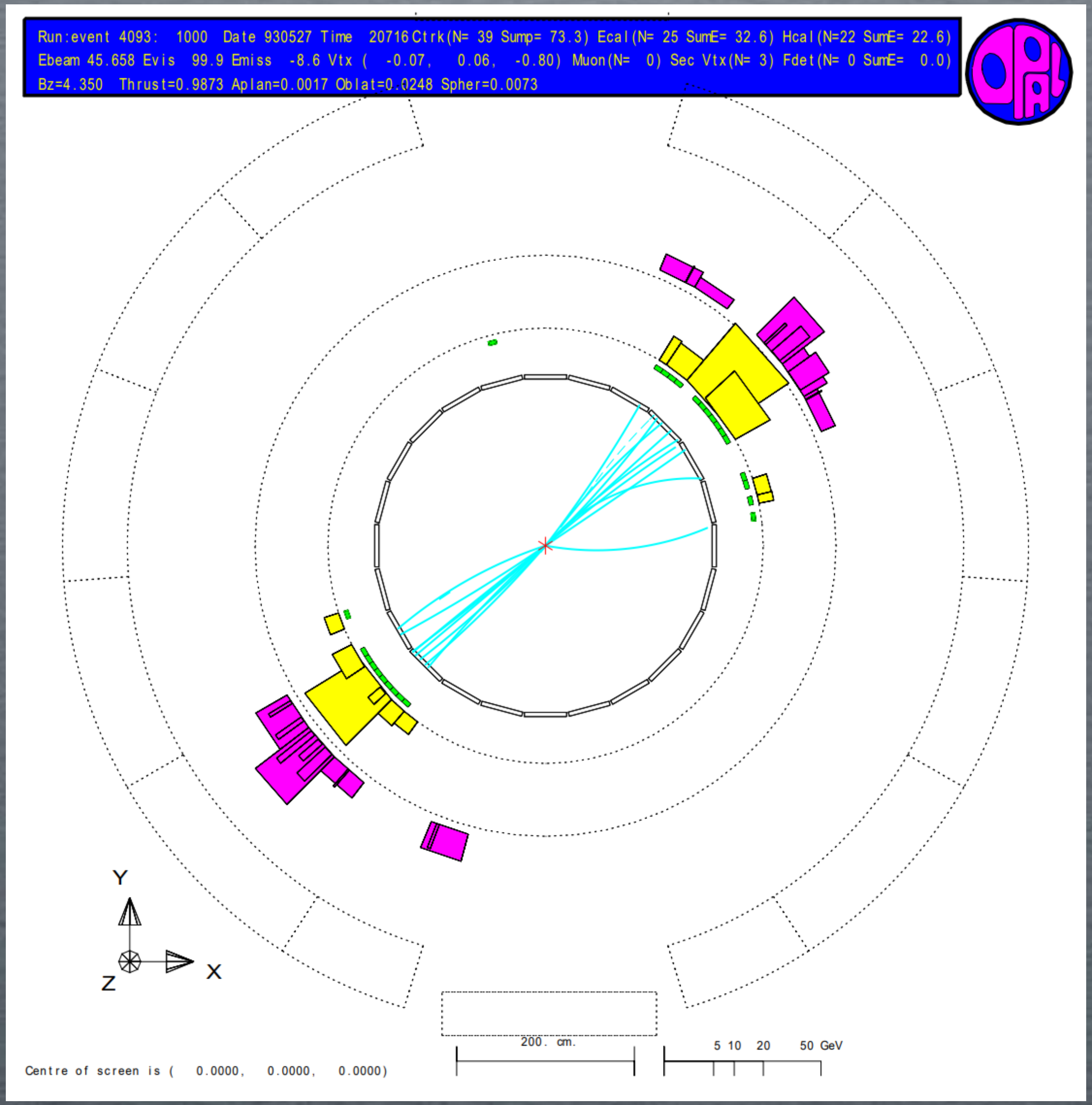
Prospects for precision QCD jet calculations



Andrea Banfi

University of Sussex

FCC-ee Workshop – CERN – 9 June 2022

Outline

- Global three-jet observables
- Pathways to reducing hadronization
- Multi-jet studies
- Lund-plane observables

Not included in this talk, but still worth thinking about

- Mean values of jet observables
- High jet multiplicities (five or more)
- Heavy-quark mass effects

Three-jet observables

Strong coupling with jets

- Jet observables constitute an important means of determination of the strong coupling
- Increase in precision of PT QCD calculations resulted in massive decrease in theory uncertainties

Two-jet rate (NNLL+NNLO)

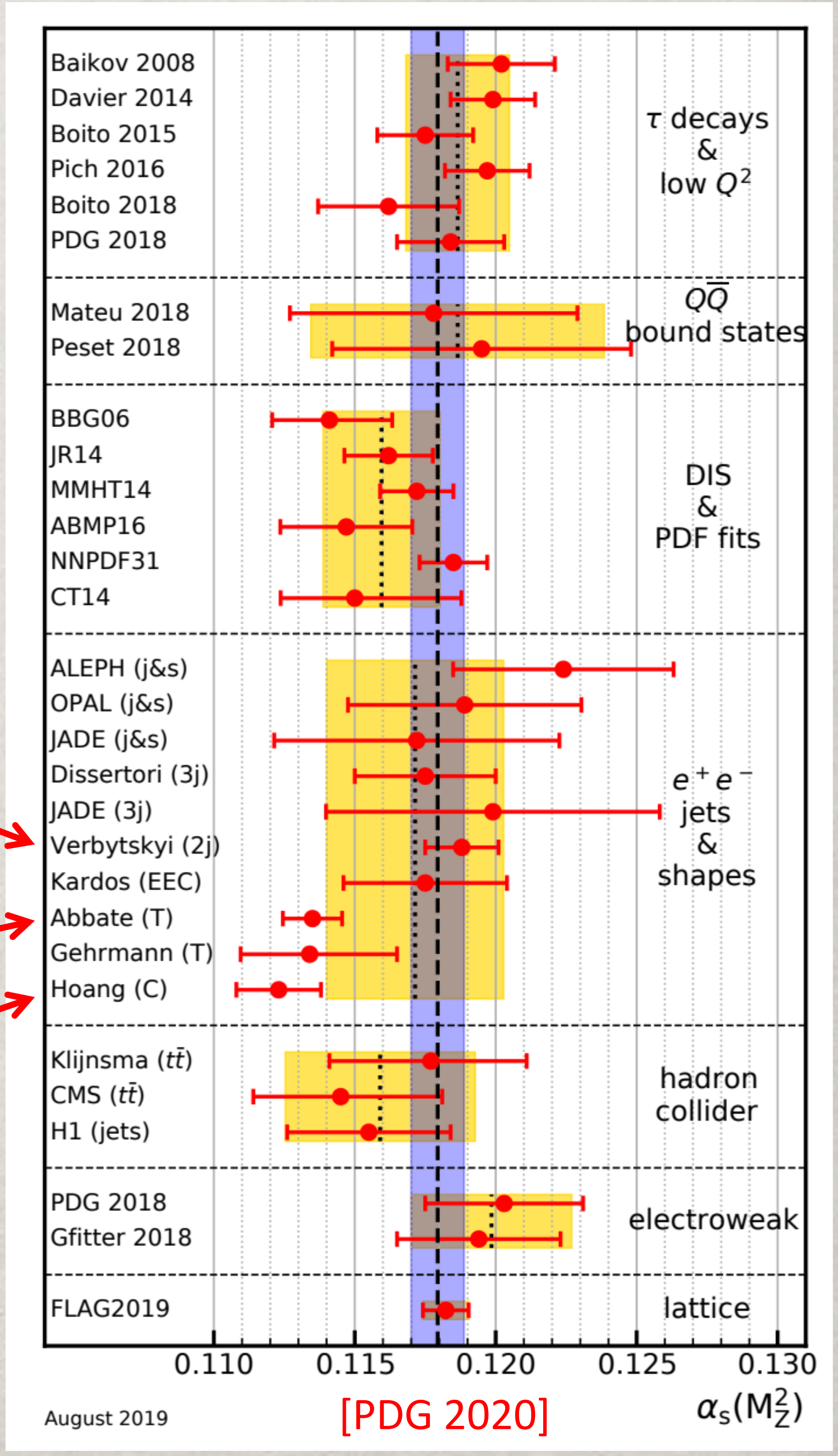
$$\alpha_s(M_Z) = 0.1188 \pm 0.0013$$

Thrust and C-parameter ((N)NNLL+NNLO)

$$\alpha_s(M_Z) = 0.1137^{+0.0034}_{-0.0027}$$

$$\alpha_s(M_Z) = 0.1123 \pm 0.0015$$

- Can we push this accuracy even further?

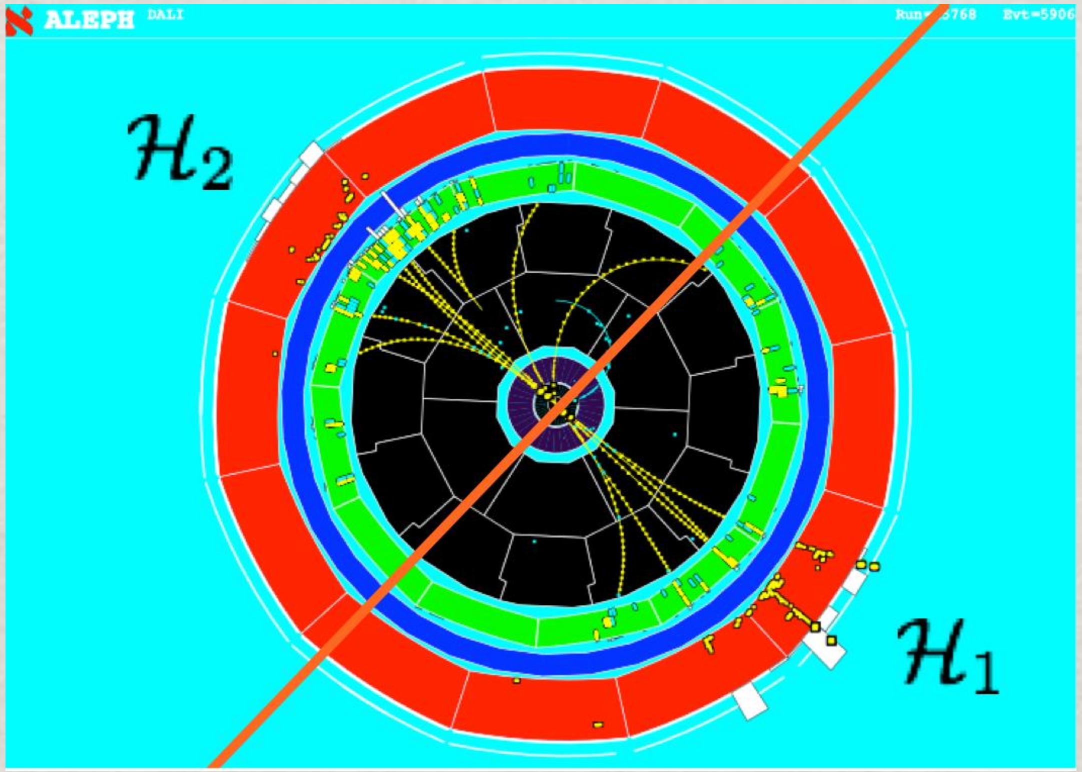


Global vs non-global

- For any final-state observable, a function of all final-state hadron momenta, we study its rate $\Sigma(v)$, the fraction of events where the observable's value is less than a threshold v
- Non-global observables are those whose rate does not restrict emissions in a selected phase-space region

global

$$\rho_H = \max\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$



non-global

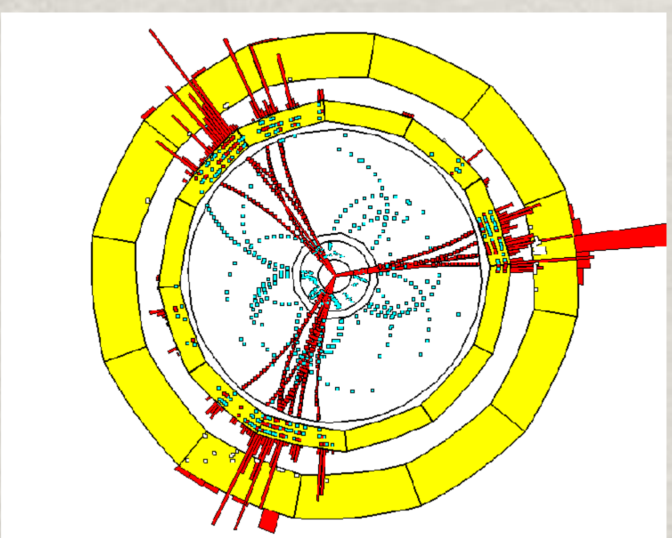
$$\rho_L = \min\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$

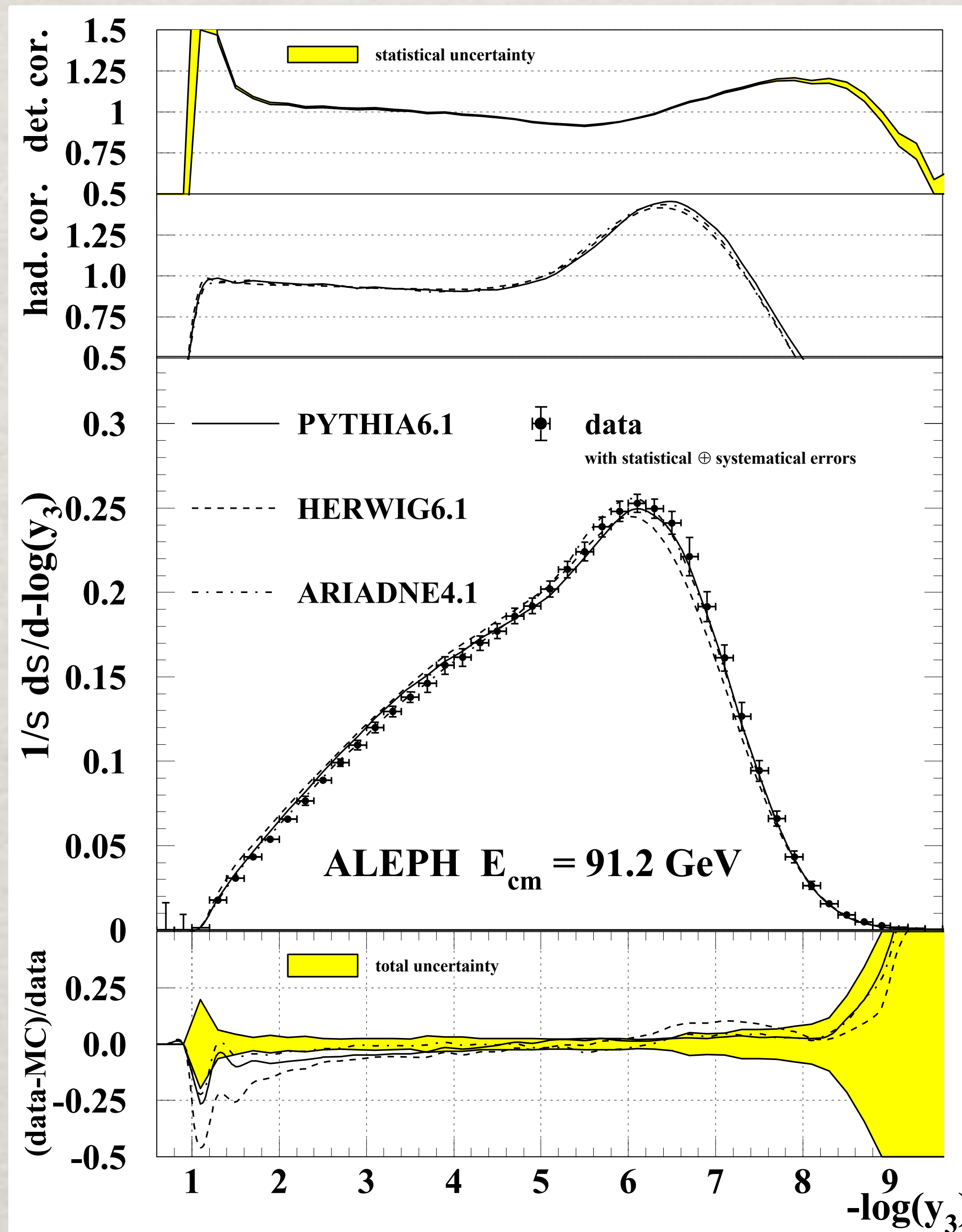
- Non-global observables are most common in hadron collisions, where particles are detected away from the beam region

Three-jet observables in QCD

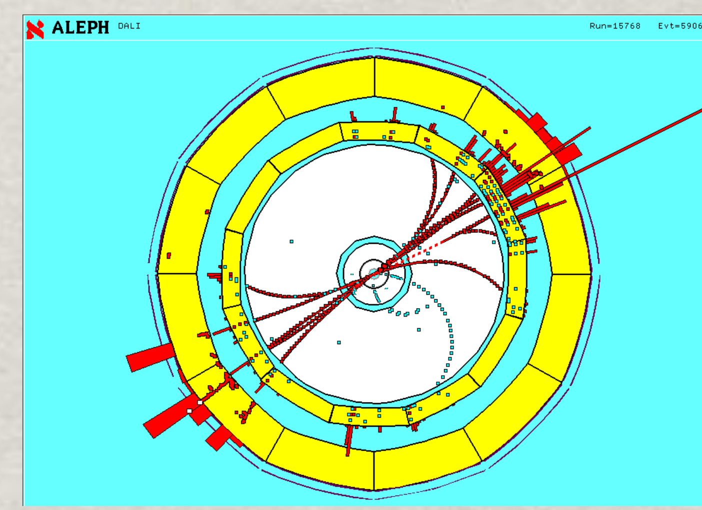
- Global three-jet observables vanish with two final-state particles, and they are different from zero with an extra emission $\Rightarrow$ directly sensitive to α_s

fixed order





resummation



$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

$$\sim \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \ldots\right]$$

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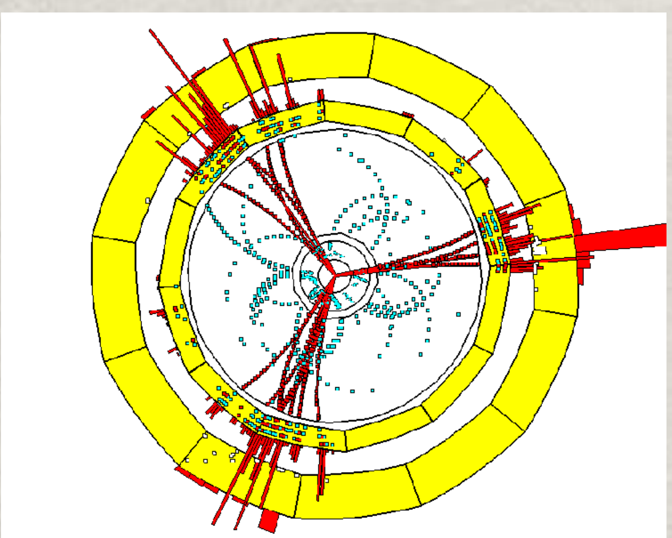

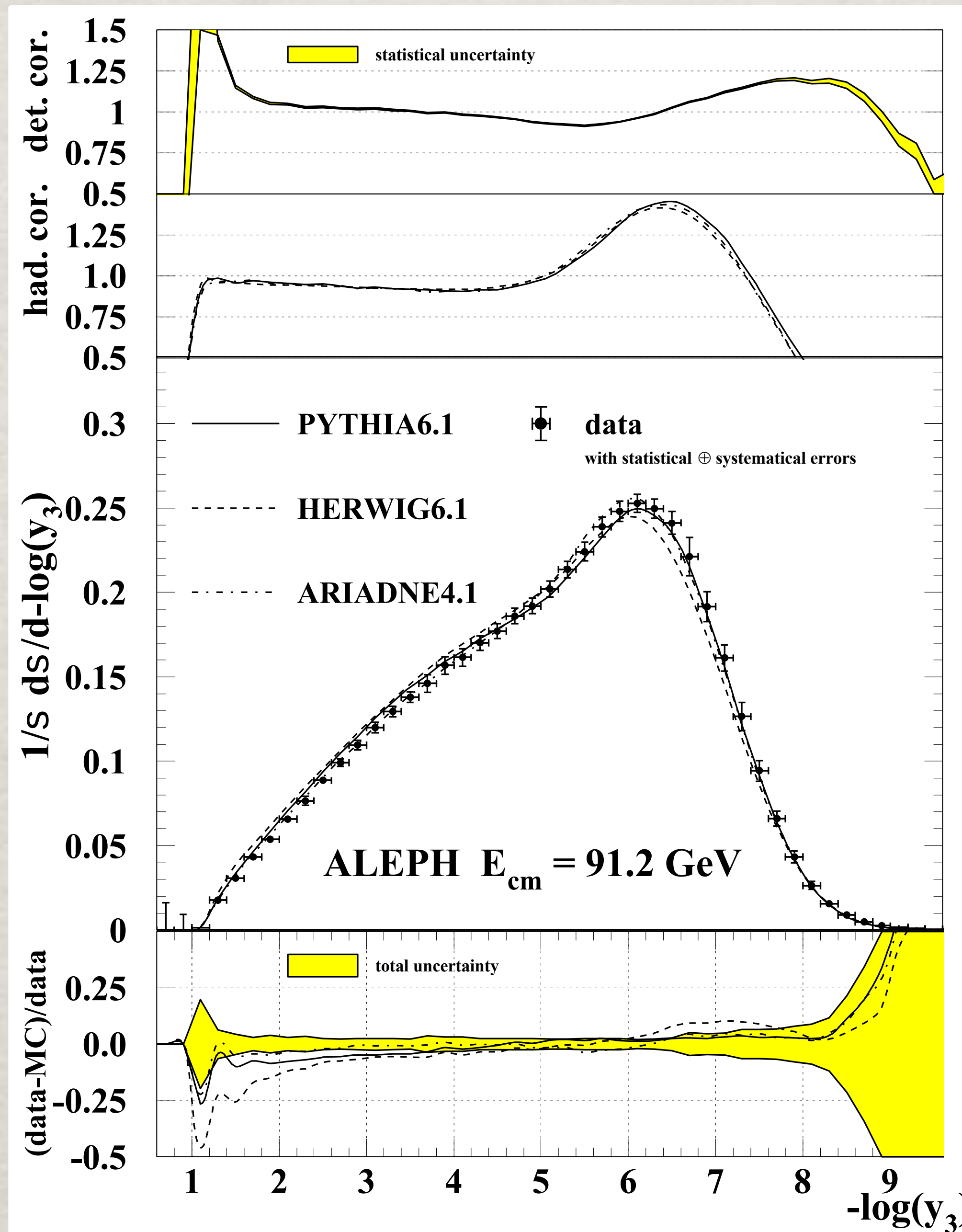

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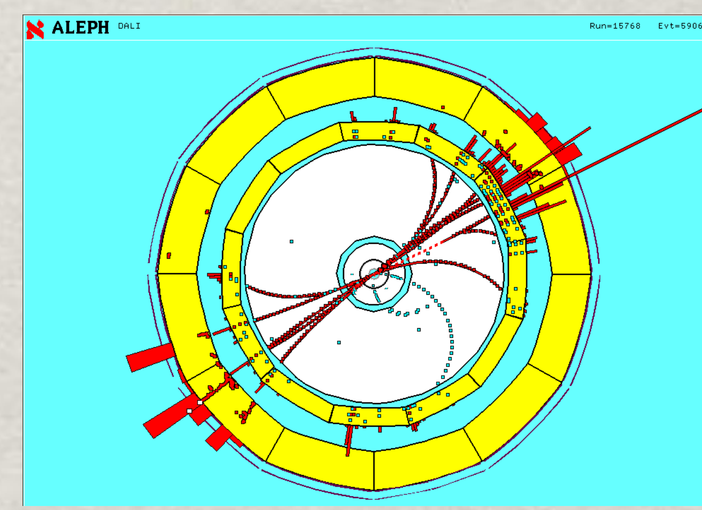


$$\sim \underbrace{\alpha_s}_{\text{LO}} + \underbrace{\alpha_s^2}_{\text{NLO}} + \underbrace{\alpha_s^3}_{\text{NNLO}} \ldots$$

$$\underbrace{e^L}_{\text{LL}} \left[\underbrace{1}_{\text{NLL}} + \underbrace{\alpha_s}_{\text{NLL}} + \underbrace{\alpha_s^2}_{\text{NNLL}} + \ldots \right]$$

