Universal relations for rotating Boson Stars

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What are Universal Relations?

In general and for BHs

- Universal Behaviour: Properties of a system deduced from a small finite set of global parameters [Yagi and Yunes, 2017].
- BH universality: This is the *no-hair theorem* [Heusler, 1998].

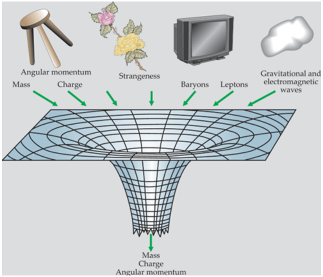

Figure: [Physics Today 62, 4, 47 (2009); https://doi.org/10.1063/1.3120896]

What are Universal Relations?

For other astrophysical objects

- Not expected for other event-horizonless objects.
- Approximate universality founded for Neutron and Quark Stars, the $I - Love - Q$ relations.
- Higher order multiples universal behaviour.
- Multiple generalizations [Doneva and Pappas, 2018].
- Not involving multipoles relations: Binding energies, oscillation modes, mass and radius [Lattimer and Prakash, 2001, Andersson and Kokkotas, 1996].

Importance of universality

Testing their validity:

- Multi-messenger observation tests [Yagi and Yunes, 2013].
- Oscillation modes measurements [Chirenti et al., 2017].

Incompressibility deviations $\rightarrow$ param estimation. [Yagi et al., 2014].

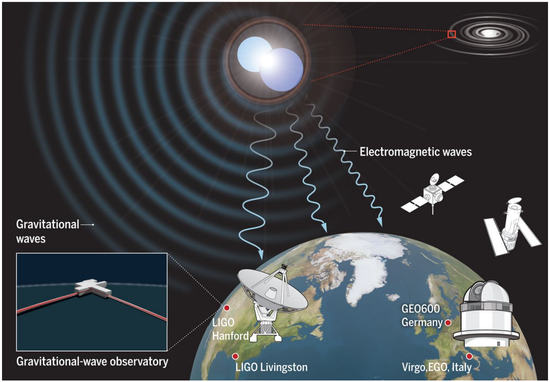

Figure: [Vitale, 2021]

Importance of universality

Assuming their validity:

- Obtaining quantities that are difficult to measure: $M, S_p, I \rightarrow Q$ [Lattimer and Schutz, 2005].
- Q-Love $\rightarrow$ breaking the spin degeneracy [Yagi and Yunes, 2017].
- EOS parameter estimation [Silva et al., 2016].

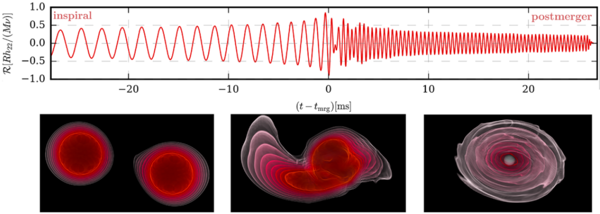

Figure: [Dietrich et al., 2021]

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What are Boson Stars (BSs)

Compact, stationary configurations of scalar field(s) bounded by gravity.

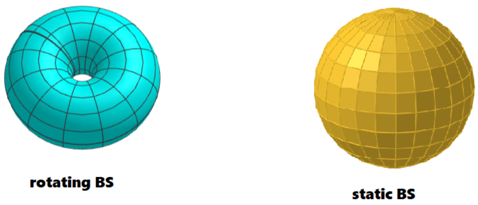


Figure: Shape of rotating (in cyan) and static (yellow) BSs.

No slowly rotating BS![Kobayashi et al., 1994].
We will need full rotation formalism!

