Comparison of eccentric numerical relativity simulations to small mass-ratio perturbation theory

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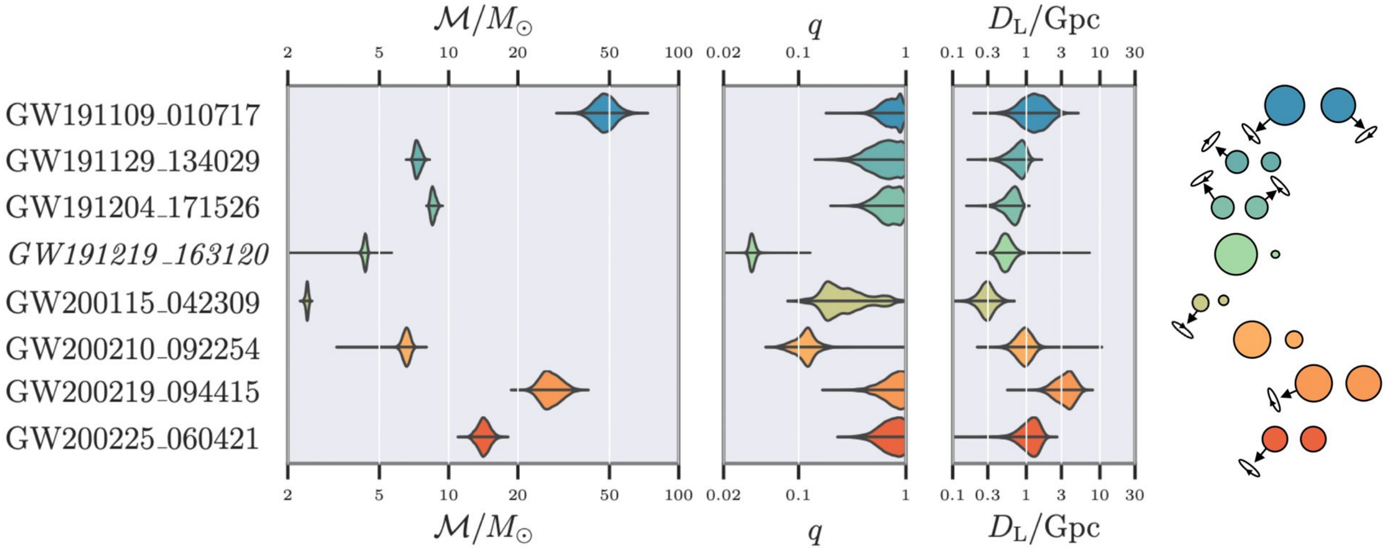
Gravitational-wave detections

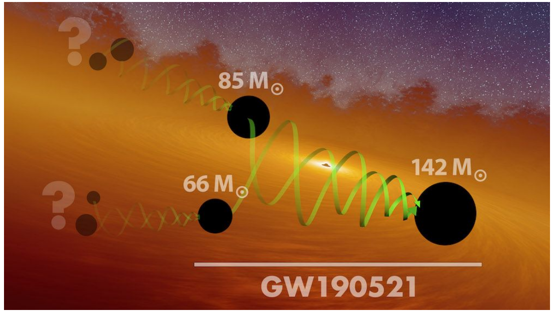
- Analysis of the third observing run (O3) recently out (Abbot+2021, Nitz+2021, Olsen+2021).
- Quasi-circular waveform models used in searches and parameter estimation studies.
- Mass-ratio, $Q=m_1/m_2$, for binary black holes (BBHs) mostly consistent with comparable masses.
- In O4 and future detectors, more detections of mass asymmetries as well as eccentric binaries.



Plot from https://www.ligo.org/science/Publication-O3bCatalog/. Credits: LIGO-Virgo-KAGRA Collaborations/Isobel Romero-Shaw/OzGrav.