Fixed-order PT calculations

Three-jet production in e^+e^- annihilation has been known at NNLO for a long time

- First-ever NNLO calculation of $e^+e^- \to 3\,\mathrm{jets}$ with the method of antennae

  [Gehrmann-De Ridder Gehrmann Glover Heinrich 0711.4711]

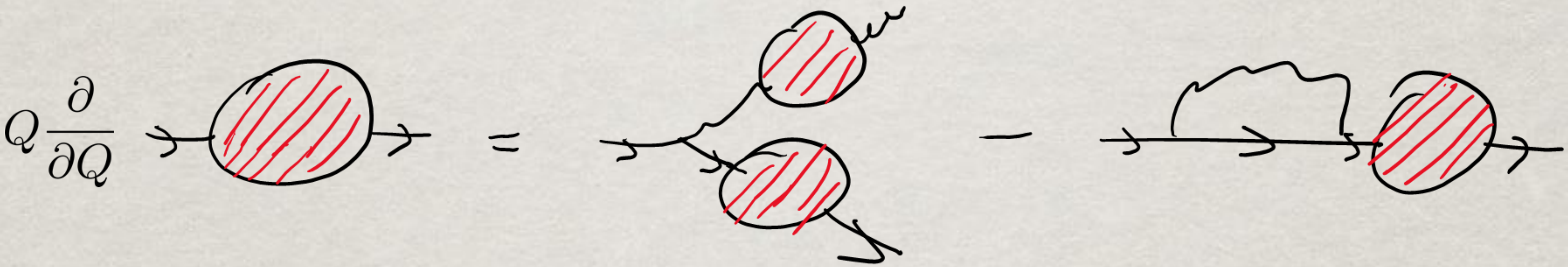
  [Weinzierl 0807.3241]

- More recent calculations using a fully local subtraction scheme (CoLoRFulNNLO) [Del Duca Duhr Kardos Somogyi Szor Trocsanyi Tulipant 1606.03453]

Fixed-order calculations are fully exclusive in all final-state particles, so they can be applied equally to global and non-global observables

Recursive IRC safety

- Event-shape distributions and jet rates measure emissions directly, so all-order resummations should track infinitely many gluon splittings. This leads to the onset of non-linear dynamics

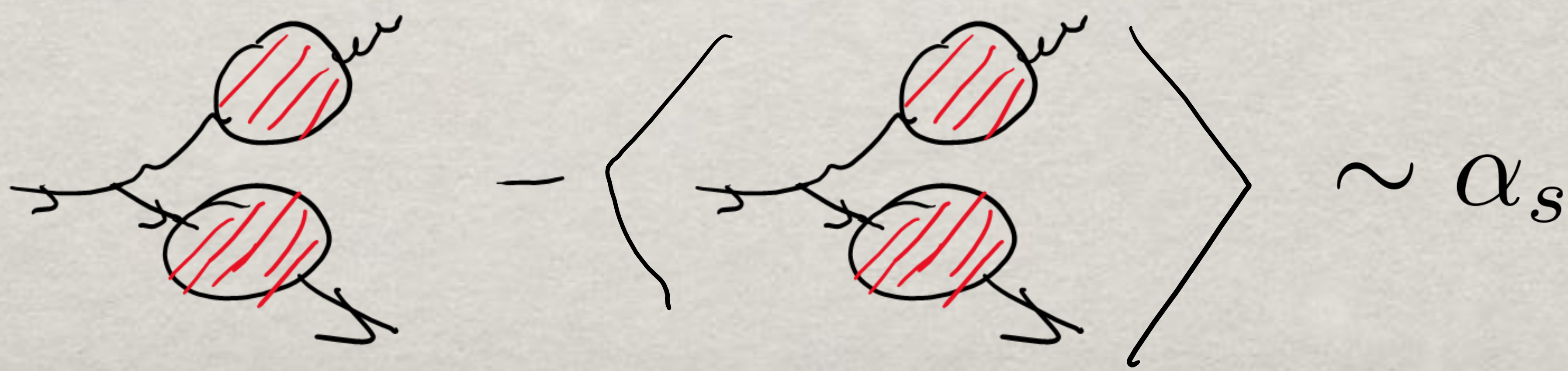


- At NLL, there are observables for which one can integrate inclusively over secondary splittings. Such observables are called rIRC safe

[Banfi Salam Zanderighi hep-ph/0407286]

- For rIRC safe observables, the difference from the inclusive approximation is at most NNLL

[AB El-Menoufi Monni 1807.11487]



rIRC-SAFE RESUMMATION

Most three-jet global event shapes and jet resolution parameters are rIRC safe. Their distributions can be resummed at very high logarithmic accuracy

- Some observables (e.g. thrust, broadening) enjoy factorisation theorems in SCET $\Longrightarrow$ NNNLL resummation [Becher Schwartz 0803.0342] [Becher Bell 1210.0580] [Hoang Kolodubrez Mateu Stewart 1501.04111]

- General semi-numerical NNLL resummation of event shapes and jet rates in e^+e^- annihilation with the ARES method [AB Monni McAslan Zanderighi 1412.2126] [AB Monni McAslan Zanderighi 1607.03111] [AB El-Menoufi Monni 1807.11487] [Arpino AB El-Menoufi 1912.09341]

- General NNLL resummation of factorisable observables in SCET with the semi-numerical program SoftServe [Bell Rahn Talbert 2004.08396]

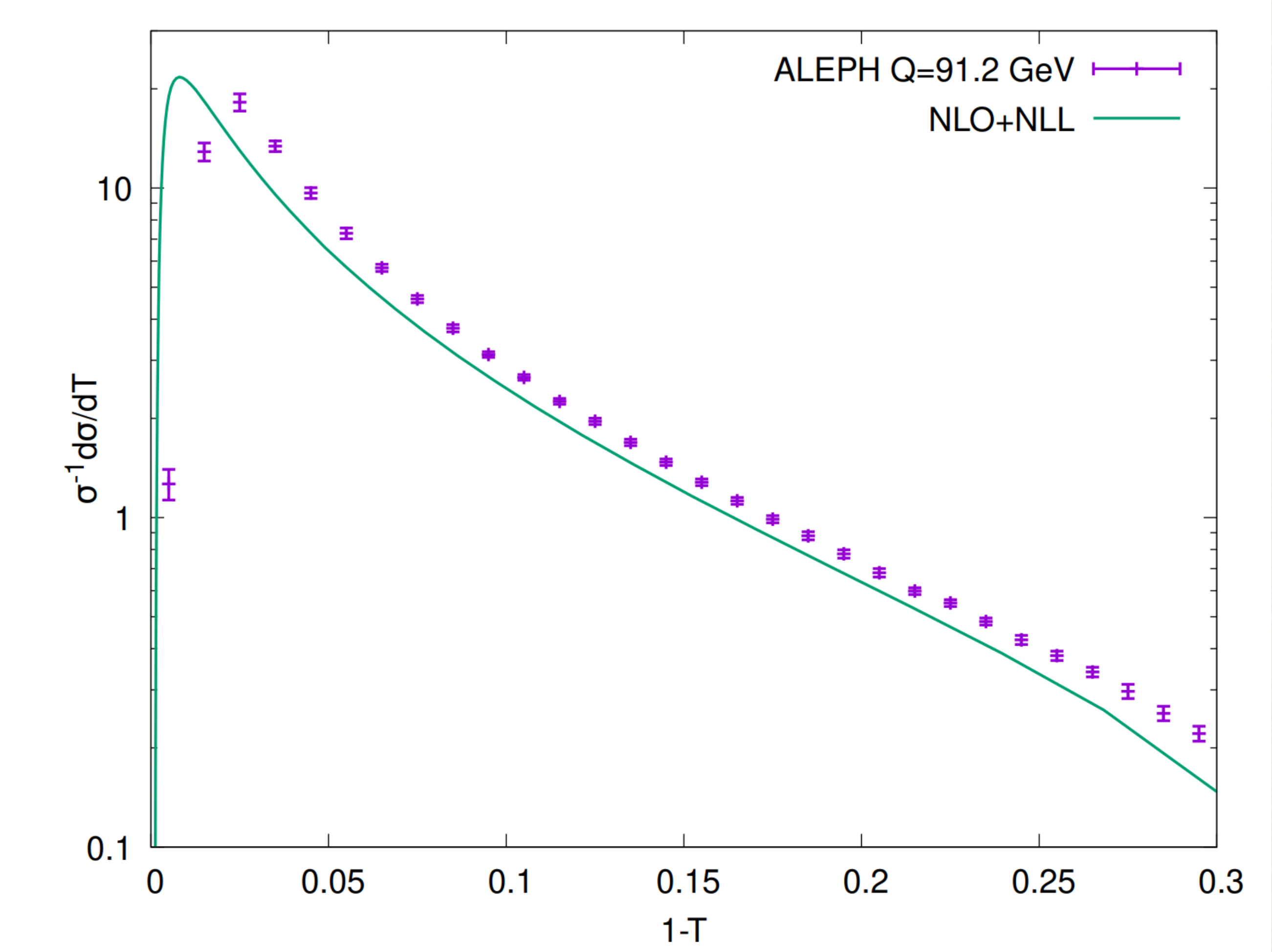
How does such an amazing precision reflect in the determination of the strong coupling?

We don't talk about

HADRONIZATION

HADRONIZATION EFFECTS

- At the energies probed so far, perturbative prediction do not agree straightaway with data



HADRONIZATION EFFECTS

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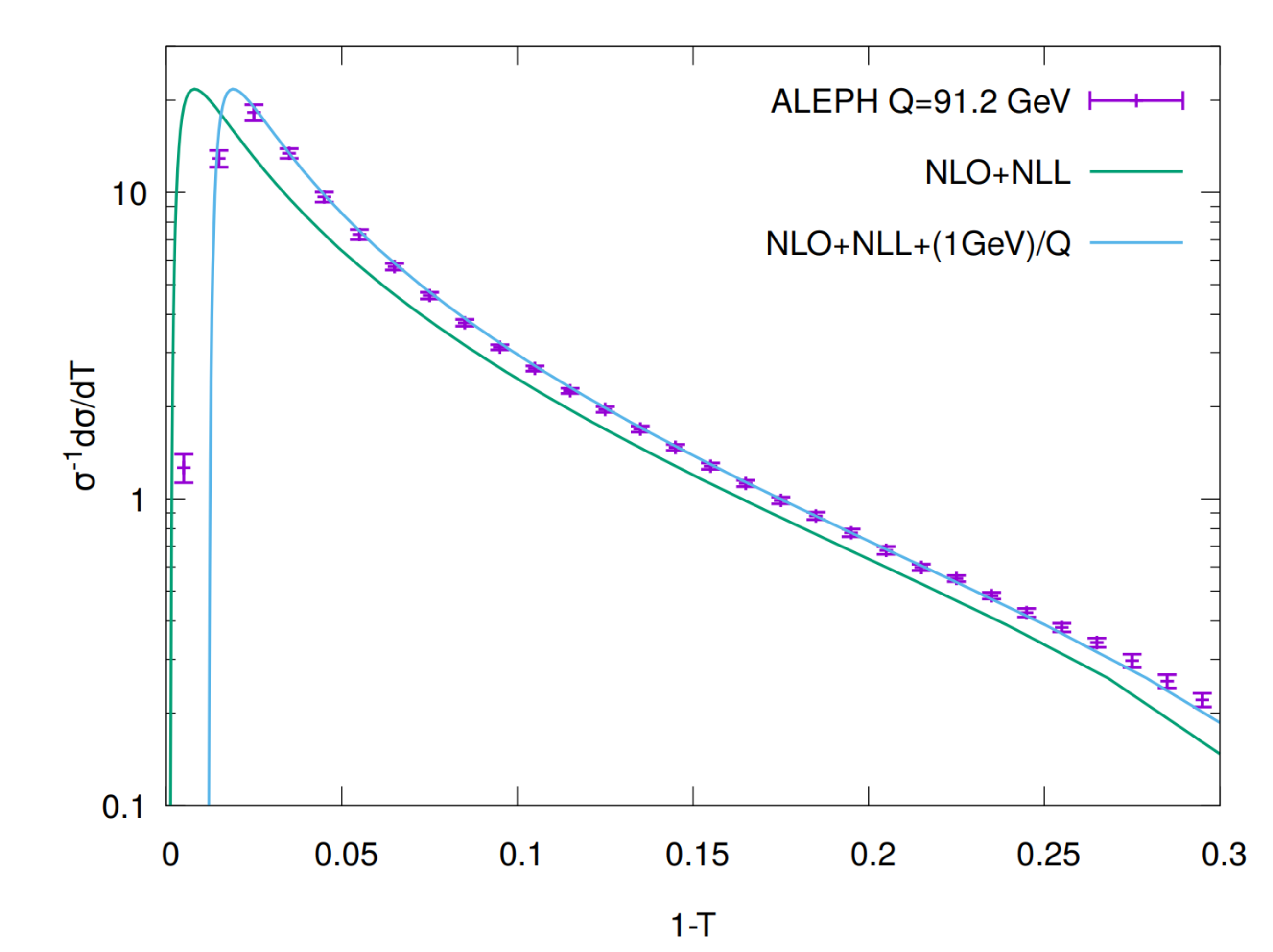

- Central hadrons with momenta ~1GeV give rise to a 1/Q suppressed shift of perturbative distributions of jet observables (~10% at LEP energies)

[Dokshitzer Webber hep-ph/9704298]

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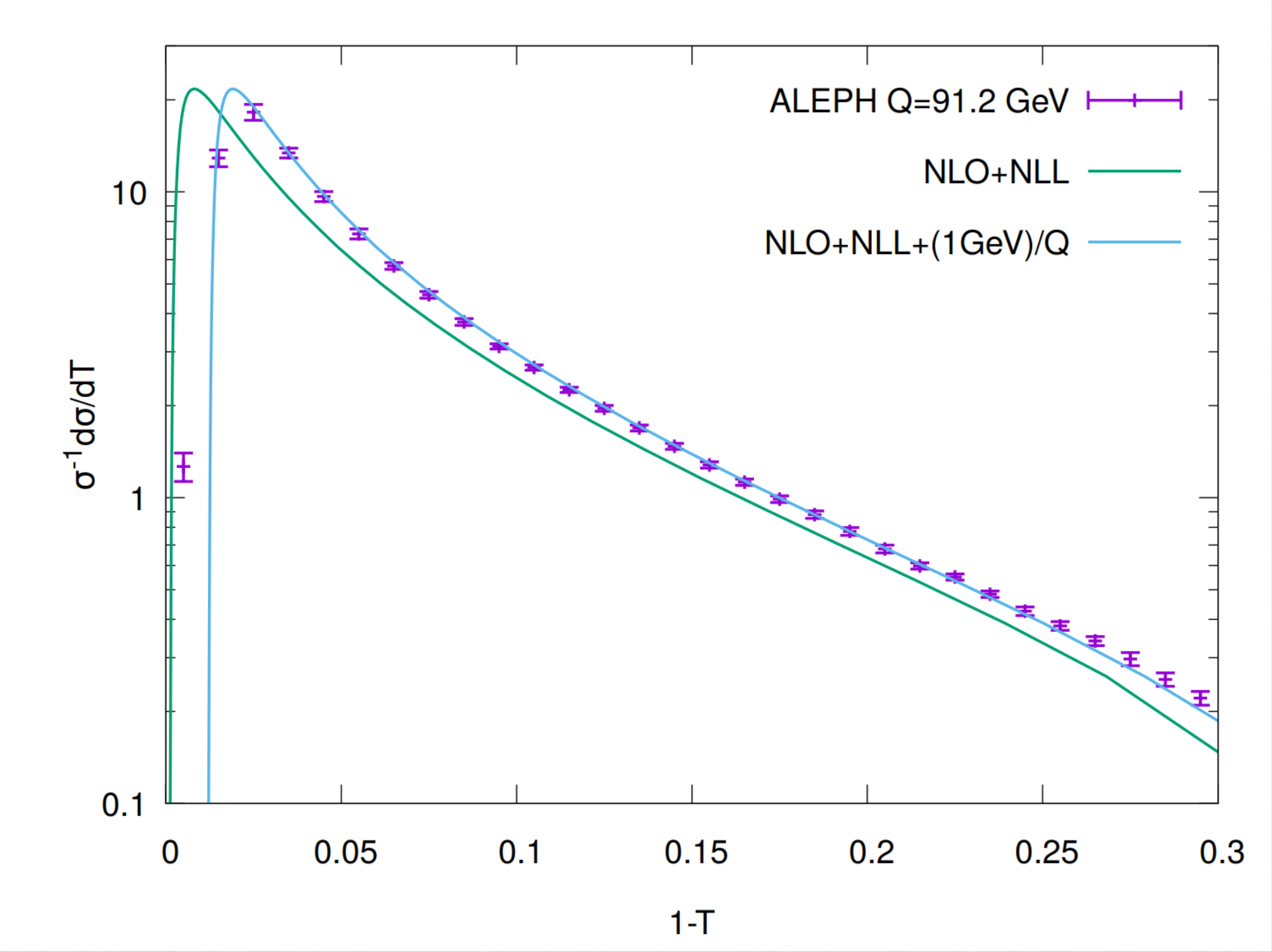

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[Dokshitzer Webber hep-ph/9704298]

- How can we generally estimate the size of the shift and even more suppressed hadronization corrections?

