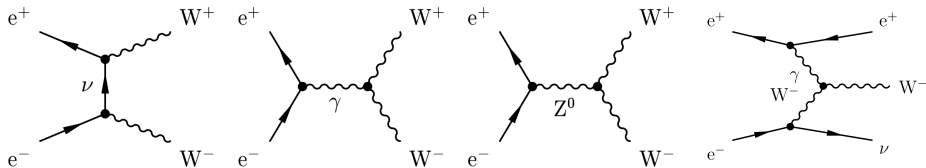# Precision Electroweak Physics with $e^+e^-$ Colliders above the Z-Pole with a Focus on $m_{\mathrm{W}}$

Graham W. Wilson

University of Kansas

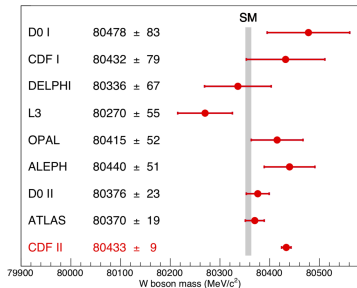June 10, 2022



$W^+W^-$ production

Single W

# Outline of Topics

# INTRODUCTION

LEP W-Boson Mass
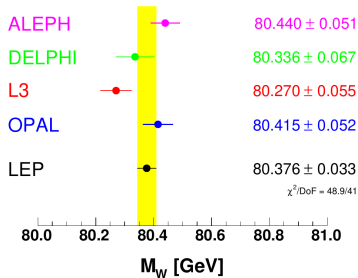
- The LEP results are based on 42 separate measurements with a **healthy** $\chi^2$.
- The LEP-combined (33 MeV), LHCb (32 MeV), D0 Run II (23 MeV), ATLAS (19 MeV) and CDF Run II (9.4 MeV) measurements have a $\chi^2/\text{DoF} = 17.1/4$, with p-value of **0.2%** for compatibility (neglecting correlations).
- So reasonably strong evidence that the ensemble of experimental results are **inconsistent with each other** independent of any SM prediction.
- The standard PDG procedure is to add a scale factor "democratically" to all measurements to parametrize our ignorance.

# PDG scale factors

(What can happen with supposed high precision measurements)
The new world average $m_W$ uncertainty should be scaled up by about 2.1 leading to an uncertainty of 15 MeV in PDG-2022 compared with 12 MeV in PDG-2020.



The charged kaon mass has been in this scale-factored state for 30 years!
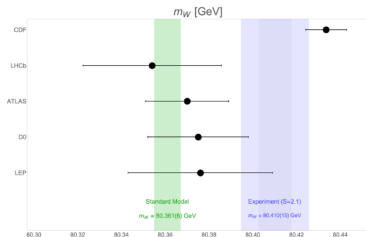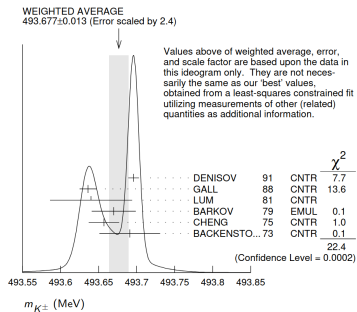


Plot from Resonaances blog (Adam Falkowski). Independently I had also done this and concluded that the scale-factored world-average is $+3.2\sigma$ off the SM value used by CDF

Perhaps one or more experiments has underestimated uncertainties. Also may be difficult to measure the same thing in $p\bar{p}$, $pp$, and $e^+e^-$ collisions.
Strong motivation to measure $m_W$ well in complementary ways in $e^+e^-$ collisions!

fully hadronic $q\bar{q}q\bar{q}$     semi-leptonic $q\bar{q}\ell\nu_\ell$     fully leptonic $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$
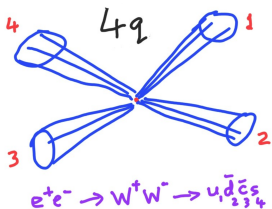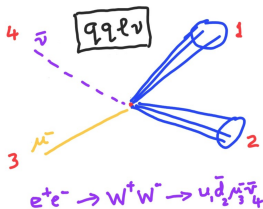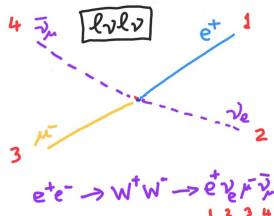
$B_h^2 = 45.4\%$     $6B_\ell B_h = 43.9\%$     $9B_\ell^2 = 10.6\%$

- Here we take $\ell = e, \mu, \tau$. Events with $\tau$ leptons are of some use even for $m_W$.
- 100% of the WW final states are potentially useful for $m_W$ in $e^+e^-$ collisions not just the 22% of the W final state used in hadron colliders.
- Much of the power of an $e^+e^-$ collider is that one measures the **mass** of the W decay products either directly or by imposing kinematic constraints.

# W Mass

$m_W$ is an experimental challenge. Especially so for hadron colliders.

There are several promising approaches to measuring $m_W$ at an $e^+e^-$ collider:

1. **Constrained Reconstruction** Kinematically-constrained reconstruction of $W^+W^-$ using constraints from **4-momentum conservation** and optionally mass-equality: the LEP2 work-horse. Primarily using $q\bar{q}\ell\nu_\ell$ events. Color reconnection disfavors use of $q\bar{q}q\bar{q}$ channel. Use $E_b$ constraint for $q\bar{q}\tau\nu_\tau$.

2. **Hadronic Mass** Direct measurement of the **hadronic mass**. This can be applied particularly to single-W events decaying hadronically or to the hadronic system in semi-leptonic $W^+W^-$ events (especially for $q\bar{q}\tau\nu_\tau$).

3. **Lepton Endpoints** The 2-body decay of each W leads to endpoints in the lepton (or jet) **energy** at $E_\ell = E_b(1 \pm \beta)/2$ where $\beta$ is the W velocity. These can be used to infer $m_W$. Can use for WW events with $\geq 1$ prompt lepton.

4. **Fully Leptonic Reconstruction** **Pseudomass** method (Apply 5 constraints).

5. **Threshold Scan** Measurement of the $W^+W^-$ cross-section near **threshold**. Uses all final states. Requires dedicated luminosity well below Higgs threshold and good control of background. ILC benefits from longitudinal polarization for background control. See P. Azzurri talk for more details.

Data-taking 1996–2000, with $\sqrt{s} =$ 161–209 GeV



LEP W-Boson Mass ($q\bar{q}\ell\nu_\ell$)

| Experiment | $M_W$(non-4q) [GeV] |
|---|---|
| ALEPH | $80.429 \pm 0.060$ |
| DELPHI | $80.339 \pm 0.075$ |
| L3 | $80.212 \pm 0.071$ |
| OPAL | $80.449 \pm 0.063$ |
| LEP | $80.372 \pm 0.036$ |

correl. with 4q = 0.20

LEP W-Boson Mass ($q\bar{q}q\bar{q}$)

| Experiment | $M_W$(4q) [GeV] |
|---|---|
| ALEPH | $80.475 \pm 0.080$ |
| DELPHI | $80.311 \pm 0.137$ |
| L3 | $80.325 \pm 0.080$ |
| OPAL | $80.353 \pm 0.083$ |
| LEP | $80.387 \pm 0.059$ |

correl. with non-4q = 0.20
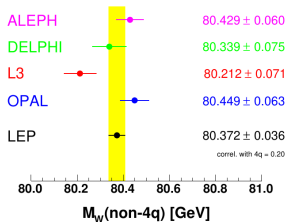
| Threshold Analysis | |
|---|---|
| Experiment | $m_W[\text{GeV}]$ |
| ALEPH | $80.20 \pm 0.34$ |
| DELPHI | $80.45^{+0.45}_{-0.41}$ |
| L3 | $80.78^{+0.48}_{-0.42}$ |
| OPAL | $80.40^{+0.46}_{-0.43}$ |

OPAL ($\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$): $80.41 \pm 0.41 \pm 0.13$ GeV

LEP W-Boson Mass ($M_W$)

| Experiment | $M_W$ [GeV] |
|---|---|
| ALEPH | $80.440 \pm 0.051$ |
| DELPHI | $80.336 \pm 0.067$ |
| L3 | $80.270 \pm 0.055$ |
| OPAL | $80.415 \pm 0.052$ |
| LEP | $80.376 \pm 0.033$ |

$\chi^2/\text{DoF} = 48.9/41$
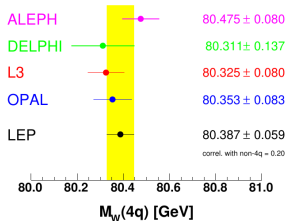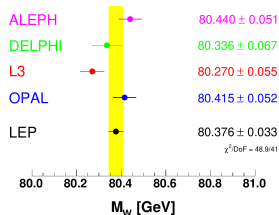
# Constrained Reconstruction of $m_W$ in WW events

$$P_s(m_W, \Gamma_W, m_{i,\text{rec}}) = S(m_W, \Gamma_W, m_i, s') \otimes ISR(s', s) \otimes R(m_i, m_{i,\text{rec}})$$

- Main LEP2 results were based on applying kinematic constraints to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$ events.
- Here 5C fit. $(E, \vec{p}) = (\sqrt{s}, \vec{0})$ and $m_{W^+} = m_{W^-}$
- OPAL used a convolution fit (CV), a reweighting MC template technique (RW) and a Breit-Wigner fit (BW). All 3 applied separately to $q\bar{q}\ell\nu_\ell$ and $q\bar{q}q\bar{q}$.
- CV fit is most powerful - uses per event resolution function.

hep-ex/0508060
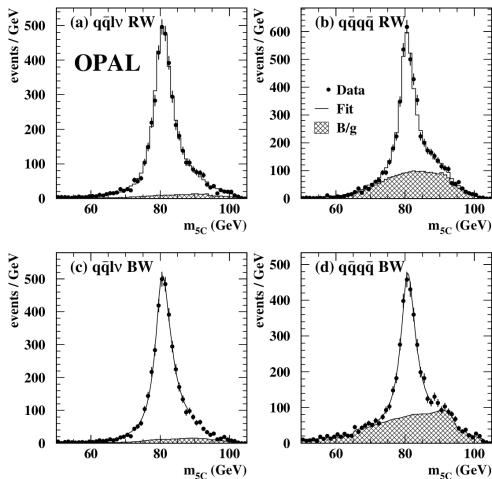
| Source | Systematic Uncertainty in MeV | | | |
|---|---|---|---|---|
| | on $m_W$ | | | on $\Gamma_W$ |
| | $q\bar{q}\ell\nu_\ell$ | $q\bar{q}q\bar{q}$ | Combined | |
| ISR/FSR | 8 | 5 | 7 | 6 |
| Hadronisation | 13 | 19 | 14 | 40 |
| Detector effects | 10 | 8 | 9 | 23 |
| LEP energy | 9 | 9 | 9 | 5 |
| Colour reconnection | – | 35 | 8 | 27 |
| Bose-Einstein Correlations | – | 7 | 2 | 3 |
| Other | 3 | 10 | 3 | 12 |
| Total systematic | 21 | 44 | 22 | 55 |
| Statistical | 30 | 40 | 25 | 63 |
| Statistical in absence of systematics | 30 | 31 | 22 | 48 |
| Total | 36 | 59 | 34 | 83 |

- $q\bar{q}q\bar{q}$ events benefit in fitted **mass resolution** from all 4 fermions being visible and detectable, but they also have **combinatorial ambiguities**.
- The color reconnection (CR) phenomenon (well established in other systems) is thought to be a severe limitation for using the $q\bar{q}q\bar{q}$ channel to progress on $m_W$ at future $e^+e^-$ colliders. LEP2 results use model with **no CR**.

## FUTURE $e^+e^-$ MEASUREMENTS OF $m_W$ ABOVE THRESHOLD

Examples will be mostly drawn from ILC.
Issues are mostly similar for other collider possibilities.

## Terra Cognita?
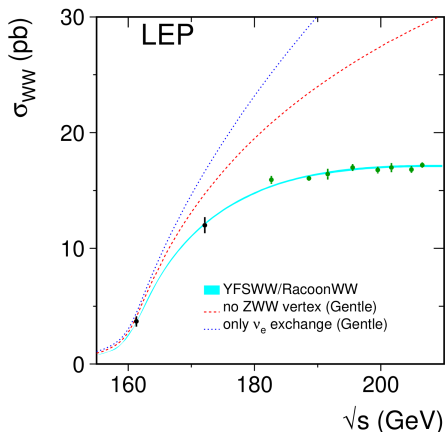


- To date only 4 time-limited years of exploration of physics substantially above the WW threshold.
- LEP2 - only about 10,000 WW events per experiment with unpolarized beams.
- Future colliders - expect $10^4 - 10^6$ times more WW events. So typical 1% precision goals of LEP2 translate to $100 - 10$ ppm in the future.
- Demands of Higgs and top physics and searches for new physics will privilege running at substantially higher energies.
- Many of the physics targets can profitably use data at a wide range of center-of-mass energies.

