Parity Violating Gravitational Waves at the end of Inflation

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Braga

Motivation

- Inflation (A. Guth, A. Linde, A. Starobinsky A.Albrecht and P. Steinhartdt)
  - Flatness, Horizon, Anisotropies in CMB, Origin of Large-Scale Structure in the Universe
  - Early phase of accelerated expansion, around 50-60 efolds
  - Unknown particle physics description
  - Connection with Standard Cosmology through reheating
  - May provide an explanation for the origin of the cosmological structure,
- Dark Matter (DM)
  - Around 85% of the observed matter
  - Stable non-relativistic non-luminous fluid
  - Also with unknown description...
  - No WIMPS so far...

Inflation: Basic Introduction

$$S_\phi = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

E.O.M.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi) \qquad\qquad p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi)$$

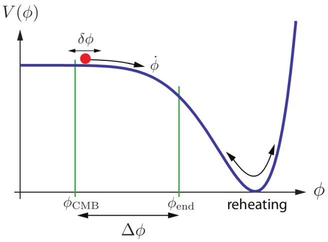
$$p_\phi = \omega\rho_\phi$$

- With dominant potential $p = -\rho_\phi \Rightarrow \omega = -1 \rightarrow$ Accelerated Expansion

$$\frac{1}{2}\dot{\phi} \ll V(\phi) \qquad \epsilon = \frac{m_P^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

$$\ddot{\phi} \ll 3H\dot{\phi} \qquad \eta = \frac{m_P^2}{8\pi}\frac{V''(\phi)}{V(\phi)}^2 \ll 1$$

$$N_e = \log\left(\frac{a_0}{a_i}\right) = \int_{a_0}^{a_e}\frac{da}{a} \simeq -\frac{1}{m_P^2}\int_{\phi_i}^{\phi_e}\frac{V(\phi)}{V'(\phi)}d\phi$$



D. Baumann 0907.5424

Inflaton-Vector Coupling $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$

- Need to end inflation and move into standard Cosmological Model
- Couple inflaton with Vector particles, here U(1) gauge fields

- Slow roll motion will lead an exponential vector production with a peak at the end of inflation

- More specifically, amplification of just one vector transverse polarization,
- Parity violation visible in GW's produced by vector contribution in source term $T_{\mu\nu}$

- Main constrain on inflaton-vector coupling from bounds on not observed non-gaussianities

Inflaton-Vector Coupling: Consequences

- Production of primordial magnetic fields G.B. Field, S.M. Carroll astro-ph/9811206.
- Dissipation mechanism to allow inflation on steep potential scenarios
  - For YM gauge fields may lead Warm Inflation scenario
    K.V. Berghaus, P.W. Graham, D.E. Kaplan 1910.07525
- Generation of dark sectors
  - If gauge particles belong to a dark sector observed dark matter relic abundance obtained in the mass range $\mu\mathrm{eV} \lesssim m \lesssim 10\,\mathrm{TeV}$
    M. Bastero Gil, J. Santiago, L. Ubaldi, R. Vega Morales 1810.07208
  - Efficient fermion pair production, Schwinger effect
    V.Domcke, Y. Ema, K. Mukaida 1910.01205
- Production of Primordial Black Holes A. Linde, S. Mooij, and E. Pajer 1212.1693, J. García-Bellido, M. Peloso, C. Unal 1610.03763
- Gravitational Wave Generation M. M. Anber, L. Sorbo 1203.5849.

Vector particle production

- Relate inflation and reheating with production of DM

$$\mathcal{S} = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2A_\mu A^\mu + \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

- If $m < H$ at the end of inflation, negligible effects on the production mechanism.
- Transverse modes EOM in conformal time, $ad\tau = dt$, (during inflation $\tau \simeq -\frac{1}{aH}$)

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm 2k\frac{\xi}{\tau}\right]A_\pm(\tau,\tau) = 0,$$

$$\xi \equiv \frac{\alpha\dot{\phi}}{2Hf} = \sqrt{\frac{\epsilon}{2}}\frac{\alpha}{f}M_{\rm Pl} \qquad \text{with} \qquad \epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{\dot{\phi}^2}{2H^2M_{Pl}^2}$$

