



Techniques for Loop Calculations in the LHC era

and

Accords and interfaces

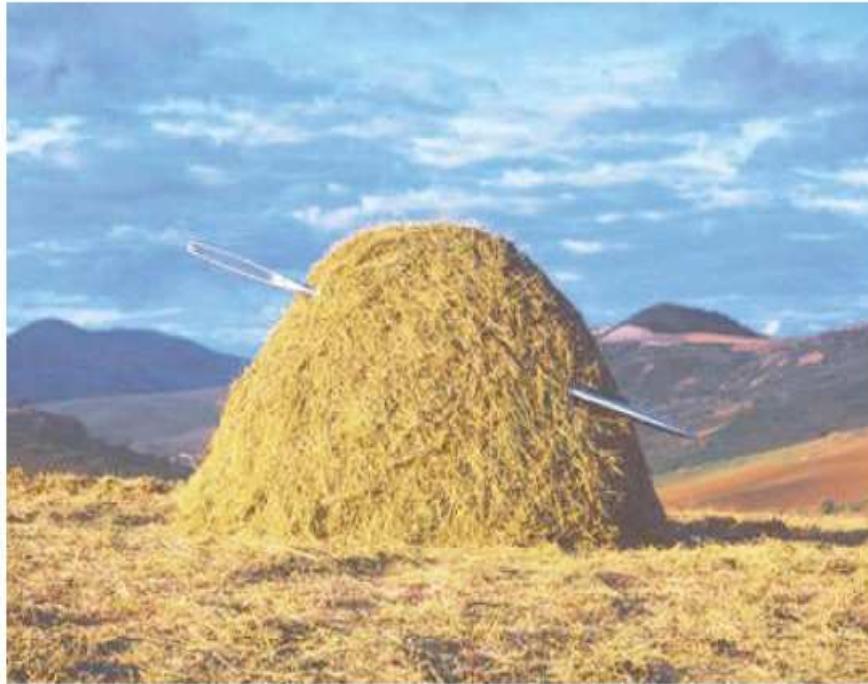
Fawzi BOUDJEMA

LAPTh-Annecy, France



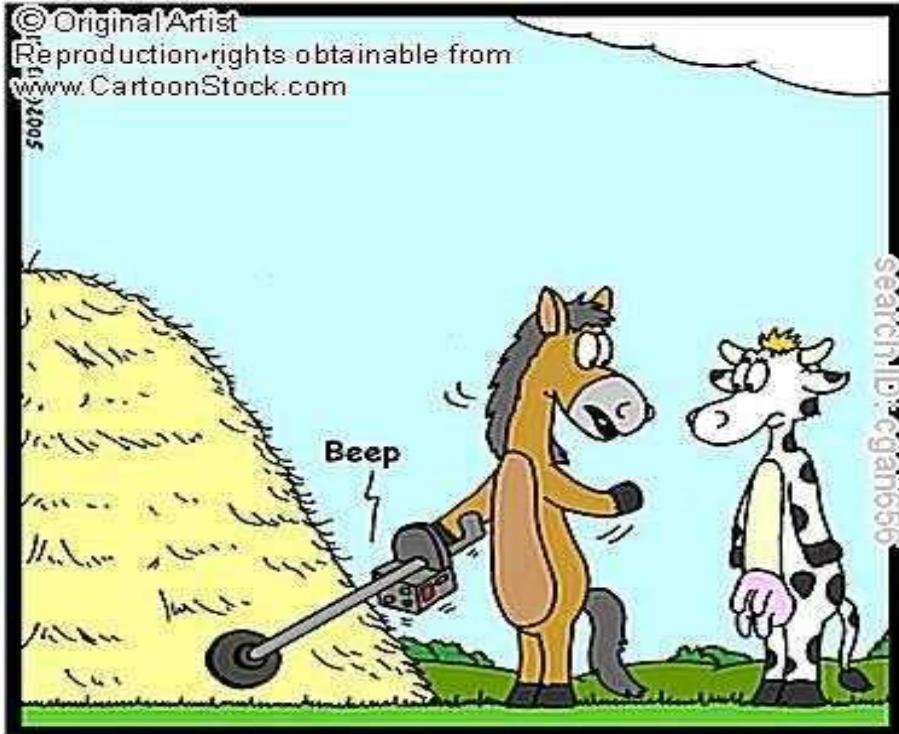
Approche du phénoménologue

What we hope for!



Magic Needle!

Approche du phénoménologue



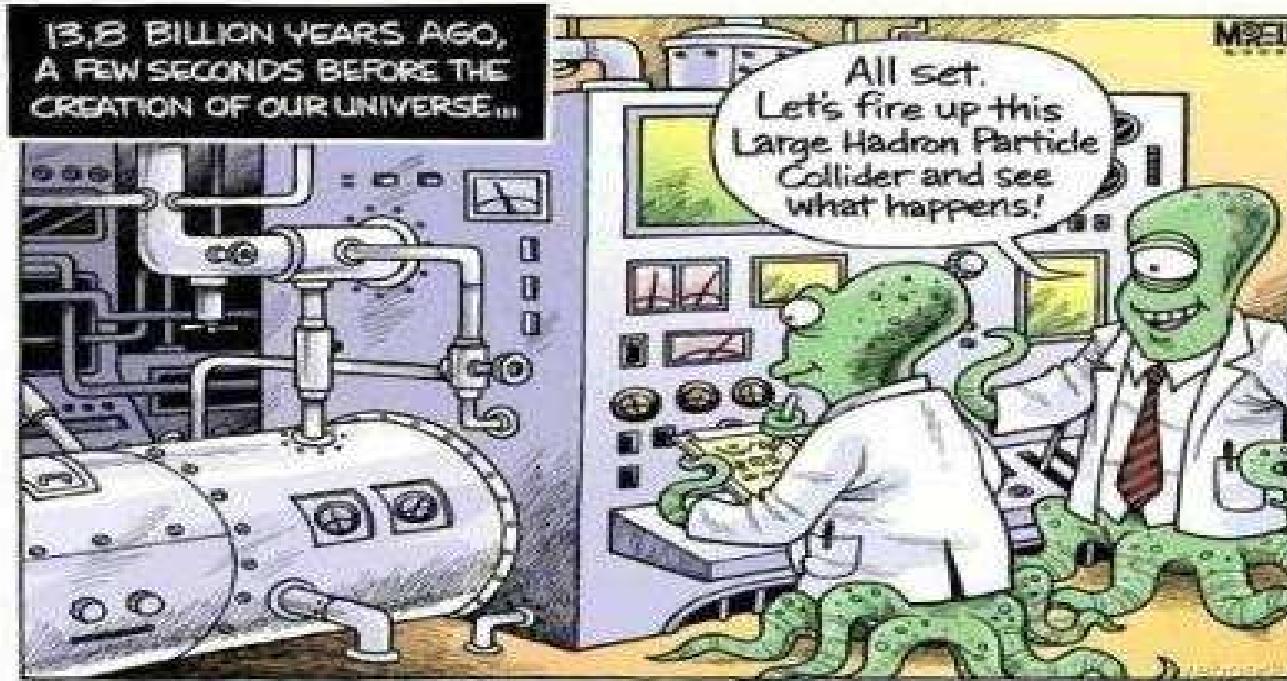
Clever Enough to find special signatures

Approche du phénoménologue

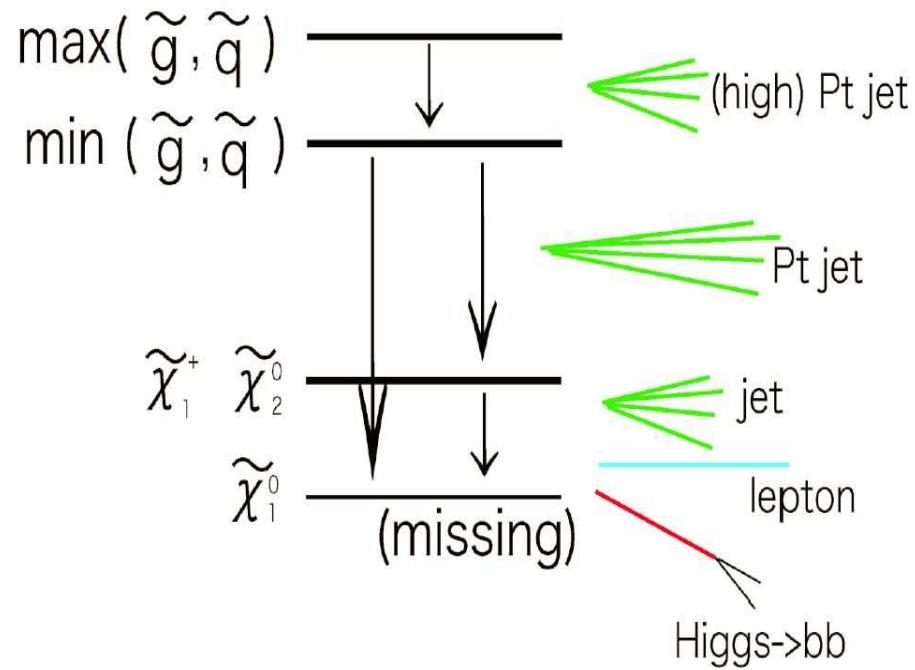


most probably count each straw to weigh the haystack precisely

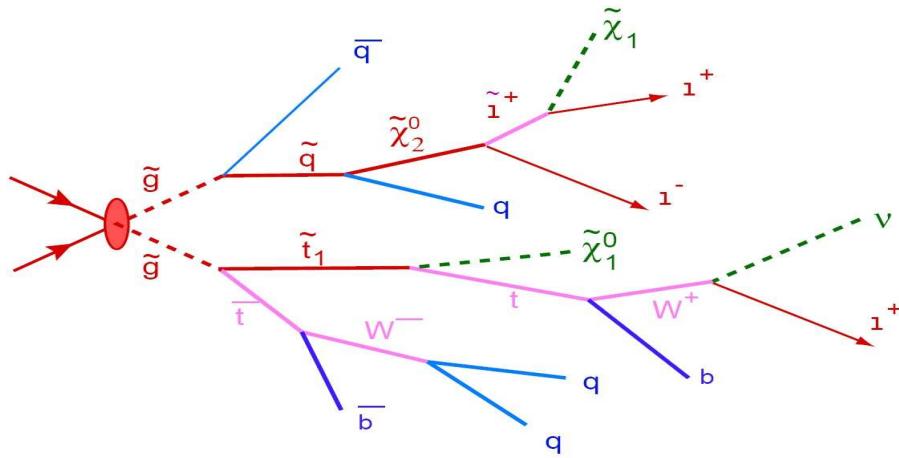
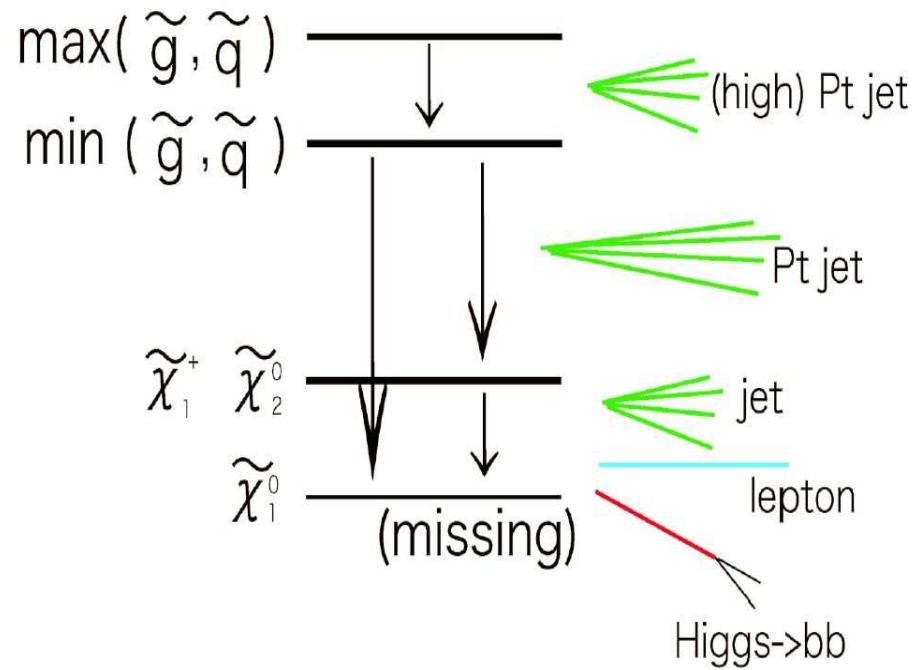
Turn on the machine!



