The stochastic gravitational wave background from close hyperbolic encounters of primordial black holes in dense clusters

Santiago Jaraba

Work in collaboration with Juan García-Bellido and Sachiko Kuroyanagi

12th Iberian Gravitational Waves Meeting, 8th June 2022

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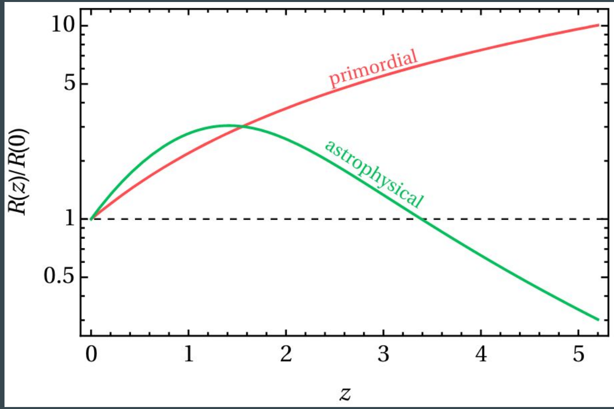
Brief summary of PBHs

M. Raidal et al.,
arXiv:1812.01930

- Definition:
  - Produced shortly after inflation, in radiation dominated era.
  - Sufficiently large density perturbations collapse into a BH.



- Key differences with ABHs
  - PBHs produced much earlier -> different merger rate evolution
    - Increasing redshift eventually makes ABH mergers vanish, while PBHs would still remain
  - PBHs generated without spin, unlike ABHs
    - Proposed mechanisms for spin induction
  - ABHs can't be generated between 50-130 $M_\odot$ (pair-instability supernova gap) or below a few $M_\odot$
    - PBH mass spectrum would be broader.

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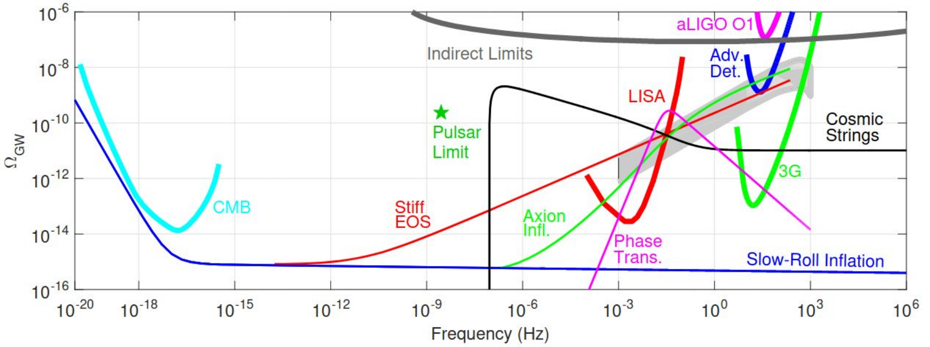
The stochastic gravitational wave background

- Detectors as LIGO/Virgo detect intense GW signals from individual BBH
- Weaker, unresolved signals would form a continuous background: SGWB
- Lots of sources would also leave an imprint in this background: very rich field!
  - Inflation model (slow-roll, axion, etc.), early universe phase transitions, cosmic strings, preheating...
- Interesting to look for this background in all possible frequencies.



LIGO and Virgo collaborations, arXiv:0910.5772

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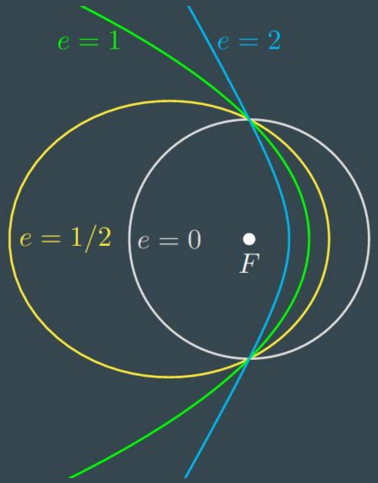
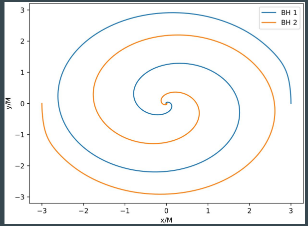
Binary black holes vs close hyperbolic encounters



