Theory aspects of top-pair production near threshold

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Precision calculations for future e^+e^- colliders: targets and tools
CERN, June 7 - 17, 2022

MB, Kiyo, Schuller, Part I 1312.4791 [hep-ph] and Part II in preparation
MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864 [hep-ph]
MB, Maier, Piclum, Rauh, 1506.06865 [hep-ph]
MB, Maier, Rauh, Ruiz-Femenia, 1711.10429

including earlier results from

MB, Schuller, Kiyo, hep-ph/0501289
MB, Kiyo, 0804.4004 [hep-ph]
MB, Jantzen, Ruiz-Femenia, 1004.2188 [hep-ph]
Jantzen, Ruiz-Femenia, 1307.4337 [hep-ph]
Marquard, Piclum, Seidel, Steinhauser, 1401.3004 [hep-ph]

...

`QQbar_threshold` code MB, Kiyo, Maier, Piclum, 1605.03010 [hep-ph]

https://www.hepforge.org/downloads/qqbarthreshold/

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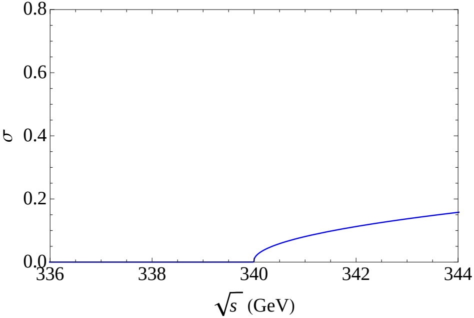
Pair production threshold – Strong Coulomb force and Weak Decay

- Ultra-precise mass measurement, order of magnitude relative to *pp*
- “Spectroscopic” measurement utilizing unique QCD dyanmics
- Free from uncertainties of interpretation of mass definition
- Sensitivity to Higgs Yukawa force, too?

Stable top quark, no strong interaction



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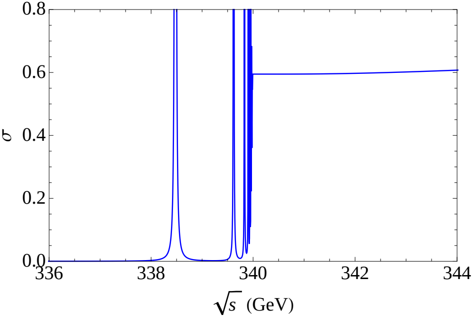
Stable top quark, with strong Coulomb force



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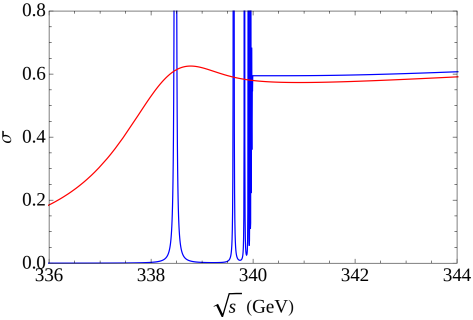
Unstable top quark, with strong Coulomb force



Challenges for theory

Scales

$$\underbrace{m_t,\, m_t v,\, m_t v^2}_{\text{QCD dynamics}}, \qquad \underbrace{m_{W,H},\, \Gamma_t}_{\text{EW}}, \qquad \underbrace{\ln m_e}_{\text{QED ISR}}$$

- Weak coupling but non-perturbative QCD (strong Coulomb force)
  - NNNLO MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864 [hep-ph]
  - NNLL (almost) Hoang, Stahlhofen, 1309.6323 [hep-ph]
- The rest of the SM: Electroweak, Higgs and QED
- Non-resonant production $e^+e^- \to W^+W^- b\bar{b}$ and cuts
- QED initial state radiation / parton distribution effects

Non-perturbative but weak coupling. Expansion in α_s and $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{\sqrt{q^2} - 2m_t}{m_t}}$, while $\alpha_s/v = O(1)$

$$R \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \Big\{ \underbrace{1\,(\text{LO})}_{\text{Fadin, Khoze}} ; \underbrace{\alpha_s, v\,(\text{NLO})}_{\text{Peskin, Strassler}}; \underbrace{\alpha_s^2, \alpha_s v, v^2\,(\text{NNLO})}_{1998}; \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3\,(\text{N3LO})}_{2005\text{-}2015}; \ldots \Big\}$$

