The GW signal from primordial magnetic fields in the PTA frequency band

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ARP, C. Caprini, A. Neronov, D. Semikoz, *Phys. Rev. D* **105**, 123502 (2022), arXiv:2201.05630
A. Neronov, ARP, C. Caprini, D. Semikoz, *Phys. Rev. D* **103**, L041302 (2021), arXiv:2009.14174

ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020), arXiv:1903.08585
ARP *et al.*, *JCAP* **04** (2022), 019, arXiv:2107.05356 (2021)

https://github.com/AlbertoRoper/GW_turbulence

Probing the early Universe with GWs: Cosmological GW background

- Why background? Individual sources are not resoluble, superposition of single events occurring in the whole Universe

- From phase transitions
  - Ground-based detectors (LVK, ET, CE) frequencies are 10–1000 Hz [see **A. Romero-Rodríguez talk**]

    Peccei-Quinn, B-L, left-right symmetries, ... $\sim 10^7, 10^8$ GeV (untested physics, SM extensions)
  - Space-based detectors (LISA) frequencies are 10^{-5}–10^{-2} Hz

    Electroweak phase transition ~ 100 GeV ($f_c \sim 10^{-5}$ Hz)
  - Pulsar Timing Array **(PTA)** frequencies are 10^{-9}–10^{-7} Hz

    Quantum chromodynamic (QCD) phase transition ~ 100 MeV ($f_c \sim 10^{-9}$ Hz)

- From inflation
  - B-modes of CMB anisotropies ($f_c \sim 10^{-18}$ Hz)
  - Can cover all f spectrum, depending on end-of-reheating T, and blue-tilted (beyond slow-roll inflation)

MHD sources in the early universe

- Magnetohydrodynamic (MHD) sources of GWs
  - Hydrodynamic turbulence from first-order phase transitions
  - **Primordial magnetic fields**
- High-conductivity of the early universe leads to a high-coupling between magnetic and velocity fields

- Other sources of GWs include
  - True vacuum bubble collisions
  - Sound waves [see **D. Weir** talk]
  - Cosmic topological defects (cosmic strings)
  - Primordial black holes [see **J. García-Bellido** talk]

Gravitational spectrum (ground-based detectors)
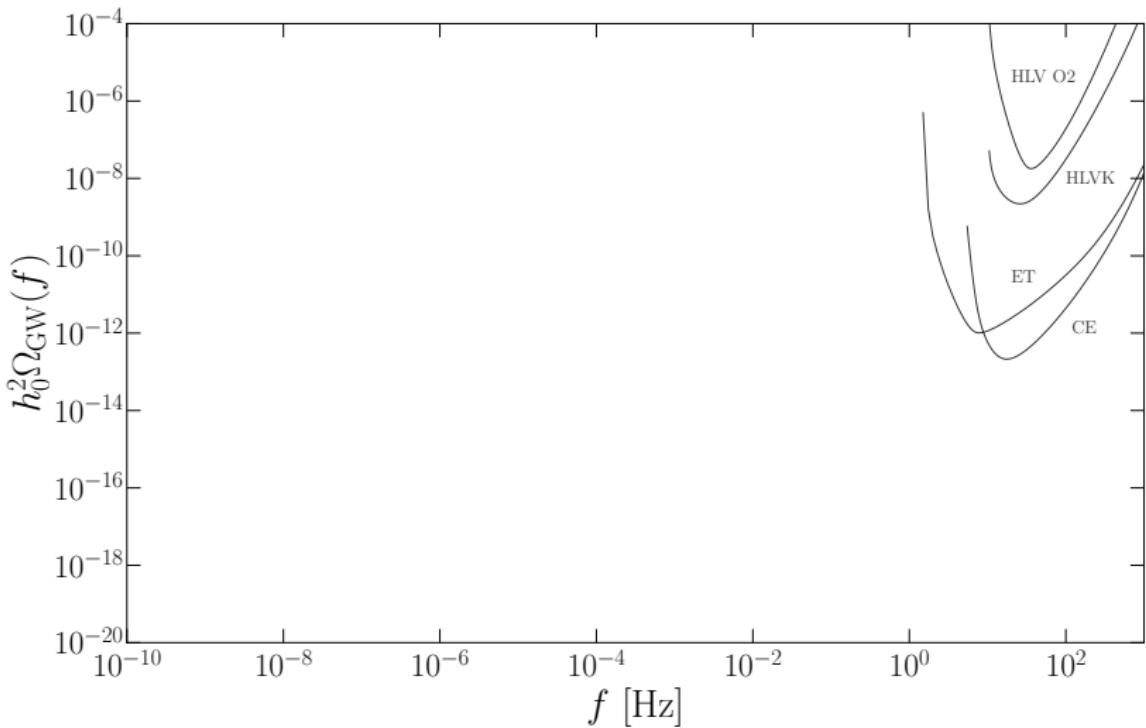
Gravitational spectrum (space-based detectors)
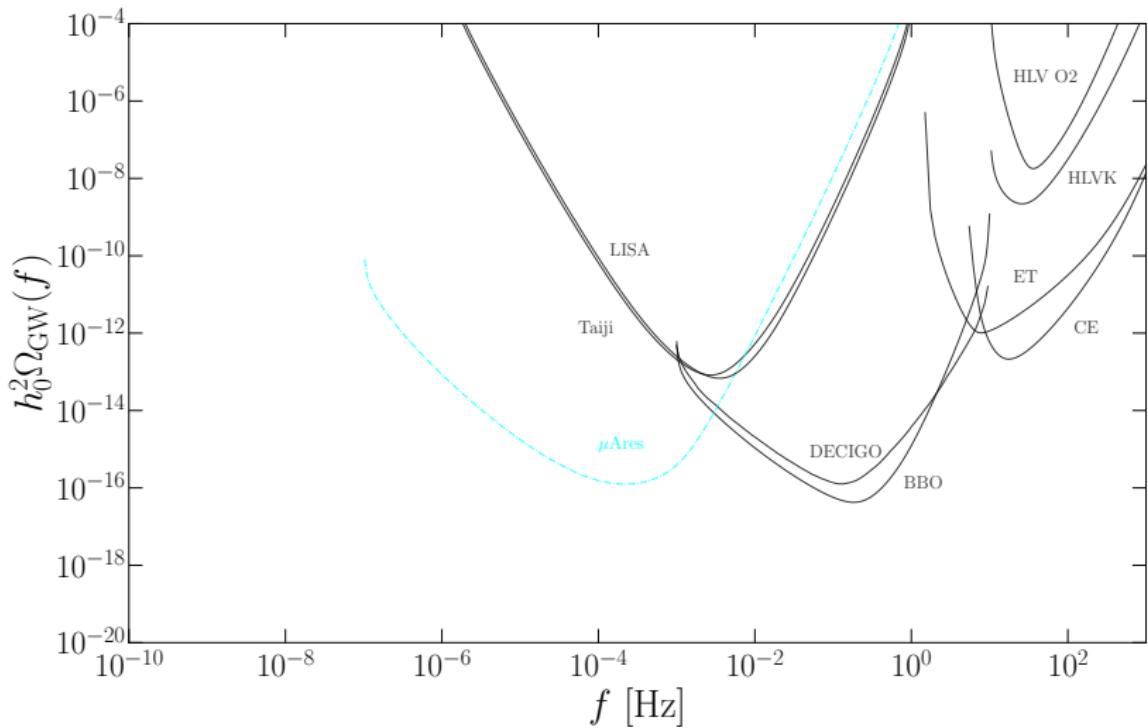
Pulsar Timing Array (PTA) collaborations

