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# On ISR in $e^+e^-$ collisions

FCC-ee Targets and Tools Workshop, CERN, 8/6/2022

Loosely based on 2203.12557 *"Initial state QED radiation aspects for future $e^+e^-$ colliders"*

This a contribution to Snowmass 2021 by various authors, where the current status of simulation codes, and/or their theoretical underpinnings, is reviewed

Shortest summary:

- Status is heterogeneous
- Readiness generally low (e.g. the codes run but do not meet precision targets)

A slightly longer summary:

- QED-specific theoretical bases are well established, and mostly already employed at LEP. However, conceptual and/or technical progress is still needed, and is being pursued

- The "recycling" of QCD perturbative techniques is limited so far; potential for growth in the future (e.g. matrix element computations, use of EFTs, collinear resummation techniques)

- Dedicated high-precision tools not precise enough

- Modern multi-purpose tools, such as PSMCs, generally poorly tested in $e^+e^-$ high-energy environments

Material covered in 2203.12557:

Theory:

- McMule
- Next-to-leading power factorisation
- YFS vs collinear factorisation generalities

Material covered in 2203.12557:

Theory:

- McMule 1909.10244, 2007.01654, 2112.07570 (Banerjee, Engel, Signer, Ulrich)
  - A modern approach (dim. reg., subtraction, massless photons) to NNLO (fixed-order) QED computations, that extends FKS to "FKS$^{\ell}$"
  - Exploits YFS-like factorisation, and employs next-to-soft stabilisation
  - So far targets low-energy applications, does not include ISR resummation

Material covered in 2203.12557:

Theory:

- McMule
- Next-to-leading power factorisation 2008.01736 (Laenen, Sinninghe Damsté, Vernazza, Waalewijn, Zoppi)
  - Diagram-based classification of jets contributing to NLP in QED

Material covered in 2203.12557:

Multi-purpose tools:

- Pythia8
- Herwig
- Sherpa
- MadGraph5_aMC@NLO
- Whizard

Dedicated tools:

- BabaYaga
- RacoonWW, Racoon4f
- KKMC-ee, KORAL[W/Z], BH[LUMI/WIDE], YSF[WW3/ZZ]

Material covered in 2203.12557:

Multi-purpose/dedicated tools:

All tools aim to give realistic descriptions of physical observables, and thus include some form of ISR/FSR resummation

For the latter, all tools adopt collinear factorisation bar for KKMC-ee, KORAL*, BH*, YSF*, and one instance of Sherpa, that adopt YSF

Among multi-purpose tools:

- Emphasis on small angle/small energy radiation (loosely speaking: parton shower, although not really [only] such): Pythia8, Herwig, Sherpa
- Emphasis on matrix element computations: MadGraph5_aMC@NLO, Whizard

At the LHC nowadays there is a strict interplay between MEGs and PSMCs. This is essentially absent thus far in $e^+e^-$ (as far as QED radiation is concerned)

Presently, the main difference for particles branching off ISR is:
PSMCs are exclusive in them, MEGs (semi-)inclusive

Consider a generic cross section, sufficiently inclusive:

$$\sigma = \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^{n} \sum_{j=0}^{n} \varsigma_{n,i,j} L^i \ell^j$$

This is symbolic, and only useful to expose the presence of:

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}\,, \qquad L = \log \frac{Q^2}{m^2}$$

Numerology: consider the production of $Z \to ll$ at:

- $\sqrt{Q^2} = m_Z$

$$\begin{aligned} L = 24.18 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} L = 0.06 \\ 0 \leq m_{ll} \leq m_Z\,, \quad \ell = 8.29 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.02 \\ m_Z - 1 \text{ GeV} \leq m_{ll} \leq m_Z\,, \quad \ell = 13.66 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.034 \end{aligned}$$

Consider a generic cross section, sufficiently inclusive:

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This is symbolic, and only useful to expose the presence of:

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}\,, \qquad L = \log \frac{Q^2}{m^2}$$

Numerology: consider the production of $Z \to ll$ at:

- $\sqrt{Q^2} = 500$ GeV

$$\begin{aligned} L = 24.59 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} L = 0.068 \\ 0 \leq m_{ll} \leq m_Z\,, \quad \ell = 1.46 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.0036 \\ m_Z - 1\ \mathrm{GeV} \leq m_{ll} \leq m_Z\,, \quad \ell = 4.51 \quad &\Longrightarrow \quad \frac{\alpha}{\pi} \ell = 0.01 \end{aligned}$$

It takes a lot of brute force (i.e. fixed-order results to some $\mathcal{O}(\alpha^n)$) to overcome the enhancements due to $L$ and $\ell$.

