

Search for gravitational waves from black hole hyperbolic encounters in LIGO-Virgo

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Motivation

- Some models predict dense black hole clusters in our universe
 - Primordial black hole models
 - Models for the centers of galaxies and globular clusters
- Black holes in dense clusters will scatter off each other in hyperbolic orbits.
- Sufficiently close scatters will emit gravitational waves (GWs) detectable by current interferometers on Earth.
- Before this work, no targeted search for close hyperbolic encounters (CHEs) had been carried in GW detectors.

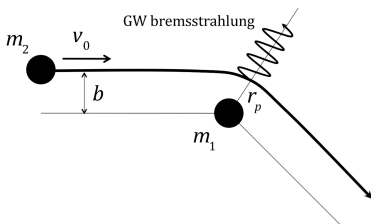


Figure 1: Schematic representation of a Hyperbolic Encounter.

Steps to search for CHE in GW detectors

- Theoretical templates of the GWs emitted by CHE.
- Characterize the signal GWs from CHE leave in laser interferometers.
- Data processing to make the CHE signal stand out over detector noise.
- Develop a trigger to select the data that might contain a CHE. Our trigger will consist on:
 - 1 Data preselection based on correlations between interferometers.
 - 2 Convolutional Neural Network to determine which correlations can come from CHE.

CHE templates

- Problem: Scattering of two gravitationally interacting masses m_1 and m_2 with spins \vec{S}_1 and \vec{S}_2 .
- No analytical solution in General Relativity (GR).
- Numerical Relativity is computationally cost prohibitive.
- We can use the Post Newtonian (PN) approximation because:
 - BHs do not get as close as in CBC (there is no merger).
 - Accurately following the phase for many cycles is not as critical.
- We take up to leading order spin effects $\rightarrow O(1/c^3) \rightarrow 1.5\text{PN}$.

GW polarizations

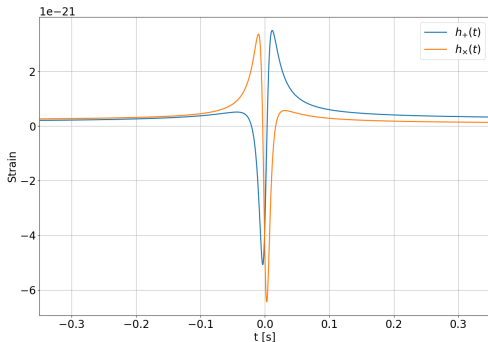


Figure 2: Representative gravitational waves polarization emitted by maximally spinning black holes with $m_1 = 20M_\odot$, $m_2 = 15M_\odot$, $b = 70GM/c^2$, $e_{t0} = 1.1$, $\Phi_0 = 0$, $\theta_1^i = 0.5$ rad, $\phi_1^i = 0.35$ rad, $\theta_2^i = 0.8$ rad, $\phi_2^i = 1$ rad. $t = 0$ represents the time of closest approach.

- Quadrupolar nature of the GWs $\rightarrow f_{\text{GW}} = 2f_{\text{orbit}}$.
- CHE perform “half” of an orbit \rightarrow GWs from CHE perform one oscillation.

Projected GWs into the detector

- The signal the GWs leave in the interferometers is simulated by projecting them into each detector and taking into account light travel time delays.

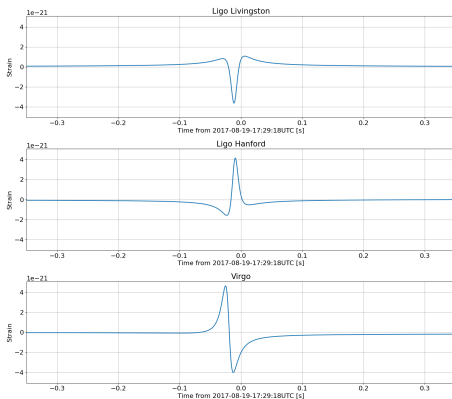


Figure 3: Result of projecting the GWs of Fig. 2 into the GW detectors, assuming that they come from $\delta = 1.0\text{rad}$, $\alpha = 3.7\text{rad}$, with $\psi = 0.2\text{rad}$ and with the periastron time taking place at 17:29:18 UTC of 2017-08-19 at the center of the Earth.