- An array of pulsars is observed to compute the delays on the time of arrival due to the presence of GWs
- International collaborations (IPTA)
    - European Pulsar Timing Array (EPTA)
    - North American Nano-Hertz Observatory for Gravitational Waves (NANOGrav)
    - Parkes Pulsar Timing Array (PPTA)
   - Indian Pulsar Timing Array Project (InPTA)
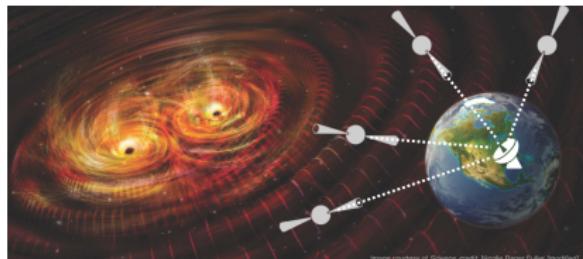
Figure: Image courtesy of Science, credit: Nicolle Rager Fuller

# NANOGrav 12.5 yr data observation[1]
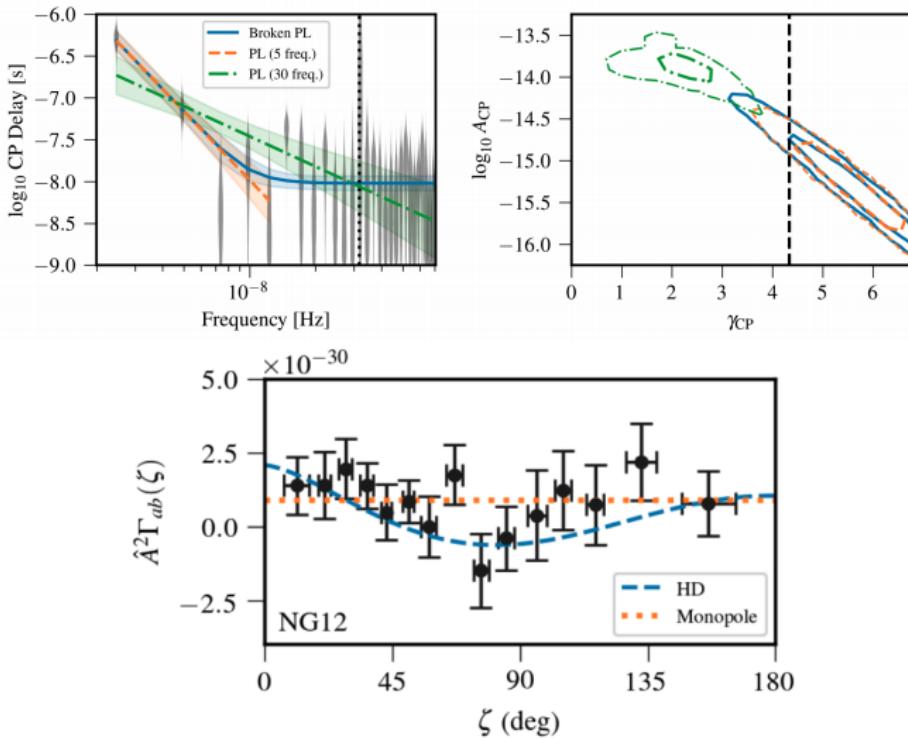
Detectability of the SGWB produced by non-helical primordial magnetic fields from the QCD phase transition by NANOGrav[2]
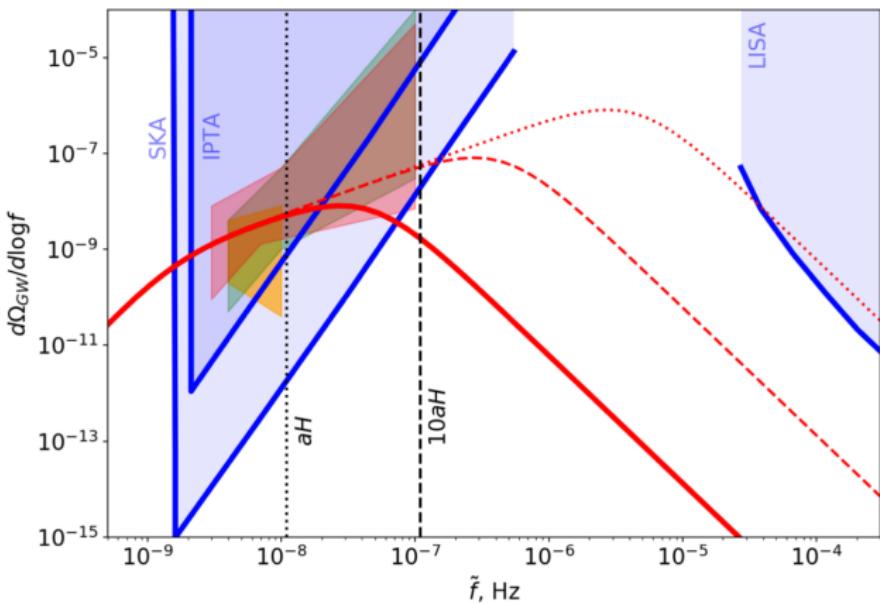
[2] A. Neronov, ARP, C. Caprini and D. Semikoz,
*Phys. Rev. D* **103**, L041302 (2021)