- Slow roll motion will lead an exponential vector production of one tranverse mode, peaking at the end of inflation

Vector particle production

- With $\dot{\phi} > 0 \to \xi > 0$, implying that the modes will develop an instability when

$$k^2 \pm 2k\frac{\xi}{\tau} = k^2 \mp 2k\xi aH < 0$$

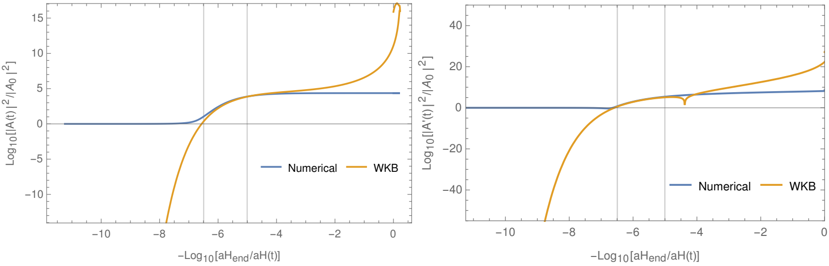
Only A_+ is amplified! and A_- will stay in vacuum

- Treating ξ as constant, appropriate during the beginning of a slow roll evolution

$$A_+(k,\tau)_{\mathrm{WKB}} \simeq \frac{1}{\sqrt{2k}}\left(\frac{-k\tau}{2\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}} \qquad \text{for} \qquad \frac{1}{8\xi} < -k\tau < 2\xi$$

- Provides a good intuition into the behavior before horizon crossing

M.M. Amber, L. Sorbo 0908.4089



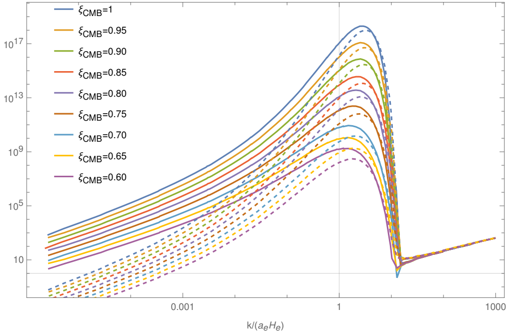
Vector production - Semi-analytical take

- Build time evolution function for each mode in 3 steps

$$A_+(k,\tau) = A_+(k,\tau_{end})T^k(\tau,\tau_{end}) \simeq \begin{cases} A_{BD}(k) & \tau < \tau_{tac} \\ A_{WKB}(k,\tau) & \tau_{tac} < \tau < \tau_h \\ A_+(k,\tau_{end}) & \tau > \tau_h \ . \end{cases}$$

Transitions are smoothed with $\tanh \delta \left(\frac{\tau}{\tau_\star} - 1 \right)$

- Obtain numerical amplitude spectrum for A' and A at the end of inflation

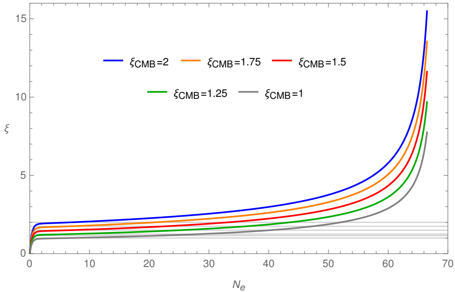


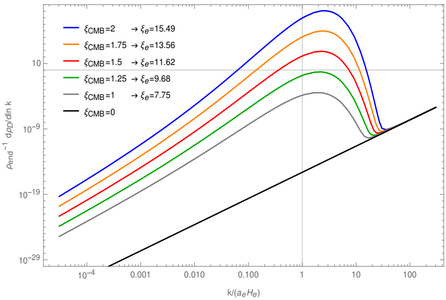
Vector production - Back-reaction

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{\alpha}{4f} F\tilde{F} \qquad \xi \equiv \frac{\alpha\dot{\phi}}{2Hf}$$