Data Processing

- Filtering:
 - Below 20Hz → large increase in seismic and thermal noise.
 - Above 800Hz → small amount of CHE signal.
 - Remove noise dominated frequencies applying 20-800Hz band-pass.
- Whitening
 - Whiten to weigh down the strain in the noisiest regions and make all frequencies have same noise.
- Q transform:
 - GWs from CHEs only perform only one oscillation → extended time-frequency structure.

Data processing example

- Inject example CHE in the detector output: $s(t) = s_{exp}(t) + h_{CHE}(t)$
- Filter, whiten and Q transform the data containing the injection.

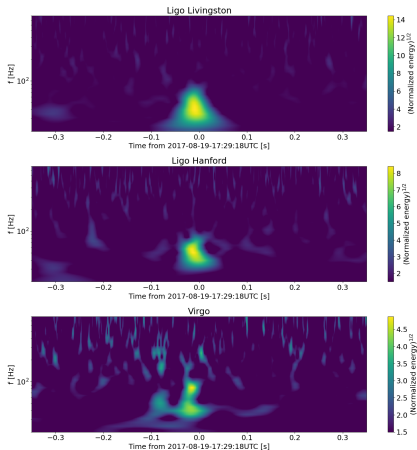


Figure 4: Square root of the normalized energy obtained by applying the data processing to the example CHE injected in the detector data.

Search for CHEs in LIGO-Virgo data

- CHEs look very similar to common blip glitches.
- Coincidences are very important to reject these glitches.
- More detectors online improve search sensitivity.
- Analyze public LIGO-Virgo O2 data with all three detectors in nominal operation → 15.3 effective days.
- Since CHEs only do one oscillation burst methods are competitive with and can outperform match filtering techniques for the search.
- Use a two step trigger to look for CHEs:
 - 1 Correlation trigger.
 - 2 Neural Network.

Correlation trigger

- We want a loose trigger to preselect events for the Neural Network.
- Look for correlations in the time-frequency domain with Pearson's coefficient:

$$r[x, y] = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{\sum_{n=0}^{N-1} (x_n - \bar{x})(y_n - \bar{y})}{\sqrt{\sum_{n=0}^{N-1} (x_n - \bar{x})^2} \sqrt{\sum_{n=0}^{N-1} (y_n - \bar{y})^2}},$$

- Trigger discriminant: $D = ar_{L1-H1} + (1 - a)r_{L1-V1}$
- In O2 we determine that for CHE signals, optimally $a \sim 0.67$
- Preselect images with $D > 0.3$

Testing correlation trigger with injections

- What fraction of CHEs can we detect as a function of their SNR?
- We inject CHEs with random parameters and run the correlation trigger over them.

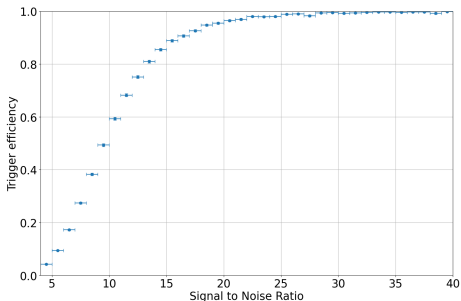


Figure 5: Trigger efficiency as a function of the signal to noise ratio of the injected events.

- We are accepting most events with SNR > 10 .

Trigger False Alarm Rate

- We estimate the number of expected noise events by running the trigger on shifted data.
- As expected, events with larger discriminant are increasingly uncommon.

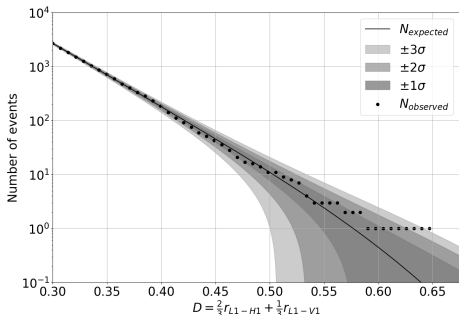


Figure 6: Number of events above a certain discriminant observed and expected.

$$\sigma = \sqrt{N_{expected}}$$

- Total preselected ($D > 0.3$) events in the data: 2704

Neural Network

- We want to classify which of the 2704 preselected images come from noise and which from CHE.
- Neural networks (NN) are mathematical models to analyze data using Machine Learning.