PPTA 15 yr data observation[3]
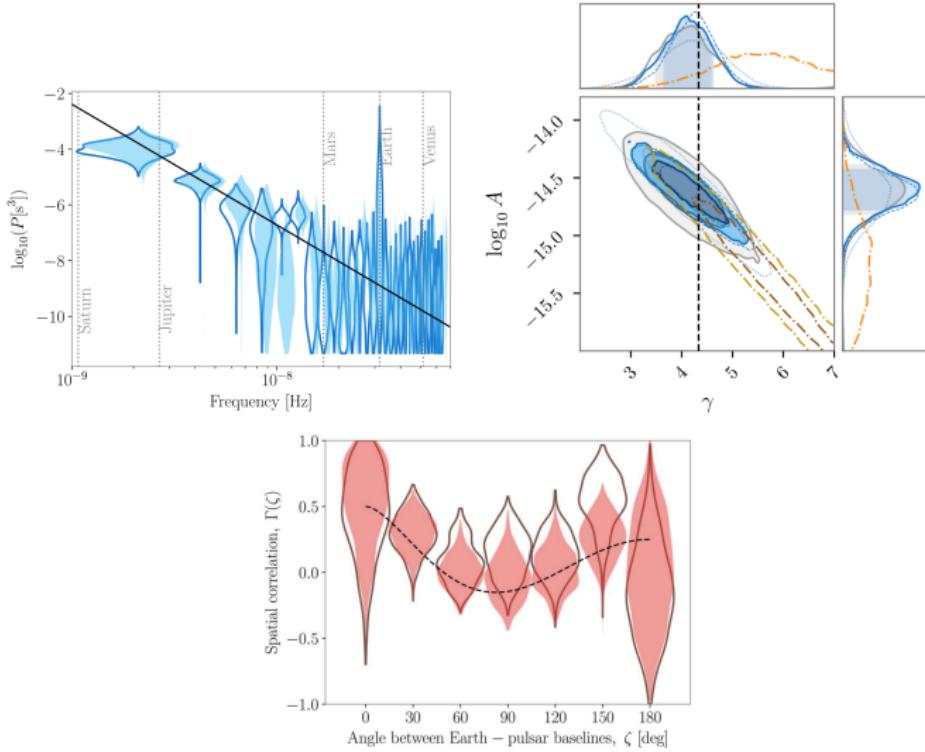
EPTA 24 yr data observation (DR 2)[4]
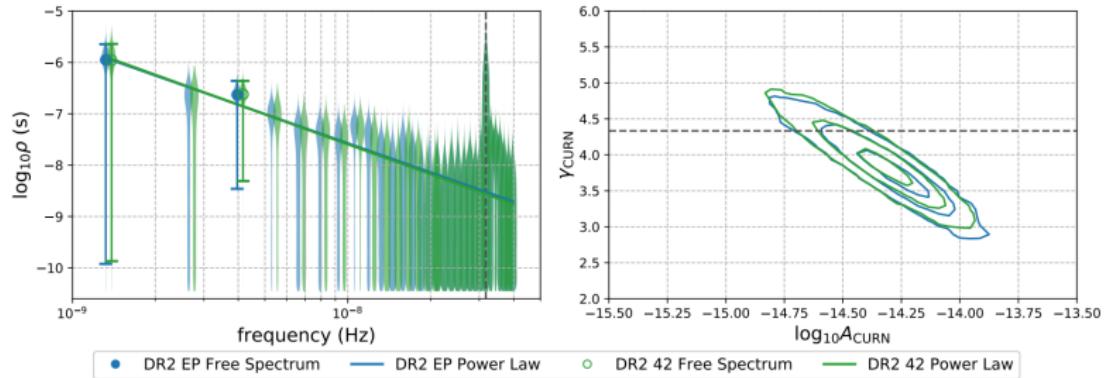
IPTA combined data observation (DR 2)[5]
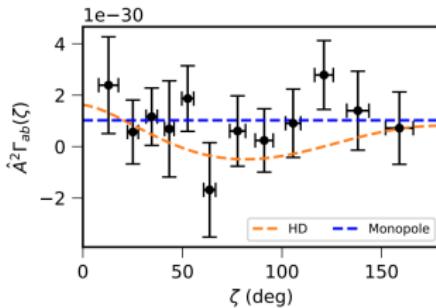