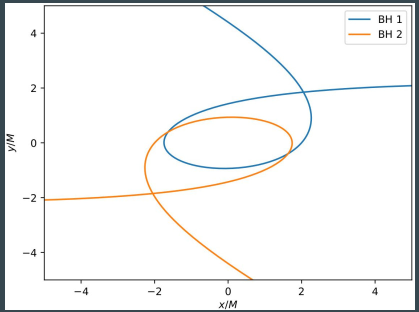
- Keplerian motion:
  - 2-body problem is described by an ellipse (e<1), parabola (e=1) or hyperbola (e>1)
- General relativity:
  - The energy loss in elliptic motion (BBH) leads to an eventual merger (unless disrupted)
  - In some “hyperbolic” encounters with e~1, BHs lose so much energy that they become bounded
  - In pure hyperbolic encounters, both BH have enough kinetic energy to overcome energy loss
  - If close enough (CHE), energy emission can still be notorious. Source of GW!

Binary black holes vs close hyperbolic encounters

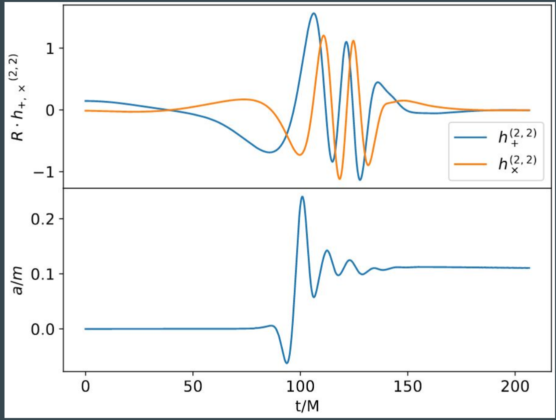
- Not only BBH have interesting dynamics...

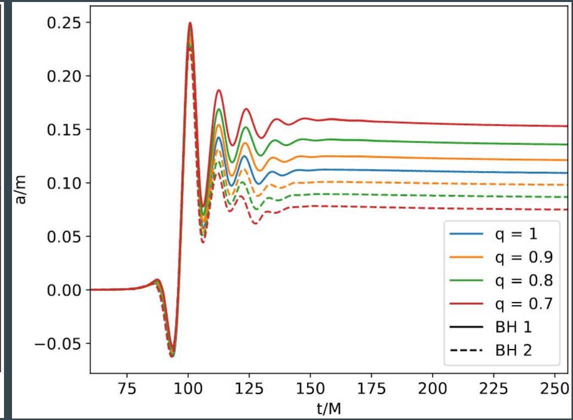


- CHE also do! The trajectories don't follow hyperbolas anymore



- An interesting effect is **spin induction**. Some PBHs could acquire it just by moving nearby others!





S. Jaraba, J. García-Bellido, arXiv:2106.01436

What about the CHE contribution to the SGWB?

J. García-Bellido, S. Jaraba, S. Kuroyanagi, arXiv:2109.11376

SGWB computation: general formalism

- The SGWB can be computed as $\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f} = \frac{1}{\rho_c}\int_0^\infty dz\,\frac{N(z)}{1+z}\frac{dE_{\rm GW}}{d\ln f_r}$
  - $f_r = (1+z)f$ frequency in source frame
  - $dE_{\rm GW}/d\ln f_r$ GW energy emission / log. frequency bin in source frame
  - $N(z) = \frac{\tau(z)}{(1+z)H(z)}$ number density of GW events at redshift z
    - $H(z)$ Hubble expansion rate
    - $\tau(z) = \iint \frac{dm_1}{m_1}\frac{dm_2}{m_2}\frac{d\tau}{d\ln m_1\, d\ln m_2}$ event rate / (unit time x comoving volume)

- Both for BBH, CHE contributions, we need an <u>energy spectrum</u> and <u>event rate</u>