# (Polarized) Cross-Sections
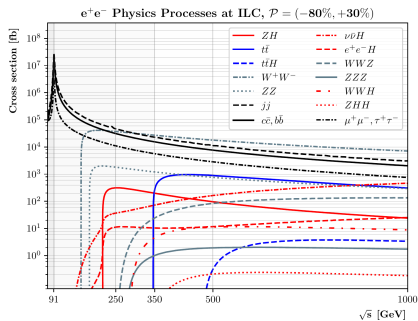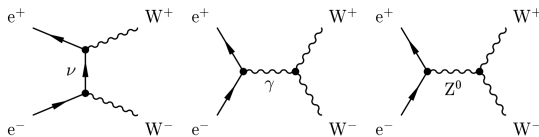


$\sigma_{WW}$ ($\sqrt{s} = 250$ GeV) $= 37$ pb
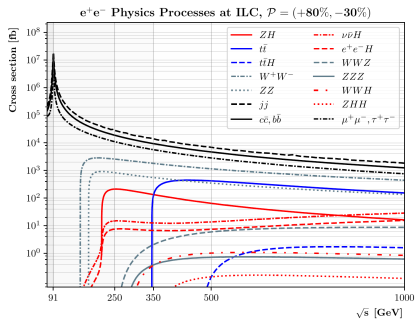
$\sigma_{WW}$ ($\sqrt{s} = 250$ GeV) $= 3$ pb

For (-80%, +30%) expect 75M W bosons per $\mathrm{ab}^{-1}$ at $\sqrt{s} = 250$ GeV.

4f final state, $ff'e^+\nu_e$ or $ff'e^-\bar{\nu}_e$ with $W \to ff'$. (CC20 diagrams for $W \to q\bar{q}$)

1   $\gamma e^+ \to u\bar{d}\bar{\nu}_e$
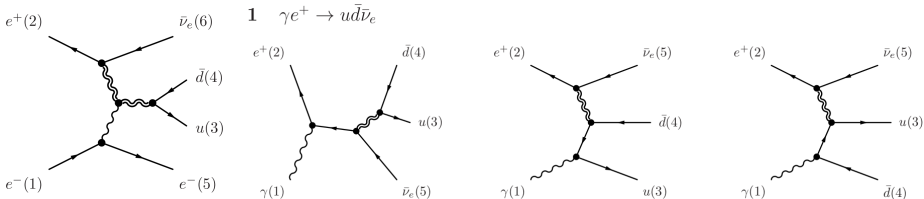


- At higher $\sqrt{s}$, opportunity to produce W and Z in t-channel processes where typically an electron has minimal $p_T$ and is undetected
- Can use hadronic W decays to reconstruct the mass
- Could use hadronic Z decays with similar kinematics for control
- Some benefit from polarization



K. Hagiwara et al. / Weak boson production
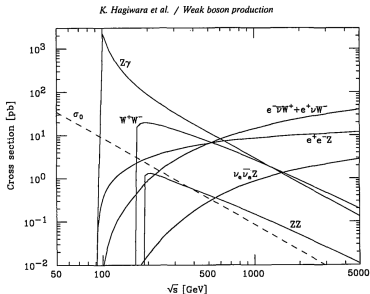
Figure 4.1: Schematic layout of the ILC in the 250 GeV staged configuration.



- $(2.0, 0.2, 4.0)$ $\mathrm{ab}^{-1}$ at $\sqrt{s} = (250, 350, 500)$ GeV
- Polarized beams (4 colliders in 1)
- Room for dedicated runs at Z $(0.1~\mathrm{ab}^{-1})$ and at WW threshold $(0.5~\mathrm{ab}^{-1})$ prior to energy upgrade (arXiv:1506.07830)
- Can upgrade to higher energies

# ILC Accelerator Parameters

See ILC paper for Snowmass for latest on ILC accelerator, detectors and physics

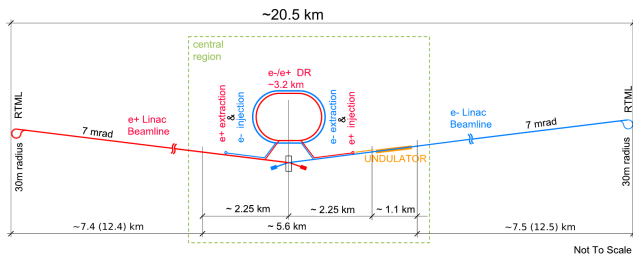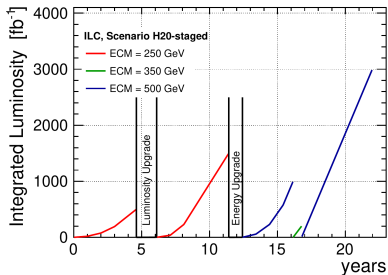| Quantity | Symbol | Unit | Initial | $\mathcal{L}$ Upgrade | Z pole | Upgrades | | |
|---|---|---|---|---|---|---|---|---|
| Centre of mass energy | $\sqrt{s}$ | GeV | 250 | 250 | 91.2 | 500 | 250 | 1000 |
| Luminosity | $\mathcal{L}$ | $10^{34}\text{cm}^{-2}\text{s}^{-1}$ | 1.35 | 2.7 | 0.21/0.41 | 1.8/3.6 | 5.4 | 5.1 |
| Polarization for $e^-/e^+$ | $P_-(P_+)$ | % | 80(30) | 80(30) | 80(30) | 80(30) | 80(30) | 80(20) |
| Repetition frequency | $f_{\text{rep}}$ | Hz | 5 | 5 | 3.7 | 5 | 10 | 4 |
| Bunches per pulse | $n_{\text{bunch}}$ | 1 | 1312 | 2625 | 1312/2625 | 1312/2625 | 2625 | 2450 |
| Bunch population | $N_e$ | $10^{10}$ | 2 | 2 | 2 | 2 | 2 | 1.74 |
| Linac bunch interval | $\Delta t_b$ | ns | 554 | 366 | 554/366 | 554/366 | 366 | 366 |
| Beam current in pulse | $I_{\text{pulse}}$ | mA | 5.8 | 8.8 | 5.8/8.8 | 5.8/8.8 | 8.8 | 7.6 |
| Beam pulse duration | $t_{\text{pulse}}$ | $\mu$s | 727 | 961 | 727/961 | 727/961 | 961 | 897 |
| Average beam power | $P_{\text{ave}}$ | MW | 5.3 | 10.5 | 1.42/2.84*) | 10.5/21 | 21 | 27.2 |
| RMS bunch length | $\sigma_z^*$ | mm | 0.3 | 0.3 | 0.41 | 0.3 | 0.3 | 0.225 |
| Norm. hor. emitt. at IP | $\gamma\epsilon_x$ | $\mu$m | 5 | 5 | 5 | 5 | 5 | 5 |
| Norm. vert. emitt. at IP | $\gamma\epsilon_y$ | nm | 35 | 35 | 35 | 35 | 35 | 30 |
| RMS hor. beam size at IP | $\sigma_x^*$ | nm | 516 | 516 | 1120 | 474 | 516 | 335 |
| RMS vert. beam size at IP | $\sigma_y^*$ | nm | 7.7 | 7.7 | 14.6 | 5.9 | 7.7 | 2.7 |
| Luminosity in top 1 % | $\mathcal{L}_{0.01}/\mathcal{L}$ | | 73 % | 73 % | 99 % | 58.3 % | 73 % | 44.5 % |
| Beamstrahlung energy loss | $\delta_{\text{BS}}$ | | 2.6 % | 2.6 % | 0.16 % | 4.5 % | 2.6 % | 10.5 % |
| Site AC power | $P_{\text{site}}$ | MW | 111 | 138 | 94/115 | 173/215 | 198 | 300 |
| Site length | $L_{\text{site}}$ | km | 20.5 | 20.5 | 20.5 | 31 | 31 | 40 |

Table 4.1: Summary table of the ILC accelerator parameters in the initial 250 GeV staged configuration and possible upgrades.

Note: $\sqrt{s}$, luminosities, polarizations, BS energy loss, power needs. Potential to run at all center-of-mass energies from 91 to 1000 GeV.

Energy recovery $e^+e^-$ colliders have received attention. Conceptual ideas include CERC (ERL boosted FCC-ee) & ERLC (Twin LC). Latest Recycling Linear Collider (ReLiC) looks very intriguing!



*Plot from ReLiC paper
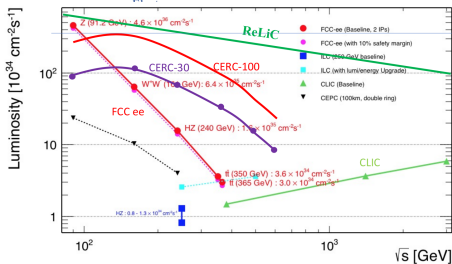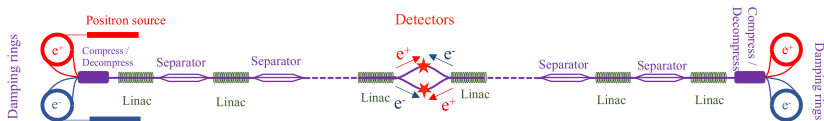
- Scope for much higher lumi and/or power savings
- Really explore HH production
- Potential for high L performance at high energy
- With polarized beams and low beamstrahlung

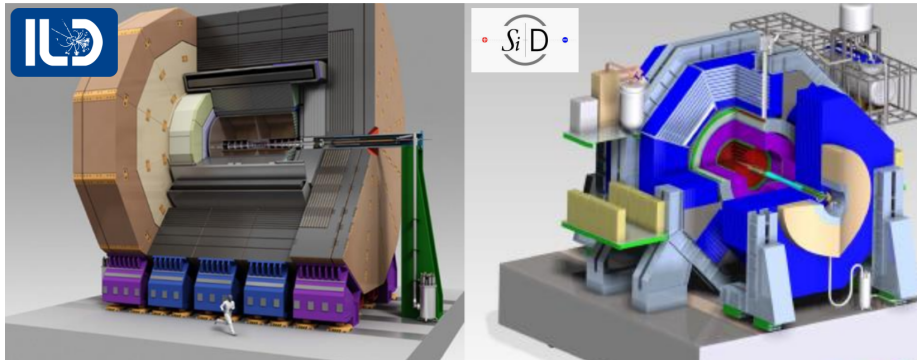Any of these machines is revolutionary compared to SLC/LEP.

# ILC Detectors

Modern detectors designed for ILC [5]

ILD = International Large Detector
(also ILD Interim Design Report (IDR) [6])

SiD = Silicon Detector



- B=3.5–5T. Particle-flow for hadronic jets. **Very hermetic**.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

# ILD Detector (See IDR)

## General Remarks
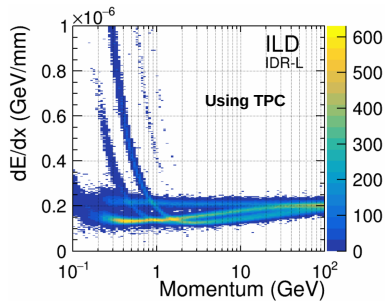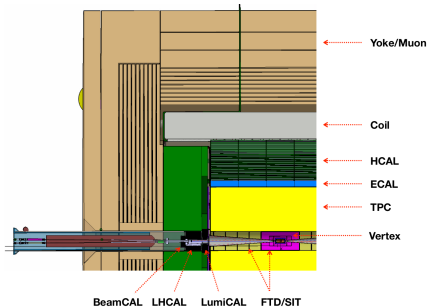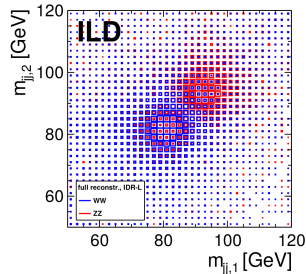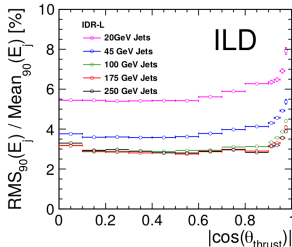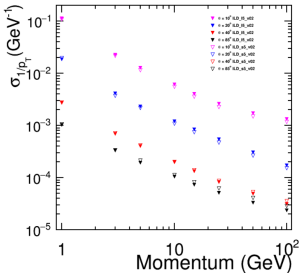
- It is not straightforward to project the performance for measurements that are probably **systematics** limited with $ab^{-1}$ data sets.
- Future $e^+e^-$ collider data sets will benefit from much **better detectors** than at LEP2, the advantages of beam **polarization** (for linear colliders) and an experimental environment conducive to precision measurement (trigger, bunch structure, hermeticity (ILC), detector material).
- Measurements of W mass, were already quite complex at LEP2. Getting to a **realistic** estimate of the eventual performance at a future $e^+e^-$ collider is not trivial.
- We can make educated guesses and identify salient issues.
- In some simpler cases, like the polarized WW threshold scan (ILC) and purely leptonic observables, we can be relatively confident of the experimental projections including systematics.

# Sensitivity to $m_{\mathrm{W}}$ at hadron and $\mathrm{e^+e^-}$ colliders

Hadron colliders rely on the $m_T(\ell, \nu)$ and $p_T(\ell)$ in leptonic decays of singly produced W bosons. In contrast, $\mathrm{e^+e^-}$ colliders can reconstruct the mass of the W boson decay products: measure directly $(m_{\mathrm{W}}, \Gamma_{\mathrm{W}})$ from the B-W lineshape.