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2) = i \int d^4x\, e^{iq\cdot x}\, \langle 0|T(j_\mu(x) j_\nu(0))|0\rangle, \qquad j^\mu(x) = [\bar{Q}\gamma^\mu(\gamma_5)Q](x)$$

Summation through Schrödinger equation

$$\text{Im}\,\Pi(E) = \frac{N_c}{2m^2} \underbrace{\sum_{n=1}^{\infty} Z_n \times \pi\delta(E_n - E)}_{\text{bound states}} + \Theta(E) \underbrace{\text{Im}\,\Pi(E)_{\text{cont}}}_{\text{continuum}}$$

$$R \equiv \frac{\sigma_{e^+e^- \to WWb\bar{b}X}}{\sigma_0} = 12\pi e_t^2 K\ \text{Im}\,\Pi(E + i\Gamma_t) + [\text{EWC + non-resonant}]$$

Tools

Multiple scales m_t, $m_t v$ integrated out in steps through a sequence of non-relativistic effective theories and the threshold expansion. See [MB, Kiyo, Schuller, arXiv:1312.4791 [hep-ph]]

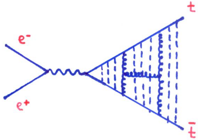
$$\mathcal{L}_{\text{QCD}}\left[Q(h,s,p),\ g(h,s,p,us)\right] \qquad \mu > m$$

$$\downarrow$$

$$\mathcal{L}_{\text{NRQCD}}\left[Q(s,p),\ g(s,p,us)\right] \qquad mv < \mu < m$$

$$\downarrow$$

$$\mathcal{L}_{\text{PNRQCD}}\left[Q(p),\ g(us)\right] \qquad \mu < mv$$



$$\sigma_{e^+e^-\to t\bar{t}X} = \frac{16\pi^2\alpha_{\text{em}}^2 e_t^2}{s}\,\text{Im}\left[\frac{N_c}{2m^2}\left(c_v\left[c_v - \frac{E}{m}\left(c_v + \frac{d_v}{3}\right)\right] G_{\text{PNRQCD}}(E) + \ldots\right)\right],$$

$$\begin{aligned}\delta_3 G_{\text{PNRQCD}}(E) &= \langle\mathbf{0}|\hat{G}_0(E)i\delta V_1 i\hat{G}_0(E)i\delta V_1 i\hat{G}_0(E)i\delta V_1 i\hat{G}_0(E)|\mathbf{0}\rangle \\ &\quad + 2\langle\mathbf{0}|\hat{G}_0(E)i\delta V_1 i\hat{G}_0(E)i\delta V_2 i\hat{G}_0(E)|\mathbf{0}\rangle + \langle\mathbf{0}|\hat{G}_0(E)i\delta V_3 i\hat{G}_0(E)|\mathbf{0}\rangle + \delta^{us}G(E)\end{aligned}$$

The effective Lagrangian

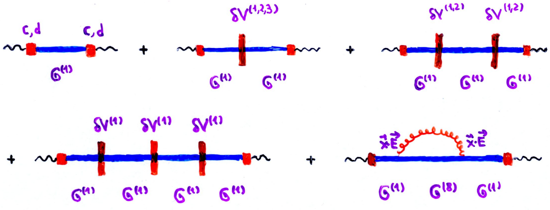
Integrating out soft fluctuations results in a spatially non-local effective Lagrangian since $[k^i]_{\rm soft} \sim [k^i]_{\rm pot}$.