It is always convenient to first improve by means of factorisation formulae:

$$d\sigma(L,\ell) = \mathcal{K}_{soft}(\ell;L)\beta(L)d\mu \quad (1)$$

$$= \mathcal{K}_{coll}(L;\ell)\otimes d\hat{\sigma}(\ell) \quad (2)$$

Use of:

(1) YFS (resummation of $\ell$)

(2) collinear factorisation (resummation of $L$)

Common features: $\mathcal{K}$ is an *all-order* universal factor; $\beta$ and $d\hat{\sigma}$ are process-specific and computed order by order

(still brute force, but to a lesser extent)

# YFS

Aim: soft resummation for:

$$\left\{ e^+(p_1) + e^-(p_2) \longrightarrow X(p_X) + \sum_{i=0}^{n} \gamma(k_n) \right\}_{n=0}^{\infty}$$

Achieved with:

$$\begin{aligned} d\sigma(L,\ell) &= \mathcal{K}_{soft}(\ell;L)\beta(L)d\mu \\ &= e^{Y(p_1,p_2,p_X)} \sum_{n=0}^{\infty} \beta_n \left(\mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^{n}\right) d\mu_{X+n\gamma} \end{aligned}$$

This is symbolic, and stands for both the EEX and CEEX approaches [hep-ph/0006359 Jadach, Ward, Was] that build upon the original YFS work [Ann.Phys.13(61)379]

EEX: exclusive (in the photons) exponentiation, matrix element level

CEEX: coherent exclusive (in the photons) exponentiation, amplitude level, including interference

# YFS

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Achieved with:

$$d\sigma(L,\ell) \;=\; e^{Y(p_1,p_2,p_X)} \sum_{n=0}^{\infty} \beta_n \left( \mathcal{R}p_1, \mathcal{R}p_2, \mathcal{R}p_X; \{k_i\}_{i=0}^{n} \right) d\mu_{X+n\gamma}$$

- $Y$ essentially universal (process dependence only through kinematics); resums $\ell$
- The soft-finite $\beta_n$ are process-specific, and are constructed by means of local subtractions involving matrix elements and eikonals (i.e. *not* BN)

$$\beta_n = \alpha^b \sum_{i=0}^{n} \alpha^i \sum_{j=0}^{i} c_{n,i,j} L^j$$

- For a given $n$, matrix elements have different multiplicities, hence the need for the kinematic mapping $\mathcal{R}$

# Collinear factorisation

Aim: collinear resummation for:

$$\left\{ k(p_k) + l(p_l) \longrightarrow X(p_X) + \sum_{i=0}^{n} a_i(k_n) \right\}_{n=0}^{\infty} \qquad a_i = e^{\pm}, \gamma \ldots$$

with initial-state particles stemming from beams:

$$(k,l) = (e^+, e^-)\,, \quad (k,l) = (e^+, \gamma)\,, \quad (k,l) = (\gamma, e^-)\,, \quad (k,l) = (\gamma, \gamma)\,, \ldots$$

Master formula:

$$\begin{aligned} d\sigma(L,\ell) &= \mathcal{K}_{coll}(L;\ell) \otimes d\hat{\sigma}(\ell) \\ \longrightarrow \quad d\sigma_{kl} &= \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2)\, \Gamma_{j/l}(z_-, \mu^2, m^2) \\ & \qquad \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2; p_X, \{k_i\}_{i=0}^{n}) \end{aligned}$$

- $\Gamma_{\alpha/\beta}$ universal (the PDF); resums $L$
- The collinear-finite $d\hat{\sigma}_{ij}$ are process-specific, and are the standard short-distance matrix elements, constructed order by order (*with* BN). May or may not include resummation of other large logs (including $\ell$)

# YFS vs collinear factorisation

Both are systematically improvable in perturbation theory:
in YFS the $\beta_n$'s (fixed-order), in collinear factorisation both the PDFs (logarithmic accuracy) and the $d\hat{\sigma}$'s (fixed-order, resummation)

+ YFS: very little room for systematics. Exceptions are the kinematic mapping $\mathcal{R}$, and the quark masses (when the quarks are radiators). Renormalisation schemes??