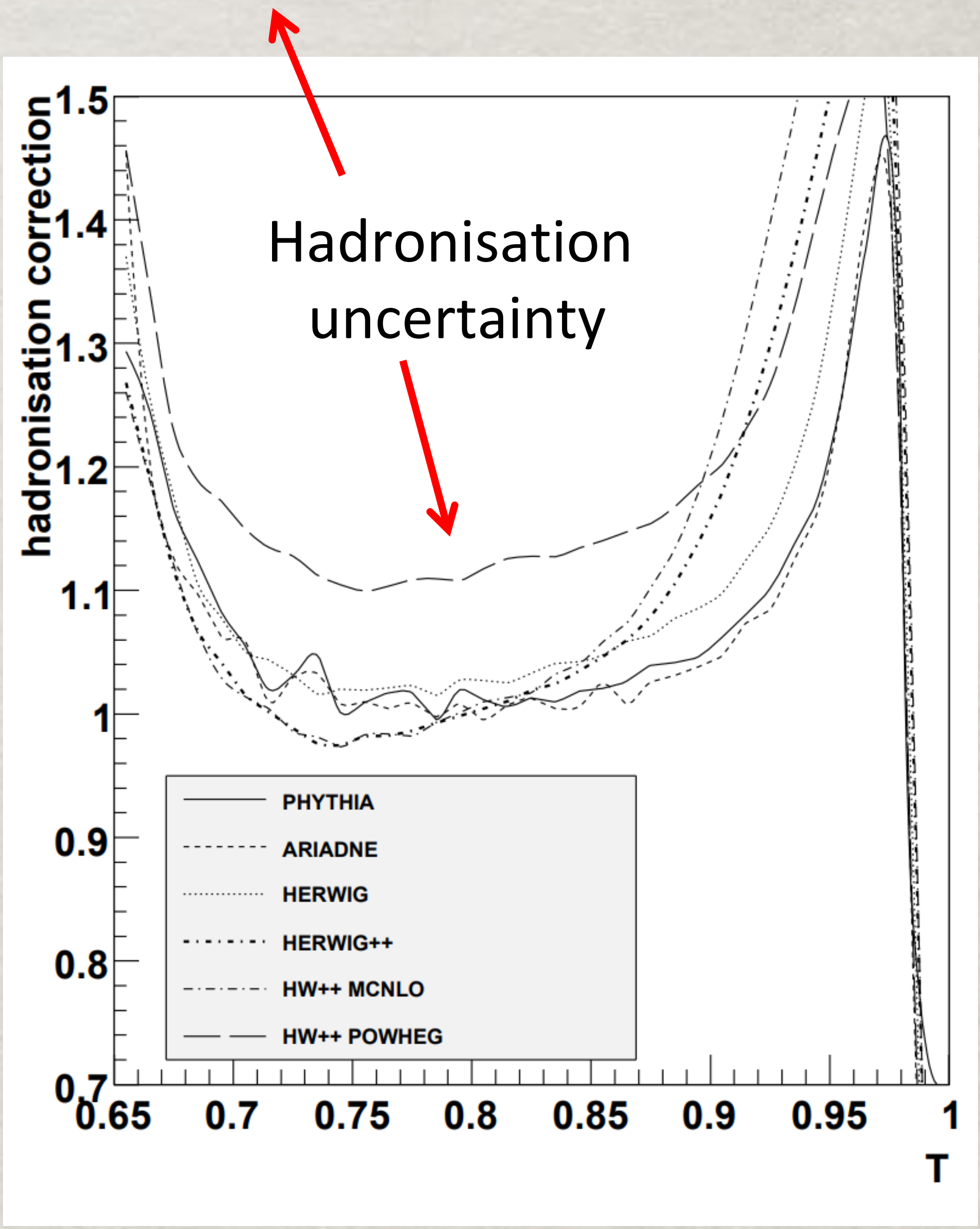
Monte-Carlo determination

- Most analyses determine hadronization corrections using Monte Carlo event generators, as the ratio between hadron- and parton-level results

[Dissertori et al 0906.3436]

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{had}) \pm 0.0035(\text{theo})$$

- With this approach one captures all hadronisation effects, including the interplay between perturbative and non-perturbative effects



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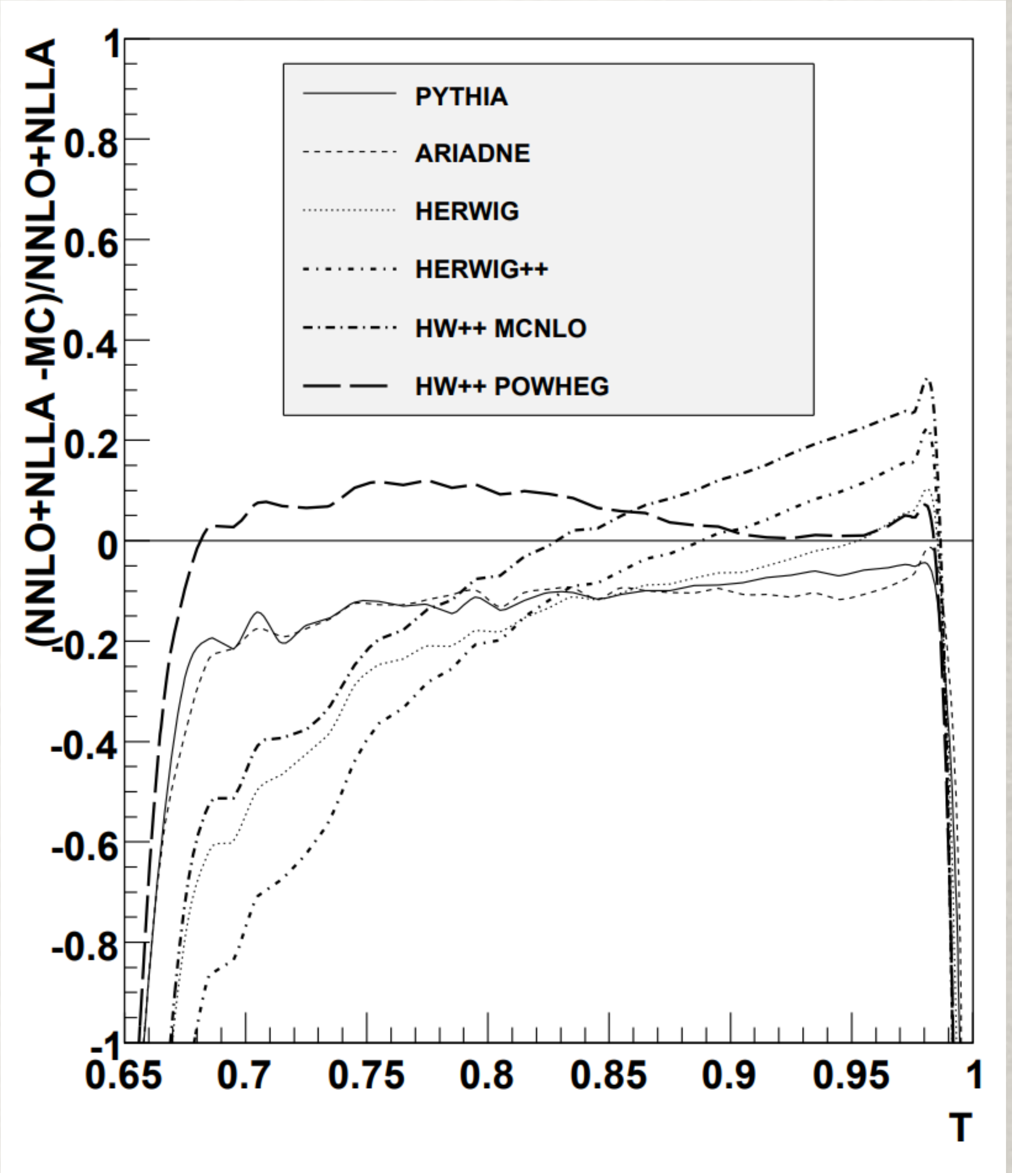

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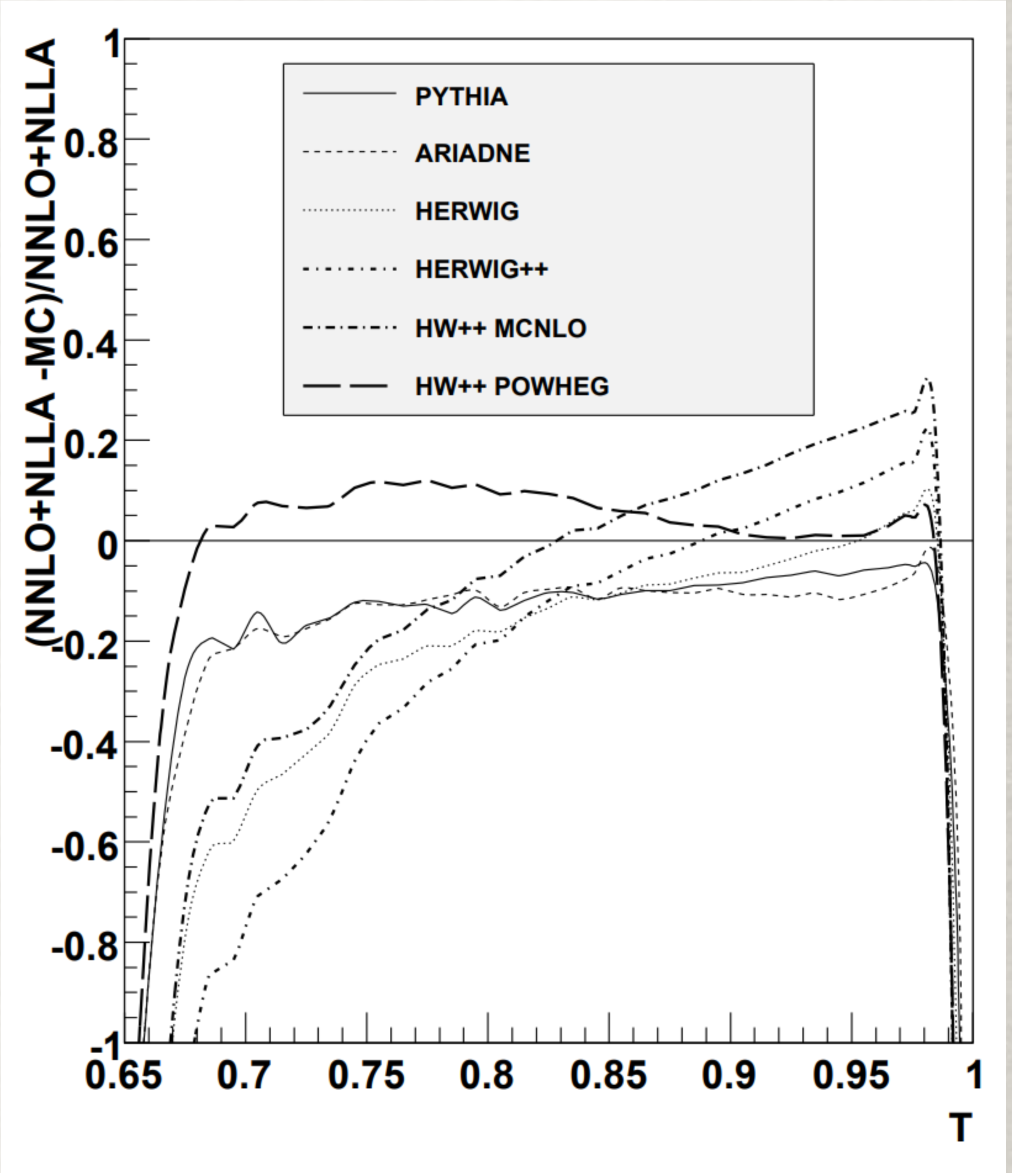
- This approach is sensible as long as perturbative QCD uncertainties dominate: is it still valid now that NNLL resummations are available?

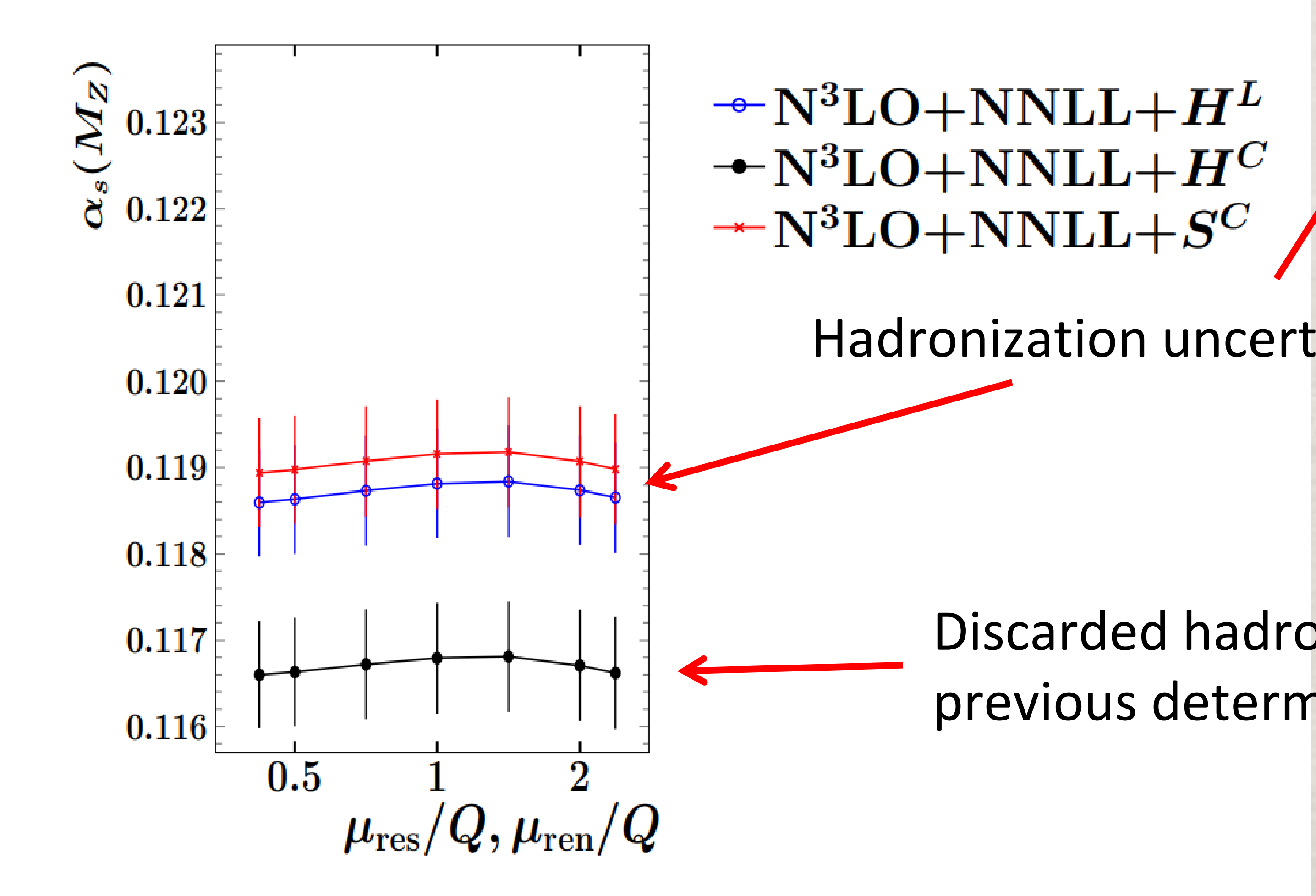


The two-jet rate

- Monte-Carlo determinations of hadronization corrections have been combined with a NNLL resummation only for the two-jet rate

[Verbytskyi et al 1902.08158]

$$\alpha_s(M_Z) = 0.1188 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0010(\text{had}) \pm 0.0006(\text{theo})$$



Hadronization uncertainty

Perturbative QCD uncertainty

Discarded hadronization model, not an issue in previous determinations due to larger PT uncertainty

- With NNLL resummations available, hadronization is the main source of uncertainty $\Rightarrow$ try to gain analytical understanding of hadronization corrections

Simultaneous PT-NP fits

- Leading 1/Q hadronization corrections can be theoretically modelled in terms of the emission of a single extra-soft gluon $\Rightarrow$ simultaneous fit of α_s and NP parameter for different event shapes

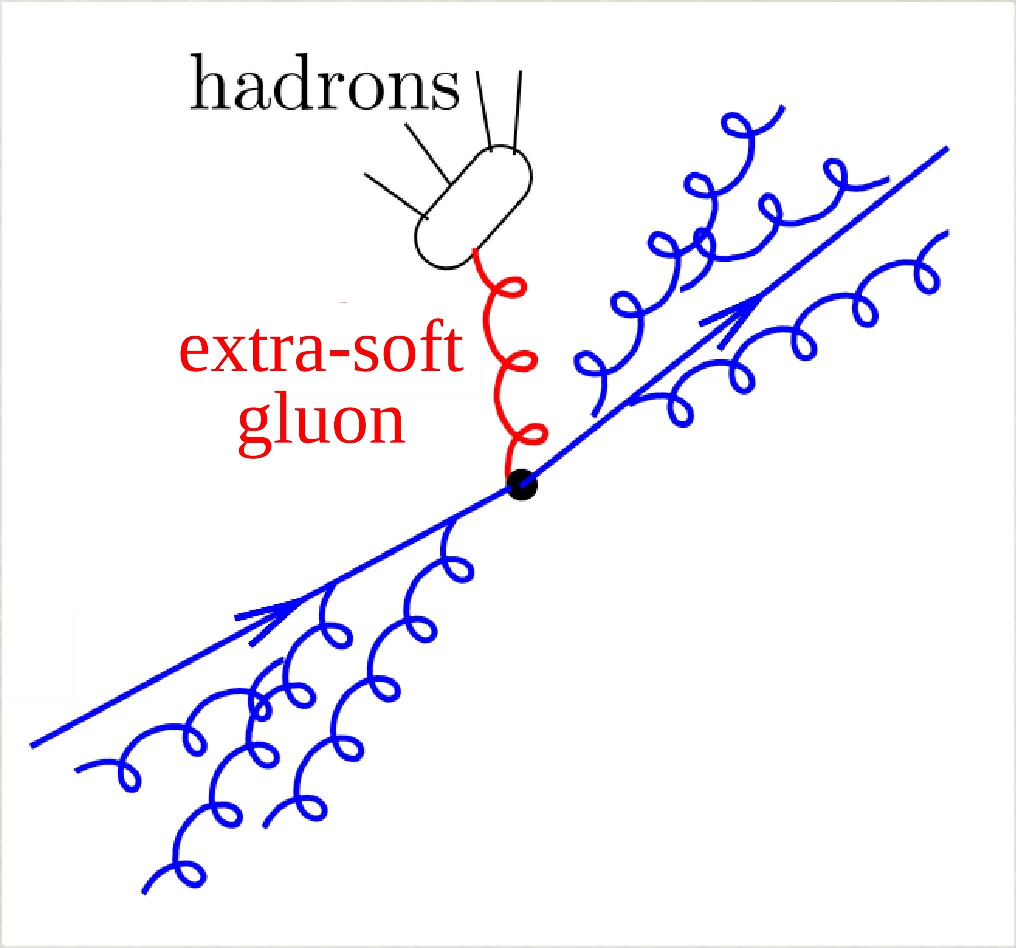
Universal (?) NP parameter

Observable dependent but calculable

$$\text{shift} = \frac{\langle k_t \rangle_{\text{NP}}}{Q} \langle c_V \rangle_{\text{PT}}$$

$$\langle c_V \rangle_{\text{PT}} \equiv \int d\eta \frac{d\phi}{2\pi} \langle h_V(\eta, \phi) \rangle$$

Average over PT configurations



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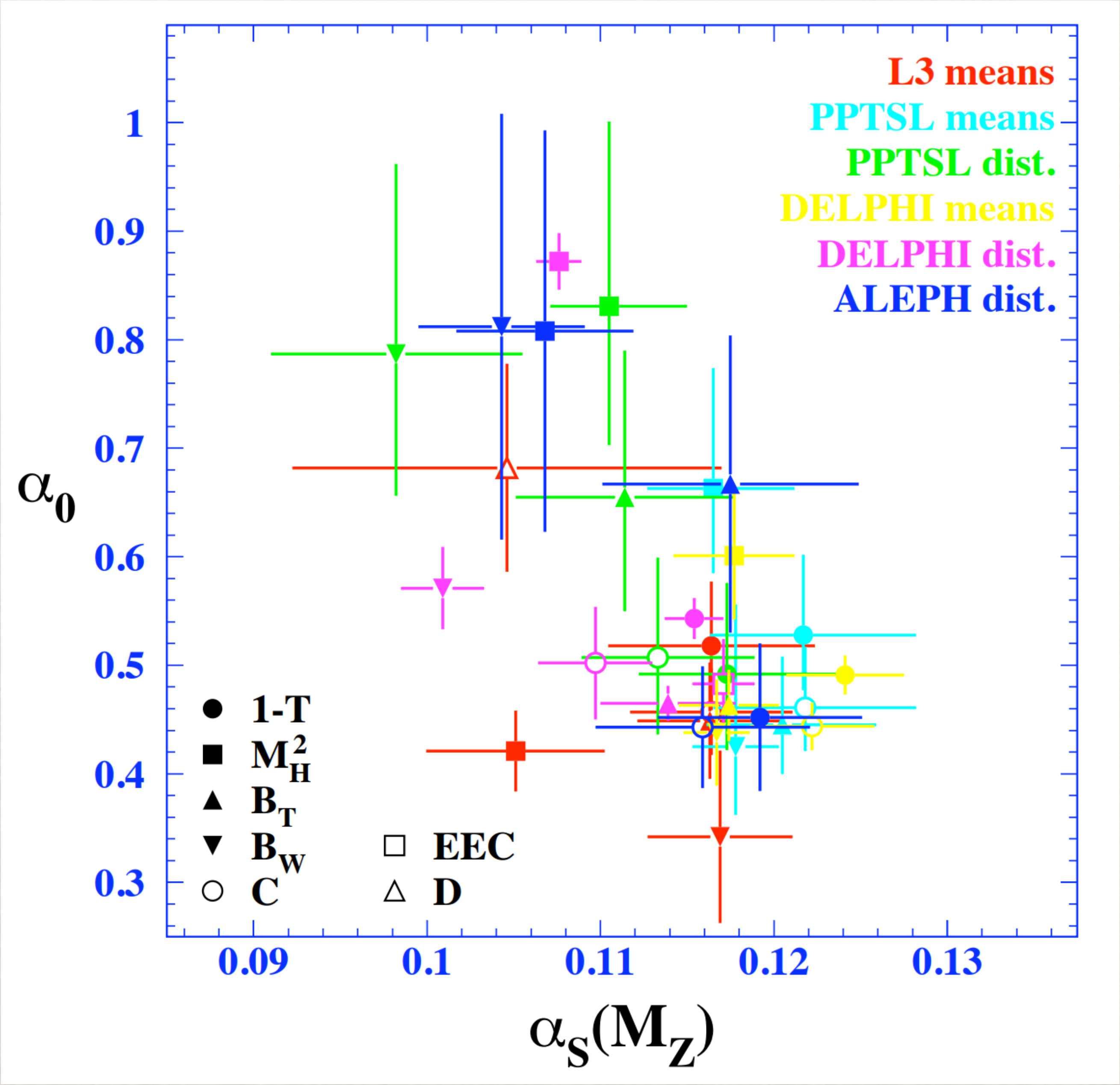


- Fits of α_s with NLL + NLO + 1/Q shift in the two-jet region are very observable dependent $\Rightarrow$ what happens at NNLL?

