Constraining ΛCDM cosmological parameters with Einstein Telescope mock data

Based on "Califano, De Martino, Vernieri, Capozziello, 2022, arxiv:2205.11221", Submitted to **JCAP**

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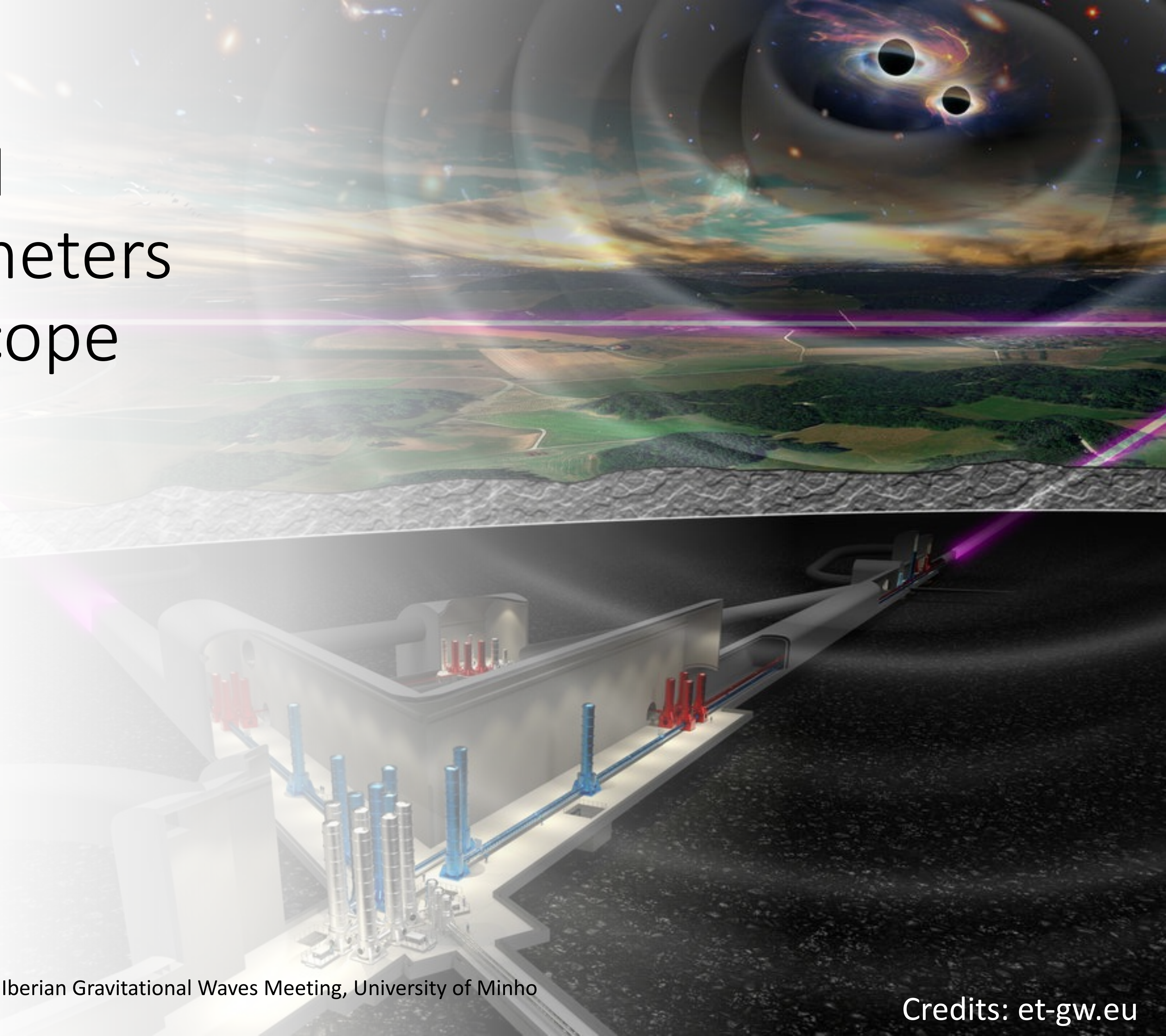

Credits: et-gw.eu

Gravitational Waves as Standard Sirens

GW observables

- With GWs detectors we can measure the amplitude and phase evolution of the signal

$$h_+ = \frac{2c}{D_L}\left(\frac{G\mathcal{M}}{c^3}\right)^{\frac{5}{3}} f^{\frac{2}{3}}(1+\cos^2 i)\cos 2\Phi(t) \qquad h_X = \frac{2c}{D_L}\left(\frac{G\mathcal{M}}{c^3}\right)^{\frac{5}{3}} f^{\frac{2}{3}}\cos i \cos 2\Phi(t)$$

- Observation of the phase gives:

$$\tau_c := \frac{f}{\dot{f}}, \qquad D_L \propto \frac{c}{(f^2\tau_c h)}, \qquad \mathcal{M} = \frac{(1+z)(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}} \propto f^{-\frac{11}{5}}\dot{f}^{\frac{3}{5}}.$$

- Binary inspiral allows for a determination of the distance to the source without any reference to the cosmic distance ladder.

