Applications of the close-limit approximation: horizonless compact objects and scalar fields

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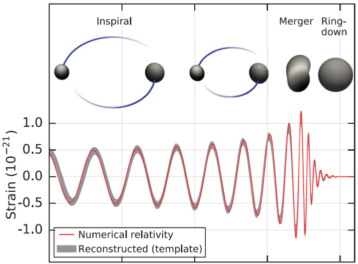
Modelling GW generation



Credit: LIGO/Virgo

Models which do not assume GR

Models which do not assume BHs

What is the close limit approximation (CLAP)?

CLAP as an alternative approach [Price&Pullin, 1994]

(i) Initial data to model last stages of the merger

(ii) Perturbative expansion in the initial separation

(iii) Small deformation of the final object

(iv) Perturbation equations to obtain GWs

Agreement with simulations?

[Anninos+,1993]

Equal mass head-on collision

Defining the $4D$ metric with BHs on Z-axis: $\xi_\ell = \frac{1}{2}\left(\frac{Z_0}{M}\right)^\ell$, for $\ell = 2, 4, 6, \ldots$

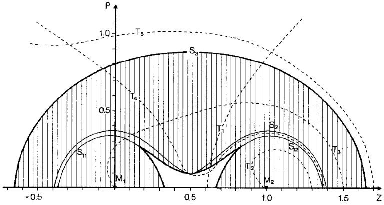
$$ds^2 = -f dt^2 + \left(f^{-1} dr^2 + r^2 d\Omega^2\right)\left[1 + \frac{4}{1 + \frac{M}{2R}} \sum_{\ell=2,4,\ldots}^{\infty} \xi_\ell \left(\frac{M}{R}\right)^{\ell+1} P_\ell\left(\cos\theta\right)\right]$$

$$R = \frac{1}{4}\left(\sqrt{r} + \sqrt{r - 2M}\right)^2, \quad f = 1 - \frac{2M}{r}$$

Initial separation

$$L = \int_{Z_1}^{Z_2} \left[1 + \frac{M}{4}\left(\frac{1}{Z_0 + Z} + \frac{1}{Z_0 - Z}\right)\right]^2 dZ$$

[Andrade, 1996]



[Bishop, 1982]

Spherically symmetric compact body with mass M and a surface at $r = r_0$,

$$r_0 = 2M(1+\epsilon), \quad \epsilon \ll 1$$

Binary ECOs

Brill-Lindquist initial data: ${}^3ds^2_{\mathrm{BL-ECO}} = \left[1 + \frac{4}{1+\frac{M}{2R}} \sum_{\ell=2,4,\ldots}^{\infty} \xi_\ell \left(\frac{M}{R}\right)^{\ell+1} P_\ell(\cos\theta)\right] \left(f^{-1}dr^2 + r^2 d\Omega^2\right)$

Decomposing the metric $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$ the only non-vanishing perturbations are h_{rr}, $h_{\theta\theta}$, $h_{\phi\phi}$

[Zerilli, 1971] [Moncrief,1974] [Cunningham+, 1979]

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial r_*^2} - \left(1 - \frac{2M}{r}\right) \frac{6\left(3M^3 + 6M^2 r + 4Mr^2 + 4r^3\right)}{r^3(3M+2r)^2}\psi = 0$$

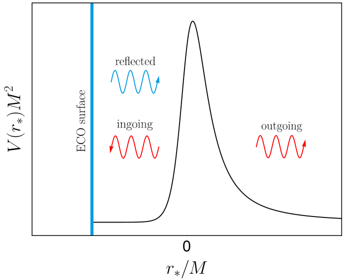
with $\psi(t_0, r) \propto \xi_2 = \frac{1}{2}\left(\frac{Z_0}{M}\right)^2$

ECOs head-on collision

(i) Collision to a single BH: same as original CLAP

(ii) Collision to a single ECO

- Exterior spacetime excitations
  [Vishveshwara, 1970] [Berti+, 2009]
- Contributions from interior: lapse very small = large delays
  [Mark+, 2017] [Ferrari&Kokkotas, 2000] [Cardoso&Pani, 2018]

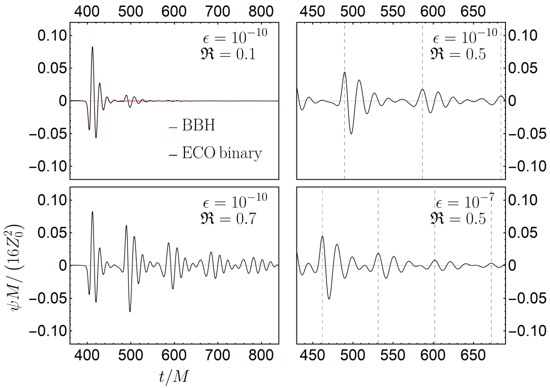


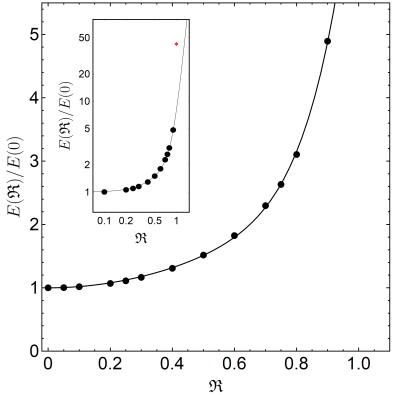
"Freezing" the inner region:
incoming waves partially reflected

$$\tilde{\phi}(r_*) \sim e^{-i\omega r_*} + \mathfrak{R} e^{i\omega r_*}$$

$\mathfrak{R}$ is the *reflectivity coefficient*.