− Collinear factorisation: systematic variations much larger. At the LL (used in phenomenology so far) a rigorous definition of uncertainties is impossible (parameters are arbitrary), and comparisons with YFS are largely fine tuned

− YFS: the computations of $\beta_n$ are not standard (EEX) and highly non-trivial (CEEX)

+ Collinear factorisation: the computations of $d\hat{\sigma}_{ij}$ are standard

Folk wisdom about collinear factorisation: while YFS is exclusive in the photons (*true* in EEX and CEEX), collinear factorisation is inclusive (*not true* in general)

Folk wisdom about collinear factorisation: while YFS is exclusive in the photons (*true* in EEX and CEEX), collinear factorisation is inclusive (*not true* in general)

Firstly, one is always exclusive in the photons (possibly) emerging from the hard process ($d\hat{\sigma}_{ij}$)

Secondly, whether one is exclusive also in the photons associated with ISR depends on the implementation of the factorisation formula

- MC integration as is: inclusive (or modelled as e.g. in Whizard)
- Integration through recursion (e.g. parton shower): exclusive.
  Examples: Pythia8, Herwig, Sherpa, Babayaga

Folk wisdom about collinear factorisation: while YFS is exclusive in the photons (*true* in EEX and CEEX), collinear factorisation is inclusive (*not true* in general)

Open problems in *precise*, exclusive $e^+e^-$ simulations: extend QCD matching techniques (MC@NLO, Powheg) to ISR QED⋆; extend logarithmic accuracy of shower to NLL; extend matching beyond NLO

Before these can be considered $\longrightarrow$

⋆LL solutions in $pp$ collisions in HORACE [see e.g. hep-ph/0609170] and Powheg [see e.g. 1302.4606]

All physics simulations based on collinear factorisation done so far are based on a LL-accurate picture

This is not tenable at high energies/high statistics:

- accuracy is insufficient (see e.g. $W^+W^-$ production)
- systematics not well defined

Step 0 was to upgrade PDFs from LL to NLL accuracy: increase of precision, and meaningful systematics, in particular factorisation-scheme dependence

## $z$-space LO+LL PDFs $(\alpha \log(E/m))^k$:

$\sim$ 1992

- $0 \leq k \leq \infty$ for $z \simeq 1$ (Gribov, Lipatov)
- $0 \leq k \leq 3$ for $z < 1$ (Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini; Skrzypek)
- matching between these two regimes
- for $e^-$

## $z$-space NLO+NLL PDFs $(\alpha \log(E/m))^k + \alpha\,(\alpha \log(E/m))^{k-1}$:

$\longrightarrow$ 1909.03886, 1911.12040, 2105.06688 (Bertone, Cacciari, Frixione, Stagnitto)

- $0 \leq k \leq \infty$ for $z \simeq 1$
- $0 \leq k \leq 3$ for $z < 1$ $\iff$ $\mathcal{O}(\alpha^3)$
- matching between these two regimes
- for $e^+$, $e^-$, and $\gamma$
- both numerical and analytical
- factorisation schemes: $\overline{\mathrm{MS}}$ and $\Delta$ (that has DIS-like features)

All physics simulations based on collinear factorisation done so far are based on a LL-accurate picture

This is not tenable at high energies/high statistics:

- accuracy is insufficient (see e.g. $W^+W^-$ production)
- systematics not well defined

Step 1 (Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao) is to include in the NLL PDFs thus obtained the ingredients necessary for sensible phenomenology, in particular:

- evolution with all fermion families (leptons and quarks), including their respective mass thresholds
- renormalisation schemes other than $\overline{\rm MS}$: $\alpha(m_Z)$ and $G_\mu$
- assess implications by studying realistic observables in physical processes

Now done (out in a few weeks) $\longrightarrow$

Sample *preliminary* results for:

$$\begin{aligned} e^+e^- &\longrightarrow q\bar{q} \\ e^+e^- &\longrightarrow t\bar{t} \\ e^+e^- &\longrightarrow W^+W^- \end{aligned}$$

with $q\bar{q}$ production (massless quarks) restricted to ISR QED radiation.
The other two are in the SM