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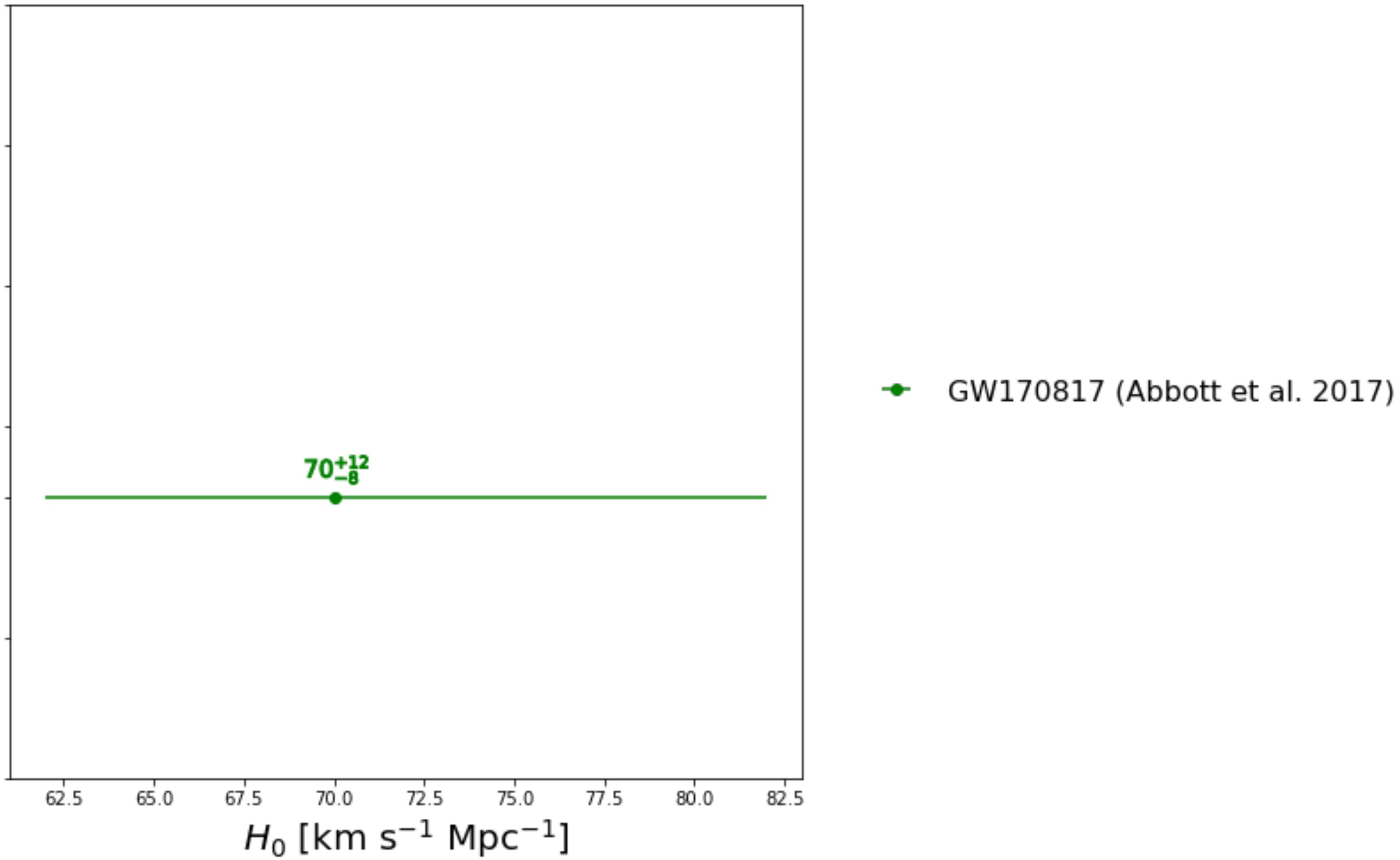
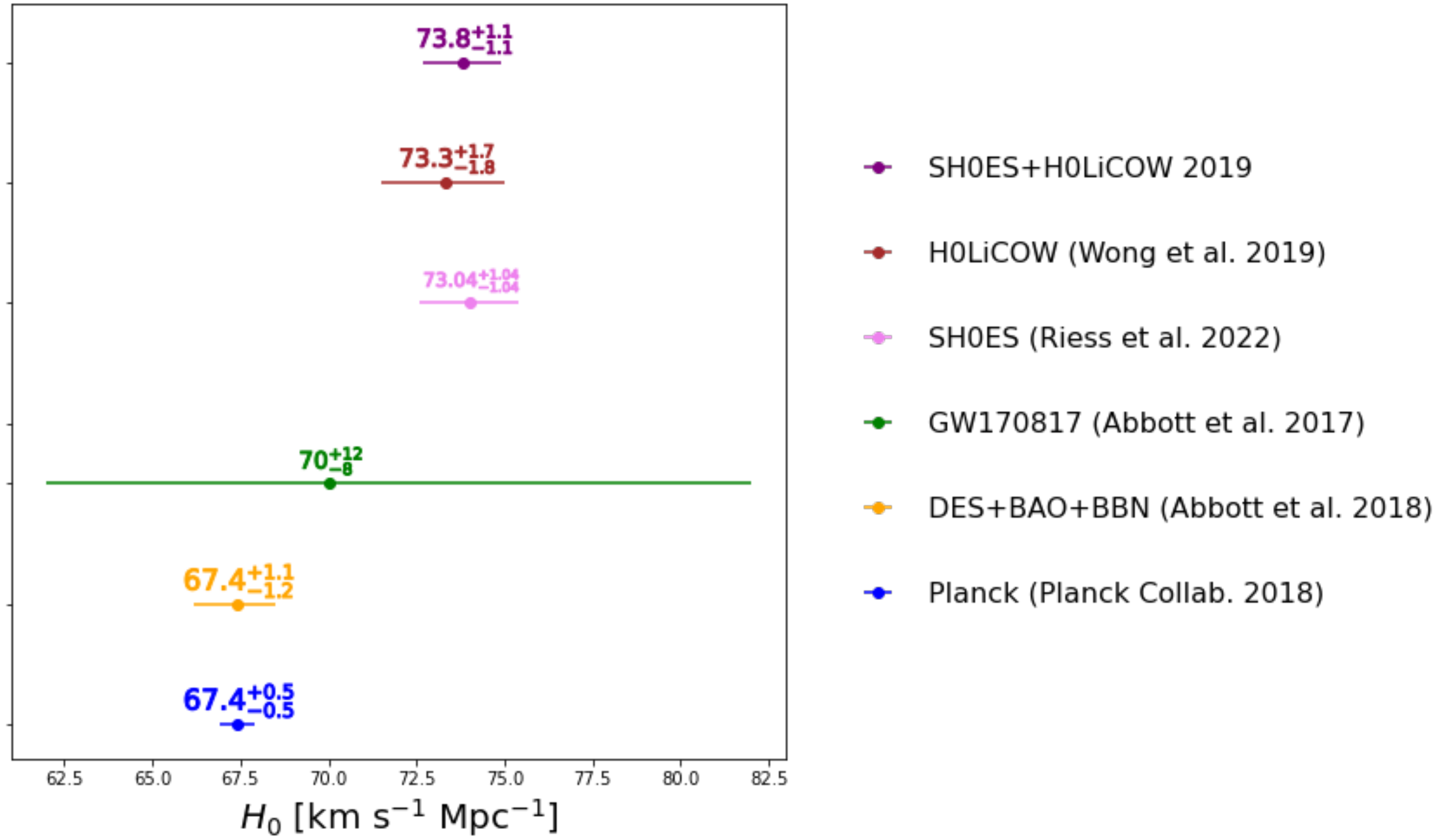
- GWs suffer of mass-redshift degeneracy.

H_0 tension



70^{+12}_{-8}
62.5
65.0
67.5
70.0
72.5
75.0
77.5
80.0
82.5
H_0 [km s^{-1} Mpc^{-1}]
GW170817 (Abbott et al. 2017)

H_0 tension



73.8+1.1 −1.1
73.3+1.7 −1.8
73.04+1.04 −1.04
70+12 −8
67.4+1.1 −1.2
67.4+0.5 −0.5
62.5
65.0
67.5
70.0
72.5
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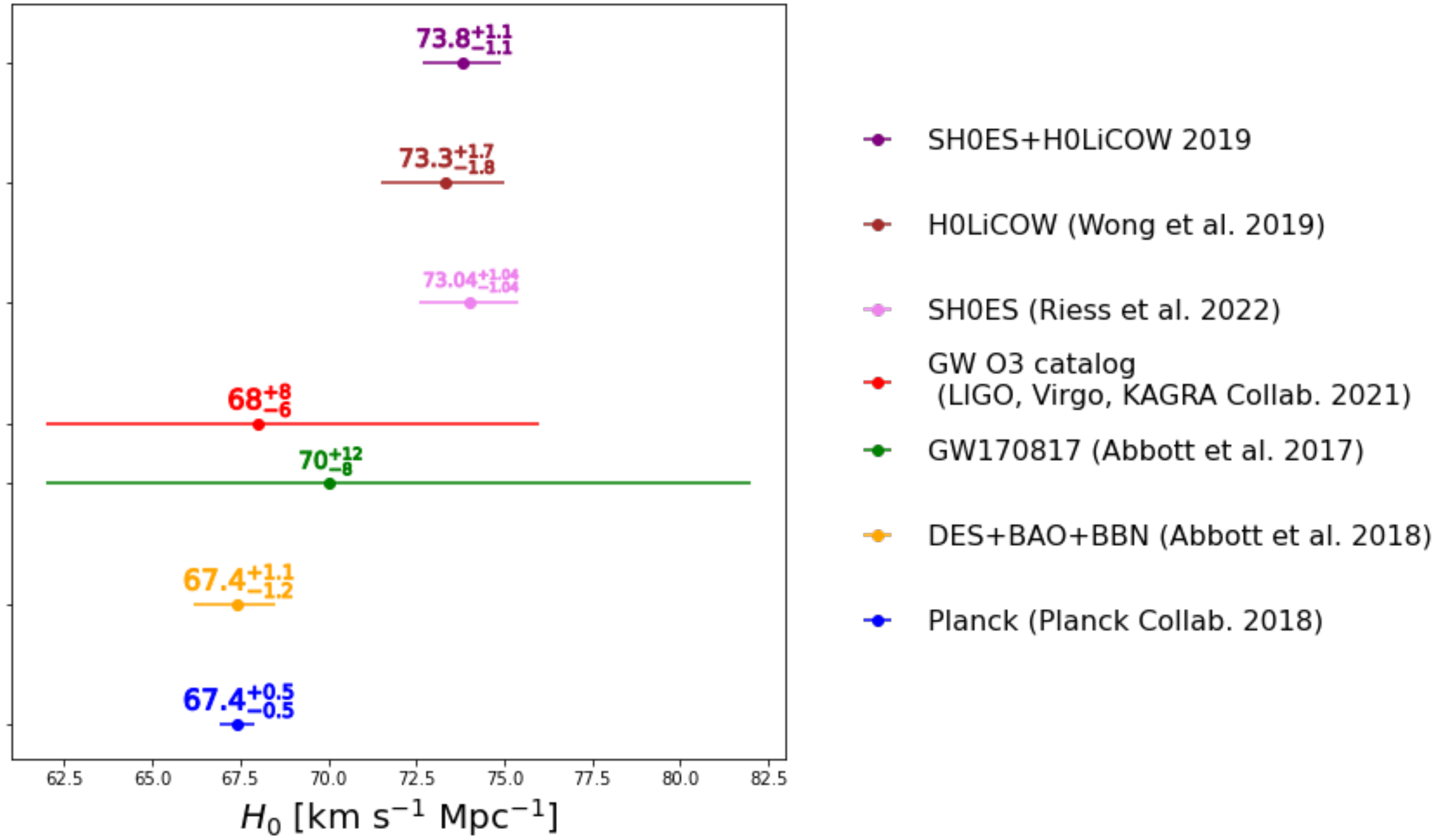
H_0 [km s^{-1} Mpc^{-1}]
SH0ES+H0LiCOW 2019
H0LiCOW (Wong et al. 2019)
SH0ES (Riess et al. 2022)
GW170817 (Abbott et al. 2017)
DES+BAO+BBN (Abbott et al. 2018)
Planck (Planck Collab. 2018)