SGWB from binary black holes

- Merger rate (PBH)

$$\frac{d\tau^{\rm BBH}}{d\ln m_1\, d\ln m_2} \approx 14.8\ {\rm yr}^{-1}{\rm Gpc}^{-3} h_{70}^4 \left(\frac{\Omega_{\rm DM}}{0.25}\right)^2 \left(\frac{\delta_{\rm loc}}{10^8}\right) \left(\frac{v_0}{10\ {\rm km/s}}\right)^{-11/7} f(m_1)\, f(m_2) \frac{M^{10/7}}{(m_1\, m_2)^{5/7}}$$

  - $\delta_{\rm loc}\, ,\, v_0$ cluster-dependent parameters
  - $f(m_i)$ logarithmic mass distributions so that $\int f(m_i) d\log(m_i) = f_{\rm PBH}$
  - $\Omega_{\rm DM}\, ,\, f_{\rm PBH}$ cosmological parameters. We assume $f_{\rm PBH} = 1$, but easy rescaling otherwise

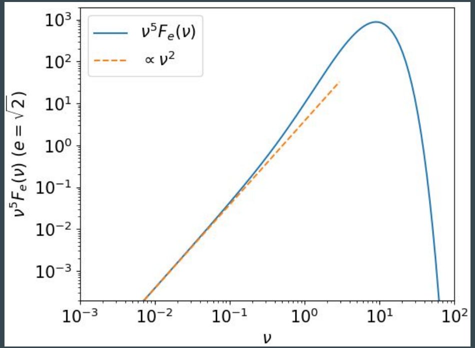
- Energy spectrum $\frac{dE^{\rm BBH}}{d\ln f_r} = \frac{(\pi G)^{2/3} m_1\, m_2}{3c^2\, M^{1/3}} f_r^{2/3} \mathcal{F}(f_r)$
  - $\mathcal{F}(f_r)$ describes the deviation from the frequency dependence of the inspiral phase $f_r^{2/3}$
  - At low frequencies, $\mathcal{F}(f_r) \approx 1$ -> characteristic slope $\Omega_{\rm BBH}(f) \propto f^{2/3}$

SGWB from close hyperbolic encounters

- Event rate (PBH)

$$\frac{d\tau^{\rm CHE}}{dm_1\,dm_2} \approx 25.4 \times 10^{-8}\ {\rm yr}^{-1}{\rm Gpc}^{-3}\,h_{70}^4 \left(\frac{\Omega_{\rm DM}}{0.25}\right)^2 \left(\frac{\delta_{\rm loc}}{10^8}\right) \frac{f(m_1)}{m_1}\frac{f(m_2)}{m_2}\frac{M^2}{m_1\,m_2}\frac{e^2-1}{(v_0/c)^3}$$

  - $v_0 = \sqrt{GM/a}$ relative asymptotic velocity
  - a, e orbital parameters. Usually we use $y = \sqrt{e^2-1}$



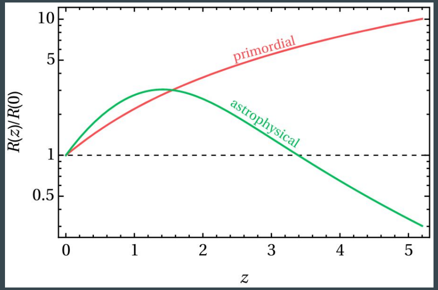
- Energy spectrum

$$\frac{dE_{\rm GW}^{\rm CHE}}{d\ln f_r} = \frac{4\pi}{45}\frac{G^3 m_1^2 m_2^2}{a^2 c^5 \nu_0}\,\nu^5 F_e(\nu)$$

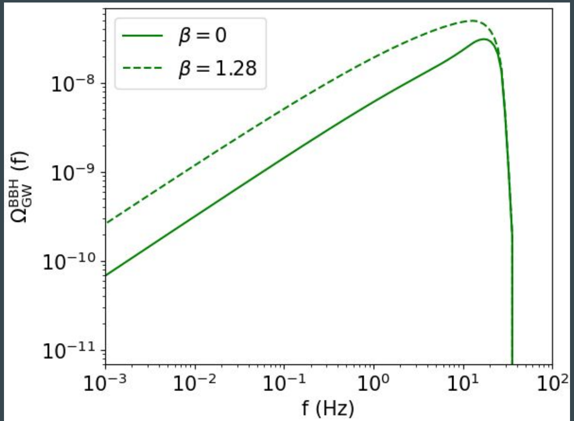
  - $\nu \equiv 2\pi\nu_0 f_r\,,\quad \nu_0^2 \equiv a^3/GM\,,\ \nu^5 F_e(\nu)$ polynomial with exponential suppression
  - At low frequencies, $\Omega_{\rm CHE} \propto f^2$. Different slope than BBH!