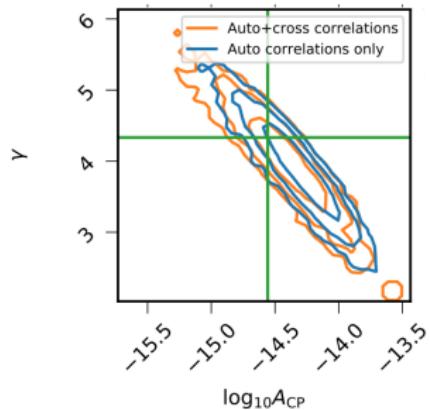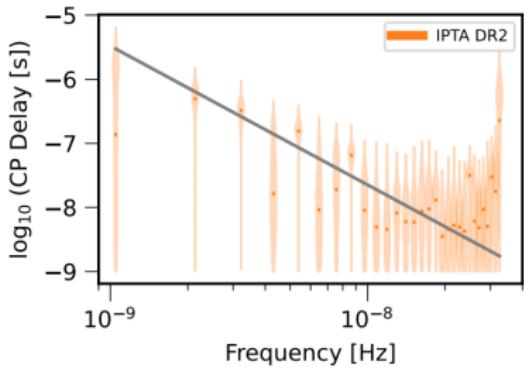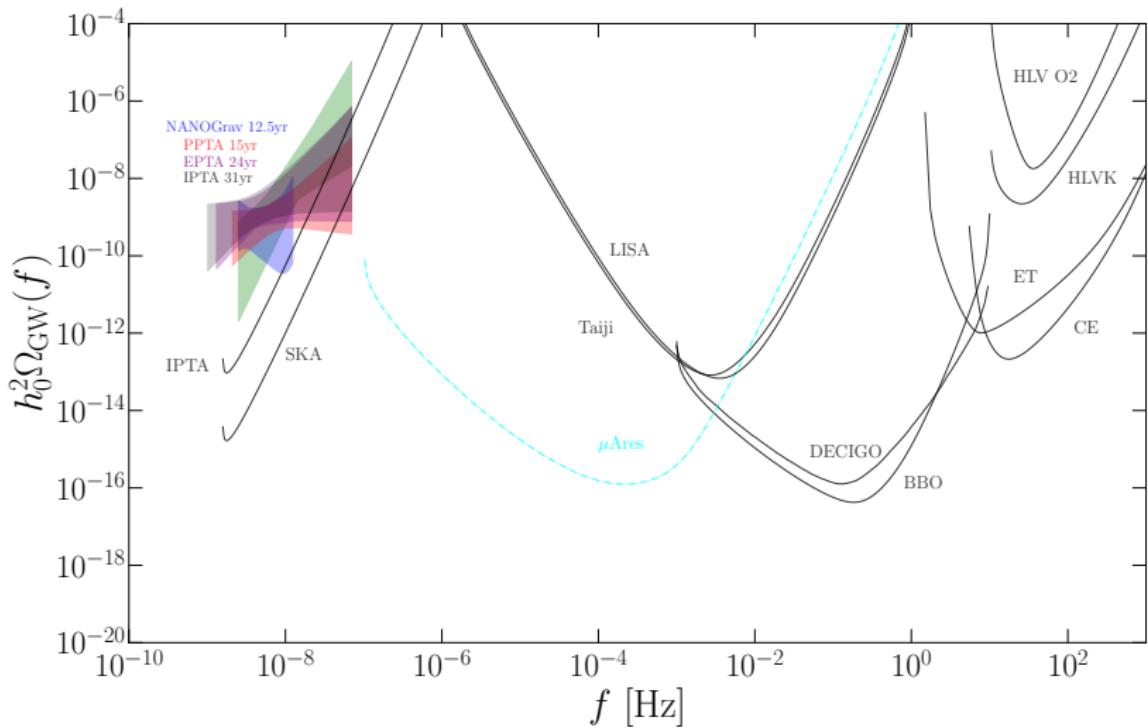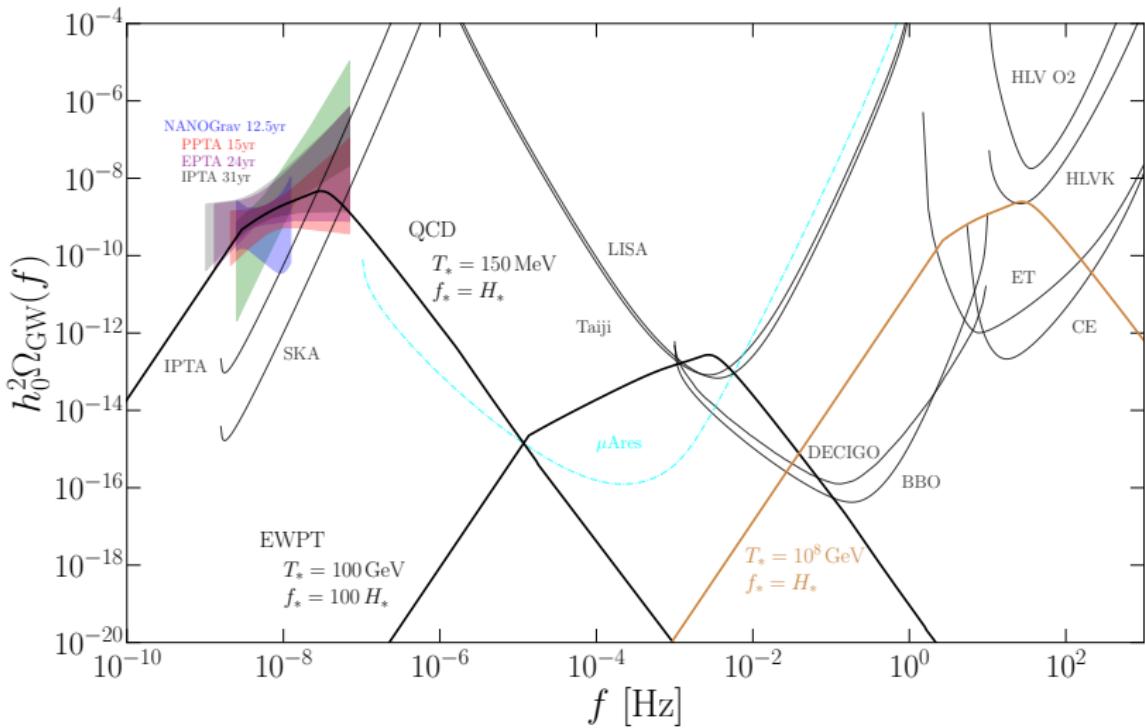
[6] ARP, C. Caprini, A. Neronov, D. Semikoz,
Phys. Rev. D **105**, 123502 (2022)

