12th Iberian Gravitational Waves Meeting

June 2022, 8th, from Braga



Lucas Pinol Instituto de Física Teórica (IFT) UAM-CSIC Based on:

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

TENSOR PRIMORDIAL NON-GAUSSIANITIES AND INDUCED ANISOTROPIES IN THE SGWB

Stochastic Gravitational Wave Background



(a) One-loop tensor power spectrum



(b) One-loop scalar-tensortensor bispectrum (c) One-loop tensor two-point function in the presence of a classical scalar source.

I will try to connect to other talks of the conference in blue

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II. Tensor PNGs

And induced anisotropies in the SGWB

Stochastic Gravitational Wave Background

I. Primordial Non-Gaussianities (PNGs) Quick introduction and definitions



Non-linearities in the sky



Sources of non-Gaussianity:

- Foreground
- Late-time evolution: lensing, etc.
- Early-time evolution: gravity, interactions, etc.
- **Initial conditions:**

Primordial non-Gaussianities from inflation

Planck CMB intensity maps

[Chen 2010]

 $T_{\rm ini}(\theta,\varphi) = T_{\rm ini}^G(\theta,\varphi) + f_{\rm NL}^{\rm local} \times \left[T_{\rm ini}^G(\theta,\varphi)\right]^2$ **<u>Reviews</u>:** [Bartolo, Komatsu, Matarrese, Riotto 2004] *Non-Gaussian if* $f_{\rm NL}^{\rm local} \neq 0$ Gaussian

PRIMORDIAL BISPECTRUM

Power spectrum = 2.10×10^{-9}

 ζ the primordial curvature perturbation

$$<\zeta_{\vec{k}_{1}}\zeta_{\vec{k}_{2}}\zeta_{\vec{k}_{3}}>=(2\pi)^{7}\,\delta^{(3)}\left(\overrightarrow{k_{1}}+\overrightarrow{k_{2}}+\overrightarrow{k_{3}}\right)\frac{A_{s}^{2}}{(k_{1}k_{2}k_{3})^{2}}\times S(k_{1},k_{2},k_{3})$$

$$\vec{k}_{3}$$

$$\vec{k}_{2}$$
Shape function

Similar definitions for tensor and mixed NGs: $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \gamma_{\vec{k}_2} \rangle$, $\langle \zeta_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle$

 k_1

$$\begin{split} \vec{k}_{1} \vec{\zeta}_{\vec{k}_{2}} \vec{\gamma}_{\vec{k}_{3}} >, & < \vec{\zeta}_{\vec{k}_{1}} \vec{\gamma}_{\vec{k}_{2}} \vec{\gamma}_{\vec{k}_{3}} >, & < \vec{\gamma}_{\vec{k}_{1}} \vec{\gamma}_{\vec{k}_{2}} \vec{\gamma}_{\vec{k}_{3}} > \\ SST & STT & TTT \end{split}$$

 γ the primordial tensor perturbation





ζ The curvature perturbation

 $<\zeta_{\vec{k}_{1}}\zeta_{\vec{k}_{2}}\zeta_{\vec{k}_{3}}>=(2\pi)^{7}\,\delta^{(3)}\left(\overrightarrow{k_{1}}+\overrightarrow{k_{2}}+\overrightarrow{k_{3}}\right)\frac{A_{s}^{2}}{(k_{1}k_{2}k_{3})^{2}}\times S(k_{1},k_{2},k_{3})$

 \vec{k}_2

 \vec{k}_3

 \vec{k}_1

Shape function

Power spectrum = 2.10×10^{-9}

Shape templates

Ex: Single-field inflation (attractor)

n $S = \frac{5}{12}(1 - n_s)S_{\text{loc}} + \frac{\epsilon}{8}S_{\text{eq}} + \dots = \text{VERY SMAL}$ s) $f_{\text{NL}}^{\text{loc}} = f_{\text{NL}}^{\text{eq}}$