$$\mathcal{L}_{\rm PNRQCD} = \psi^\dagger \Big(iD_0 + i\frac{\Gamma_t}{2} + \frac{\boldsymbol{\partial}^2}{2m} + \frac{\boldsymbol{\partial}^4}{8m^3}\Big)\psi + \chi^\dagger \Big(iD_0 - i\frac{\Gamma_t}{2} - \frac{\boldsymbol{\partial}^2}{2m} - \frac{\boldsymbol{\partial}^4}{8m^3}\Big)\chi$$
$$+ \int d^{d-1}\mathbf{r}\, \Big[\psi^\dagger\psi\Big](x+\mathbf{r})\, \Big(-\frac{\alpha_s C_F}{r} + \delta V(r,\boldsymbol{\partial})\Big)\, \Big[\chi^\dagger\chi\Big](x)$$
$$- g_s\psi^\dagger(x)\boldsymbol{x}\boldsymbol{E}(t,\mathbf{0})\psi(x) - g_s\chi^\dagger(x)\boldsymbol{x}\boldsymbol{E}(t,\mathbf{0})\chi(x)$$

- The leading-order Coulomb potential is part of the unperturbed Lagrangian. The asymptotic states correspond to the composite field $[\psi^\dagger\chi](\boldsymbol{R},\boldsymbol{r})$ with free propagation in the cms coordinate. The propagation in the relative coordinate is determined by the Coulomb Green function $G_c^{(1)}(\boldsymbol{r},\boldsymbol{r}';E)$
- Perturbations consist of kinetic energy corrections, perturbation potentials, and ultrasoft gluon interactions.

The NNNLO contribution to the PNRQCD correlation function is

$$G^{(3)} = -G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_1 G_c^{(1)} + 2G_c^{(1)}\delta V_1 G_c^{(1)}\delta V_2 G_c^{(1)} - G_c^{(1)}\delta V_3 G_c^{(1)} + \delta G_{\rm us}$$



where

$$G_c^{(1,8)}(\mathbf{r},\mathbf{r}',E) = \frac{my}{2\pi} e^{-y(r+r')} \sum_{l=0}^{\infty} (2l+1)(2yr)^l (2yr')^l P_l\left(\frac{\mathbf{r}\cdot\mathbf{r}'}{rr'}\right) \sum_{s=0}^{\infty} \frac{s!L_s^{(2l+1)}(2yr)L_s^{(2l+1)}(2yr')}{(s+2l+1)!(s+l+1-\lambda)}$$

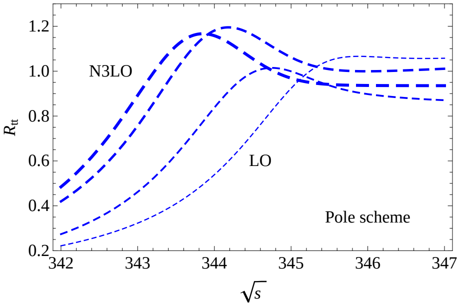
$$y = \sqrt{-m(E+i\epsilon)},\ \lambda = \frac{m\alpha_s}{2y} \times \{C_F\,(\text{singlet});\, C_F - C_A/2\,(\text{octet})\}$$

For singlet need only $l = 0$, for octet only $l = 1$.

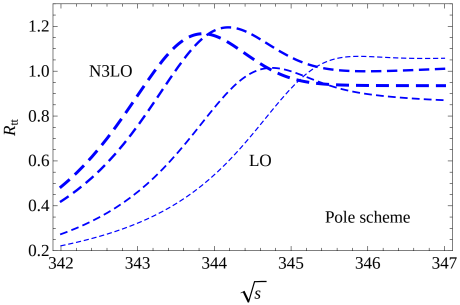
3rd order ingredients

- Bound state quantities (S-wave)
  - E_n – Kniehl, Penin, Smirnov, Steinhauser (2002); MB, Kiyo, Schuller (2005); Penin, Sminrov, Steinhauser (2005)
  - $|\psi_n(0)|^2$ – MB, Kiyo, Schuller (2007); MB, Kiyo, Penin (2007)
- Matching coefficients
  - a_3 – Anzai, Kiyo, Sumino (2009); Smirnov, Sminrov, Steinhauser (2009)
  - c_3 – Marquard, Piclum, Seidel, Steinhauser (2014) [2009]
- Continuum (PNRQCD correlation function)
  - ultrasoft – MB, Kiyo (2008)
  - potential – MB, Kiyo, [Schuller], in preparation [2007]
  - P-wave – MB, Piclum, Rauh (2013)

Note: logarithmically enhanced 3rd order terms known before or resummed [Hoang et al. 2001-2013; Pineda et al. 2002-2007]. But non-log terms are as large in individual terms.
2nd order available since end of 1990s.