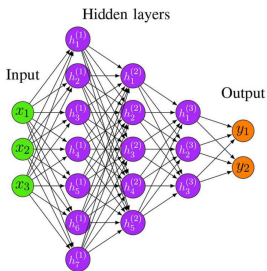


Figure 7: NN sketch. [F. Emmert-Streib (2020)]

- NN input: pixels of the image to be classified.
- NN output: probability of the image to be a CHE.
- Hidden layers:
 - Used for internal calculations only.
 - Contain free parameters to fit training data.

- We use one of the most refined architectures for image classification: Residual Convolutional Neural Network

Image samples

- Put the normalized energy of the three detectors in the same image.
- Image duration: 0.7s



(a) CHE image.



(b) Noise image.



(c) Data Image.

Figure 8: Examples of images used to train and validate our neural network as well as to make predictions on the data.

Neural Network training and performance

- Training the NN means finding the internal parameters such that:
 - If input is a CHE image \rightarrow return 1
 - If input is a noise image \rightarrow return 0
- NN output is then interpreted as CHE probability.
- Train the NN on 64028 noise images and 45356 CHE images.
- We consider an image a CHE candidate if $p_{\text{CHE}} > 0.9$

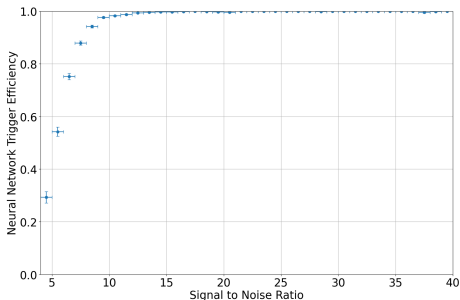


Figure 9: NN trigger efficiency as a function of the signal to noise ratio of the CHE test events.

- We are able to detect most CHEs with signal to noise ratio above 5

Neural Network results on Data

- Distribution of data with CHE probability can be compared with expectation if only noise was present.

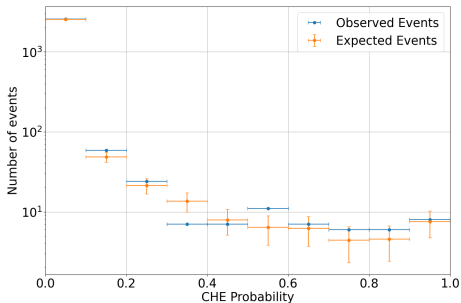


Figure 10: Number of observed events and expected noise events as a function of CHE probability given by the NN. The number of expected noise events is the number of events we would expect to see if the data only contained noise.

$$\Delta N_{\text{expected}} = \sqrt{N_{\text{expected}}}$$

- Consistently more events than expected at high CHE probability.
- At $p_{\text{CHE}} > 0.9 \rightarrow$ We expect 7.5 ± 2.7 noise events and we observe 8.

Most significant events

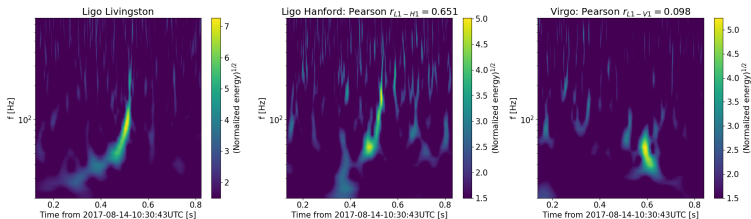


Figure 11: Event with $p_{\text{CHE}} = 0.997$ and correlation discriminant $D = 0.469$. This event is GW170814, the coalescence of two black holes of masses $31M_{\odot}$ and $25M_{\odot}$.