Waveforms and echoes





$$\Delta t_{\rm ECO} \sim 4.3\,|\log\epsilon|\,M \qquad \frac{E}{M} \approx 10^{-6}\left(6.14 + \mathfrak{R}^2(1.29 + 3.26\mathfrak{R}^6)\right)\frac{256 Z_0^4}{M^4}$$

CLAP for scalar fields

Equations of motion

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\left(\nabla\Phi\right)^2\right]$$

$$G_{\mu\nu} = \frac{1}{2}T_{\mu\nu}, \quad \Box\Phi = 0$$

Ansätze

Spacetime : $\Box\Phi = 0 \Longrightarrow \Box = \Box^{(0)} + Z_0^2\,\Box^{(1)} + \mathcal{O}(Z_0^3)$

Scalar : $\Phi = \dfrac{\psi_{\ell\,m}\,(t,r)\,Y^{\ell\,m}\,(\theta,\phi)}{r} + Z_0^2\sum_{\ell'\neq\ell}\dfrac{\psi_{\ell'\,m}\,(t,r)\,Y^{\ell'\,m}\,(\theta,\phi)}{r}$

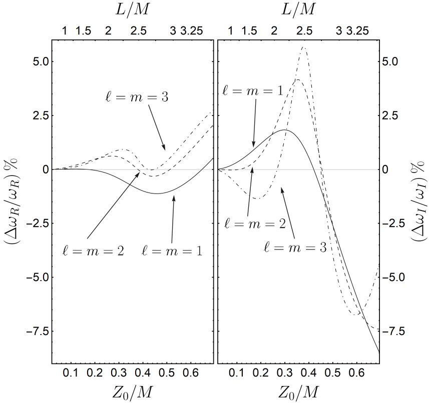
Master equation

$$\frac{\partial^2\psi_{lm}}{\partial t^2} + \frac{\partial^2\psi_{lm}}{\partial r^2}\left(U_0 + Z_0^2\tilde{U}_0\right) + \frac{\partial\psi_{lm}}{\partial r}\left(U_1 + Z_0^2\tilde{U}_1\right) + \psi_{lm}\left(W_0 + Z_0^2 W_1\right) = 0$$

Can a BH binary grow small scalars?

When does the ringdown start?

$$\frac{\Delta\omega_{R/I}}{\omega_{R/I}} = \frac{\omega_{R/I} - \omega_{R/I}^{(\mathrm{Schw})}}{\omega_{R/I}^{(\mathrm{Schw})}}$$



Follow-up

Inspiral of equal-mass, compact, horizonless objects

BHs collision at the speed of light

Binary BH waveforms in alternative theories, e.g. Einstein-scalar-Gauss-Bonnet

Effects of linear momentum on dynamical scalarization

...

Appendix

Initial data

Set of $3D$ spatial hypersurfaces Σ_t

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\gamma_{ij}\beta^i dt dx^j + \gamma_{ij}dx^i dx^j$$

$$\mathcal{H}^{\rm GR} \equiv {}^3R + K^2 - K_{ij}K^{ij} = 16\pi\rho \text{ (Hamiltonian constraint)}$$

Not unique solution for BH binaries

[Brill&Lindquist, 1963] [Bowen&York, 1980] [...]

Brill Lindquist

$${}^3ds^2_{\rm BL} = \varphi^4_{\rm BL}\left(dR^2 + R^2 d\Omega^2\right) \implies \nabla^2\varphi_{\rm BL} = 0$$

$$\varphi_{\rm BL} = 1 + \frac{m_1}{2|\boldsymbol{R} - \boldsymbol{R}_1|} + \frac{m_2}{2|\boldsymbol{R} - \boldsymbol{R}_2|} = 1 + \frac{M}{2R} + \sum_{\ell=1}^{\infty} \xi_\ell \left(\frac{M}{R}\right)^{\ell+1} P_\ell\left(\cos\theta\right)$$

$$\xi_\ell = \left\{\left(\frac{R_1}{M}\right)^\ell \frac{m_1}{2M} + \left(\frac{R_2}{M}\right)^\ell \frac{m_2}{2M}\right\}$$

Initial data for ECOs

$$^3ds^2_{\mathrm{ECO}} = \varphi^4\, {}^3d\eta^2 = \varphi^4\left(dR^2 + R^2 d\Omega^2\right)$$

Hamiltonian constraint becomes

$$\rho = \frac{\varphi^{-5}}{2\pi}\left[\frac{3M}{2R_0^3}\Theta\left(R_0 - R\right)\right]$$

$$\nabla^2_\eta \varphi = \frac{3M}{2R_0^3}\Theta\left(R_0 - R\right)$$

Conformal factor for ECO

$$\varphi = 1 + \frac{M}{2}\left(\frac{3}{2R_0} - \frac{R^2}{2R_0^3}\right) + \frac{M}{2}\left(\frac{1}{R} + \frac{R^2}{2R_0^3} - \frac{3}{2R_0}\right)\Theta\left(R - R_0\right)$$

Toy model