CDF Run II
2.4M $W \to \mu\nu_\mu$ decays
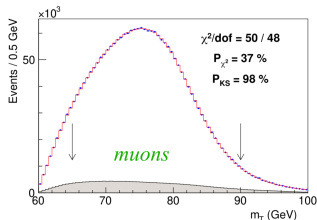
$m_{\mathrm{W}}(m_T) = 80\,446.1 \pm 9.2 \pm 7.3$ MeV
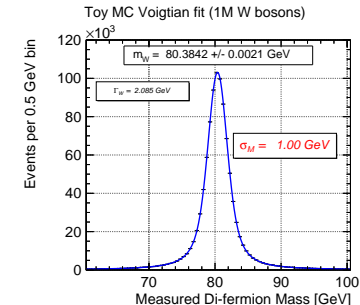
Fit with Breit-Wigner $\otimes$ Gaussian

Ultimate sensitivity of a future $\mathrm{e^+e^-}$ collider depends on the techniques, channels, mass resolution, and statistics. Could achieve the same $m_{\mathrm{W}}$ stat. sensitivity as this CDF plot with **only** 2.2% of the W decays for $\sigma_M = 1.0$ GeV (optimistic).

# Intrinsic $m_W$ Sensitivity from Lineshape



Toy MC Voigtian fit (1M W bosons)

$m_W = 80.3842$ +/- 0.0021 GeV

$\Gamma_W = 2.085$ GeV

$\sigma_M = 1.00$ GeV



Toy MC Voigtian fit (1M W bosons)

$m_W = 80.3854$ +/- 0.0036 GeV

$\Gamma_W = 2.085$ GeV

$\sigma_M = 2.50$ GeV



Toy MC Voigtian fit (1M W bosons)

$m_W = 80.3878$ +/- 0.0051 GeV

$\Gamma_W = 2.085$ GeV

$\sigma_M = 4.00$ GeV
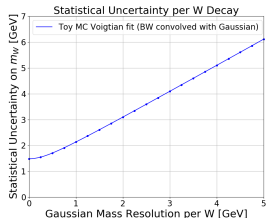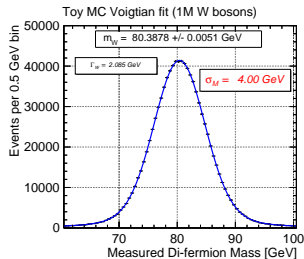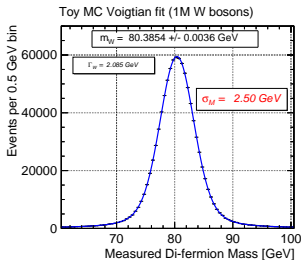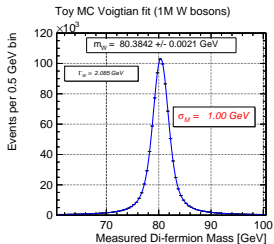


Basic sensitivity

$$\sigma_{m_W} = \frac{f(\sigma_M, \Gamma_W)}{\sqrt{N_W}}$$

We will use both:

- Per decay $m_W$ estimators ($m_{ij}$).

- Per event estimators: average mass, $\frac{1}{2}(m_{12} + m_{34})$ or $m_{5C}$ $\{N_{WW}\}$
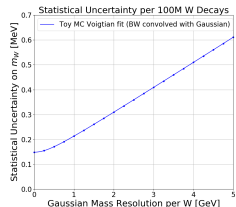


Scaled to ILC-like statistics

To a very good approximation, the distribution of the averaged mass, follows the same Breit-Wigner distribution. So apply the same curve to WW events.
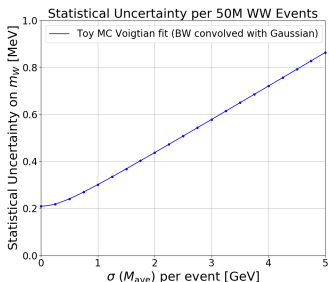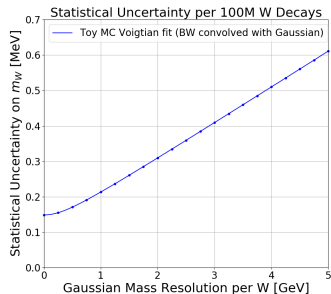




| $\sigma_M$ (GeV) | $\Delta m_W$ (MeV) | $\Delta\Gamma_W^a$ (MeV) | $\Delta\Gamma_W^b$ (MeV) |
|---|---|---|---|
| 1.0 | 0.21 | 0.41 | 0.63 |
| 2.5 | 0.35 | 0.63 | 1.0 |
| 4.0 | 0.50 | 0.89 | 1.6 |

- Fits with 100M W decays and 1, 2 or 3 parameters fitted ($m_W$, $\Gamma_W$, $\sigma_M$).
- Statistical uncertainties only. Note that individual W's and event-averaged masses will have very different resolutions (some excellent).

# Beamstrahlung

Beam-beam interaction leads to energy loss (radiated photons).
Two main issues (more important as $\sqrt{s}$ increases).

1. worsening of the validity of the kinematic constraints (similar to ISR).
2. presence of "overlay" particles from concurrent soft $\gamma\gamma$ and e$\gamma$ collisions



Whizard 3.0.3

$\sqrt{s} = 250$ GeV, e$^-$ e$^+$ → q q $\mu^-$ $\nu$, P=(-0.8,0.3)
- No ISR
- ISR
- ISR + ILC BES & Beamstrahlung



| | h31 | |
|---|---|---|
| Entries | 684993 | |
| Mean x | 124.52 | |
| Mean y | 124.519 | |
| Std Dev x | 1.04084 | |
| Std Dev y | 1.035 | |
| Integral | 564579 | |
| 0 | 0 | 0 |
| 55410 | 564579 | 0 |
| 9438 | 55566 | 0 |
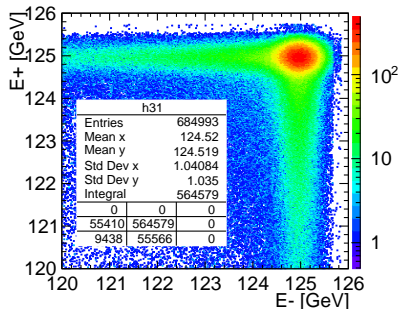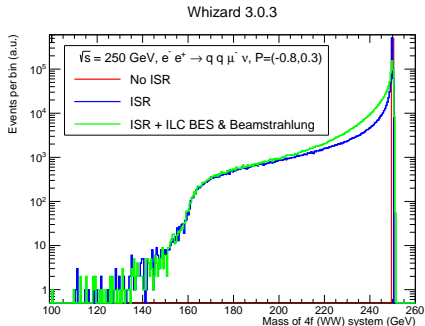
- Idealized: $< M > = 250.0$ GeV
- ISR only: $< M > = 242.9$ GeV
- ISR+BES+BS: $< M > = 240.3$ GeV

Need to use medium-angle Bhabhas and $e^+e^- \to \mu^+\mu^-$ to measure the luminosity spectrum (essentially the beam structure functions).

# $\sqrt{s_p}$ Method for Absolute Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.
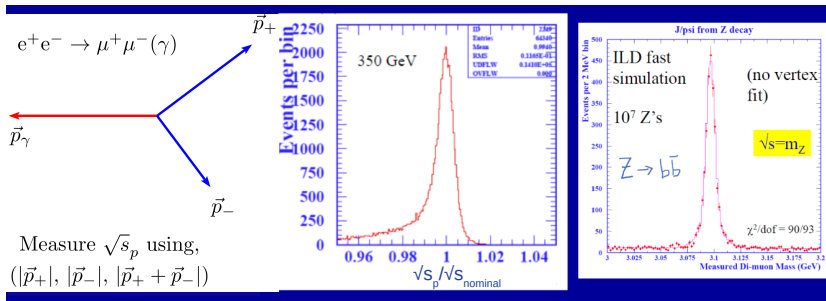


Tie detector $p$-scale to particle masses (know $J/\psi$, $\pi^+$, p to 1.9, 1.3, 0.006 ppm)

Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on $p$-scale per 1.2M $J/\psi \rightarrow \mu^+\mu^-$ ($4 \times 10^9$ hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and $e^+e^-$ (and ... 4l etc). (Links to more details in backup)
- relies on excellent modeling of QED effects (**ISR** and FSR)
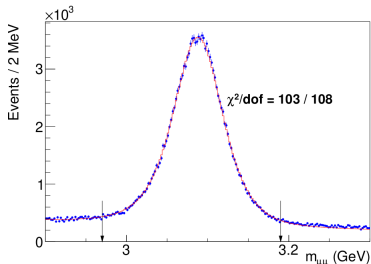
| Source | $J/\psi$ (ppm) | $\Upsilon$ (ppm) | Correlation (%) |
|---|---|---|---|
| QED | 1 | 1 | 100 |
| Magnetic field non-uniformity | 13 | 13 | 100 |
| Ionizing material correction | 11 | 8 | 100 |
| Resolution model | 10 | 1 | 100 |
| Background model | 7 | 6 | 0 |
| COT alignment correction | 4 | 8 | 0 |
| Trigger efficiency | 18 | 9 | 100 |
| Fit range | 2 | 1 | 100 |
| $\Delta p/p$ step size | 2 | 2 | 0 |
| World-average mass value | 4 | 27 | 0 |
| Total systematic | 29 | 34 | 16 ppm |
| Statistical NBC (BC) | 2 | 13(10) | 0 |
| Total | 29 | 36 | 16 ppm |
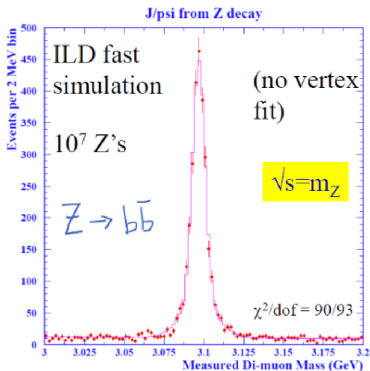
Much better mass resolution at ILC. Can measure momentum scale to 1 ppm stat.
with 4.2B hadronic Z's. Systematics should be better than CDF (eg. no trigger).
Previous "conservative" estimate of 10 ppm for ILC seems too conservative.

# Fallback approach: $\sqrt{s}_A$ Center-of-Mass Energy Method

RDP does NOT work for $E_b$ for 100km circular colliders for $\sqrt{s} \gtrsim 170$ GeV.
Use radiative return events to the Z using only precision angular measurements.

Assume one photon recoiling from $\mu^+\mu^-$

$$x_\gamma \equiv \frac{E_\gamma}{E_b} = 1 - \frac{m_{12}^2}{s}$$

At $\sqrt{s} = 250$ GeV,
$x_\gamma = 0.867$, $E_\gamma = 108$ GeV,
for $m_{12} = m_Z$.

Write $m_{12}^2/s = f(\theta_1, \theta_2)$.
Then assume, $m_{12} = m_Z$.



$e^+ e^- \to Z (\gamma) \to \mu^+\mu^-(\gamma)$
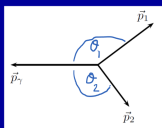
GWW – MPI 96
LEP Collabs.

Hinze & Moenig

Photon often not detected.
Use muon angles to (photon/beam-axis).
Requires precision polar angle.

$$\sqrt{s} = m_Z \sqrt{\frac{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1 + \theta_2)}{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1 + \theta_2)}}$$

Statistical error per event of order $\Gamma/M = 2.7\%$

Acceptance degrades quickly at high $\sqrt{s}$
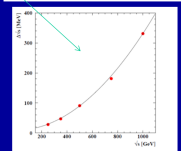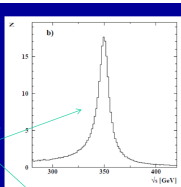
- uses $m_Z$ and is limited in ultimate precision by its knowledge (23 ppm today).
- can also use $e^+e^-$, and even $\tau^+\tau^-$ decays of the Z (maybe also $Z \to q\bar{q}$)
- per event uncertainty poor given $\Gamma_Z$.
- again need excellent ISR/FSR modeling.

Most recent study in K. Moenig talk and proceedings from LCWS05.

# Kinematic Reconstruction of $q\bar{q}\ell\nu_\ell$ Events
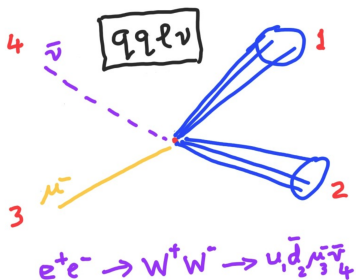


- Measure 4-vectors of jet-1, jet-2, and the charged lepton.
- Estimate measurement uncertainties for these parameters for each event.
- Adjust the measured parameters, and the 3 unmeasured parameters defining the neutrino momentum, to simultaneously minimize the fit $\chi^2$, while satisfying the imposed hard constraints.
- The constraints are 4-momentum conservation (4C / 1 dof fit) or 4-momentum conservation plus $m_{12} = m_{34}$ (mass-equality) (5C / 2 dof fit).

These result in either one fitted event mass, $M$, for 5C fits, or two fitted di-fermion masses, $m_{12}$ and $m_{34}$, for 4C fits and the fitted uncertainties and the corresponding $\chi^2$ p-values to test these hypotheses.

# Constrained Fits

Some ideas and progress

1. Photon radiation treatment in kinematic fits (M. Beckmann, B. List and J. List) arXiv:1006.0436 Applied to $q\bar{q}q\bar{q}$ at $\sqrt{s} = 500$ GeV.
2. Jet specific energy resolution studies (Wilson, IWLC 2010).
3. "ErrorFlow" studies: parametrizing jet uncertainties (A. Ebrahimi thesis)
4. Kinematic Fitting for Particle Flow Detectors at Future Higgs Factories (Y.Radkhorrami, J.List), arXiv:2111.14775
5. Kinematic reconstruction at FCC-ee* (M. Béguin thesis) - also near threshold.