How do we compute these signals?

- Direct numerical simulations using the PENCIL CODE[7] to solve:
    - Relativistic MHD equations adapted for radiation-dominated era (after electroweak symmetry is broken)
    - Gravitational waves equation

- Numerical simulations are necessary to solve MHD dynamics and to take into account the exact unequal time correlator of the GW sourcing term (analytical estimates require simplifying assumptions).

- The assumption that the two-point correlations of the magnetic stress are constant in time accurately explains the numerical results when the eddy-turnover time is large (slow turbulent evolution) for initially present nonhelical fields[8].

[7] Pencil Code Collaboration, JOSS **6**, 2807 (2020), https://github.com/pencil-code/

[8] ARP, C. Caprini, A. Neronov, D. Semikoz,
Phys. Rev. D **105**, 123502 (2022)

MHD description

Right after the electroweak phase transition we can model the plasma using continuum MHD

- Charge-neutral, electrically conducting fluid
- Relativistic magnetohydrodynamic (MHD) equations
- Radiation-dominated Universe

$$p = \rho c^2 / 3,$$

  i.e. $w = 1/3$ (ultrarelativistic EoS)

- Friedmann–Lemaître–Robertson–Walker metric

$$g_{\mu\nu} = \mathrm{diag}\{-1, a^2, a^2, a^2\}$$

Contributions to the stress-energy tensor

$$T^{\mu\nu} = \left(p/c^2 + \rho\right) U^\mu U^\nu + p g^{\mu\nu} + \pi^{\mu\nu} + F^{\mu\gamma} F^\nu{}_\gamma - \frac{1}{4} g^{\mu\nu} F_{\lambda\gamma} F^{\lambda\gamma},$$

- From fluid motions
  $T_{ij} = \left(p/c^2 + \rho\right) \gamma^2 u_i u_j + p \delta_{ij}$

  - Ultrarelativistic EoS:
    $p = \rho c^2/3$
  - Viscous stresses:
    $\pi_{ij} = 2\nu(p/c^2 + \rho)S_{ij}$

- From magnetic fields:
  $T_{ij} = -B_i B_j + \delta_{ij} B^2/2$

- 4–velocity $U^\mu = \gamma(c, u^i)$

- 4–potential $A^\mu = (\phi/c, A^i)$

- 4–current $J^\mu = (c\rho_e, J^i)$

- Faraday tensor
  $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Conservation laws

$$T^{\mu\nu}{}_{;\nu} = 0$$

We assume subrelativistic motions:

$$\gamma^2 \sim 1 + (v/c)^2 + \mathcal{O}(v/c)^4$$

Relativistic MHD equations are reduced to[9]

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3} \left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho \right) + \frac{1}{\rho} \left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2 \right],$$

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{3} \mathrm{u} \left(\boldsymbol{\nabla} \cdot \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \ln \rho \right) - \frac{\boldsymbol{u}}{\rho} \left[\boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) + \eta \boldsymbol{J}^2 \right]$$

$$- \frac{1}{4} \boldsymbol{\nabla} \ln \rho + \frac{3}{4\rho} \boldsymbol{J} \times \boldsymbol{B} + \frac{2}{\rho} \boldsymbol{\nabla} \cdot (\rho \nu \boldsymbol{S}) + \mathcal{F},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \boldsymbol{J} + \mathcal{E}), \quad \boldsymbol{J} = \boldsymbol{\nabla} \times \boldsymbol{B},$$

for a flat expanding universe with comoving and normalized $p = a^4 p_{\mathrm{phys}}, \rho = a^4 \rho_{\mathrm{phys}}, B_i = a^2 B_{i,\mathrm{phys}}, u_i$, and conformal time t ($\mathrm{d}t = a \mathrm{d}t_c$).