Simultaneous PT-NP fits

- Most accurate determinations of α_s with event shapes arise from simultaneous fits of 1/Q hadronization corrections

Thrust (NNLL+NNLO)

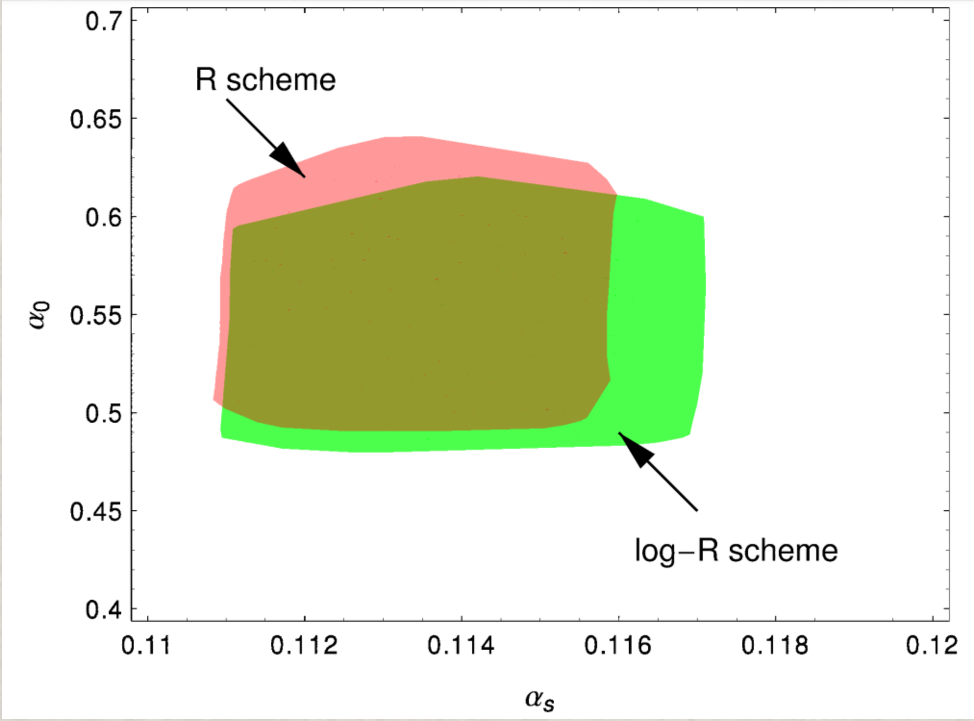
[Gehrmann Luisoni Monni 1210.6945]

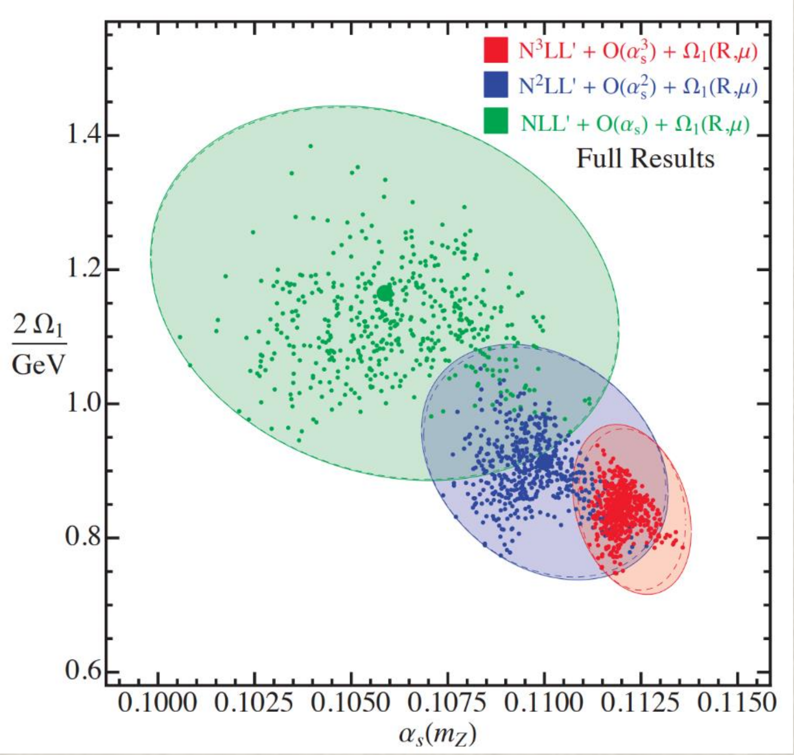
C-parameter (NNNLL+NNLO)

[Hoang Kolodubrez Mateu Stewart 1501.04111]

$$\alpha_s(M_Z) = 0.1137^{+0.0034}_{-0.0027} \qquad \alpha_s(M_Z) = 0.1123 \pm 0.0015$$

$$\alpha_0(2\,\mathrm{GeV}) = 0.524^{+0.096}_{-0.044} \sim \text{shift} \sim \Omega_1 = 0.421 \pm 0.063\,\mathrm{GeV}$$



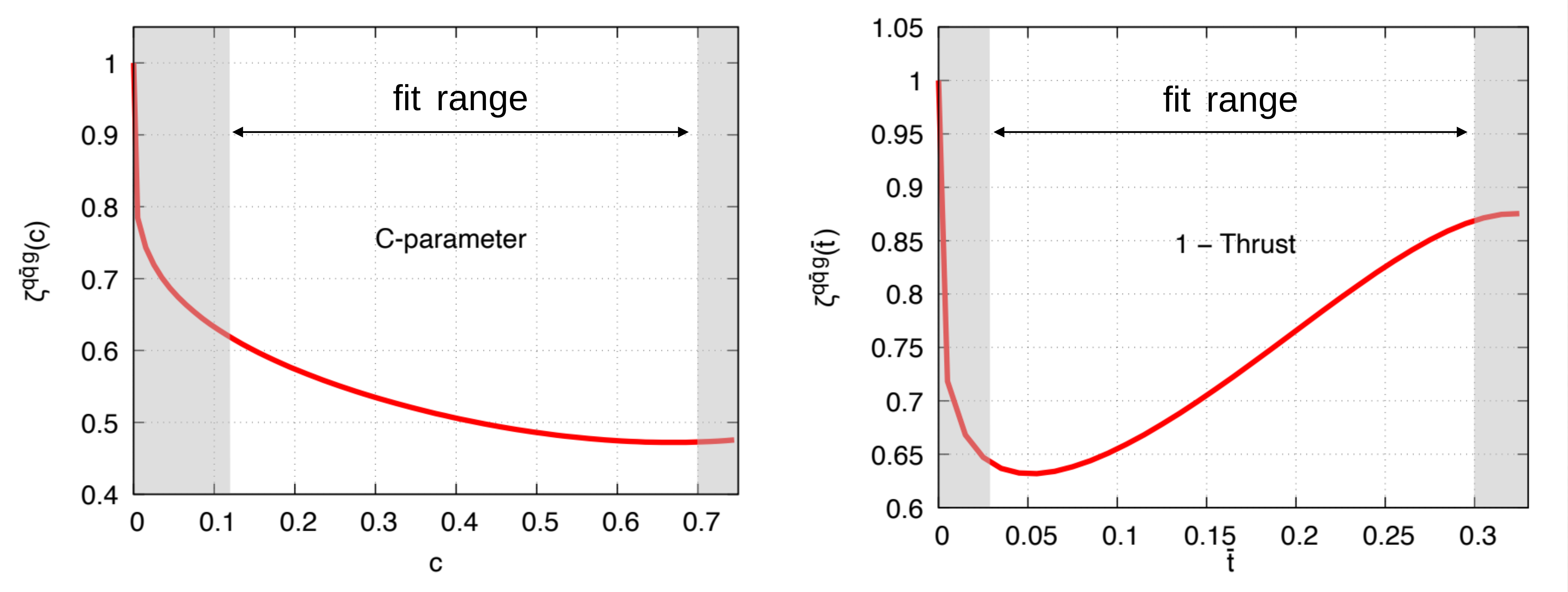


- Both fits assume that the shift in the fit range is the same as in the two-jet region, where 1-T an C are very small $\Rightarrow$ is this justified?

PT-NP interplay

- The 1/Q shift depends on the observable's value in the fit range $\Rightarrow$ extra 3-4% uncertainty in the determination of α_s [Luisoni Monni Salam 2012.00622]
- It is possible to calculate analytically the deviation $\zeta(v)$ of the shift from the two-jet limit $\zeta(0) = 1$ (see talk by P. Nason)

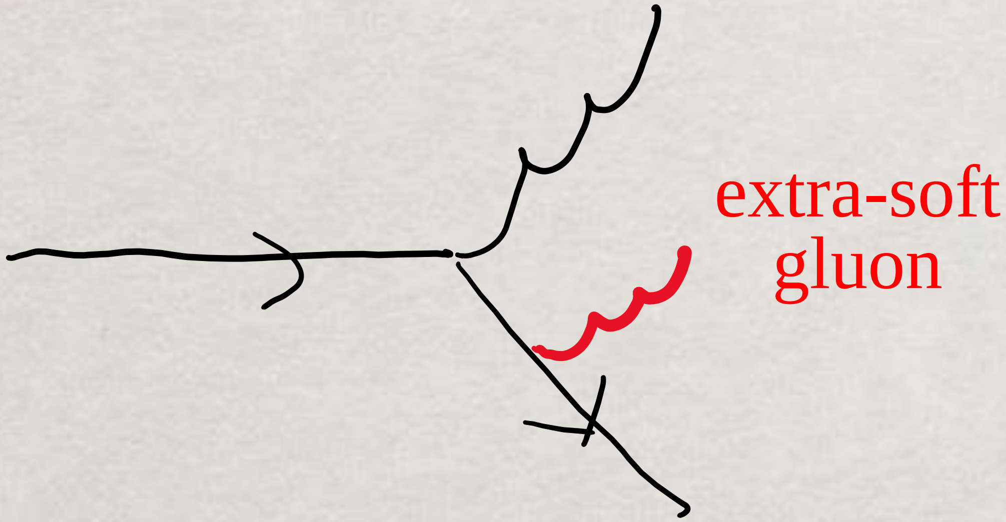
[Caola Ferrario-Ravasio Limatola Melnikov Nason Ozcelik 2204.02247]

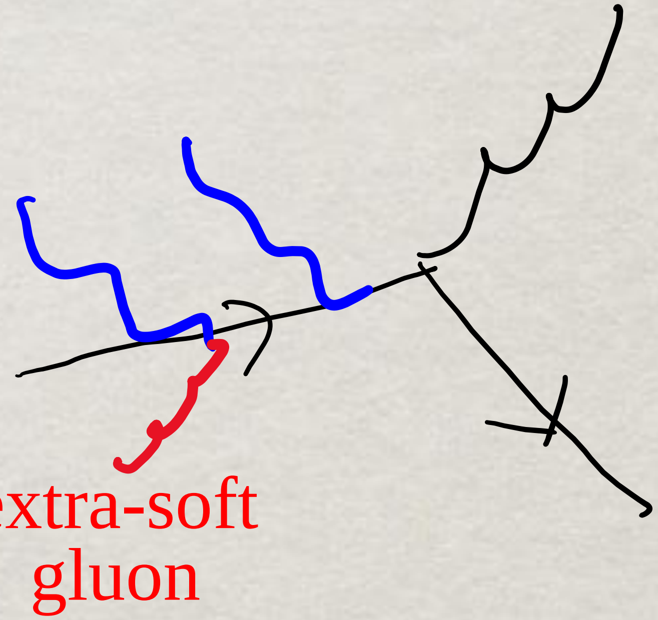


- New frontier for precision: calculation of the 1/Q shift in the three-jet region for all event shapes

PT-NP interplay

- The calculation of the shift in the three-jet region can be applied equally to the wide-jet broadening





- For the total jet broadening, one needs to account for the displacement of one of the hard partons from the thrust axis due to multiple soft-collinear emissions

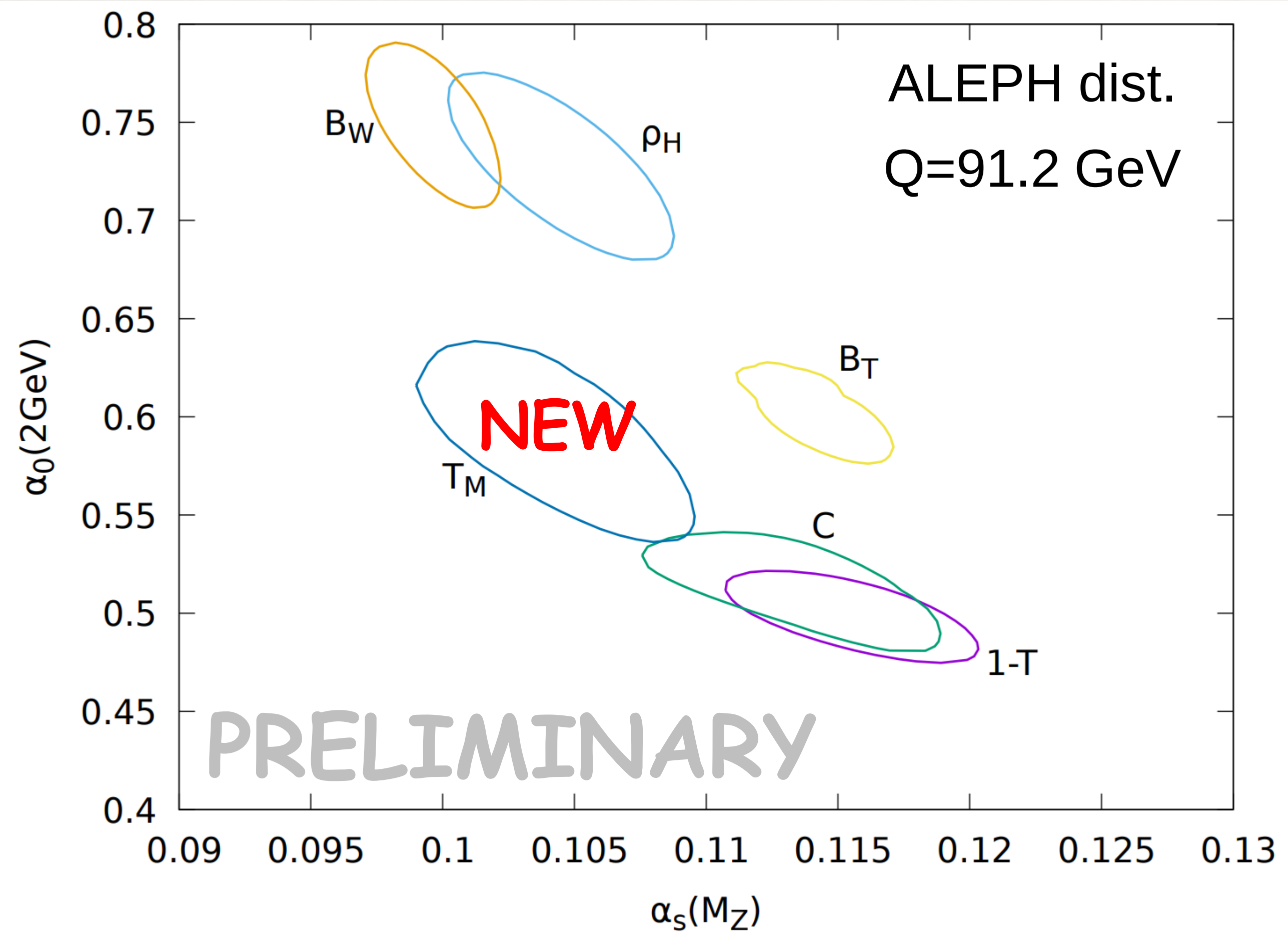
[Dokshitzer Marchesini Salam hep-ph/9812487]

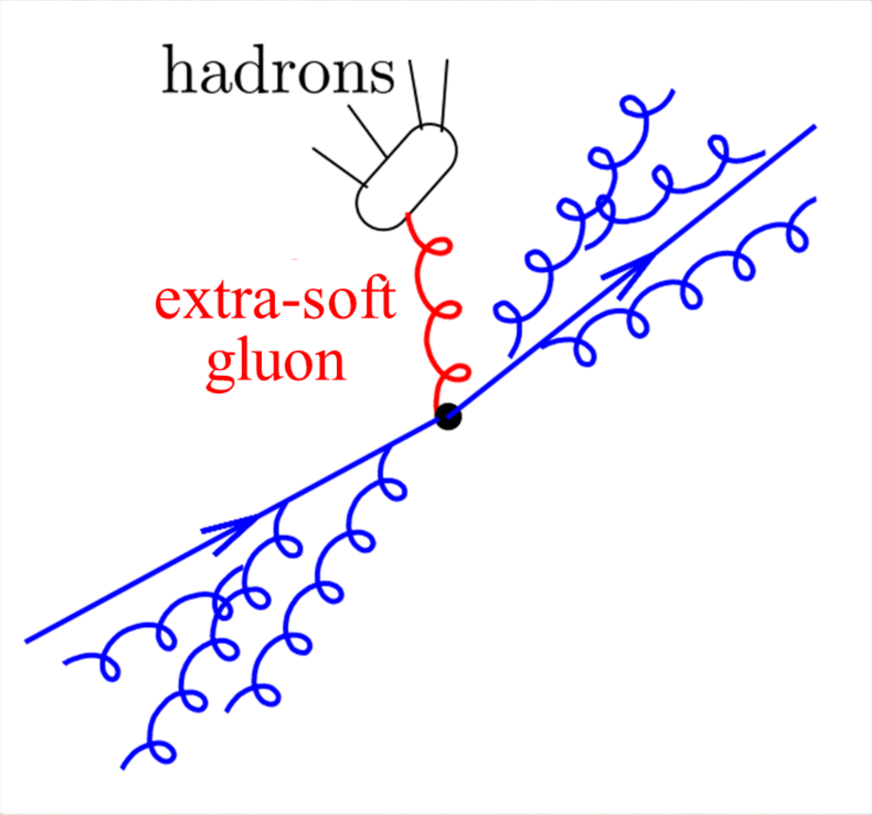
- For a generic event shape, even in the two-jet region, one needs to compute the shift in the presence of multiple soft-collinear emissions

PT-NP interplay

- In the two-jet region, the shift can be computed by considering a single extra-soft gluon accompanied by an arbitrary number of soft and collinear gluons: these can be simulated with a Monte-Carlo procedure

[AB El-Menoufi Wood 22xx.yyyyy]





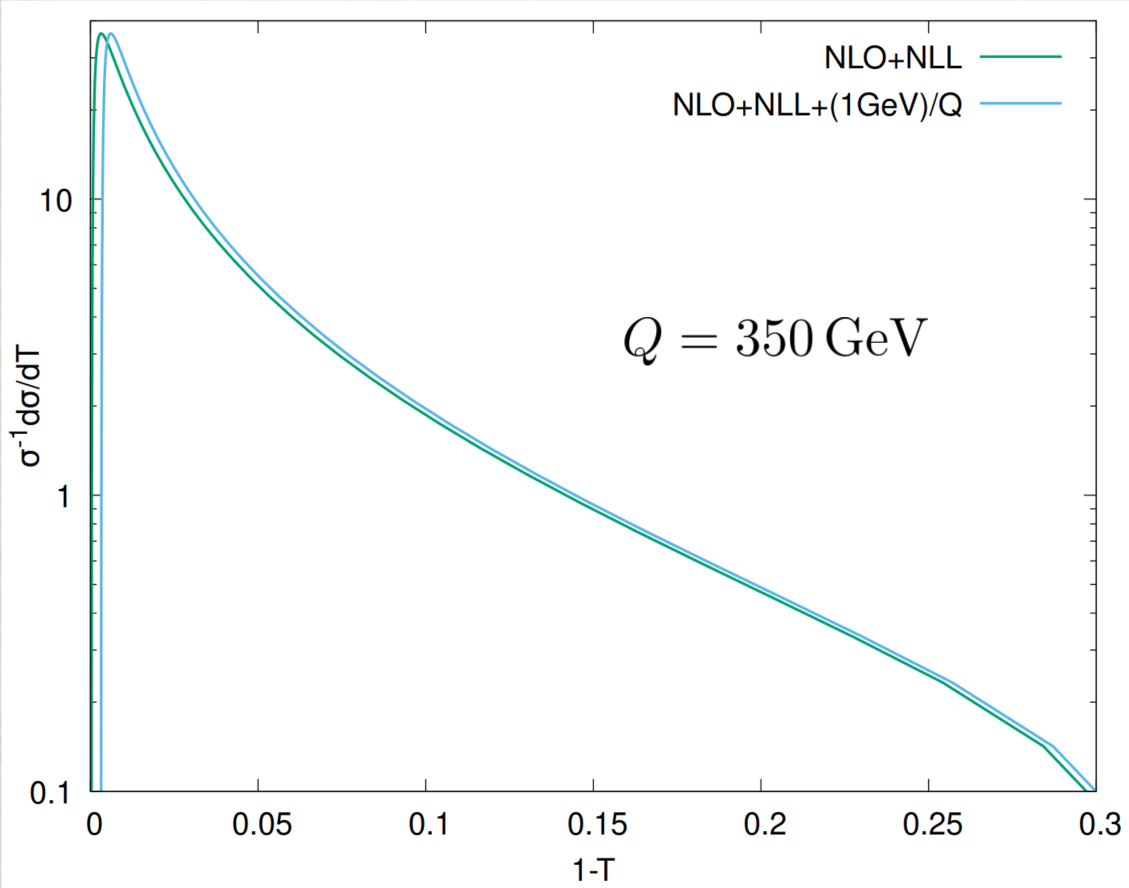
- Beyond NLL accuracy, new **perturbative** configurations have to be considered $\Rightarrow$ not clear whether 1/Q corrections correspond to a global shift

- Open question: If NNLL+1/Q programme is successful, what is the theoretical uncertainty associated to higher power corrections?