- Typically one neglects back-reaction term in inflaton evolution, if $\xi \lesssim \mathcal{O}(10)$

$$\rho_D = \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle = \frac{1}{4\pi^2 a^4} \int_0^\infty dk\, k^2 \left(|\partial_\tau A_+(k,\tau)|^2 + k^2\, |A_+(k,\tau)|^2 \right)$$





Vector production - Back-reaction

Taking $\left\langle F\tilde{F}\right\rangle = S_{EB} = \langle \bar{E}\cdot\bar{B}\rangle$,

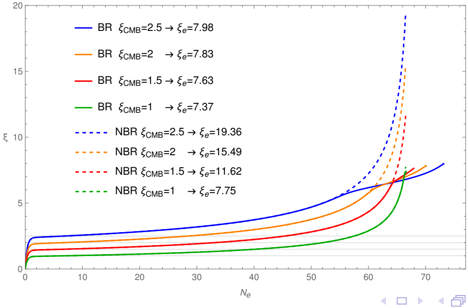
$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = \dot{\phi}\frac{\alpha}{f}S_{EB}$$

$$\dot{\rho}_A + 3H(\rho_A + p_A) = -\dot{\phi}\frac{\alpha}{f}S_{EB}$$

$$\dot{S}_{EB} = \dot{\bar{E}}\cdot\bar{B} + \bar{E}\cdot\dot{\bar{B}} = -4H\,S_{EB} - \frac{\alpha}{f}\dot{\phi}\,|\bar{B}|$$

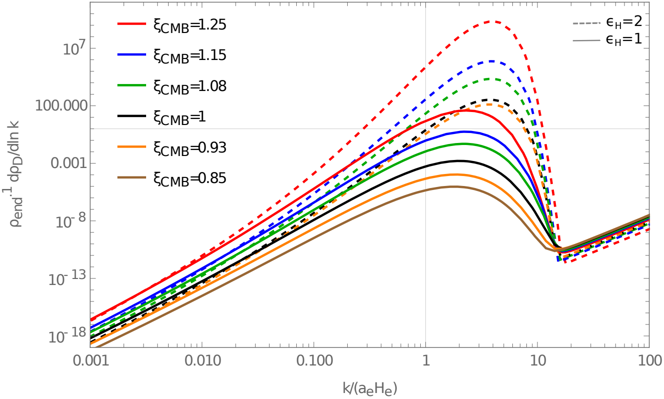
- We then use the WKB approximation to “source” the dissipation term

$$\langle|\bar{B}|^2\rangle = \frac{1}{8\pi^3 a^4}\int d^3k\, k^2 \sum_{\lambda=\pm} |A_{+\,WKB}|^2$$



Vector production - Back-reaction

- One needs to check if back-reaction is being well reproduced
- Furthermore we see amplification until $\epsilon_H = 2$, not just until $\epsilon_H = 1$



Gravitational Waves

- Looking into the tensor modes

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right]$$

- From Einstein's equation

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \Delta h_{ij} = \frac{2}{M_P^2}\Pi_{ij}{}^{lm} T^{EM}_{lm}$$

$$h_\pm(\mathrm{k}) = -\frac{2H^2}{M_P^2}\int d\tau' G_k\left(\tau, \tau'\right)\tau'^2 \int \frac{d^3\mathrm{q}}{(2\pi)^{3/2}}\Pi_\pm^{lm}(\mathrm{k})\times$$
$$\times\left[A'_l\left(\mathrm{q}, \tau'\right) A'_m\left(\mathrm{k}-\mathrm{q}, \tau'\right) - \varepsilon_{lab} q_a A_b\left(\mathrm{q}, \tau'\right)\varepsilon_{mcd}\left(k_c - q_c\right) A_d\left(\mathrm{k}-\mathrm{q}, \tau'\right)\right]$$

Where $G_k\left(\tau, \tau'\right)$ is the retarded Green function for the operator $d^2/d\tau^2 - (2/\tau)d/d\tau + k^2$

$$G_k\left(\tau, \tau'\right) = \frac{1}{k^3\tau'^2}\left[\left(1 + k^2\tau\tau'\right)\sin k\left(\tau - \tau'\right) + k\left(\tau' - \tau\right)\cos k\left(\tau - \tau'\right)\right]$$