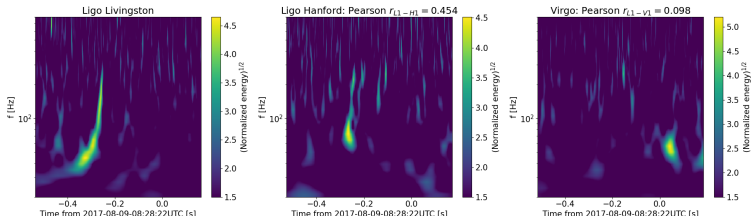


Figure 12: Event with $p_{\text{CHE}} = 0.991$ and correlation discriminant $D = 0.337$. This event is GW170809, the coalescence of two black holes of masses $35M_{\odot}$ and $24M_{\odot}$.

Possible CHEs

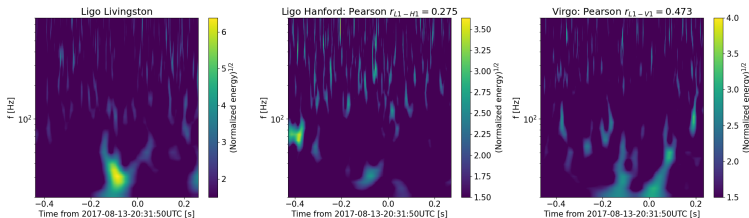


Figure 13: Event with $p_{\text{CHE}} = 0.980$ and correlation discriminant $D = 0.341$.

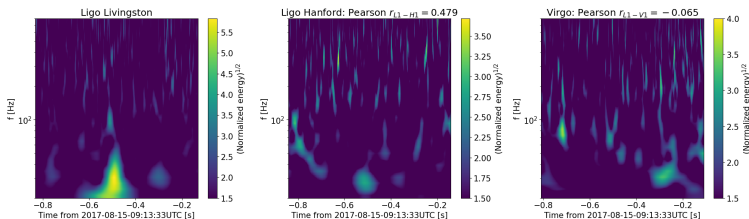


Figure 14: Event with $p_{\text{CHE}} = 0.976$ and correlation discriminant $D = 0.321$.

Conclusions

- Determined GWs emitted by CHEs and characterized how they look in real GW detectors.
- Looked for CHEs in LIGO-Virgo data using burst methods and machine learning.
- Found 2 of the 3 binary black hole mergers contained in the data analyzed.
- Obtained 6 candidate CHEs on which to perform further analysis.
- The number of events is consistent with just noise.

Backup Slides

Hamiltonian Formulation of the problem

- Hamiltonian of the system:

$$H(\vec{r}, \vec{p}, \vec{S}_1, \vec{S}_2) = H_N(\vec{r}, \vec{p}) + H_{1\text{PN}}(\vec{r}, \vec{p}) + H_{\text{SO}}(\vec{r}, \vec{p}, \vec{S}_1, \vec{S}_2) + O\left(\frac{1}{c^4}\right),$$

where

$$H_N(\vec{r}, \vec{p}) = \frac{p^2}{2} - \frac{1}{r},$$

$$H_{1\text{PN}}(\vec{r}, \vec{p}) = \frac{1}{c^2} \left(\frac{1}{8}(3\eta - 1)(p^2)^2 - \frac{1}{2} \left[(3 + \eta)p^2 + \eta(\hat{n} \cdot \vec{p})^2 \right] \frac{1}{r} + \frac{1}{2r^2} \right),$$

$$H_{\text{SO}}(\vec{r}, \vec{p}, \vec{S}_1, \vec{S}_2) = \frac{1}{c^2 r^3} (\vec{r} \times \vec{p}) \cdot \vec{S}_{\text{eff}},$$