Redshift dependence of event rates

M. Raidal et al., arXiv:1812.01930

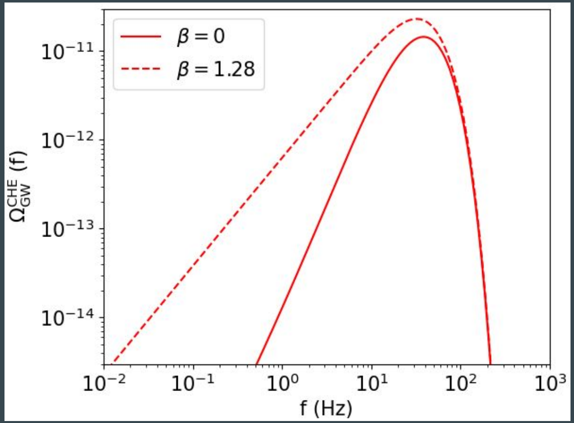


- Previous event rates were assumed to be constant in redshift.
- We can add a $(1+z)^\beta$ dependence to match the figure for PBH evolution. $\beta \approx 1.28$
- Change for BBH: overall amplitude, no drastic shape change
- Change for CHE: low-f tail slope modified! Sensitivity to event rate evolution!



$$\alpha = \frac{2}{3}$$

$$\alpha = \min\left\{2, \frac{5}{2} - \beta\right\}$$



Differences and detection

- BBH contribution will be dominant in LIGO-LISA freqs. and eventually be detected.
- It is possible to produce a detectable CHE contribution for certain parameter sets.
- CHE contribution easier to produce the higher the frequency. Potential for UHF GW?

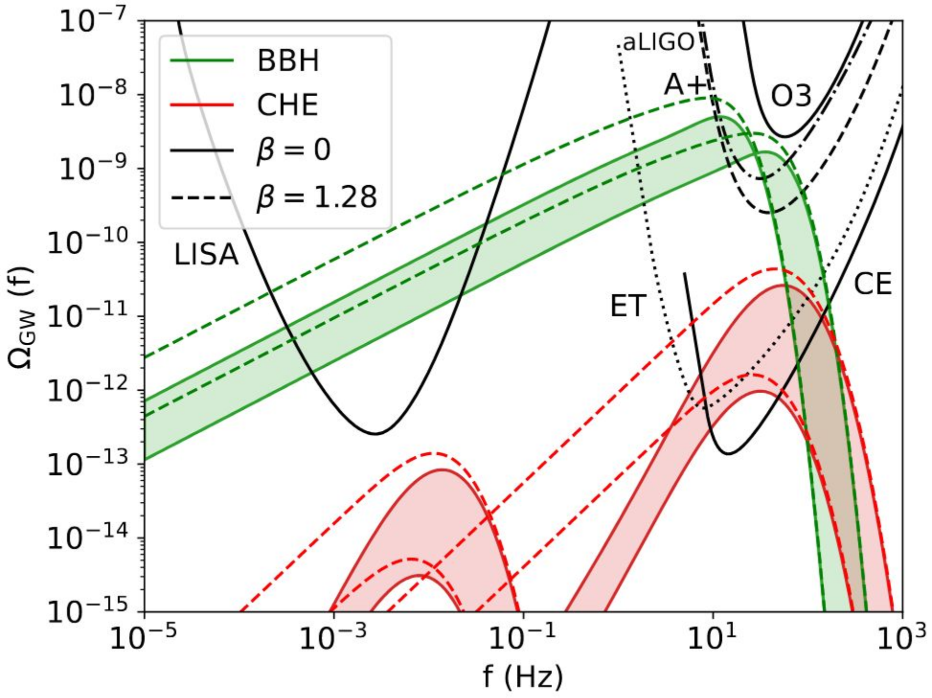


Figure: all parameters have log-normal distributions, σ=1. Median for masses ~100 $M_{\odot}$.