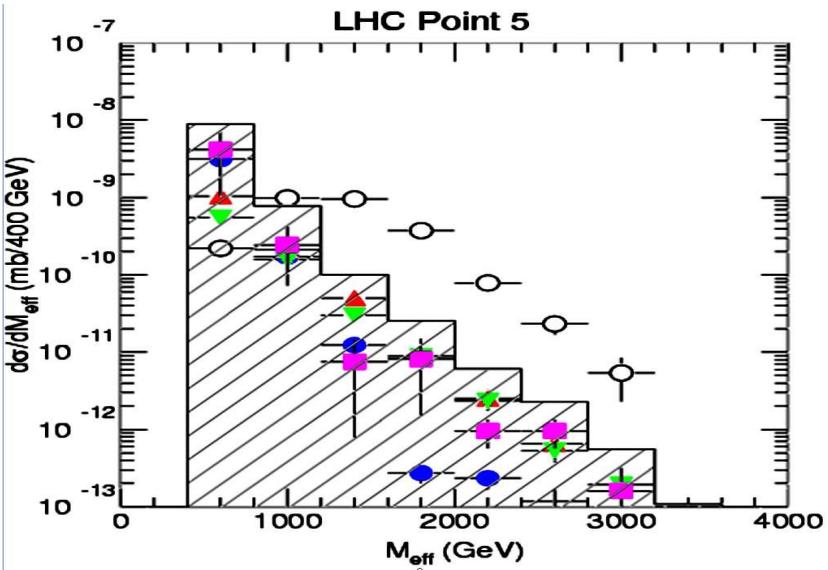
in 1998 we were told to expect an early SUSY discovery



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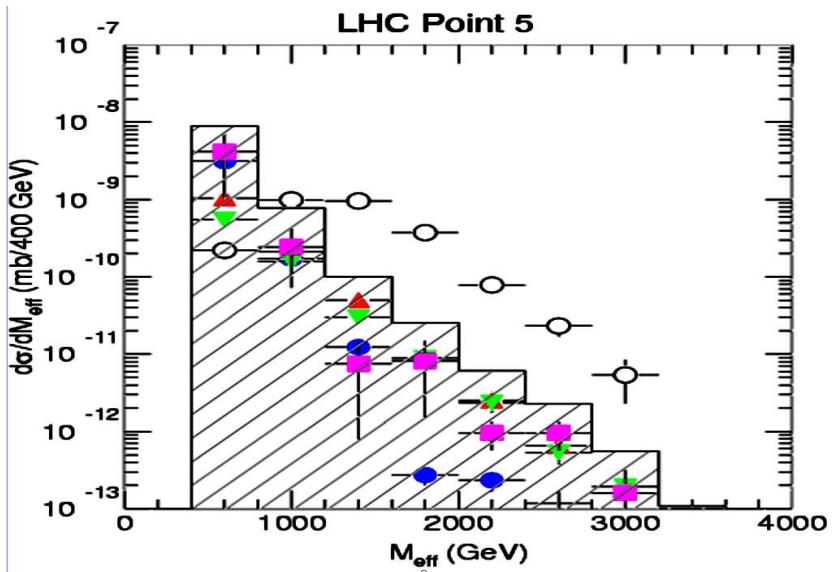


ATLAS TDR (same with CMS)

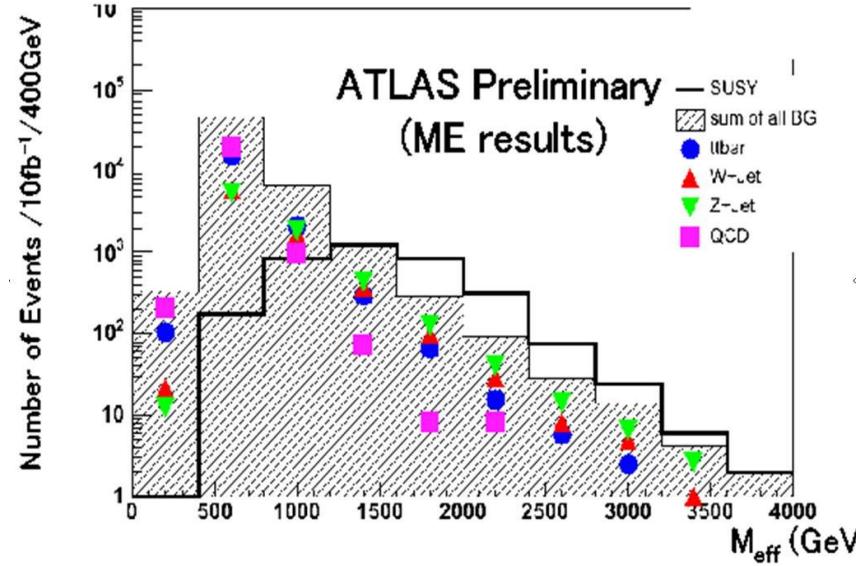


ATLAS TDR 98
(mSUGRA point, PreWMAP)

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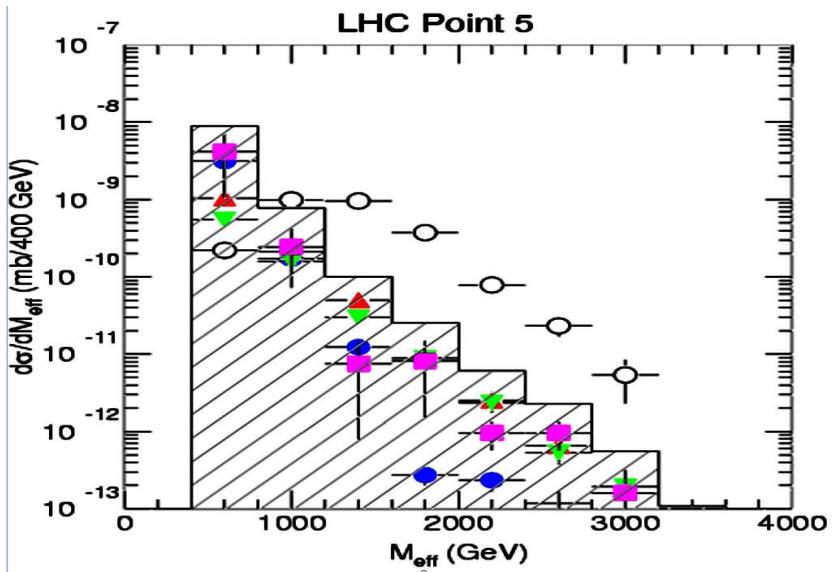


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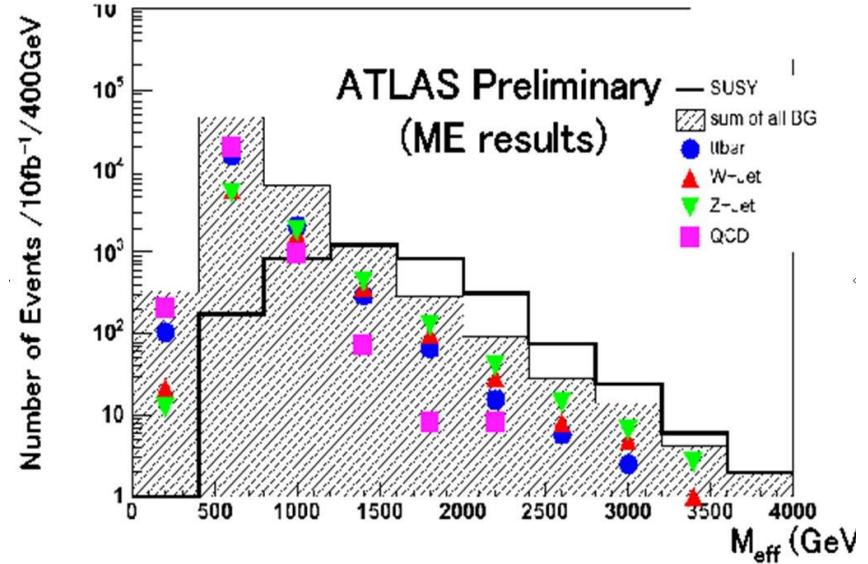


ATLAS 2006

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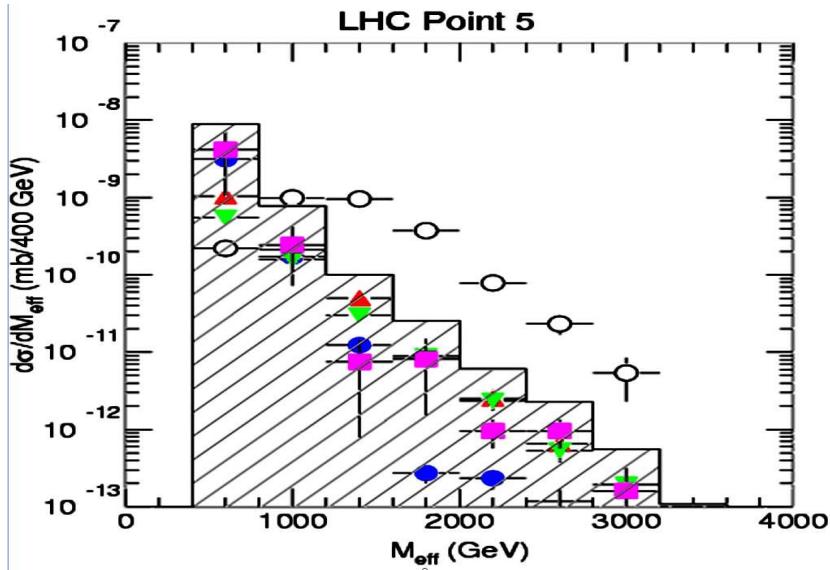
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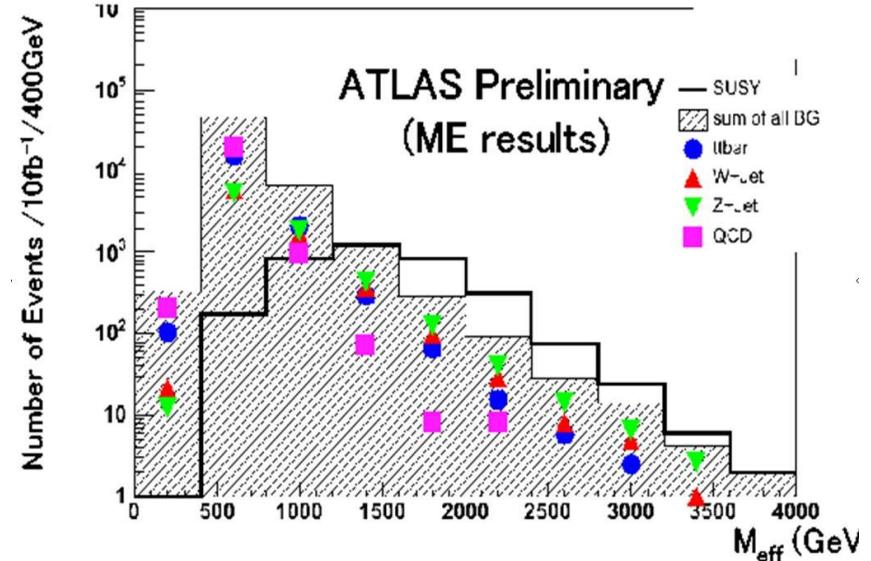
ATLAS 2006

What happened?

ATLAS TDR (same with CMS)



ATLAS TDR 98
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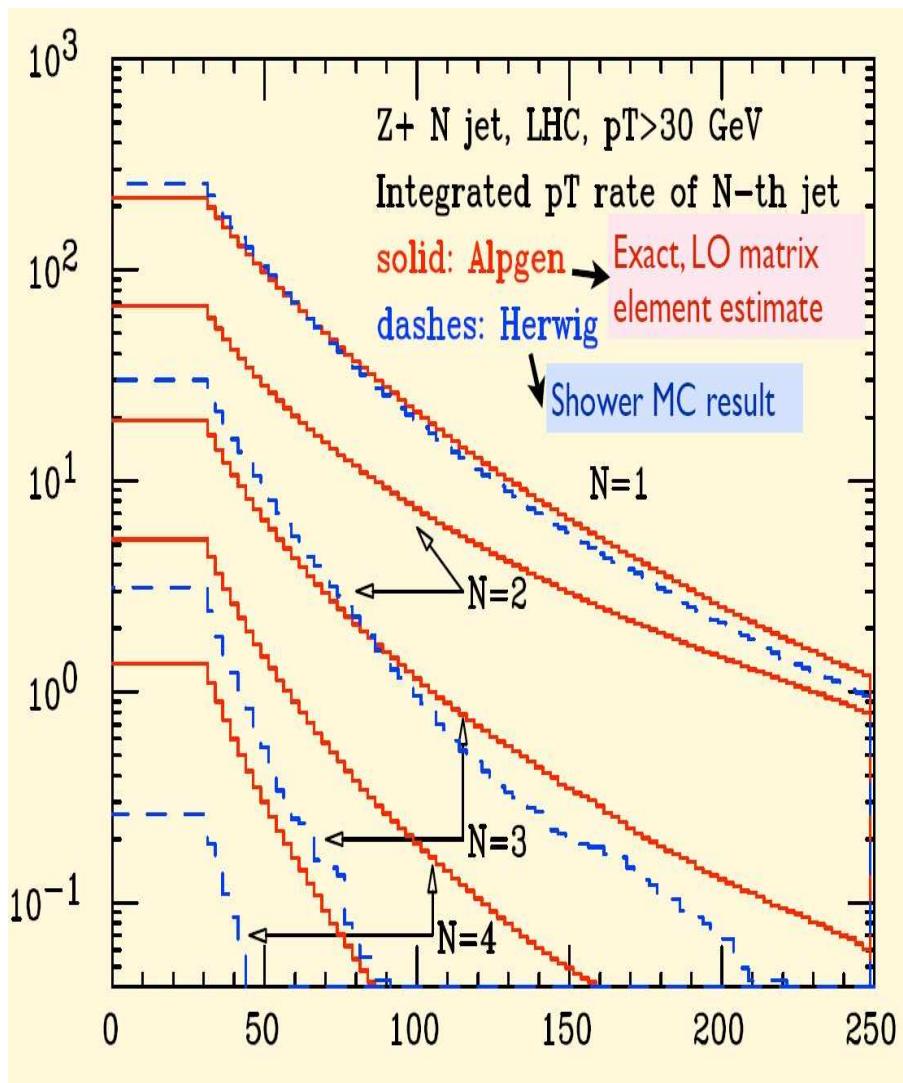


ATLAS 2006

QCD and SM processes can also produce hard jets! and these are/were lacking in PS/MC

ME vs PS: Limitations of PS

- PS do not describe hard jets
- ME do but in practice can not produce as many jets as PS
- ME evaluates the complete set of all diagrams/configurations: costly
- some real progress has been made in interfacing ME with PS