Gravitational Waves - WKB approximation

- Taking $A_- = 0$ and $A_+ = A_+(k,\tau)_{\rm WKB}$ and $-k\tau \to 0$ the correlation function gives

$$\langle h_+(\mathrm{k}) h_+(\mathrm{k}')\rangle \simeq 8.6\times 10^{-7}\,\frac{H^4}{M_P^4}\,\frac{e^{4\pi\xi}}{\xi^6}\,\frac{\delta(\mathrm{k}+\mathrm{k}')}{k^3}$$
$$\langle h_-(\mathrm{k}) h_-(\mathrm{k}')\rangle \simeq 1.8\times 10^{-9}\,\frac{H^4}{M_P^4}\,\frac{e^{4\pi\xi}}{\xi^6}\,\frac{\delta(\mathrm{k}+\mathrm{k}')}{k^3}$$

Scale invariant spectra for both the left- and the right-handed tensor modes

- Joining with usual amplification of vacuum fluctuations in de Sitter space (solutions of the homogeneous part)

$$\mathcal{P}^{t,+} = \frac{H^2}{\pi^2 M_P^2}\left(1 + 8.6\times 10^{-7}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$
$$\mathcal{P}^{t,-} = \frac{H^2}{\pi^2 M_P^2}\left(1 + 1.8\times 10^{-9}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

- Chirality parameter

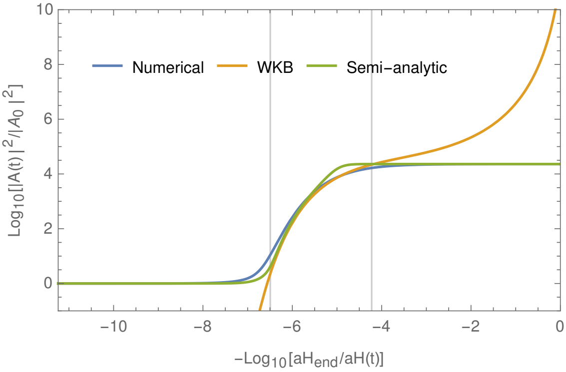
$$\Delta\chi = \frac{4.3\times 10^{-7}\,\frac{e^{4\pi\xi}}{\xi^6}\,\frac{H^2}{M_P^2}}{1 + 4.3\times 10^{-7}\,\frac{e^{4\pi\xi}}{\xi^6}\,\frac{H^2}{M_P^2}}$$