Conclusions

- Different frequency dependencies -> both contributions can be disentangled
- CHE component sensitive to event rate evolution, unlike BBH
  - Key to distinguishing between PBH and ABH sources!
- Some values of orbital parameters can make the CHE contribution detectable
- Possible extensions of this work:
  - Detailed modelling of ABH contribution
  - More detailed clustering profile of PBHs or orbital parameter distributions

Thank you for your attention!

Backup slide: full expressions of Omega_GW

$$\Omega_{\rm GW}^{\rm BBH}(\mathcal{F}=1) \approx 2.39\times 10^{-13}\, h_{70} \times \left(\frac{\Omega_{\rm DM}}{0.25}\right)^2 \left(\frac{\delta_{\rm loc}}{10^8}\right)\left(\frac{v_0}{10\ {\rm km/s}}\right)^{-11/7}\left(\frac{f}{\rm Hz}\right)^{2/3} \times \int dm_1\, dm_2\, \frac{f(m_1)\, f(m_2)\, (m_1+m_2)^{23/21}}{(m_1\, m_2)^{5/7}}$$

$$\Omega_{\rm GW}^{\rm CHE}(f) \approx 9.81\times 10^{-13}\, h_{70}\left(\frac{\Omega_{\rm M}}{0.3}\right)^{-1/2}\left(\frac{\Omega_{\rm DM}}{0.25}\right)^2 \times \left(\frac{\delta_{\rm loc}}{10^8}\right)\left(\frac{a}{0.1\ {\rm AU}}\right)\left(\frac{f}{10{\rm Hz}}\right)^2\left(\frac{y}{0.01}\right) \times \int \frac{dm_1}{100M_\odot}\,\frac{dm_2}{100M_\odot} f(m_1)\, f(m_2)\, e^{-2x_0\xi(y)}\, \tilde{I}\,[y, x_0]$$

$$\xi(y) = y - \tan^{-1} y, y = \sqrt{e^2-1}$$

$$\tilde{I}\,[y,\, x_0] \simeq \frac{2x_0^{5/2-\beta}}{(2\xi)^{3/2+\beta}}\,\frac{1}{y(1+y^2)^2} \times \left[2(1-y^2+4y^4)\,\xi^2\,\Gamma\left(-\frac{1}{2}+\beta,\, 2x_0\xi\right) + 3y^3(-1+3y^2)\,\xi\,\Gamma\left(\frac{1}{2}+\beta,\, 2x_0\xi\right) + 3y^6\,\Gamma\left(\frac{3}{2}+\beta,\, 2x_0\xi\right)\right]$$

Backup slide: peak expressions for CHE

$$\Omega_{\rm GW}^{\rm CHE}(f_{\rm peak}) \approx 3.6\times 10^{-13}\, h_{70} \times \left(\frac{\Omega_{\rm M}}{0.3}\right)^{-1/2} \left(\frac{\Omega_{\rm DM}}{0.25}\right)^{2} \left(\frac{\delta_{\rm loc}}{10^8}\right) \left(\frac{a}{0.1\ {\rm AU}}\right)^{-2} \times \left(\frac{y}{0.01}\right)^{-5} \frac{m_1}{100M_\odot}\frac{m_2}{100M_\odot}\frac{m_1+m_2}{200M_\odot}$$

$$\Omega_{\rm GW}^{\rm CHE}(f_{\rm peak}) \approx 4.4\times 10^{-13}\, h_{70} \times \left(\frac{\Omega_{\rm M}}{0.3}\right)^{-1/2} \left(\frac{\Omega_{\rm DM}}{0.25}\right)^{2} \left(\frac{\delta_{\rm loc}}{10^8}\right) \left(\frac{f_{\rm peak}}{50\ {\rm Hz}}\right)^{4/3} \times \left(\frac{y}{0.01}\right)^{-1} \frac{m_1}{100M_\odot}\frac{m_2}{100M_\odot}\left(\frac{m_1+m_2}{200M_\odot}\right)^{1/3}$$

$$f_{\rm peak} \simeq 43{\rm Hz}\left(\frac{y}{0.01}\right)^{-3}\left(\frac{M}{200M_\odot}\right)^{1/2}\left(\frac{a}{0.1\ {\rm AU}}\right)^{-3/2}$$