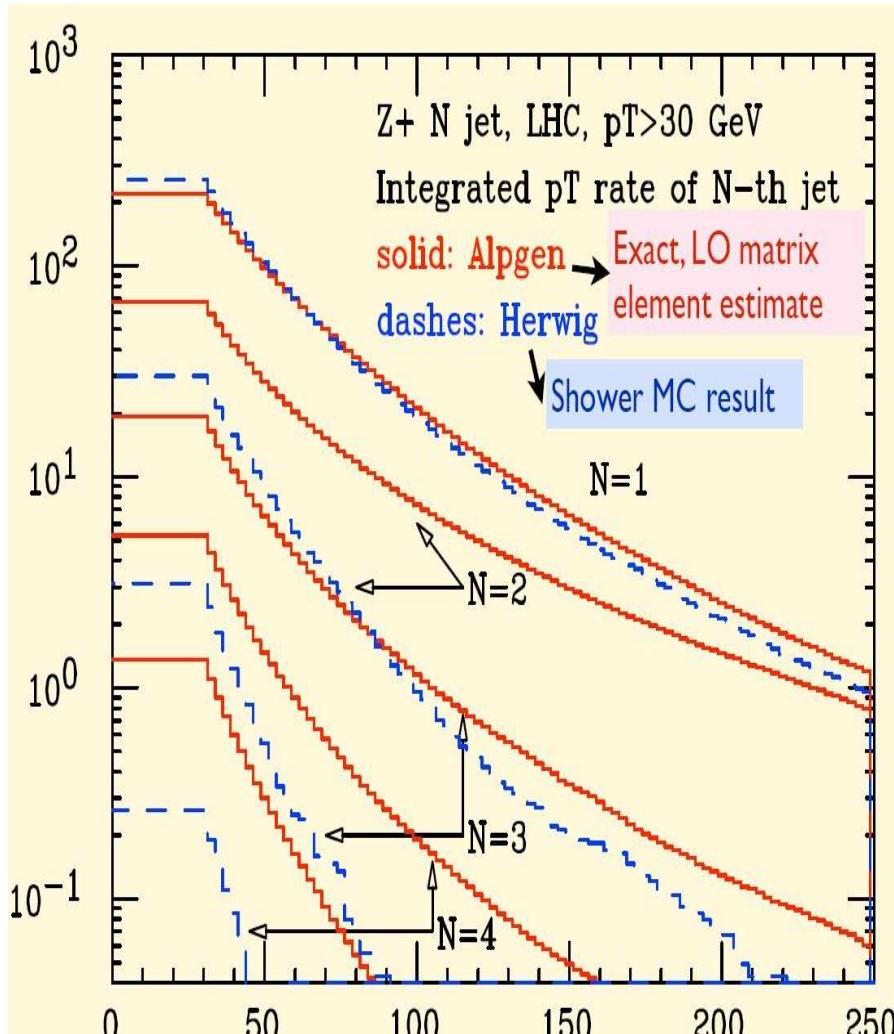
$$\frac{d\sigma_{ME}}{dx_1 dx_2} \propto \left| \text{[diagram]} + \text{[diagram]} \right|^2$$

$$\frac{d\sigma_{PS}}{dx_1 dx_2} \propto \left| \text{[diagram]} \right|^2 + \left| \text{[diagram]} \right|^2$$

Still, all of this at leading order although very recent improvement MC/@NLO and POWHEG (higher multiplicity + NLO-Hamilton+Nason)

ME vs PS: Limitations of PS

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- ME do but in practice can not produce as many jets as PS
- ME evaluates the complete set of all diagrams/configurations: costly
- some real progress has been made in interfacing ME with PS
- Importance of NLO multileg, normalisation, etc...**



$$\frac{d\sigma_{ME}}{dx_1 dx_2} \propto \left| \text{diagram } 1 + \text{diagram } 2 \right|^2$$

$$\frac{d\sigma_{PS}}{dx_1 dx_2} \propto \left| \text{diagram } 1 \right|^2 + \left| \text{diagram } 2 \right|^2$$

$$\sin^2 \theta_W$$

- From measured W/Z masses:

$$s_W^2 = s_M^2 = 1 - M_W^2/M_Z^2 \rightarrow$$

$$s_M^2 = .22164 \pm .00079 \text{ (i.e. } .4\% \text{ precision)}$$



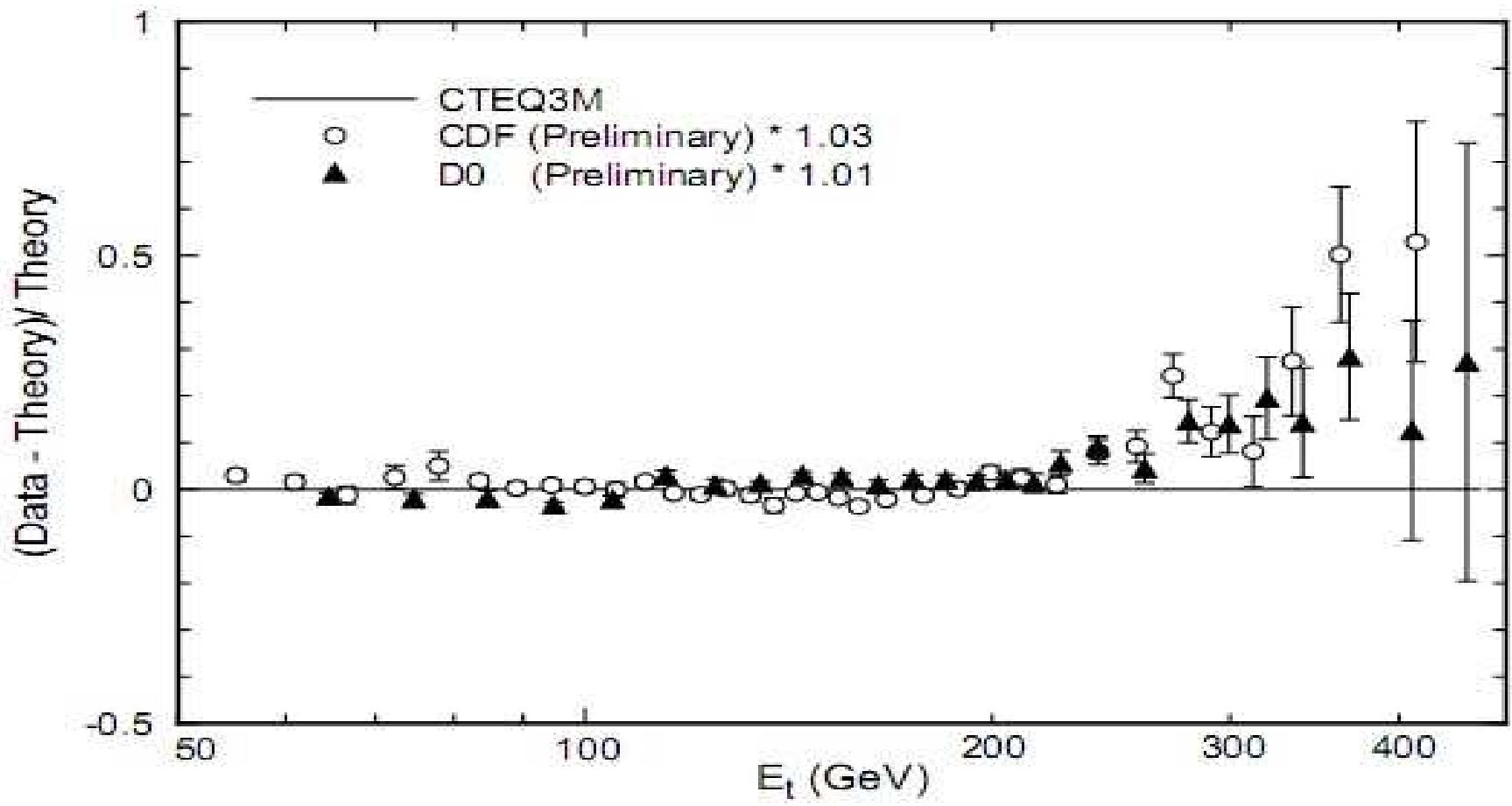
$10\sigma!!!$

- From Leptonic Z observables, e.g:

$$A_{LR}^e = \frac{2(1-4s_{\text{eff}}^2)}{1+(1-4s_{\text{eff}}^2)^2} \rightarrow$$

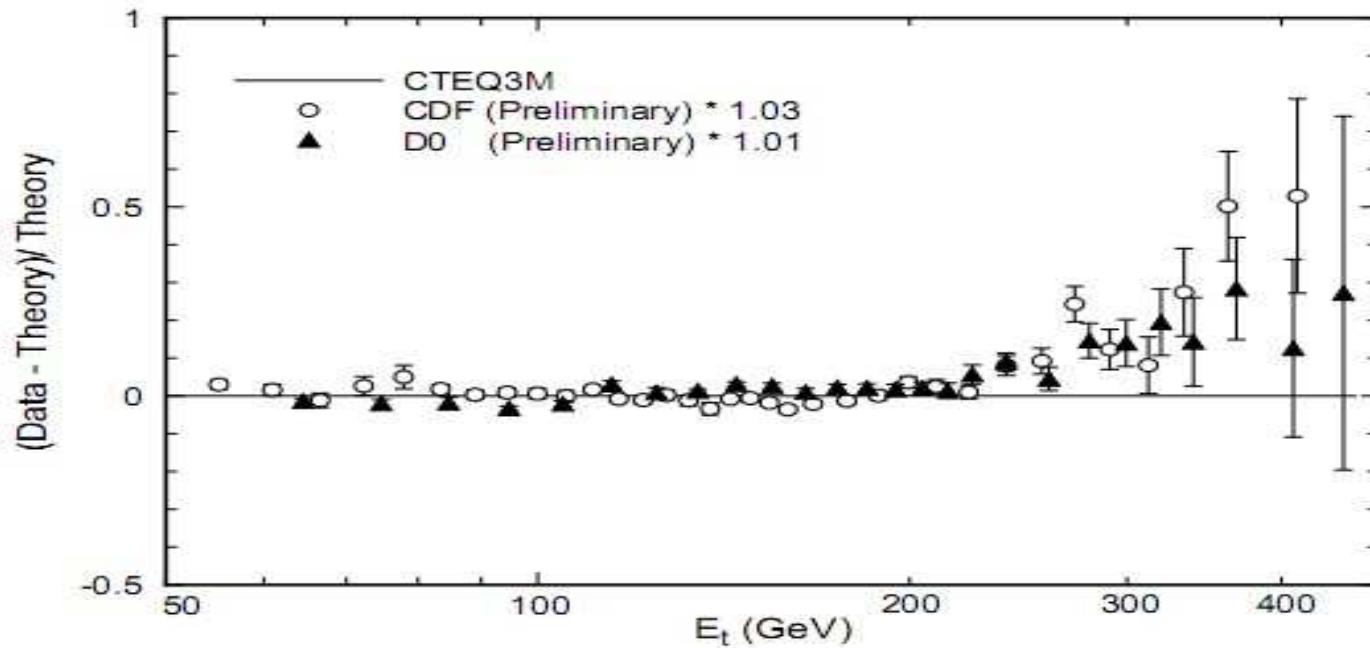
$$s_{\text{eff},l}^2 = .23159 \pm .00018 \text{ (i.e. } .08\% \text{ precision)}$$

● Tevatron jet cross section at large E_T



Discrepancies that were not

• Tevatron jet cross section at large E_T

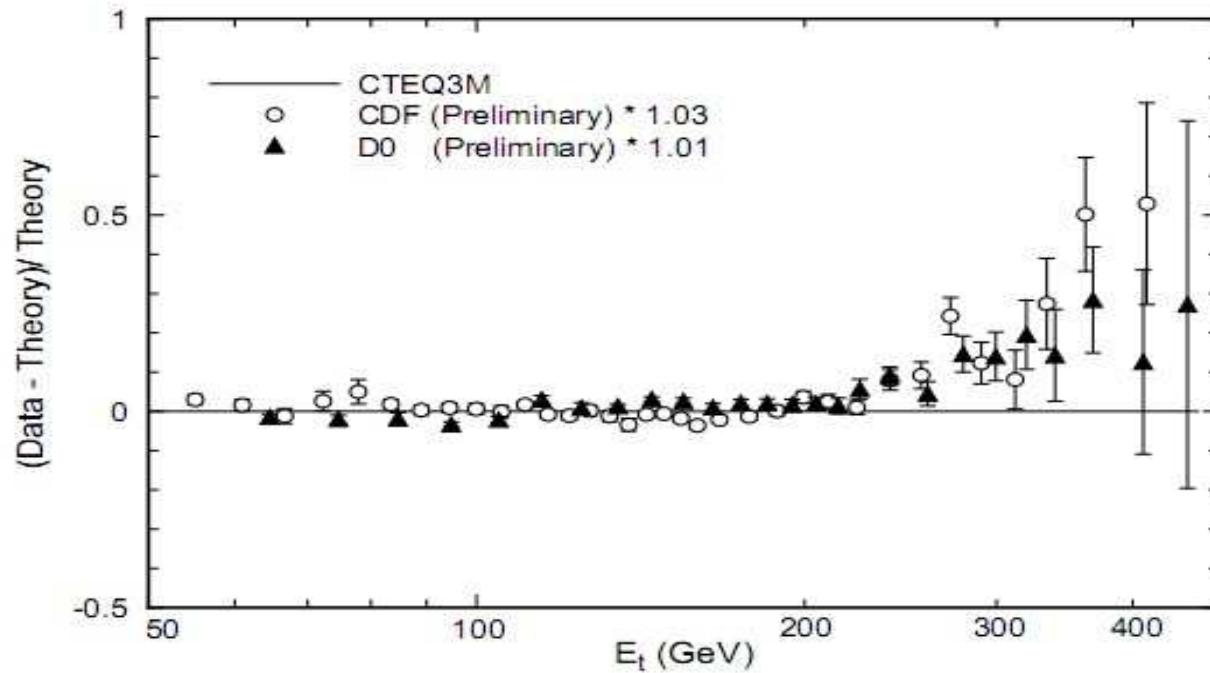


⇒ new physics?