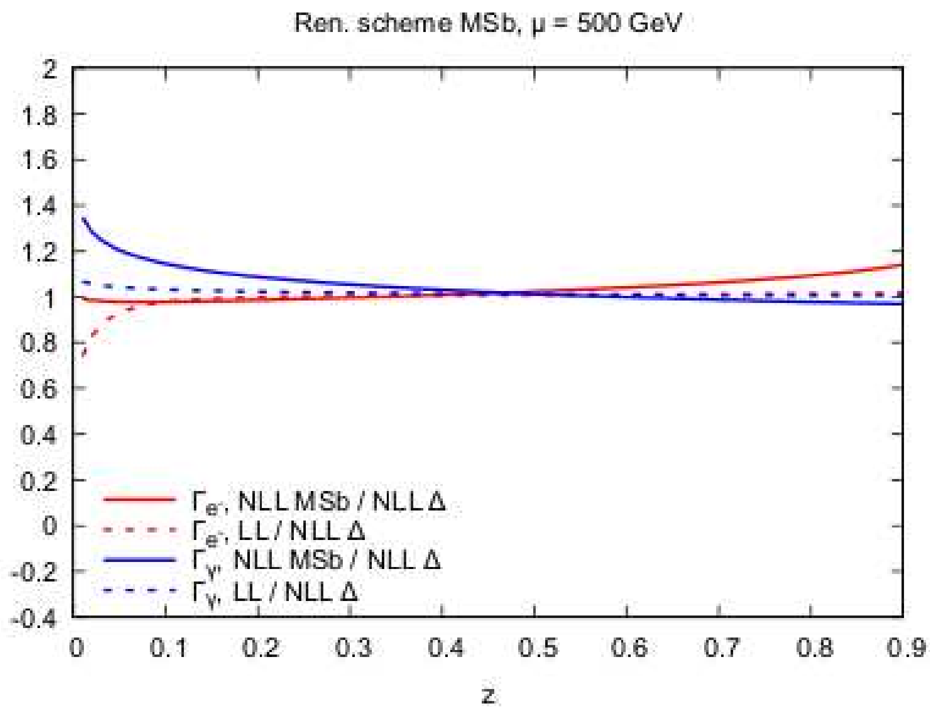
NLO accuracy, automated generation with MG5_aMC@NLO
(this version not yet public)

What is plotted:

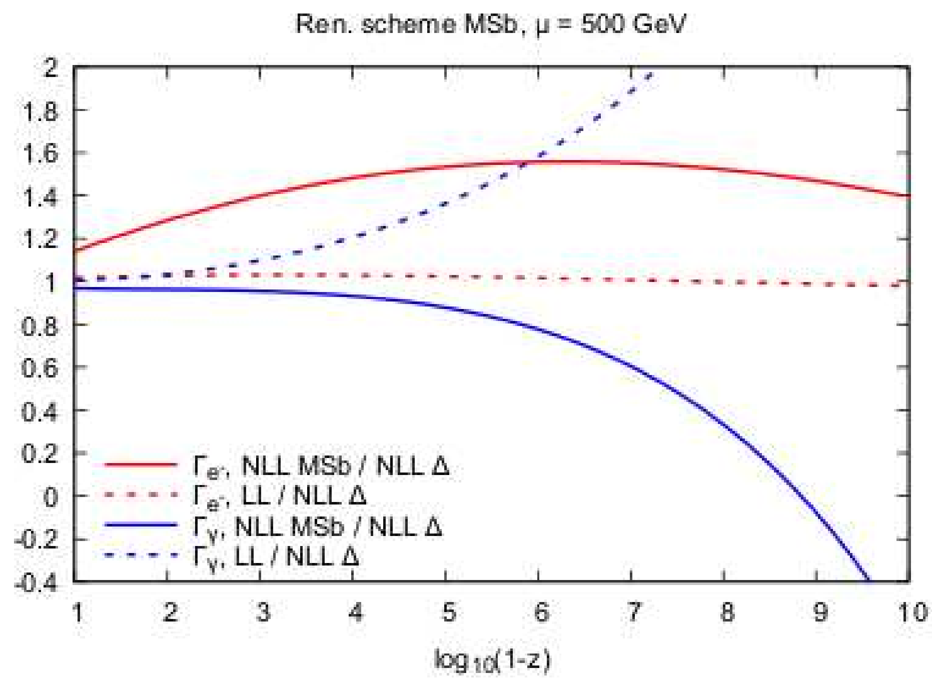
$$\sigma(\tau_{min}) = \int d\sigma \, \Theta\left(\tau_{min} \le \frac{M_{p\bar{p}}^2}{s}\right) , \qquad p = q\,,t\,,W^+$$

$\tau_{min} \sim 1$ is sensitive to soft emissions (not resummed)