N3LO
LO
Pole scheme
R_{tt}
$\sqrt{s}$
1.2
1.0
0.8
0.6
0.4
0.2
342
343
344
345
346
347

Mass scheme issues



- Pole mass cannot be determined with an accuracy better than $\mathcal{O}(\Lambda_{QCD})$ [MB, Braun, 1994; Bigi et al., 1994].
  Pole mass renormalon seads to spurious shifts in the peak position of the $t\bar{t}$ cross section [MB, 1998]
- Solution ("Kill two birds with one stone"): intermediate mass definition m_{PS}, which can be related precisely to the $\overline{\rm MS}$ mass ($\rightarrow$ top Yukawa coupling) AND avoids large, spurious corrections to the cross section.

Mass scheme issues

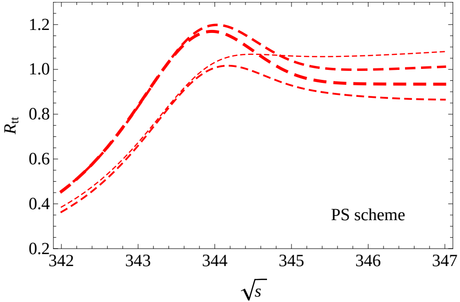
- Potential-subtracted mass [MB, 1998]

$$m_{\rm PS}(\mu_f) \equiv m_{\rm pole} + \frac{1}{2}\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\,\tilde{V}_{\rm Coulomb}(\vec{q}\,)$$

$$m_{\rm PS}(\mu_f) - \overline{m}(\overline{m}) = \underbrace{[m_{\rm PS}(\mu_f) - m_{\rm pole}]}_{\text{known to } \mathcal{O}(\mu_f\alpha_s^4)\ \text{[hep-ph/0501289]}} + \underbrace{[m_{\rm pole} - \overline{m}(\overline{m})]}_{\text{known to } \mathcal{O}(m_t\alpha_s^4)\ \text{[1502.01030]}}$$

Conversion precision $\approx 20\,$MeV. Cancellation of large perturbative contributions from the IR.

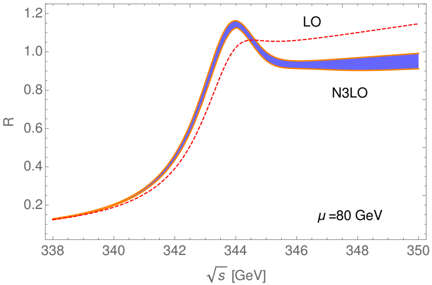
In the following use $m_{t,\rm PS}(20\,\text{GeV}) = 171.5\,\text{GeV}$.



NNNLO QCD result

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]
Photon exchange and Z-vector coupling only.

$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.23$,
$\mu = (50\ldots 80 \ldots 350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.



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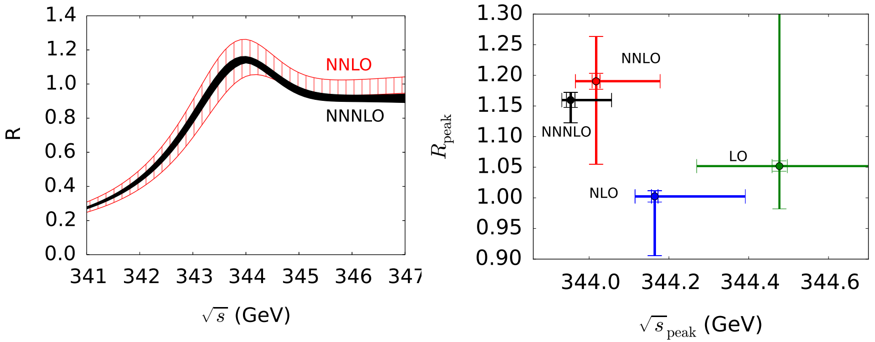


Position shift: 310 MeV (LO to NLO) 150 MeV (to NNLO) 64 MeV (to NNNLO)
Improvement of factor 3 in uncertainty in peak height.