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OBSERVATIONAL CONSTRAINTS

 ζ the primordial curvature perturbation, γ the primordial tensor perturbation

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 $\rightarrow f_{\rm NL}^{\rm sq} = -0.9 \pm 5.1$ [Planck 2018]

Shape function

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$$<\zeta_{\vec{k}_{1}}\zeta_{\vec{k}_{2}}\zeta_{\vec{k}_{3}}>=(2\pi)^{7} \delta^{(3)}(\vec{k}_{1}+\vec{k}_{2}+\vec{k}_{3})\frac{A_{s}^{2}}{(k_{1}k_{2}k_{3})^{2}}\times S(k_{1},k_{2},k_{3})$$

$$\rightarrow f_{\rm NL}^{\rm sq}=-0.9\pm5.1 \quad \text{[Planck 2018]} \quad \vec{k}_{2}$$
Shape function
$$\vec{k}_{1}$$

12

for tensor and mixed NGs:
$$<\zeta_{\vec{k}_{1}}\zeta_{\vec{k}_{2}}\gamma_{\vec{k}_{3}}>, < <\zeta_{\vec{k}_{1}}\gamma_{\vec{k}_{2}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}_{1}}\gamma_{\vec{k}_{2}}\gamma_{\vec{k}_{3}}>, < \gamma_{\vec{k}_{1}}\gamma_{\vec{k}_{2}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}_{1}}\gamma_{\vec{k}_{2}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_{\vec{k}}\gamma_{\vec{k}_{3}}\gamma_{\vec{k}_{3}}>, < <\gamma_{\vec{k}}\gamma_$$

OBSERVATIONAL CONSTRAINTS

 ζ the primordial curvature perturbation, γ the primordial tensor perturbation





Stochastic Gravitational Wave Background

II. Tensor PNGs And induced anisotropies in the SGWB



(a) One-loop tensor power spectrum



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(c) One-loop tensor two-point function in the presence of a classical scalar source.

THE SGWB: ~30 DECADES OF SCALES

At CMB scales, we have the strong constraint $A_t = r_{\star} A_s < 10^{-11}$

But much more freedom at smaller scales (higher frequencies)



THE STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

Many sources! Astrophysical, cosmological... How to disentangle them? Ivan Martin Vilchez

DISTINCTIVE FEATURES OF THE SGWB

Frequency profile

$$\overline{\Omega}_{GW}(f) = \Omega_0 \left(\frac{f}{f_*}\right)^{\mathbf{n}_{GW}(f)}$$



Having access to several orders of magnitude in frequency can help

[Auclair *et al.*, LISA CWG 2022]



 10^{-1}

 10^{0}



[Many many works, sorry for not showing yours]

DISTINCTIVE FEATURES OF THE SGWB

Antonio Manso

Often in the context of a Cherns-Simon term

Chirality

• Gauge fields: $g(\chi)F^{a\mu\nu} \tilde{F}^a_{\mu\nu} \in \mathcal{L}$

[Anber, Sorbo 2010, 2011] [Barnaby, Peloso 2011] [Dimastrogiovanni, Peloso 2013] [Adshead, Martinec, Wyman 2013] [Dimastrogiovanni, Fasiello, Fujita 2016] [Watanabe, Komatsu 2020]

• Beyond GR: $g(\chi)R^{\mu\nu} \tilde{R}_{\mu\nu} \in \mathcal{L}$

[Bartolo, Orlando 2017, 2018]

Unstable polarisation that sources **chiral** GWs:

$$\gamma_L \gg \gamma_R$$

Chirality $\chi = \frac{|P_{\gamma}^L - P_{\gamma}^R|}{P_{\gamma}^{tot}}$ can be measured

Also the possibility of other modes in the GWs:

Josu Aurrekoetxea

DISTINCTIVE FEATURES OF THE SGWB

Anisotropies:

 $\Omega_{GW}(f,\hat{n}) = \overline{\Omega}_{GW}(f) \left(1 + \boldsymbol{\delta}_{GW}(f,\hat{n})\right)$

$$\boldsymbol{a}_{\ell,\boldsymbol{m}} = \int \mathrm{d}\Omega \ Y_{\ell,\boldsymbol{m}}(\hat{n}) \boldsymbol{\delta}_{\boldsymbol{GW}}(\hat{\boldsymbol{n}})$$
$$\boldsymbol{C}_{\ell} = \frac{1}{2\ell+1} \sum_{\boldsymbol{m}} \boldsymbol{a}_{\ell,\boldsymbol{m}}^{*} \boldsymbol{a}_{\ell,\boldsymbol{m}}$$

Different sources give different anisotropies



[Bartolo et al., LISA Cosmology Working Group 2022]

SEVERAL SOURCES OF ANISOTROPIES

- GWs signal from astrophysical sources expected to be anisotropic [Cusin et al. 2017, 2018, 2019]
 [Bertacca et al. 2019]
 [Bellomo et al. 2021]
- Cosmological background propagates through structures → anisotropic
 [Alba, Maldacena 2015]
 [Contaldi et al. 2016]
 [Bartolo et al. 2018, 2019] These anisotropies inherit a non-Gaussian statistics from propagation
 [Domcke, Jinno, Rubira 2020]
- Primordial NGs also induce anisotropies:
 [Jeong, Kamionkowski 2012] Anisotropies of the LSS from the same effect
 [Brahma, Nelson, Chandera 2013]
 [Dimastrogiovanni et al. 2014, 2015, 2021]

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 [Dimastrogiovanni et al. 2014, 2015, 2021]

Squeezed: $k_S \gg k_L$

PNG-INDUCED ANISOTROPIES

• The idea:

Consider the modulation of **two short modes** by a **long one**:

Seen from far away the signal is anisotropic

 $\delta_{LSS}(k_S, \hat{n}); \delta_{GW}(f_S, \hat{n}) \propto \langle Y_S Y_S \rangle_{X_L} \propto \langle Y_S Y_S X_L \rangle$ $f_{NL,sq}^{YYX} \text{ from inflation: PNG}$ [Jeong, Kamionkowski 2012]

Here Y can be a scalar (anisotropies of the LSS) or a tensor (anisotropies of the SGWB)

Also X can be a scalar (modulation by a scalar mode) or a tensor (modulation by a tensor mode)

[Dimastrogiovanni, Fasiello, LP 2022] *ArXiv:2203.17192*

• Formal derivations with the **in-in formalism**:

$$\left\langle \hat{\mathcal{O}}(t) \right\rangle = \left\langle 0 \left| \left. \bar{T} \left(e^{i \int_{-\infty^+}^t \mathrm{d}t' \hat{H}^I_{\mathrm{int}}(t')} \right) \hat{\mathcal{O}}^I(t) T \left(e^{-i \int_{-\infty^-}^t \mathrm{d}t'' \hat{H}^I_{\mathrm{int}}(t'')} \right) \left| \left. 0 \right\rangle \right. \right\rangle$$

Rigorous quantum computation of correlation functions in cosmology ≠ Scattering amplitudes in particle physics

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

- Formal derivations with the in-in formalism:
 - We look for interactions between small and large scales $\rightarrow f_{\text{NL},\gamma\gamma\gamma}^{\text{sq}}$ and $f_{\text{NL},\gamma\gamma\zeta}^{\text{sq}} \rightarrow \langle \gamma_{S}\gamma_{S}X_{L}\rangle$

with $X_L \in \{\gamma_L, \zeta_L\}$

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

- Formal derivations with the in-in formalism:
 - ♦ We look for interactions between small and large scales → $f_{NL,\gamma\gamma\gamma}^{sq}$ and $f_{NL,\gamma\gamma\zeta}^{sq}$
 - * A long-wavelength source J_L can be treated classicaly and has negligible derivatives:

$$\hat{J}_{L} = J_{L}(\tau)\hat{a}_{\vec{k}} + J_{L}^{*}(\tau)\hat{a}_{-\vec{k}}^{\dagger} \xrightarrow{}_{-k\tau \to 0} J_{L}^{\text{cl}}(\tau)\left(\hat{a}_{\vec{k}} + \hat{a}_{-\vec{k}}^{\dagger}\right) ; \quad \left(\partial_{i}J_{L}^{\text{cl}}, \partial_{t}J_{L}^{\text{cl}}\right) \text{ are negligible}$$

$$\left[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}^{\dagger}\right] = (2\pi)^{3}\delta^{(3)}(\vec{k} - \vec{k}') \qquad \qquad \boldsymbol{b}_{\vec{k}} \quad \left[b_{\vec{k}}, b_{\vec{k}'}^{\dagger}\right] = 0$$

J can be X or not, here we keep it generic to treat multifield scenarios, e.g. $X = \zeta$ and $J = \sigma$

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

- Formal derivations with the in-in formalism:
 - ♦ We look for interactions between small and large scales $\rightarrow f_{\text{NL},\gamma\gamma\gamma}^{\text{sq}}$ and $f_{\text{NL},\gamma\gamma\zeta}^{\text{sq}}$
 - * A long-wavelength source J_L can be treated classicaly and has negligible derivatives:
 - * A **3-pt interaction** involving J_L becomes a **2-pt interaction** times a classical source J_L^{cl}



[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

 $<\gamma_{\vec{k}_1}\gamma_{\vec{k}_2}>_{J^{\mathrm{cl}}}$

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 2-pt functions in the presence of a classical source are now defined:

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 - 2-pt functions in the presence of a classical source are now defined
 - We compute both diagrams with the in-in formalism and are therefore able to relate them:

$$\langle \gamma_{\vec{k}_{1}} \gamma_{\vec{k}_{2}} \rangle_{J^{\text{cl}}} \stackrel{=}{|\vec{k}_{1} + \vec{k}_{2}| \ll k_{1,2}} \int d^{3}\vec{q} \ \delta^{(3)} (\vec{q} + \vec{k}_{1} + \vec{k}_{2}) \frac{B_{\text{sq}}^{\gamma\gamma X} (\vec{k}_{1}, \vec{k}_{2}, \vec{q})}{P_{JX}(q)} J^{\text{cl}}(\vec{q})$$

Non-diagonal part, $\vec{k}_1 + \vec{k}_2 \neq \vec{0}$, of the 2-pt function does not vanish \rightarrow anisotropies

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

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 - 2-pt functions in the presence of a classical source are now defined
 - We compute both diagrams with the in-in formalism and are therefore able to relate them:

$$\langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{\boldsymbol{J}^{\text{cl}}} = \int d^3 \vec{q} \ \delta^{(3)} \left(\vec{q} + \vec{k}_1 + \vec{k}_2 \right) P_{\gamma}(k_1) \boldsymbol{f}_{\text{NL}, \boldsymbol{\gamma} \boldsymbol{\gamma} \boldsymbol{X}}^{\text{sq}} \left(\vec{k}_1, \vec{k}_2, \vec{q} \right) \boldsymbol{J}^{\text{cl}}(\vec{\boldsymbol{q}})$$

 $\boldsymbol{J^{\mathrm{cl}}(\vec{\boldsymbol{q}})} \text{ is a statistical quantity} \to \text{ so is } \langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{J^{\mathrm{cl}}} \to \langle \delta(\hat{n}_1) \delta(\hat{n}_2) \rangle \propto \langle J^{\mathrm{cl}}(\vec{q}) J^{\mathrm{cl}}(\vec{q}') \rangle \neq 0$

MULTIFIELD MODELS WITH LARGE ANISOTROPIES

- Spin-2 EFT of inflation: $\sigma_{ij} = \partial_i \partial_j \sigma^{(0)} + \sigma_{ij}^{(2)}$ [Bordin *et al.* 2018]
- $\rightarrow \sigma^{(2)}$ couples linearly to γ and can make the tilt blue: $n_t > 0$ for half of parameter space ($\dot{c}_2 < 0$)

We compute anisotropies explicitly and find: |<

$$: \sqrt{\langle \delta_{GW}^2(k_S, \hat{n}) \rangle} \sim \frac{\langle \gamma \gamma \zeta \rangle (k_S, k_S, k_L)}{P_{\gamma}(k_S) P_{\zeta \sigma^{(0)}}(k_L)} \sqrt{\mathcal{P}_{\sigma^{(0)}}(k_L)}$$



(a) Mixed scalar-tensor-tensor bispectrum.