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[9] A. Brandenburg, *et al.*, *Phys. Rev. D* **54**, 1291 (1996)

GWs equation for an expanding flat Universe

- Assumptions: isotropic and homogeneous Universe

- Friedmann–Lemaître–Robertson–Walker (FLRW) metric $\gamma_{ij} = a^2 \delta_{ij}$

- Tensor-mode perturbations above the FLRW model:

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}^{\mathrm{phys}} \right), \quad |h_{ij}^{\mathrm{phys}}| \ll |g_{ij}|$$

- GW equation is[10]

$$\left(\partial_t^2 - \frac{a''}{a} - c^2 \nabla^2 \right) h_{ij} = \frac{16\pi G}{a c^2} (T_{ij}^{\mathrm{TT}} + \cancel{t_{ij}})$$

  - h_{ij} are rescaled $h_{ij} = a h_{ij}^{\mathrm{phys}}$
  - Comoving spatial coordinates $\nabla = a \nabla^{\mathrm{phys}}$
  - Conformal time $\mathrm{d}t = a \, \mathrm{d}t_c$
  - Comoving stress-energy tensor components $T_{ij} = a^4 T_{ij}^{\mathrm{phys}}$
  - Radiation-dominated epoch such that $a'' = 0$
  - Nonlinear leading-order term $t_{ij} \propto \partial_i h_{lm}^{\mathrm{TT}} \partial_j h_{lm}^{\mathrm{TT}}$ omitted[11]

[10] L. P. Grishchuk, *Sov. Phys. JETP* **40**, 409 (1974)

[11] Y. He, ARP and A. Brandenburg, *submitted to Phys. Rev. D*, arXiv:2110.14456 (2021)

Numerical results for decaying MHD turbulence[12]

Initial conditions

- Initial stochastic non-helical magnetic field
- Batchelor spectrum, i.e., $E_{\mathrm{M}} \propto k^4$ for small $k < k_*$
- Kolmogorov spectrum in the inertial range, i.e., $E_{\mathrm{M}} \propto k^{-5/3}$

$$kB_i = \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right) g_j \sqrt{2E_{\mathrm{M}}(k)},$$

$$E_{\mathrm{M}}(k) = E_{\mathrm{M}}^* \left(\frac{17}{5}\right)^{1/\alpha} \frac{(k/k_*)^4}{\left[1 + \frac{12}{5}(k/k_*)^{\alpha(4+5/3)}\right]^{1/\alpha}}; \quad \alpha = 2$$

[12] A. Brandenburg *et al.*, *Phys. Rev. D* **96**, 123528 (2017)
ARP *et al.*, *Phys. Rev. D* **102**, 083512 (2020)
ARP *et al.*, *JCAP* **04** (2022), 019
ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022)

Numerical results for decaying MHD turbulence

Free parameters on the initial conditions

- Magnetic energy density at t_* is a fraction of the radiation energy density, $\Omega_{\mathrm{M}} = \mathcal{E}_{\mathrm{M}}/\mathcal{E}_{\mathrm{rad}}^* = \frac{1}{2}B_0^2 \leq 0.1$ (BBN limit).

- Spectral peak k_*, normalized by H_*/c, is given by the characteristic scale of the sourcing turbulence (as a fraction of the Hubble radius) and should be $k_* \geq 2\pi$ by causality.

- Time t_* at which the magnetic field is generated, corresponding to the temperature scale T_* (e.g., $T_* \sim 150$ MeV at the QCD phase transition).

Numerical results for decaying MHD turbulence[13]

run	Ω_M^*	$k_* \mathcal{H}_*^{-1}$	$\mathcal{H}_* \delta t_e$	$\mathcal{H}_* \delta t_{fin}$	$\Omega_{GW}^{num}(k_{GW})$	$[\Omega_{GW}^{env}/\Omega_{GW}^{num}](k_{GW})$	n	$\mathcal{H}_* L$	$\mathcal{H}_* t_{end}$	$\mathcal{H}_* \eta$
A1	9.6×10^{-2}	15	0.176	0.60	2.1×10^{-9}	1.357	768	6π	9	10^{-7}
A2	–	–	–	–	–	–	768	12π	9	10^{-6}
E1	8.1×10^{-3}	6.5	1.398	2.90	5.5×10^{-11}	1.184	512	4π	8	10^{-7}
E2	–	–	–	–	–	–	512	10π	18	10^{-7}
E3	–	–	–	–	–	–	512	20π	61	10^{-7}
E4	–	–	–	–	–	–	512	30π	114	10^{-7}
E5	–	–	–	–	–	–	512	60π	234	10^{-7}
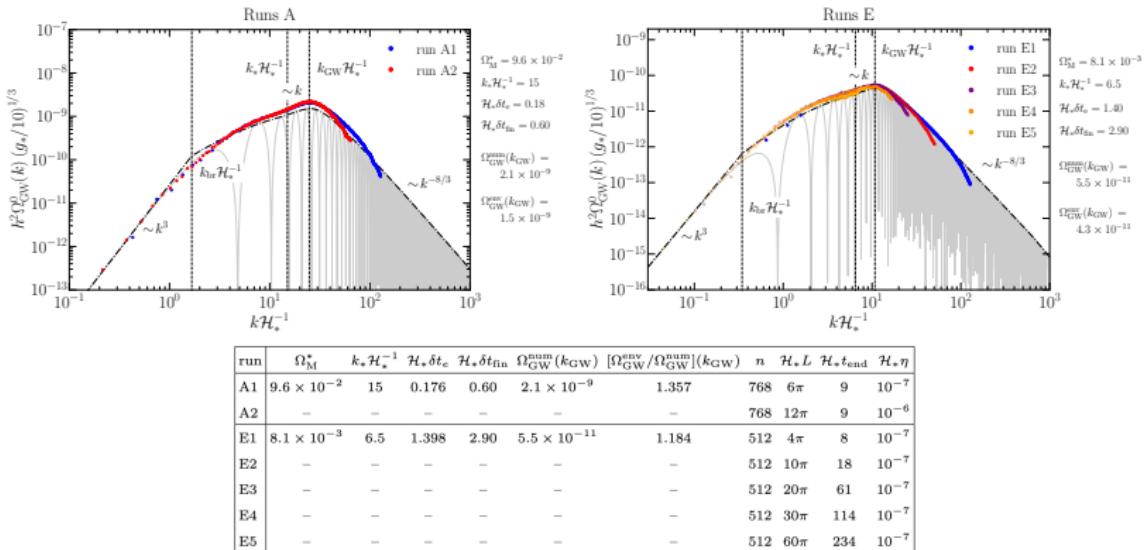
[13] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022)