- To get the equations of motion we use Poisson's brackets:

$$\{r_i, p_j\} = \delta_{ij},$$

$$\{S_{1i}, S_{1j}\} = \epsilon_{ijk} S_{1k},$$

$$\{S_{2i}, S_{2j}\} = \epsilon_{ijk} S_{2k}.$$

Solution for the orbit

- After some manipulation, the equations of motion can be efficiently solved to obtain the orbit.
- The orbit will depend on:
 - Black hole masses m_1, m_2
 - Black hole initial spins \vec{S}_1, \vec{S}_2
 - Initial eccentricity e_{t0}
 - Impact parameter b
 - Initial orbital azimuthal angle Φ_0
 - Orbital inclination angle Θ

$$b = 70 \text{ GM}/c^2 \quad v_{\max} = 0.36 c$$

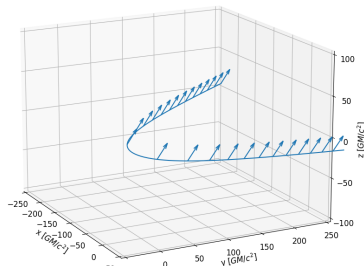


Figure 15: Example of an orbit for maximally spinning black holes with $m_1 = 20M_\odot$, $m_2 = 15M_\odot$, $b = 70 \text{ Gm}/c^2$, $e_{t0} = 1.1$, $\Phi_0 = 0$, $\theta_1^i = 0.5 \text{ rad}$, $\phi_1^i = 0.35 \text{ rad}$, $\theta_2^i = 0.8 \text{ rad}$, $\phi_2^i = 1 \text{ rad}$. The arrow represents \vec{S}_{eff} .

GWs derived from the orbit

- GWs can be computed from the orbit.
- Use formula with u up to leading order spin effects:

$$\begin{aligned}
 h_{\times} = & 4 \frac{Gm\eta}{c^4 R^2} \left\{ -(\mathbf{p} \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{n})z + (\mathbf{p} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v}) - \frac{\delta}{c} \left[([(3(N \cdot \mathbf{n})\dot{r} - (N \cdot \mathbf{v})](\mathbf{q} \cdot \mathbf{n}) - 3(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{v}))(\mathbf{p} \cdot \mathbf{n}) - 3(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{v}))z \right. \right. \\
 & + 2(\mathbf{p} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v})(N \cdot \mathbf{v}) + \frac{1}{6c^2} [6(1 - 3\eta)(N \cdot \mathbf{v})^2(\mathbf{p} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{v}) + [(6\eta - 2)(N \cdot \mathbf{v})^2(\mathbf{q} \cdot \mathbf{n}) + (48\eta - 16)(N \cdot \mathbf{v})(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{v})] \\
 & \times (\mathbf{p} \cdot \mathbf{n}) + (48\eta - 16)(N \cdot \mathbf{v})(N \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{n}) + ((-14 + 42\eta)(N \cdot \mathbf{n})^2 - 4 + 6\eta)(\mathbf{q} \cdot \mathbf{v})(\mathbf{p} \cdot \mathbf{v})z + (-9\eta + 3)(\mathbf{q} \cdot \mathbf{v}) \\
 & \times (\mathbf{p} \cdot \mathbf{v})v^2 + (29 + (7 - 21\eta)(N \cdot \mathbf{n})^2)(\mathbf{q} \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{n})z^2 + ((-9\eta + 3)(N \cdot \mathbf{n})^2 - 10 - 3\eta)(\mathbf{q} \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{n})zv^2 + [(-36\eta + 12) \\
 & \times (N \cdot \mathbf{v})(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{n}) + ((15 - 45\eta)(N \cdot \mathbf{n})^2 + 10 + 6\eta)(\mathbf{q} \cdot \mathbf{v})](\mathbf{p} \cdot \mathbf{n}) + [(15 - 45\eta)(N \cdot \mathbf{n})^2 + 10 + 6\eta](\mathbf{p} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{n})\dot{r}z \\
 & \left. + ((45\eta - 15)(N \cdot \mathbf{n})^2 - 9\eta + 3)(\mathbf{q} \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{n})\dot{r}^2z \right] + \frac{z^2(\mathbf{q} \cdot \mathbf{n})}{c^2} [X_2\chi_2(\mathbf{p} \cdot (\mathbf{s}_2 \times N)) - X_1\chi_1(\mathbf{p} \cdot (\mathbf{s}_1 \times N))] \Big\}, \\
 h_{+} = & 2 \frac{Gm\eta}{c^4 R^2} \left\{ [((\mathbf{q} \cdot \mathbf{n})^2 - (\mathbf{p} \cdot \mathbf{n})^2)z + (\mathbf{p} \cdot \mathbf{v})^2 - (\mathbf{q} \cdot \mathbf{v})^2] - \frac{\delta}{2c} [((N \cdot \mathbf{n})\dot{r} - (N \cdot \mathbf{v}))z(\mathbf{p} \cdot \mathbf{n})^2 - 6z(N \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{v}) + (-3(N \cdot \mathbf{n})\dot{r} \right. \\
 & + (N \cdot \mathbf{v})z(\mathbf{q} \cdot \mathbf{n})^2 + 6z(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{v}) + 2((\mathbf{p} \cdot \mathbf{v})^2 - (\mathbf{q} \cdot \mathbf{v})^2)(N \cdot \mathbf{v}) + \frac{1}{6c^2} [6(N \cdot \mathbf{v})^2((\mathbf{p} \cdot \mathbf{v})^2 - (\mathbf{q} \cdot \mathbf{v})^2)(1 - 3\eta) \\
 & + [(6\eta - 2)(N \cdot \mathbf{v})^2(\mathbf{p} \cdot \mathbf{n})^2 + (96\eta - 32)(N \cdot \mathbf{v})(N \cdot \mathbf{n})(\mathbf{p} \cdot \mathbf{v})(\mathbf{p} \cdot \mathbf{n}) + (-6\eta + 2)(N \cdot \mathbf{v})^2(\mathbf{q} \cdot \mathbf{n})^2 + (-96\eta + 32)(N \cdot \mathbf{v}) \\
 & \times (N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{n}) + [(-14 + 42\eta)(N \cdot \mathbf{n})^2 - 4 + 6\eta](\mathbf{p} \cdot \mathbf{v})^2 + [(-42\eta + 14)(N \cdot \mathbf{n})^2 + 4 - 6\eta](\mathbf{q} \cdot \mathbf{v})^2]z \\
 & + ((-9\eta + 3)(\mathbf{p} \cdot \mathbf{v})^2 + (-3 + 9\eta)(\mathbf{q} \cdot \mathbf{v})^2)v^2 + [(29 + (7 - 21\eta)(N \cdot \mathbf{n})^2)(\mathbf{p} \cdot \mathbf{n})^2 + [-29 + (21\eta - 7)(N \cdot \mathbf{n})^2](\mathbf{q} \cdot \mathbf{n})^2]z^2 \\
 & + (((-9\eta + 3)(N \cdot \mathbf{n})^2 - 10 - 3\eta)(\mathbf{p} \cdot \mathbf{n})^2 + ((-3 + 9\eta)(N \cdot \mathbf{n})^2 + 10 + 3\eta)(\mathbf{q} \cdot \mathbf{n})^2)zv^2 + ((-36\eta + 12)(N \cdot \mathbf{v})(N \cdot \mathbf{n}) \\
 & \times (\mathbf{p} \cdot \mathbf{n})^2 + ((-90\eta + 30)(N \cdot \mathbf{n})^2 + 20 + 12\eta)(\mathbf{p} \cdot \mathbf{v})(\mathbf{p} \cdot \mathbf{n}) + (-12 + 36\eta)(N \cdot \mathbf{v})(N \cdot \mathbf{n})(\mathbf{q} \cdot \mathbf{n})^2 + (90\eta - 30)(N \cdot \mathbf{n})^2 \\
 & - 12\eta - 20)(\mathbf{q} \cdot \mathbf{v})(\mathbf{q} \cdot \mathbf{n})\dot{r}z + [((45\eta - 15)(N \cdot \mathbf{n})^2 - 9\eta + 3)(\mathbf{p} \cdot \mathbf{n})^2 + ((15 - 45\eta)(N \cdot \mathbf{n})^2 - 3 + 9\eta)(\mathbf{q} \cdot \mathbf{n})^2]z^2 \Big\} \\
 & + \frac{z^2}{c^2} [(\mathbf{p} \cdot \mathbf{n})(X_2\chi_2[\mathbf{p} \cdot (\mathbf{s}_2 \times N)] - X_1\chi_1[\mathbf{p} \cdot (\mathbf{s}_1 \times N)]) + (\mathbf{q} \cdot \mathbf{n})(X_1\chi_1[\mathbf{q} \cdot (\mathbf{s}_1 \times N)] - X_2\chi_2[\mathbf{q} \cdot (\mathbf{s}_2 \times N)]] \Big\}
 \end{aligned}$$