L. Sorbo 1101.1525

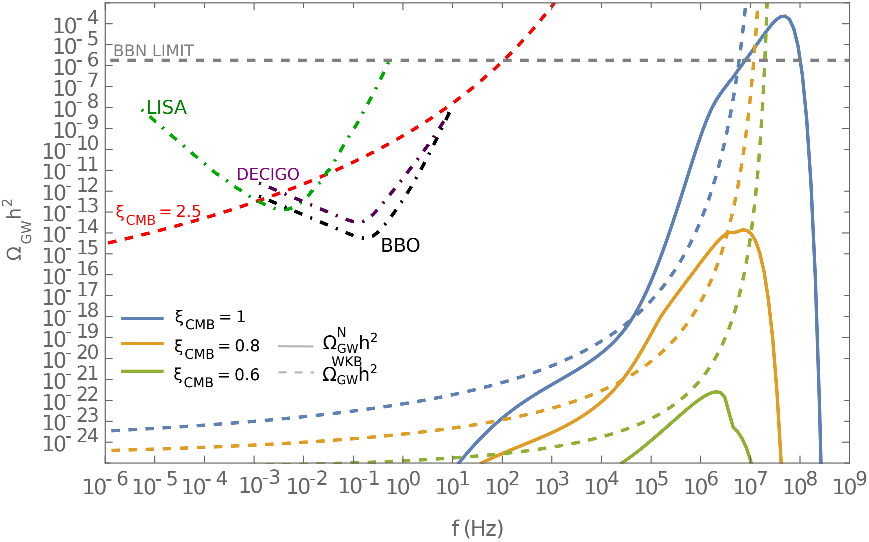
Gravitational Waves - Next steps

Introduce in

$$h_{\pm}(\mathrm{k}) = -\frac{2H^2}{M_P^2}\int d\tau' G_k\left(\tau,\tau'\right)\tau'^2 \int \frac{d^3\mathrm{q}}{(2\pi)^{3/2}}\Pi_{\pm}^{lm}(\mathrm{k})\times$$
$$\times\left[A_l'\left(\mathrm{q},\tau'\right)A_m'\left(\mathrm{k}-\mathrm{q},\tau'\right) - \varepsilon_{lab}q_a A_b\left(\mathrm{q},\tau'\right)\varepsilon_{mcd}\left(k_c-q_c\right)A_d\left(\mathrm{k}-\mathrm{q},\tau'\right)\right]$$



Gravitational Waves - Preleminary



$$\Omega^N_{GW}h^2 = \frac{\Omega_{R\,0}h^2}{24}\frac{k^3}{2\pi^2}\langle h_+ h_+\rangle \qquad \Omega^{WKB}_{GW}h^2 = 1.5\times 10^{-7}\frac{H^4}{m_P^4}\frac{e^{4\pi\xi}}{\xi^6}$$

N.Bartolo et al 1610.06481

Conclusions

- Mechanisms to address early universe puzzles may be tested with gravitational waves

- Exponential production of gauge fields during inflation may lead to detectable gravitational wave spectrum

- It may be even detectable by LISA, but one needs to refine the analysis
- A_+ to be estimated in the valid regime, full back-reaction needs to be optimized

- Important signal for early universe dynamics in parity violations
  - Chiral gravitational waves are produced without requiring modifications in the gravity theory
  - Tensor modes produced during inflation leave an imprint in the CMB through the B-modes (parity odd) yet to be detected.
  - Non vanishing correlations with E-modes and T fluctuations (parity even) will point towards a parity violation

BACKUP-SLIDES

Vector production - Back-reaction - Gradient Expansion

- Consider the vacuum expectation values of bilinear electromagnetic functions in coordinate space to include all physically relevant modes

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\langle \mathrm{E}\cdot \mathrm{rot}^n\, \mathrm{E}\rangle \qquad \dot{\mathcal{E}}^{(n)} + (n+4)H\mathcal{E}^{(n)} - 2\,\frac{\alpha}{f}\,\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = \left[\dot{\mathcal{E}}^{(n)}\right]_b$$

$$\mathcal{G}^{(n)} = -\frac{1}{a^n}\langle \mathrm{E}\cdot \mathrm{rot}^n\, \mathrm{B}\rangle \qquad \dot{\mathcal{G}}^{(n)} + (n+4)H\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - \frac{\alpha}{f}\,\dot{\phi}\,\mathcal{B}^{(n)} = \left[\dot{\mathcal{G}}^{(n)}\right]_b$$

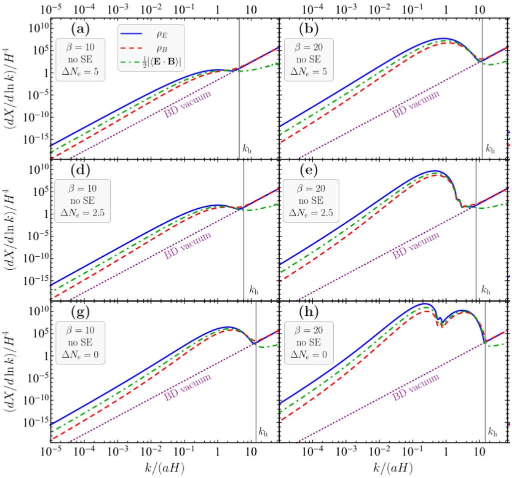
$$\mathcal{B}^{(n)} = \frac{1}{a^n}\langle \mathrm{B}\cdot \mathrm{rot}^n\, \mathrm{B}\rangle \qquad \dot{\mathcal{B}}^{(n)} + (n+4)H\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = \left[\dot{\mathcal{B}}^{(n)}\right]_b$$