- BLL - do simplified study of $q\bar{q}q\bar{q}$ reconstruction at $\sqrt{s} = 500$ GeV without "overlay".
- Shown is the average di-jet mass and its resolution (Voigtian fit).
- 4j+$\gamma$ method adds an ISR photon as an additional "measured" object with large error
- Estimate 1.35 GeV mass resolution for 52% of events.
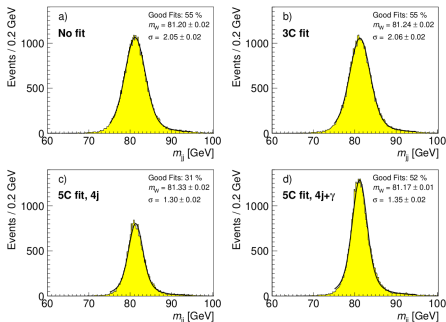
# Toy study of constrained fitting for $q\bar{q}\ell\nu_\ell$ (ILC250)

- Looked at $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ events generated with Whizard 3.0.3.
- 3 configurations examined: no ISR, ISR only, ISR + ILC-BES&BS
- Used jet energy and angular resolution parametrization from D. Ward and W. Yan (from 2009). Neglected jet masses. $m_{had}$ resolution $\approx 2.4$ GeV.
- Used APLCON (V. Blobel) implementation
- Treat neutrino as unmeasured. Both 4C and 5C fits (1 dof & 2 dof).
- Method works perfectly with no ISR.
- Lots of room for improvement by using event-by-event fitted uncertainties.
- Issues with BLL photon method – may not work for $q\bar{q}\ell\nu_\ell$? (less constraints)
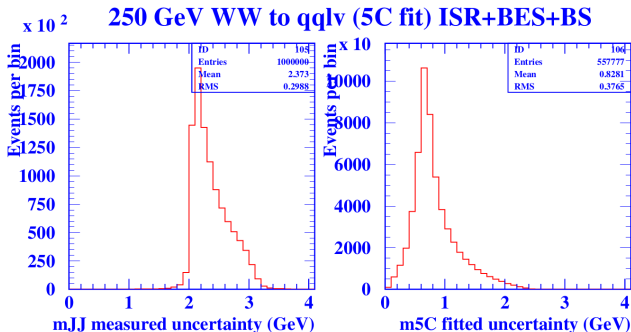


250 GeV WW to qqlv (5C fit) ISR+BES+BS

Successful fits defined as converging and having $p_{\text{fit}} > 0.02$
(Residual $= m_{\text{estimate}} - m_{\text{generator}}$)



250 GeV WW to qqlv (4C fit) ISR only
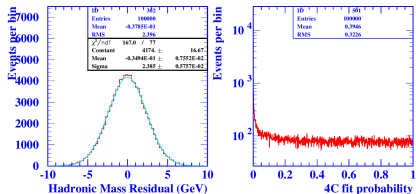


250 GeV WW to qqlv (5C fit) ISR only
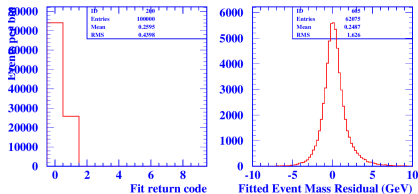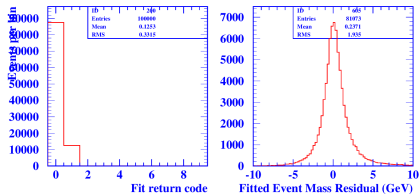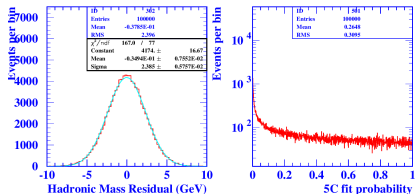
$\varepsilon_{\text{fit}} = 81\%$, "$\sigma$"=1.94 GeV

$\varepsilon_{\text{fit}} = 62\%$, "$\sigma$"=1.63 GeV

Successful fits defined as converging and having $p_{\text{fit}} > 0.02$
(Residual = $m_{\text{estimate}} - m_{\text{generator}}$)
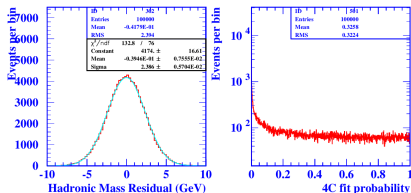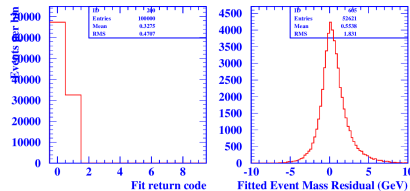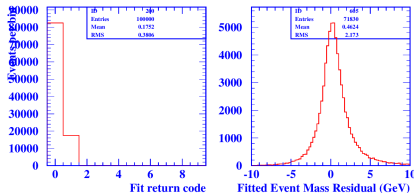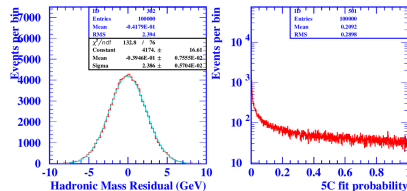


$\varepsilon_{\text{fit}} = 72\%$, "$\sigma$"=2.17 GeV          $\varepsilon_{\text{fit}} = 55\%$, "$\sigma$"=1.83 GeV

*On average*, the fit does not appear to improve much over the hadronic mass
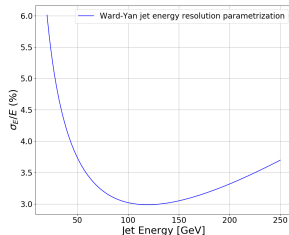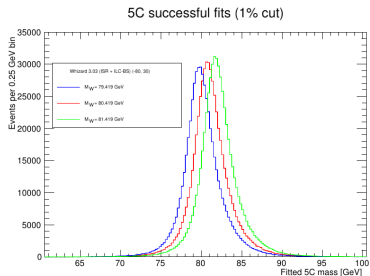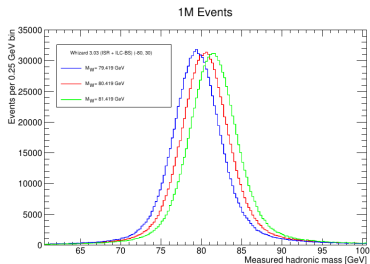
Scaled to ILC 250 scenario of 2 $\mathrm{ab}^{-1}$ 45/45/5/5 ILC scenario (11M WW). Template fit: constrained fit uncertainties not yet used, background neglected.

| Case | $\Delta m_{\mathrm{W}}^{\mathrm{stat}}$ |
|---|---|
| $m_{\mathrm{had}}$ | 1.04 MeV |
| 5C mass (fit OK) | 0.87 MeV |
| 5C mass / $m_{\mathrm{had}}$ | **0.79** MeV |

# Jet Specific Energy Resolution

- Great deal of potential for better energy resolution knowledge for each idiosyncratic jet.

- PFA response depends on charged/EM/neutral hadron (NH) fractions.

- Trivially based on NHs (see examples)

- But also by for example fitting $\pi^0 \to \gamma\gamma$

Full sim+reco study (from IWLC 2010)



Here, the events are divided into 3 distinct classes based on reconstructed neutral hadron energy.
A) ENH < 2 GeV
B) 2 < ENH < 10 GeV
C) ENH > 10 GeV.
Significant differences based on MEASURED quantity: can be exploited immediately. Rather Gaussian distributions.

rms90 values for |cosθ| < 0.7 and 45 GeV jets

uu, dd, ss at 91 GeV

2.36±0.02% (16%/√E) A)

uu, dd, ss at 91 GeV

3.45±0.02% (23%/√E) B)

uu, dd, ss at 91 GeV

4.76±0.03% (32%/√E) C)
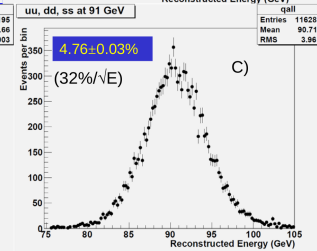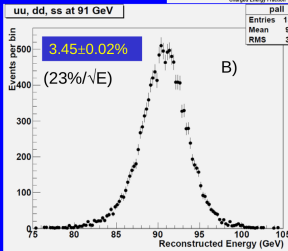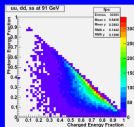
Use better jet energy resolution knowledge and fit uncertainties to improve $m_W$.

W→ qq Gen. Mass Difference





Sensitivity to $m_W$ with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**
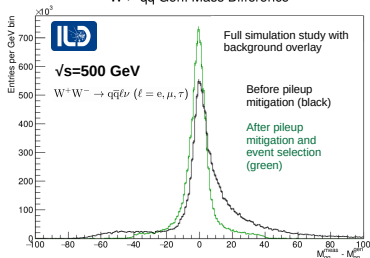
- **Hadronic mass** study, J. Anguiano (KU).

- Stat. $\Delta m_W = 2.4$ MeV for $1.6$ ab$^{-1}$ (-80%, +30%).

- Can be improved, but $m_{had}$-only measurement likely limited by JES systematic

- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set
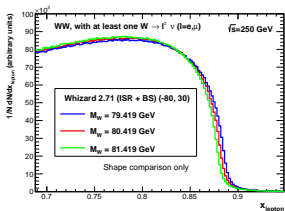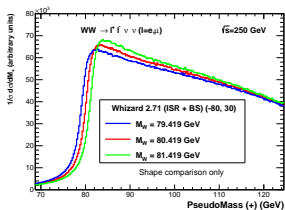
- Stat. $\Delta m_W = 4.4$ MeV for 2 ab$^{-1}$ (45,45,5,5) at $\sqrt{s} = 250$ GeV

- **Leptonic observables** (shape-only): $M_+$, $M_-$, $x_\ell \equiv E_\ell/E_b$ . Exptl. systematics small.

# $m_{\mathrm{W}}$ Measurement Using Leptons

One complementary method for measuring $\mathrm{M_W}$ at LEP was the measurement by OPAL (hep-ex/020326) using $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$ events. Results were modest. Limited by the integrated luminosity of 0.67 fb$^{-1}$ (unpolarized), and the poor momentum resolution ($\Delta p/p$). ILC will be much better for L, P and $\Delta p/p$. Disadvantages: higher $\sqrt{s}$ and beamstrahlung.

Method uses lepton $\vec{p}$ measurement:

- The prompt (e, $\mu$)-lepton energy spectrum in ee, $\mu\mu$, e$\mu$, e$\tau$, $\mu\tau$ events with endpoints at $E_\pm = \frac{1}{2}\, E_{\mathrm{b}}(1 \pm \beta)$. Can also apply to $q\bar{q}e\nu_e$ and $q\bar{q}\mu\nu_\mu$.
- The positive pseudo-mass ($M_+$) solution in ee, $\mu\mu$, e$\mu$ events.

Latter assumes 4-momentum conservation, equal (l-$\nu$) masses, and guesses that the neutrinos are in the same plane as the di-lepton.

$$
\begin{aligned}
M_\pm^2 \;=\; & \frac{2}{|\vec{p}_\ell + \vec{p}_{\ell'}|^2}\Big((P\,\vec{p}_{\ell'} - Q\,\vec{p}_\ell)\cdot(\vec{p}_\ell + \vec{p}_{\ell'}) \\
& \pm\sqrt{|\vec{p}_\ell \times \vec{p}_{\ell'}|^2[|\vec{p}_\ell + \vec{p}_{\ell'}|^2(E_{\mathrm{b}} - E_\ell)^2 - (P+Q)^2]}\Big),
\end{aligned}
\tag{1}
$$

where

$$
P = E_{\mathrm{b}}E_\ell - E_\ell^2 + \frac{1}{2}m_\ell^2, \qquad Q = -E_{\mathrm{b}}E_{\ell'} - \vec{p}_\ell\cdot\vec{p}_{\ell'} + \frac{1}{2}m_{\ell'}^2.
$$
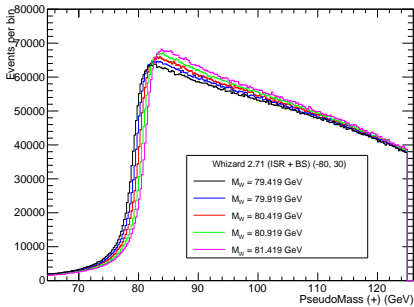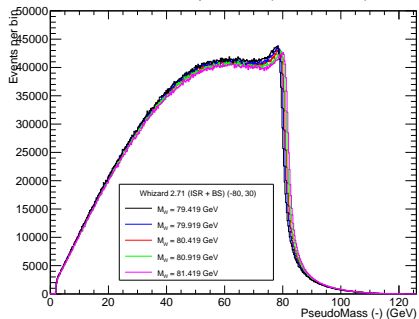
- Study just uses changes in the shape. The total cross sections should be relatively insensitive to $m_W$ well above threshold (depends on SM parameter scheme implementation though ....).
- Plots are at generator level (no detector smearing).
- Find that **both** pseudomasses are sensitive to $m_W$.