Antenna Patterns

$$F_{+}(\theta, \phi, \psi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi ,$$
$$F_{\times}(\theta, \phi, \psi) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi .$$

- θ and ϕ give the direction of the GW in the frame in which the arms of the interferometer are in the x and y axis.
- ψ measures the polarization of the wave.
- Detectors have different locations and orientations $\rightarrow \theta, \phi$ and ψ are different in each one for a GW coming from a location in the sky with right ascension α and declination δ .
- GWs arrive at different times in different locations due to the light travel time.

Injected signal

- Current detectors are dominated by noise.
- Data processing suppresses the noise, enhancing the signal.

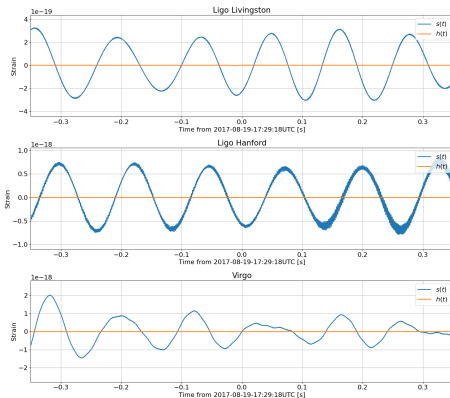


Figure 16: In blue, result $s(t)$ of injecting the example GWs into the experimental strain of the detectors. In orange, the GW signal $h(t)$. The noise is so dominating that the features of the gravitational wave signal can not be seen.

CHE frequency dependence

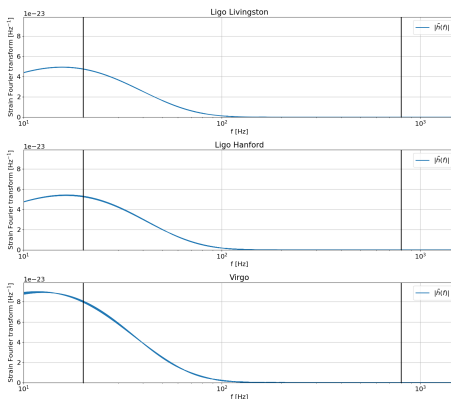


Figure 17: Fourier transform of the gravitational wave shown in Fig. 3. We have marked with black lines the 20-800Hz frequency window we are using to band-pass the strain.

Filtering

- Below 20Hz and above 800Hz data is dominated by noise.
- Remove noise dominated frequencies applying 20-800Hz band-pass.

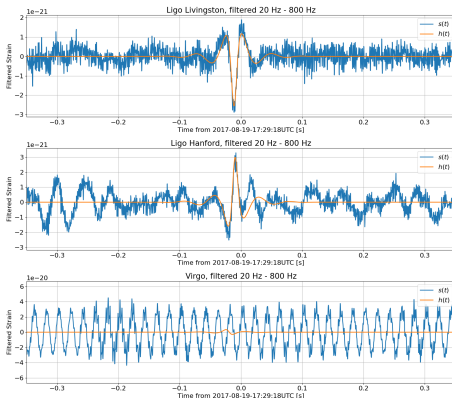


Figure 18: Result of band-pass filtering the example signals within a 20-800Hz window.

Whitening

- The signal in frequencies with more noise is less significant.
- Whiten to make all frequencies have same noise $\tilde{s}(f) = \frac{\tilde{s}(f)}{\sqrt{S_n(f)}}$.

