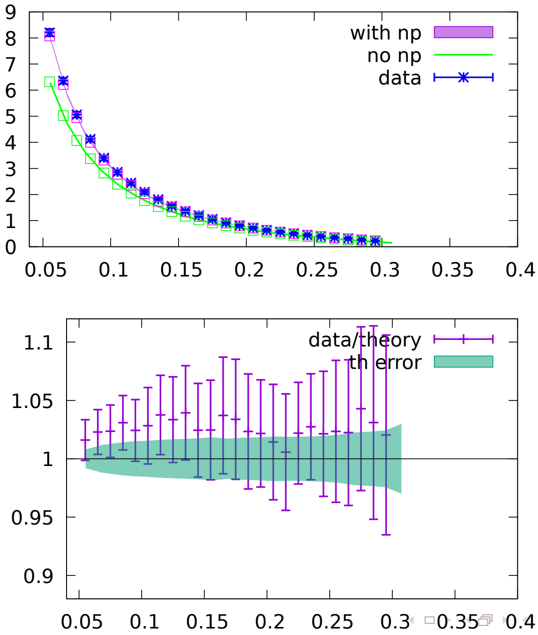
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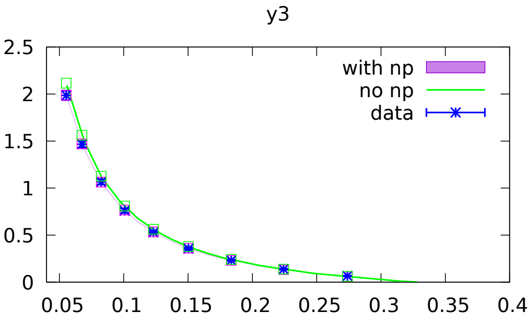

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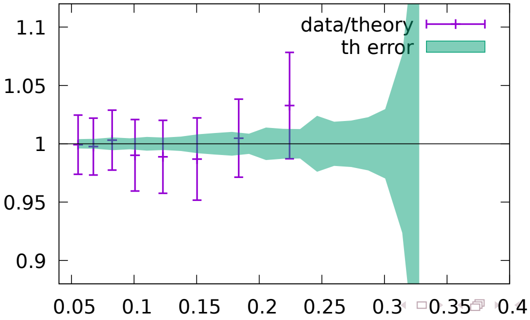

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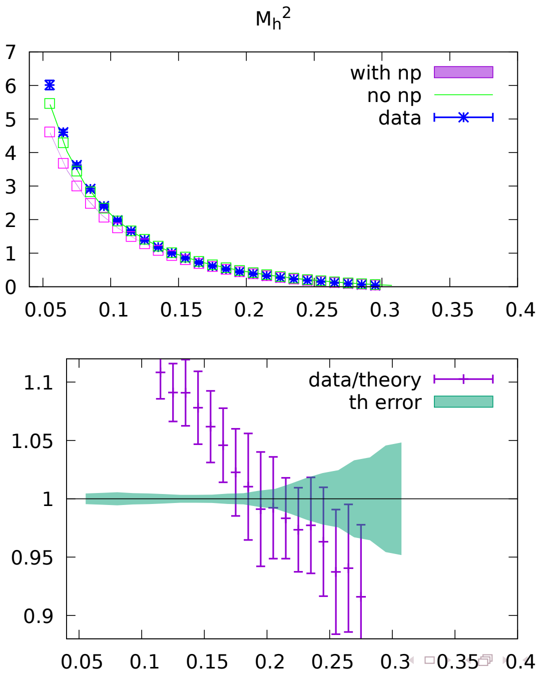
We found it impossible to obtain a good fit of the heavy-jet mass, and of the jet mass differences. The following figures display their distributions compared to data, using the same optimal parameter obtained from the fit to C, t and y_3



M_h^2
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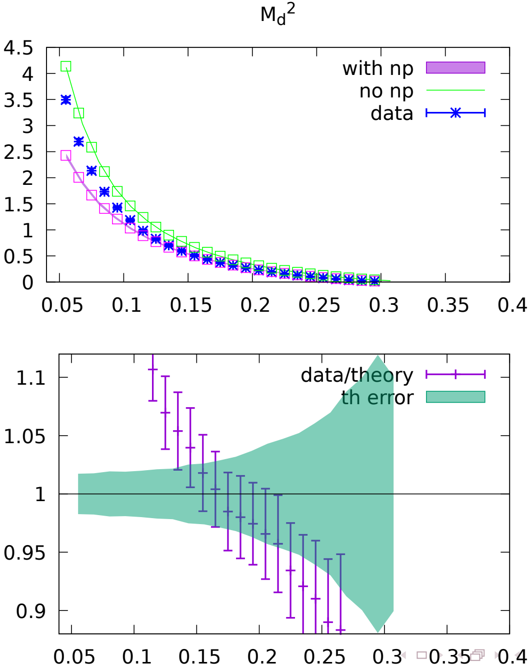

M_d^2
with np
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th error

- We obtain a good simultaneous fit to C, t and y_3, yielding values of α_s compatible with the world average.
- The heavy jet mass and jet mass difference cannot be fitted in any reasonable way, even as individual distributions
- Notice that the non-perturbative correction for both the heavy jet mass and the mass difference has the opposite sign than for C and t. This is also the case for y_3, that, however, has a very small correction.

Opportunity for future e^+e^- colliders

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})
FCC-ee-Z	4	88-95	150
FCC-ee-W	2	158-162	12
FCC-ee-H	3	240	5
FCC-ee-tt	5	345-365	1.5

I estimate the following Z/γ^* hadronic cross sections:

E_{CM}	σ (nb)	Num. had. events
91.2	33.1	5.0×10^{12}
160	0.026	0.31×10^{9}
240	0.009	0.45×10^{8}
350	0.0039	0.58×10^{7}

Even at the highest energy the number of events is not distant from what was collected at LEP1 (16×10^6 events).

Opportunity for future e^+e^- colliders

- We can expect a (hard to quantify) reduction of the systematic error with respect to the LEP1 case.
- We can expect a negligible statistical error at the higher energies.

Opportunity for future e^+e^- colliders

- Would an N^3LO calculation useful?
  An N^3LO calculation would be by itself of great value. For example, to see if the factorial growth associated to renormalons becomes visible.
  From a practical viewpoint, by looking at the theoretical uncertainty and at the present experimental uncertainty, I would say yes, provided enough is understood on power corrections.
- Power corrections are large on the Z peak (of order 0.1%). Perturbative uncertainties are of order 0.02%. At the highest energy of 350 GeV, NP corrections should be a factor of 4 smaller, becoming of the order of higher order uncertainties.

Opportunity for future e^+e^- colliders

- Something new has being recently understood on power corrections. The theoretical consequences of these findings have yet to be fully explored. Hopefully these finding will bring in better agreement between power correction estimates obtained with shower Monte Carlo and those obtained from analytic methods generators.
- The impact of N^{3}LO calculations for shape variables will strongly depend upon the development of our understanding of NP effects.
- This understanding can also be tested now, by using preserved LEP data.
- Depending upon these developments, the availability of high statistics data at higher energies may allow for a high precision determination of α_s at high energy, to be contrasted with the low energy determination of the Lattice approach.