$\sqrt{s}$=250 GeV. $\mu^- \nu \tau^+ \nu$ (Whizard SM)
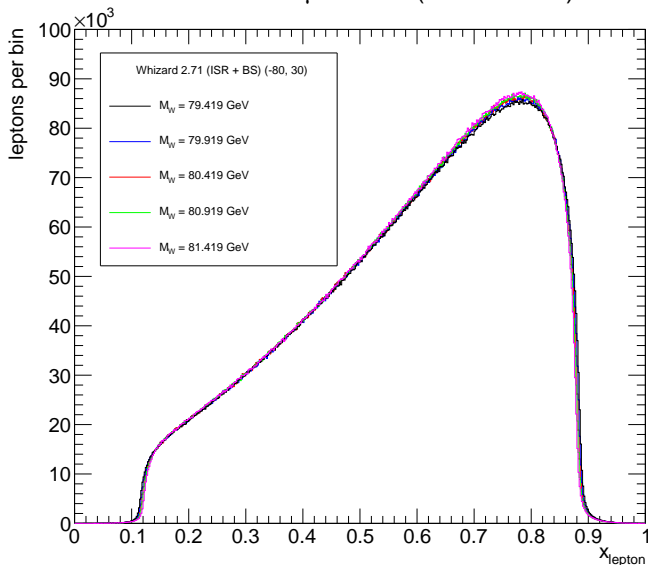
# Estimated $m_W$ statistical uncertainties from leptons

Use 2.0 ab$^{-1}$ with all beam polarizations (45%/45%/5%/5%) at generator level at $\sqrt{s} = 250$ GeV incl. beamstrahlung. Detector resolution neglected ($\sigma \ll \Gamma_W$). Estimates based on ensemble test fits.

1. $M_+$: 1.50M prompt dilepton events = 8.8 MeV
2. $M_-$: 1.50M prompt dilepton events = 11.2 MeV
3. Pseudomasses combined: 1.50M prompt dilepton events = 6.9 MeV (assuming uncorrelated)
4. Endpoints: 4.50M leptons (from dileptons)= 11.0 MeV
5. Combined: Fully leptonic (M and endpoints) = 5.9 MeV (neglects possible correlation (+11% in OPAL case))
6. Semi-leptonic endpoints (12.6M leptons) = 6.6 MeV
7. Grand total = 4.4 MeV

Fully hadronic channel has huge statistical power, but thought plagued by color reconnection (CR) systematics.

Christiansen and Sjöstrand (arXiv:1506.09085) show that CR effects could be diagnosed using W mass measurements at various $\sqrt{s}$.

**Table 2** Systematic W mass shifts at center-of-mass energies of 240 and 350 GeV, respectively. The $\langle \delta \overline{m}_W \rangle$ is the mass shift in the CR models relative to the no-CR result. The Monte Carlo statistical uncertainty is 5 MeV

| Method | $\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm}$ = 240 GeV) | | | | | | |
|---|---|---|---|---|---|---|---|
| | SK-I | SK-II | SK-II$'$ | GM-I | GM-II | GM-III | CS |
| 1 | +95 | +29 | +25 | −74 | +400 | +104 | +9 |
| 2 | +87 | +26 | +24 | −68 | +369 | +93 | +8 |
| 3 | +95 | +30 | +26 | −72 | +402 | +105 | +10 |
| Method | $\langle \delta \overline{m}_W \rangle$ (MeV) ($E_{cm}$ = 350 GeV) | | | | | | |
| | SK-I | SK-II | SK-II$'$ | GM-I | GM-II | GM-III | CS |
| 1 | +72 | +18 | +16 | −50 | +369 | +60 | +4 |
| 2 | +70 | +18 | +15 | −50 | +369 | +60 | +4 |
| 3 | +71 | +18 | +16 | −50 | +369 | +60 | +3 |

But this is not really at all well established and very model dependent.
Note that jet reconstruction in the 4q channel normally tries to reduce the potential size of such effects

# Updated $m_W$ Prospects Tables

## 1: Polarized threshold scan

| $\Delta M_W$ [MeV] | LEP2 | ILC | ILC | ILC |
|---|---|---|---|---|
| $\sqrt{s}$ [GeV] | 161 | 161 | 161 | 161 |
| $\mathcal{L}$ [fb$^{-1}$] | 0.040 | 100 | 480 | 500 |
| $P(e^-)$ [%] | 0 | 90 | 90 | 80 |
| $P(e^+)$ [%] | 0 | 60 | 60 | 30 |
| statistics | 200 | 2.4 | 1.1 | |
| background | | 2.0 | 0.9 | |
| efficiency | | 1.2 | 0.9 | |
| luminosity | | 1.8 | 1.2 | |
| polarization | | 0.9 | 0.4 | |
| systematics | 70 | 3.0 | 1.6 | |
| experimental total | 210 | 3.9 | 1.9 | 3.0 |
| beam energy | 13 | 0.4 | 0.4 | 0.4 |
| theory | - | 1.0 | 1.0 | 1.0 |
| total | 210 | 4.0 | 2.2 | 3.2 |

Table 10: Current and preliminary anticipated uncertainties in the measurement of $M_W$ at $e^+e^-$ colliders close to $WW$ threshold.

## 2: $q\bar{q}\ell\nu_\ell$

| $\Delta M_W$ [MeV] | LEP2 | ILC | ILC | ILC |
|---|---|---|---|---|
| $\sqrt{s}$ [GeV] | 172-209 | 250 | 350 | 500 |
| $\mathcal{L}$ [fb$^{-1}$] | 3.0 | 2000 | 200 | 4000 |
| $P(e^-)$ [%] | 0 | 80 | 80 | 80 |
| $P(e^+)$ [%] | 0 | 30 | 30 | 30 |
| beam energy | 9 | 0.4 | 0.55 | 0.8 |
| luminosity spectrum | N/A | 1.0 | 1.4 | 2.0 |
| hadronization | 13 | 1.3 | 1.3 | 1.3 |
| radiative corrections | 8 | 1.2 | 1.5 | 1.8 |
| detector effects | 10 | 1.0 | 1.0 | 1.0 |
| other systematics | 3 | 0.3 | 0.3 | 0.3 |
| total systematics | 21 | 2.3 | 2.7 | 3.3 |
| statistical | 30 | 0.75 | 2.8 | 0.9 |
| total | 36 | 2.4 | 3.9 | 3.4 |

Table 6: Current and preliminary estimated experimental uncertainties in the measurement of $M_W$ at $e^+e^-$ colliders from kinematic reconstruction in the $q\bar{q}\ell\nu_\ell$ channel with $\ell = e, \mu$.

- Changes wrt Snowmass 2013
- Update with current ILC run plan integrated luminosities
- Halve beam energy uncertainty (10 ppm $\rightarrow$ 5 ppm)
- Include guessed theory uncertainty in threshold total

## 3: Hadronic mass

| $\Delta M_W$ [MeV] | ILC | ILC | ILC | ILC |
|---|---|---|---|---|
| $\sqrt{s}$ [GeV] | 250 | 350 | 500 | 1000 |
| $\mathcal{L}$ [fb$^{-1}$] | 2000 | 200 | 4000 | 2000 |
| $P(e^-)$ [%] | 80 | 80 | 80 | 80 |
| $P(e^+)$ [%] | 30 | 30 | 30 | 30 |
| jet energy scale | 3.0 | 3.0 | 3.0 | 3.0 |
| hadronization | 1.5 | 1.5 | 1.5 | 1.5 |
| pileup | 0.5 | 0.7 | 1.0 | 2.0 |
| total systematics | 3.4 | 3.4 | 3.5 | 3.9 |
| statistical | 0.75 | 2.0 | 0.5 | 0.5 |
| total | 3.5 | 4.0 | 3.5 | 3.9 |

Table 8: Preliminary estimated experimental uncertainties in the measurement of $M_W$ at $e^+e^-$ colliders from direct reconstruction of the hadronic mass in single-W and WW events where one W decays hadronically. Does not include WW with $q\bar{q}\ell\nu_\ell$ where $\ell = e, \mu$.

Workshop thought: what about using $E_\gamma$ in $WW\gamma$ for $m_W$? $E_\gamma^{\mathrm{thresh}}$ is 65.3/72.5 GeV at 240/250 GeV. Need $E_\gamma^{\mathrm{thresh}}$ scale to 9 ppm for 1 MeV systematic - challenging!

# Triple Gauge Couplings Introduction

In general WW$\gamma$ and WWZ coupling described by 14 independent complex couplings. See Hagiwara et al (1987) for details.

- LEP2 analyses focused on 3 couplings (assumed real): $g_1^Z$, $\lambda_\gamma$ and $\kappa_\gamma$.
- Main sensitivity from WW. Mostly $q\bar{q}\ell\nu_\ell$ but also $q\bar{q}q\bar{q}$ and $\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$.
- $q\bar{q}\ell\nu_\ell$ has the advantage that except for the jet-charge ambiguity, all 5 angles can be determined.
- Requires fully differential measurements.



**ALEPH**
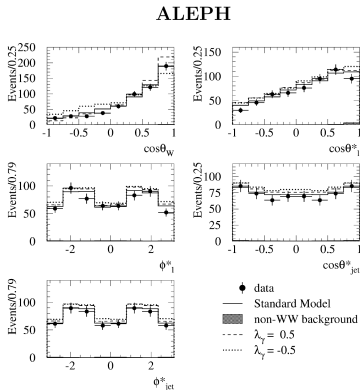
WW $\to q\bar{q}\ell\nu_\ell$ ($\ell = e, \mu$ channels)

# Triple Gauge Couplings
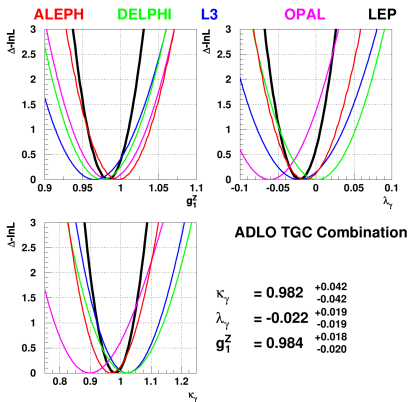


- LEP2 - only about 10,000 WW events per experiment with unpolarized beams.
- Future colliders - expect $10^4 - 10^6$ times more WW events. So typical few% precision of LEP2 translates to few $10^{-4}$ in the future.
- Higher $\sqrt{s}$ and polarization very helpful.
- In addition to WW, TGC-induced single W, $\gamma$, Z relevant too.
- ILC studies emphasize simultaneous measurement of beam polarization and TGC parameters using several processes.

**ADLO TGC Combination**

$\kappa_\gamma$ = 0.982 $^{+0.042}_{-0.042}$

$\lambda_\gamma$ = -0.022 $^{+0.019}_{-0.019}$

$g_1^Z$ = 0.984 $^{+0.018}_{-0.020}$
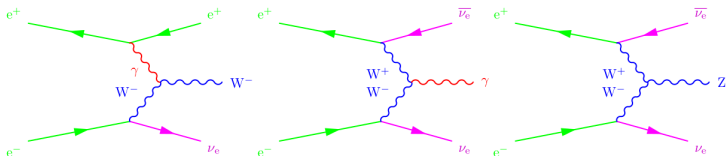
# ILC TGC (and Polarization) Study

See Robert Karl's thesis (and its appendices) for more details.
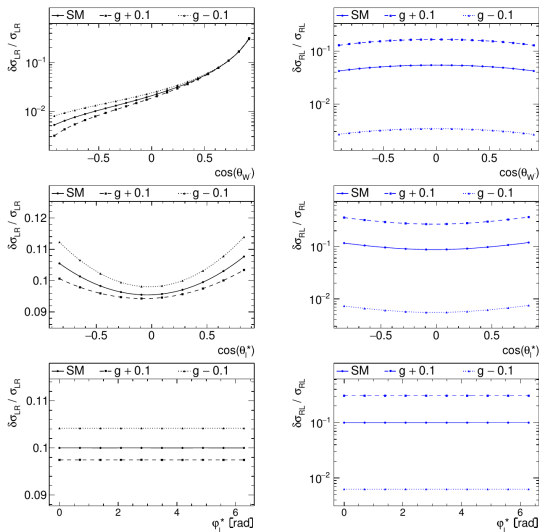


- Example with $g_1^Z$ sensitivity
- $q\bar{q}\ell\nu_\ell$ with $\ell = \mu$ for ILC250
- Clearly different chiral cross-sections show different sensitivity.
- Note $\sigma_{LR} \gg \sigma_{RL}$

# ILC TGC (and Polarization) Study

$q\bar{q}\ell\nu_\ell$ with $\ell = e^+$ for ILC250 (Note also have $\ell = e^-$ events)
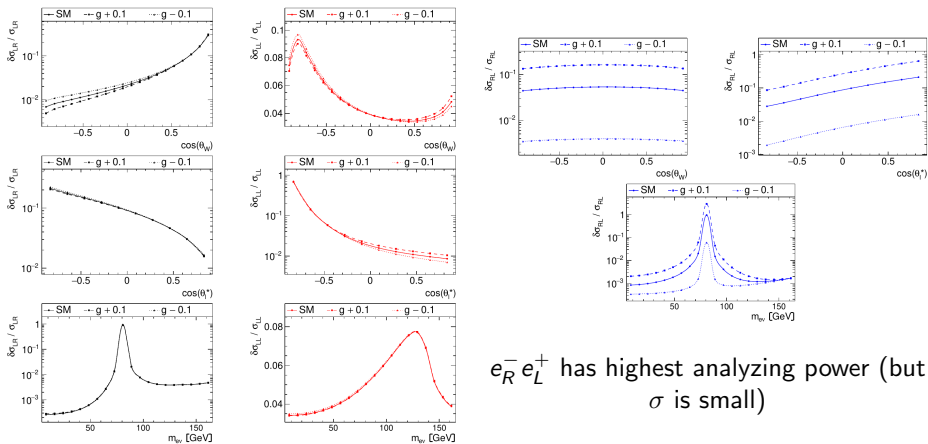


$e_R^- e_L^+$ has highest analyzing power (but $\sigma$ is small)