Analytical model

- Assumption: magnetic field evolution ($\delta t_{\rm e} \sim 1/(v_{\rm A} k_*)$) is slow compared to the GW dynamics ($\delta t_{\rm GW} \sim 1/k$) at all $k \lesssim k_*$

- We can derive an analytical expression for the envelope over the oscillations[14] of $\Omega_{\rm GW}(k)$

$$
\Omega_{\rm GW}(k, t_{\rm fin}) \approx 3 \left(\frac{k}{k_*}\right)^3 \Omega_{\rm M}^{*\,2} \frac{\mathcal{C}(\alpha)}{\mathcal{A}^2(\alpha)} \, p_\Pi\left(\frac{k}{k_*}\right)
$$
$$
\times \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\,\delta t_{\rm fin} < 1, \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\,\delta t_{\rm fin} \geq 1. \end{cases}
$$

[14] ARP et al., Phys. Rev. D **105**, 123502 (2022)

Analytical model

- The effective duration of the source t_{fin} that determines the position of the k^3 break is not given by the analytical model.
- We expect it to be a function of the eddy turnover time t_e
- From the simulations, we obtain the following empirical fit:

Using PTA to constrain the primordial magnetic field[15]

$T_* = 150$ MeV, $g_* = 15$

- NANOGrav broken PL
- NANOGrav single PL
- PPTA
- EPTA
- IPTA

$h^2 \Omega_{GW}^0(f)$

f [Hz]
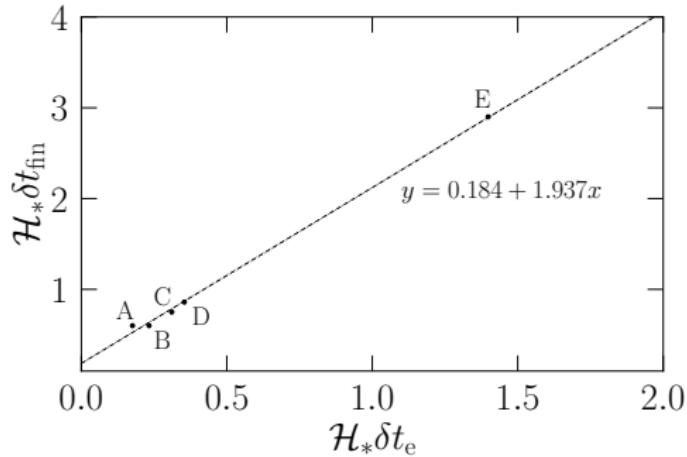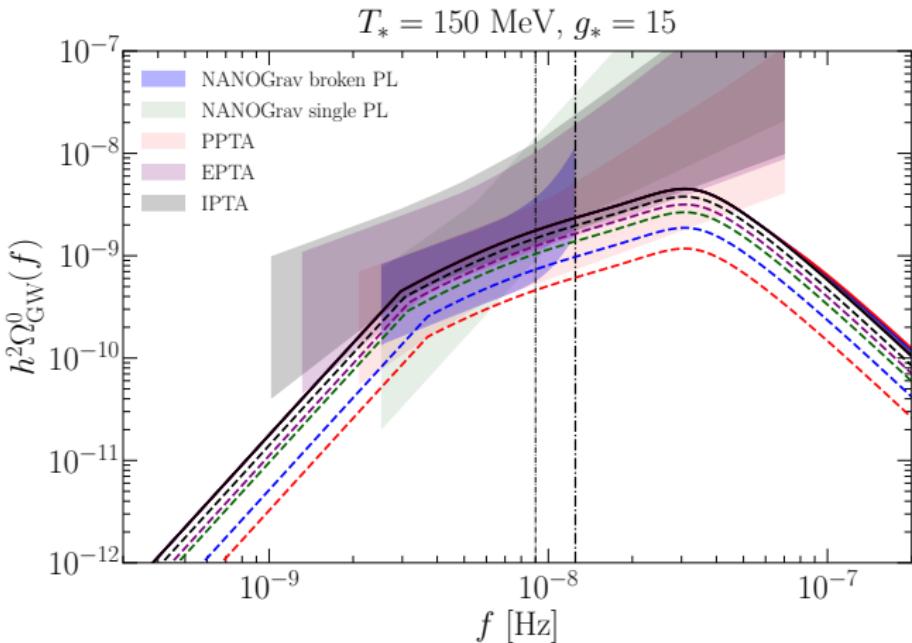
[15] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022)

Using PTA to constrain the primordial magnetic field[16]