Discrepancies that were not

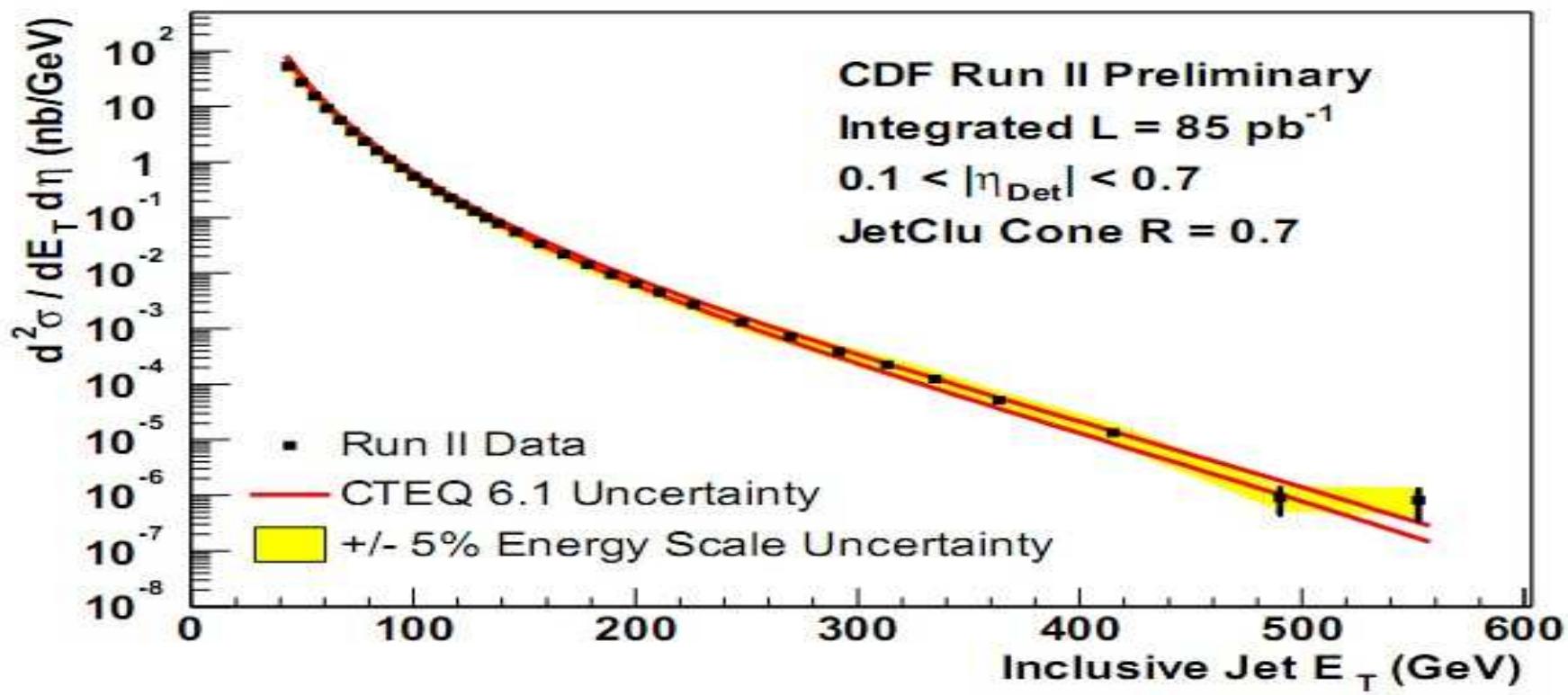
• Tevatron jet cross section at large E_T



$$\mathcal{L}_{\text{BSM}} \supseteq \frac{\tilde{g}^2}{M^2} \bar{\psi} \gamma^\mu \psi \bar{\psi} \gamma_\mu \psi \quad \Rightarrow \quad \frac{\text{data} - \text{theory}}{\text{theory}} \propto \tilde{g}^2 \frac{E_T^2}{M^2}$$

Discrepancies that were not

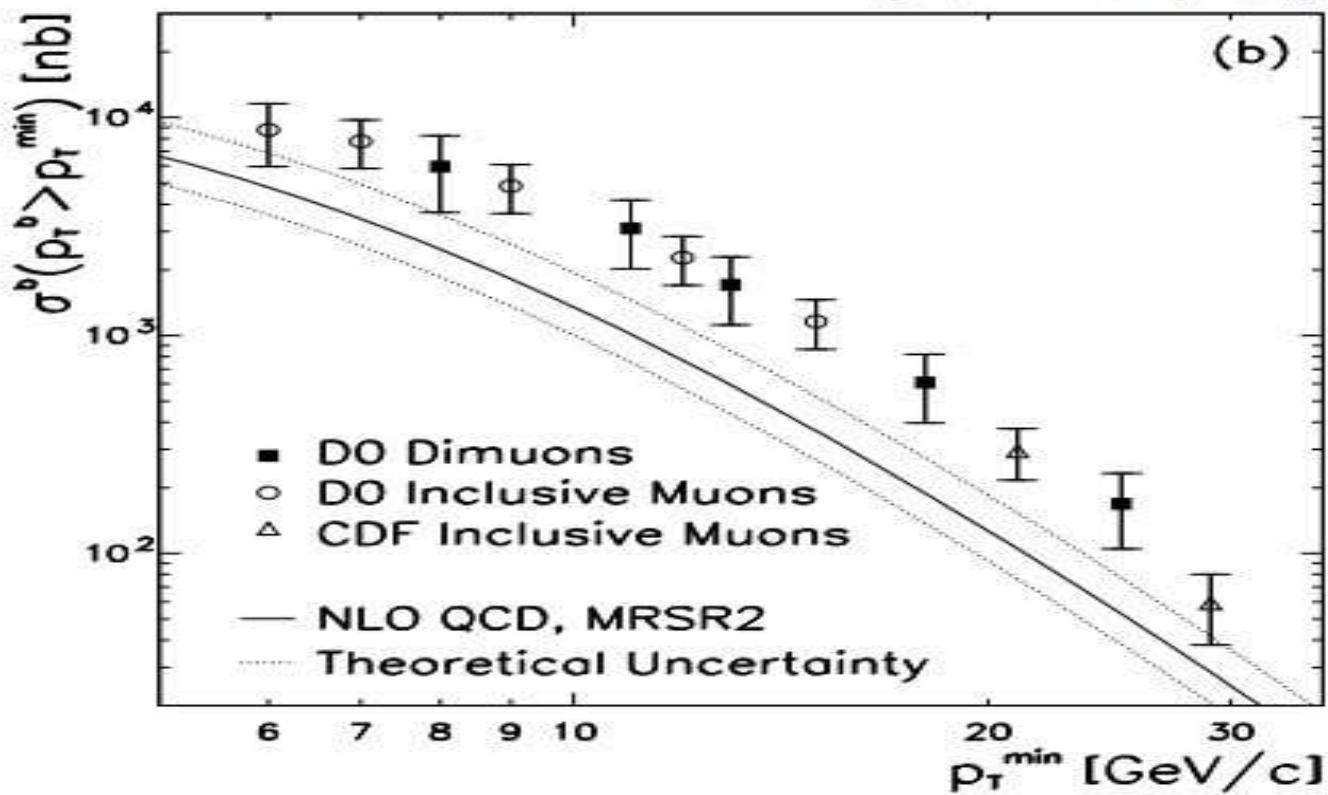
• Tevatron jet cross section at large E_T



⇒ just QCD... (uncertainty in gluon pdf at large x)

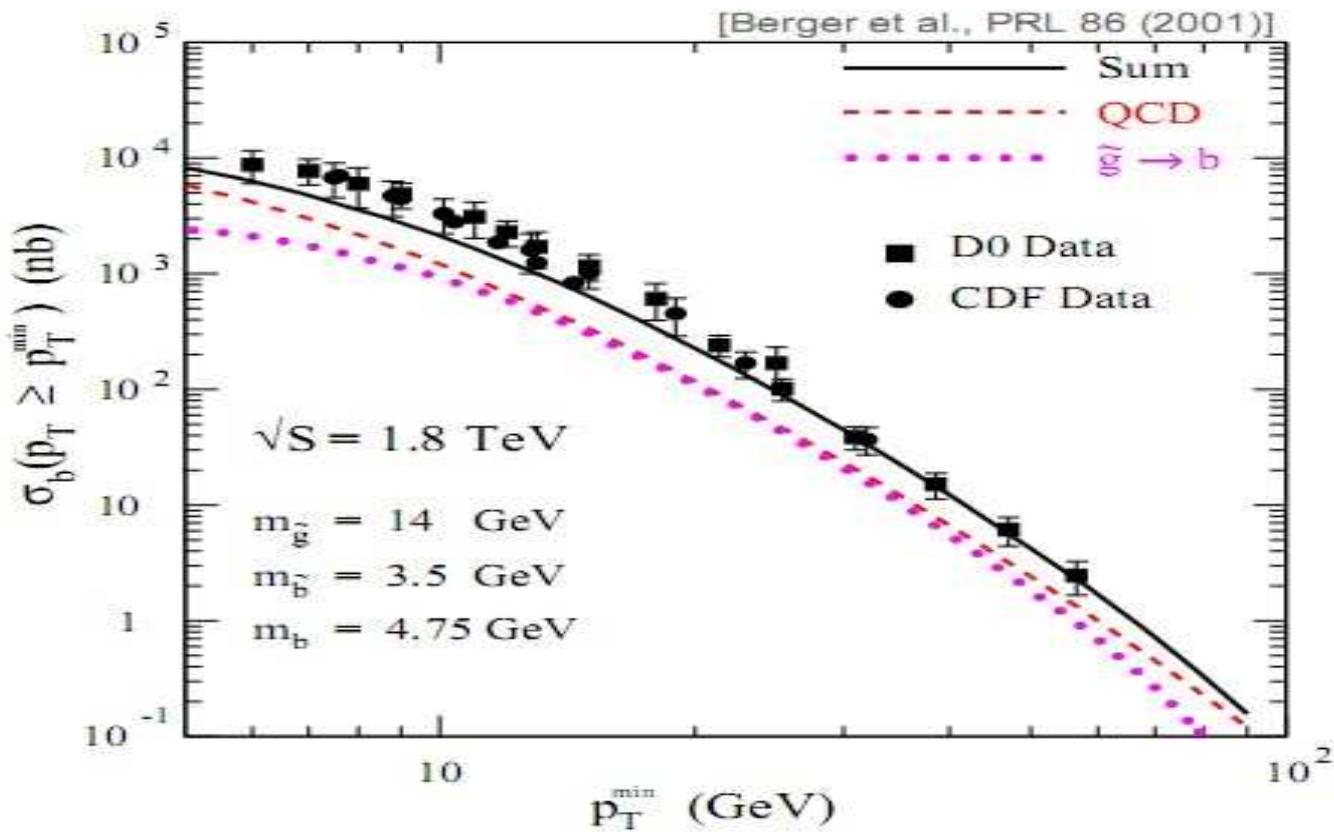
► Tevatron bottom quark cross section

[DO, PLB 487 (2000)]



⇒ new physics?

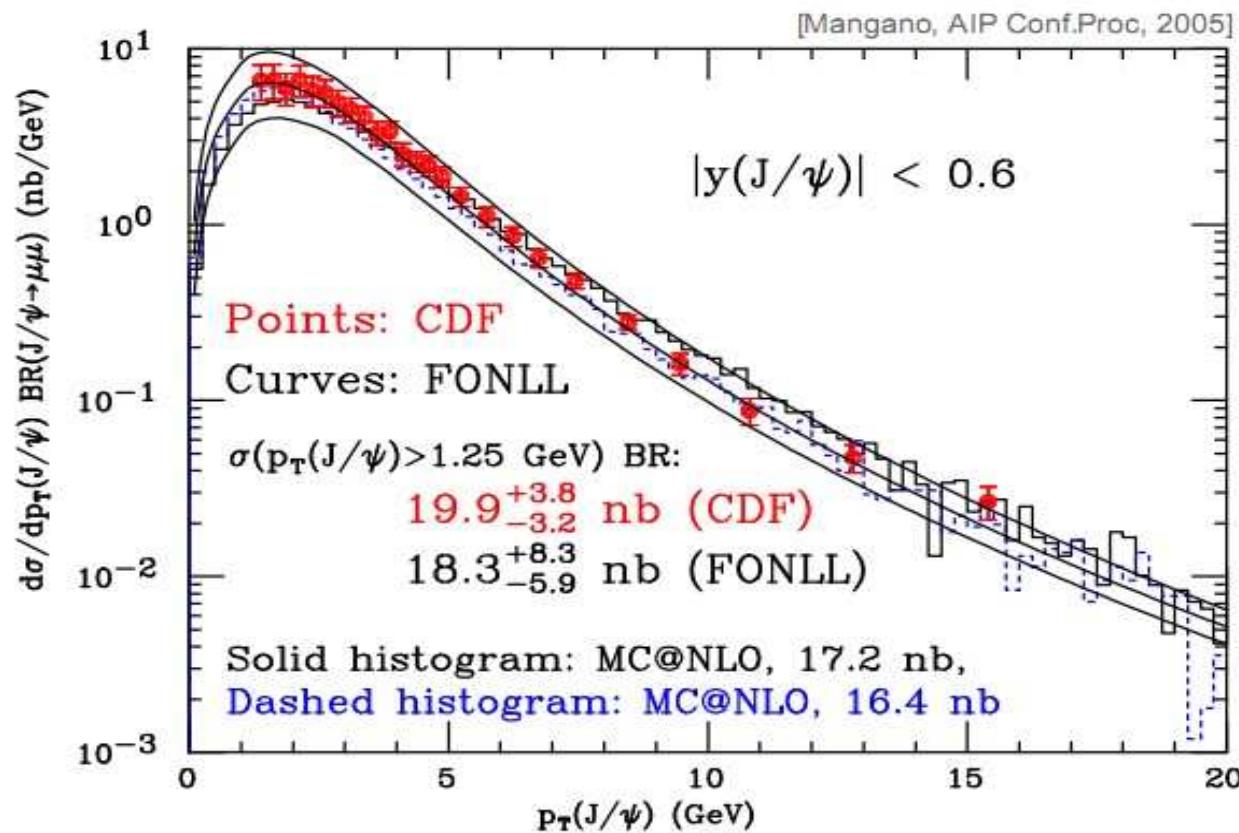
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⇒ light gluino/sbottom production?