H_0 tension


73.8$^{+1.1}_{-1.1}$
73.3$^{+1.7}_{-1.8}$
73.04$^{+1.04}_{-1.04}$
68$^{+8}_{-6}$
70$^{+12}_{-8}$
67.4$^{+1.1}_{-1.2}$
67.4$^{+0.5}_{-0.5}$
62.5 65.0 67.5 70.0 72.5 75.0 77.5 80.0 82.5
H_0 [km s^{-1} Mpc^{-1}]
SH0ES+H0LiCOW 2019
H0LiCOW (Wong et al. 2019)
SH0ES (Riess et al. 2022)
GW O3 catalog (LIGO, Virgo, KAGRA Collab. 2021)
GW170817 (Abbott et al. 2017)
DES+BAO+BBN (Abbott et al. 2018)
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Third generation detectors: Einstein Telescope

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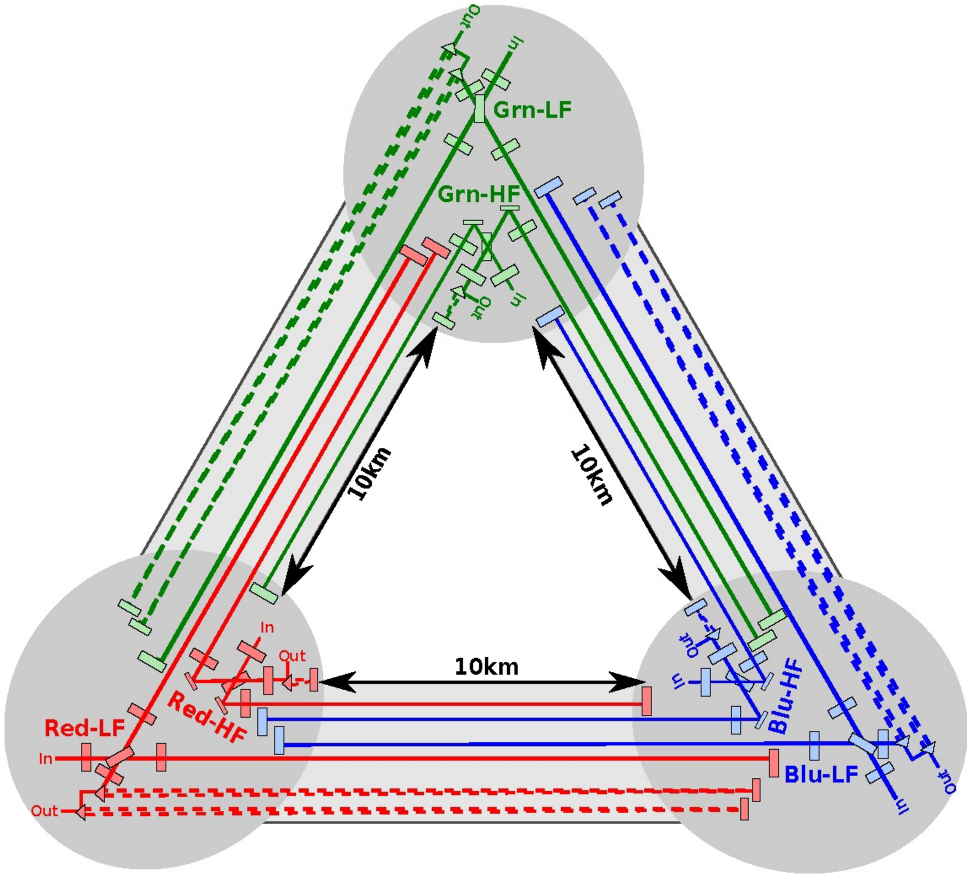

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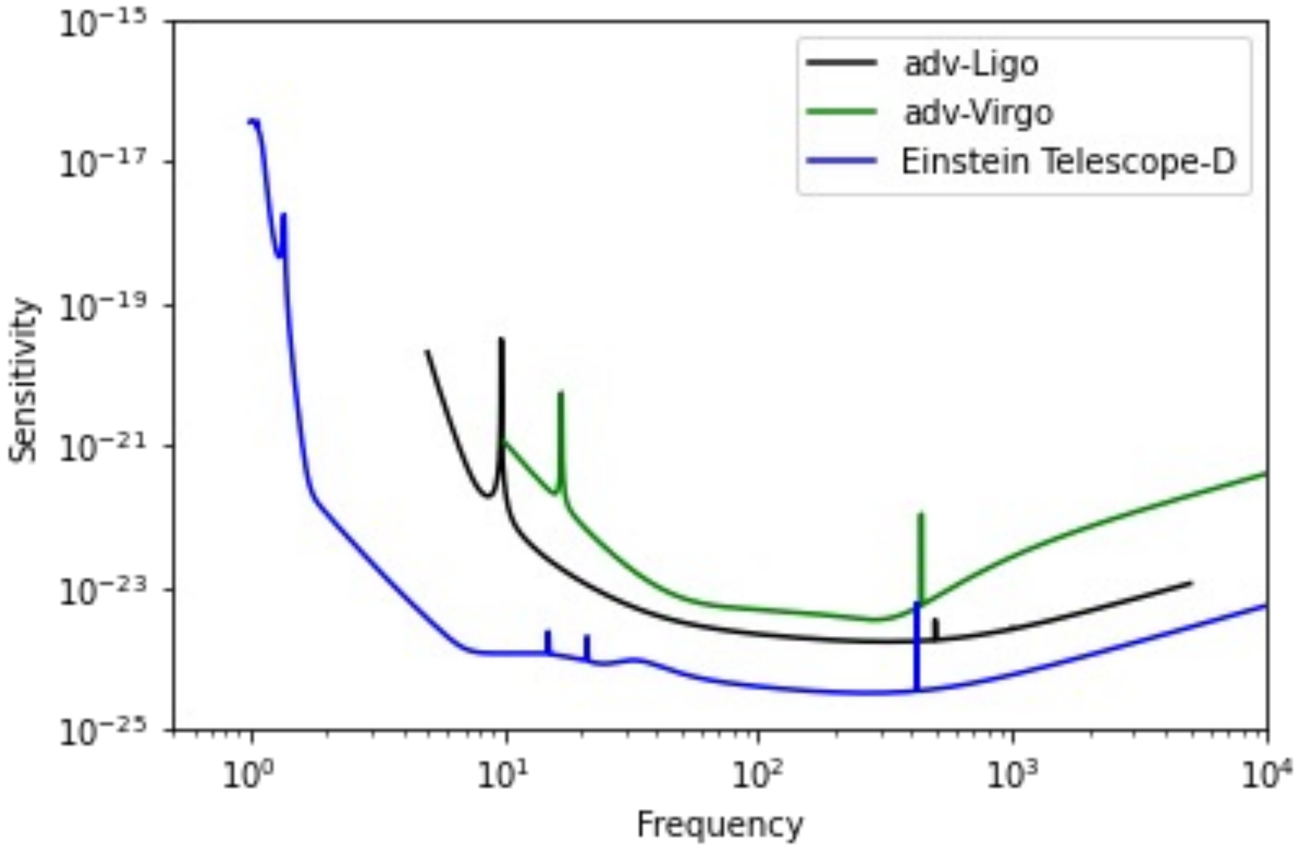


Construction of mock sources catalog

- We assume a fiducial cosmological model Λ-CDM

$$H_0 = 67.66\ km\ s^{-1}\ Mpc^{-1} \quad \Omega_{k,0} = 0.00 \quad \Omega_{\Lambda,0} = 0.6889$$

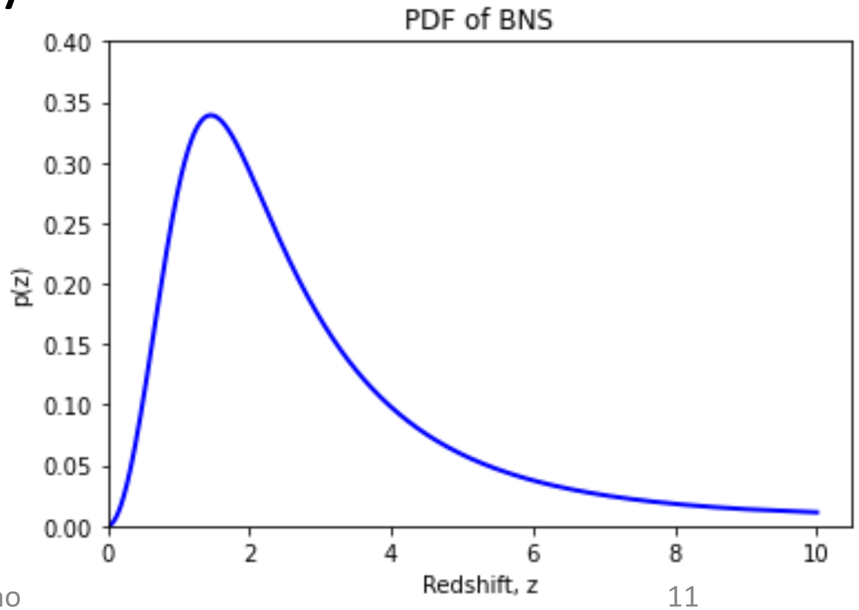
- Given a cosmology the theoretical luminosity distance will be

$$D_L^{th}(z) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz'}{\sqrt{(1-\Omega_{k,0}-\Omega_{\Lambda,0})(1+z')^3 + \Omega_{k,0}(1+z')^2 + \Omega_{\Lambda,0}}}$$