# Dependence of PDFs on factorisation scheme




$z < 1$

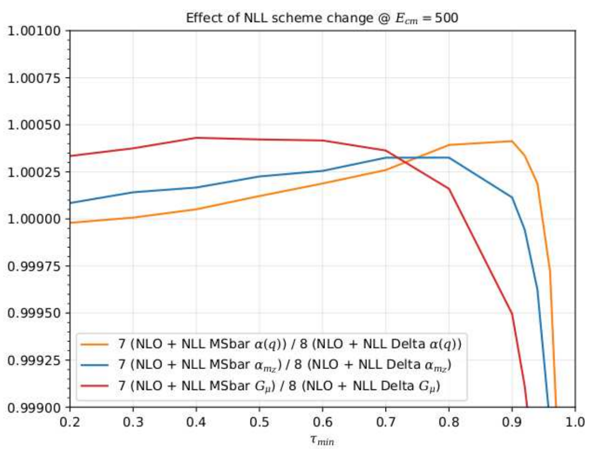



$z \simeq 1$

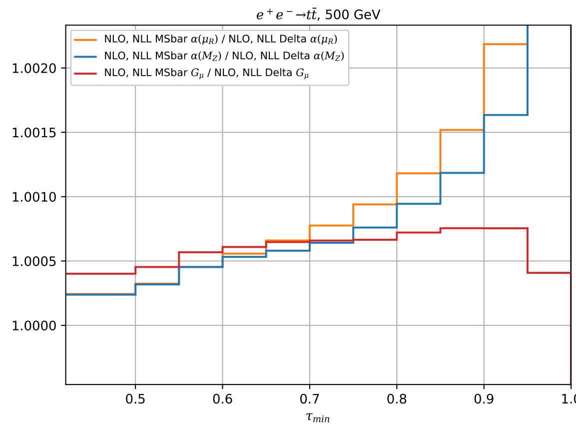
Very large dependence at the NLL at $z \to 1$ ($\mathcal{O}(1)$); this is particularly significant *(but unphysical!)* since the electron has an integrable divergence there

Electron at NLL in the Delta scheme close to the LL result (differences of $\mathcal{O}(5\%)$)

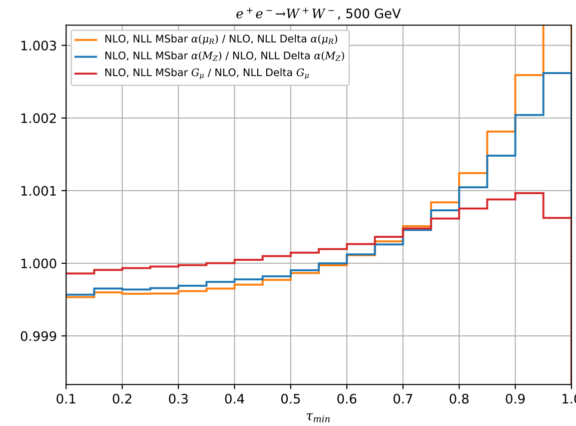
# Dependence of observables on factorisation scheme




$q\bar{q}$




$t\bar{t}$



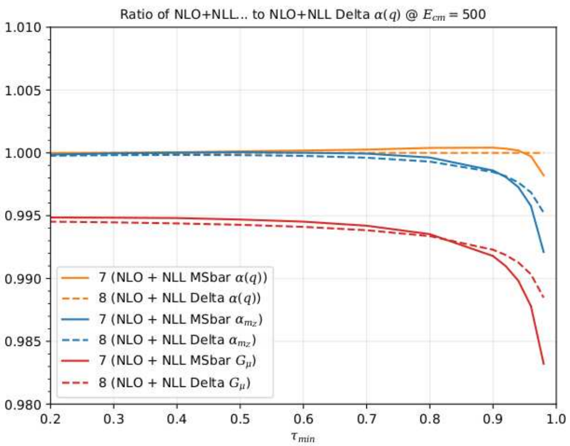

$W^+W^-$

$\mathcal{O}(1)$ differences for PDFs down to $\mathcal{O}(10^{-4} - 10^{-3})$ for observables

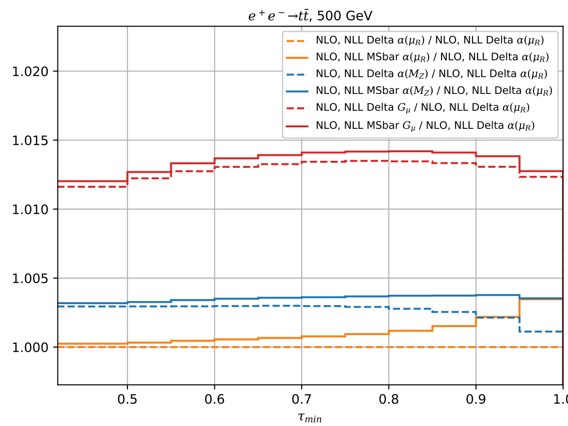
In the $\overline{\mathrm{MS}}$ scheme, huge cancellations between PDFs and short-distance cross sections