From QCD $t\bar{t}$ threshold to a realistic prediction

- Axial-vector Z-coupling (not a non-QCD effect), P-wave production
  N3LO [MB, Piclum, Rauh, 2013] [starts at NNLO]

- QED effects
  N3LO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NLO, Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010]

- Electroweak matching coefficients absorptive parts [Hoang, Reisser, 2004] and electroweak corrections in general [Guth, Kühn, 1992]
  NNLO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NNLO] $[\alpha_{\rm EW} \sim \alpha_s^2]$

- Higgs contributions [Eiras, Steinhauser, 2006; MB, Maier, Piclum, Rauh, 2015]
  N3LO [starts at NNLO]

- Non-resonant contributions: $\sigma_{e^+e^-\to W^+W^-b\bar{b}_{\rm nonres}}(\mu_w)$
  Mostly inclusive, possibly invariant mass cuts.
  NNLO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NLO, MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011, Jantzen, Ruiz-Femenia, 2013; Ruiz-Femenia, 2014]]

- QED initial state radiation
  Formally NNLO, but large logs of m_e. Effectively LO. [MB, Maier, Rauh, Ruiz-Femenia, 2017]

Systematics of Yukawa coupling effects

Power counting of new parameters m_H, λ_t:

- $m_H \sim m_t$ or $m_H \sim m_t v$ [Strassler, Peskin, 1991]
- $\lambda_t \sim \alpha_s$ or $\lambda_t \sim \alpha_{\rm EW} \sim \alpha_s^2$

Adopt $m_H \sim m_t$ and $\lambda_t = y_t^2/(4\pi)$ as electroweak coupling.

Higgs-exchange Yukawa potential is effectively local

$$\frac{y_t^2}{\vec{q}^{\,2} + m_H^2} \to \frac{y_t^2}{m_H^2} \qquad \Rightarrow \qquad \delta\sigma \propto -\frac{y_t^2}{m_H^2}\,\mathrm{Im}[G_0(E)^2]_{\overline{\rm MS}}$$

N3LO effect relative to $-g_s^2/\vec{q}^{\,2}$.

Need only single insertion of Higgs-exchange tree-level potential into Coulomb Green function.
Formally not the dominant Yukawa coupling effect.

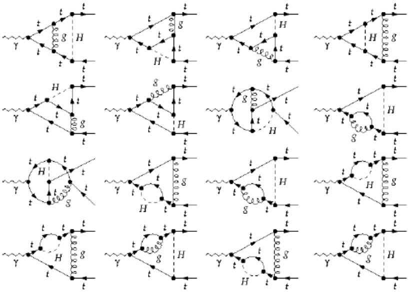
Short-distance Yukawa coupling effects

Modification of the $t\bar{t}$ production vertex

NNLO – 1-loop correction to vector-current matching [Guth, Kühn, 1991]
N3LO – Mixed 2-loop Higgs-QCD correction to vector-current matching [Eiras, Steinhauser, 2006]

$$c_v = 1 - 0.103|_{\alpha_s} \underbrace{-0.022|_{\alpha_s^2} + 0.031|_{y_t^2}}_{\text{NNLO}} \underbrace{-0.070|_{\alpha_s^3} - 0.019|_{y_t^2\alpha_s}}_{\text{NNNLO}} + \ldots,$$

Local approximation of the Higgs potential required for consistent cancellation of factorization scale dependence.

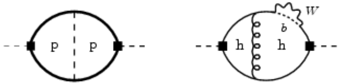


Finite-width divergence and scale-dependence

The pure-QCD calculation in the (P)NRQCD framework is technically inconsistent from NNLO. Uncancelled $1/\epsilon$ poles.