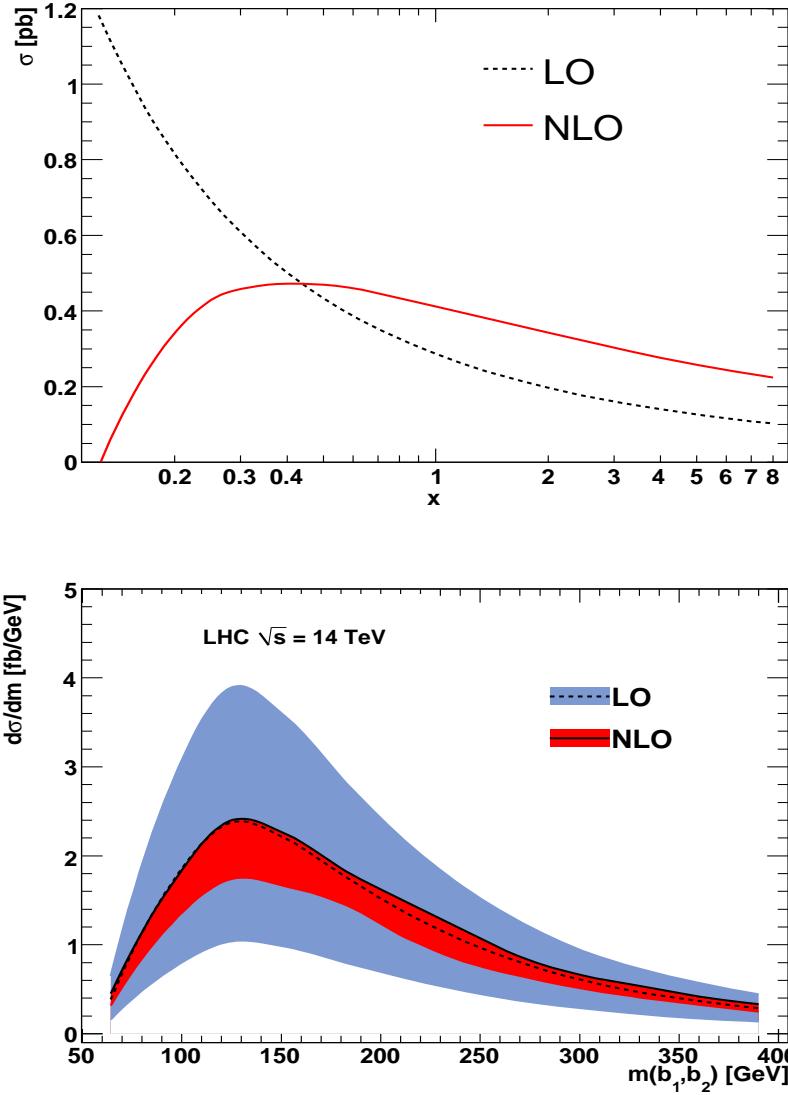
Discrepancies that were not

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⇒ just QCD... (α_s , low- x gluon pdf, $b \rightarrow B$ fragmentation & exp. analyses)

Need for NLO: example from Thomas and Annecy friends, 4b at NLO



The dependence of the LO and NLO prediction of $pp(q\bar{q}) \rightarrow b\bar{b}b\bar{b} + X$ at the LHC ($\sqrt{s} = 14$ TeV) on the renormalisation scale $\mu_R = x\mu_0$ with $\mu_0 = \sqrt{\sum_j p_T^2(b_j)}$. The factorisation scale is fixed to $\mu_F = 100$ GeV.

Invariant mass (m_{bb}) distribution of the two leading b -quarks . The LO/NLO bands are obtained by varying the renormalisation scale μ_R between $\mu_0/4$ and $2\mu_0$ with $\mu_0 = \sqrt{\sum_j p_T^2(b_j)}$. The full (dashed) line shows the NLO (LO) prediction for the value $\mu_R = \mu_0/2$.

what NLO brings

- LO predictions only qualitative, due to poor convergence of perturbative expansion $\alpha_s \sim 0.1 \rightarrow$ NLO can be $\mathcal{O}(30 - 100)\%$
- First prediction of normalization of cross-sections is at NLO less sensitivity to unphysical input scales (renormalization,factorization)
- more physics at NLO
 - parton merging to give structure in jets
 - more species of incoming partons enter at NLO
 - initial state radiation effects
- a prerequisite for more sophisticated calculations which match NLO with parton showers

Usual procedure and normalisation with data.....

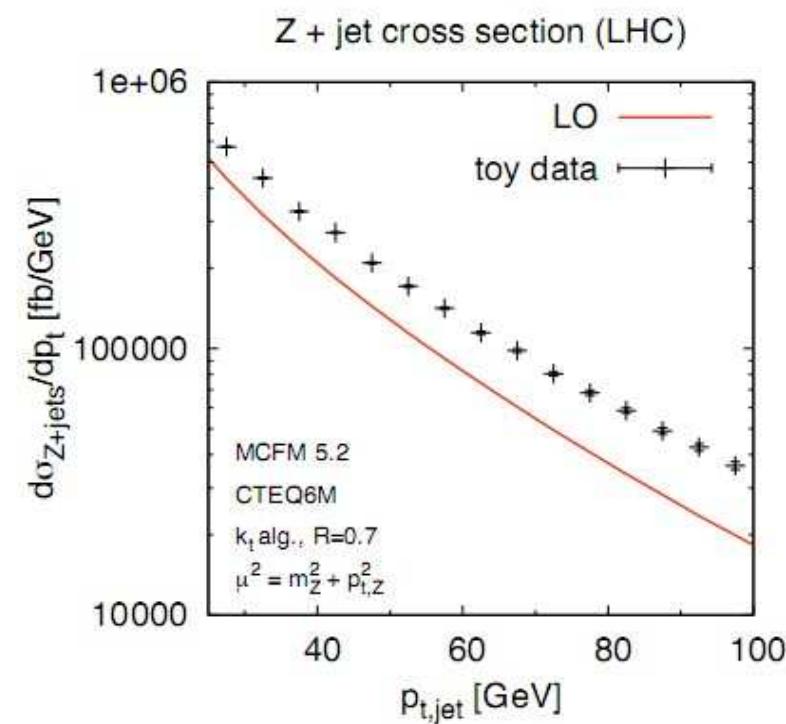
- Stage 1: get control sample in low pt region (little SUSY contamination)
- Stage 2: once LO is validated using data, trust it in signal region

Example for Salam, Zanderighi et al, high Use W+1 jet known at NLO to see how good this works

Is NLO really needed?

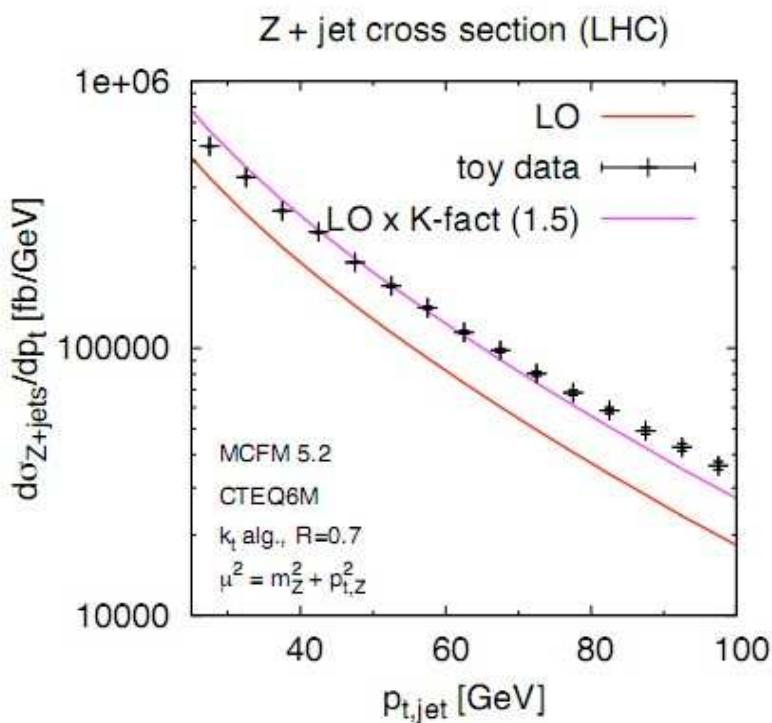
Stage I:

get control sample (K-factor)



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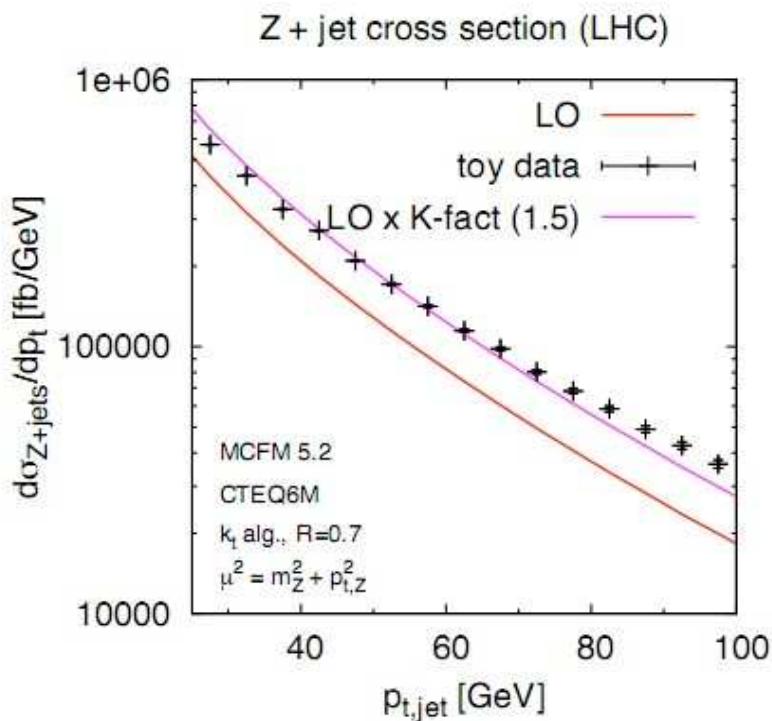
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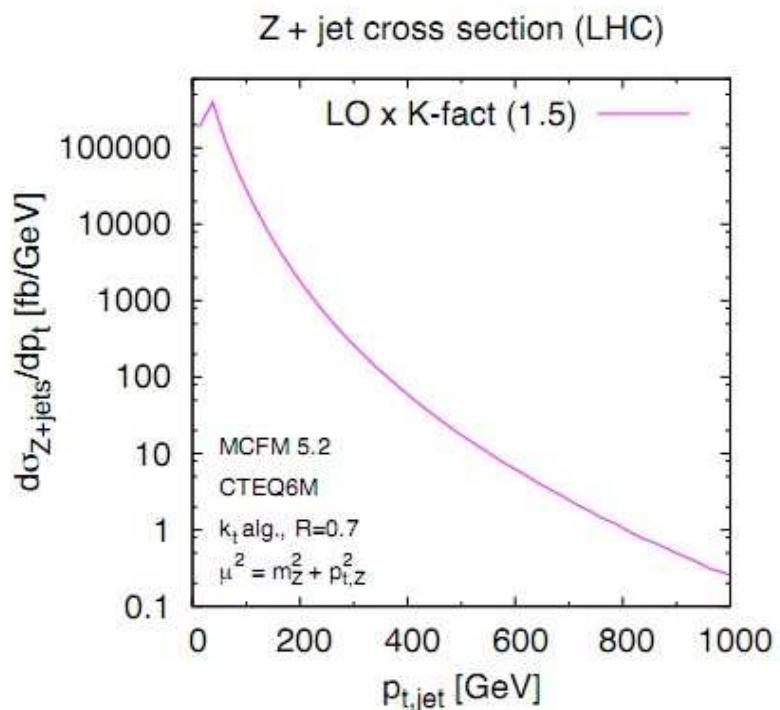
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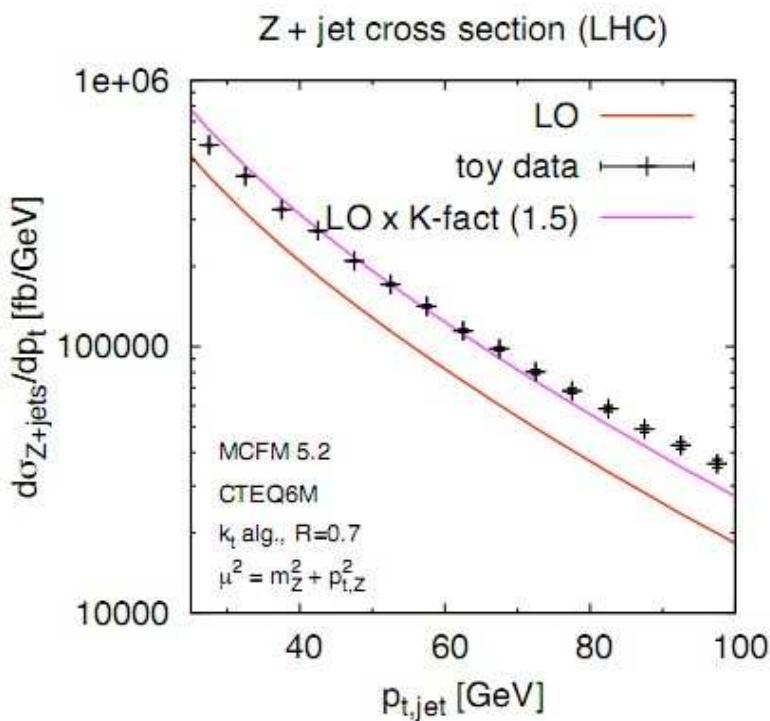
Stage 2:

extrapolate to the signal region

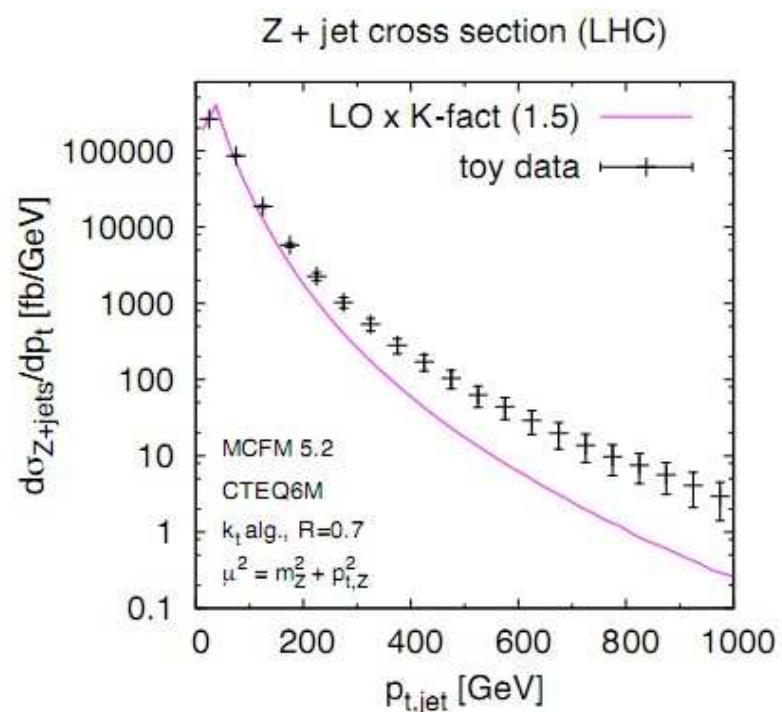


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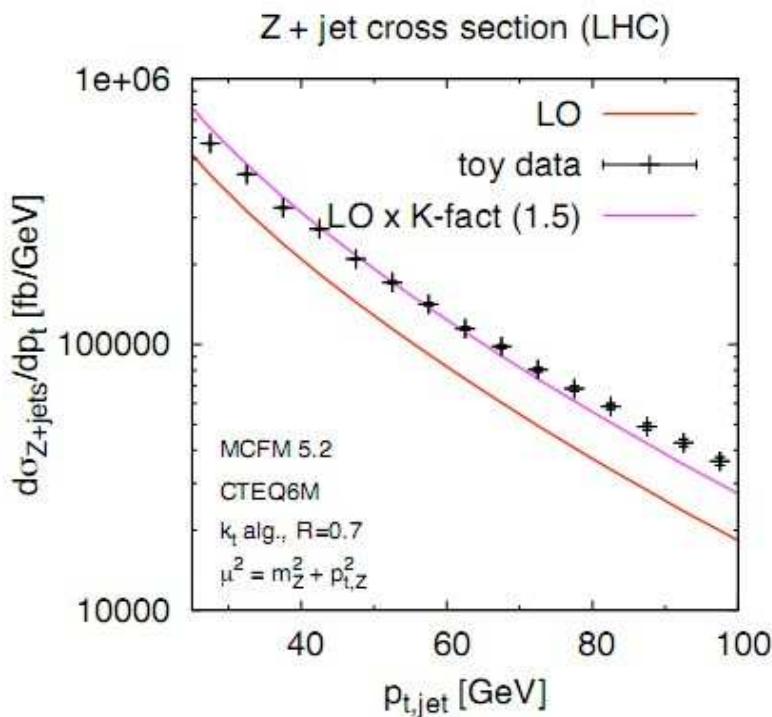
Stage 2:
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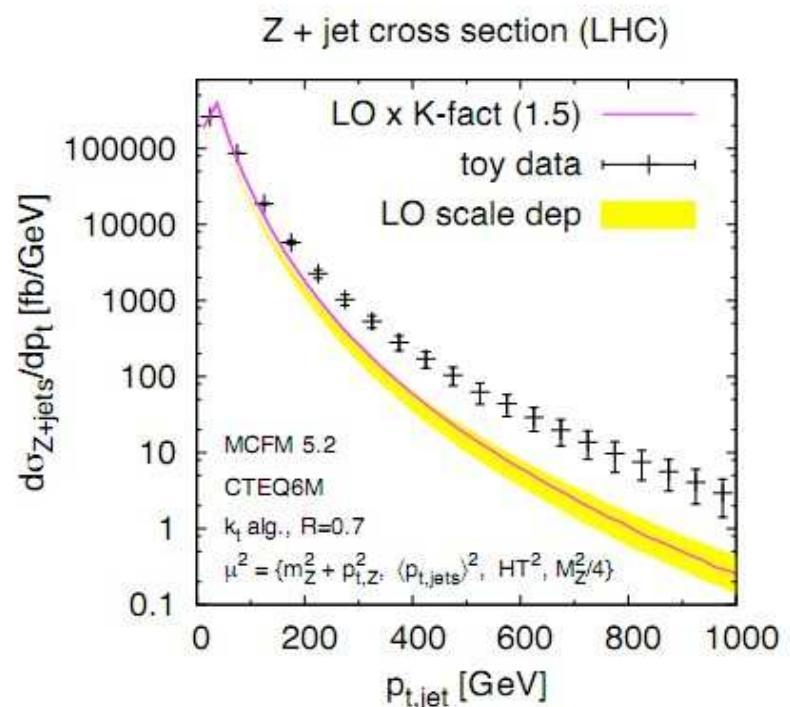
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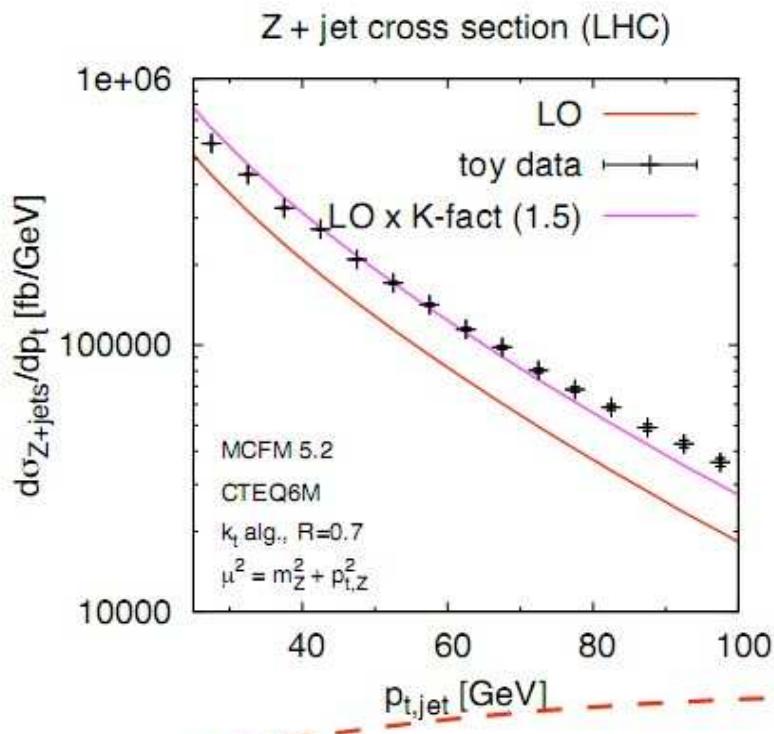
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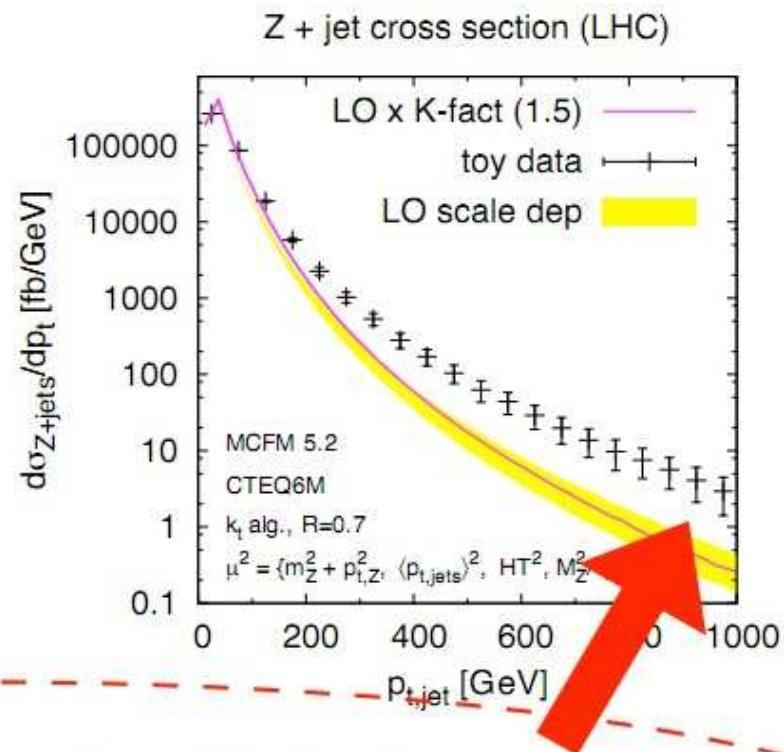


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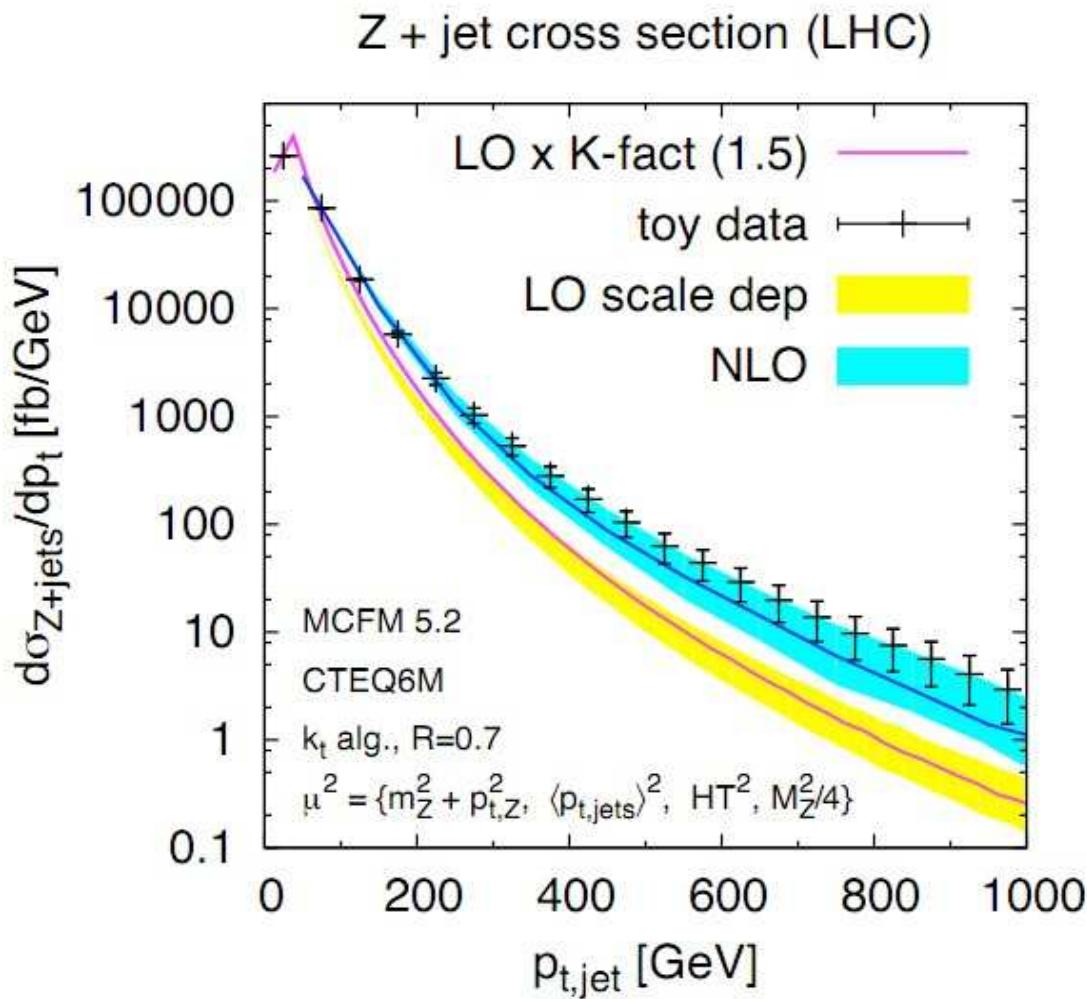


Stage 2:
extrapolate to the signal region



Factor 10 excess. 6σ deviation.
Discovery ??

No, just plain NLO QCD...



NB: source of large K-factor understood [soft Z radiated from hard jets]

See Butterworth, Davison, Salam, Rubin '08

The dreamer's wishlist for NLO processes

Single boson	Diboson	Triboson	Heavy flavor
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		$b\bar{b} t\bar{t}$
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Les Houches 2009 Experimenter's Wishlist

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \rightarrow VV\text{jet}$	$WW\text{jet}$ completed by Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi.
2. $pp \rightarrow \text{Higgs+2jets}$	$ZZ\text{jet}$ completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier
3. $pp \rightarrow VVV$	ZZZ completed by Lazopoulos/Melnikov/Petriello and WWZ by Hankele/Zeppenfeld (see also Binoth/Ossola/Papadopoulos/Pittau)
4. $pp \rightarrow t\bar{t} b\bar{b}$	relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek calculated by the Blackhat/Sherpa and Rocket collaborations
5. $pp \rightarrow V+3\text{jets}$	
Calculations remaining from 2005,	completed since
6. $pp \rightarrow t\bar{t}+2\text{jets}$	relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek
7. $pp \rightarrow VV b\bar{b}$,	relevant for $\text{VBF} \rightarrow H \rightarrow VV$, $t\bar{t}H$
8. $pp \rightarrow VV+2\text{jets}$	relevant for $\text{VBF} \rightarrow H \rightarrow VV$ VBF contributions calculated by (Bozzi/(/)Jäger/Oleari/Zeppenfeld
NLO calculations added to list in 2007	
9. $pp \rightarrow bbbb$	$q\bar{q}$ channel calculated by Golem collaboration
NLO calculations added to list in 2009	
10. $pp \rightarrow V+4\text{ jets}$	top pair production, various new physics signatures
11. $pp \rightarrow Wb\bar{b}j$	top, new physics signatures
12. $pp \rightarrow tt\bar{t}\bar{t}$	various new physics signatures
Calculations beyond NLO added in 2007	
13. $gg \rightarrow W^*W^* \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \rightarrow t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and $Z/\gamma+\text{jet}$	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Slow Progress for NLO from 1 to 4

$pp \rightarrow W + 0jet$	1978	Altarelli, Ellis, Martinelli
$pp \rightarrow W + 1jet$	1989	Arnold, Ellis, Reno
$pp \rightarrow W + 2jets$	2002	Arnold, Ellis
$pp \rightarrow W + 3jets$	2009	BlackHat+Sherpa; Ellis, Melnikov, Zanderighi

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$pp \rightarrow t\bar{t}b\bar{b}$ Bredenstein, Denner, Dittmaier, Pozzorini

Bevilacqua, Czakon,

$pp \rightarrow t\bar{t} + 2jets$ Bevilacqua, Czakon, Papadopoulos, Pittau, Worek

$pp \rightarrow 4b$ Golem (Binoth, Guillet, Greiner, Guffanti, Reiter, Reuter..)