Behaviour qualitatively similar for different renormalisation schemes

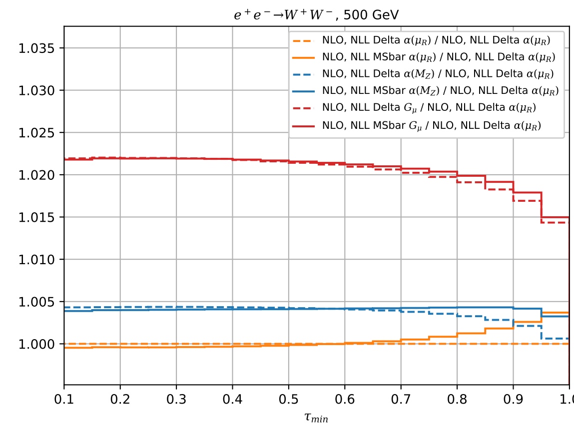
# Factorisation vs renormalisation scheme dependence




$q\bar{q}$




$t\bar{t}$

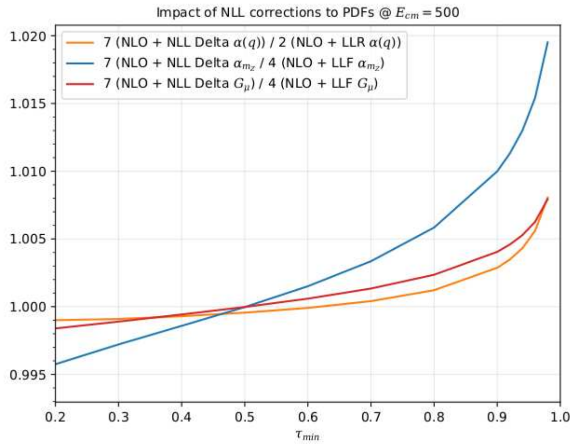



$W^+W^-$

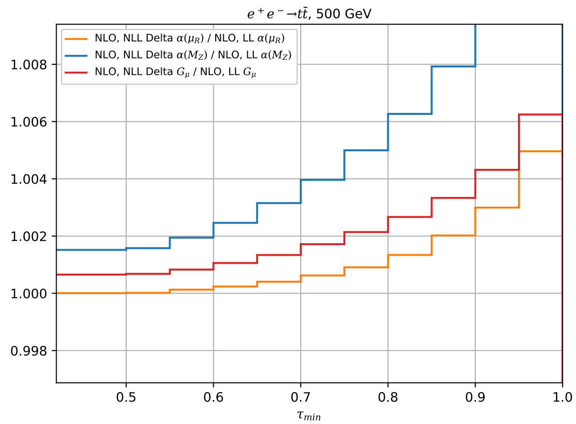
Renormalisation-scheme dependence much larger than factorisation-scheme dependence, with process-dependent pattern

Depending on the precision, renormalisation scheme is an informed choice; factorisation scheme always induces a systematic