- We extract the redshift from a normalized probability distribution

$$p(z) = \frac{R_z(z)}{\int_0^{10} R_z(z)\,dz}, \qquad R_z(z) = \frac{R_m(z)}{1+z}\frac{dV(z)}{dz},$$

$$R_m(z) = R_0 \cdot \int_{t_{min}}^{t_{max}} R_f[t(z)-t_d]\,P(t_d)\,dt_d\,,$$



- To generate the synthetic signals of GWs, we assume the NS mass distribution be uniform in the interval $[1, 2.5]\ M_{\odot}$.
- We select sky angles θ and ϕ from an isotropic distribution, and orientation angle i and the polarization ψ from uniform distribution.
- We estimate the SNR ρ for the ET.

$$\rho = \sqrt{\sum_i (\rho^{(i)})^2}$$

$$\rho_i^2 = \frac{5}{6} \frac{\left[G\ M_{c,obs}\right]^{\frac{5}{3}} F_i^2(\theta, \phi, \psi, i)}{c^3 \pi^{\frac{4}{3}} d_L^2(z)} \int_{f_{lower}}^{f_{max}} \frac{f^{\frac{7}{3}}}{S_{h,i}(f)} df$$

- We retain an event if $\rho > \rho^{threshold}$.

- We extracted d_L from a Gaussian distribution $\mathcal{N}\left(d_L^{fid}, \sigma_{d_L}\right)$

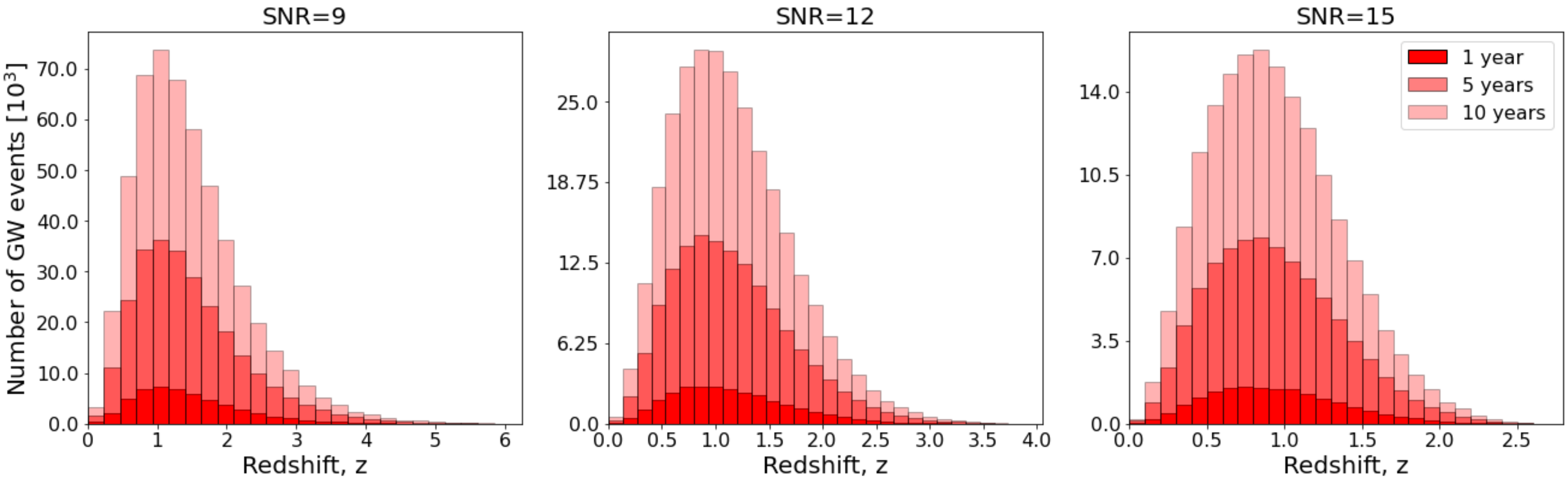
$$\sigma_{d_L} = \sqrt{\left(\sigma_{inst}^2 + \sigma_{lens}^2 + \sigma_{pec}^2\right)}$$

$$\sigma_{instr} = \frac{2}{\rho} d_L(z)$$

$$\sigma_{lens} = F_{delens}(z) \left[0.066 \left(\frac{1-(1+z)^{-0.25}}{0.25}\right)^{1.8} d_L(z)\right]$$

$$\sigma_{pec} = \left[1 + \frac{c(1+z)^2}{H(z) d_L(z)}\right] \frac{\sqrt{\langle v^2 \rangle}}{c} d_L(z)$$

GW events distribution



Selection of Electromagnetic Counterpart

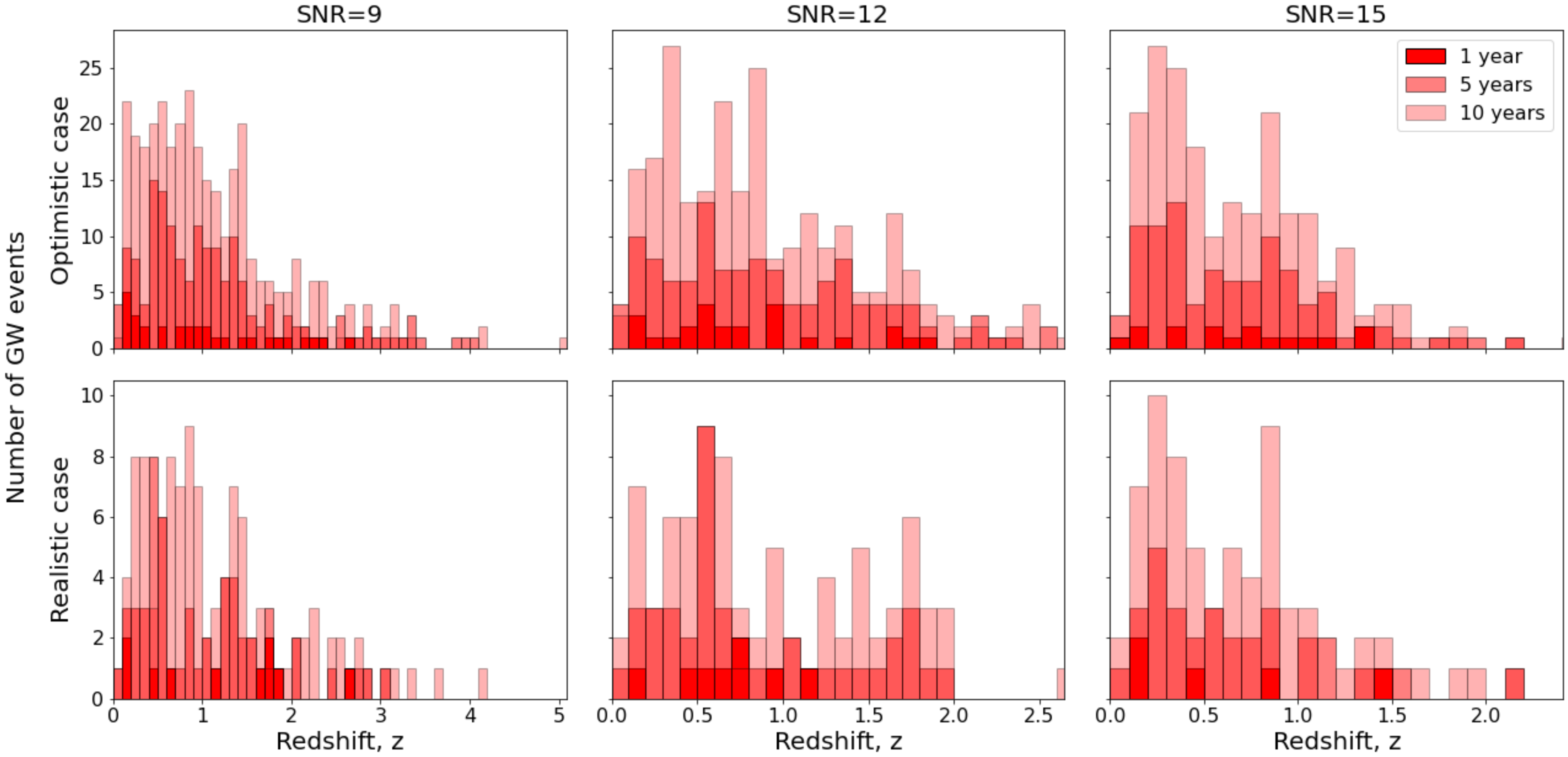
- We estimate the flux for the coincident short GRB

$$F(\theta_V) = \frac{L(\theta_V)(1+z)}{4\pi d_L^2 k(z) b}$$

$$b = \frac{\int_{1\ keV}^{10^4 keV} EN(E)dE}{\int_{E_1}^{E_2} N(E)dE} \qquad k(z) = \frac{\int_{E_1}^{E_2} N(E)dE}{\int_{E_1(1+z)}^{E_2(1+z)} N(E)dE}$$

- We record the combined event if $F(\theta_V) > F^{threshold} \left(= 0.2 \frac{ph}{cm^2 s}\right)$ for THESEUS satellite.
- We estimate a rate of combined detection of $5 \div 31$ events /year.