Pathways to Squeezing hadronization

Hadronization at future colliders

- At future lepton colliders, hadronization corrections to two-jet observables will be way smaller than at LEP1 $\Rightarrow$ 1 jet ~ 1 parton



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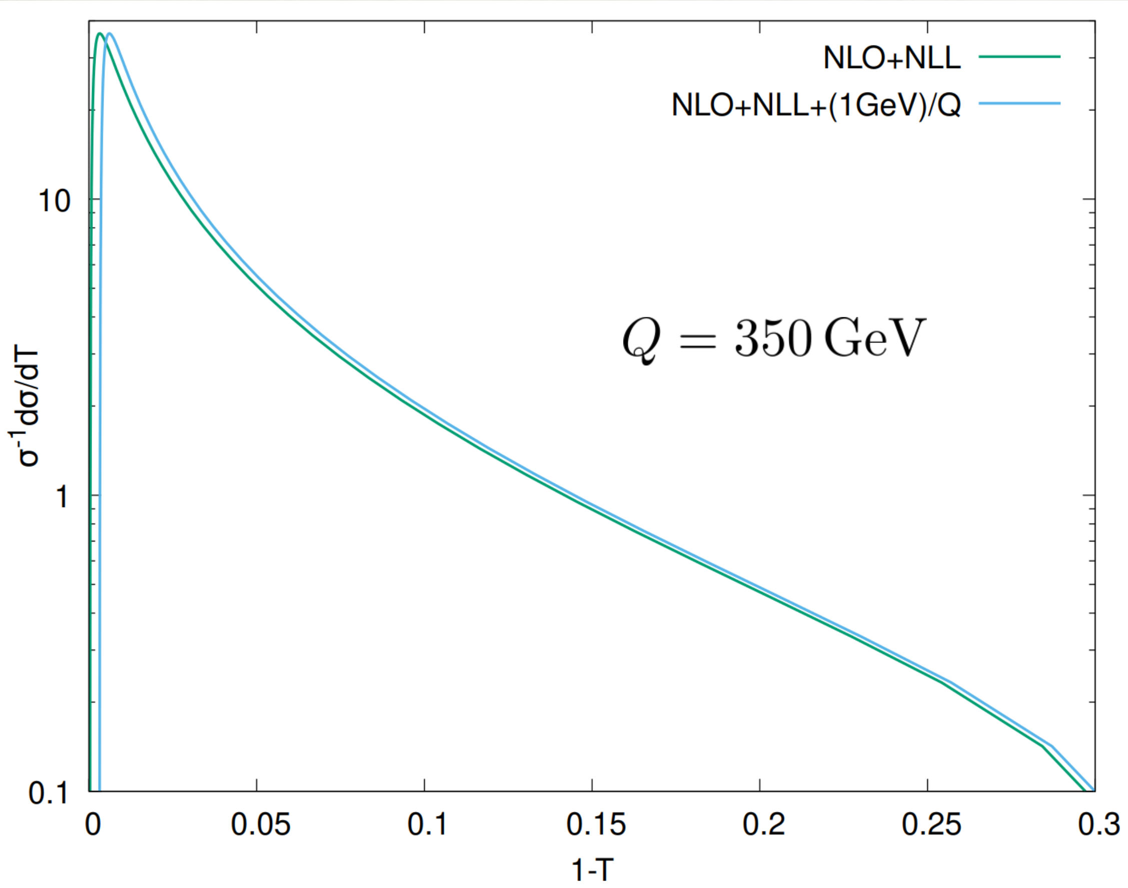
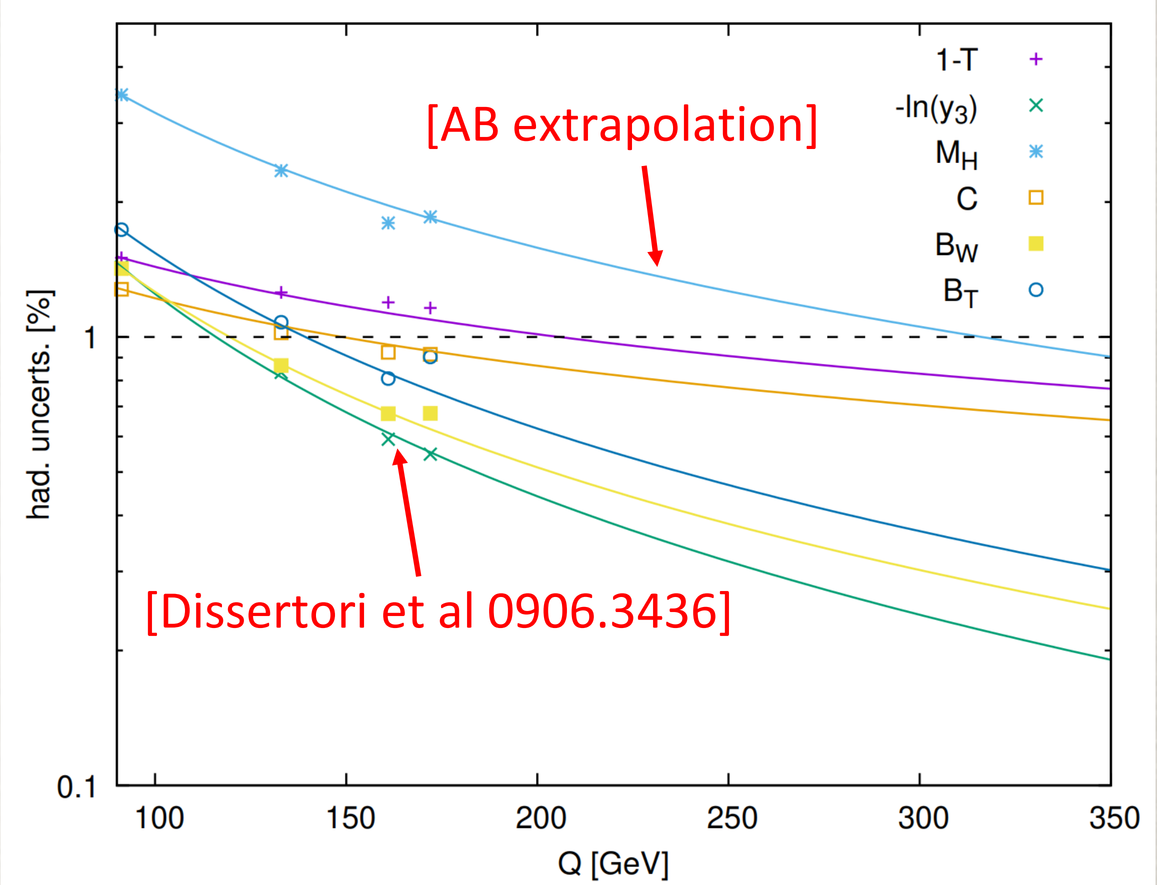


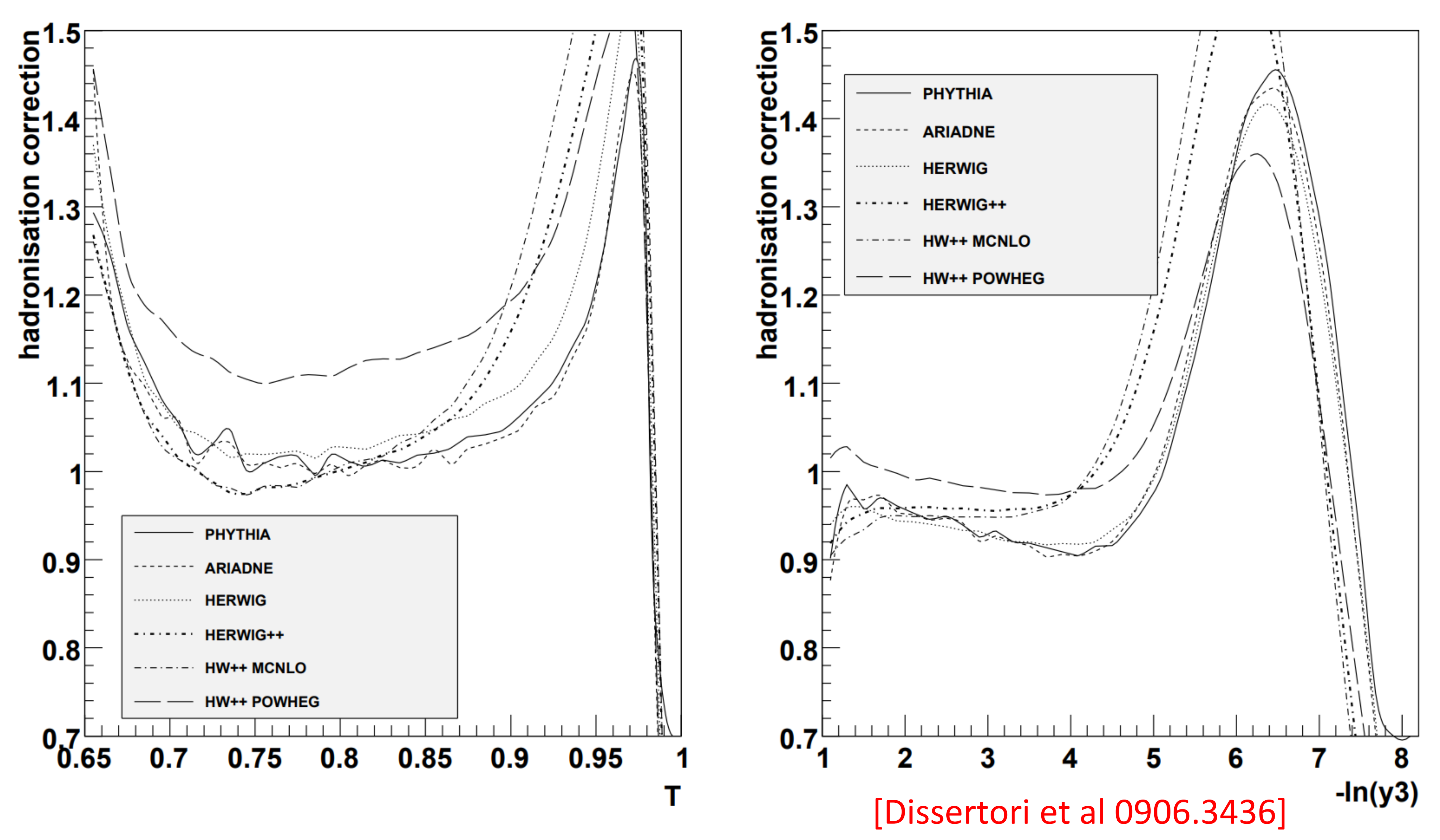


Two-fold advantage for fits of the strong coupling

- Monte-Carlo hadronization corrections would have a reduced impact in the error on α_s $\Rightarrow$ perturbative uncertainties (less than %) dominant
- Negligible impact of subleading hadronization corrections $\Rightarrow$ more reliable determination of NP parameter(s) of leading 1/Q corrections

The two-jet rate

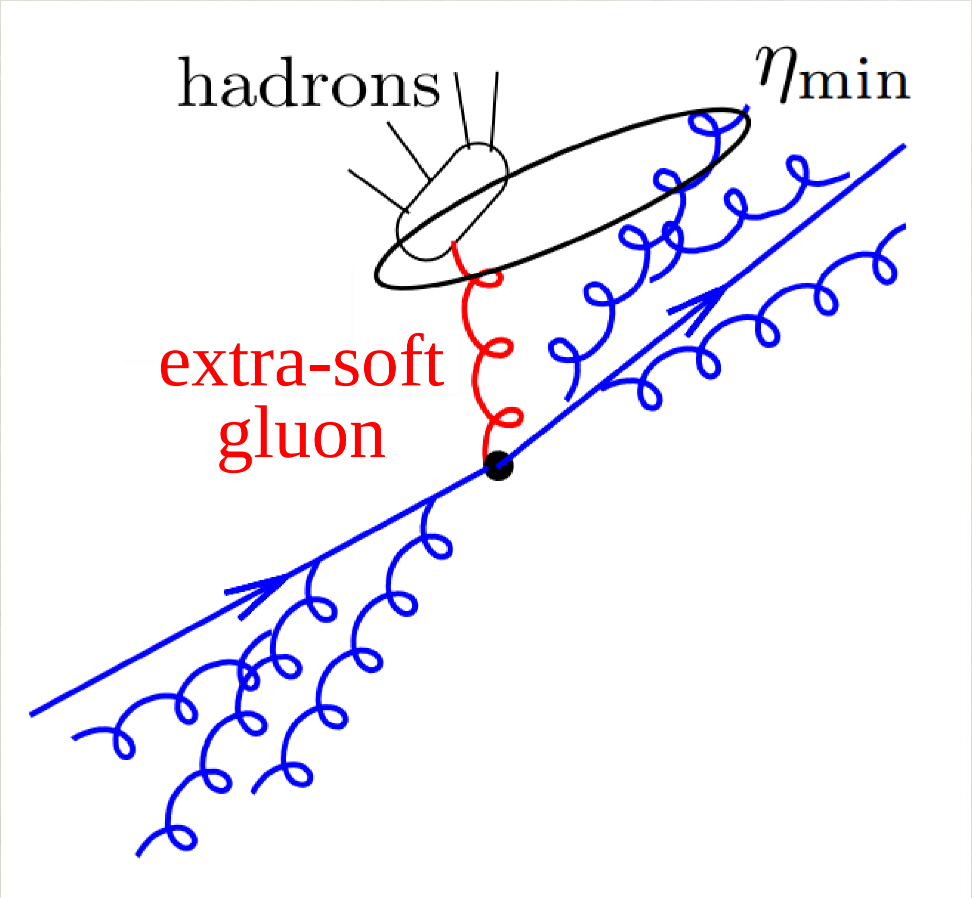
- The two-jet rate shows smaller hadronization corrections than event-shape distributions $\Rightarrow$ can we understand why?



[Dissertori et al 0906.3436]

The two-jet rate

- Without any perturbative emissions, hadronization corrections are $1/Q^2$
- In Durham algorithm, in the two-jet region, the presence of extra PT radiation might cause an extra-soft gluon to be clustered with PT soft-collinear gluons



computable with MC procedure

$$\text{shift} = \frac{\langle k_t \rangle_{\text{NP}}}{Q} \langle c_{y_3} \rangle_{\text{PT}}$$

$$\langle c_{y_3} \rangle_{\text{PT}} \sim \frac{k_{\text{t,jet}}}{Q} \int_0^{\eta_{\min}} d\eta$$

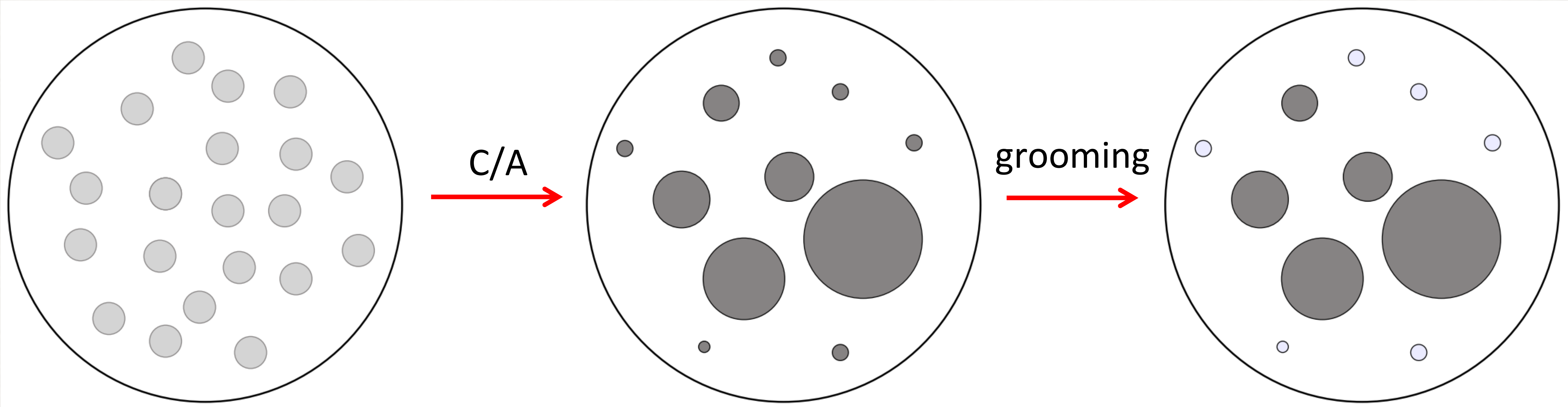
$$\frac{k_{\text{t,jet}}}{Q} \sim \sqrt{y_3} \qquad \eta_{\min} \sim \ln\left(\frac{1}{\sqrt{y_3}}\right)$$

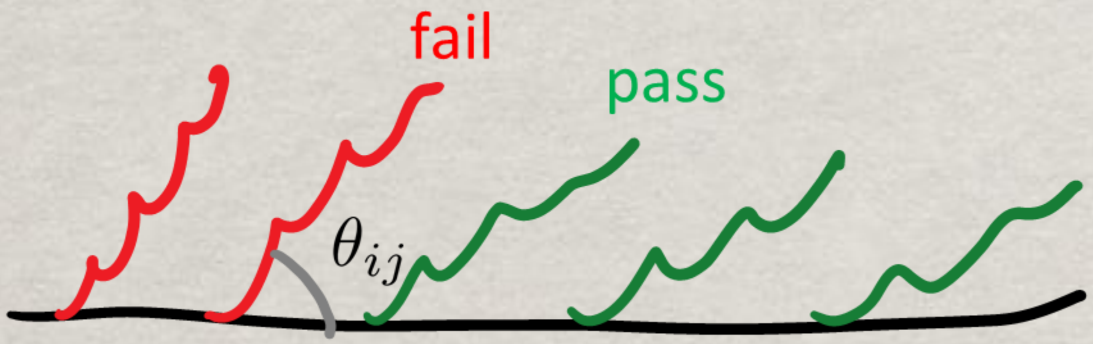
suppressed in the two-jet limit

- In the Cambridge algorithm, no clusterings between widely separated objects are allowed $\Rightarrow$ implication for 1/Q hadronization corrections?

Soft drop

- Groomers (mMDT, soft drop) are designed to clean jets from softer constituents [Larkoski Marzani Soyez Thaler 1402.0007]





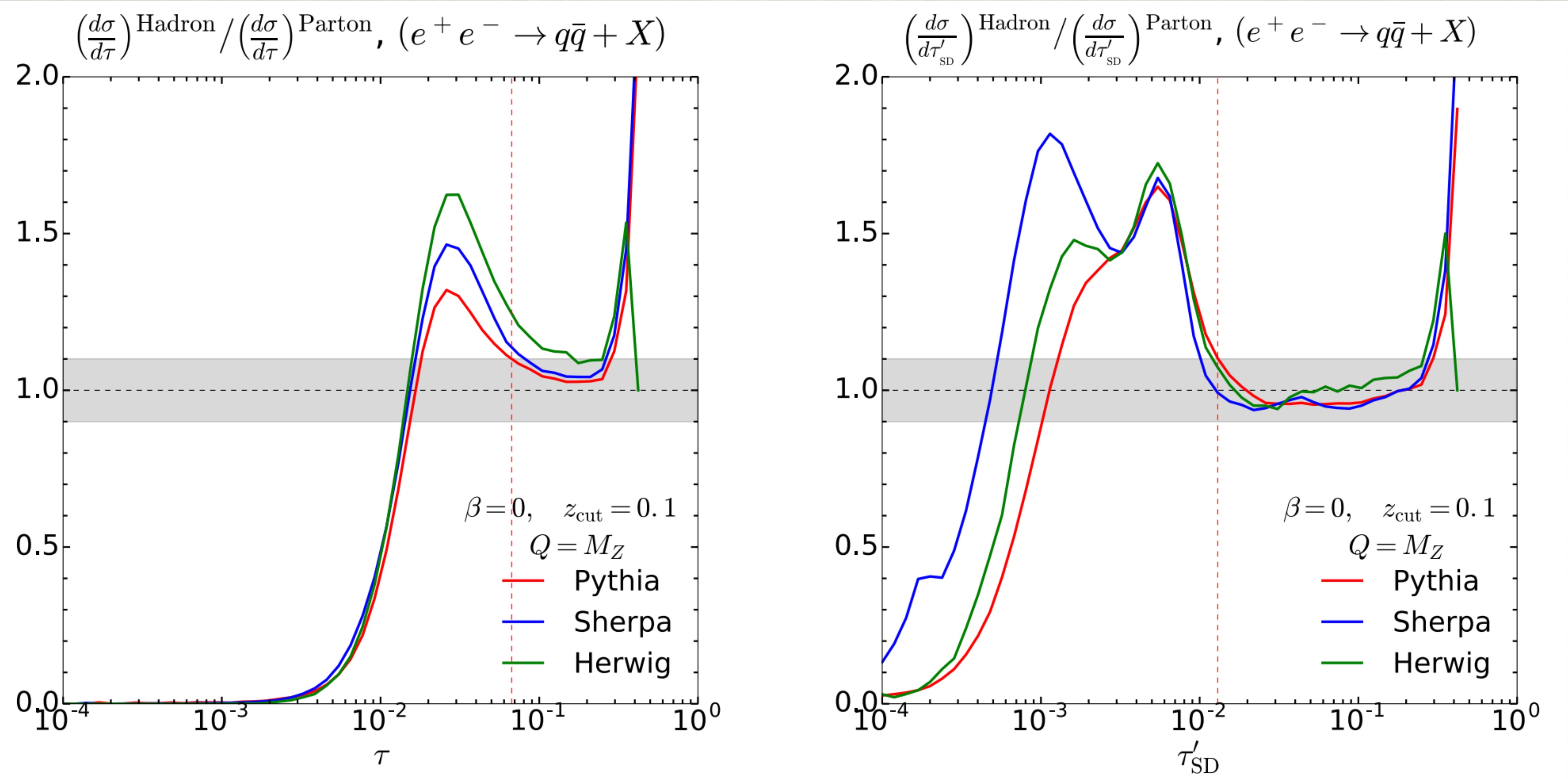
$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\rm cut} \left(\frac{1-\cos\theta_{ij}}{1-\cos R}\right)^{\beta/2}$$

$\beta = 0$: mMDT

Soft-drop thrust

- Grooming procedures can be applied to jet observables in order to eliminate soft large-angle hadrons
- Example: soft-drop thrust, computed on hadrons that survive a soft-drop procedure

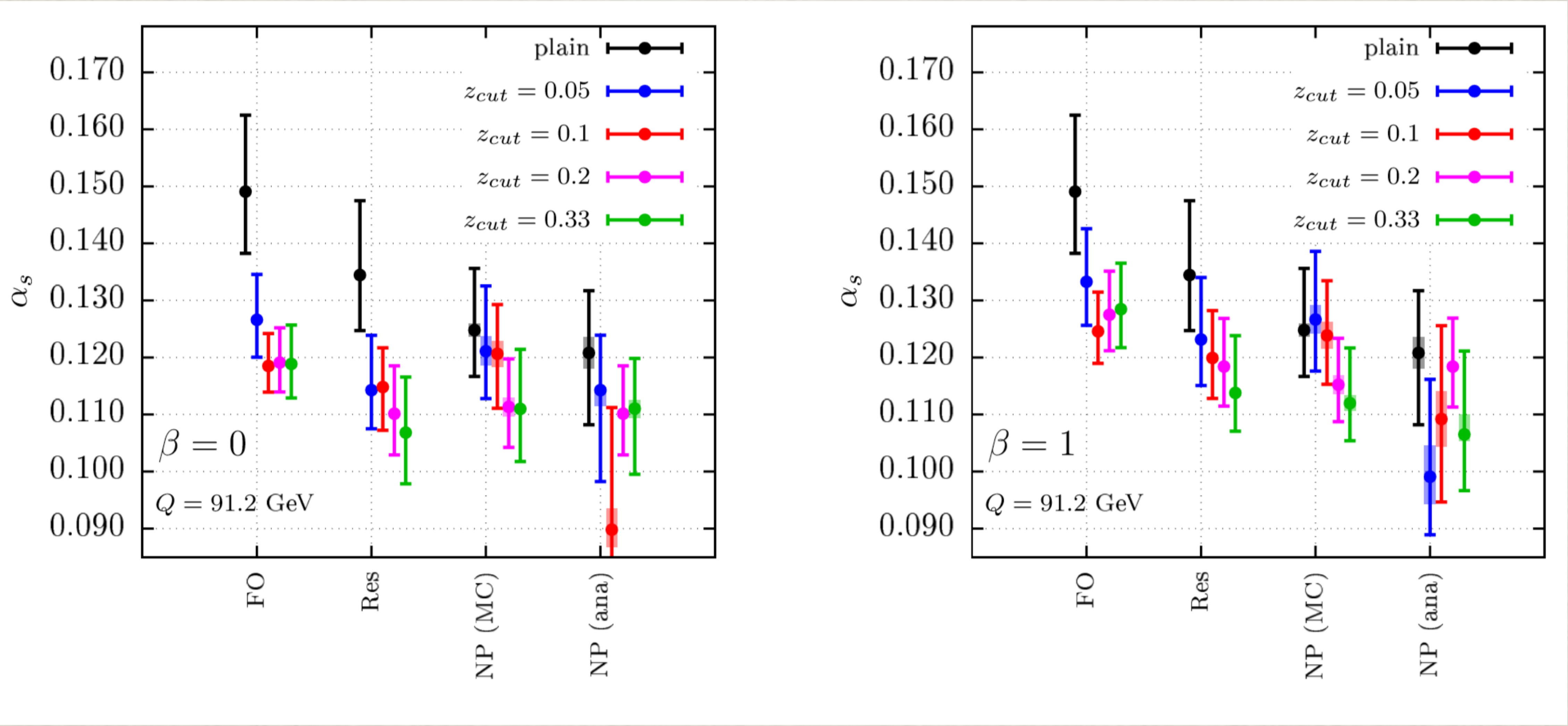
[Baron Marzani Theeuwes 1803.04719]