Bridging the mass-ratio gap

q=1



LVK

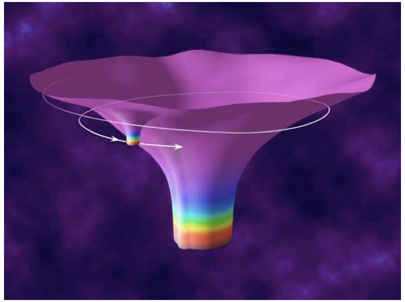
NASA

Intermediate mass BH

$(10 + 1000)M_\odot$

$(10^3 + 10^6)M_\odot$

q=0



EMRI

$(10 + 10^6)M_\odot$

Numerical relativity (NR)

q $\gtrsim$ 20

?

Small-mass-ratio approximation (SMR)

expansion in symmetric mass-ratio ν = q / (1+q)^2

Bridging the mass-ratio gap

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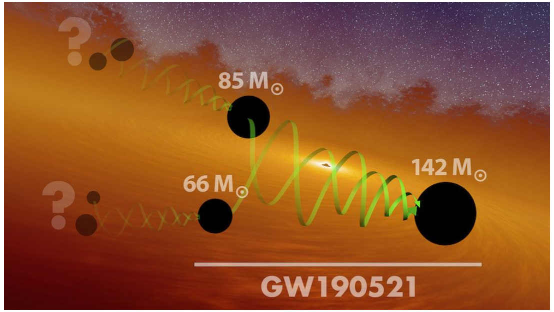

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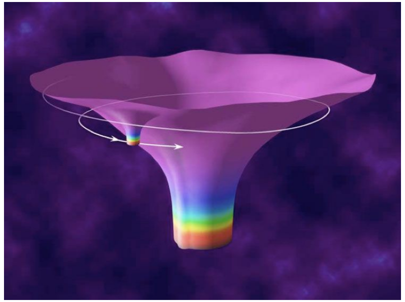

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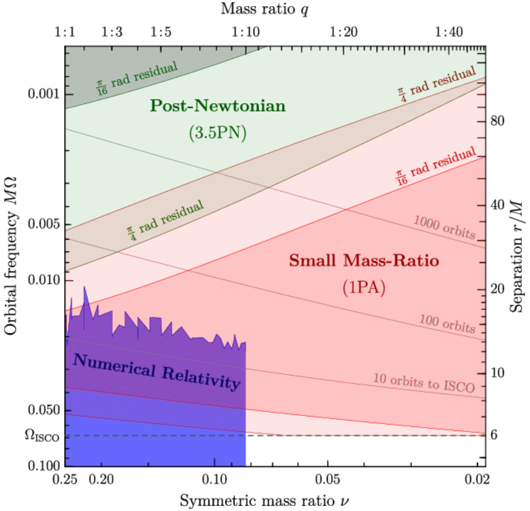


Small-mass-ratio approximation (SMR)

expansion in symmetric mass-ratio ν = q / (1+q)^2

- Recently [VdMeent&Pfeiffer2020] proposed method to combine information from NR and SMR theory to bridge the gap between both (quasi-circular non-spinning).

- This work extends on eccentric non-spinning BBHs.

Small Mass-Ratio (SMR) evolutions

- Equations of motion as a perturbative series in symmetric mass-ratio $\boldsymbol{\nu} = m_1 m_2 /(m_1+m_2)^2$.

- At zero-th order, just geodesics around Kerr. Geodesic frequencies are known analytically.

$$\frac{dq_r}{dt} = \Omega_r, \qquad \frac{d\phi}{dt} = \Omega_\phi.$$

- At next order energy dissipation drives inspiral. Orbit-averaged fluxes on the right-hand-side are functions of eccentricity, e, and semi-latus rectum, p,