Multileg One-loop: electroweak digression

- End of 80's:

- Applications to LEP 1: 1-loop to $Z \rightarrow f\bar{f}$

- Labour of many years and many groups

- Essentially 2-point and 3-point vertex functions (some 4-points, 2-loop for self-energies)

- few ten's of diagrams

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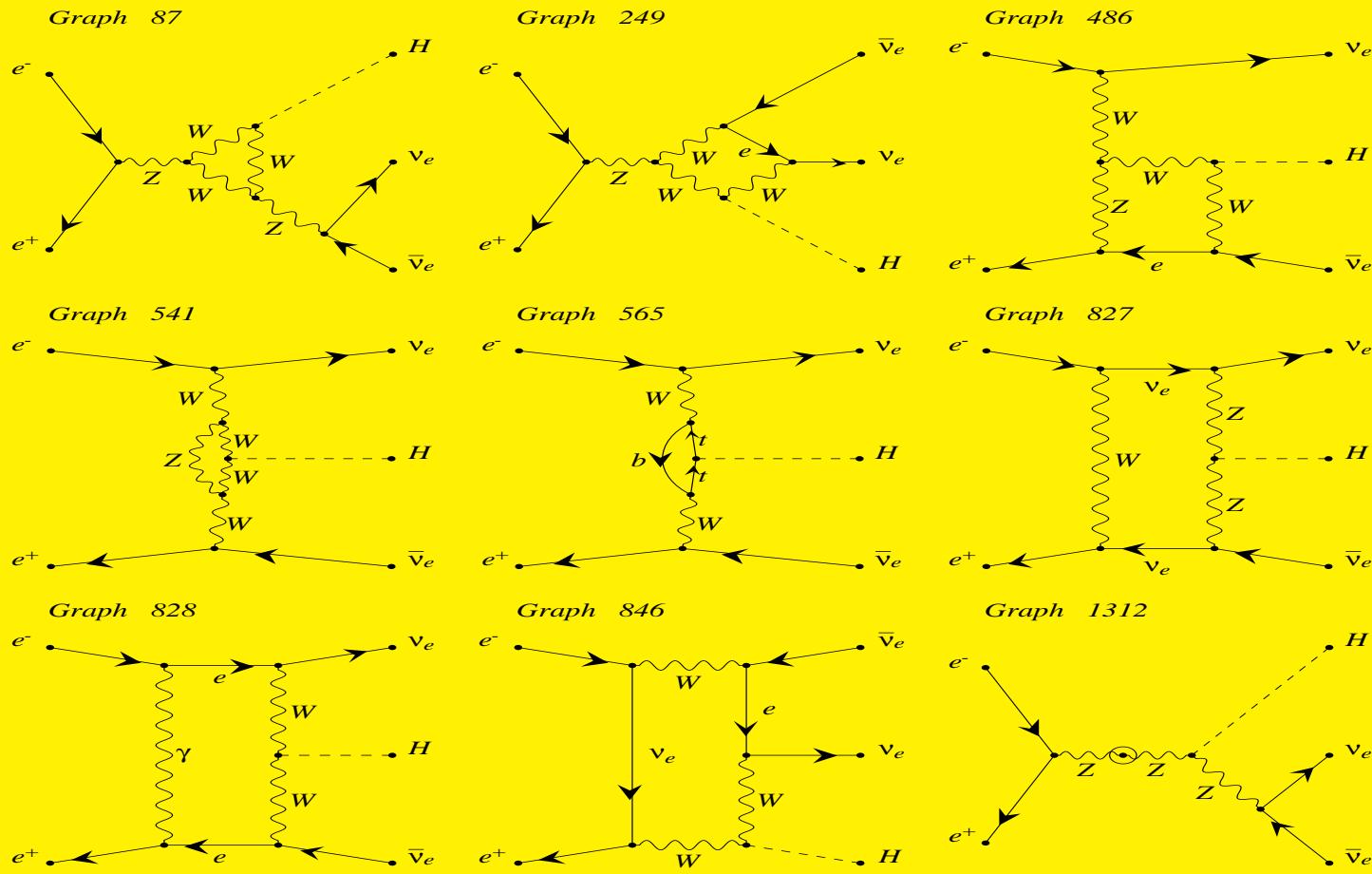
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- up to 2009

- $e^+e^- \rightarrow \nu\nu HH$ Boudjema et al.,

- $e^+e^- \rightarrow 4f$ Denner et al.,

Part A: Virtual Corrections, Loops



produced by GRACEFIG

à la Feynman

The Feynmanians, Textbook

Feynman Recipe, knitting with vertices and propagators

Draw all possible types of diagrams	topology
Figure out which particles can run on each type of diagram	combinatorics
Translate diagrams into expressions applying the Feynman rules contract indices, take traces, (multiply add blocks)	data-base look up algebra
Collect and write up the results as a computer code integrate over phase space	programming coding/computing
run the program to get numerical values	waiting!

Matrix Elements Generation and Automation: Feynman diagrams

Automation of LO calculations of (partonic) processes $2 \rightarrow N$ are now automatised including integration, for (say) $N < 8$ based on different methods

- ALPGEN (not quite Feynman)
- CompHEP/CalcHEP
- Grace
- HELAS/PHEGAS
- MADGRAPH/MADEVENT
- O'Mega/WHIZARD
- SHERPA/Amegic

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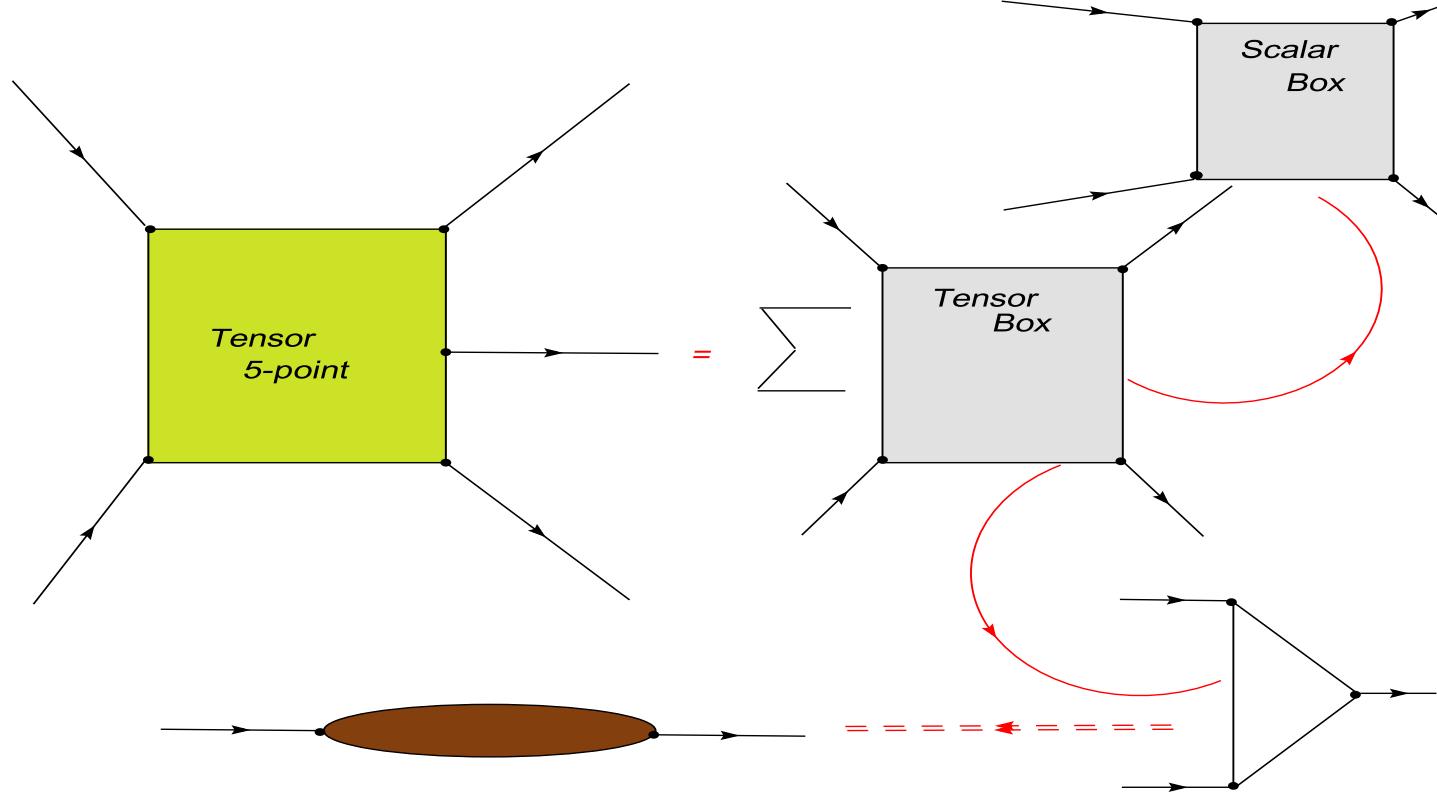
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the more particles one deals with a (Feynman) diagrammatic calculations is costly as the number of diagrams grows $N!$

#gluons	2	3	4	5	6	7	8
#diagrams	4	25	220	2485	34300	0.5M	80M

Loop Integrals and Reduction

$$\underbrace{T_{\mu\nu\cdots\rho}^{(N)}}_M = \int \frac{d^n l}{(2\pi)^n} \frac{l_\mu l_\nu \cdots l_\rho}{D_0 D_1 \cdots D_{N-1}}, \quad M \leq N$$



- Tensor integrals and scalar integrals with $N > 4$ reduced to scalars $N = 2, 3, 4$
- ex. rank 4 box need to solve a system of 15×15 equations. System involves, Gram determinants that may lead to severe instabilities

The Feynmanians, Textbook

$$\begin{aligned}
 C_\mu &= \frac{(2\pi\mu)^{(4-n)}}{i\pi^2} \int d^n l \frac{l_\mu}{(l^2 - m_1^2 + i\varepsilon)(l + r_1)^2 - m_2^2 + i\varepsilon(l + r_2)^2 - m_3^2 + i\varepsilon} \\
 &= r_{1\mu} C_1 + r_{2\mu} C_2
 \end{aligned}$$

use $2q.r = (q+r)^2 - (q^2 + r^2)$
 with $f_i = r_i^2 - m_{i+1}^2 + m_1^2$

$$\begin{pmatrix} 2r_1^2 & 2r_1 \cdot r_2 \\ 2r_1 \cdot r_2 & 2r_2^2 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} B_0(r_2^2, m_1^2, m_2^2) - B_0((r_1 - r_2)^2, m_2^2, m_3^2) - f_1 C_0 \\ B_0(r_1^2, m_1^2, m_2^2) - B_0((r_1 - r_2)^2, m_2^2, m_3^2) - f_2 C_0 \end{pmatrix}$$

Inversion, solution in terms of scalar functions (B_0, C_0) involve the Gram determinant

$$\det G_2 = \det(2r_i r_j), i, j = 1, 2$$

If Gram Det small, numerical instability. Worse for higher rank tensors.

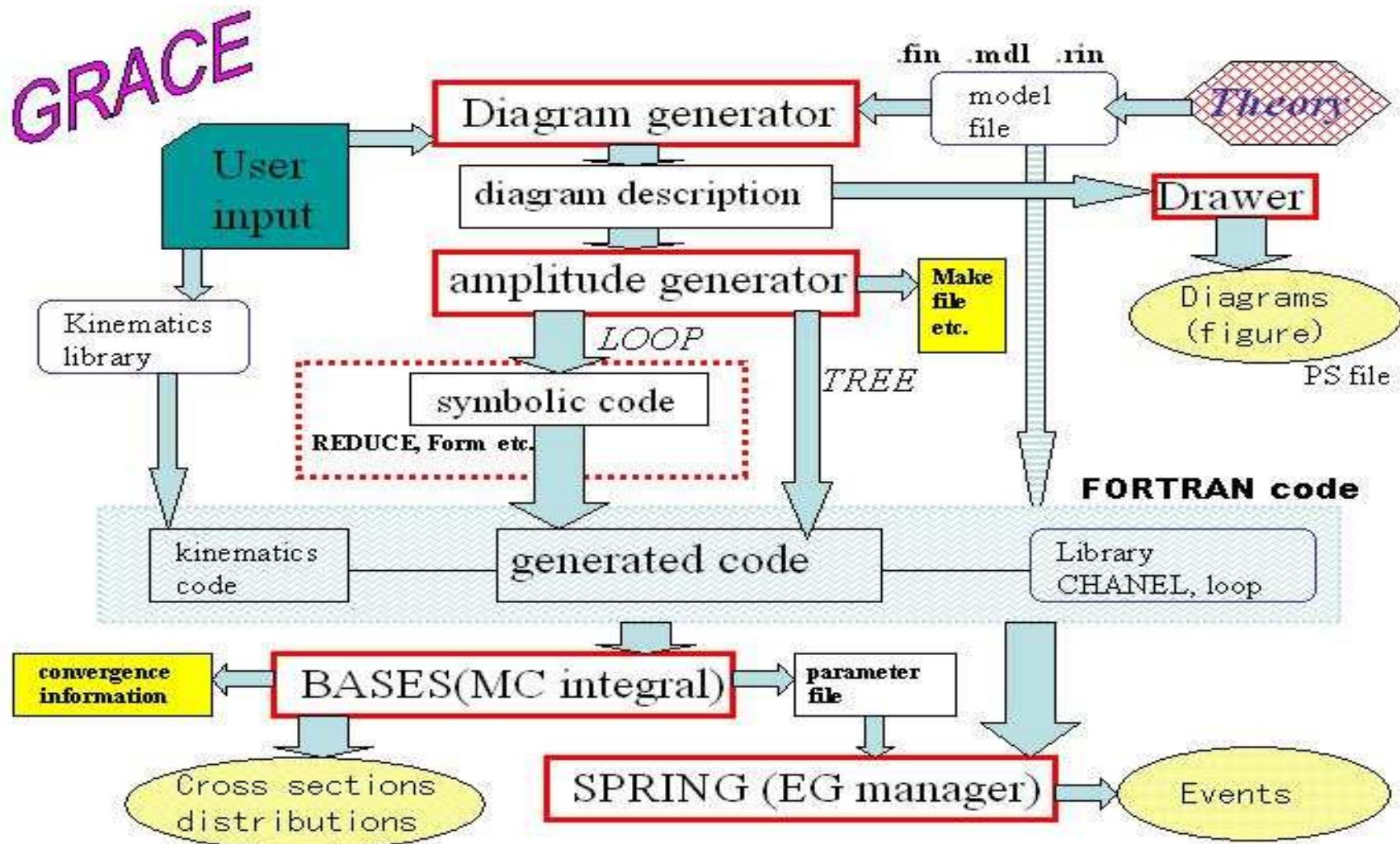
for Gram=0, use segmentation, FB 2005

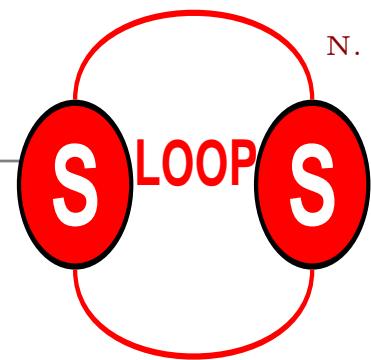
CPU of the various N-point

Process	6-point	5-point	4-point	3-point	Others
$e^+e^- \rightarrow e^+e^- H$	-	33%	11%	47%	9%
nbr of Feyn. dia.		20	44	348	98
$e^+e^- \rightarrow \nu\bar{\nu} HH$	67%	13%	10%	8%	2%
nbr of Feyn. dia.	74	218	734	1804	586

Perhaps that Passarino Veltman no longer adequate for present day purposes, although many optimisations have been implemented. Many developments recently,...