SOFT-DROP THRUST

- Determination of α_s using soft-drop thrust distribution at NLL+NLO and pseudo-data generated by SHERPA

[Marzani Reichelt Schumann Soyez Theeuwes 1906.1504]

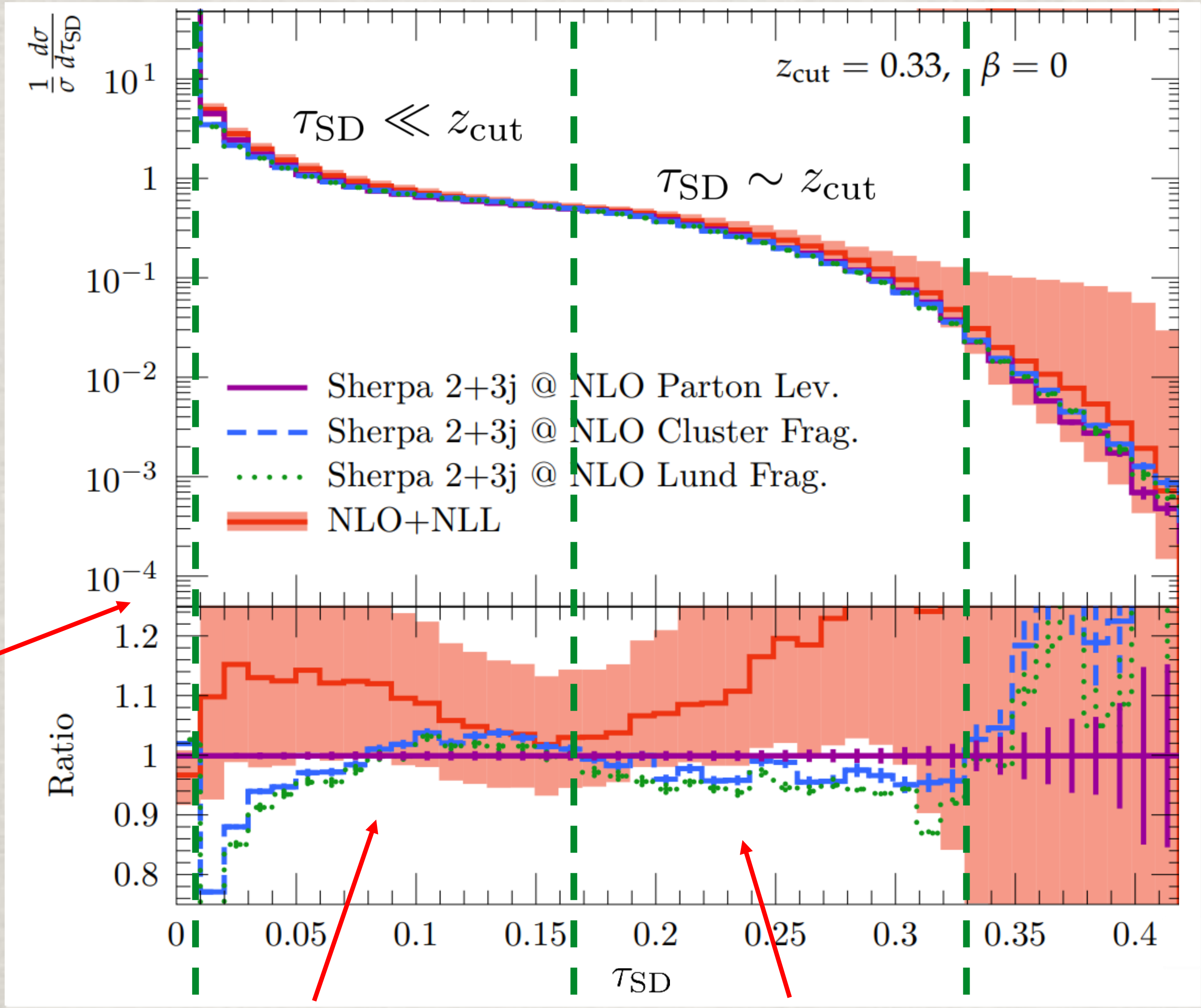


- At this accuracy, PT uncertainties still dominate $\Rightarrow$ NNLL resummation?

SOFT-DROP THRUST: ACCURACY

- The soft-drop distribution shows more features with respect to plain thrust

[Marzani Reichelt Schumann Soyez Theeuwes 1906.1504]



hadronization

$$\sim \frac{1}{Q}\left(\frac{1}{Q z_{cut}}\right)^{\frac{1}{1+\beta}}$$

NNLO

NNNLL resummation

[Kardos Larkoski Trocsanyi 2002.00942, 2002.05730]

Non-global resummation

[Benkendorfer Larkoski 2108.02779]

- Non-global resummations have been recently pushed to NNLL

[AB Dreyer Monni 2104.06416 , 2111.02413]

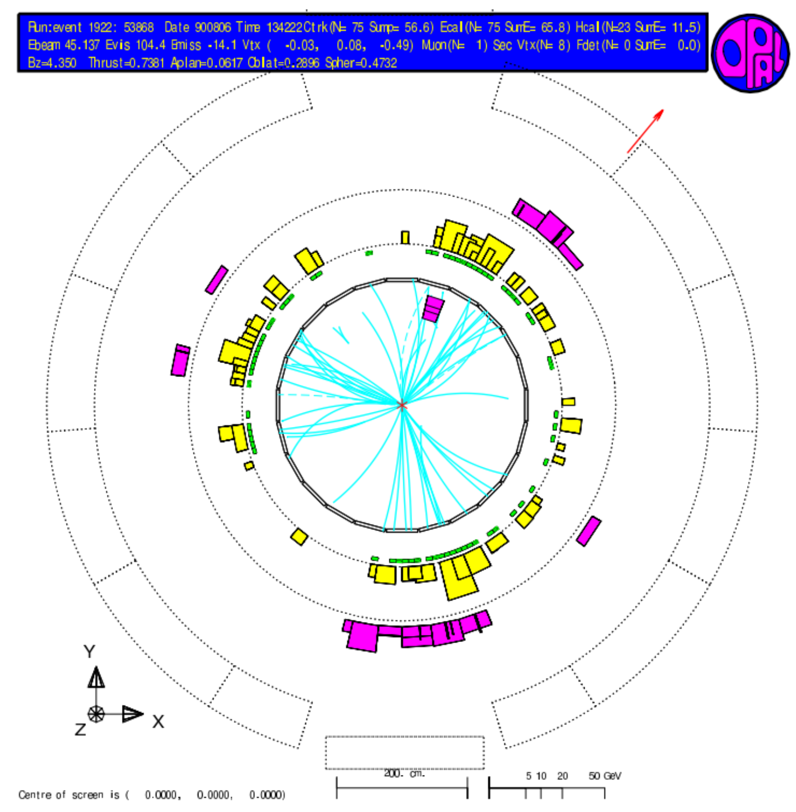
[Becher Rau Xu 2112.02108]

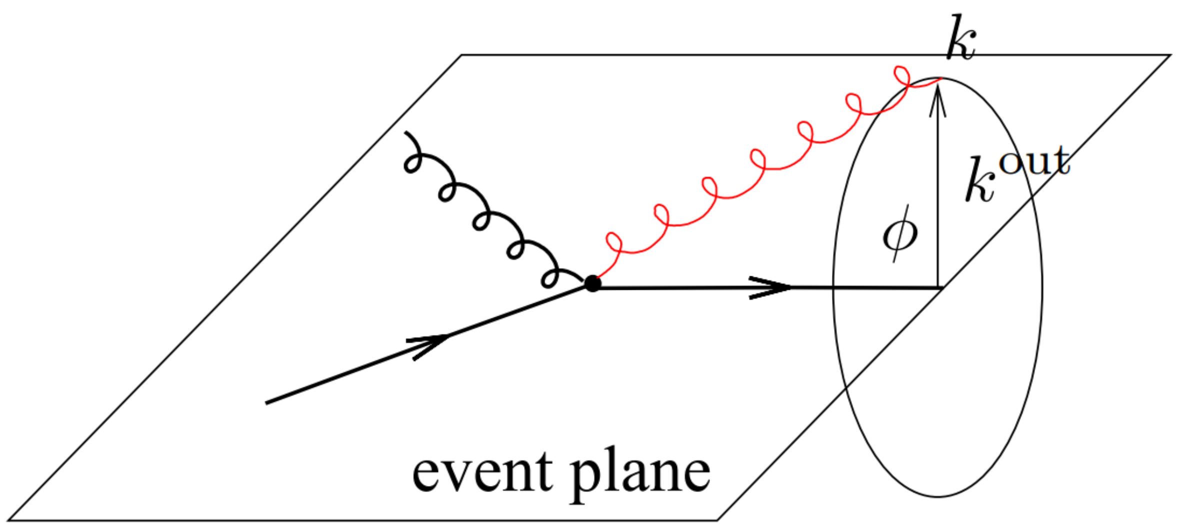
Multi-jet studies

Four-jet event shapes

- Near-to-planar four-jet event shapes (e.g. D-parameter) could be used to probe hadronization effects in gluon jets

[AB Dokshitzer Marchesini Zanderighi hep/ph 0104162]

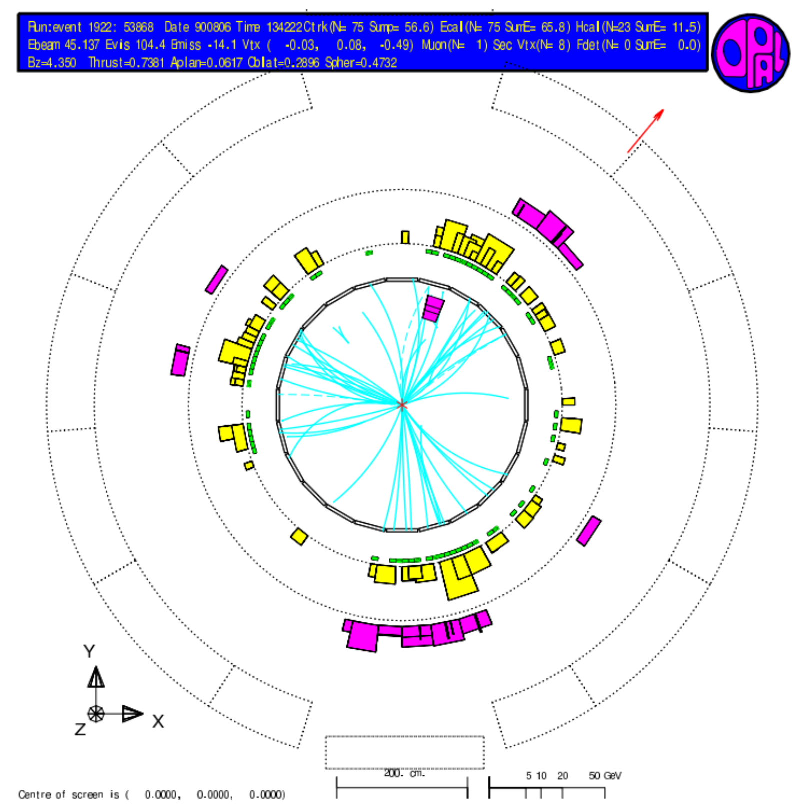


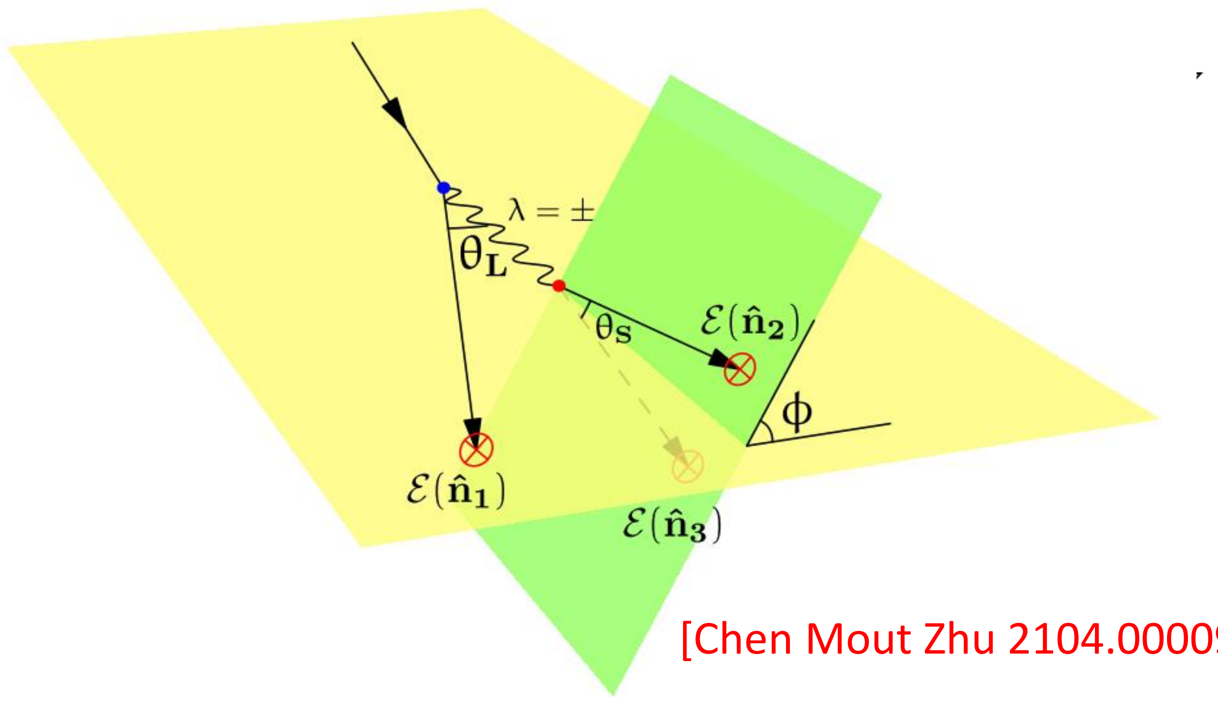


Four-jet event shapes

- Near-to-planar four-jet event shapes (e.g. D-parameter) could be used to probe hadronization effects in gluon jets

[AB Dokshitzer Marchesini Zanderighi hep/ph 0104162]





[Chen Mout Zhu 2104.00009]

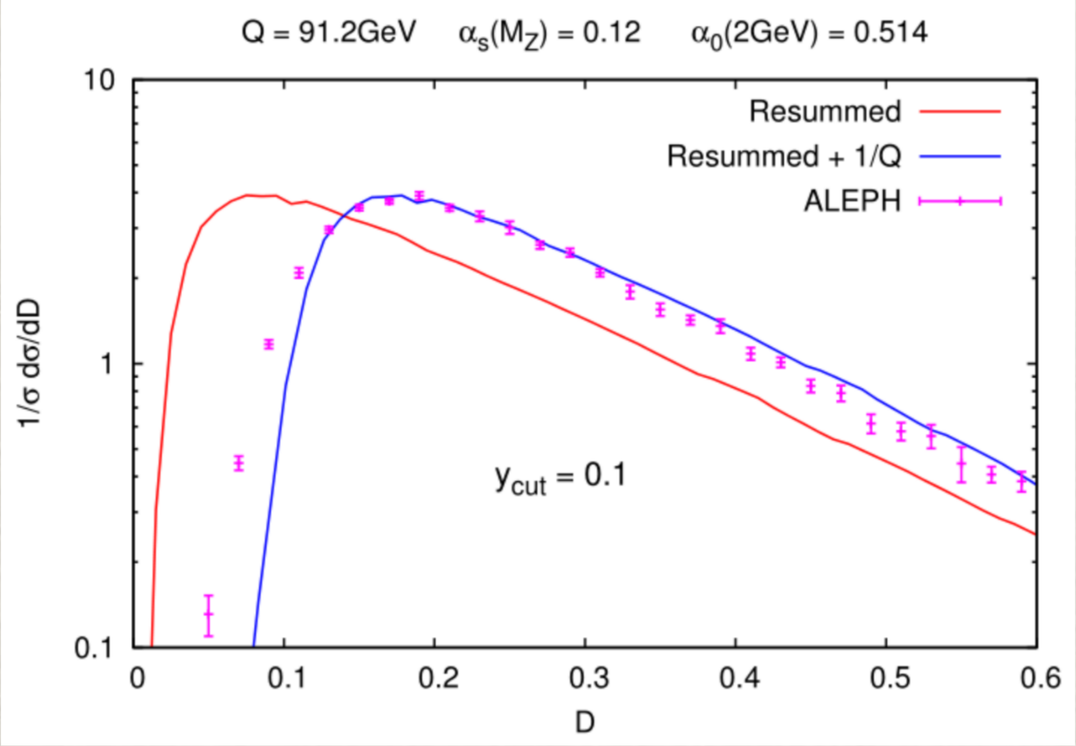
- Starting at NNLL accuracy, we can probe spin-correlation effects in collinear gluon splitting

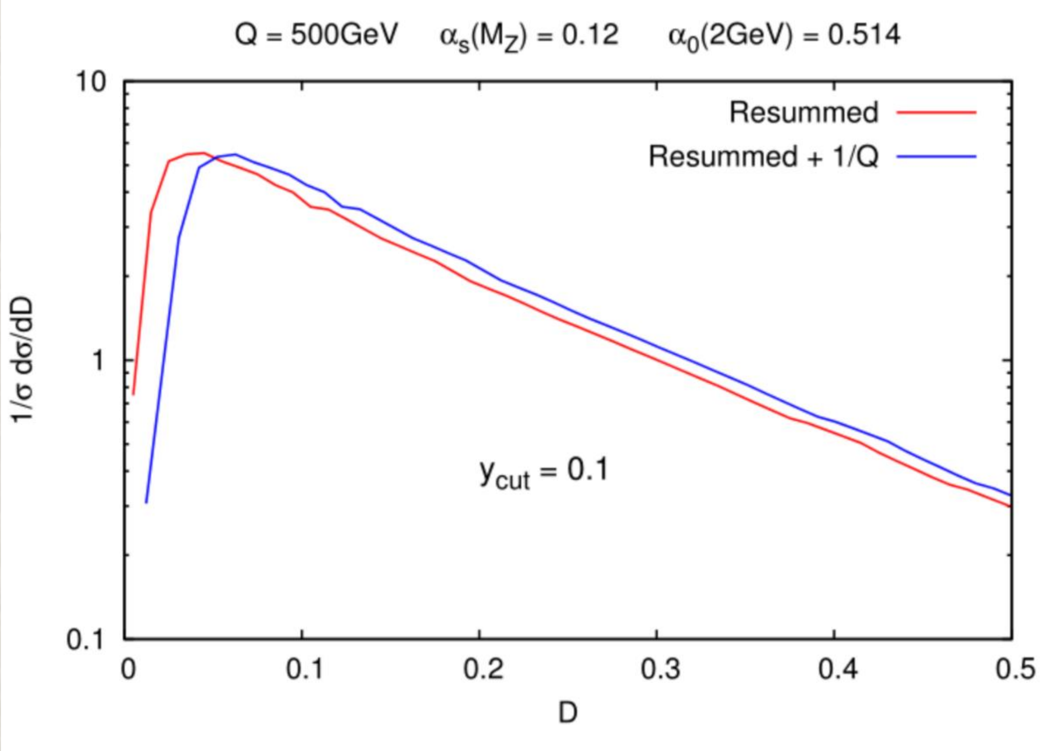
[Arpino AB El-Menoufi 1912.09341]

D-PARAMETER: HADRONISATION

- Hadronization corrections in three-jet events are very large at LEP (twice as large as in two-jet events due to radiating gluon) $\Rightarrow$ fits of leading hadronisation corrections problematic

[AB 0706.2722]





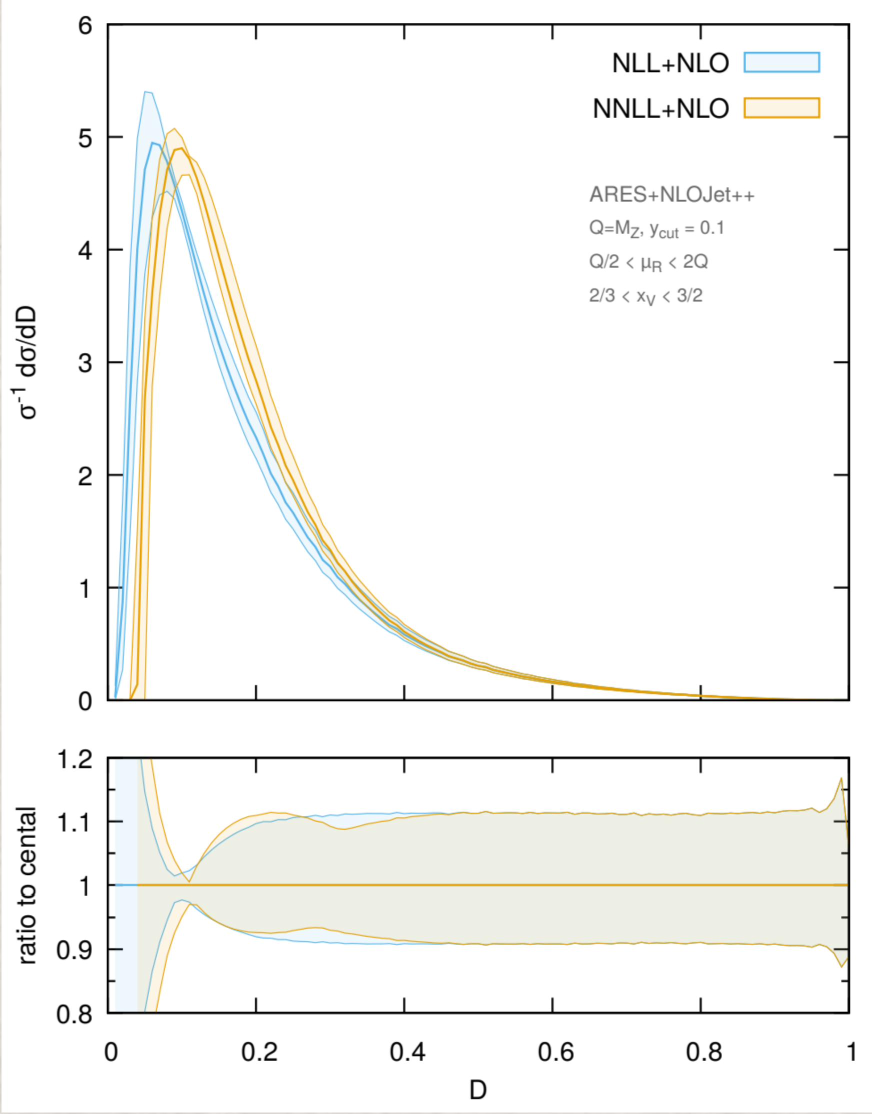
- Hadronization effects in three-jet observables at future e^+e^- colliders are as large as those for two-jet event shapes at LEP $\Rightarrow$ new tests of leading hadronization corrections?

