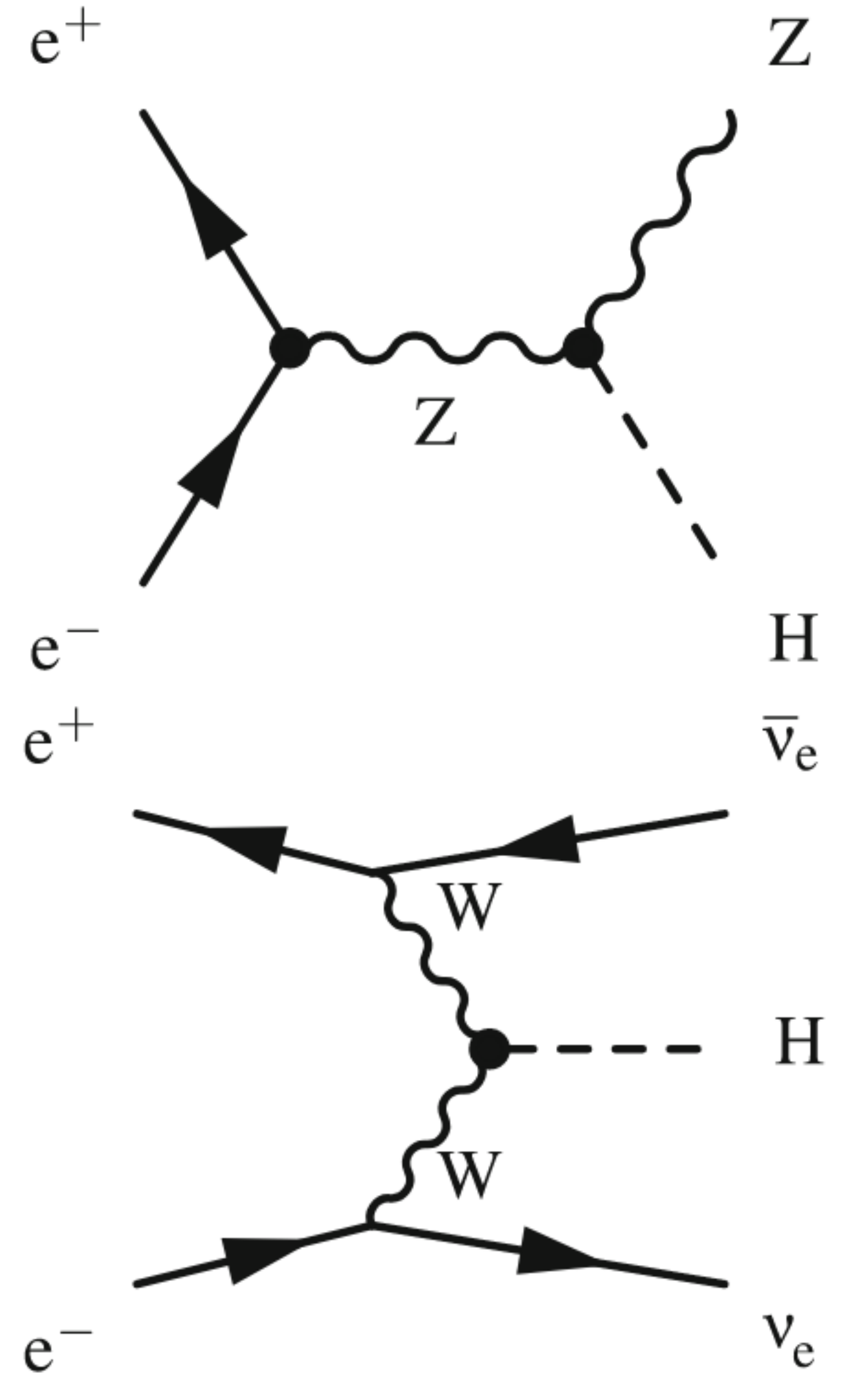
Higgs production and decay at e+e- colliders: theoretical status and challenges

Li Lin Yang
Zhejiang University

Higgs production at e+e- colliders

Higgs-strahlung (ZH)



W-fusion (WWH)

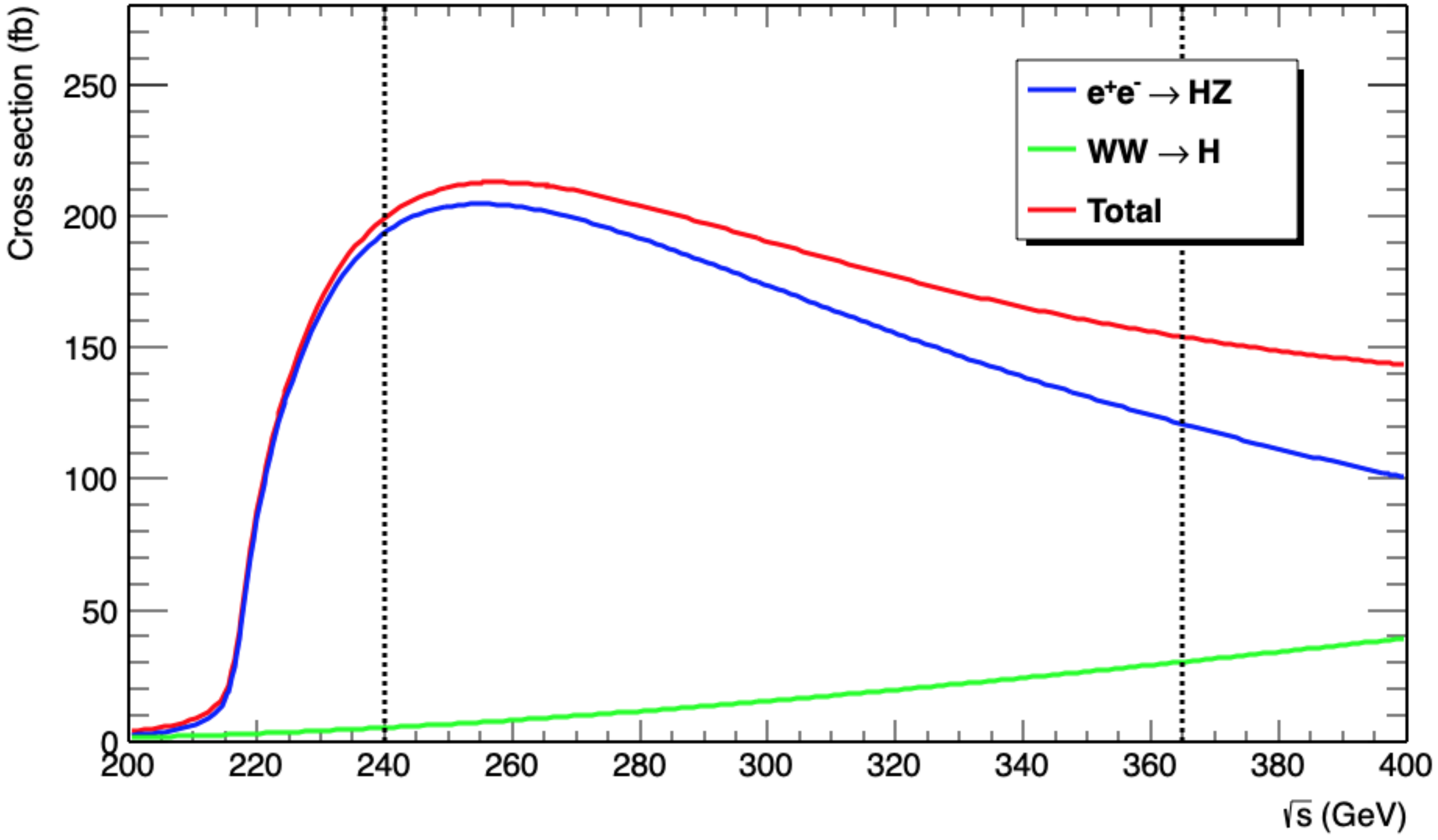


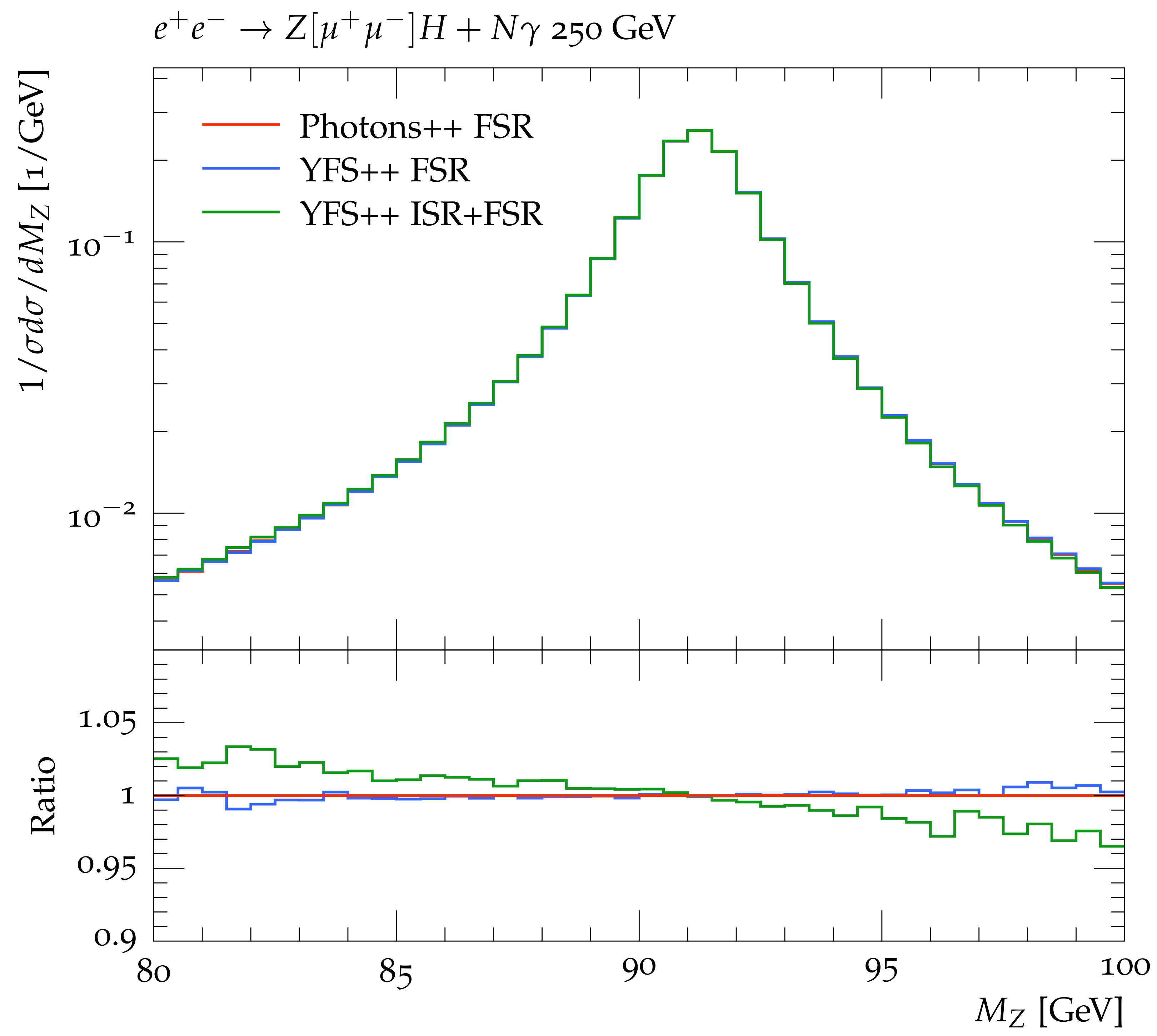
Figure from 2106.15438

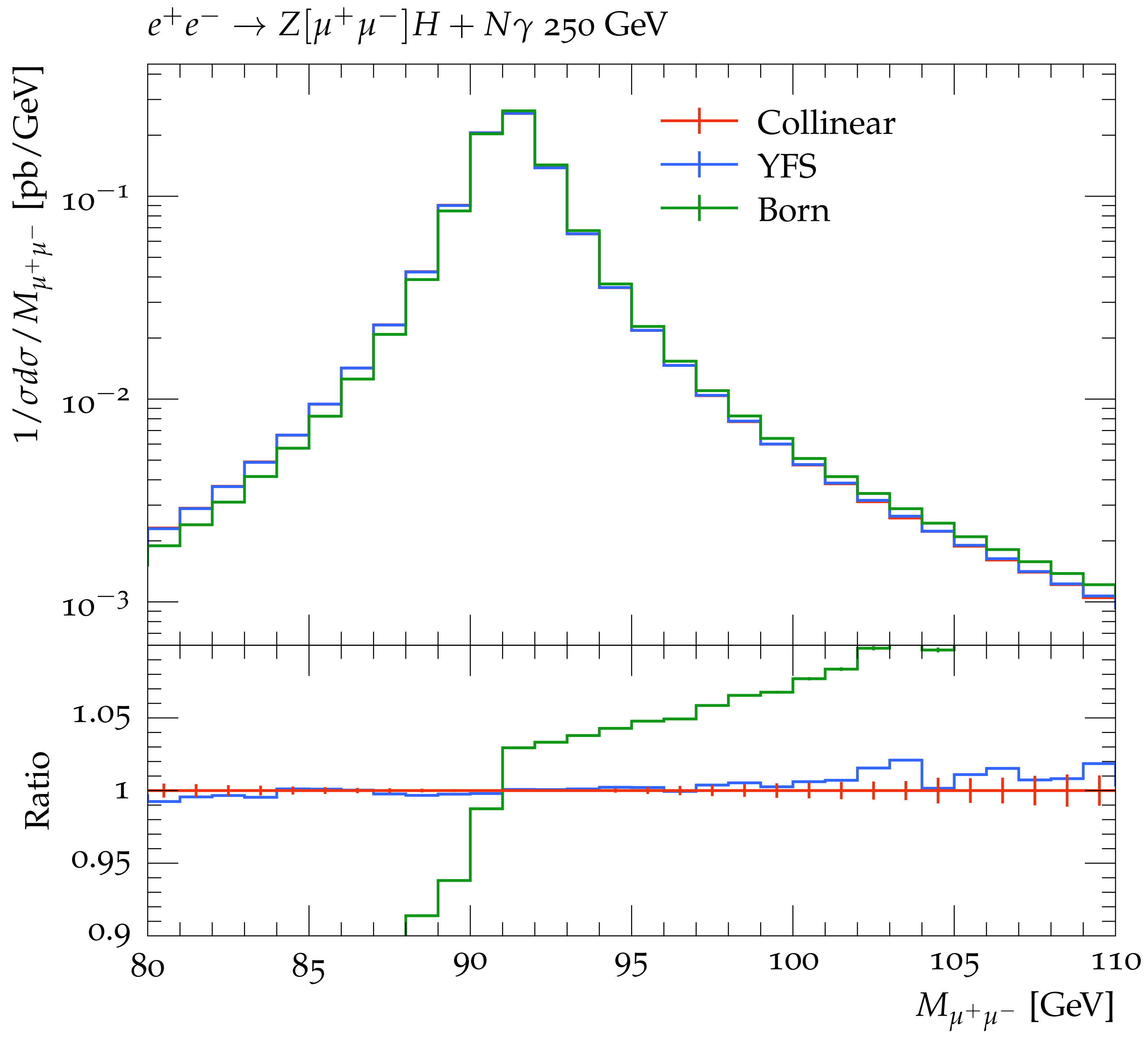
NLO EW + QED radiations built in Monte Carlo event generators

QED ISR and FSR effects

Critically re-examined very recently
(Talk by Stefano Frixione)

Blümlein, Schönwald: 2202.08476
Krauss, Price, Schönherr: 2203.10948
Frixione et al.: 2203.12557
and many more references therein





Figures from 2203.10948

Mixed QCD-EW corrections to ZH

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

the $\alpha(m_Z)$ scheme.