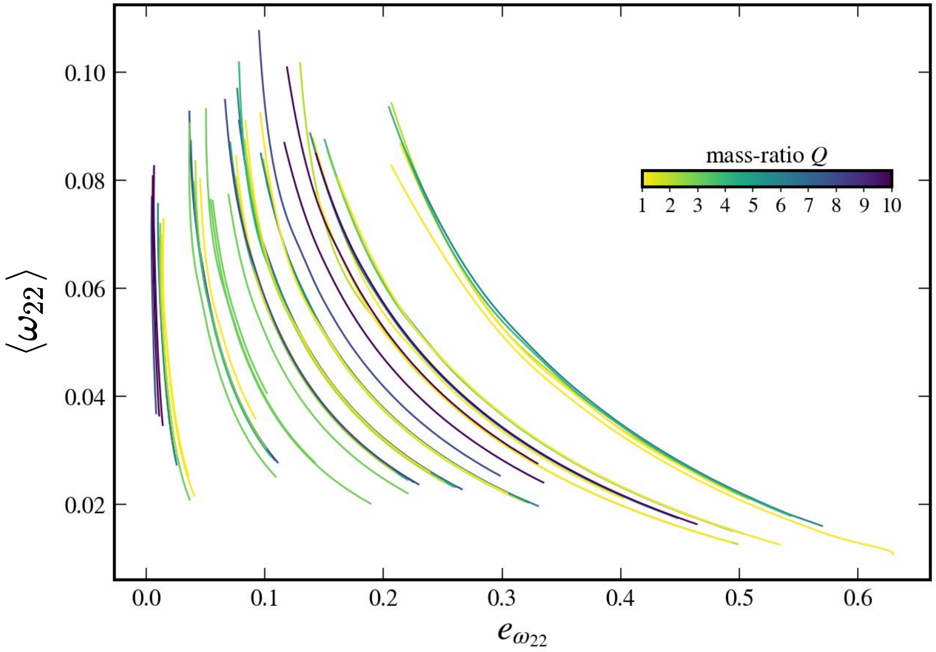
$$\frac{dp}{dt} = \nu\langle F_p \rangle, \qquad \frac{de}{dt} = \nu\langle F_e \rangle.$$

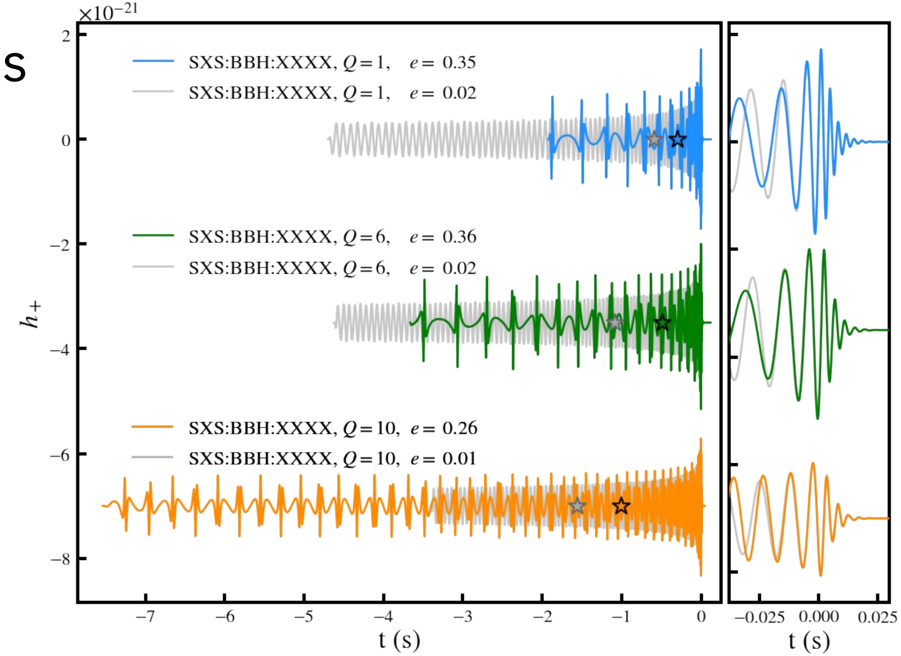
- The fluxes are determined numerically at any (p,e) value using a frequency domain Teukolsky code.