# ILC TGC projections

| | $E_{CMS}$[GeV] | $e^+e^- \to \mu\nu q\bar{q}$ | $e^+e^- \to e\nu q\bar{q}$ | comb. |
|---|---|---|---|---|
| $\Delta P_{e^-}^-$ [$10^{-4}$] | 250 | 9.8 | 9.67 | 9.46 |
| | 500 | 16.5 | 13.1 | 12.7 |
| $\Delta P_{e^-}^+$ [$10^{-4}$] | 250 | 4.83 | 4.59 | 4.03 |
| | 500 | 7.17 | 5.83 | 5.08 |
| $\Delta P_{e^+}^-$ [$10^{-4}$] | 250 | 4.03 | 4.02 | 3.96 |
| | 500 | 6.95 | 6.58 | 6.37 |
| $\Delta P_{e^+}^+$ [$10^{-4}$] | 250 | 5.94 | 5.87 | 5.79 |
| | 500 | 9.97 | 8.38 | 8.22 |
| TGC | $E_{CMS}$[GeV] | $e^+e^- \to \mu\nu q\bar{q}$ | $e^+e^- \to e\nu q\bar{q}$ | comb. |
| $\Delta g$ [$10^{-4}$] | 250 | 36.6 | 15.3 | 13.2 |
| | 500 | 6.95 | 3.98 | 3.38 |
| $\Delta \kappa$ [$10^{-4}$] | 250 | 45.1 | 18.6 | 15.8 |
| | 500 | 6.85 | 4.54 | 3.58 |
| $\Delta \lambda$ [$10^{-4}$] | 250 | 61.4 | 22.4 | 21.1 |
| | 500 | 15.4 | 6.09 | 5.73 |

**Table 10.11:** *Polarisation and aTGC precisions as obtained from the $\mu\nu q\bar{q}$ and $e\nu q\bar{q}$ final state individually and combined. The $\mu\mu q\bar{q}$ and $f\bar{f}$ final state are additionally included for each measurement. Results are given for 250 GeV and 500 GeV for 1 ab$^{-1}$ with a sharing of 45%(40%) for the opposite-sign and 5%(10%) for the like-sign configurations at 250(500) GeV, respectively. A global selection efficiency of 60% and a global purity of 80% is considered but no uncertainties on all three quantities are taken into account.*

- Uses also $\mu^+\mu^-$q$\bar{q}$ and f$\bar{f}$ final states (for poln.)
- Standard ILC running assumptions are a factor of 2 (250) and 4 (500) higher integrated luminosities.
- So combined uncertainties of (1.7, 1.8, 2.8) $\times 10^{-4}$ for ($g_1^Z$, $\kappa_\gamma$ and $\lambda_\gamma$) respectively.
- $q\bar{q}\ell\nu_\ell$ with $\ell = e^\pm$ VERY IMPORTANT.
- Study relies on accurate modeling of the single-W process – may be theoretically and experimentally challenging.
- Expect further improvement with OO and additional channels/variables.

# Two-Fermion Processes

2f is very important for several reasons

1. High cross-section
2. Probe for new physics at highest center-of-mass energy $e^+e^-, \mu^+\mu^-, \tau\tau, q\overline{q}$
3. Radiative return process, $e^+e^- \to Z\gamma$.
4. Also $e^+e^- \to \nu\bar{\nu}\gamma$ for radiative neutrino counting etc.
5. Need excellent modeling for these important backgrounds.
6. Use for in situ beam polarization measurement
7. Use for in situ center-of-mass energy calibration and luminosity spectrum determination
8. Use Bhabhas and $e^+e^- \to \gamma\gamma$ for relative and absolute luminosity.
9. $e^+e^- \to q\overline{q}$ and WW statistics of order $10^8$ or more, so could benefit from luminosity to 0.01% or even better.

# Multi-Fermion Processes

- Most of the WW like physics requires 4f generators.
- Many signals and backgrounds likely need to be done 4f final-state by 4f final-state.
- A pet peeve / your homework assignment. Can you use your favorite general purpose event generator to estimate how well one could measure $m_{\mathrm{W}}$ using cross-sections and polarization asymmetries vs $\sqrt{s}$? (Usual answer No – scheme dependence: $m_{\mathrm{W}}$ may be both a kinematic parameter and a coupling parameter).
- The $q\bar{q}\ell\nu_\ell$ channel with $\ell = e$ is very important. But also needs care at low $q^2$. Much specialized work at LEP2 - but need translation to usable tools.

Unpolarized cross-sections
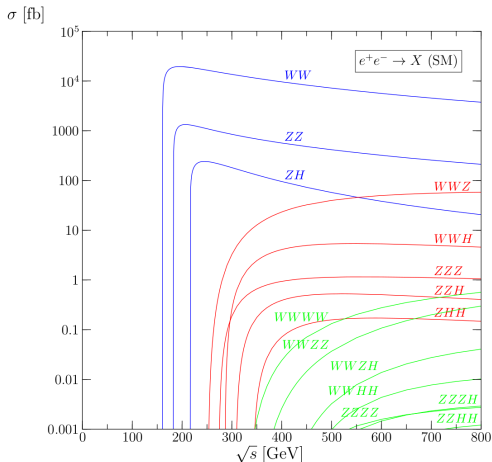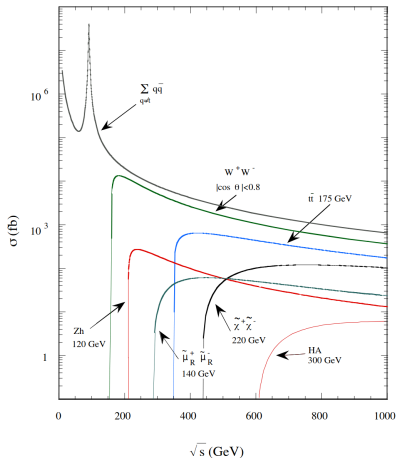


**Study electroweak physics with 2f, 4f, 6f, 8f.** Ranging from the Z-pole to $\sqrt{s}$ conceivably around 1 TeV. **Expect ZH, WWZ, WWH, ZZZ, ZZH, ZHH, tt, WWWW, WWZZ, ttH within reach of 500 GeV collider.**

# Theory/Modeling Wish List

(From an experimentalist with an ILC orientation and LEP experience)
All calculations/MC generators ideally should

1. include beam polarization (in principle arbitrary orientations)
2. include beamstrahlung (more relevant to MC generator)
3. publish standardized predictions for realistic observables

Physics:

1. ISR modeling especially ISR photons with $p_T$
2. HO effects likely to affect distributions -eg. FSR, electroweak radiation, low-mass resonance emission
3. treatment of tau polarization
4. pesky specialized processes that need some care: examples two-photon interactions, special Bhabha topologies (TEEGG-like)
5. For linear colliders - good model for "overlay" events. (likely to be measured in data though).
6. Advances in hadronization modeling. Future $e^+e^-$ collider data likely to impact this a lot.
7. I suspect the $q\bar{q}\ell\nu_\ell$ channel with $\ell = e$ is of special importance.

Aside - we experimentalists really need to demonstrate that the electron detector performance is not too degraded compared with muons.

# Summary

- Several methods to measure the W mass with precision of a few MeV.

- Systematics are to some extent complementary.

- Estimate overall experimental uncertainty of 2.0 MeV for ILC while not neglecting systematics. This could be reduced further to about 1.5 MeV combined with dedicated $0.5 \ \mathrm{ab}^{-1}$ run at threshold.

- Constrained reconstruction - very promising - but needs more detailed study.

- Experimental strategies for controlling systematics associated with $\sqrt{s}$, polarization, luminosity spectrum are worked out.

- Momentum scale is key for $\mu^+\mu^-$ based measurements of $\sqrt{s}$ in collision. Enabled by precision low material tracker. Can also open up precision polarized Z-pole running program for ILC

- Semi-leptonic ($q\bar{q}\ell\nu_\ell$) events are a key element to measuring $m_{\mathrm{W}}$, TGCs, and beam polarization and likely lead to stringent demands on modeling accuracy.

- Two-fermion events are very important too.

- An accelerator is needed. Let's work together to make it possible.

# Recent studies related to $\sqrt{s}_p$ method

- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $K^0_S$, $\Lambda$, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $K^0_S$, $\Lambda$ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or $J/\psi$ statistics).

### Recently,

- Several talks on $\sqrt{s}_p$ and $\sqrt{s}$ issues. Latest ones, ILCX, ILC-WG3 and ILC-MDI
- Includes a more careful look at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$. Include crossing angle, full simulation and reconstruction with ILD, track error matrices, vertex fitting, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- Also a look at colliding beam-energy/interaction-vertex correlations and more of a focus on $dL/d\sqrt{s}$ issues.
- Prospects for Z lineshape with a polarized scan including energy systematics.

# $m_W$ from cross-section close to threshold



$\sigma_t \sim \beta$   $\sigma_s \sim \beta^3$

$$\Delta M_{sys}^{bkgd} = 470 \text{ MeV} \left[ \frac{\Delta \sigma}{1 \text{ pb}} \right]$$

$$\Delta M_{\mathrm{stat}} = \left| \frac{\mathrm{d}\sigma}{\mathrm{d}M} \right|^{-1} \Delta\sigma = \left| \frac{\mathrm{d}\sigma}{\mathrm{d}M} \right|^{-1} \sqrt{\frac{\sigma}{\varepsilon\mathcal{L}}} = \frac{K}{\sqrt{\varepsilon\mathcal{L}}}$$



**W Mass Sensitivity**

No beam-strahlung (0,0)
With ILC beam-strahlung (0,0)
With BS (-80%,0)
With BS (-80%,+30%)
With BS (-90%,+60%)
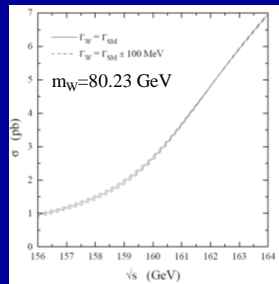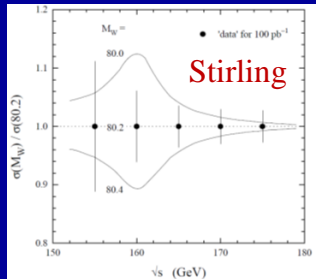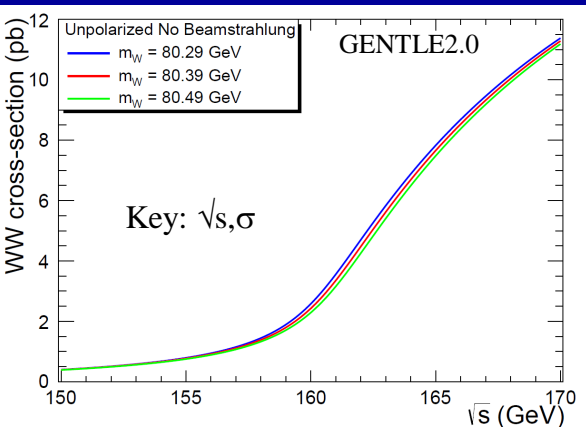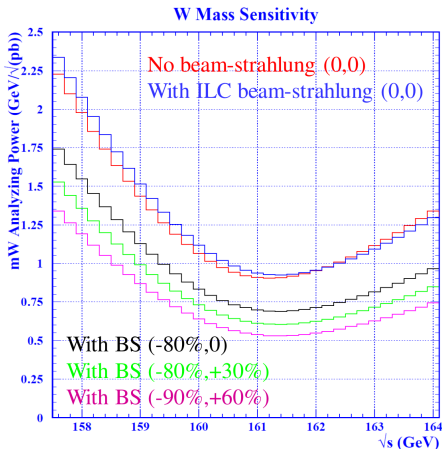
- Following Stirling, Nucl. Phys. B456 (1995) 3
- Plot shows $K = \sqrt{\sigma} \left| \frac{\mathrm{d}\sigma}{\mathrm{d}M} \right|^{-1}$
- For $\varepsilon = 100\%$, $\mathcal{L} = 100$ fb$^{-1}$ and (-80%, +30%) polarizations, find $\Delta M_{\mathrm{stat}} = 1.9$ MeV at the optimum
- Polarization of $\mathrm{e}^-$ and $\mathrm{e}^+$ beams at ILC (necessarily with beamstrahlung) offers **much** better sensitivity per unit of integrated luminosity than the LEP-like unpolarized case

# ILC Polarized Threshold Scan
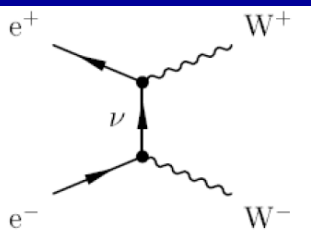


Use (-+) helicity combination of e⁻ and e⁺ to enhance WW.

Use (+-) helicity to suppress WW and measure background.

Use (--) and (++) to control polarization (also use 150 pb Z-like events)

**GENTLE 2.0 with ILC 161 beamstrahlung\***

Each set of curves has $m_W$ = 80.29, 80.39, 80.49 GeV.

With |P| = 90% for e⁻ and |P| = 60% for e⁺.