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
240	252.0	228.6	231.5
250	252.0	227.9	230.8
300	190.0	170.7	172.9
350	135.6	122.5	124.2
500	60.12	54.03	54.42

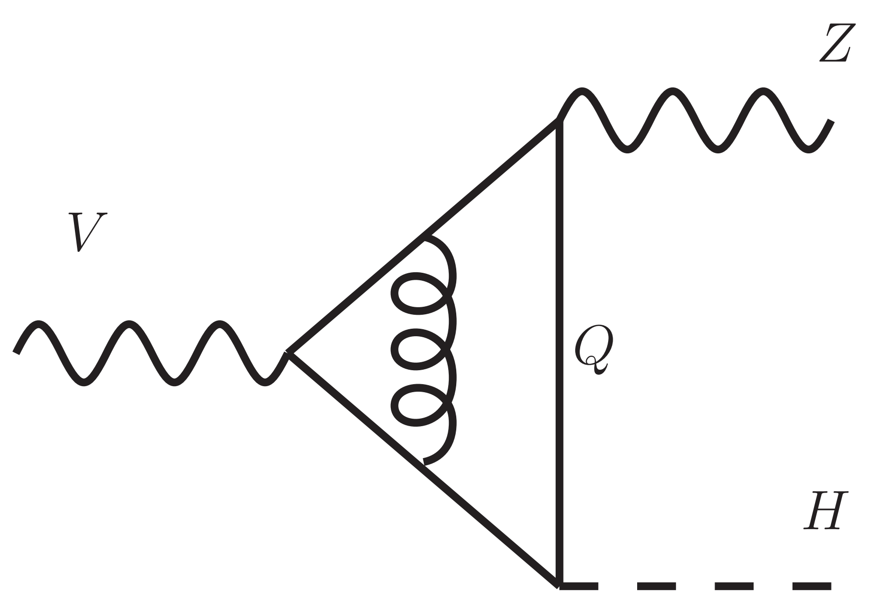
Corrections at the level of ~1%: non-negligible compared to the ~0.3% experimental accuracy

Sun, Feng, Jia, Sang: 1609.03995

$\sqrt{s}$	Schemes	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
240	$\alpha(0)$	223.14 ± 0.47	229.78 ± 0.77	$232.21^{+0.75+0.10}_{-0.75-0.21}$
	$\alpha(M_Z)$	252.03 ± 0.60	$228.36^{+0.82}_{-0.81}$	$231.28^{+0.80+0.12}_{-0.79-0.25}$
	G_μ	239.64 ± 0.06	$232.46^{+0.07}_{-0.07}$	$233.29^{+0.07+0.03}_{-0.06-0.07}$

Residue dependence on renormalization schemes

Calculation methods back then



Bottleneck was the two-loop triangle integrals

→ Purely numeric evaluation with sector decomposition

Private code of 1508.02512 (employed by 1609.03955)

FIESTA/CubPack (employed by 1609.03995)

Slow; bad convergence around or above $2m_Q$ threshold

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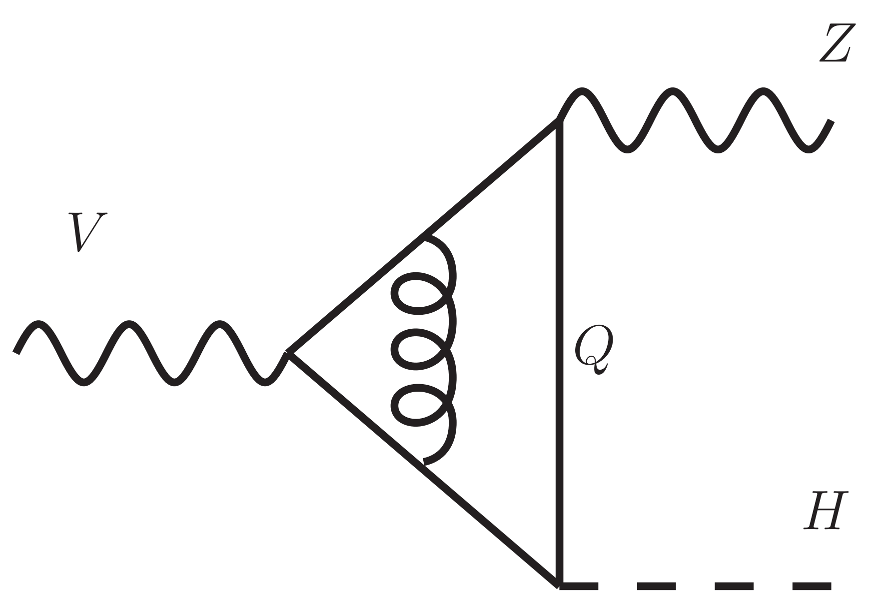

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Alternative method: $1/m_t$ expansion

Gong, Li, Xu, **LLY**, Zhao: 1609.03955

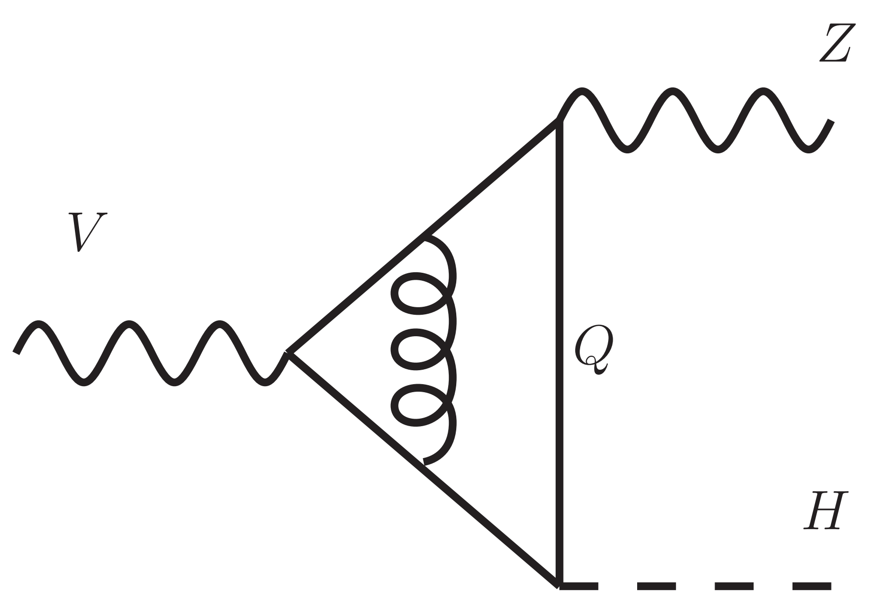
$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%
300	80.0%	15.2%	2.1%	1.1%
350	69.7%	12.6%	2.7%	2.1%
500	137%	18.6%	17.3%	31.1%

Good approximation for low energies: analytic expressions easy to implement in Monte-Carlo

Not valid for high energies…

A new calculation for the HZV two-loop diagrams

Wang, Xu, **LLY**: 1905.11463



$$x = -\frac{q^2}{4m_Q^2}$$

$$y = -\frac{p_Z^2}{4m_Q^2}$$

$$z = -\frac{p_H^2}{4m_Q^2}$$

Constructed a canonical basis of master integrals

$$d\vec{f}(x,y,z;\epsilon) = \epsilon\, dA(x,y,z)\, \vec{f}(x,y,z;\epsilon)$$

$$= \epsilon \sum_i A_i\, d\log(\alpha_i)\, \vec{f}(x,y,z;\epsilon)$$

Alphabet contains 4 kinds of square roots

$$\sqrt{x(x+1)} \quad \sqrt{y(y+1)} \quad \sqrt{z(z+1)} \quad \sqrt{x^2+y^2+z^2-2xy-2yz-2zx}$$

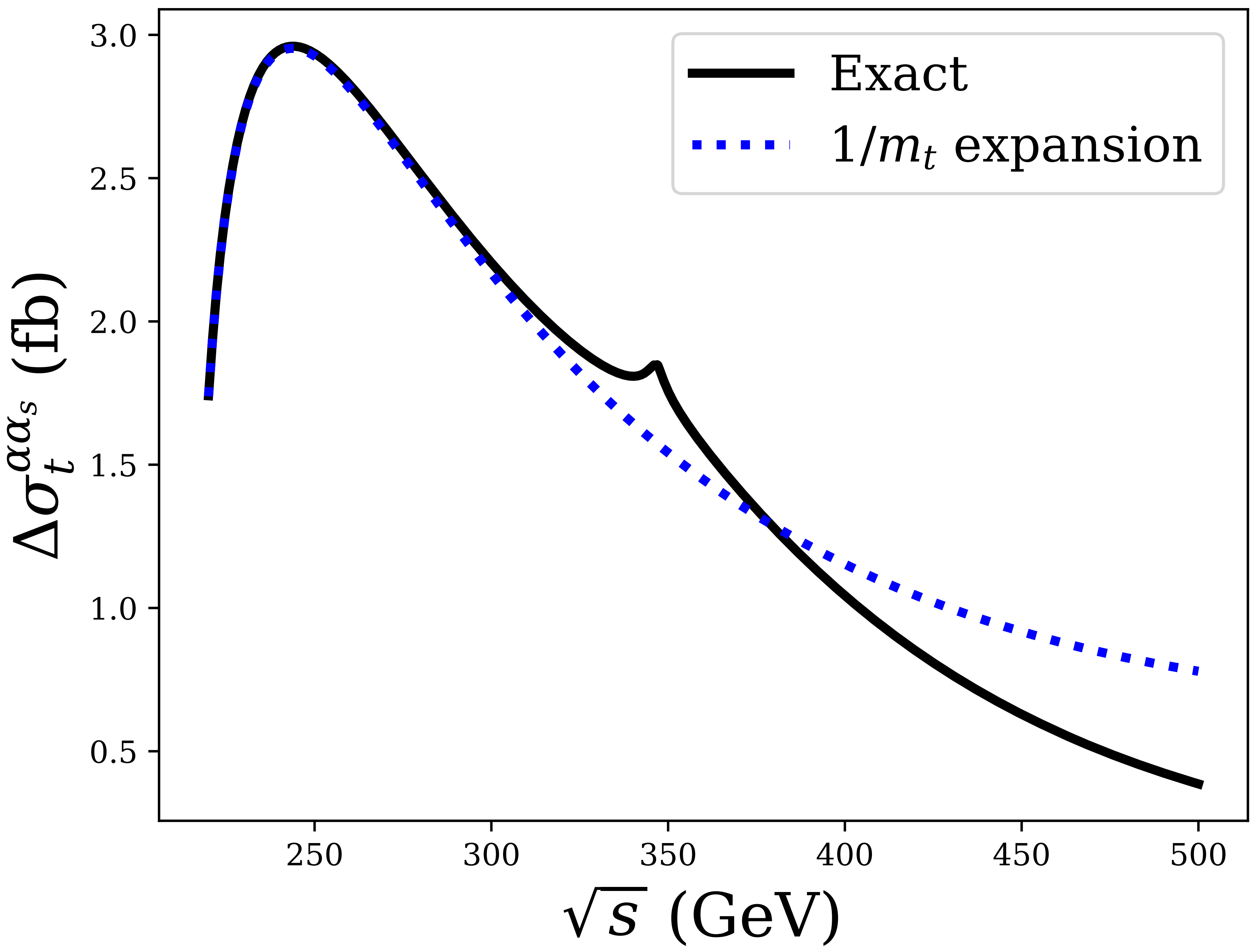
Solutions up to weight-3 written in terms of GPLs

Weight-4 parts expressed as one-fold integrals (not ideal, but usable)

A new calculation for the HZV two-loop diagrams

Wang, Xu, **LLY**: 1905.11463

The new result works well for all kinematic configurations



NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections
to ZH cross section