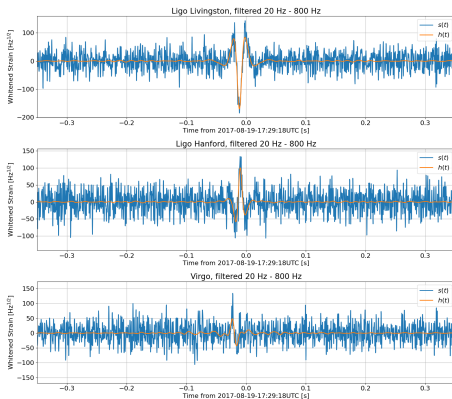


Figure 19: Result of whitening the example filtered signals.

3σ correlated event

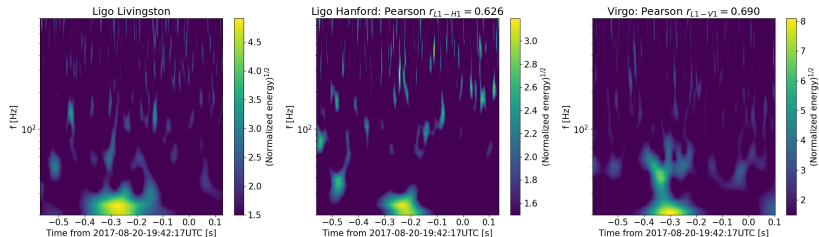


Figure 20: Event with the highest value of the trigger discriminant found in the analyzed data, with a value $D = 0.647$. From our false alarm rate analysis we expect an event like this every 240 days but we got one in 15.3 days of data.

Other CHE candidates I

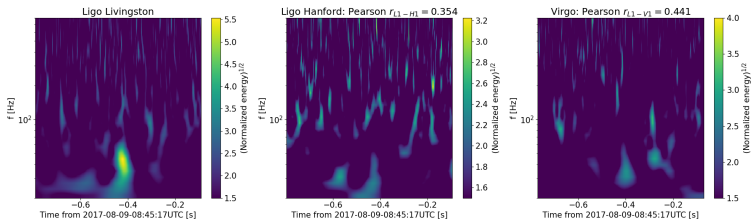


Figure 21: Event with $p_{\text{CHE}} = 0.974$ and correlation discriminant $D = 0.382$.

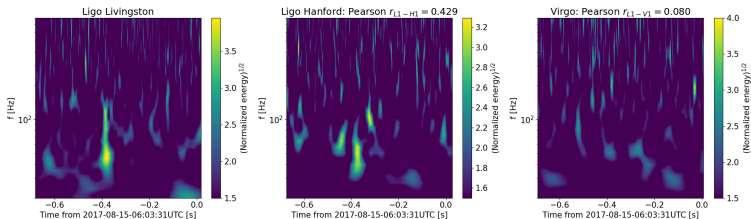


Figure 22: Event with $p_{\text{CHE}} = 0.942$ and correlation discriminant $D = 0.314$.

Other CHE candidates II

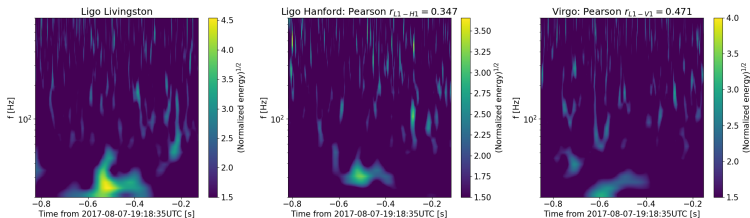


Figure 23: Event with $p_{CHE} = 0.927$ and correlation discriminant $D = 0.388$.

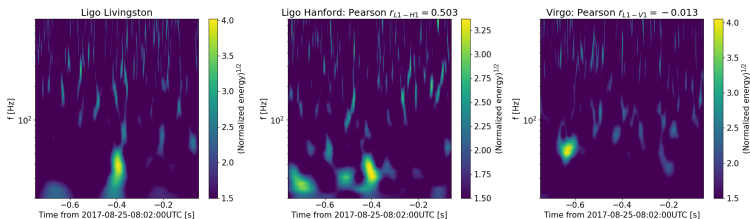


Figure 24: Event with $p_{CHE} = 0.914$ and correlation discriminant $D = 0.337$.