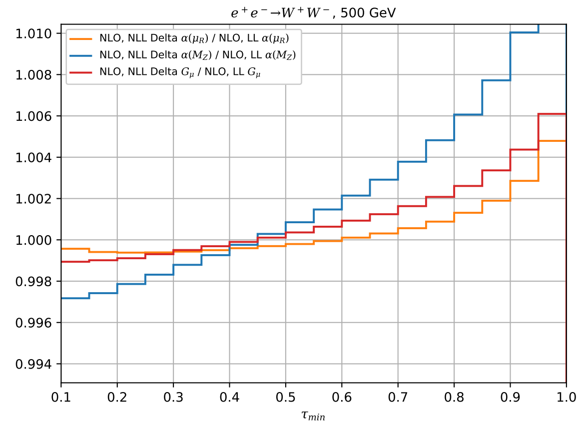
# NLL vs LL




$q\bar{q}$




$t\bar{t}$

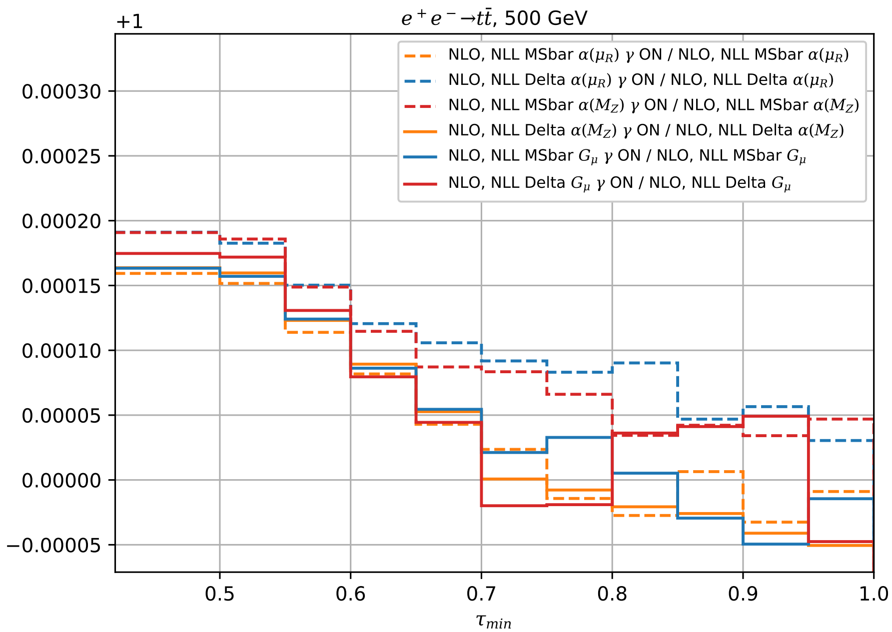



$W^+W^-$

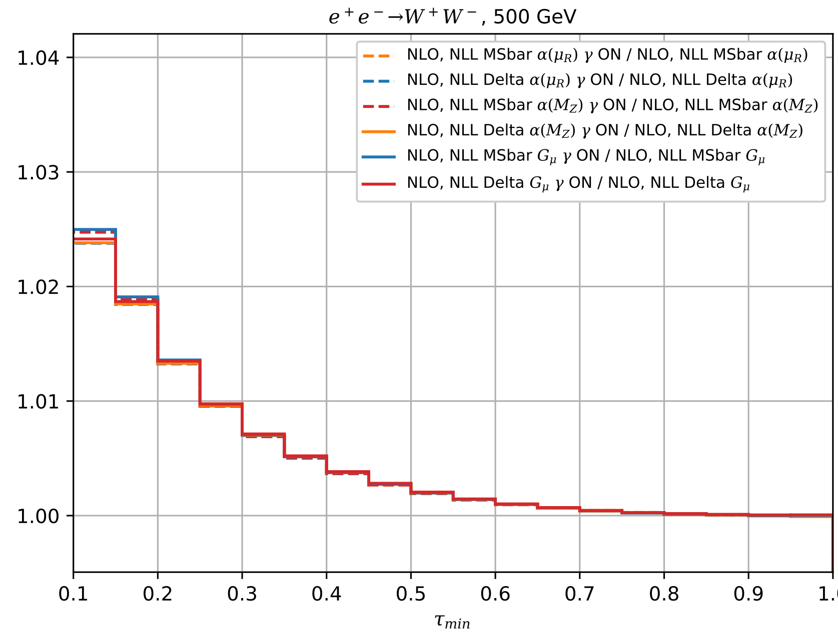
Effects are non trivial

Pattern dependent on the process (and on the observable) as well as on the renormalisation scheme

# Impact of $\gamma\gamma$ channel




$t\bar{t}$




$W^+W^-$

Essentially independent of factorisation and renormalisation schemes: a genuine physical effect

Utterly negligible for $t\bar{t}$, significant for $W^+W^-$ – process dependence is not surprising

Plan:

- Make this version of PDFs public, independent of MadGraph5_aMC@NLO
- Upgrade the public version of MadGraph5_aMC@NLO, to include the capabilities shown in previous plots
- Include resummation of soft non-collinear logs in PDFs
- Longer term: go to NNLL (hard but feasible; QCD gives blueprint)

# TOOLS

# Pythia8

1410.3012, 2203.11601

Ask, Bierlich, Christiansen, Corke, Desai, Gellersen, Helenius, Ilten, Lönnblad, Mrenna, Prestel, Preuss, Rasmussen, Sjöstrand, Skands, Utheim, Verheyen

- Three choices of parton shower
  1. "simple shower": dipole-based, positive, backward evolution with LL electron PDF
  2. Vincia: $p_T$-ordered shower based on antenna ($2 \to 3$ splitting kernels, not partitioned). Fully coherent and positive but with bad multiplicity scaling; approximate solution based on discarding opposite-charge dipoles
  3. DIRE: indefinite-sign dipoles; $2 \to 3$ soft kernels matched to $1 \to 2$ collinear ones. In principle allows for interference effects and merging, but not studied
- Very flexible and apt to modelling uncertainties
- No simulation of beamstrahlung

# Sherpa

0811.4622, 1905.09127, 0810.5071, 2203.10948

Bothmann, Chahal, Gleisberg, Höche, Krause, Krauss, Kuttimalai, Liebschner, Napoletano, Price, Schönherr, Schulz, Schumann, Siegert, Winter

- Two choices for ISR/FSR
  1. Dipole parton shower with LL electron PDF. Dipoles are restricted to same-charge ones so as to have positive-definite results
  2. EEX YFS. ISR including up to $\mathcal{O}(\alpha^3 L^3)$, FSR for leptons up to NLO EW and NNLO QED. All are hard coded (i.e. not automated)
- Merging capabilities are in-house (not yet exploited here?)
- No simulation of beamstrahlung (interface to CIRCE foreseen)