[16] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022)
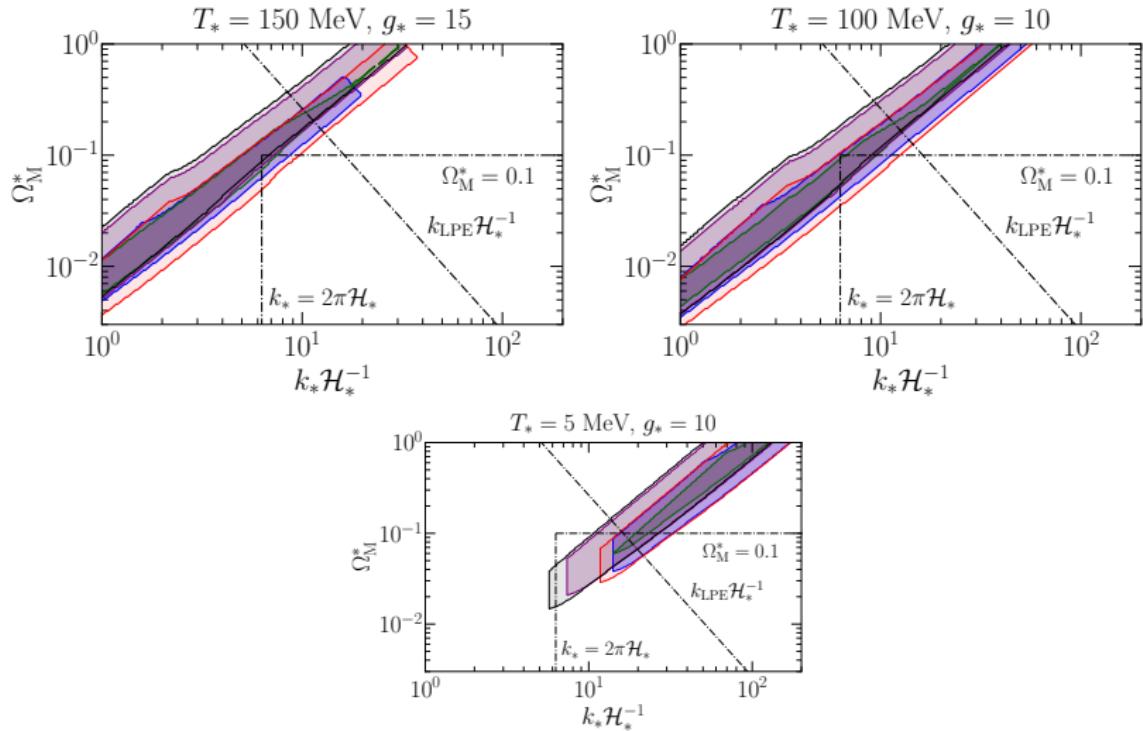
Using PTA (and LISA) to constrain the primordial magnetic field[17]
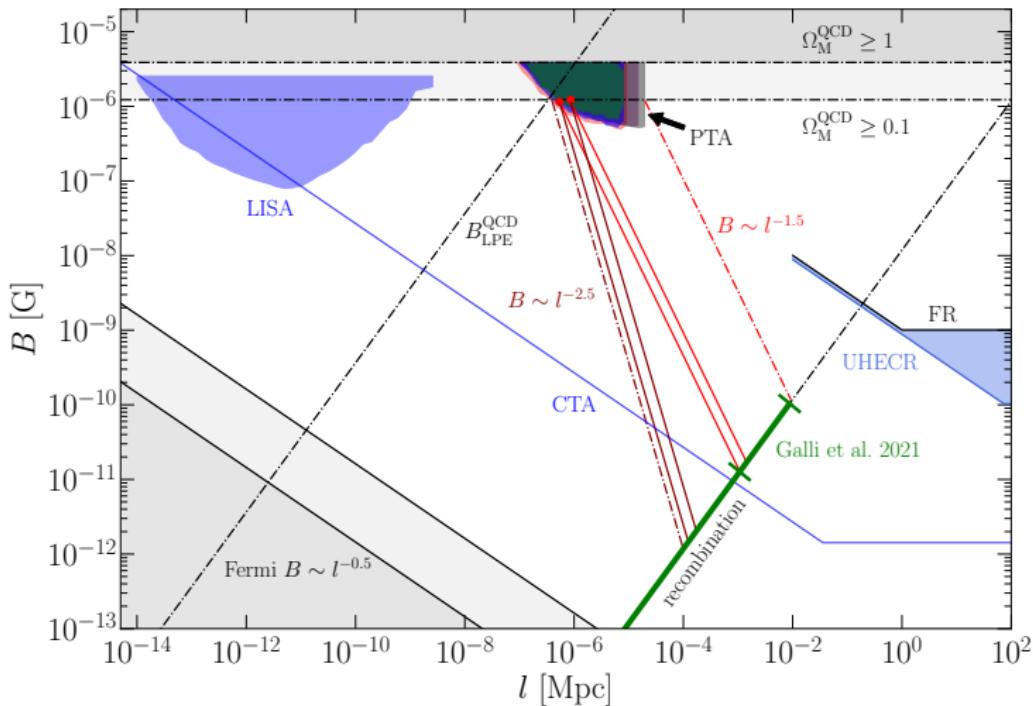
[17] ARP *et al.*, *Phys. Rev. D* **105**, 123502 (2022)

Conclusions

- We have performed simulations of MHD turbulence to study the stochastic GW background produced by primordial magnetic fields during the radiation-dominated era (e.g., at the scale of the QCD phase transition)

- The dynamical evolution of the magnetic fields is slow compared to the time scales of the SGWB production, which allows to assume constant magnetic stresses

- The analytical model derived works very well to predict the SGWB shape and amplitude, especially for magnetic fields with low eddy turnover times

- The SGWB presents a f^3 spectrum at low frequencies and a transition towards f below the spectral peak at $f_{\mathrm{GW}} > f > 1/\delta t_{\mathrm{fin}}$

- The effective duration of the source cannot be predicted from the analytical model, so we have used numerical simulations to obtain a fit as a function of the eddy turnover time

- We have constrained the parameters of the magnetic field using the analytical model compared to the observations from the PTA collaborations:

$$2 < T_*/\mathrm{MeV} < 200, \quad \Omega_{\mathrm{M}}^* > 0.01, \quad l_* > 0.1\,\mathcal{H}_*$$

- The derived model has also been applied to LISA and it can be applied to other detectors, e.g., Einstein Telescope

The End
Thank You!

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github.com/AlbertoRoper/GW_turbulence
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