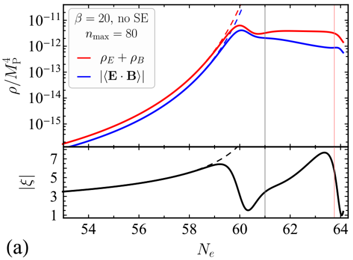
- Truncation condition from properties of spectral decomposition $\mathcal{X}^{n+1} \simeq \left(\frac{k_h}{a}\right)^2 \mathcal{X}^{n-1}$ with $k_h = 2\,\xi\, a\, H$

- Boundary terms are introduced to estimate the contribution from the modes that become classical as they cross the horizon, constantly growing during inflation

E.V Gorbar, K. Schimtz, O.O.Sobol, S.I. Vilchinskii 2109.01651

Vector production - Back-reaction - Gradient Expansion





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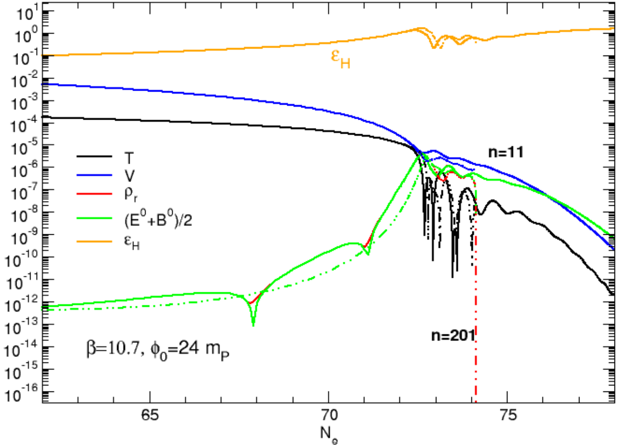
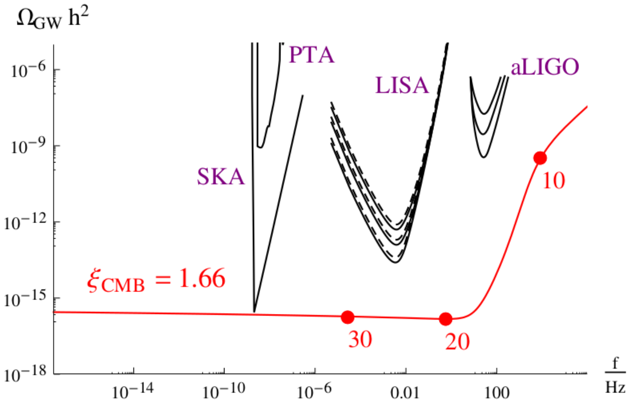
$$\beta = \frac{\alpha}{f} m_p$$

Vector production - Back-reaction - Gradient Expansion



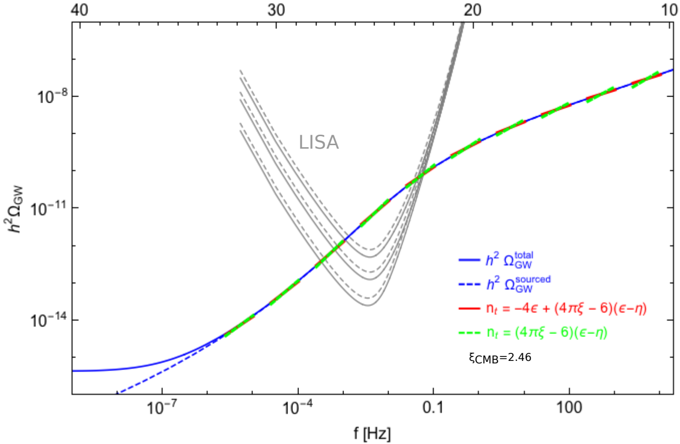
Gravitational Waves - WKB approximation



J. García-Bellido, M. Peloso, C. Unal 1610.03763

$$\Omega_{GW} h^2 = 1.5 \times 10^{-7} \frac{H^4}{m_P^4} \frac{e^{4\pi\xi}}{\xi^6}$$

Gravitational Waves - WKB approximation



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