- Finite-width divergences (overall log divergence, already at NNLO):

$$[\delta G(E)]_{\text{overall}} \propto \frac{\alpha_s}{\epsilon} \cdot E$$



Since $E = \sqrt{s} - 2m_t + i\Gamma$, the divergence survives in the imaginary part:

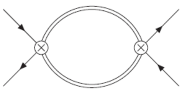
$$\text{Im}\,[\delta G(E)]_{\text{overall}} \propto m_t \times \frac{\alpha_s \alpha_{ew}}{\epsilon} \quad \Rightarrow \quad \ln(\mu_w/(m_t E))$$

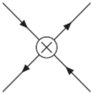
- Electroweak effect. Must consider $e^+e^- \to W^+W^-b\bar{b}$.

$$\sigma_{e^+e^-\to W^+W^-b\bar{b}} = \underbrace{\sigma_{e^+e^-\to[t\bar{t}]_{\text{res}}}(\mu_w)}_{\text{pure (PNR)QCD}} + \sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$$

Non-resonant starts at NLO (overall linear divergence) [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]. Finite-width scale dep must cancel. Need consistent dim reg calculation.

Unstable particle EFT provides a systematic expansion of the amplitude in powers of Γ/m. [MB, Chapovsky, Signer, Zanderighi, 2003]





Resonant contributions

Production of an on-shell, non-relativistic $t\bar{t}$ pair and subsequent decay $t \to W^+ b$. Effective non-relativistic propagator contains on-shell width.

Non-resonant contributions

All-hard region. Off-shell lines. Full theory diagrams expanded around $s = 4m_t^2$. No width in propagators.

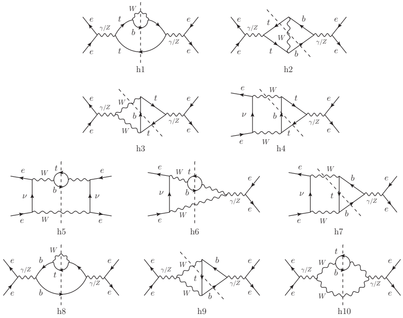
$$i\mathcal{A} = \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0)\, i\mathcal{O}_p^{(l)}(x)] | e^- e^+ \rangle + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

$$\mathcal{O}_p^{(v,a)} = \bar{e}_{c_2} \gamma_i (\gamma_5) e_{c_1} \, \psi_t^\dagger \sigma^i \chi_t \qquad \mathcal{O}_{4e}^{(k)} = \bar{e}_{c_1} \Gamma_1 e_{c_2} \, \bar{e}_{c_2} \Gamma_2 e_{c_1},$$

$$\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \mathrm{Im}\left[C_{4e}^{(k)}\right] \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

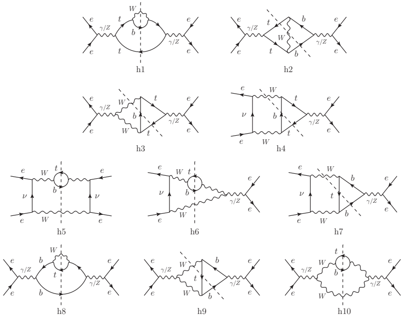
Separately divergent and factorization ("finite-width") scale-dependent.

Non-resonant corrections at NLO & NNLO



Equivalent to the dimensionally regulated $e^+e^- \to bW^+\bar{t}$ process with $\Gamma_t = 0$, expanded in the hard region around $s = 4m_t^2$.

NNLO: $\mathcal{O}(\alpha_s)$ corrections to this process
[MB, Maier, Rauh, Ruiz-Femenia, 2017]



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$$\int_{\Delta^2}^{m_t^2} dp_t^2\, (m_t^2 - p_t^2)^{\frac{d-3}{2}} H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

$$p_t^2 \equiv (p_b + p_{W+})^2$$

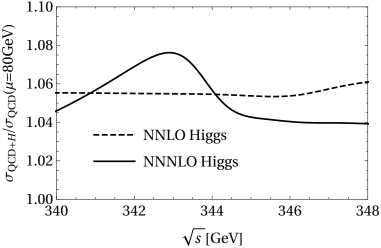
$$H_1\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right) \overset{p_t^2 \to m_t^2}{\rightarrow} \text{const}\times \frac{1}{(m_t^2 - p_t^2)^2}$$