(b) Tensor two-point function in the presence of a classical scalar source.

[Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

MULTIFIELD MODELS WITH LARGE ANISOTROPIES

adiabatic

entropic

 $\gg 1$

- Supersolid inflation: two fundamental scalar fluctuations (ζ_n, R_{π_0}) [Celoria *et al.* 2021]
- $\rightarrow R_{\pi_0}$ couples **quadratically** to γ and can make the tilt blue: $n_t = 2(n_s^{en} 1) > 0$

We compute anisotropies explicitly and find: $\sqrt{\langle \delta_{GW}^2(k_S, \hat{n}) \rangle} \sim f_{NL,sq}^{\gamma\gamma\zeta}(k_S, k_S, k_L) \left(\sqrt{\frac{\mathcal{P}_{\zeta_n} \mathcal{P}_{R_{\pi_0}}}{\mathcal{P}_{\zeta_n R_{\pi_0}}}} \right) \frac{A_s^{1/2}}{k_L}$

 $\mathcal{O}(\mathbf{1})$



(b) One-loop scalar-tensortensor bispectrum



(c) One-loop tensor two-point function in the presence of a classical scalar source. [Dimastrogiovanni, Fasiello, LP 2022] ArXiv:2203.17192

CONCLUSION

 \succ Primordial NGs contain much more information than a single number $f_{\rm NL}^{\rm local}$

> We do not know much about tensor primordial NGs

Squeezed tensor primordial NGs survive in the form of induced anisotropies in the SGWB, allowing for a new probe of the inflationary field content and interactions

> Formidable opportunity to use the nonlinear Universe as a particle detector

$$\delta_{\rm GW}(\hat{n}) \propto \langle \gamma_{\vec{k}_1} \gamma_{\vec{k}_2} \rangle_{\zeta^{\rm cl}} = \int d^3 \vec{q} \ \delta^{(3)} \left(\vec{q} + \vec{k}_1 + \vec{k}_2 \right) P_{\gamma}(k_1) f_{\rm NL,sq}^{\gamma\gamma\zeta} \left(\vec{k}_1, \vec{k}_2, \vec{q} \right) \boldsymbol{\zeta^{\rm cl}}(\vec{\boldsymbol{q}})$$

<u>Note</u>: $\boldsymbol{\zeta^{cl}}(\vec{q})$ is the seed of anisotropies in the CMB! Cross-correlations, *e.g.*, $\langle \delta_{GW} \delta T \rangle \neq 0$

*l***-DEPENDENCE**

Anisotropies:

 $\Omega_{GW}(f,\hat{n}) = \overline{\Omega}_{GW}(f) (1 + \boldsymbol{\delta}_{GW}(f,\hat{n}))$

$$\boldsymbol{a}_{\ell,\boldsymbol{m}} = \int \mathrm{d}\Omega \ Y_{\ell,\boldsymbol{m}}(\hat{n}) \boldsymbol{\delta}_{\boldsymbol{G}\boldsymbol{W}}(\hat{\boldsymbol{n}})$$
$$\boldsymbol{C}_{\ell} = \frac{1}{2\ell+1} \sum_{\boldsymbol{m}} \boldsymbol{a}_{\ell,\boldsymbol{m}}^{*} \boldsymbol{a}_{\ell,\boldsymbol{m}}$$



[Dimastrogiovanni et al. 2021]

Scalar PNGs The rich multifield phenomenology