Numerical Relativity (NR) simulations

- We produced 52 new eccentric non-spinning simulations with the Spectral Einstein Code (SpEC), numerical relativity (NR) code :

$$Q = 1/q = m_1/m_2 = [1 - 10], \quad e_{\omega_{22}} = [0.01 - 0.7]$$





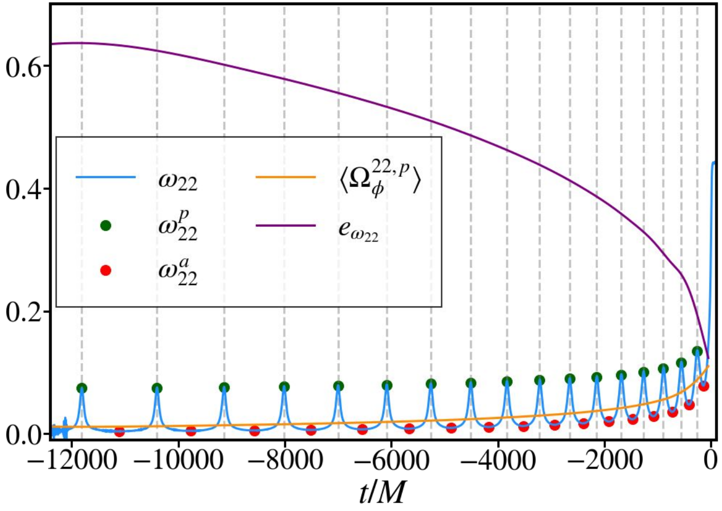
- Long simulations, [20-50] GW cycles.
- 3 different resolutions for each simulation.
- Typical wall clock times:
  - q =1 : ~ 5-10 days
  - q=10 : ~ 2-3 months.

Calculation of frequencies, eccentricity, and fluxes

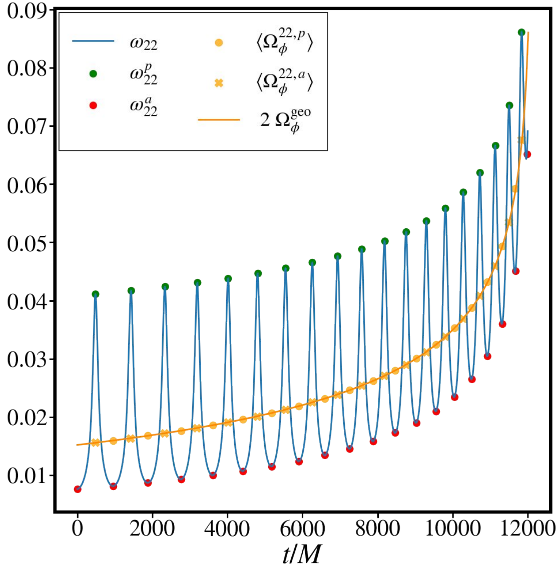
- Need coordinate-invariant definition of eccentricity which is applicable to both NR and SMR.
- Define eccentricity from the frequency of the quadrupole GW (2,2)-mode. $\omega_{22} = \frac{d}{dt}\mathrm{Arg}[h_{22}(t)], \quad e_{\omega_{22}} = \frac{\omega_{22,p}^{1/2} - \omega_{22,a}^{1/2}}{\omega_{22,p}^{1/2} + \omega_{22,a}^{1/2}},$ $\omega_{22,p}$: ω_{22} at periastron. $\omega_{22,a}$: ω_{22} at apastron.

$$\langle \omega_{22} \rangle \equiv \langle \Omega_\phi^{22} \rangle_i = \frac{1}{t_{max}^{i+1} - t_{max}^{i}} \int_{t_{max}^{i}}^{t_{max}^{i+1}} \omega_{22}(t) dt, \quad \langle \dot{E}_{GW} \rangle_i = \frac{1}{t_{max}^{i+1} - t_{max}^{i}} \int_{t_{max}^{i}}^{t_{max}^{i+1}} \dot{E}_{GW}(t) dt.$$