Example 6 points in √s.
78% (-+),
17% (+-)
2.5% (--),
2.5% (++)

**Need 10 ppm error on √s to target 2 MeV on mW**
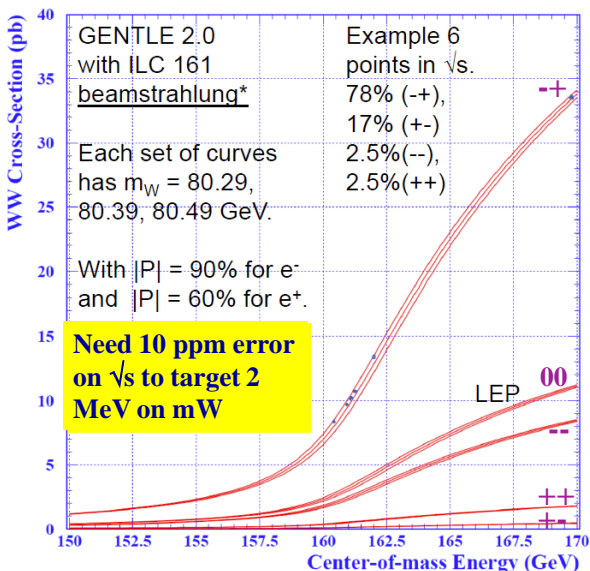
Experimentally very robust. Measure pol., bkg. in situ

# ILC Polarized Scan Counting Experiment

Example: 6 point scan (index i), (90% e-, 60% e+ polarization) with -+, +-, ++ and - - helicity combinations (index k)
Count events in 3 WW candidate categories (lvlv, qqlv, qqqq – index j) with expectation $\mu_{ijk}$ and one Z-like category (radiative return and f fbar) with expectation $\nu_{ik}$.

**96 event counts**

Data could also be taken with other helicity combinations (00, -0,+0,0-,0+ ) if warranted. (eg. further checks of polarization model)

| $\sqrt{s}$ (GeV) | L (fb$^{-1}$) | $f$ | $\lambda_{e-} \cdot \lambda_{e+}$ | $N_{ll}$ | $N_{lh}$ | $N_{hh}$ | $N_{RR}$ |
|---|---|---|---|---|---|---|---|
| 160.6 | 4.348 | 0.7789 | −+ | 2752 | 11279 | 12321 | 926968 |
| | | 0.1704 | +− | 20 | 67 | 158 | 139932 |
| | | 0.0254 | ++ | 2 | 19 | 27 | 6661 |
| | | 0.0254 | −− | 21 | 100 | 102 | 8455 |
| 161.2 | 21.739 | 0.7789 | −+ | 16096 | 67610 | 73538 | 4635245 |
| | | 0.1704 | +− | 98 | 354 | 820 | 697141 |
| | | 0.0254 | ++ | 37 | 134 | 130 | 33202 |
| | | 0.0254 | −− | 145 | 574 | 622 | 42832 |
| 161.4 | 21.739 | 0.7789 | −+ | 17334 | 72012 | 77991 | 4639495 |
| | | 0.1704 | +− | 100 | 376 | 770 | 697459 |
| | | 0.0254 | ++ | 28 | 104 | 133 | 33556 |
| | | 0.0254 | −− | 135 | 553 | 661 | 42979 |
| 161.6 | 21.739 | 0.7789 | −+ | 18364 | 76393 | 82169 | 4636591 |
| | | 0.1704 | +− | 81 | 369 | 803 | 697851 |
| | | 0.0254 | ++ | 43 | 135 | 174 | 33271 |
| | | 0.0254 | −− | 146 | 618 | 681 | 42689 |
| 162.2 | 4.348 | 0.7789 | −+ | 4159 | 17814 | 19145 | 927793 |
| | | 0.1704 | +− | 16 | 62 | 173 | 138837 |
| | | 0.0254 | ++ | 10 | 28 | 43 | 6633 |
| | | 0.0254 | −− | 46 | 135 | 141 | 8463 |
| 170.0 | 26.087 | 0.7789 | −+ | 63621 | 264869 | 270577 | 5560286 |
| | | 0.1704 | +− | 244 | 957 | 1447 | 838233 |
| | | 0.0254 | ++ | 106 | 451 | 466 | 40196 |
| | | 0.0254 | −− | 508 | 2215 | 2282 | 50979 |

Table 7: Illustrative example of the numbers of events in each channel for the standard 100 fb$^{-1}$ 6-point ILC scan with 4 helicity combinations.

Fit essentially includes experimental systematics. Main one: **background** determination.

| Fit parameter | Value | Error |
|---|---|---|
| $m_W$ (GeV) | 80.388 | $3.77 \times 10^{-3}$ |
| $f_l$ | 1.0002 | $0.924 \times 10^{-3}$ |
| $\varepsilon$ (lvlv) | 1.0004 | $0.969 \times 10^{-3}$ |
| $\varepsilon$ (qqlv) | 0.99980 | $0.929 \times 10^{-3}$ |
| $\varepsilon$ (qqqq) | 1.0000 | $0.942 \times 10^{-3}$ |
| $\sigma_B$ (lvlv) (fb) | 10.28 | 0.92 |
| $\sigma_B$ (qqlv) (fb) | 40.48 | 2.26 |
| $\sigma_B$ (qqqq) (fb) | 196.37 | 3.62 |
| $A_{LR}^B$ (lvlv) | 0.15637 | 0.0247 |
| $A_{LR}^B$ (qqlv) | 0.29841 | 0.0119 |
| $A_{LR}^B$ (qqqq) | 0.48012 | $4.72 \times 10^{-3}$ |
| $|P(e^-)|$ | 0.89925 | $1.27 \times 10^{-3}$ |
| $|P(e^+)|$ | 0.60077 | $9.41 \times 10^{-4}$ |
| $\sigma_Z$ (pb) | 149.93 | 0.052 |
| $A_{LR}^Z$ | 0.19062 | $2.89 \times 10^{-4}$ |

Note 125 inv fb/yr now feasible!
(1908.08212, Yokoya, Kubo, Okogi).
2-point scan estimates

| $|P(e^-)|$ | $|P(e^+)|$ | 100 fb$^{-1}$ | 500 fb$^{-1}$ |
|---|---|---|---|
| 80 % | 30 % | 6.02 | **2.88** |
| 90 % | 30 % | 5.24 | 2.60 |
| 80 % | 60 % | 4.05 | 2.21 |
| 90 % | 60 % | 3.77 | 2.12 |

Total $m_W$ experimental uncertainty (MeV)

High $|P(e^+)|$ very helpful!

Example 6-point ILC scan with 100 fb$^{-1}$

$$\Delta m_W (\text{MeV}) = 2.4 \, (\text{stat}) \oplus 3.1 \, (\text{syst}) \oplus 0.4 \, (\sqrt{s}) \oplus \text{theory}$$

($\sqrt{s}$ uncertainty revised to 5 ppm given recent developments)

# Fits to W Lineshape (M, Γ, $\sigma_M$)

Higgs factory machines like ILC likely systematics dominated for $m_W$ and $\Gamma_W$. Statistical uncertainties for $m_W$ and $\Gamma_W$ for $10^7$ W bosons.

| $\sigma_M$ (GeV) | $\Delta m_W$ (MeV) | $\Delta\Gamma_W^a$ (MeV) | $\Delta\Gamma_W^b$ (MeV) |
|:---:|:---:|:---:|:---:|
| 1.0 | 0.67 | 1.3 | 2.0 |
| 2.0 | 0.98 | 1.7 | 2.7 |
| 2.5 | 1.1 | 2.0 | 3.2 |
| 3.0 | 1.3 | 2.3 | 3.7 |
| 4.0 | 1.6 | 2.8 | 5.0 |

Estimated from a simple parametric fit of the Breit-Wigner lineshape convolved with a range of constant Gaussian experimental mass resolutions, $\sigma_M$. The $m_W$ uncertainty is evaluated with a one parameter fit with the width and mass resolution fixed. The corresponding uncertainties on the $\Gamma_W$ width are evaluated either with the mass resolution fixed and known perfectly from a 2-parameter fit ($\Gamma_W^a$), or more realistically, from a 3-parameter fit ($\Gamma_W^b$) that also fits for the mass resolution.

Voigtian Fit of 10M W

$\Gamma = 2.0859 \pm 0.0037$

$\sigma = 2.9986 \pm 0.0020$

$M = 80.3874 \pm 0.0013$
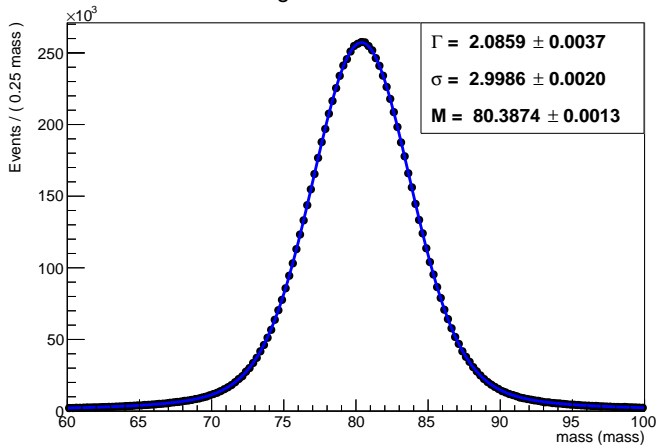
I had wrongly assumed that one needed to know $\sigma$ very well to extract $\Gamma$, but this is not the case. Of course with no constraint on $\sigma$, the uncertainty on $\Gamma$ is larger. In reality, $\sigma$ varies from W to W. So for a similar approach to work, one needs well understood event by event errors. Use by categorizing events with varying quality levels.

# Kinematic Reconstruction in Fully Leptonic Events

See Appendix B of Hagiwara et al., Nucl. Phys. B. 282 (1987) 253 for full production and decay 5-angle reconstruction in fully leptonic events ($\ell\nu_\ell\ell'\bar{\nu}_{\ell'}$) without taus as motivated by TGC analyses.

The technique applies energy and momentum conservation. One solves for the anti-neutrino 3-momentum, decomposed into its components in the dilepton plane, and out of it. Additional assumptions are:

- the energies of the two W's are equal to $E_b$, so m(W$^+$) = m(W$^-$).
- a specified value for $m_W$

$$\vec{p}_{\bar{\nu}} = a\, \vec{p}_\ell + b\, \vec{p}_{\ell'} + c\, \vec{p}_\ell \times \vec{p}_{\ell'}$$

By specifying, $m_W$, one can find $a$, $b$ and $c^2$, so there are two solutions.

The alternative pseudomass technique, does not assume $m_W$, but sets $c = 0$, and similarly has two solutions ($a_+$, $b_+$) and ($a_-$, $b_-$).

# Hadronization Systematics

## How does a W, Z, H, t decay hadronically?

Models like PYTHIA, HERWIG etc have been tuned extensively to data. Not expected to be a complete picture.

Inclusive measurements of **identified particle rates** and **momenta spectra** are an essential ingredient to describing hadronic decays of massive particles.

ILC could provide comprehensive measurements with up to 1000 times the published LEP statistics and with a much better detector with Z running.

High statistics with W events.

## Why?

Measurements based on hadronic decays, such as hadronic mass, jet directions underlie much of what we do in energy frontier experiments.

Key component of understanding jet energy scales and resolution.

Important to also understand flavor dependence: u-jets, d-jets, s-jets, c-jets, b-jets, g-jets.

# Momentum Scale Calibration (essential for $\sqrt{s}$)

Most obvious: use $J/\psi \rightarrow \mu^+\mu^-$. Event rate limited unless sizeable Z running.

| Particle | $n_{Z\mathrm{had}}$ | Decay | BR (%) | $n_{Z\mathrm{had}} \cdot$ BR | $\Gamma/M$ | PDG ($\Delta M/M$) |
|----------|------|-------|--------|------|------|------|
| $J/\psi$ | 0.0052 | $\mu^+\mu^-$ | 5.93 | 0.00031 | $3.0 \times 10^{-5}$ | $1.9 \times 10^{-6}$ |
| $K_S^0$ | 1.02 | $\pi^+\pi^-$ | 69.2 | 0.71 | $1.5 \times 10^{-14}$ | $2.6 \times 10^{-5}$ |
| $\Lambda$ | 0.39 | $\pi^-$p | 63.9 | 0.25 | $2.2 \times 10^{-15}$ | $5.4 \times 10^{-6}$ |
| $D^0$ | 0.45 | $K^-\pi^+$ | 3.88 | 0.0175 | $8.6 \times 10^{-13}$ | $2.7 \times 10^{-5}$ |
| $K^+$ | 2.05 | various | - | - | $1.1 \times 10^{-16}$ | $3.2 \times 10^{-5}$ |
| $\pi^+$ | 17.0 | $\mu^+\nu_\mu$ | 100 | - | $1.8 \times 10^{-16}$ | $2.5 \times 10^{-6}$ |

Candidate particles for momentum scale calibration and abundances in Z decay

Sensitivity of mass-measurement to $p$-scale ($\alpha$) depends on daughter masses and decay

$$m_{12}^2 = m_1^2 + m_2^2 + 2p_1 p_2 \left[ (\beta_1\beta_2)^{-1} - \cos\psi_{12} \right]$$

| Particle | Decay | $< \alpha >$ | max $\alpha$ | $\sigma_M/M$ | $\Delta p/p$ (10 MZ) | $\Delta p/p$ (GZ) | PDG limit |
|----------|-------|------|------|------|------|------|------|
| $J/\psi$ | $\mu^+\mu^-$ | 0.99 | 0.995 | $7.4 \times 10^{-4}$ | 13 ppm | 1.3 ppm | 1.9 ppm |
| $K_S^0$ | $\pi^+\pi^-$ | 0.55 | 0.685 | $1.7 \times 10^{-3}$ | 1.2 ppm | 0.12 ppm | 38 ppm |
| $\Lambda$ | $\pi^-$p | 0.044 | 0.067 | $2.6 \times 10^{-4}$ | 3.7 ppm | 0.37 ppm | 80 ppm |
| $D^0$ | $K^-\pi^+$ | 0.77 | 0.885 | $7.6 \times 10^{-4}$ | 2.4 ppm | 0.24 ppm | 30 ppm |

Estimated momentum scale statistical errors ($p = 20$ GeV)

Use of $J/\psi$ would decouple $\sqrt{s}$ determination from $m_Z$ knowledge.
Opens up possibility of improved $m_Z$ measurements.