# Madgraph5_aMC@NLO

1405.0301, 1804.10017, 2108.10261

Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Pagani, Shao, Stelzer, Torrielli, Zaro, Zhao

An automated matrix element generator that works with any Feynrules-generated model

- NLO for both QCD, EW (and mixed), and whatever is in the model (e.g. EFTs); NLO EW with massless initial states leptons not public yet
- ISR by means of collinear factorisation
- Kinematics of final-state photons: exact in matrix elements, inclusive in ISR photons
- Beam dynamics simulations directly as a parametrisation of GuineaPig results
- Several options for QCD matching and merging both LO and NLO (CKKW-L, MLM, MC@NLO, FxFx)

# Whizard

0708.4233, hep-ph/0102195

Kilian, Ohl, Reuter; Bach, Brass, Bredt, Moretti, Nejad, Fleper, Rothe, Schmidt, Sekulla, Speckner, Shim, Staub, Stienemeier, Weiss

An automated matrix element generator with models built in or imported from Feynrules

- NLO for both QCD and EW (and mixed). NLO EW with massless initial states leptons not public yet
- ISR by means of collinear factorisation (LL PDFs)
- Kinematics of final-state photons: exact in matrix elements, ISR single-photon distribution generated from $p_T$ logarithmic scaling
- Beam dynamics simulations through the CIRCE interface to GuineaPig
- NLO QCD matching with Powheg

# BabaYaga

hep-ph/0003268, hep-ph/0103117, 0801.3360

Balossini, Bignamini, Carloni Calame, Lunardini, Montagna, Nicrosini, Piccinini

Dedicated $e^+e^- \longrightarrow e^+e^-\,,\mu^+\mu^-\,,\gamma\gamma$ simulations

- ISR by means of collinear factorisation, with evolution equations solved recursively at the LL
- Matched to NLO QED matrix elements; relation of matching to MC@NLO and Powheg unclear
- Kinematics of final-state photons: fully exclusive; angular spectra of photons determined by eikonals
- Developed with $\mathcal{O}(10\ \mathrm{GeV})$ cm energies in mind. Now considering extension to NNLO QED and inclusion of EW effects

# RacoonWW, Racoon4f

hep-ph/0006307, hep-ph/0209330, hep-ph/9904472, hep-ph/0502063, hep-ph/0505042

Denner, Dittmaier, Roth, Wackeroth, Wieders

Dedicated $e^+e^- \longrightarrow W^+W^-$, $e^+e^- \longrightarrow (W^{+*}W^{-*} \longrightarrow)4f$ simulations

- Based on exact NLO QED charged-current matrix elements (RacoonWW uses DPA for virtuals, and includes Coloumb-singularity effects)
- ISR by means of collinear factorisation (LL PDFs)
- Necessary future improvements would likely require to: exploit higher-order near-threshold EFT [0707.0773, 0807.0102 (Actis, Beneke, Falgari, Schwinn, Zanderighi)] predictions; separation of fermionic channels; differential QCD corrections

# KKMC-ee, KORAL[W/Z], BH[LUMI/WIDE], YSF[WW3/ZZ]

hep-ph/0006359, hep-ph/9912214, 1307.4037, hep-ph/9906277, hep-ph/9705430, +various Comp.Phys.Comm 1992-2001

Jadach, Placzek, Richter-Was, Skrzypek, Ward, Was

Dedicated $e^+e^- \longrightarrow 2f\,, e^+e^-\,, W^+W^-\,, ZZ$ simulations

- All based on YSF
- All implement YSF EEX, except KKMC-ee which has both EEX and CEEX
- $\beta_n$ terms implemented to various degrees of accuracy, the highest being $\mathcal{O}(\alpha^2 L^2)$ (CEEX KKMC-ee) and $\mathcal{O}(\alpha^3 L^3)$ (EEX KKMC-ee)
- Plans: move to CEEX and include EW corrections, translate to C++ (done for KKMC-ee)

# Outlook

Ready we are not, but there is plenty of time...

One hopes that many of the techniques that have been, and are being, developed for the LHC can be ported to QED. This is reasonable, but not a given

New lines of research in QED are being explored. Their impact on the physics of future $e^+e^-$ colliders will become more clear in the next few years

My thanks to:

S. Catani, S. Jadach, E. Laenen, A. Price, J. Reuter, A. Signer,
G. Stagnitto, A. Vicini, B. Ward, M. Zaro

for discussions, providing material,. . .