What are Boson Stars (BSs)

EKG system equations:

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = 8\pi T_{\alpha\beta}, \quad (1)$$

$$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = \frac{dV}{d|\Phi|^2}\Phi, \quad (2)$$

where $R_{\alpha\beta}$ is the Ricci tensor and $T_{\alpha\beta}$ is the canonical Stress-Energy tensor of the scalar field,

$$T_{\alpha\beta} = 2\nabla_{(\alpha}\Phi^*\nabla_{\beta)}\Phi - 2g_{\alpha\beta}\left[g^{\mu\nu}\nabla_{(\mu}\Phi^*\nabla_{\nu)}\Phi + V\left(|\Phi|^2\right)\right]. \quad (3)$$

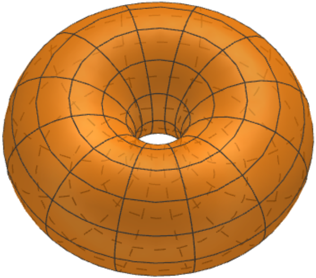
Rotating BSs

Rotating compact objects $\rightarrow$ axial symmetry [Herdeiro and Radu, 2015],

$$ds^2 = -e^{2\nu}dt^2 + e^{2\beta}r^2\sin^2\theta\left(d\psi - \frac{W}{r}dt\right)^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2), \quad (4)$$

where ν, α, β and W are functions dependent only on r, θ.
The consistent ansatz for the scalar field is

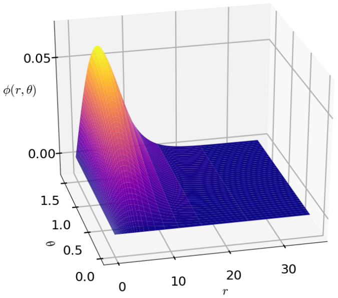
$$\Phi(t, r, \theta, \psi) = \phi(r, \theta)e^{-i(wt+n\psi)}, \quad w \in \mathbb{R}, \quad n \in \mathbb{Z}. \quad (5)$$



Numerics:

EKG numerical integration.

- Set of five coupled, non-linear, partial differential equations (1)-(2).
- Solver FIDISOL/CADSOL [Schönauer and Weiβ, 1989].
- Equations discretisation. (401×40), (x, θ) grid, $0 \leq x \leq 1$ and $0 \leq \theta \leq \pi/2$, and compactified radius from $r \in [0, \infty)$ to $x \in [0, 1]$
- Boundary conditions: Asymptotic flatness, axial symmetry, reflection on the rotation axis, vanishing scalar field on the rotation axis.



Models:

Name	$V(\phi)$
Mini-BS, BS_{Mass}	$V_{\mathrm{Mass}} = \mu^2\phi^2$
BS_{Quartic}	$V_{\mathrm{Quartic}} = \mu^2\phi^2 + \lambda/2\phi^4$
BS_{Halo}	$V_{\mathrm{Halo}} = \mu^2\phi^2 - \alpha\phi^4$
BS_{HKG}	$V_{\mathrm{HKG}} = \mu^2\phi^2 - \alpha\phi^4 + \beta\phi^6$
BS_{Sol}	$V_{\mathrm{Sol}} = \mu^2\phi^2(1-(\phi^2/\phi_0^2))^2$
BS_{Log}	$V_{\mathrm{Log}} = f^2\mu^2 \ln\left(\phi^2/f^2 + 1\right)$
$BS_{\mathrm{Liouville}}$	$V_{\mathrm{Liouville}} = f^2\mu^2\left(\exp\{\phi^2/f^2\} - 1\right)$
BS_{Axion}	$V_{\mathrm{Axion}} = \frac{2\mu^2 f^2}{B}\left(1 - \sqrt{1 - 4B\sin^2(\phi/2f)}\right)$

Table: BS potentials analyzed in the current work.

Models:

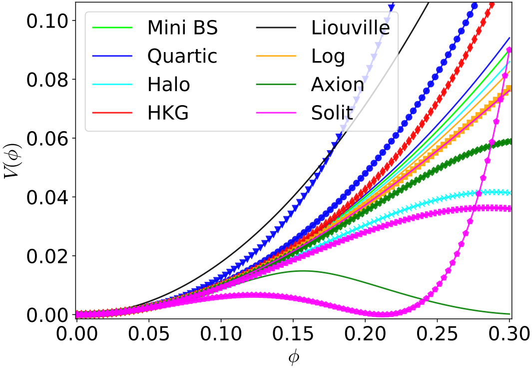


Figure: Form of the potentials $V(\phi)$, in a range relevant for our simulations, i.e $\phi \in [0, 0.29]$. Each colour denotes a different model, and different symbols correspond to different parameter values within a model.