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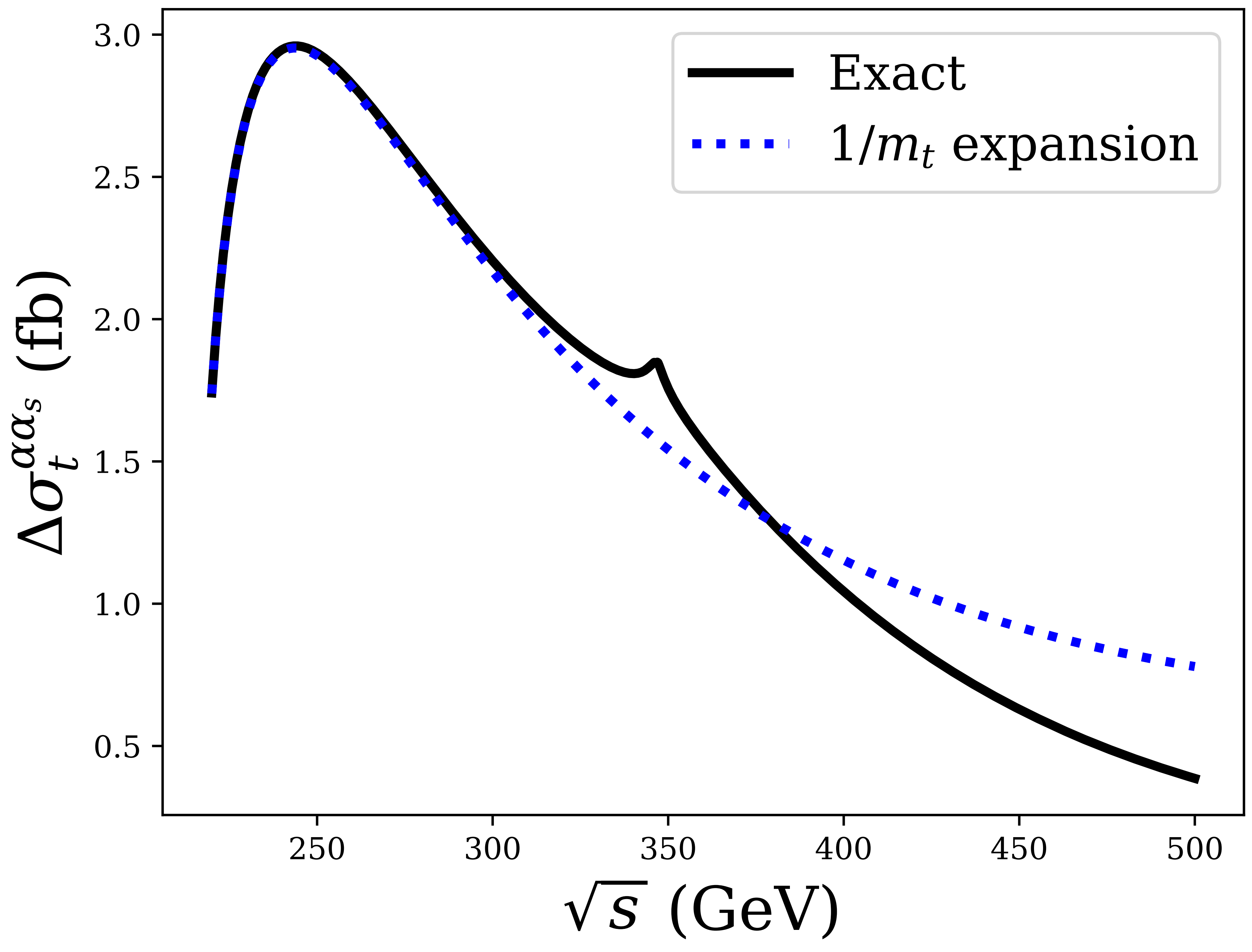
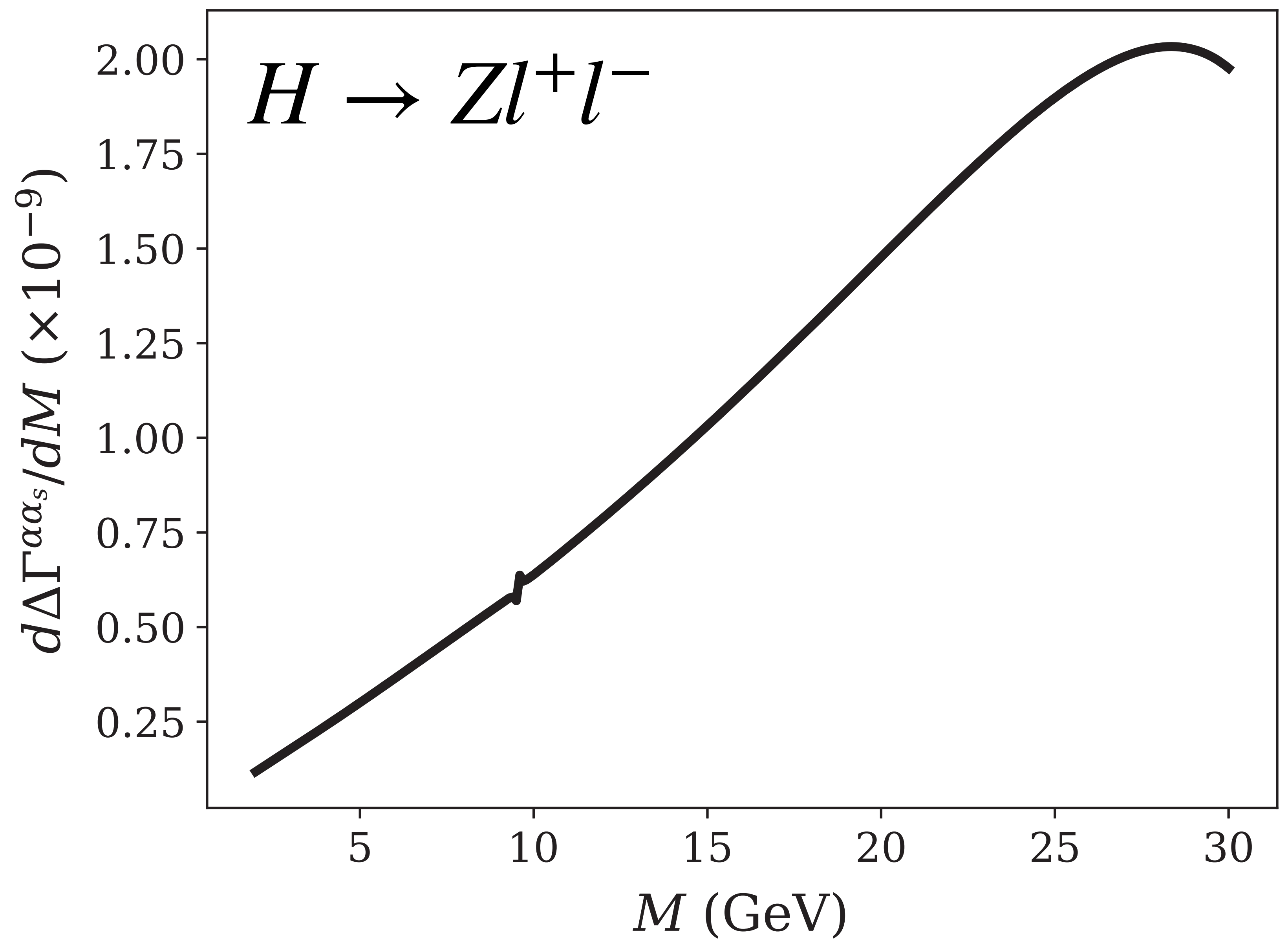


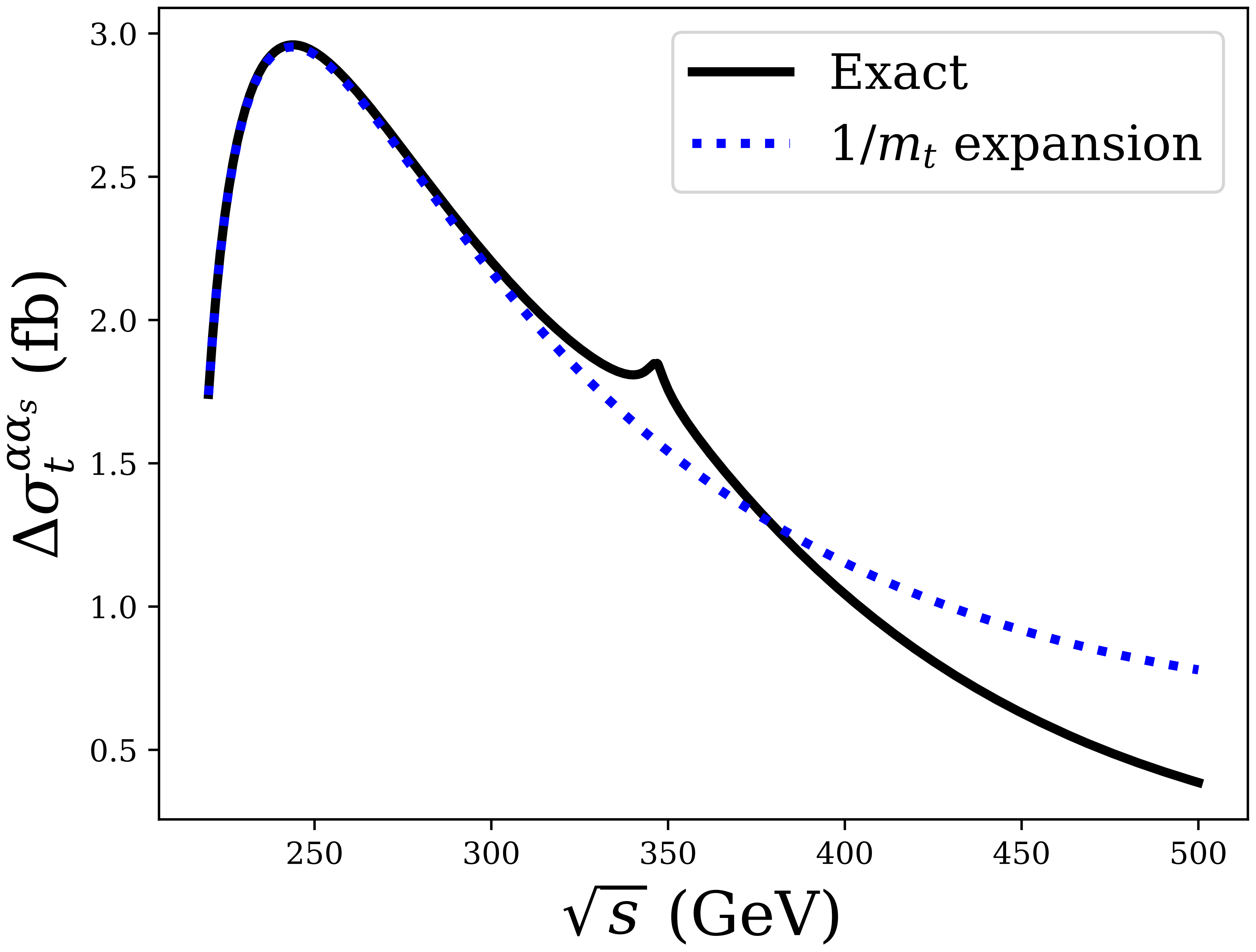
NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections to ZH cross section

Also for bottom quark loops



Bottom contribution to the M_{ll} distribution

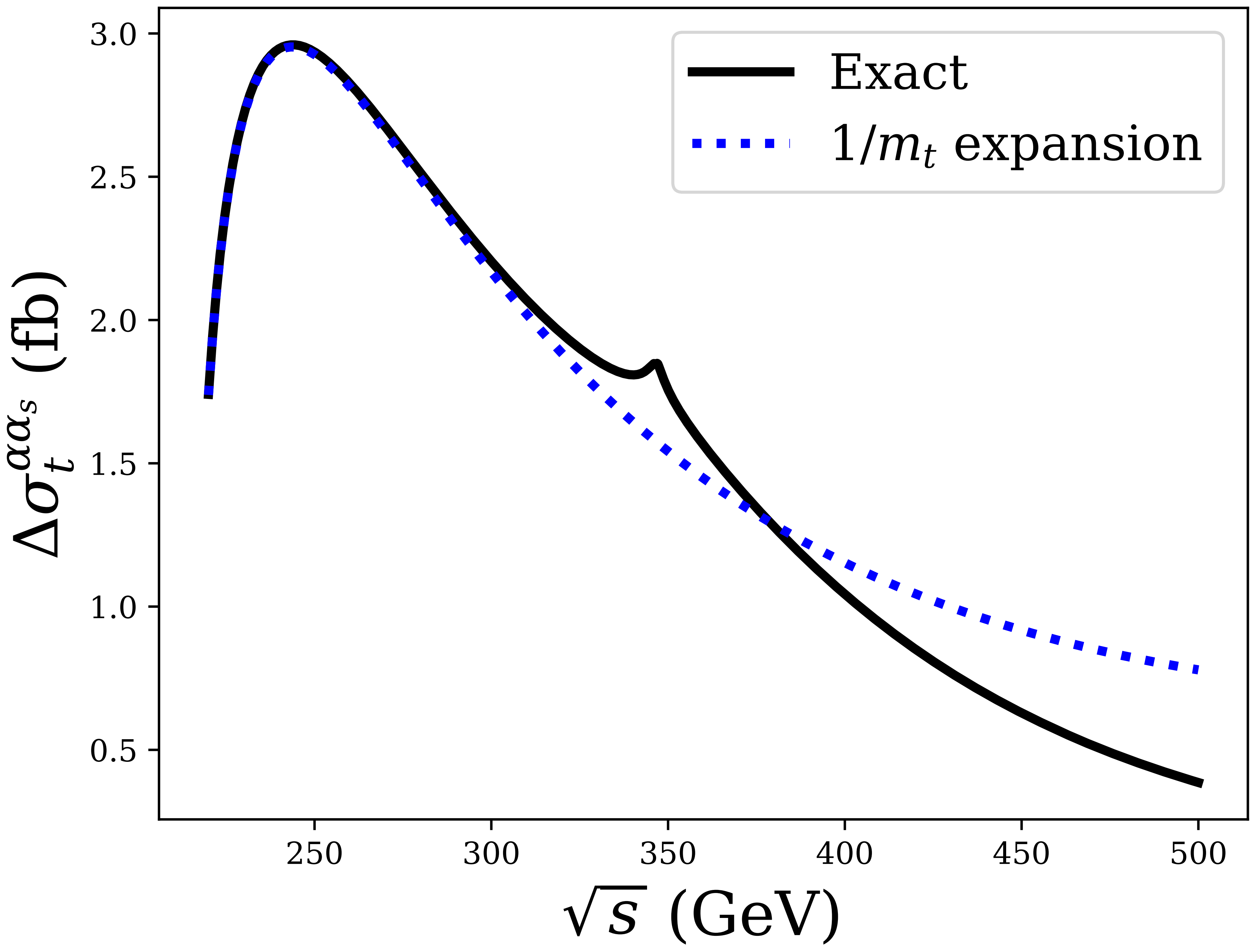
A different expansion?



While the $1/m_t$ expansion does not work at high energies, we have found that an expansion in the limit $m_H^2, m_Z^2 \ll s, m_t^2$ works pretty well!

Easier (than the exact result) to implement in Monte Carlo (available but unpublished)

A different expansion?