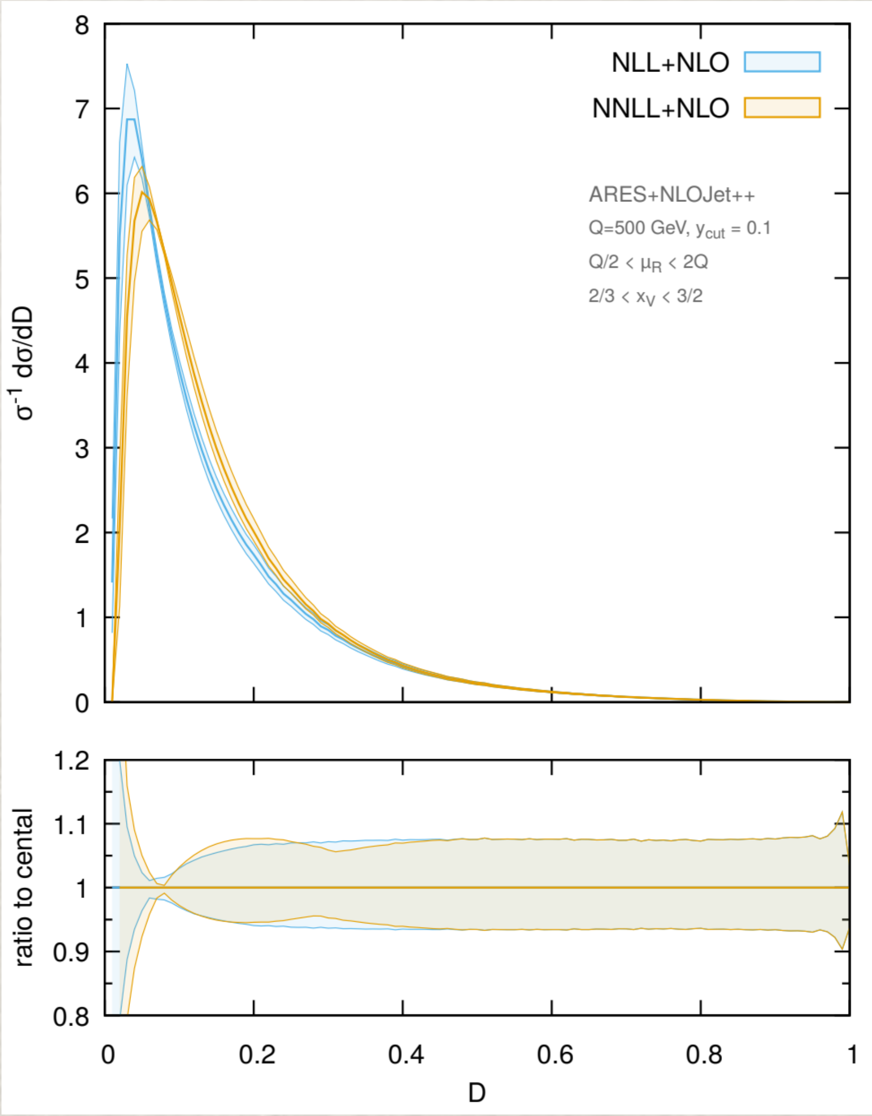
- Reaching the same accuracy as three-jet observables require a 1→ 4 NNLO calculation (within reach given recent progress in 2→ 3 calculations)

[Czakon Mitov Poncelet 2106.05331]

D-PARAMETER: PHENOMENOLOGY

- No significant reduction of theory uncertainties moving from NLL to NNLL due to third jet being mostly soft ⇒ increase y_{cut} ? [Arpino AB El-Menoufi 1912.09341]

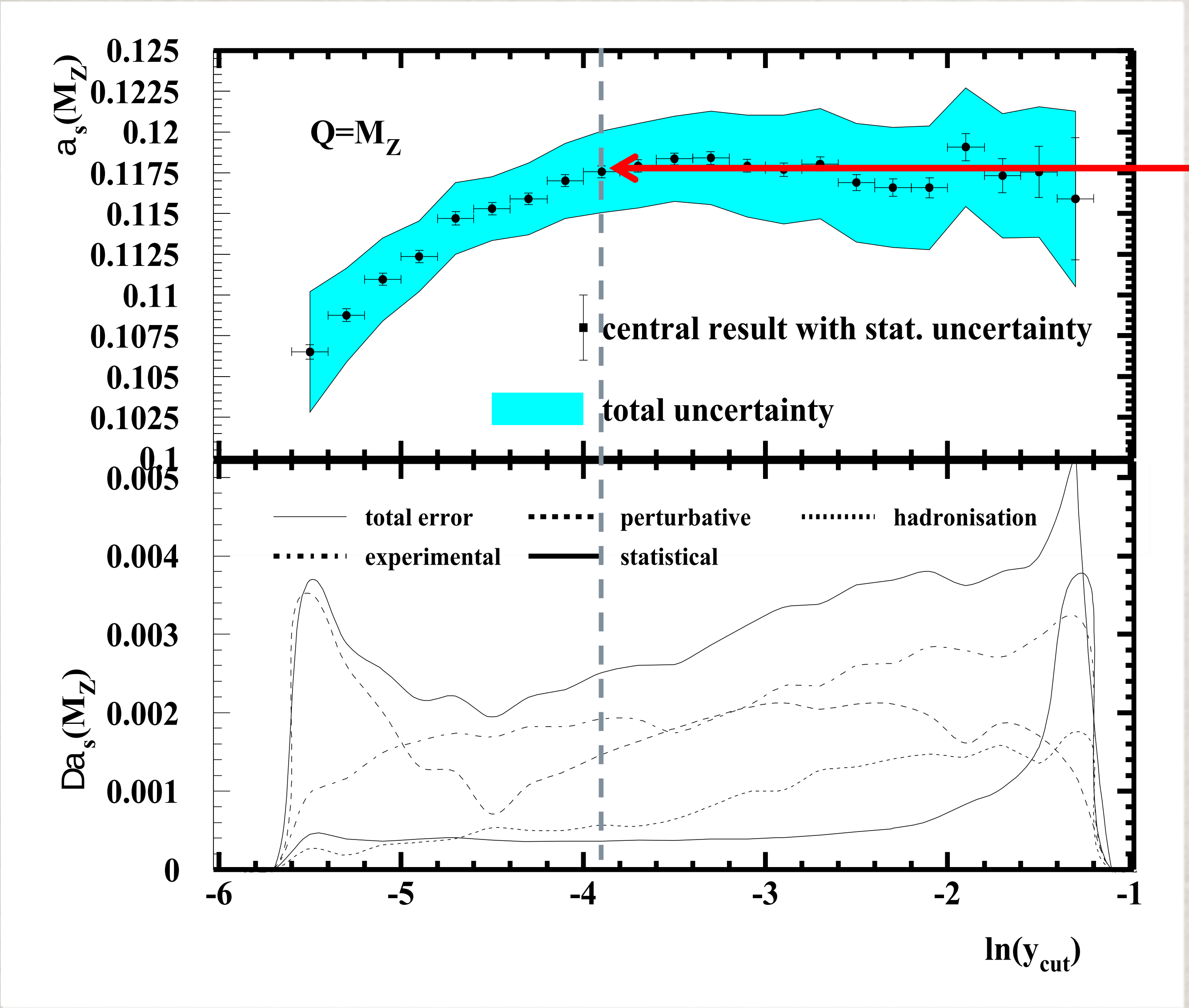




- Sending $y_{\mathrm{cut}} \to 0$ corresponds to studying four-jet event shapes in the two-jet limit. D-parameter known at (almost) NLL⇒NNLL? [Larkoski Procita 1810.06563]

Three-jet rate

- Competitive determination of α_s using ALEPH LEP1 data for the three-jet rate compared to NNLO [Dissertori Gehrmann Gehrmann Glover Heinrich Stenzel '09]



$$\alpha_s(M_Z) = 0.1175 \pm 0.0020\,(\mathrm{exp}) \pm 0.0015\,(\mathrm{theo})$$

- Hadronisation effects are very small in the fit range
- Experimental uncertainties dominated by systematics

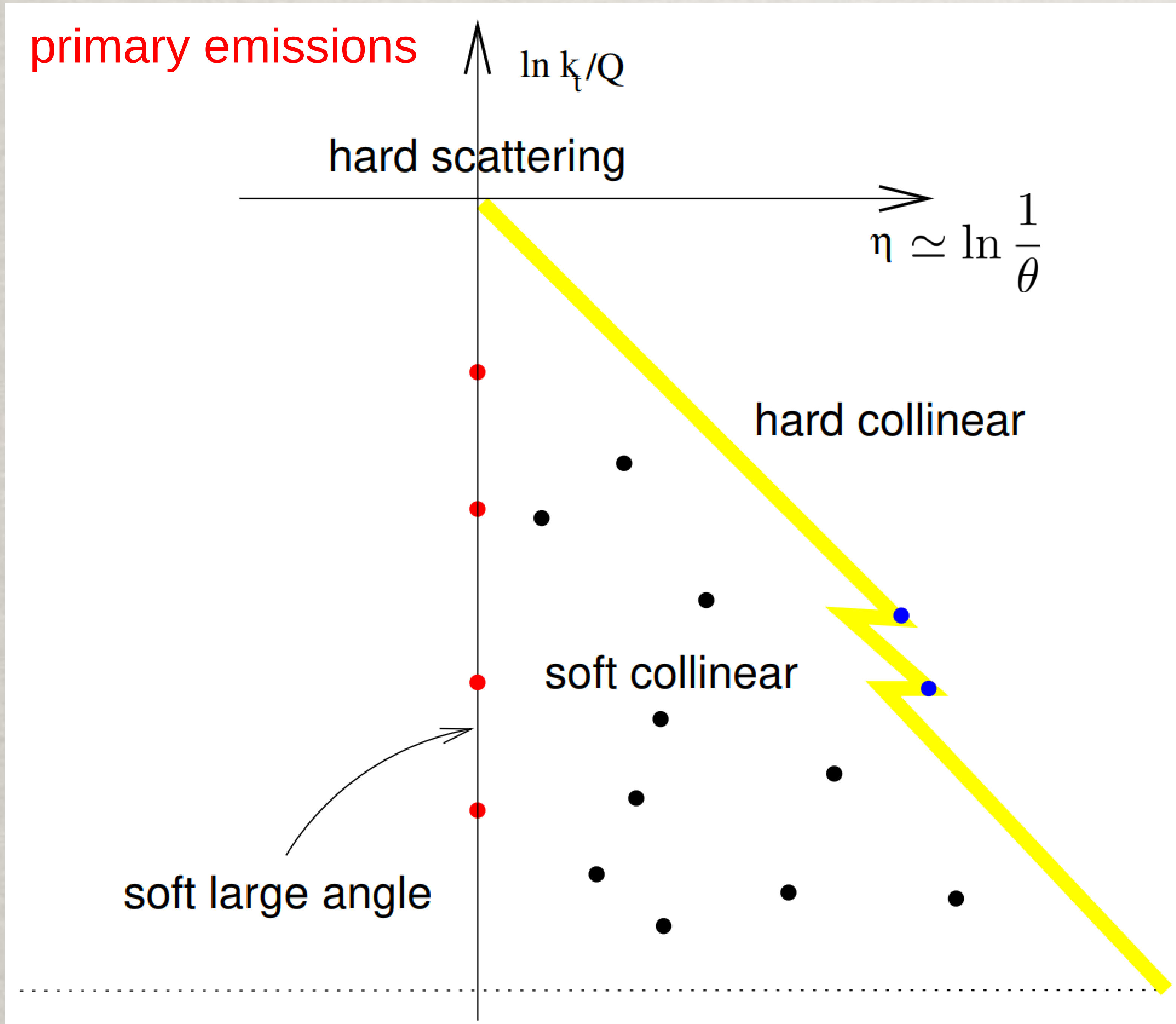
- Resummation needed for $\ln y_{\mathrm{cut}} \lesssim -4.5$: same problem as resumming four-jet event shapes in the two-jet limit

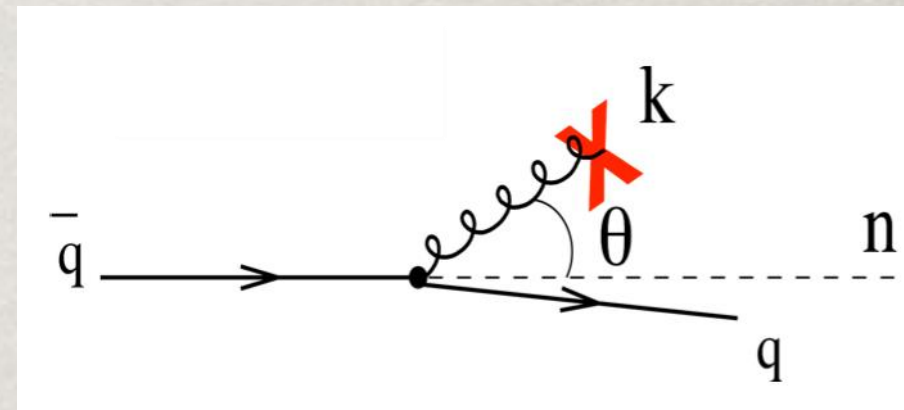
Lund-plane observables

The Lund plane

- The Lund plane is a very useful way to represent emissions in QCD

[Andersson Gustafson Lonnblad Pettersson Z. Phys. C43 (1989) 625]





Sudakov decomposition

$$P_1 = \frac{Q}{2}(1, \vec{n}) \qquad P_2 = \frac{Q}{2}(1, -\vec{n})$$

$$k = z^{(1)} P_1 + z^{(2)} P_2 + k_t$$

$$\eta = \ln\left(\frac{z^{(1)}}{z^{(2)}}\right)$$

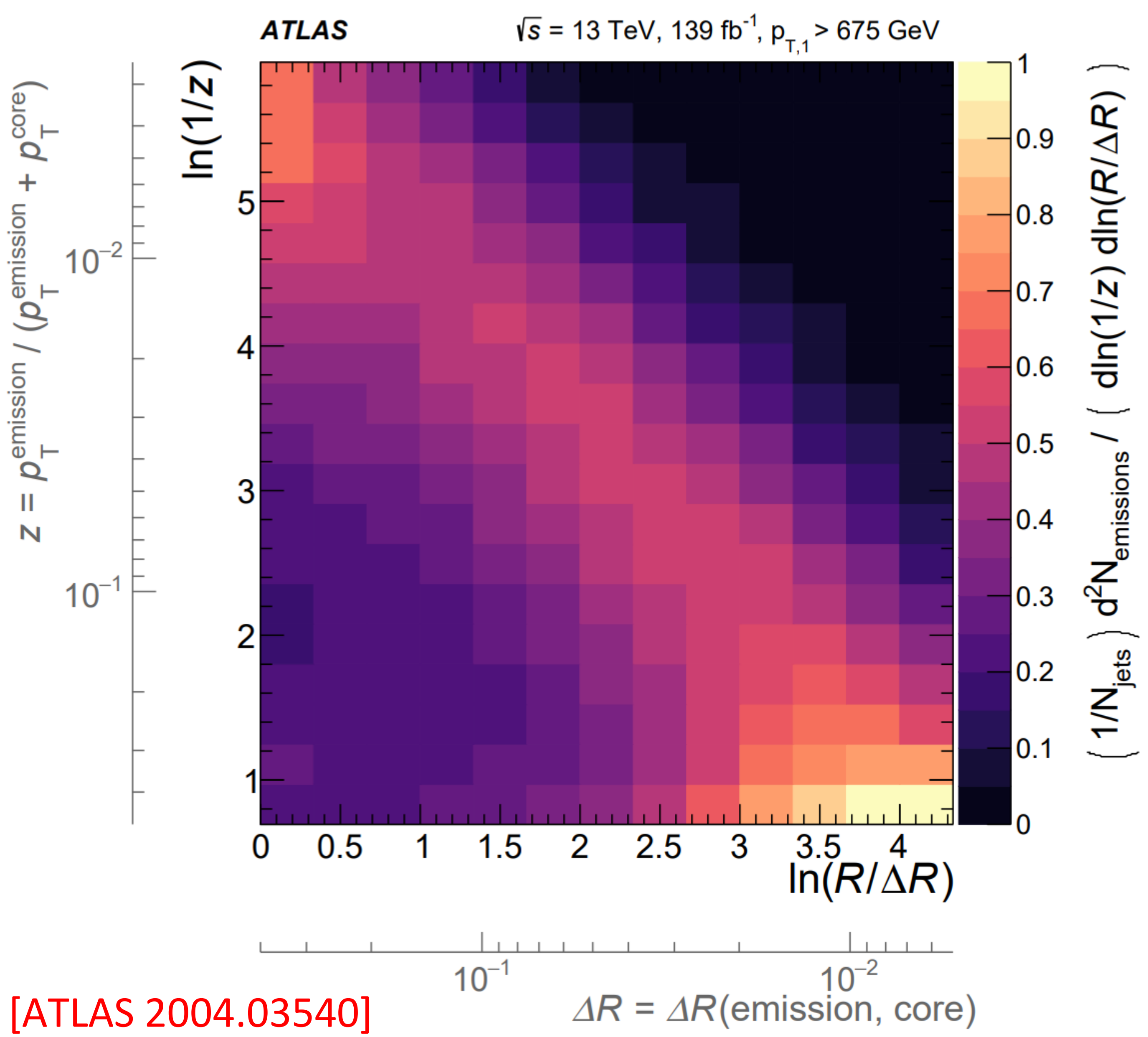
The Lund-plane density

- Primary emissions in the Lund plane can be defined in an IRC safe way by reclustering a jet with the C/A algorithm and following the harder branch

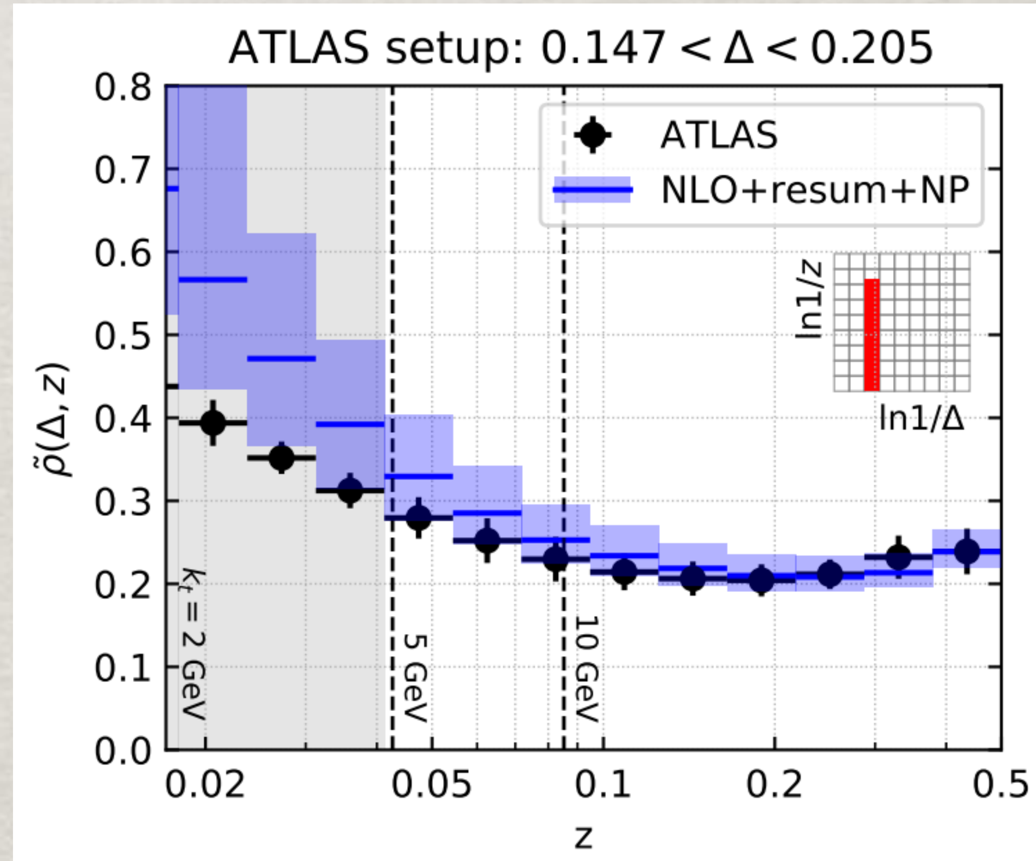
primary Lund-plane density

$$\rho(\eta, k_t) = \frac{1}{N_{\text{jets}}} \frac{dn_{\text{emsn}}}{d\eta\, d\ln k_t} \simeq \frac{2\alpha_s C_F}{\pi}$$

directly sensitive to α_s



[ATLAS 2004.03540]