GW events with detected short GRB



Analysis

- The analysis relies on Bayes' theorem

$$\underbrace{p(H_0, \Omega_{k,0}, \Omega_{\Lambda,0} | \boldsymbol{d})}_{\text{Posterior}} \propto \underbrace{\pi(H_0, \Omega_{k,0}, \Omega_{\Lambda,0})}_{\text{Prior}} \underbrace{p(\boldsymbol{d} | H_0, \Omega_{k,0}, \Omega_{\Lambda,0})}_{\text{Likelihood}}$$

- We choose an uninformative prior on the cosmological parameters

$$\pi(H_0) = \mathcal{U}(50, 90)$$

$$\pi(\Omega_{k,0}) = \mathcal{U}(-1,1)$$

$$\pi(\Omega_{\Lambda,0}) = \mathcal{U}(0,1)$$

Case I: Bright Sirens

- We include in the single-event likelihood the selection effects $\rho > \rho^t$, $F(\theta_V) > F^t$

Mandel et al., MNRAS (2019)

$$p(d_i|H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) = \frac{\int p(d_i|D_L, H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) p_{pop}(D_L|H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) dD_L}{\int p_{det}(D_L, H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) p_{pop}(D_L|H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) dD_L}$$

$$\Longrightarrow \quad p_{pop}(D_L|H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) = \delta\left(D_L^{th}(H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) - D_L\right)$$

$$\Longrightarrow \quad p(d_i|D_L, H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) \propto \exp -\frac{1}{2}\frac{(d_i - D_L)^2}{\sigma_{d_i}^2}$$

$$\Longrightarrow \quad p_{det}(D_L, H_0, \Omega_{k,0}, \Omega_{\Lambda,0}) = \int_{\substack{\rho>\rho^t \\ F>F^t}} p(d_i|D_L, H_0, \Omega_{k,0}, \Omega_{\Lambda,0})\, dd_i$$

Case II: Dark Sirens

- When we cannot extract the redshift information from electromagnetic signal, we have to marginalize the posterior over the redshift.

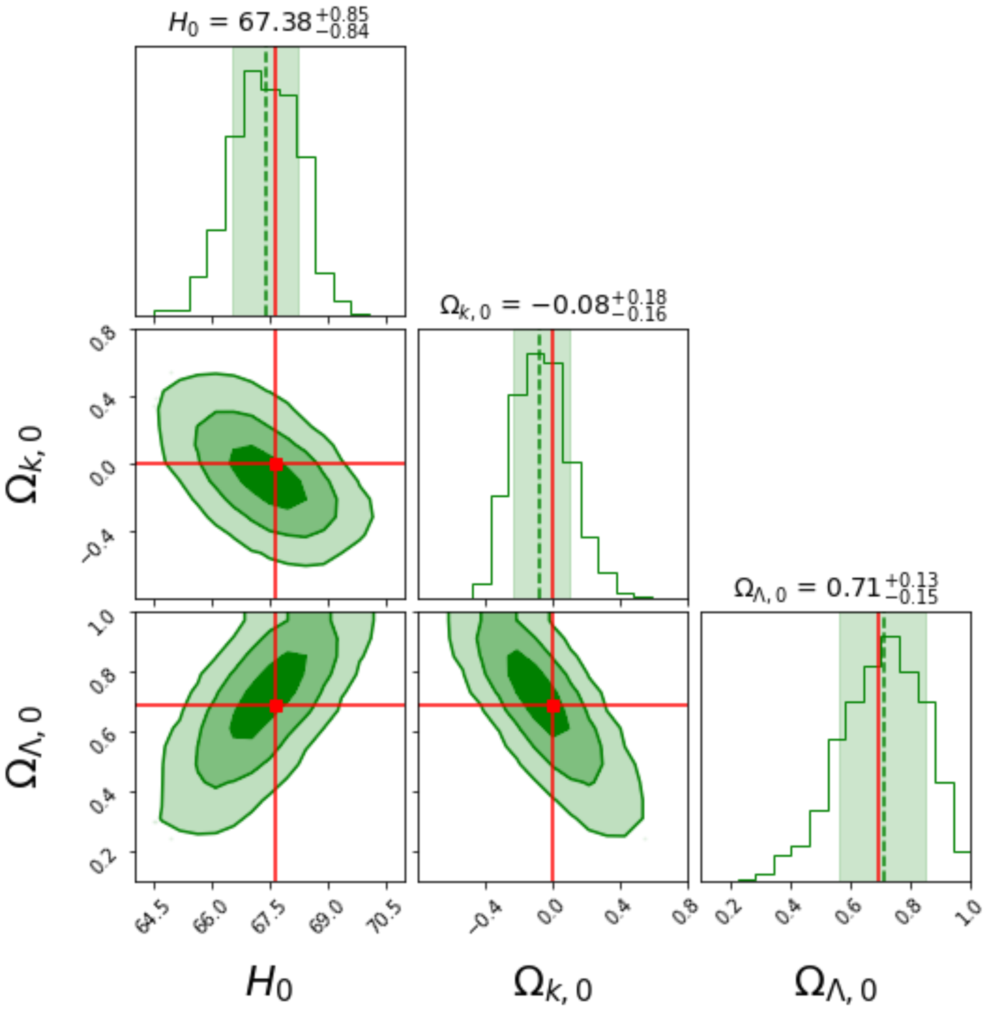
$$p(d_i|H_0,\Omega_{k,0},\Omega_{\Lambda,0}) = \int_0^{z_{max}} p\left(d_i|d_L^{th}(z,H_0,\Omega_{k,0},\Omega_{\Lambda,0})\right) \underbrace{p_{obs}(z)}_{p(z|\rho > \rho^t)} dz$$

Ding et al., JCAP (2019)