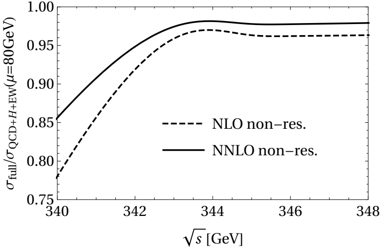
Linearly IR divergent at NLO. Finite in dim reg.
Log divergent at NNLO

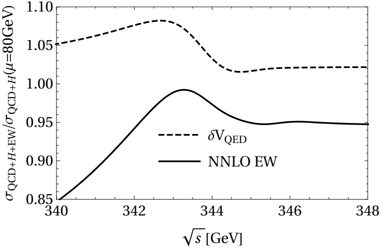
Can impose invariant mass cuts on top decay products. $\Delta^2 = M_W^2$ for inclusive cross section.
EFT works differently for loose and wide cuts [Actis, MB, Falgari, Schwinn, 2008]
Here: wide cuts

Size of non-QCD effects

- QCD = NNNLO QCD including P-wave
- NNNLO Higgs (top-Yukawa)
- NNLO electroweak+QED
- NNLO non-resonant







NNNLO (QCD+Higgs) + NNLO (EW+QED+non-resonant)

[MB, Maier, Piclum, Rauh, 2015; MB, Maier, Rauh, Ruiz-Femenia, 1710.10429]

Inclusive $e^+e^- \to W^+W^- b\bar{b}$ cross section

$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.2229$,
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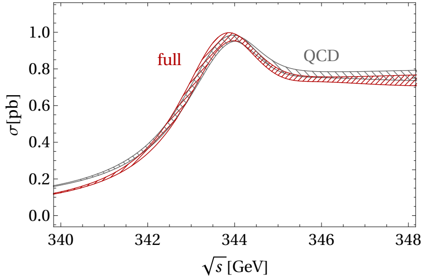
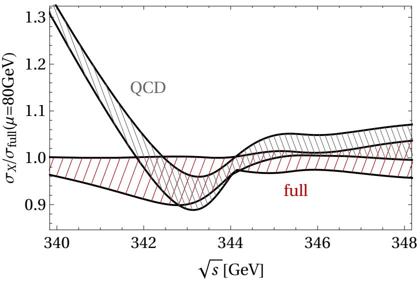
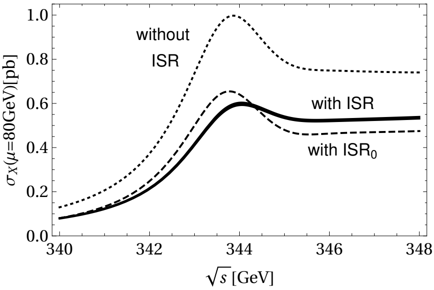




Should be part of the theoretical prediction, because separation from the “partonic cross section” depends on a factorization scheme.

[MB, Maier, Rauh, Ruiz-Femenia, 1710.10489]



QED parton distributions at large-x in LL approximation (as detailed in the LEP-1 Yellow report)

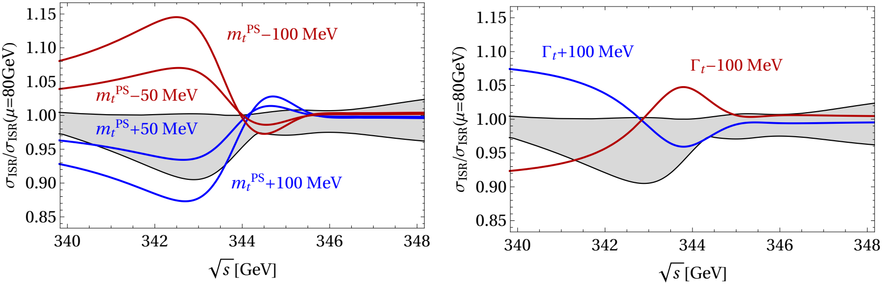
$\hookrightarrow$ major theoretical uncertainty
can be improved (see Frixione’s talk)

Shaded band: Relative scale uncertainty

$$\frac{\delta\sigma}{\sigma} = \pm(2\ldots3.5)\%$$

Superimposed: Variation with shifted top mass or width input normalized to reference.