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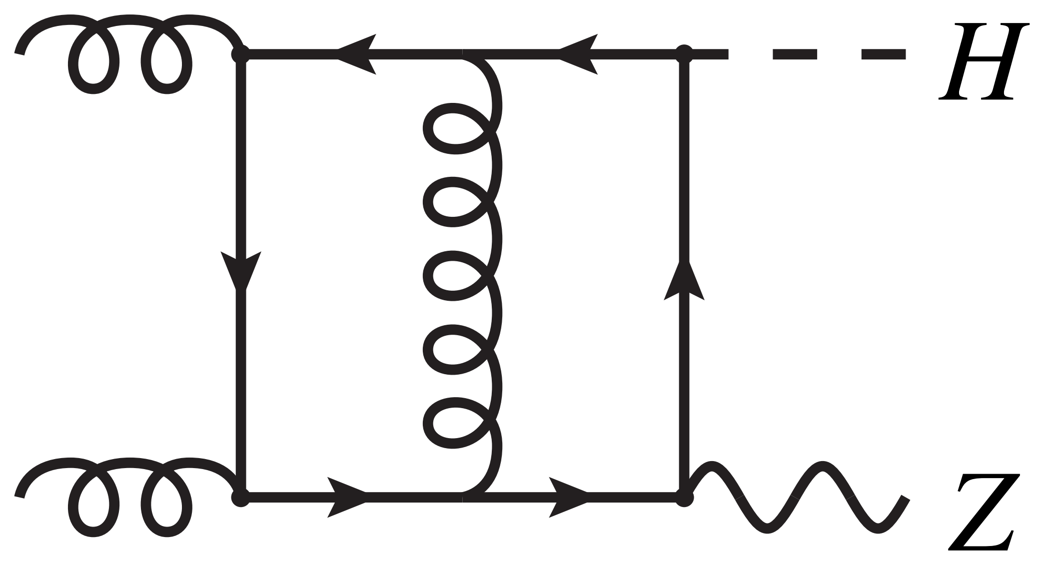
Similar ideas have been successfully applied to $gg \to HH$ and $gg \to ZH$

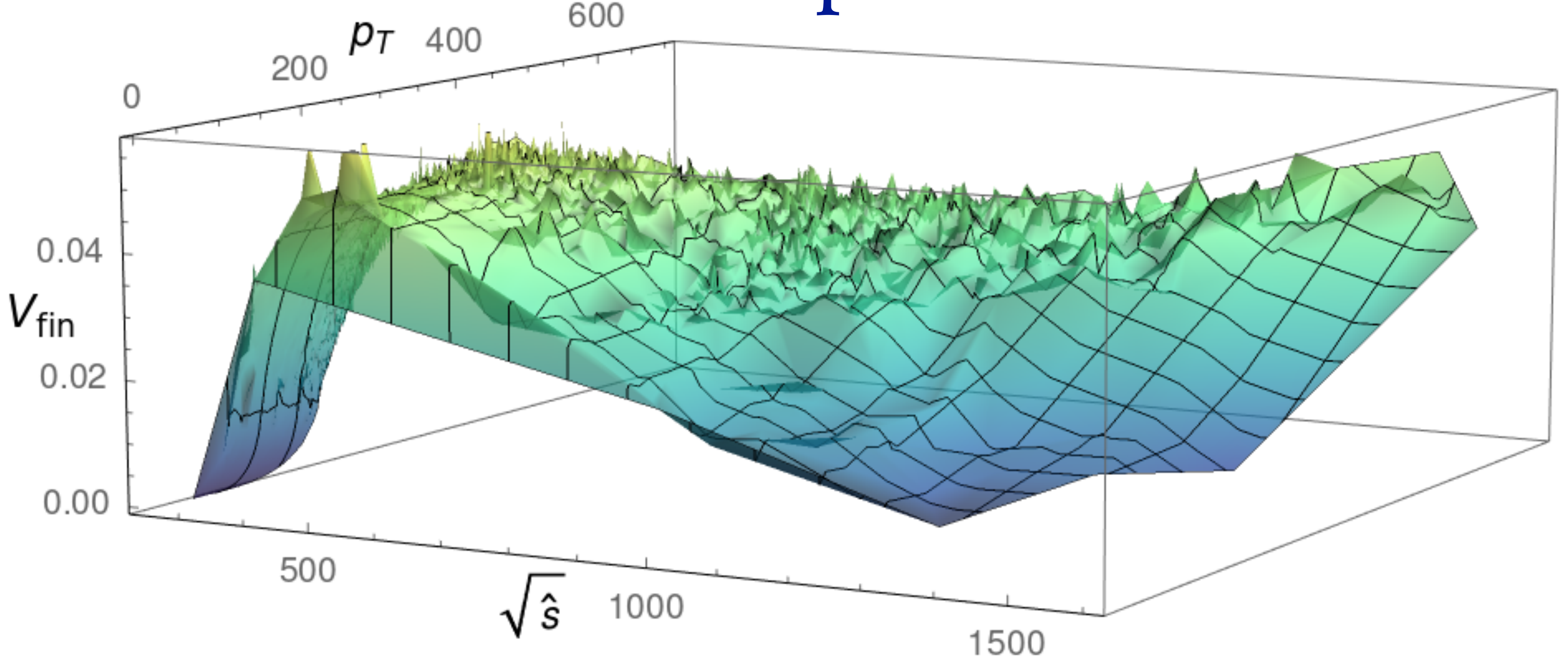


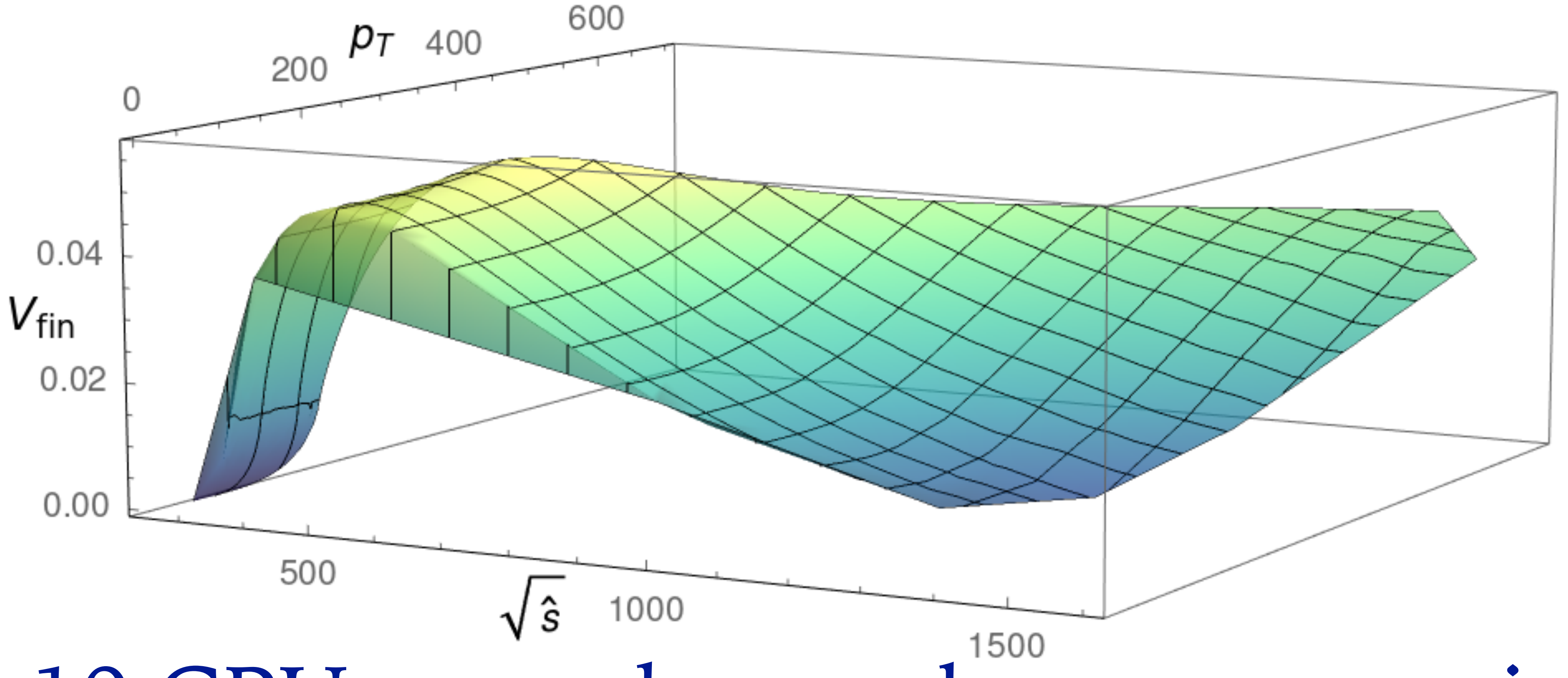
Xu, **LLY**: 1810.12002
Wang, Wang, Xu, Xu, **LLY**: 2010.15649
Wang, Xu, Xu, **LLY**: 2107.08206

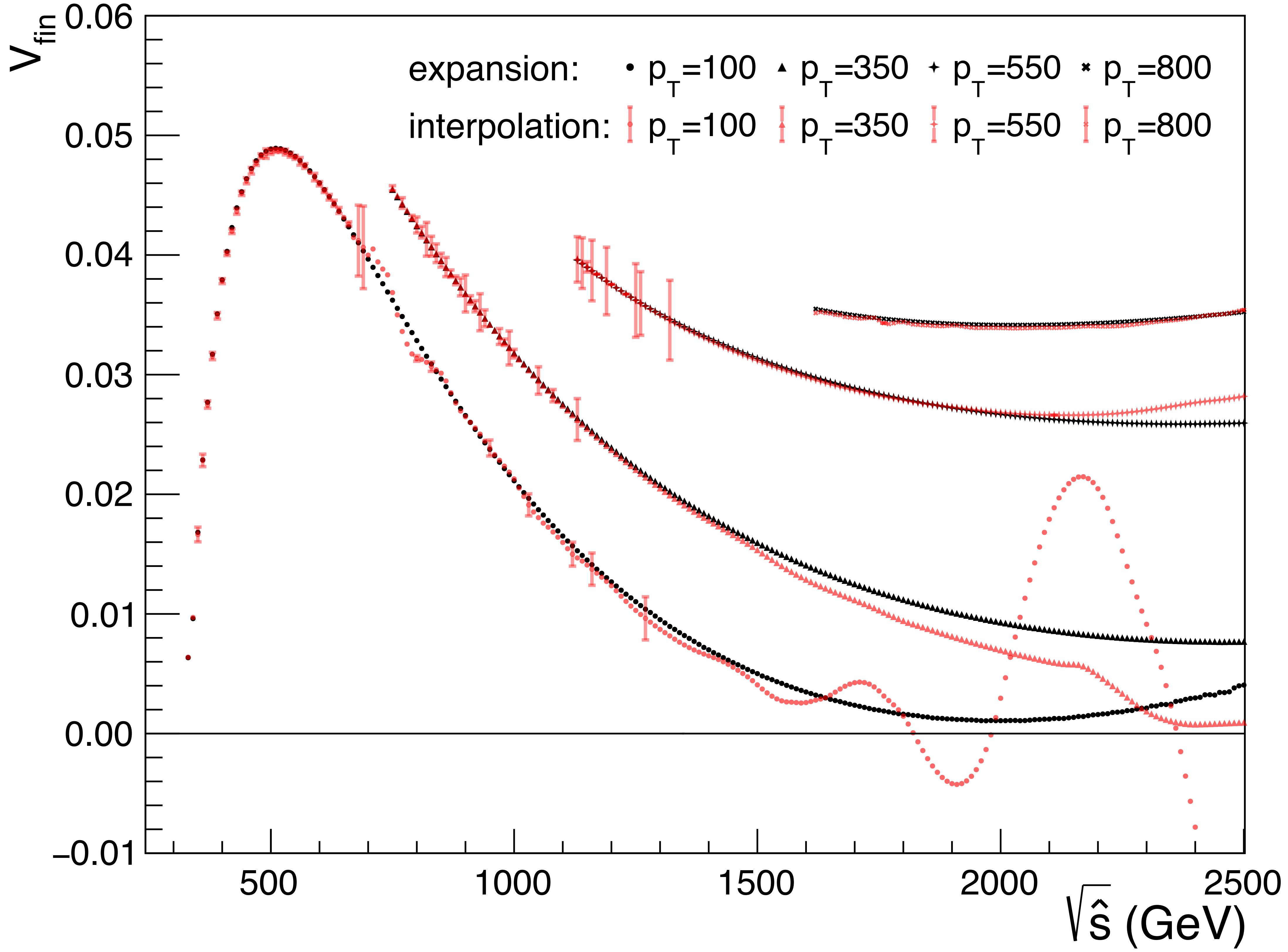
Off-topic: gg→HH

Wang, Wang, Xu, Xu, **LLY**: 2010.15649

7 GPGPU hours per phase space point with sector decomposition







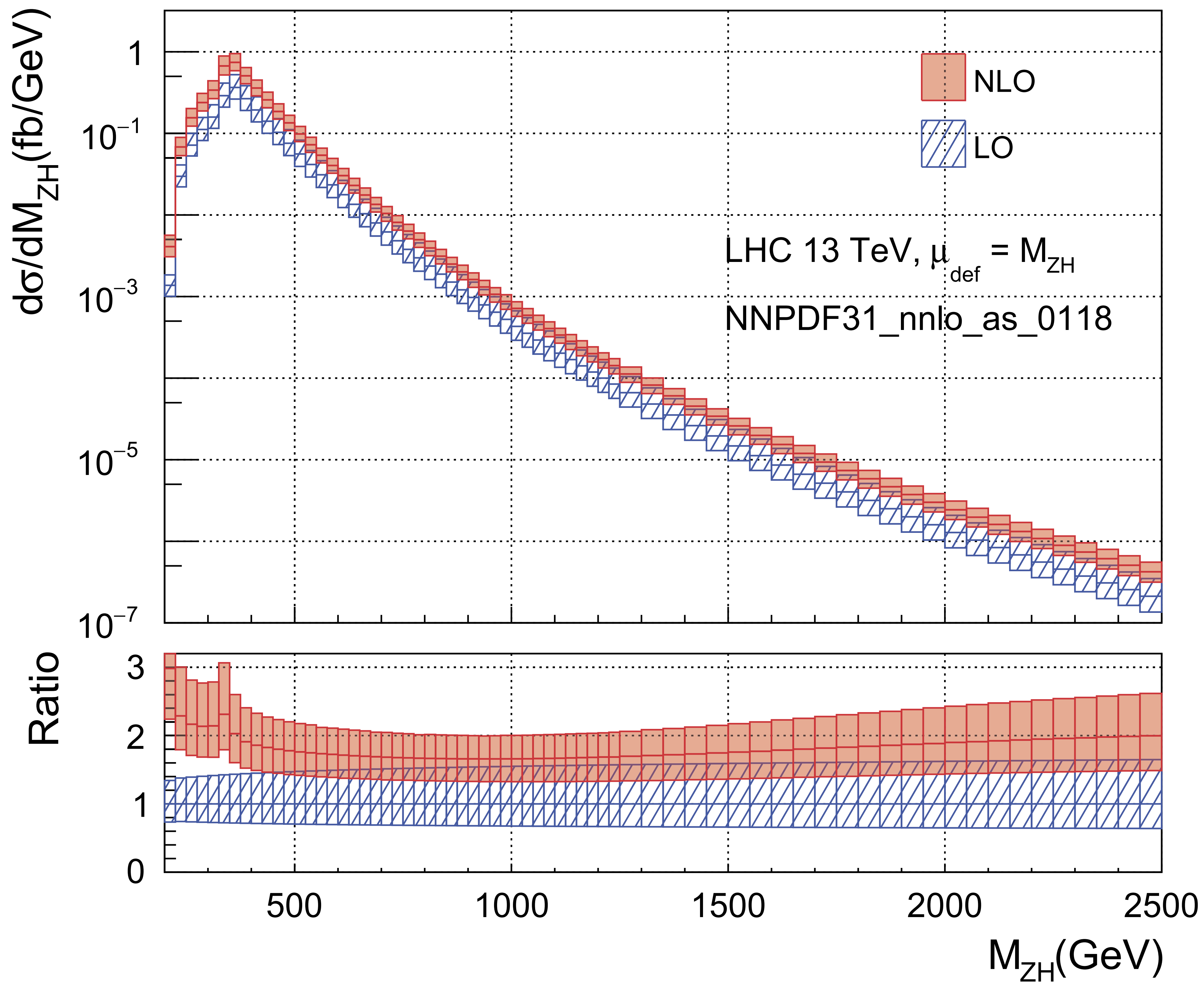
10 CPU seconds per phase space point with small-mass expansion