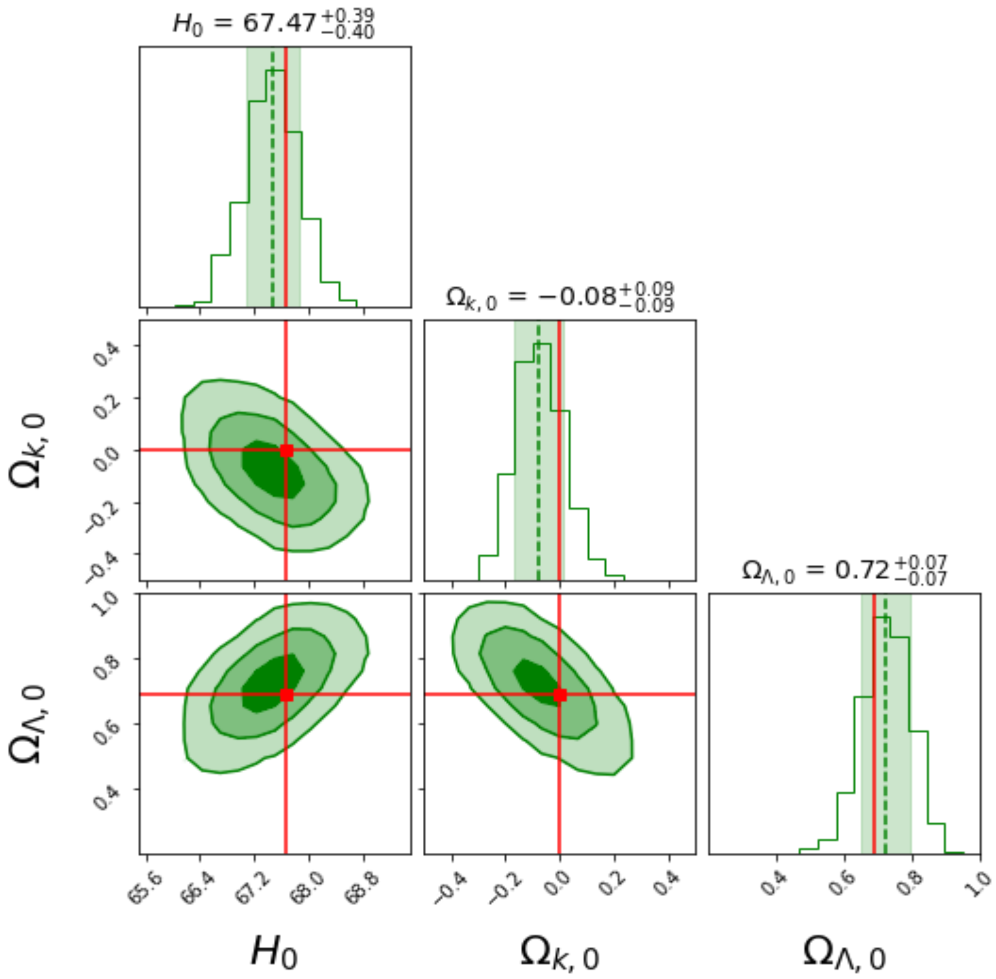
Results

Case I / MCMC sampling

SNR= 9, years = 10, Realistic Case

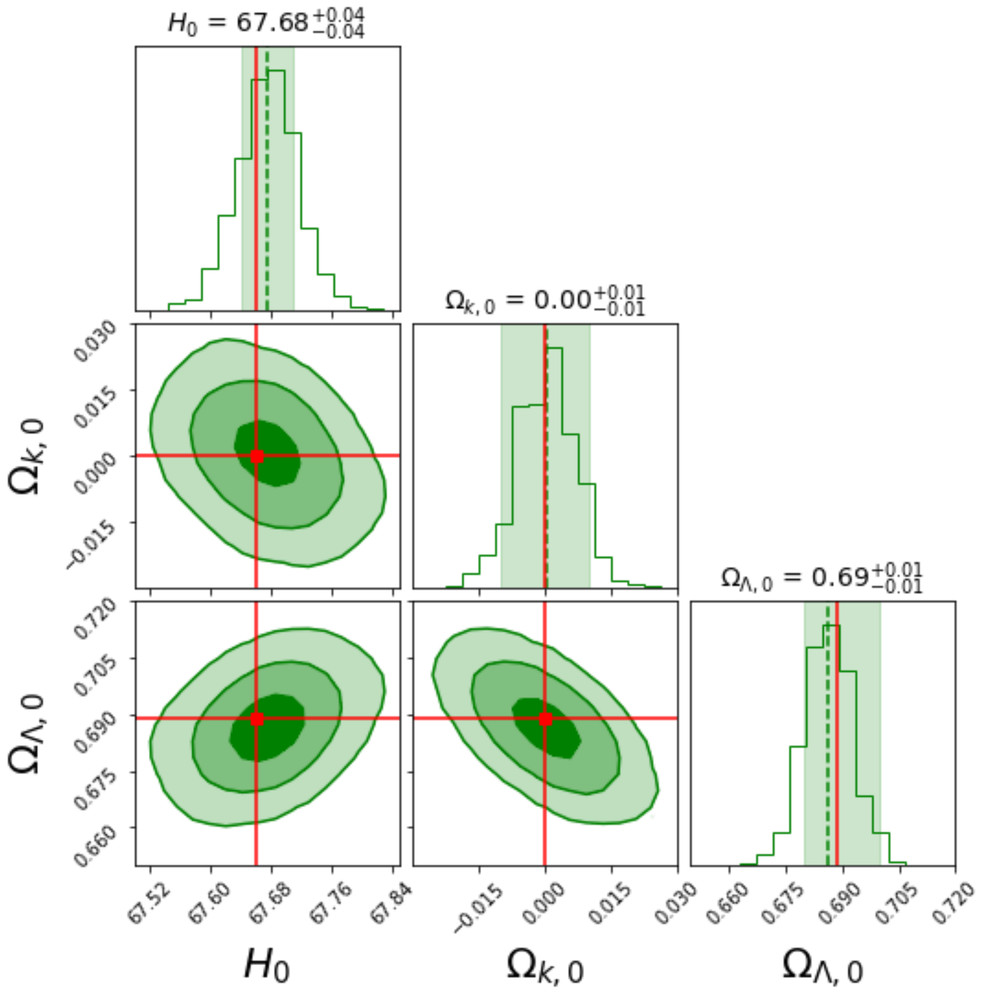


SNR= 9, years = 10, Optimistic Case

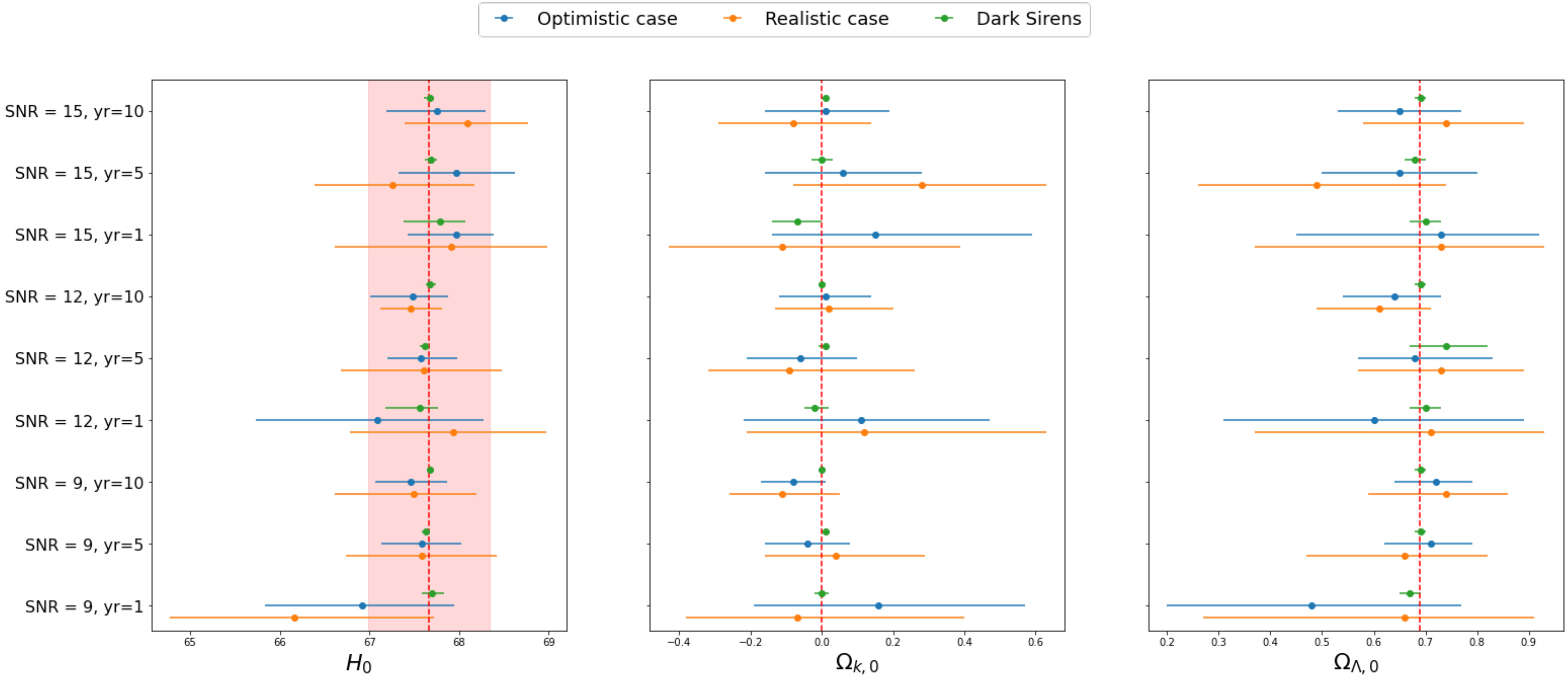


Case II / MCMC sampling

SNR= 9, years = 10

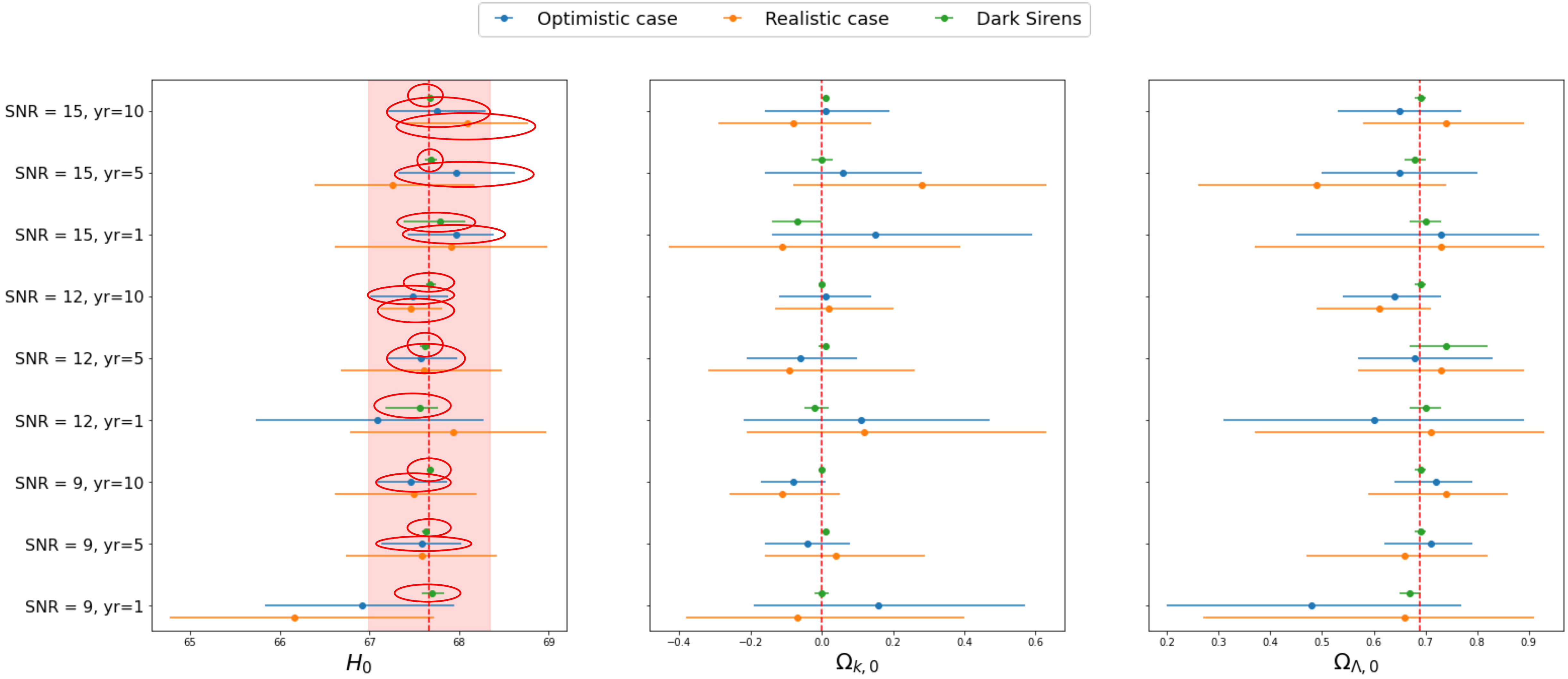


Results



Optimistic case
Realistic case
Dark Sirens
SNR = 15, yr=10
SNR = 15, yr=5
SNR = 15, yr=1
SNR = 12, yr=10
SNR = 12, yr=5
SNR = 12, yr=1
SNR = 9, yr=10
SNR = 9, yr=5
SNR = 9, yr=1