NR : $m_1/m_2 = 6$



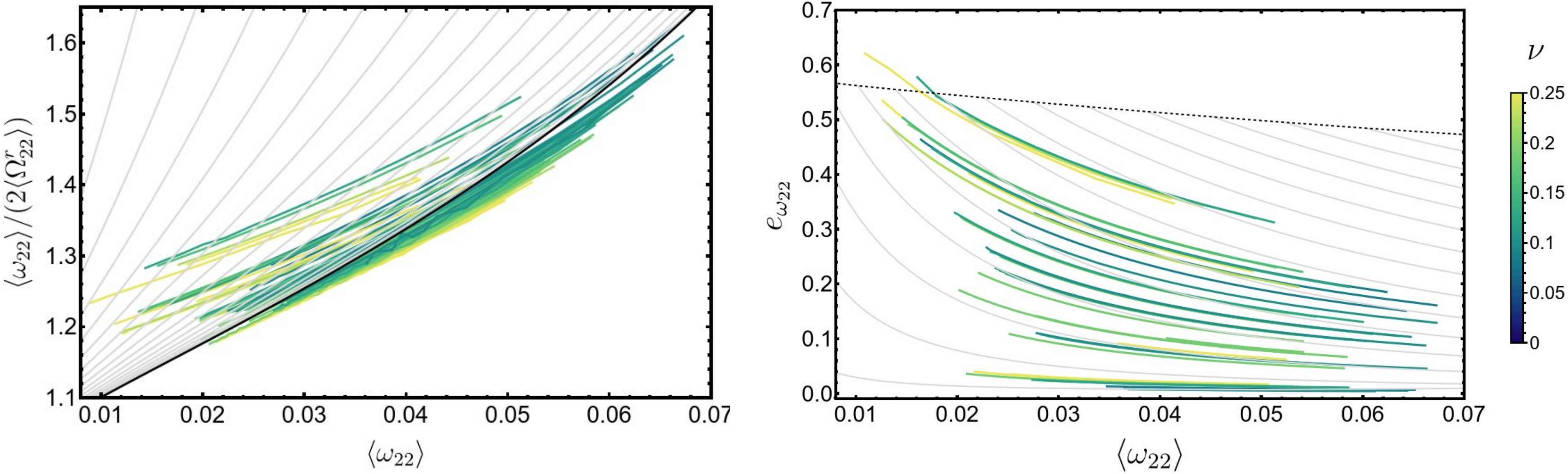
SMR : $m_1/m_2 = 1$



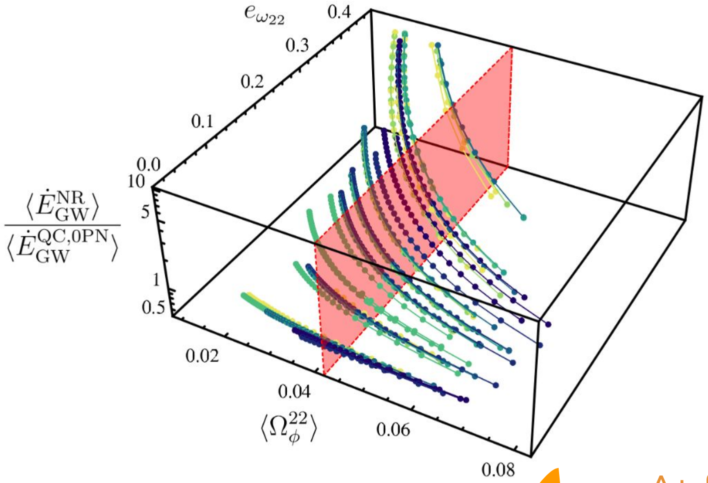
- Use orbit-average procedure to extract frequencies and fluxes from NR simulations [Lewis+2017].
- Also use periastron-passages to compute additional orbit-averaged quantities.