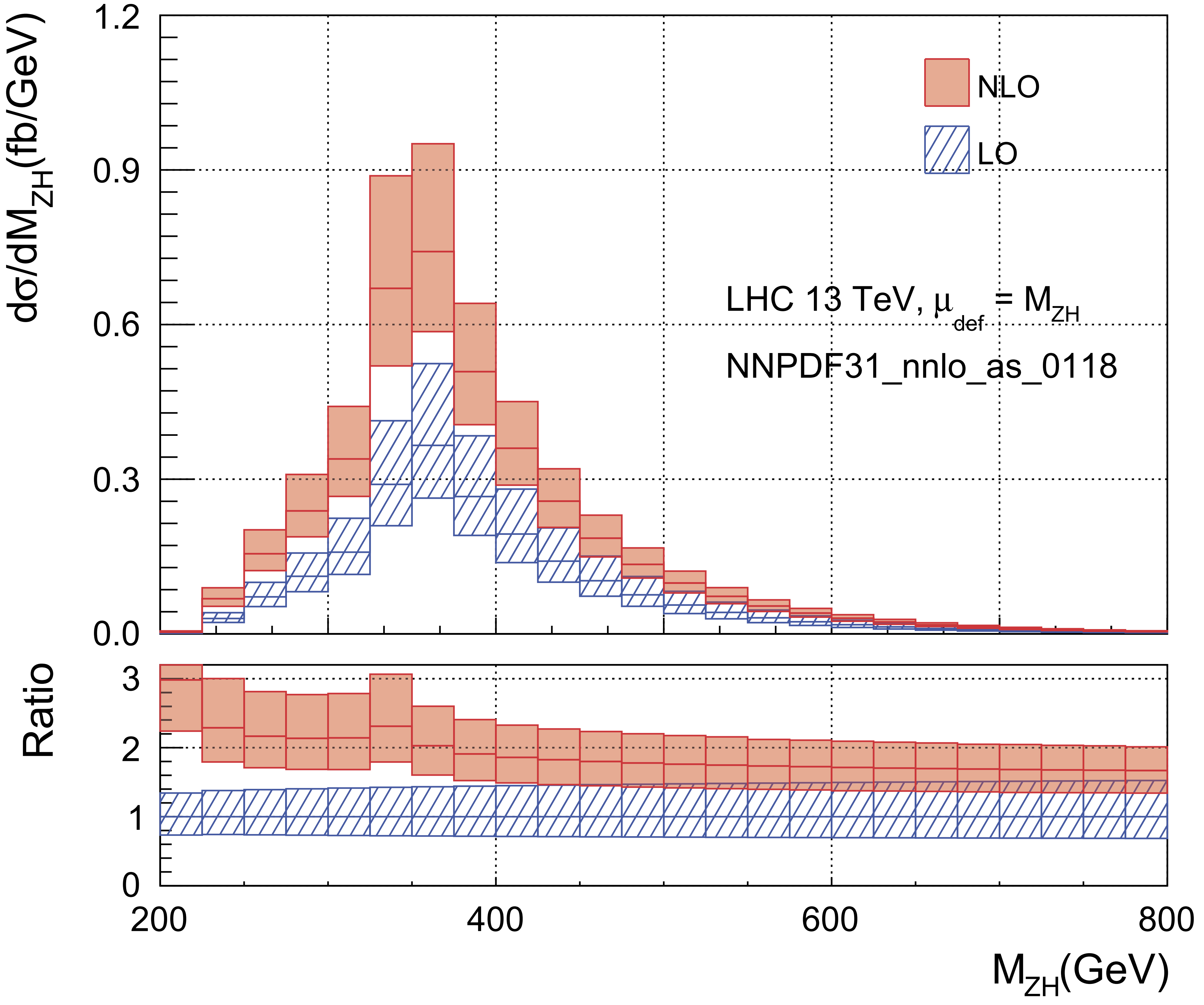
Off-topic: gg→ZH

Wang, Xu, Xu, **LLY**: 2107.08206

NLO predictions for both total and differential cross sections including top quark mass dependence

$$\sigma_{pp\to ZH} = 882.9^{+3.5\%}_{-2.5\%}\ \text{fb}$$



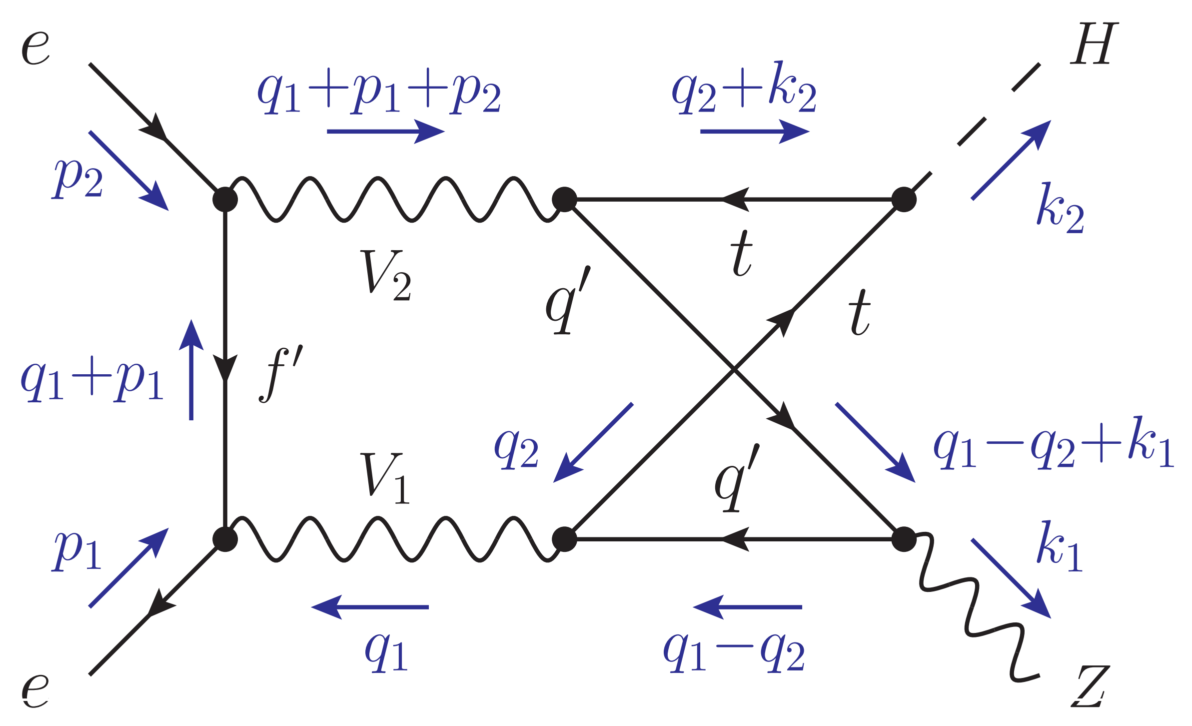


Towards two-loop EW corrections to ZH

A must to match the ~0.3% experimental accuracy

A rather challenging task: ~20000 diagrams, a lot of physical scales   Li, Wang, Wu: 2012.12513

Evaluation of a class of double boxes with a top quark loop   Song, Freitas: 2101.00308



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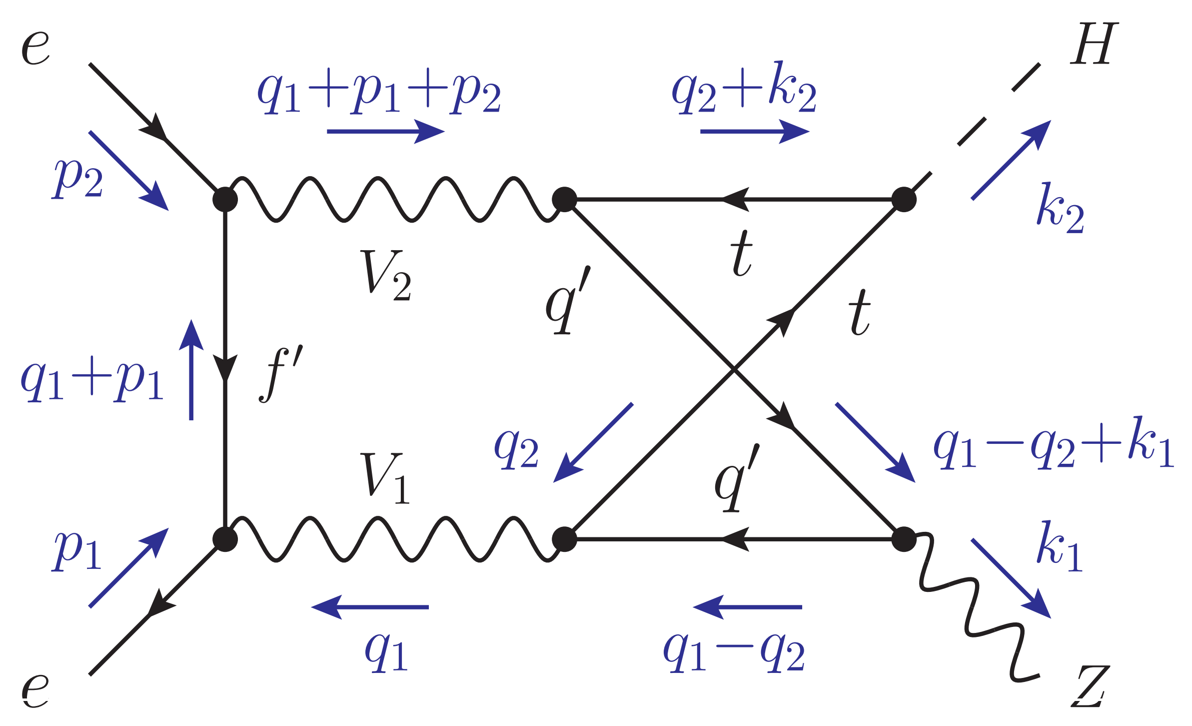

Further development of computational techniques required!

→ Talks covering both analytic and numeric methods

e.g.: Canonical differential equations in both GPL sectors and elliptic sectors

Numeric solutions (pySecDec, DiffExp, AMFlow, …)

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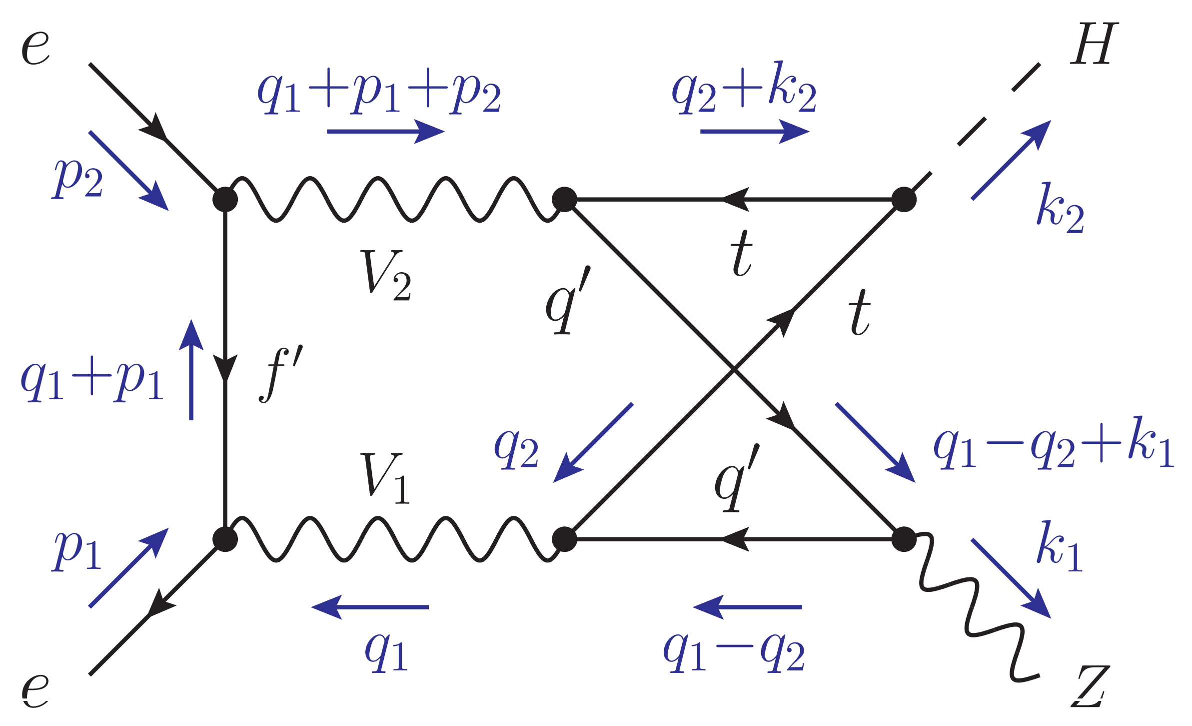

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Perhaps some kind of approximate result is good enough

→ Vague thought: asymptotic expansion in the limit $m^2_{\text{everything}} \ll s, m_t^2$?

Construction of canonical bases

As an addition to existing methods (probably covered by Andreas von Manteuffel), we proposed a novel approach to construct canonical Feynman integrals

Chen, Jiang, Xu, **LLY**: 2008.03045
Chen, Jiang, Ma, Xu, **LLY**: 2202.08127

d-log integrals in the generalized loop-by-loop Baikov representation (a larger vector space than the space of Feynman integrals)

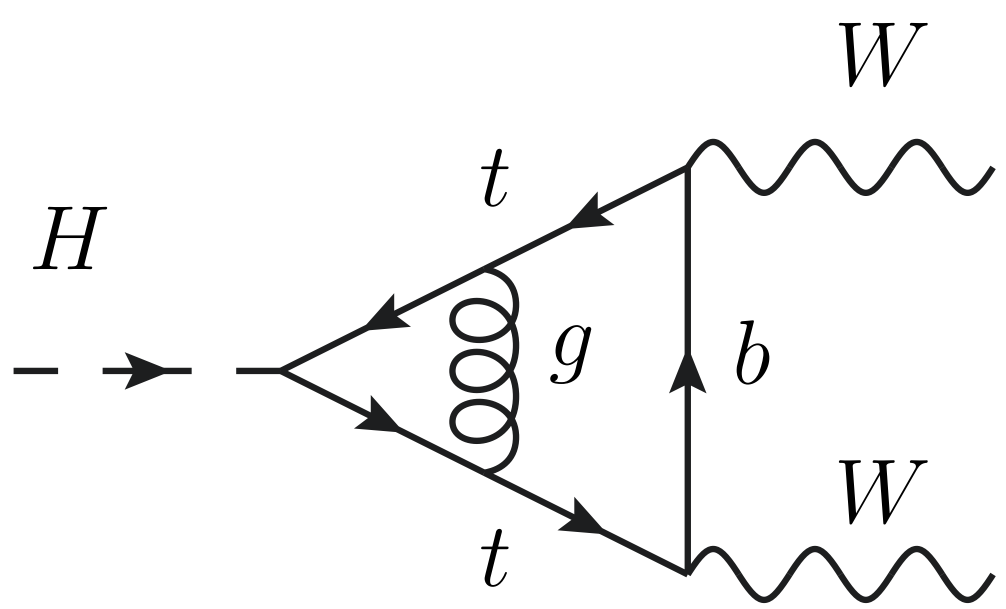
$$\int_{\mathscr{C}} \left[G(z)\right]^{\epsilon} \bigwedge_{j=1}^{n} d\log f_j(z) \longrightarrow \int \left[\prod_{i=1}^{L} \frac{d^d k_i}{i\pi^{d/2}}\right] \frac{1}{z_1^{a_1} z_2^{a_2} \cdots z_N^{a_N}}$$