65
66
67
68
69
H_0
−0.4
−0.2
0.0
0.2
0.4
0.6
$\Omega_{k,0}$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\Omega_{\Lambda,0}$

Results



Optimistic case
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Dark Sirens
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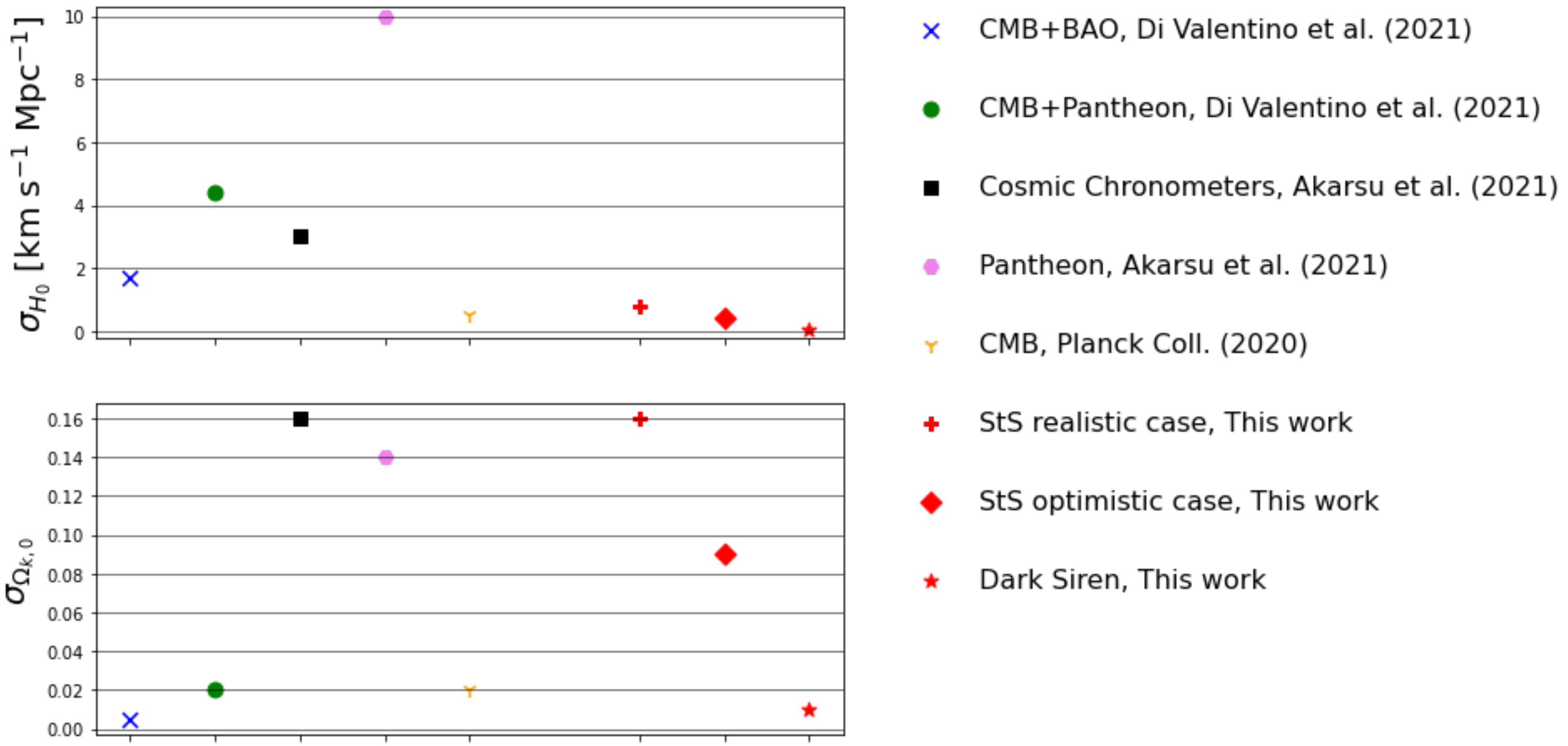
SNR = 12, yr=1
SNR = 9, yr=10
SNR = 9, yr=5
SNR = 9, yr=1
65
66
67
68
69
H_0
−0.4
−0.2
0.0
0.2
0.4
0.6
$\Omega_{k,0}$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
$\Omega_{\Lambda,0}$