[MB, Maier, Piclum, Rauh, 2015; MB, Maier, Rauh, Ruiz-Femenia, 2017]



Sensitivity to y_t and α_s

- Add

$$\Delta\mathcal{L} = -\frac{c_{\rm NP}}{\Lambda^2}(\phi^\dagger\phi)(\bar{Q}_3\widetilde{\phi}t_R) + \text{h.c.}$$

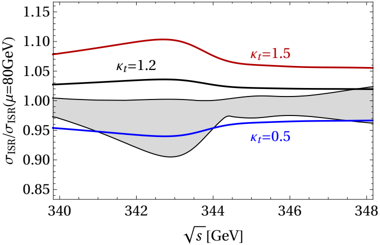
to the SM Lagrangian

$$\kappa_t \equiv \frac{y_t}{\sqrt{2}m_t/v} = 1 + \frac{c_{\rm NP}}{\Lambda^2}\frac{v^3}{\sqrt{2}m_t}$$

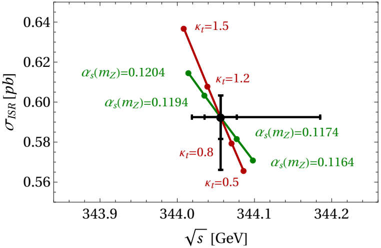
Treat top mass and Yukawa coupling as independent parameters.

- Caveat: In the framework of SMEFT there are many more and possibly more important anomalous coupling effects.

[MB, Maier, Rauh, Ruiz-Femenia, 2017]



Peak position of (κ_t, α_s) vs. th. uncertainty



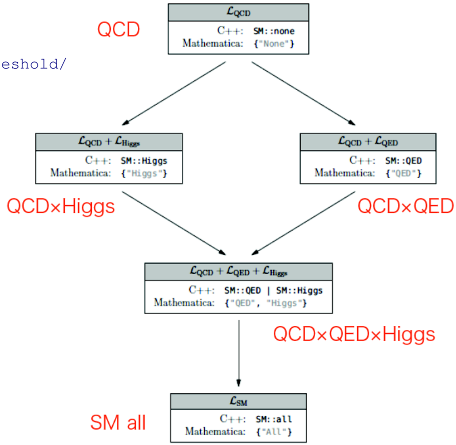
QQbar_threshold code

All results implemented in the code QQbar_threshold available from

https://www.hepforge.org/downloads/qqbarthreshold/

[MB, Kiyo, Maier, Piclum, 1605.03010]

- C++ code
  Mathematica interface via Mathlink
- Possibility to choose different models (QCD, SM,)
- Also for bottom, bound state energy and decay, mass scheme conversion.



QQbar_threshold code

examples/Mathematica/xsection_1.m

```
Needs["QQbarThreshold`"];

LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
With[
   {
     mu = 50.,
     mtPS = 168.,
     width = 1.4,
     muWidth = 350.,
     order = "N3LO"
   },
   Do[
      Print[
         sqrts, "\t",
         TTbarXSection[sqrts, {mu, muWidth}, {mtPS, width}, order]
      ],
      {sqrts, 330., 345., 1.}
   ]
];
```

Evaluation time per cross section point approx 5ms

Summary

I $e^+e^- \to t\bar{t}X$ cross section near threshold now computed at NNNLO in (PNR)QCD + top-Yukawa effects

- Sizeable 3rd order corrections and reduction of theoretical uncertainty to about $\pm 3\%$.

II Realistic predictions for $e^+e^- \to W^+W^-b\bar{b}$ near top-pair threshold

- NNLO available, including cuts invariant mass cuts.

III Parameter dependences $(m_t, \Gamma_t, y_t, \alpha_s)$ can be studied.

- (m_t, Γ_t) with unrivaled accuracy.
- y_t with 20% accuracy from threshold already challenging.

IV Further requirements:

- ISR / QED PDF's for $x \to 1$ with NLL evolution
- N4LO QCD would be reassuring, but appears prohibitive.

https://www.hepforge.org/downloads/qqbarthreshold/