Project into the space of Feynman integrals using intersection theory

$$\langle \varphi | = \sum_{i=1}^{\nu} c_i \langle e_i |$$

Mixed QCD-EW corrections to WWH

Di Vita, Mastrolia, Primo, Schubert: 1702.07331
Ma, Wang, Xu, **LLY**, Zhou: 2105.06316
Wang, **LLY**, Zhou: 2112.04122



The two-loop amplitude can be written in a fully-analytic form (involving a lot of weight-4 GPLs)

$H \to Wl\nu$

$\alpha(m_Z)$	LO	NLO EW	NNLO QCD-EW
$\Gamma\ (10^{-5}\ \text{GeV})$	4.597	4.474	4.518

G_μ	LO	NLO EW	NNLO QCD-EW
$\Gamma\ (10^{-5}\ \text{GeV})$	4.374	4.524	4.531

$e^+e^- \to \nu\bar{\nu}H$

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	$\delta\sigma_{\text{NNLO}}$ (fb)
250	7.88	0.010
350	30.6	0.040
500	74.8	0.101

Rather small corrections

Note: did not consider mixing with $Z(\to \nu\bar{\nu}) + H$

In the future: two-loop EW? Perhaps only some approximations…

The numeric evaluation of GPLs

In all the above calculations one needs numeric evaluations of a large amount of GPLs

The algorithm has been well-known for many years

Gehrmann, Remiddi: hep-ph/0111255
Vollinga, Weinzierl: hep-ph/0410259
Ablinger, Blümlein, Schneider: 1302.0378

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handyG: newer implementation using double-precision or quad-precision numbers, aimed for usage in Monte-Carlo Naterop, Signer, Ulrich: 1909.01656

The numeric evaluation of GPLs

The algorithm is recursive: one transforms the target GPL to a sum of so-called “convergent” GPLs, which can be evaluated by series expansion

A problem of **numerically** recursive implementations: to evaluate a single GPL, sometimes a transformed GPL needs to be computed for many many times!

- Greatly slows down the computation speed
- May lose accuracy due to repeated floating-point cancellations

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We have encountered such situations in the calculation of $e^+e^- \to \nu\bar{\nu}H$: in general handyG can evaluate a weight-4 GPL in far less than a second, but sometimes it takes several seconds

$$\text{e.g.:} \quad \mathtt{G}(1.0025, 0.989, 0.45, 0.89 + 0.24\mathtt{i}; 1)$$

The problem becomes much worse at higher weights: at three-loops one needs weight-6

FastGPL

https://github.com/llyang/FastGPL

A re-implementation of the algorithm: hybrid analytic/numeric

The reduction to convergent GPLs are (mostly) done in a Mathematica package (to be released)

```
<< reduceGPL`
map[{1, 0, 1, 1}, 3]
{a, 0, b, b}
There is no any artificial divergence!
{a, 0, b, c}
There is artificial divergence when c=x!
We need to rescale indices and argument of GPLs!
complex<double> G4_a0bc_b(complex<double> a, complex<double> b, complex<double> c, int sa, int sb, int sc, double x) {
a=a/x;
b=b/x;
c=c/x;
x=1.;
if(b==c) {
const vector<complex<double>> sy = {G({a, b}, {sa, sb}, x), G({a}, {sa}, x), G({0, a/b, 1}, 1), G({b}, {sb}, x)};
complex<double> res= sy[1]*sy[2] - sy[2]*sy[3] + sy[0]*G({0, 1}, 1) - sy[1]*G({0, 0, 1}, 1) + sy[3]*G({0, a/b, x/b}, 1) + G({0, a/b,
  0, 1}, 1) - G({0, a/b, x/b, 1}, 1) + G({0, a/b}, 1)*(-sy[0] + G({0, b}, {1, sb}, x)) + G({a, 0, 0, b}, {sa, 1, 1, sb}, x);
return res;
 }
else {
const vector<complex<double>> sy = {Log(b, sb), G({a}, {sa}, x), G({c/b}, 1), G({a,
  c}, {sa, sc}, x), G({0, a}, {1, sa}, x), G({0, a/b, c/b}, 1), G({0, c/b, a/b}, 1), G({c/b, 0, a/b}, 1)};
complex<double> res= sy[0]*(sy[2]*sy[4] - sy[5] - sy[6] - sy[7]) + sy[3]*G({0, c/b}, 1) + 2.*G({0, 0, a/b, c/b}, 1) + 2.*G({0, 0, c/b, a/b},
  1) + G({0, a/b, 0, c/b}, 1) + G({0, a/b, c/b, x/b}, 1) + 2.*G({0, c/b, 0, a/b}, 1) + G({0, c/b, a/b, x/b}, 1) + 2.*G({c/b, 0, 0,
  a/b}, 1) + G({c/b, 0, a/b, x/b}, 1) + G({0, a/b}, 1)*(-sy[3] + G({0, c}, {1, sc}, x)) + sy[2]*(G({0, 0, a}, {1, 1, sa}, x) - G({a,
  0, c}, {sa, 1, sc}, x)) + G({a, 0, 0, c}, {sa, 1, 1, sc}, x) + (-(sy[0]*sy[1]*sy[2]) - sy[2]*sy[4] + sy[5] + sy[6] + sy[7])*Log(-x,
  sb) + (sy[1]*sy[2]*pow(sy[0], 2.))/2. + sy[1]*(-sy[6] - sy[7] - G({0, 0, c/b}, 1) + sy[2]*(-G({0, 0}, {1, 1}, x) - 2.*Zeta(2)));
if(c!=x) res += (-sy[5] + G({0, a/b, x/b}, 1))*G({c}, {sc}, x);
return res;
 }
}
```

Generate numeric codes automatically

↓

The FastGPL library (up to weight-4 well-tested, up to weight-6 implemented)

Aiming at fast evaluations using double-precision numbers

Comparison of speed

Wang, **LLY**, Zhou: 2112.04122

	t_f (s)	t_h (s)	t_h/t_f
$\texttt{G}(1.0025, 0.989, 0.45, 0.89 + 0.24\texttt{i}; 1)$	0.006	2.2	~ 400
$\texttt{G}(0.998, 1.0545 + 0.127\texttt{i}, 0.91 + 0.25\texttt{i}, -0.226; 1)$	0.004	1.5	~ 400
$\texttt{G}(-1.04, -0.97, 0.25, -0.84 + 0.45\texttt{i}; 1)$	0.004	1.1	~ 300

Table 2: Average evaluation times of several GPLs which require many iterations.

	$0aBC$	$0abC$	$0abc$	$00aB$	$00ab$
t_f (ms)	0.22	0.25	0.20	0.08	0.05
t_h (ms)	3.1	5.8	4.5	1.3	0.80
t_h/t_f	~ 14	~ 23	~ 23	~ 17	~ 16

	$ABCD$	$abCD$	$abcD$	$abcd$
t_f (ms)	0.22	0.47	0.50	0.42
t_h (ms)	1.7	7.4	11.0	9.1
t_h/t_f	~ 7.5	~ 16	~ 22	~ 22

Table 3: Average evaluation times of a few categories of weight-4 GPLs.

$e^+e^- \to \nu\bar{\nu}H$

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	$\delta\sigma_{\mathrm{NNLO}}$ (fb)	t_f (h)	t_h (h)	t_h/t_f
250	7.88	0.010	0.45	8.60	~ 19
350	30.6	0.040	0.51	9.02	~ 18
500	74.8	0.101	0.52	9.24	~ 18

10000 sample phase-space points

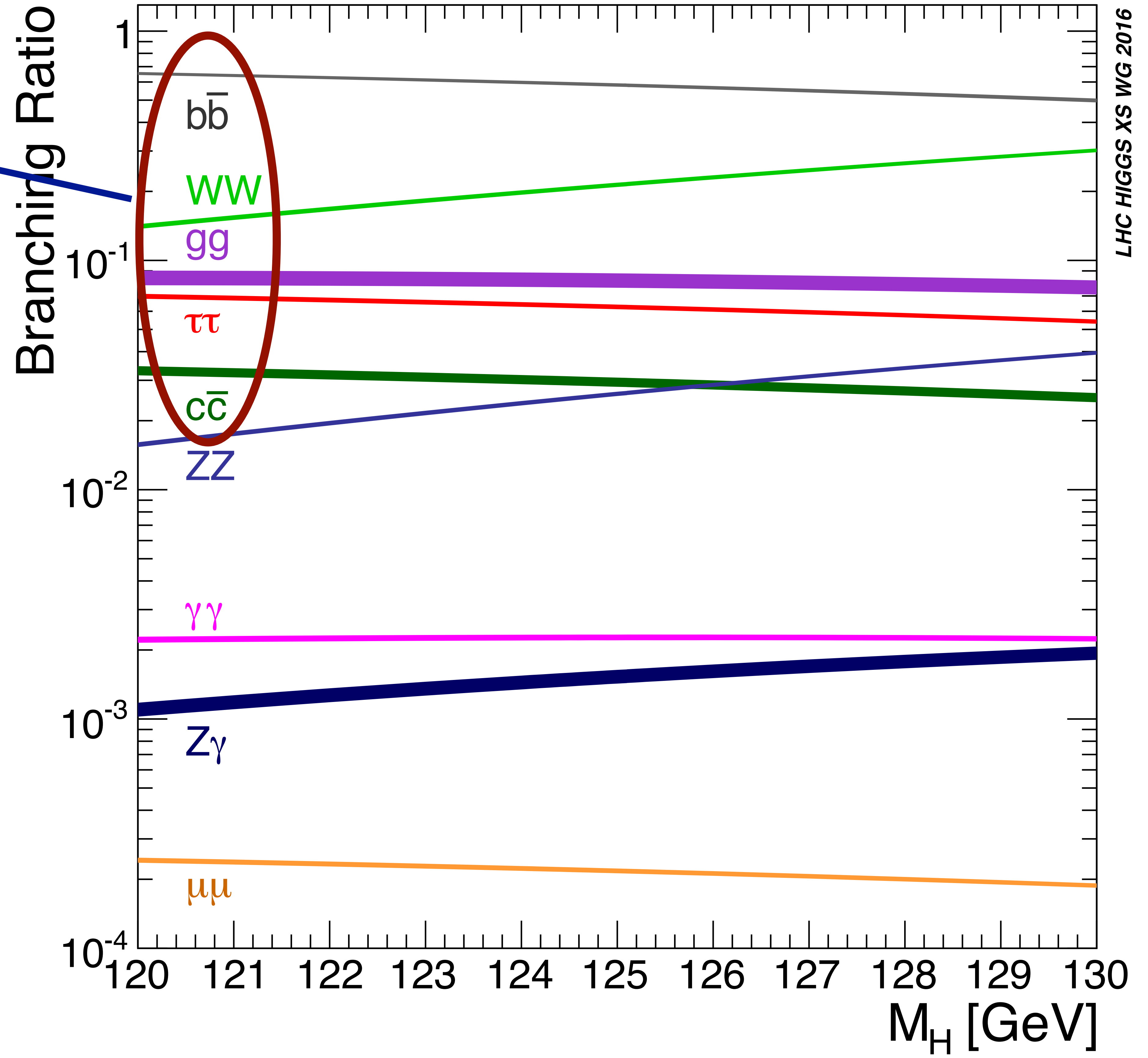
several thousand GPLs per point

FastGPL is faster in general, and is much faster for special cases

Preliminary tests show that the speed-boost is much larger at weight-6

Higgs decay

Weakness of the LHC



The hadronic channels

$H \to b\bar{b}$	Important for *HZZ* and $Hb\bar{b}$ couplings
$H \to gg$	Probes new particles running in the loop
$H \to c\bar{c}$	Unique window to charm Yukawa

Partial widths

Freitas (2021) and references therein

- $H \to q\bar{q}$
  - $\mathcal{O}(\alpha_s^4)$ in the limit of massless quarks
  - $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$ and partial $\mathcal{O}(\alpha^2)$
- $H \to gg$

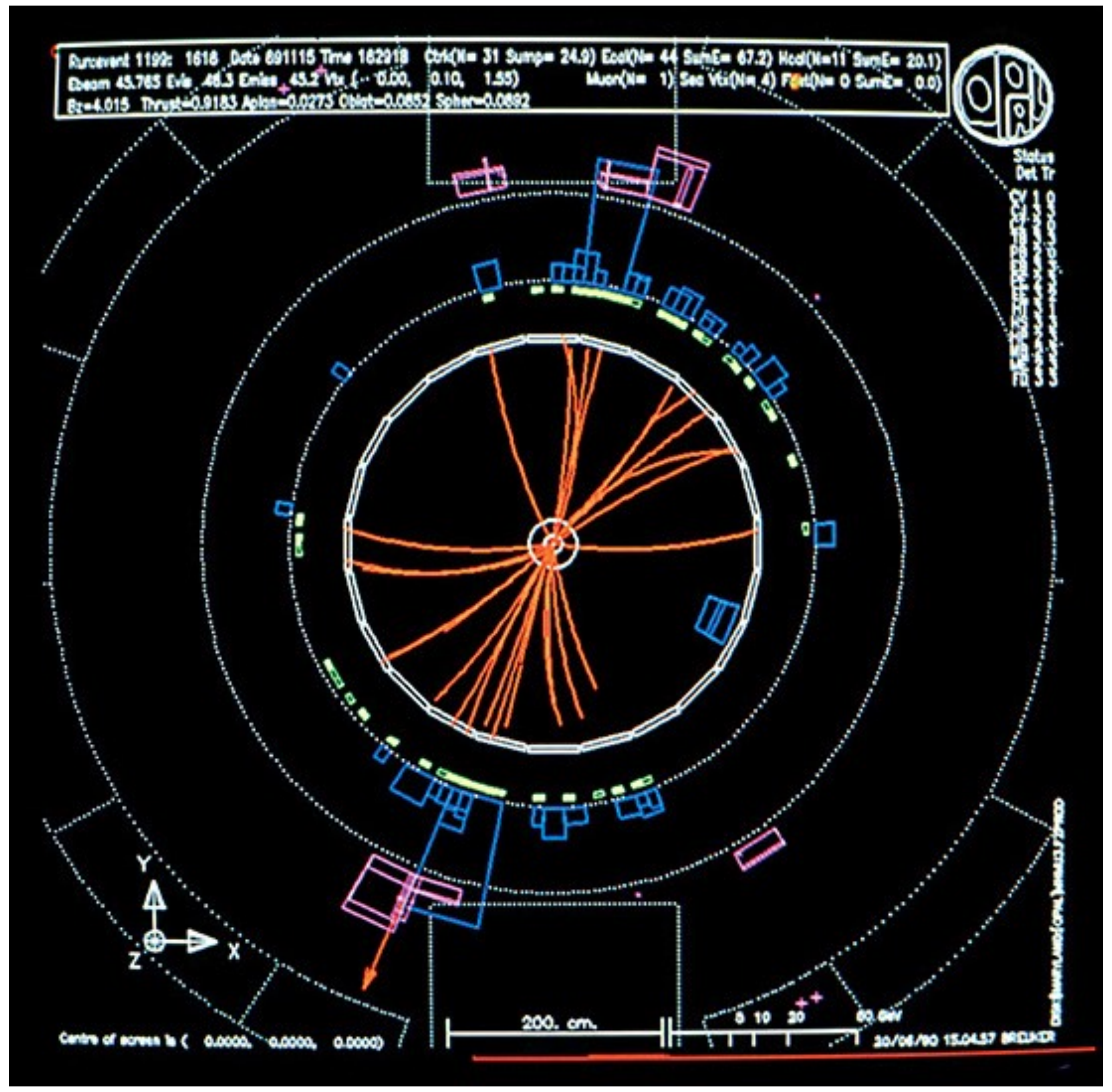
  Herzog et al.: 1707.01044
  - $\mathcal{O}(\alpha_s^4)$ with infinite m_t $\quad \Gamma_{\mathrm{N^4LO}}(H \to gg) = \Gamma_0 \left(1.844 \pm 0.011_{\,\mathrm{series}} \pm 0.045_{\,\alpha_s(M_Z),1\%}\right)$
  - $\mathcal{O}(\alpha_s^2)$ with $1/m_t$ expansion
  - $\mathcal{O}(\alpha_s^2)$ three-loop form factor with full m_t dependence (hence also bottom loop)