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Multipolar Structure and global properites

[Pappas et al., 2019, Butterworth and Ipser, 1976, Fodor et al., 1989]

$$ds^2 = -e^{2\nu}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + B^2 e^{-2\nu} r^2 \sin^2\theta\,(d\psi - \omega dt)^2 \,. \quad (6)$$

Multipolar expansion:

$$\begin{aligned}
\nu &= \sum_{l=0}^{\infty} \bar{\nu}_{2l}(r) P_{2l}(\cos\theta), \qquad \bar{\nu}_{2l}(r) = \sum_{k=0}^{\infty} \frac{\nu_{2l,k}}{r^{2l+1+k}}, \\
\omega &= \sum_{l=0}^{\infty} \bar{\omega}_{2l-1}(r) \frac{dP_{2l-1}(\cos\theta)}{d\cos\theta}, \quad \bar{\omega}_{2l-1}(r) = \sum_{k=0}^{\infty} \frac{\omega_{2l-1,k}}{r^{2l+1+k}} \\
B &= 1 + \sum_{l=0}^{\infty} \bar{B}_{2l}(r)\, T_{2l}^{\frac{1}{2}}(\cos\theta), \quad \bar{B}_{2l}(r) = \frac{B_{2l}}{r^{2l+2}}, \qquad (7)
\end{aligned}$$

where $P_l(\cos\theta)$ and $T_l^{\frac{1}{2}}(\cos\theta)$ are the Legendre and Gegenbauer.

Multipolar Structure and global properites

Multipole moments as combinations of the expansion coefficients in (7) [Pappas and Sotiriou, 2015]:

$$\begin{aligned} M &= -\nu_{0,0}, \qquad\qquad J = \frac{\omega_{1,0}}{2}, \\ Q &= \frac{4}{3} B_0 \nu_{0,0} + \frac{\nu_{0,0}^3}{3} - \nu_{2,0}, \end{aligned} \tag{8}$$

Less than 2% discrepancy between the multipole calculation and the corresponding Komar integrals [Komar, 1963].

$$M = \int_0^\infty dr \int_0^\pi d\theta r^2 \sin\theta e^{\nu+2\alpha+\beta} \left(T_t^t - \frac{1}{2} T \right), \tag{9}$$

$$J = \int_0^\infty dr \int_0^\pi d\theta r^2 \sin\theta e^{\nu+2\alpha+\beta} T_\psi^t. \tag{10}$$

Multipolar Structure and global properites

Moment of inertia

- Moment of inertia for Rigid Rotating Bodies, for example NSs:

$$I = \frac{J}{\Omega}, \text{ where } \Omega = \frac{d\psi}{dt} = \frac{u^\psi}{u^t} \quad (11)$$

- Strong-coupling assumption proposed in [Ryan, 1997] $\rightarrow \partial_r \phi$ and $\partial_\theta \phi$ are neglected. $T^{\mu\nu} \rightarrow$ perfect fluid.

Multipolar Structure and global properites

- Use the Noether current, global U(1) symmetry of the lagrangian.

$$j^{\mu} = \frac{i}{2}\sqrt{|g|}g^{\mu\nu}\left[\Phi^{*}\nabla_{\nu}\Phi - \Phi\nabla_{\nu}\Phi^{*}\right]. \tag{12}$$

- Differential angular velocity as,

$$\Omega = \frac{j^{\psi}}{j^{t}} = \frac{wg^{\psi t} - ng^{\psi\psi}}{wg^{tt} - ng^{t\psi}} = \frac{W}{r} + \frac{ne^{2(\nu-\beta)}}{r^{2}\left(w - \frac{nW}{r}\right)\sin^{2}\theta}. \tag{13}$$

  Agreement with [Ryan, 1997] expressions.