[Lifson Salam Soyez 2007.06578]

Sub-jet multiplicity

- It is possible also to compute the Lund-plane subjet multiplicity in terms of successive refinements of double-logarithmic accuracy

[Medves Soto-Ontoso Soyez 2205.02861]

$$\langle N(\alpha_s, L)\rangle \simeq \langle N(\alpha_s, 0)\rangle \left[\underbrace{h_1(\alpha_s L^2)}_{\text{DL}} + \underbrace{\sqrt{\alpha_s} h_2(\alpha_s L^2))}_{\text{NDL}} + \underbrace{\alpha_s h_3(\alpha_s L^2)}_{\text{NNDL}} + \ldots\right] \qquad L \equiv -\ln y_{\text{cut}}$$

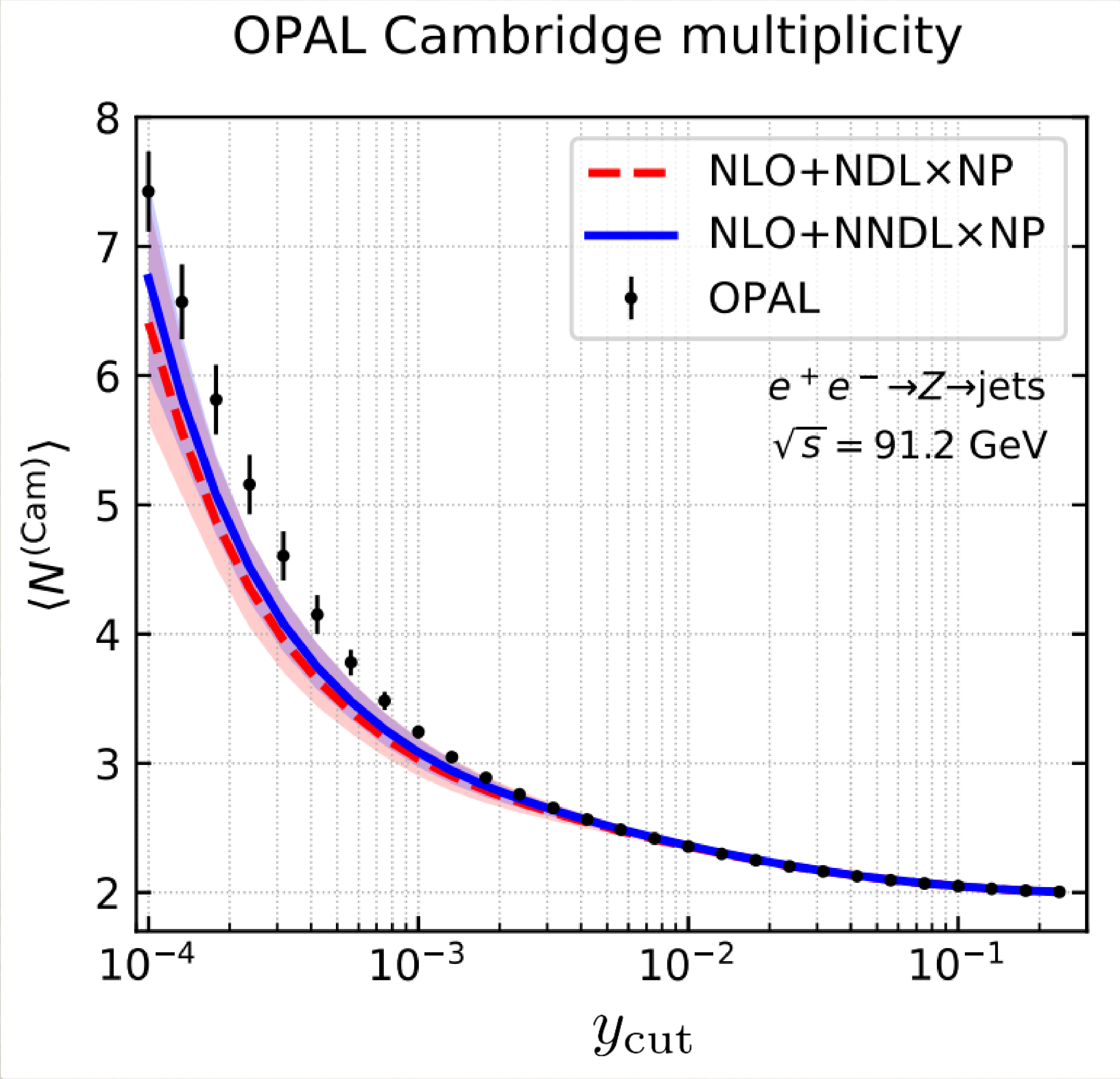
- Reinterpret old LEP data in terms of Lund-plane observables?

[ALEPH 2111.09914]

Note: even at lower logarithmic accuracies, resummations for Lund-plane observables are technically very challenging, and require a massive use of semi-numerical techniques for single-logarithmic resummations

[Dasgupta Salam hep-ph/0104277]

[Dasgupta Dreyer Salam Soyez 1411.5182]



Concluding remarks

Pushing the uncertainty in the detemination of the strong coupling below percent level involves challenges of various kind

- Computational: matching of hadronization corrections in two- and three-jet regions
- Conceptual: resummation of four-jet observables in the two-jet limit
- Technical: precision calculations for Lund-plane and jet-substructure observable

Programme of physics that could be performed in the next few years

- Global fit of three-jet event shapes at NNLL+NNLO+1/Q
- NNLL+NNLO calculation of three-jet rate and soft-drop thrust
- NNLL+NNLO calculations for Lund-plane observables (and what about hadronization?)

Concluding remarks

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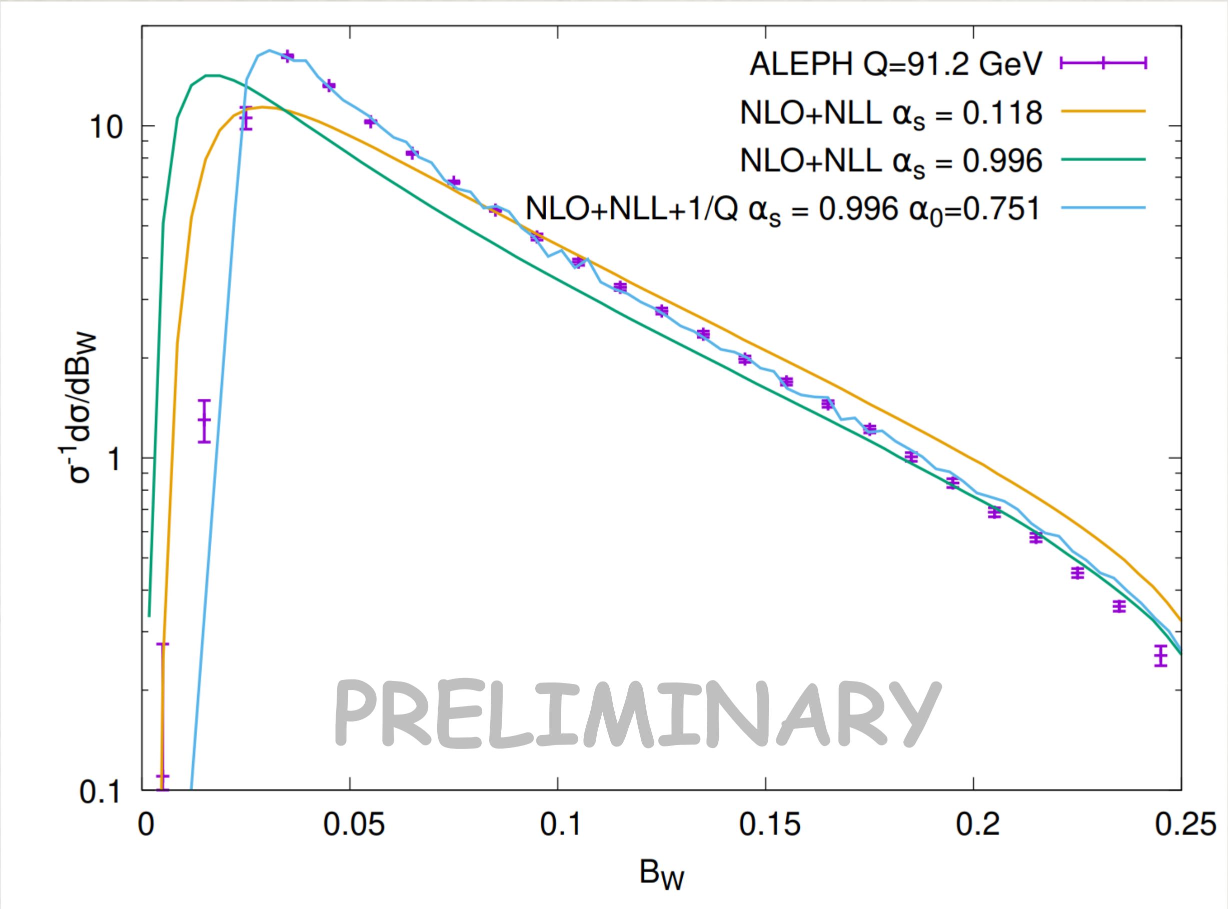
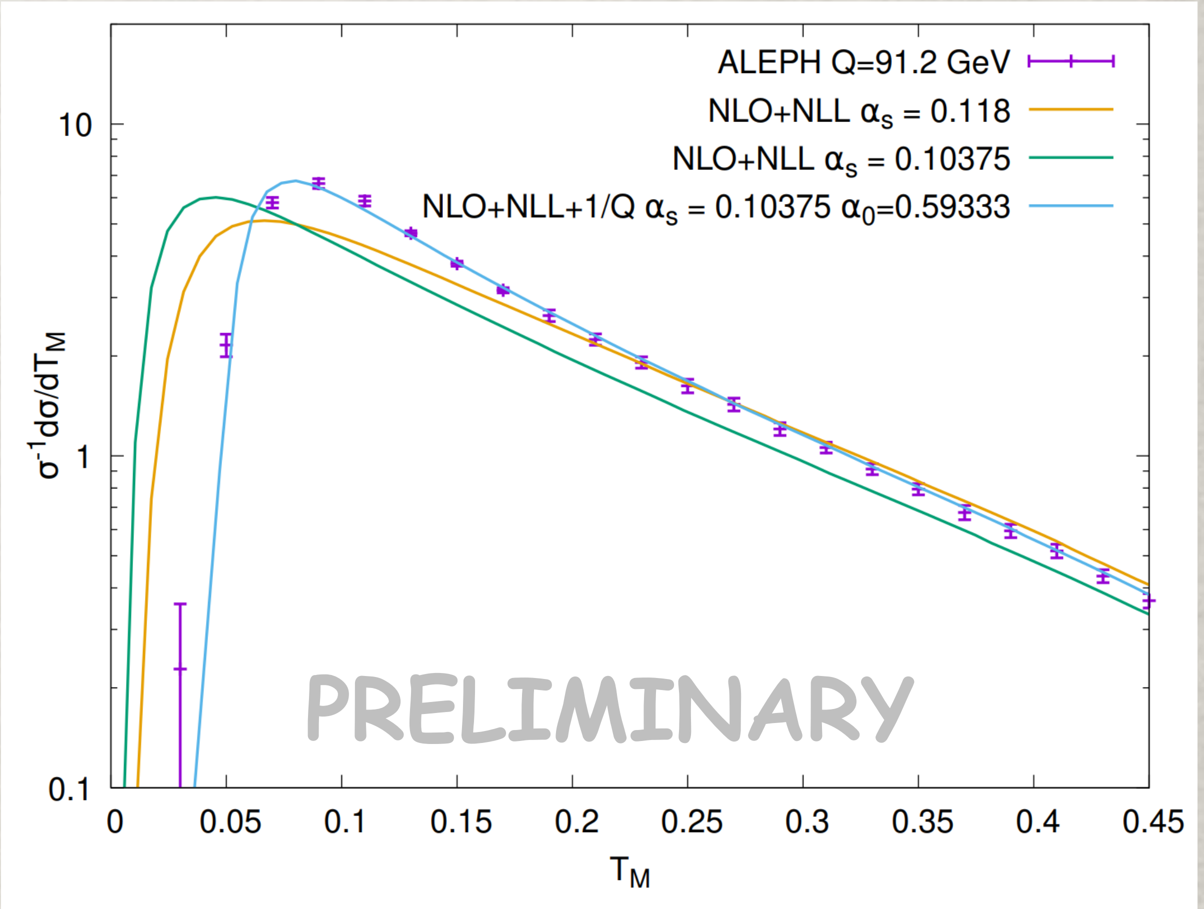
Thank you for your attention!

Extra

PT-NP interplay

- In the three-jet region, the shift for B_W and T_M becomes negative for values of α_s in line with the world average



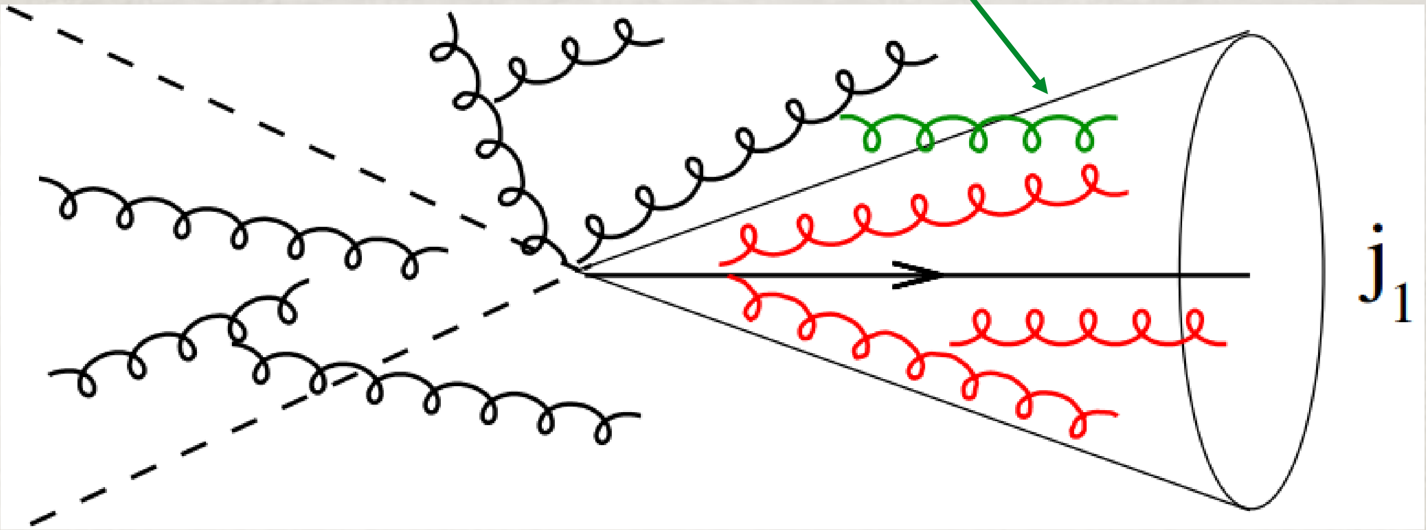


- Matching the shift to the three-jet region will most likely have a huge impact on simultaneous fits of α_s and NP parameter α_0

Non-global logarithms

- Non-global logarithms (NGLs) arise whenever measurements are restricted to limited regions of phase space, e.g. single-jet mass distribution
- They originate when softest emission in a correlated cascade of soft gluons enters the measurement region

[Dasgupta Salam hep-ph/0104277]

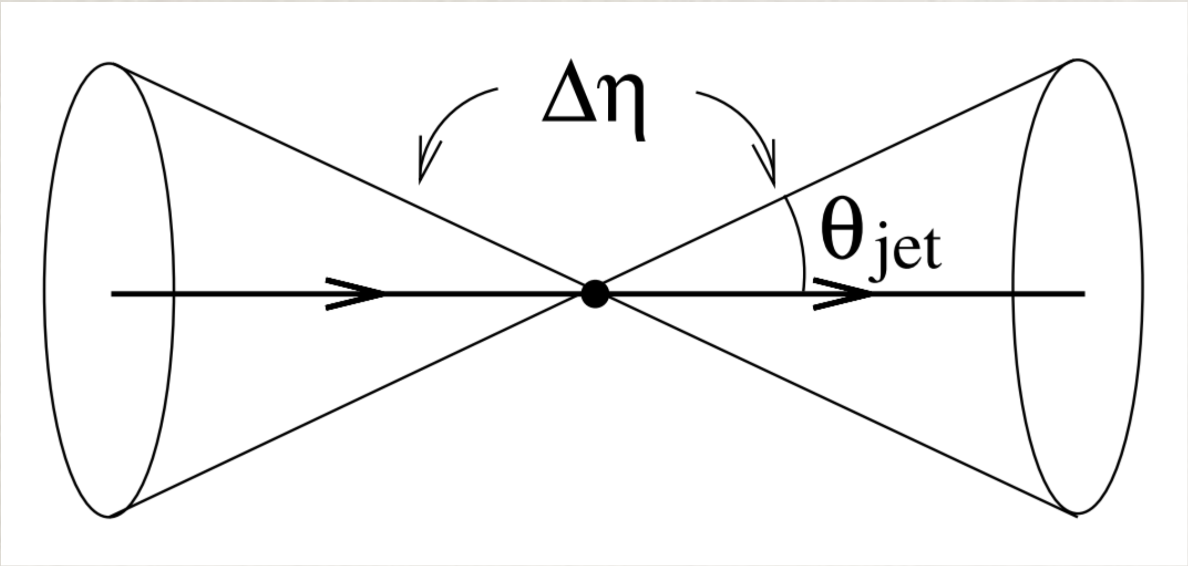


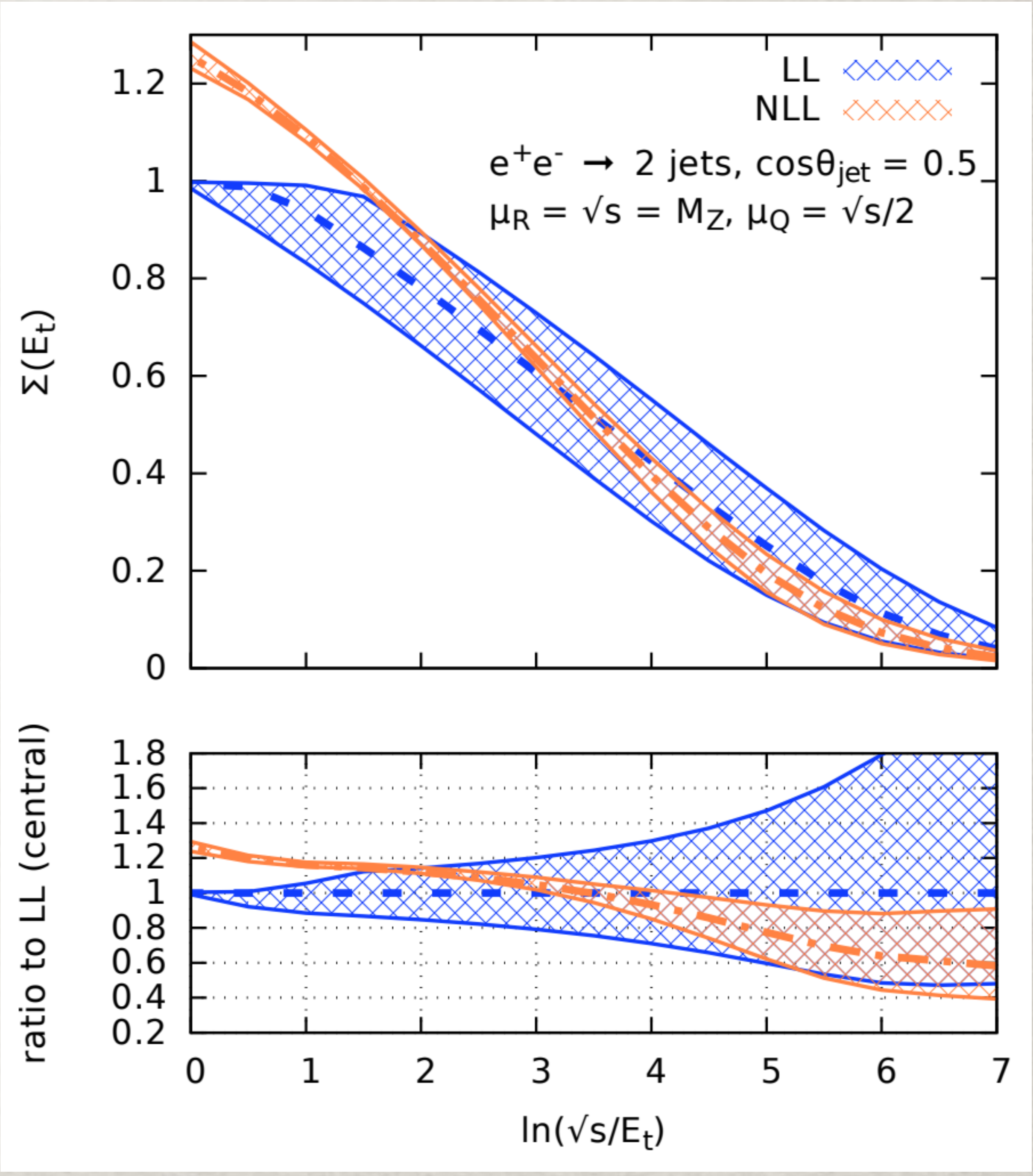
- Non-global logarithms are due to soft emissions at large angles, hence leading logarithms are single logarithms $\alpha_s^n L^n$
- Non-global observables are not rIRC safe: at LL accuracy, and in the large-N_c limit, their NGLs are resummed via the non-linear BMS equation

[AB Marchesini Smye hep-ph/0206076]

Resummation of NL NGLs

- It is possible to write a NL evolution equation for non-global logarithms and solved it numerically via a MC procedure [AB Dreyer Monni 2104.06416 , 2111.02413] [see also Becher Rau Xu 2112.02108]





- Impressive reduction of theoretical uncertainties from LL to NLL
- Resummation of any event shape and jet rate at NNLL accuracy is now possible (in the large N_c limit)
- Non-global observables for more precise measurements of α_s?