    Czakon, Niggetiedt: 2001.03008
  - $\mathcal{O}(\alpha)$ EW corrections

Event shapes

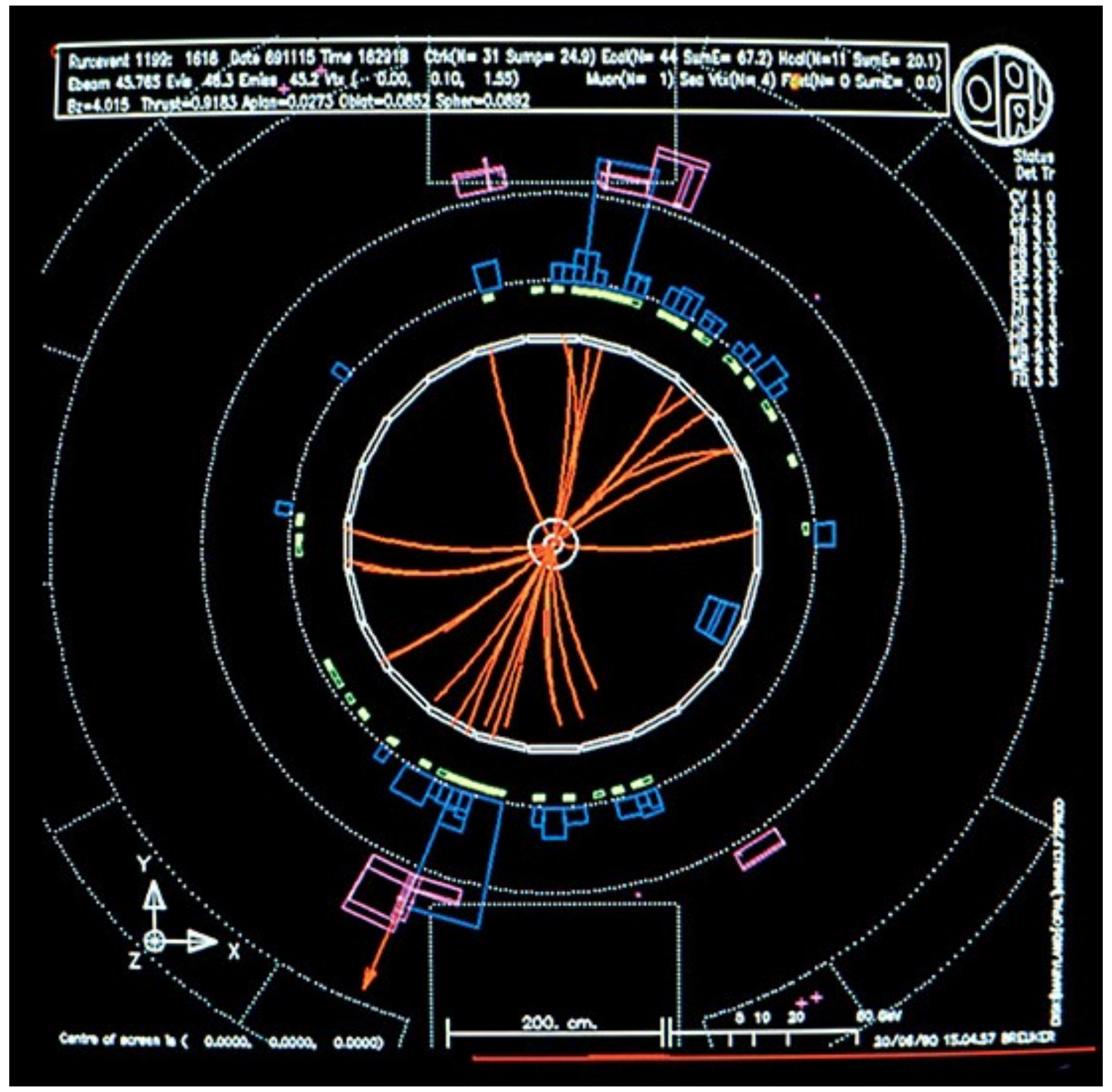
Event shapes provide more information than the total rates

- ➤ Discrimination between quark and gluon final states
- ➤ Probing kinematic dependence of the Hgg vertex
- ➤ New-physics enhanced light-quark Yukawa couplings?



Event shapes

Event shapes provide more information than the total rates



- ➤ Discrimination between quark and gluon final states
- ➤ Probing kinematic dependence of the Hgg vertex
- ➤ New-physics enhanced light-quark Yukawa couplings?

I'll focus on one particular variable: thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$

$$\tau = 1 - T$$

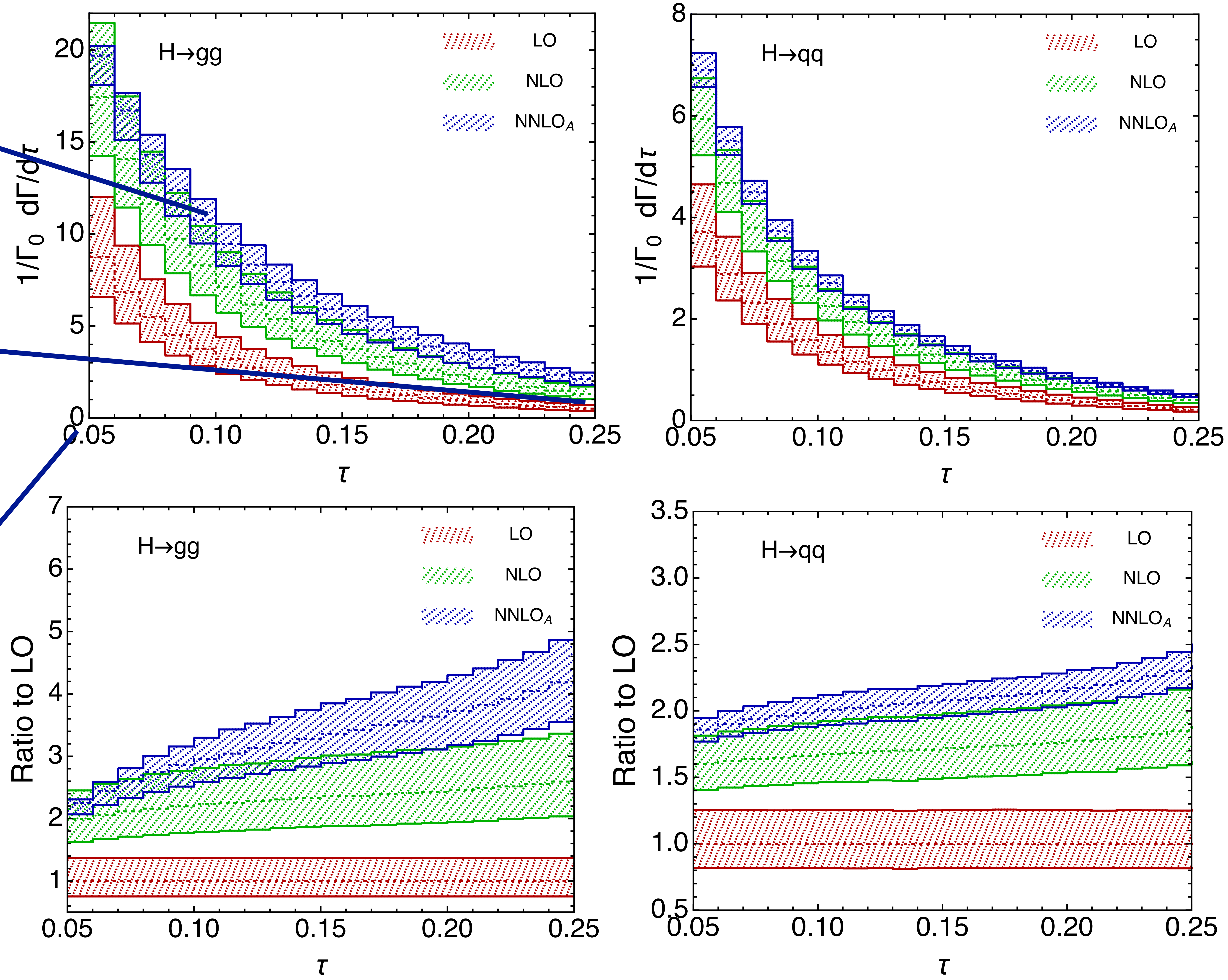
Fixed-order predictions for thrust distribution

Gao, Gong, Ju, **LLY**: 1901.02253

Large corrections, especially in the gluon channel; N^3LO needed?

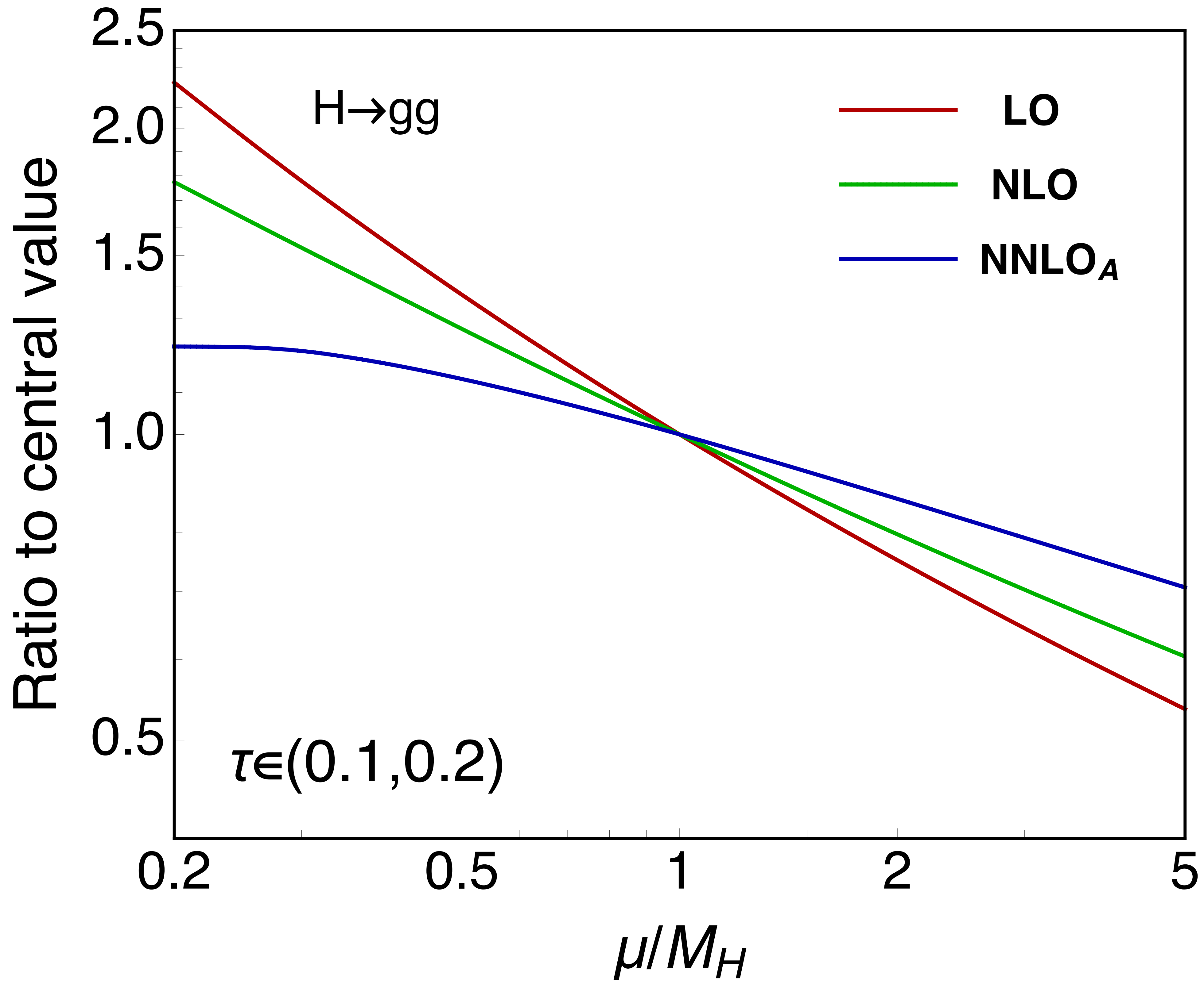
Soft-collinear approximation not valid for larger τ; a full NNLO calculation required!