Comparison with the other sources



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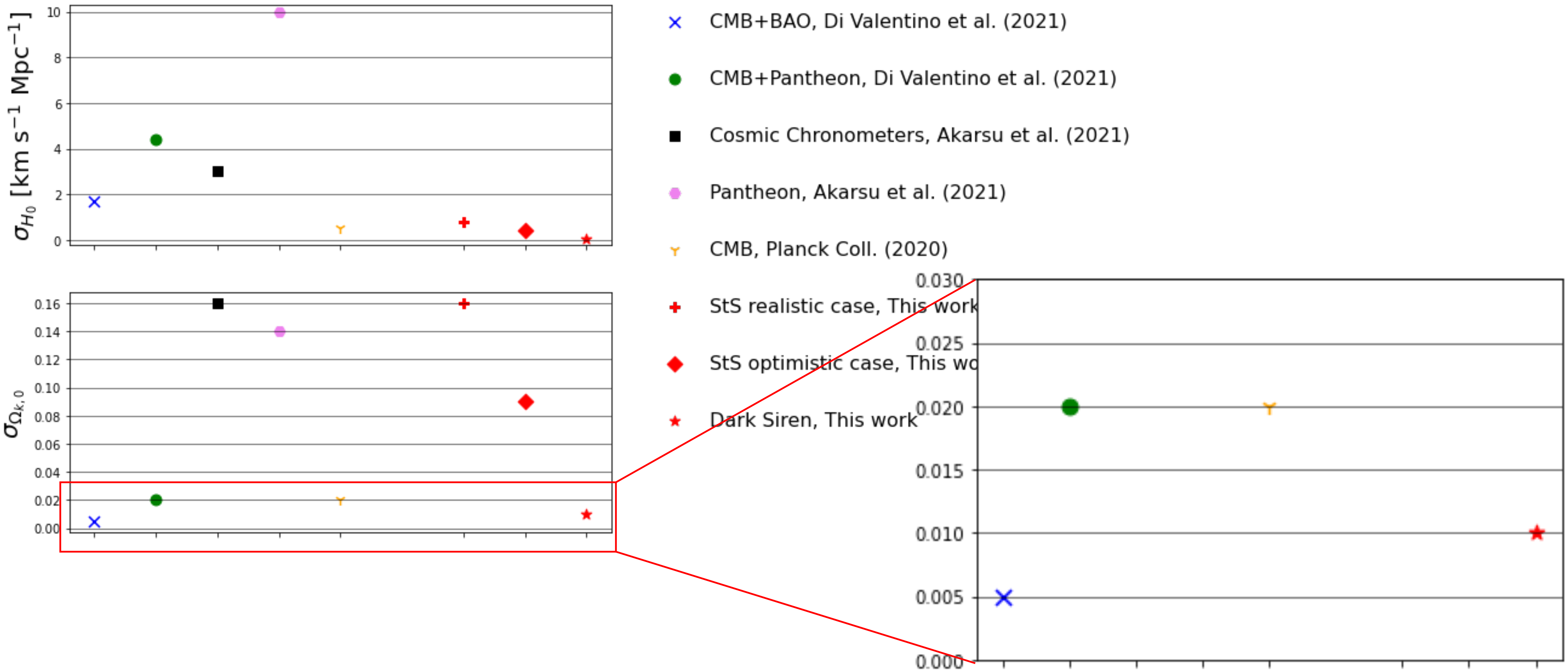

Conclusions

- With the Einstein Telescope, we could achieve an accuracy on the Hubble constant less than 1%.

- The accuracy on $\Omega_{k,0}$ is

$$\sigma_{\Omega_{k,0}} = 0.09 \text{ with Bright Sirens}$$
$$\sigma_{\Omega_{k,0}} = 0.01 \text{ with Dark Sirens}$$

- The accuracy on $\Omega_{\Lambda,0}$ is

$$\sigma_{\Omega_{\Lambda,0}} = 0.07 \text{ with Bright Sirens}$$
$$\sigma_{\Omega_{\Lambda,0}} = 0.01 \text{ with Dark Sirens}$$

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$$\sigma_{\Omega_{\Lambda,0}} = 0.01 \text{ with Dark Sirens}$$

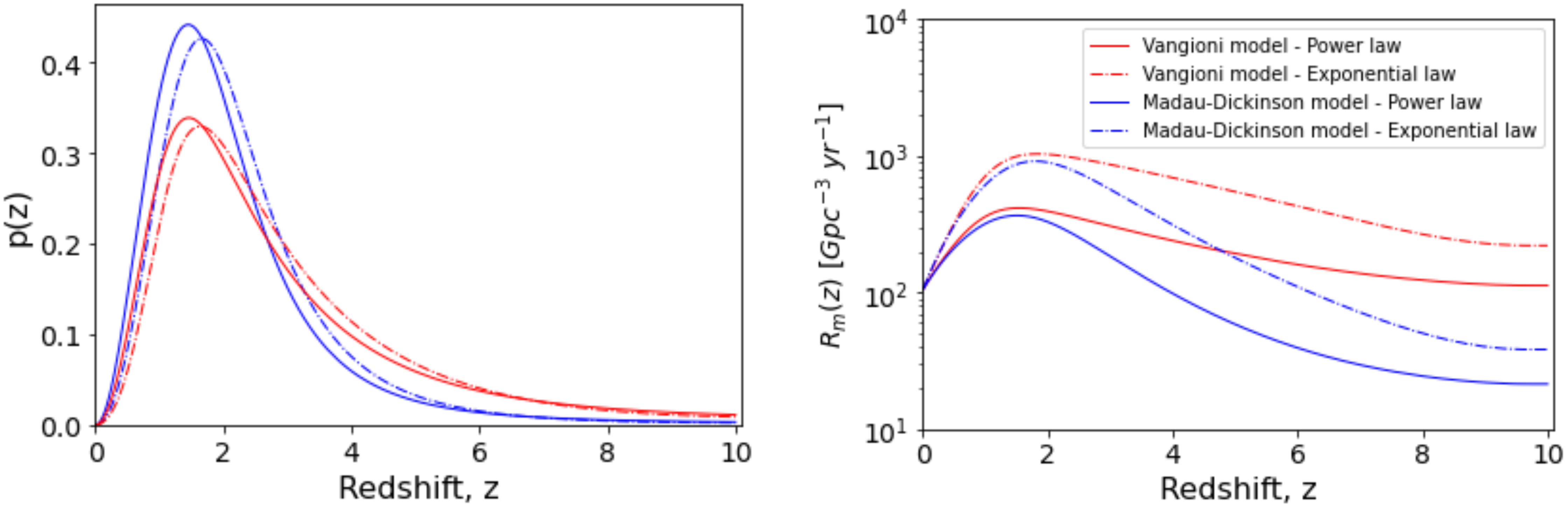
Thank you!

Question?

Extra: Impact of different assumptions

- Star Formation Rate:
  - Vangioni Model $R_f(z) = \frac{\nu a \ \exp(b(z-z_m))}{a-b+b \ \exp(a(z-z_m))}$ Vangioni et al.,MNRAS, 2015
  - Madau-Dickinson model $R_f(z) = \frac{(1+z)^{\alpha}}{1+\left[\frac{(1+z)}{C}\right]^{\beta}}$ Madau and Dickison, ARAA,2014

- t_d probability distribution:
  - Power law $P(t_d) = t_d^{-1}$
  - Exponential $P(t_d) = \tau^{-1} \exp\left(-\frac{t_d}{\tau}\right)$



0.4
0.3
0.2
0.1
0.0
p(z)
0
2
4
6
8
10
Redshift, z
10^4
10^3
10^2
10^1
$R_m(z)$ [Gpc^{-3} yr^{-1}]
Vangioni model - Power law
Vangioni model - Exponential law
Madau-Dickinson model - Power law
Madau-Dickinson model - Exponential law
0
2
4
6
8
10
Redshift, z

SNR=9, years=10

GW + EM events

MODEL	# events	H_0	$\Omega_{k,0}$	$\Omega_{\Lambda,0}$
Baseline model	332	$67.47^{+0.39}_{-0.40}$	$-0.08^{+0.08}_{-0.09}$	$0.72^{+0.07}_{-0.07}$
Model 1	603	$67.18^{+0.34}_{-0.32}$	$0.01^{+0.07}_{-0.07}$	$0.65^{+0.06}_{-0.06}$
Model 2	271	$67.48^{+0.30}_{-0.30}$	$-0.09^{+0.09}_{-0.10}$	$0.71^{+0.07}_{-0.07}$
Model 3	536	$67.20^{+0.27}_{-0.28}$	$0.01^{+0.08}_{-0.07}$	$0.65^{+0.05}_{-0.06}$

Dark Sirens

MODEL	# events	H_0	$\Omega_{k,0}$	$\Omega_{\Lambda,0}$
Baseline model	521552	$67.68^{+0.04}_{-0.04}$	$0.00^{+0.01}_{-0.01}$	$0.69^{+0.01}_{-0.01}$
Model 1	1143212	$67.64^{+0.04}_{-0.04}$	$0.00^{+0.01}_{-0.01}$	$0.69^{+0.01}_{-0.01}$
Model 2	443560	$67.62^{+0.05}_{-0.05}$	$0.01^{+0.01}_{-0.01}$	$0.68^{+0.01}_{-0.01}$
Model 3	966659	$67.68^{+0.04}_{-0.04}$	$-0.01^{+0.01}_{-0.01}$	$0.68^{+0.01}_{-0.01}$

Table 6: The *baseline* model adopts the *Vangioni model* for the SFR and the *power law* form of the time delay distribution; **Model 1** is based on the *Vangioni model* for the SFR and the *exponential distribution* of the time delay distribution; **Model 2** is based on the *Madau - Dickison model* for the SFR and the *power law* form of the time delay distribution; **Model 3** is based on the *Madau - Dickison model* for the SFR and the *exponential distribution* of the time delay distribution.

Open Issue: Redshift Information

- Host galaxy identification
- Cross-Correlation

→

- Source sky localizzation error,
- σ_{D_L} error
- overlapping sky area between GW sources and galaxy surveys
- the accurate redshift estimation of galaxies

❖ Coincendent short GRB →

❖ Only 0.1% of GW events could have a detected countepart

- Prior information on redshift →

- We need to assume astrophysical model for the $R_m(z)$

➢ Tidal deformation →

Messenger and Read, PRL, 2012

➢ We need high precision in the signal analysis

➢ It depends on neutron star equation of state