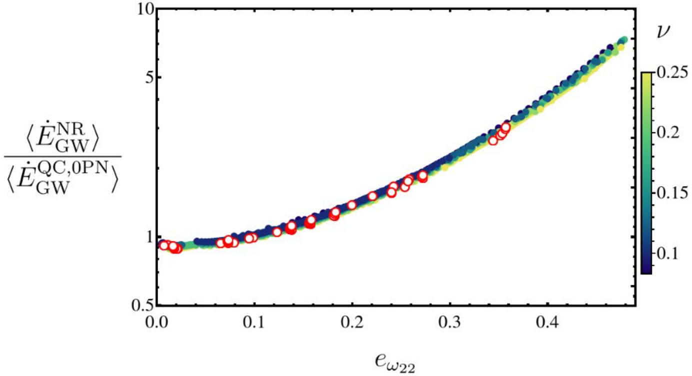
Comparing NR with SMR

- To compare NR with SMR, one must map between "NR configuration" and "SMR geodesic" in a gauge invariant manner.
- The two characteristic frequencies (orbital & radial motion) are not suitable, because the frequencies of NR often fall outside the range spanned by geodesic results.
- We identify NR with SMR by (a) same orbit-averaged orbital frequency, $\langle \Omega_\phi^{22} \rangle$, and (b) same eccentricity $e_{\omega_{22}}$.



Comparison between SMR and NR results. Energy flux

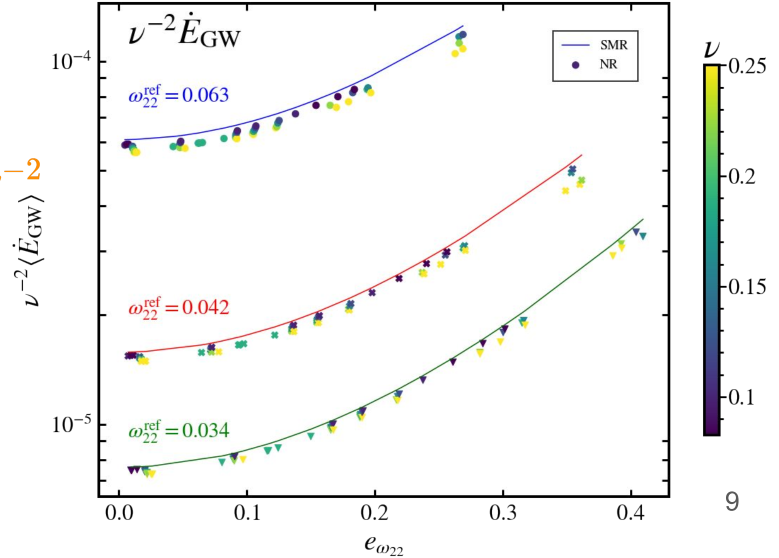




$$\langle \dot{E}^{\rm QC,0PN}_{\rm GW}\rangle = \frac{32}{5}\nu^2 x^5$$

At fixed $\langle \Omega^{\phi}_{22}\rangle_{\rm ref}$,

accounting for $\dot{E} \propto \nu^{-2}$

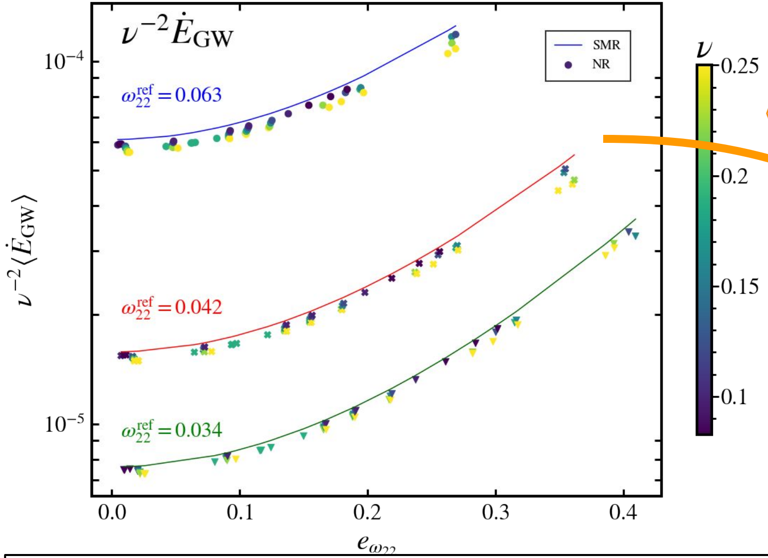


Interpretation:

→ NR and leading-order SMR agree (≲ 10 %)

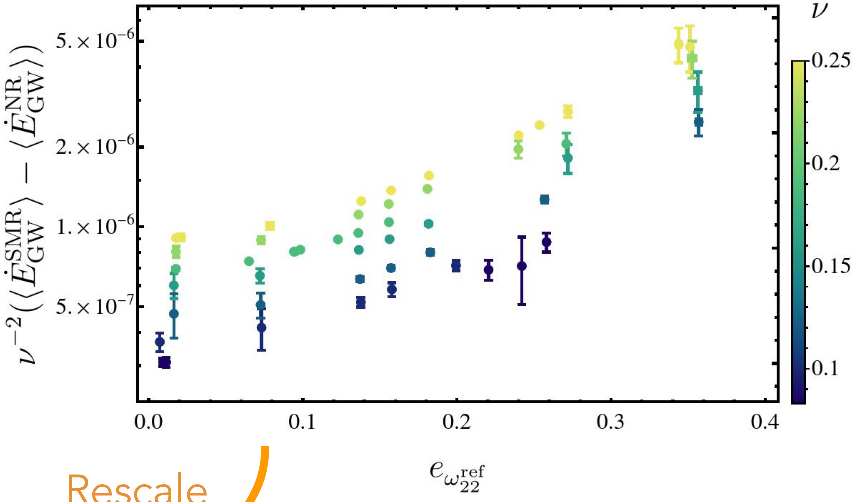
ν^2 also known as 0-PA

Data at fixed $\langle \Omega_{22}^{\phi} \rangle_{\text{ref}}$

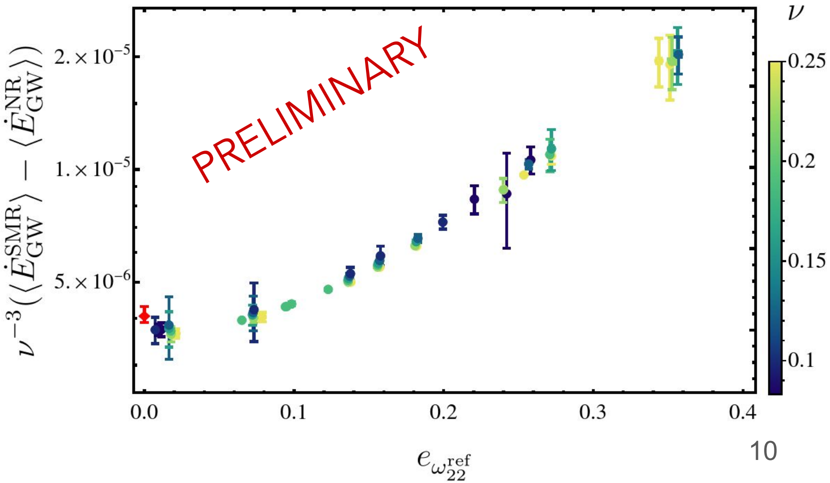


Difference

$\langle \dot{E}_{\text{GW}}^{\text{SMR}} \rangle - \langle \dot{E}_{\text{GW}}^{\text{NR}} \rangle$



Rescale by ν



Interpretation:

→ Collapsed curve gives next-to-leading order contribution

ν^3 also known as 1-PA → *unknown before*

→ Nearly vanishing spread: next-to-next-to-leading-order contribution is small

ν^4 also known as 2-PA → *insignificant*

Conclusions and future work

- Presented new 52 eccentric ($\geq$ 20 orbits) non-spinning simulations with $e_{\omega 22} \leq 0.7$ and Q=[1-10] .

- Developed tools to map eccentric SMR configurations and NR simulations.

- Analyze energy and angular momentum fluxes, and periastron advance.

- Our NR-SMR comparisons indicate that:
    - At LO 0-PA (ν^2) good agreement.
    - At NLO 1-PA (ν^3) collapsed curve indicates contribution small (unknown before) → NNLO 2-PA (ν^4) probably small.

Visualization by H. Pfeiffer.