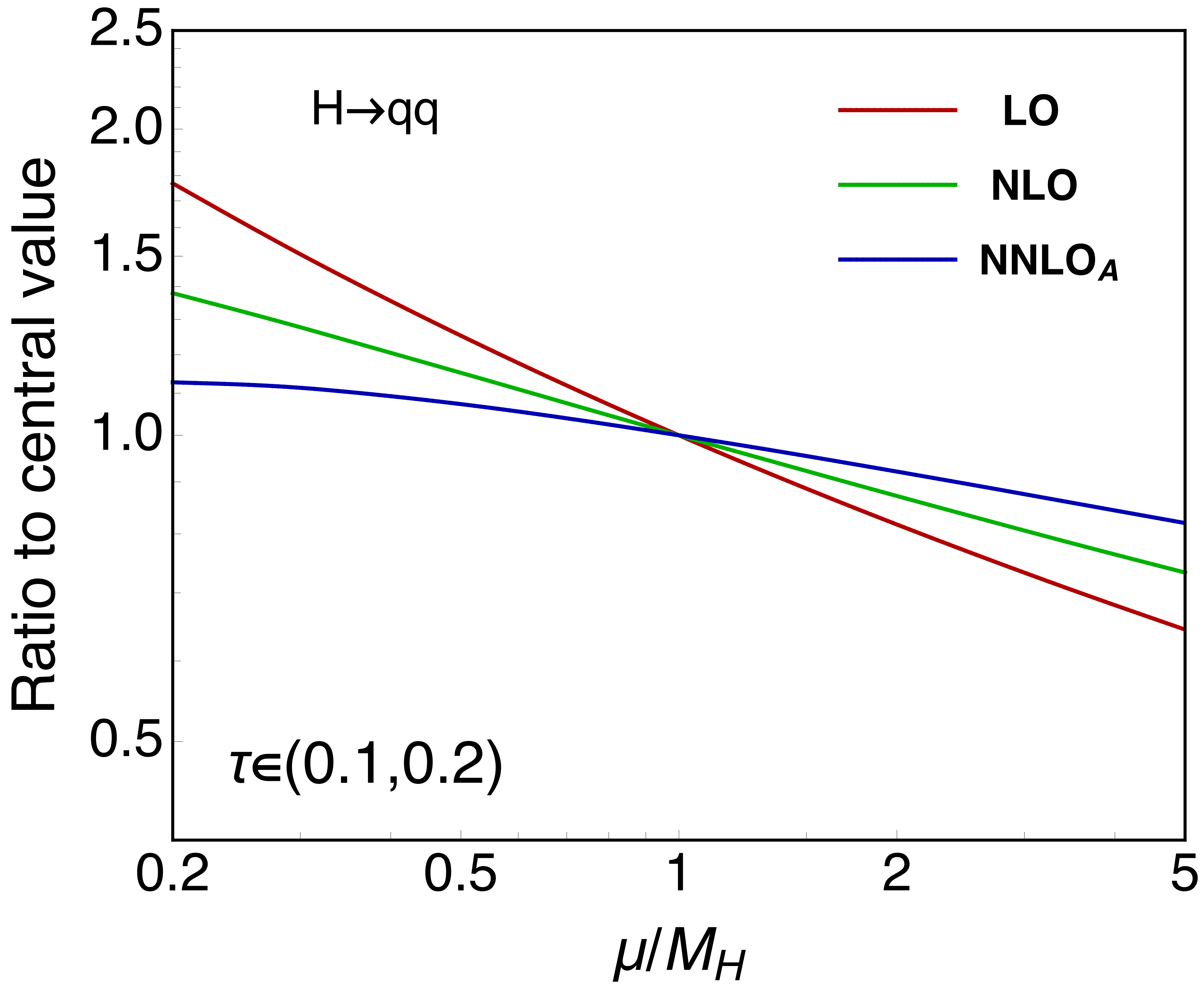
Parton shower and/or resummation needed for smaller τ



Scale dependence

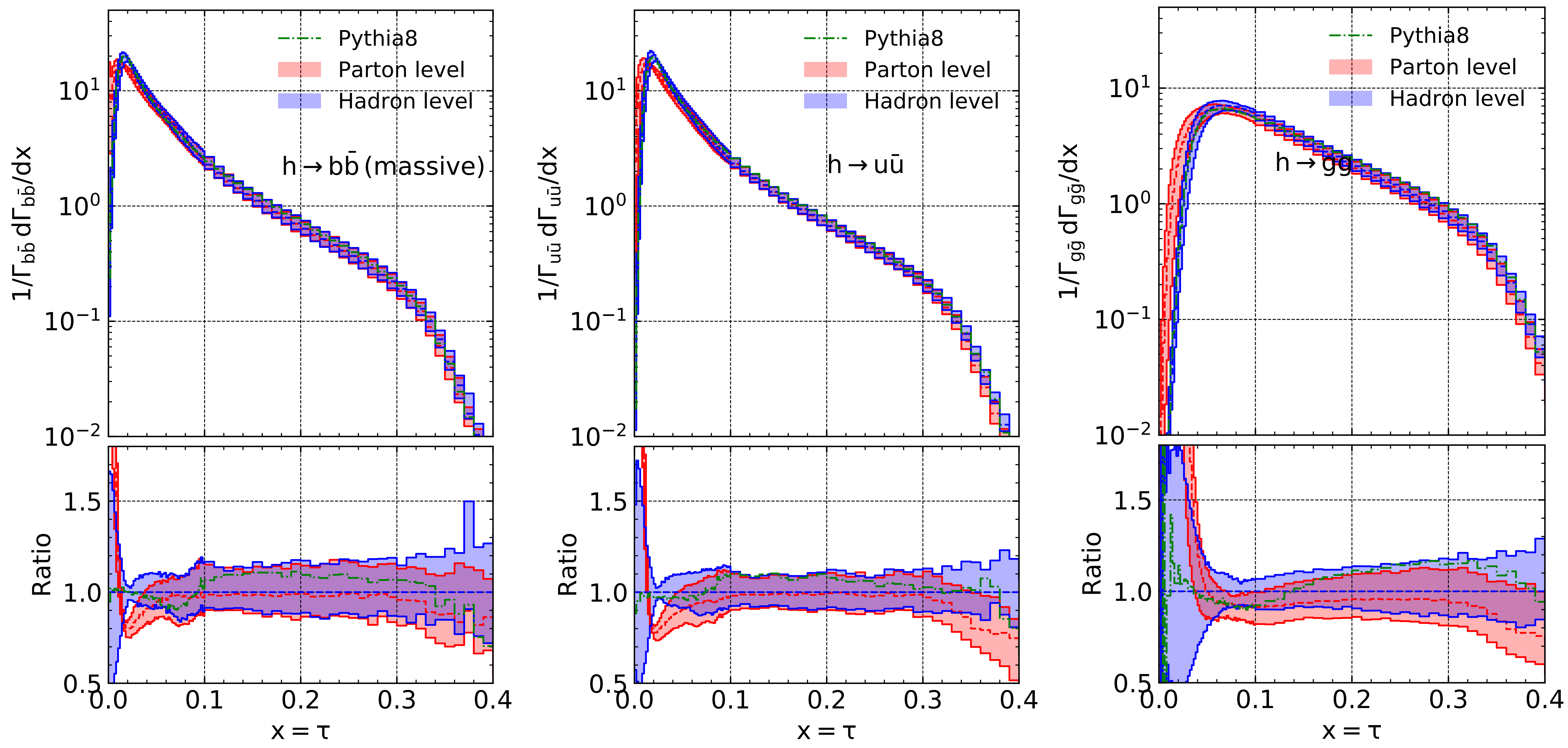
Gao, Gong, Ju, **LLY**: 1901.02253





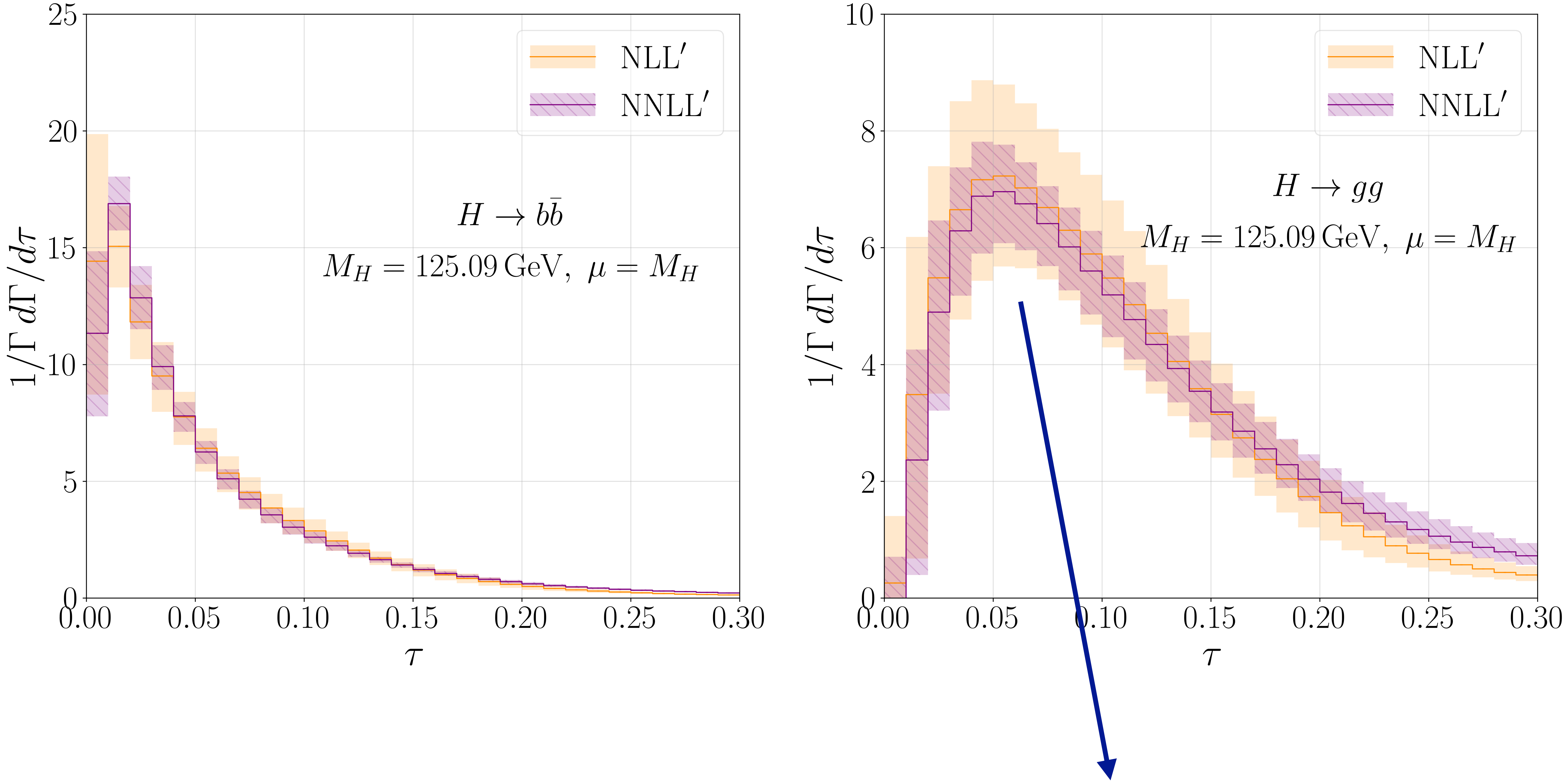
Matched with parton shower

Hu, Sun, Shen, Gao: 2101.08916



Resummed predictions

Alioli et al.: 2009.13533



Large uncertainties in the gluon channel; N^3LL or N^3LL' needed?

Towards N^3LL' thrust resummation

	hard, jet, soft functions	hard, jet, soft anomalous dimensions	cusp anomalous dimension, beta function
NNLL'	2-loop	2-loop	3-loop
N3LL	2-loop	3-loop	4-loop
N3LL'	3-loop	3-loop	4-loop
		available	available

available except the non-logarithmic term of the 3-loop soft function

The 3-loop soft function

The non-logarithmic term of the 3-loop soft function for quarks was extracted from the numeric result of EERAD3

$$c_3^S = 2s_3 + 691 = -19988 \pm 1440\,(\text{stat.}) \pm 4000\,(\text{syst.})$$

Brüser, Liu, Stahlhofen: 1804.09722

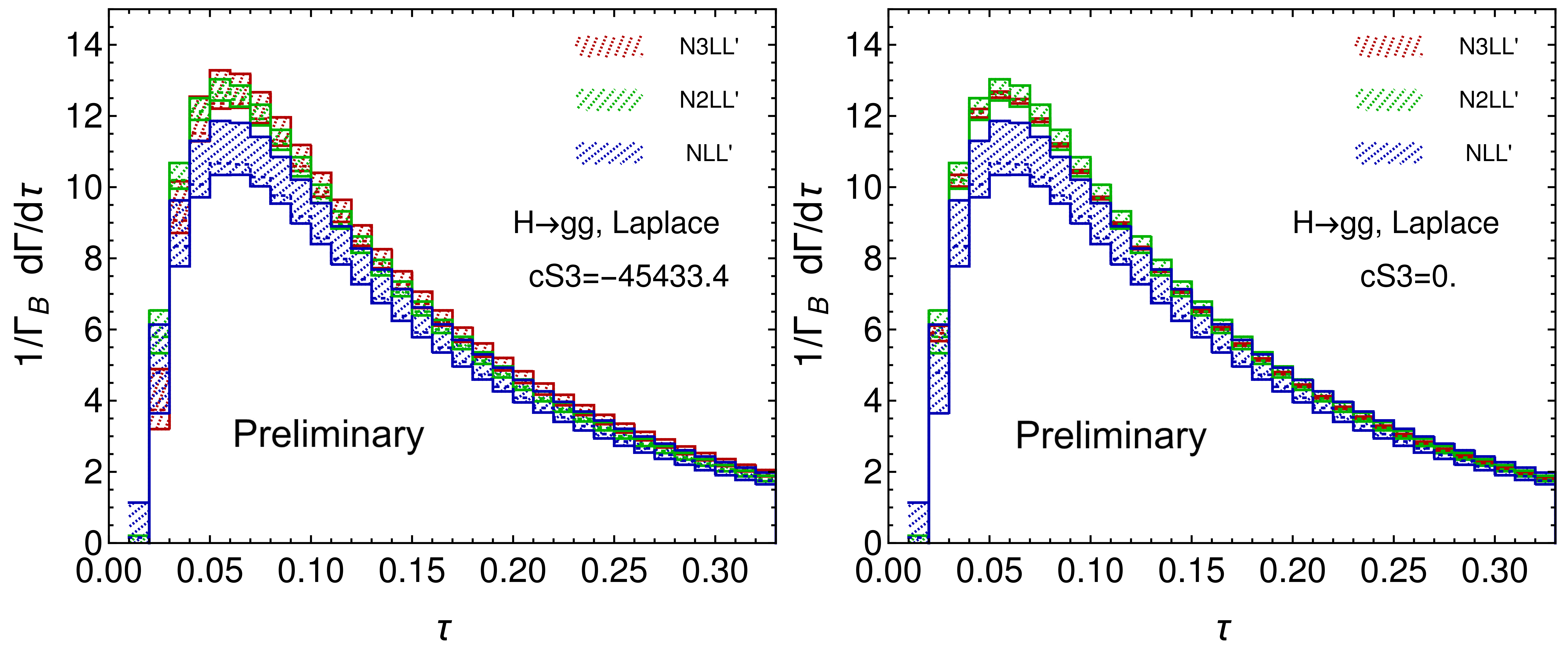
With a Casimir scaling , the corresponding term for gluons

$$c_3^S \sim -45000$$

A rather large constant term, one might worry about convergence!

Especially it multiplies $\alpha_s(\mu_s)$ at the low scale $\mu_s \sim \tau m_H$

Towards N³LL' thrust resummation



Preliminary result shows that the non-logarithmic term of the 3-loop soft function has a large impact!

We may want to know its precise value…

Thank you!