# Full Simulation + Kalman Filter

10k "single particle events"

Work in progress –
likely need to pay
attention to issues
like energy loss
model and FSR.

**Preliminary
statistical precision
similar.**
More realistic
material, energy loss
and multiple
scattering.



ILD Full Simulation (20 GeV prompt J/ψ)

ILD_o1_v5

$m_{gen}$ = 3.09688 GeV

No vertex fit
nor constraint

RMS = 0.004701 ± 0.000034
Mean = 3.096637 ± 0.000049
Entries = 9327

Γ = 0.00289 ± 0.00013
μ = 3.096737 ± 0.000041
σ = 0.002506 ± 0.000077

$\chi^2$/dof = 85/77

-46±13 ppm

Di-Muons per 0.5 MeV

Di-Muon Mass (GeV)
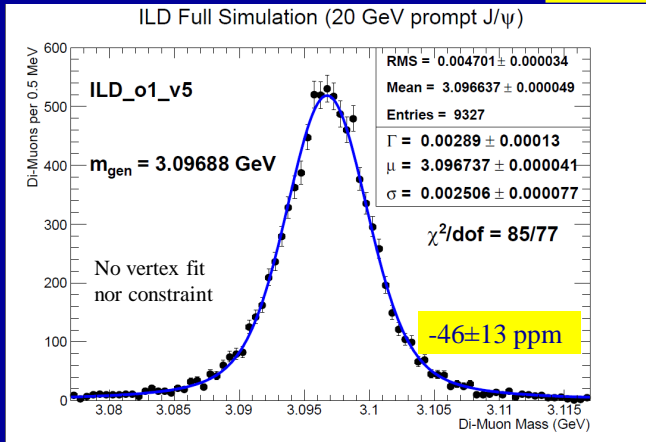
Empirical Voigtian fit.

Need consistent material model in simulation AND reconstruction

# m_W Prospects

1. Polarized Threshold Scan
2. Kinematic Reconstruction
3. Hadronic Mass

Method 1: Statistics limited.

Method 2: With up to 1000 the LEP statistics and much better detectors. Can target factor of 10 reduction in systematics.

Method 3: Depends on di-jet mass scale. Plenty Z's for 3 MeV.

**Table 1:**

| $\Delta M_W$ [MeV] | LEP2 | ILC | ILC |
|---|---|---|---|
| $\sqrt{s}$ [GeV] | 161 | 161 | 161 |
| $\mathcal{L}$ [fb$^{-1}$] | 0.040 | 100 | 480 |
| $P(e^-)$ [%] | 0 | 90 | 90 |
| $P(e^+)$ [%] | 0 | 60 | 60 |
| statistics | 200 | 2.4 | 1.1 |
| background | | 2.0 | 0.9 |
| efficiency | | 1.2 | 0.9 |
| luminosity | | 1.8 | 1.2 |
| polarization | | 0.9 | 0.4 |
| systematics | 70 | 3.0 | 1.6 |
| experimental total | 210 | 3.9 | 1.9 |
| beam energy | 13 | 0.8 | 0.8 |
| theory | - | (1.0) | (1.0) |
| total | 210 | 4.0 | 2.1 |

**Table 2:**

| $\Delta M_W$ [MeV] | LEP2 | ILC | ILC | ILC |
|---|---|---|---|---|
| $\sqrt{s}$ [GeV] | 172-209 | 250 | 350 | 500 |
| $\mathcal{L}$ [fb$^{-1}$] | 3.0 | 500 | 350 | 1000 |
| $P(e^-)$ [%] | 0 | 80 | 80 | 80 |
| $P(e^+)$ [%] | 0 | 30 | 30 | 30 |
| beam energy | 9 | 0.8 | 1.1 | 1.6 |
| luminosity spectrum | N/A | 1.0 | 1.4 | 2.0 |
| hadronization | 13 | 1.3 | 1.3 | 1.3 |
| radiative corrections | 8 | 1.2 | 1.5 | 1.8 |
| detector effects | 10 | 1.0 | 1.0 | 1.0 |
| other systematics | 3 | 0.3 | 0.3 | 0.3 |
| total systematics | 21 | 2.4 | 2.9 | 3.5 |
| statistical | 30 | 1.5 | 2.1 | 1.8 |
| total | 36 | 2.8 | 3.6 | 3.9 |

**Table 3:**

| $\Delta M_W$ [MeV] | ILC | ILC | ILC | ILC |
|---|---|---|---|---|
| $\sqrt{s}$ [GeV] | 250 | 350 | 500 | 1000 |
| $\mathcal{L}$ [fb$^{-1}$] | 500 | 350 | 1000 | 2000 |
| $P(e^-)$ [%] | 80 | 80 | 80 | 80 |
| $P(e^+)$ [%] | 30 | 30 | 30 | 30 |
| jet energy scale | 3.0 | 3.0 | 3.0 | 3.0 |
| hadronization | 1.5 | 1.5 | 1.5 | 1.5 |
| pileup | 0.5 | 0.7 | 1.0 | 2.0 |
| total systematics | 3.4 | 3.4 | 3.5 | 3.9 |
| statistical | 1.5 | 1.5 | 1.0 | 0.5 |
| total | 3.7 | 3.7 | 3.6 | 3.9 |

See Snowmass document for more details
Bottom-line: 3 different methods with prospects to measure mW with error < 5 MeV

# OPAL $m_W$ Systematics

| Source | qq̄ℓν | | | qq̄qq̄ $p_{2.5}$ | | | qq̄qq̄ $J_0$ | $\kappa_{-0.5}$ | Comb. |
|---|---|---|---|---|---|---|---|---|---|
| | CV | RW | BW | CV | RW | BW | CV | CV | CV |
| Jet energy scale | 7 | 1 | 2 | 4 | 4 | 4 | 5 | 4 | 6 |
| Jet energy resolution | 1 | 1 | 1 | 0 | 1 | 3 | 1 | 0 | 0 |
| Jet energy linearity | 9 | 9 | 12 | 2 | 2 | 4 | 2 | 1 | 6 |
| Jet angular resolution | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Jet angular bias | 4 | 4 | 4 | 7 | 7 | 6 | 6 | 7 | 5 |
| Jet mass scale | 10 | 7 | 6 | 5 | 11 | 3 | 5 | 5 | 8 |
| Electron energy scale | 9 | 6 | 8 | - | - | - | - | - | 6 |
| Electron energy resolution | 2 | 2 | 6 | - | - | - | - | - | 1 |
| Electron energy linearity | 1 | 1 | 2 | - | - | - | - | - | 1 |
| Electron angular resolution | 0 | 0 | 0 | - | - | - | - | - | 0 |
| Muon energy scale | 8 | 7 | 7 | - | - | - | - | - | 6 |
| Muon energy resolution | 2 | 2 | 3 | - | - | - | - | - | 1 |
| Muon energy linearity | 2 | 2 | 2 | - | - | - | - | - | 1 |
| Muon angular resolution | 0 | 0 | 0 | - | - | - | - | - | 0 |
| WW event hadronisation | 14 | 8 | 16 | 20 | 26 | 18 | 6 | 19 | 16 |
| Colour reconnection | - | - | - | 41 | 41 | 32 | 125 | 228 | 14 |
| Bose-Einstein correlations | - | - | - | 19 | 18 | 21 | 35 | 64 | 6 |
| Photon radiation | 11 | 11 | 10 | 9 | 8 | 8 | 9 | 9 | 10 |
| Background hadronisation | 2 | 1 | 2 | 20 | 12 | 32 | 17 | 24 | 8 |
| Background rates | 1 | 0 | 5 | 6 | 2 | 7 | 4 | 7 | 3 |
| LEP beam energy | 8 | 9 | 9 | 10 | 11 | 10 | 10 | 10 | 9 |
| Modelling discrepancies | 4 | 0 | 0 | 15 | 0 | 0 | 10 | 11 | 8 |
| Monte Carlo statistics | 2 | 3 | 3 | 2 | 3 | 3 | 2 | 2 | 2 |
| Total systematic error | 28 | 22 | 29 | 58 | 56 | 56 | 133 | 240 | 32 |
| Statistical error | 56 | 58 | 64 | 60 | 64 | 73 | 51 | 73 | 42 |
| Total error | 63 | 62 | 70 | 83 | 85 | 92 | 142 | 251 | 53 |

# Fit the Event Counts to Model Expectations

$$x \equiv |P(\mathrm{e}^-)|, \ y \equiv |P(\mathrm{e}^+)|$$

Event count expectations:

$$\mu_{ijk} = \left( f_S^k(x,y,A_{LR}^{WW}) \, \sigma_i(m_W, \alpha_S) \, \varepsilon_j B_j + g_B^k(x,y,A_{LR}^B) \, \sigma_B^j \right) f_l L_{ik}$$

$$\nu_{ik} = g_Z^k(x,y,A_{LR}^Z) \, \sigma_Z^i \, f_l L_{ik}$$

Signal, background, and Z-control sample spin factors:

$$f_S^{-+}(x,y,A) = 1 + xy + A(x+y)$$

$$f_S^{+-}(x,y,A) = 1 + xy - A(x+y)$$

$$f_S^{++}(x,y,A) = 1 - xy - A(x-y)$$

$$f_S^{--}(x,y,A) = 1 - xy + A(x-y)$$

$$g_{B,Z}^{-+}(x,y,A) = 1 + xy + A(x+y)$$

$$g_{B,Z}^{+-}(x,y,A) = 1 + xy - A(x+y)$$

$$g_{B,Z}^{++}(x,y,A) = 1 - xy - A(x-y)$$
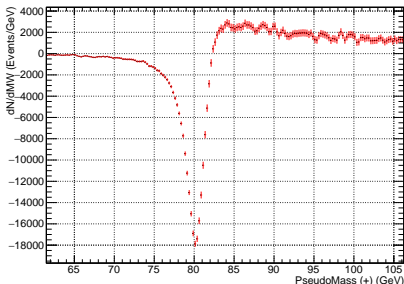
$$g_{B,Z}^{--}(x,y,A) = 1 - xy + A(x-y)$$

Set A=0.99 for WW (estimate of 0.992 (Wopper), 0.988 (Racoon))
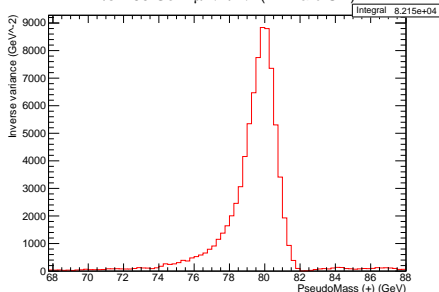
# Positive PseudoMass (10M events sensitivity) (-80,+30)

Estimate mass sensitivity bin-by-bin by using (here INCLUDE BS) ...

$$\Delta m_{\mathrm{W}} = |\frac{d\sigma}{dm_{\mathrm{W}}}|^{-1}\Delta\sigma \text{ or } \Delta m_{\mathrm{W}} = |\frac{dN}{dm_{\mathrm{W}}}|^{-1}\Delta N$$



Then, can estimate overall statistical uncertainty on $m_{\mathrm{W}}$ from

$$\Delta m_{\mathrm{W}} = \sqrt{1/\sum\frac{1}{\sigma_i^2}}$$

Here $\Delta m_{\mathrm{W}} = 1.0/\sqrt{82150}\,\mathrm{GeV} = 3.5$ MeV

**Direct reconstruction of $M_W$ and $\Gamma_W$**
**Semi-leptonic** decay channel $WW \to q\bar{q}\ell\nu$

*Study at 162.6 GeV, 240 GeV and 365 GeV*
**Only the muon decay**
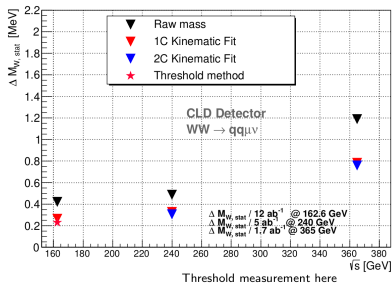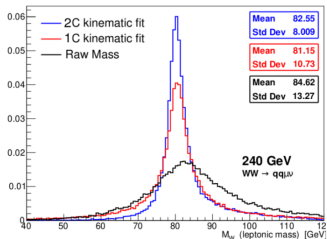
@162.6 GeV $\Delta\Gamma_W(1C) = 0.35$ MeV
@240 GeV $\Delta\Gamma_W(2C) = 0.68$ MeV
@365 GeV $\Delta\Gamma_W(2C) = 1.56$ MeV

Threshold measurement here

Full FCC-ee luminosity

Marina Béguin          W study at FCC-ee          November 18, 2019     9 / 15

At 240 GeV, find $\Delta m_W$ (stat.) of 0.31 MeV for $2 \times 5$ ab$^{-1}$.
Equivalent to 0.69 MeV for 2 ab$^{-1}$ total. (ILC250 2 ab$^{-1}$ estimate is 0.79 MeV)

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