- For $\Omega = \Omega(r, \theta)$ differentially rotating systems :

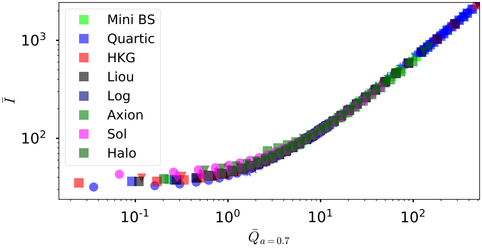
$$I = \int_{0}^{\pi}\int_{0}^{\infty}\frac{j(r,\theta)}{\Omega(r,\theta)}r^{2}\sin\theta e^{\nu+2\alpha+\beta}drd\theta, \tag{14}$$

Universal Relations for Rotating BSs

Standard dimensionless *reduced multipole moments* [Yagi and Yunes, 2013],

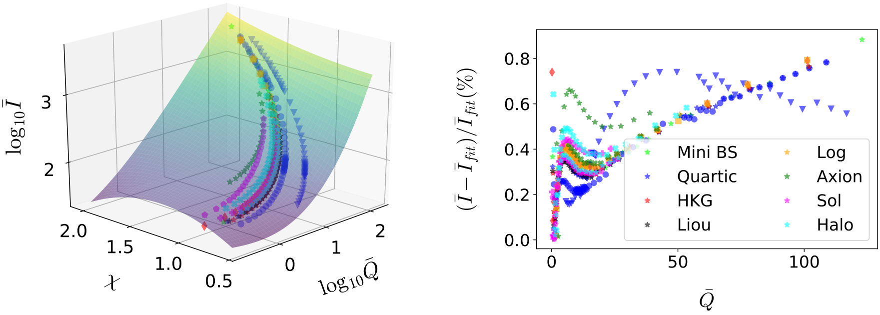
$$\bar{I} = \frac{I}{M_{99}^3}, \quad \bar{Q} = \frac{Q}{M_{99}^3 \chi^2}, \; \chi = \frac{J}{M_{99}^2}, \tag{15}$$

M_{99} is 99% of the total mass, and χ the dimensionless spin parameter. The $I - Q$ maximum difference is about 20%, improved by scaling with a power of the spin-parameter $IQ\chi^a$, with $a \sim 0.7$.



I-χ-Q UR

- We should take into account the χ, as in [Pappas and Apostolatos, 2014].
- Deviation less than a 1%!

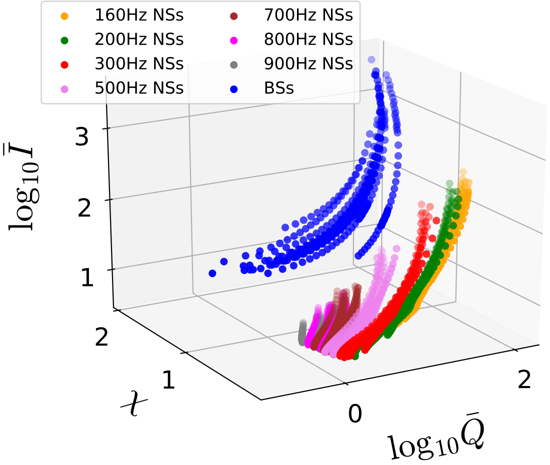


(a) (b)

NSs-BSs Comparison

NS data is clearly different from BSs

NSs simulations were done with RNS package [Stergioulas, 1992].



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Conclusions

- BSs in the astrophysical scenario, both for theoretically than observational reasons [Bustillo et al., 2021].
- For rotating BSs we have found a $I - \chi - Q$ relation that is satisfied up to 1% precission.
- The existence of such URs may become useful in the analysis of GWs, in the search of dark matter candidates and in the further understanding of the strong gravity regime of General Relativity.

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