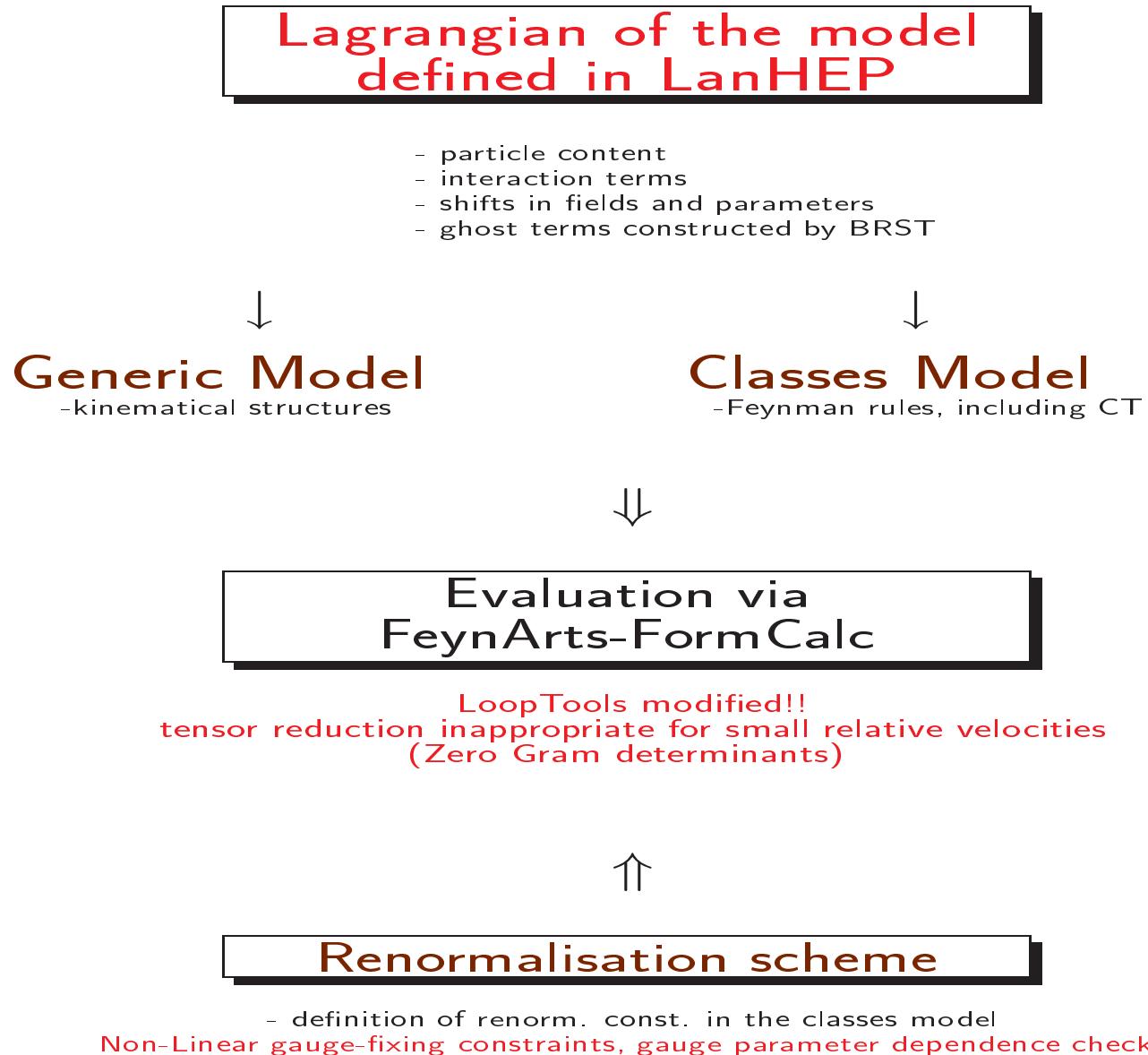
Conducive to automation





- Need for an automatic tool for susy calculations, for Colliders and Dark Matter, On-Shell scheme
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes
- Used for SM one-loop multi-leg: new powerful loop libraries (with Ninh Le Duc)

Strategy: Exploiting and interfacing modules from different codes



```

vector
A/A: (photon, gauge),
Z/Z: ('Z boson', mass MZ = 91.1875, gauge),
'W+/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar H/H:(Higgs, mass MH = 115).

transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
'W+'->'W+'*(1+dZW/2), 'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
'W+.f'->'W+.f'*(1+dZWf/2), 'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f', (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).

lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
where
lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a -
i*g/2*taupm^a^b^c{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.

```

Gauge fixing and BRS transformation

```

let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).

```

```

vector
A/A: (photon, gauge),
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```

Output of Feynman Rules
with Counterterms !!

```

M$CouplingMatrices = {

(*----- H H -----*)
C[ S[3], S[3] ] == - I *
{
{ 0 , dZH },
{ 0 , MH^2 dZH + dMHsq }
},
(*----- W+.f W-.f -----*)
C[ S[2], -S[2] ] == - I *
{
{ 0 , dZWf },
{ 0 , 0 }
},
(*----- A Z -----*)
C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
{ 0 , 0 },
{ 0 , dZZA },
{ 0 , 0 }
},

(*----- H H H -----*)
C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
{ 2 MH^2 , 3 MH^2 dZH - 2 MH^2 / SW dSW - MH^2 / MW^2 dMWSq
},
(*----- H W+.f W-.f -----*)
C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
{ 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf - 2 MH^2 / SW dSW - MH^2
},
(*----- W-.C A.c W+ -----*)
C[ -U[3], U[1], V[3] ] == - I EE *
{
{ 1 },
{ - nla }
},

```

```

vector
A/A: (photon, gauge),
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```

```

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```

```

transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
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```

```

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```

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```

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

```

```

lterm -'Z.C'*brst(G_Z).

```

```

RenConst[ dMHSq ] := ReTilde[SelfEnergy[ppt["H"] -> ppt["H"], MH]]

```

```

RenConst[ dZH ] := -ReTilde[DSelfEnergy[ppt["H"] -> ppt["H"], MH]]

```

```

RenConst[ dZZf ] := -ReTilde[DSelfEnergy[ppt["Z.f"] -> ppt["Z.f"], MZ]]
RenConst[ dZWf ] := -ReTilde[DSelfEnergy[ppt["W+.f"] ->

```

```

M$CouplingMatrices = {
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{
{ 0 , dZWf },
{ 0 , 0 }
},
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C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
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{ 2 MH^2 , 3 MH^2 dZH - 2 MH^2 / SW dSW - MH^2 / MW^2 dMWSq
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C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
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},
(*----- W-.C A.c W+ -----*)
C[ -U[3], U[1], V[3] ] == - I EE *
{
{ 1 },
{ - nla }
},

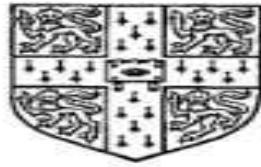
```

The Unitarians (Reformed, Modern)

THE ANALYTIC S-MATRIX

BY

R. J. EDEN P. V. LANDSHOFF D. I. OLIVE
J. C. POLKINGHORNE



CAMBRIDGE
AT THE UNIVERSITY PRESS
1966

The Unitarians

Properties of the S-Matrix

The Unitarians

Properties of the S-Matrix

- **Analyticity** Scattering amplitudes are determined by their poles and branch-cuts
- **Unitarity** The residues at the poles and the branch-cut are products of subamplitudes that are simpler (less legs and less loops)

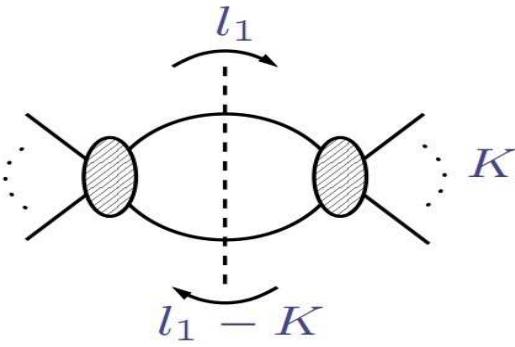
The Unitarians

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Unitarity and cutting rules (on-shell conditions)

- Ordinary Unitarity (optical theorem, Cutkosky's rule): put **two particles on-shell**, turn one-loop into products of trees $S = 1 + iT, SS^\dagger = 1 \implies 2ImT = -i(T - T^\dagger) = T^\dagger T$



$$\frac{1}{(l_i^2 - m_i^2 + i0)} \rightarrow \delta(l_i^2 - m_i^2)$$

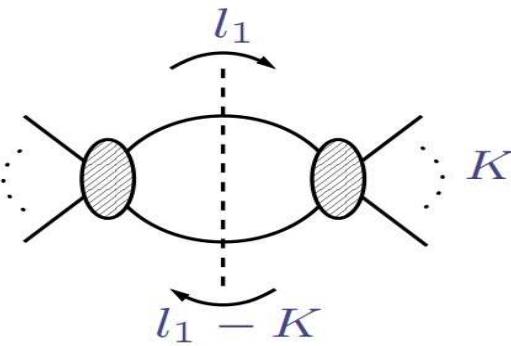
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$$\frac{1}{(l_i^2 - m_i^2 + i0)} \rightarrow \delta(l_i^2 - m_i^2)$$

- **Generalised Unitarity:** put (1), 3, 4 particles on-shell, multiple products of trees

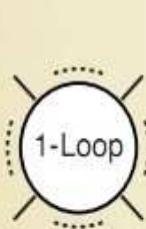
The Unitarians

Tree-amplitudes vs *Feynman diagrams*

amplitudes are **gauge invariant** on-shell objects

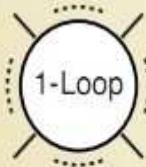
that corresponds to (large) sums of **off-shell Feynman diagrams**

Usual Passarino Veltman


$$= \sum_{10^2-10^3} \int d^D \ell \frac{\ell^\mu \ell^\nu \ell^\rho \dots}{D_1 D_2 \dots D_n} = c_4 \text{ (square loop)} + c_3 \text{ (triangle loop)} + c_2 \text{ (circle loop)} + c_1 \text{ (empty circle)}$$

in fact **each tensor integral** (there can be a few of these for each Feynman graph) is reduced to a set of **scalar integrals**, (from gauge theories to scalar field theory!)

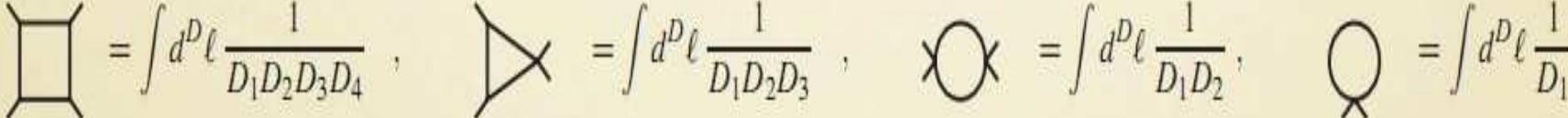
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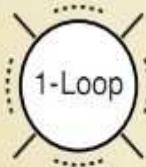
in fact **each tensor integral** (there can be a few of these for each Feynman graph) is reduced to a set of **scalar integrals**, (from gauge theories to scalar field theory!)

The Set or Basis is known (looked up): **Master integrals**



$$\text{square loop} = \int d^D \ell \frac{1}{D_1 D_2 D_3 D_4}, \quad \text{triangle loop} = \int d^D \ell \frac{1}{D_1 D_2 D_3}, \quad \text{circle loop} = \int d^D \ell \frac{1}{D_1 D_2}, \quad \text{Q loop} = \int d^D \ell \frac{1}{D_1}$$

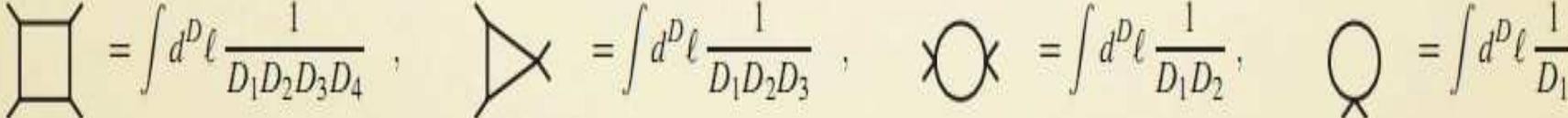
Usual Passarino Veltman



$$= \sum_{10^2-10^3} \int d^D \ell \frac{\ell^\mu \ell^\nu \ell^\rho \dots}{D_1 D_2 \dots D_n} = c_4 \text{ (square loop)} + c_3 \text{ (triangle loop)} + c_2 \text{ (circle loop)} + c_1 \text{ (empty circle)}$$

in fact **each tensor integral** (there can be a few of these for each Feynman graph) is reduced to a set of **scalar integrals**, (from gauge theories to scalar field theory!)

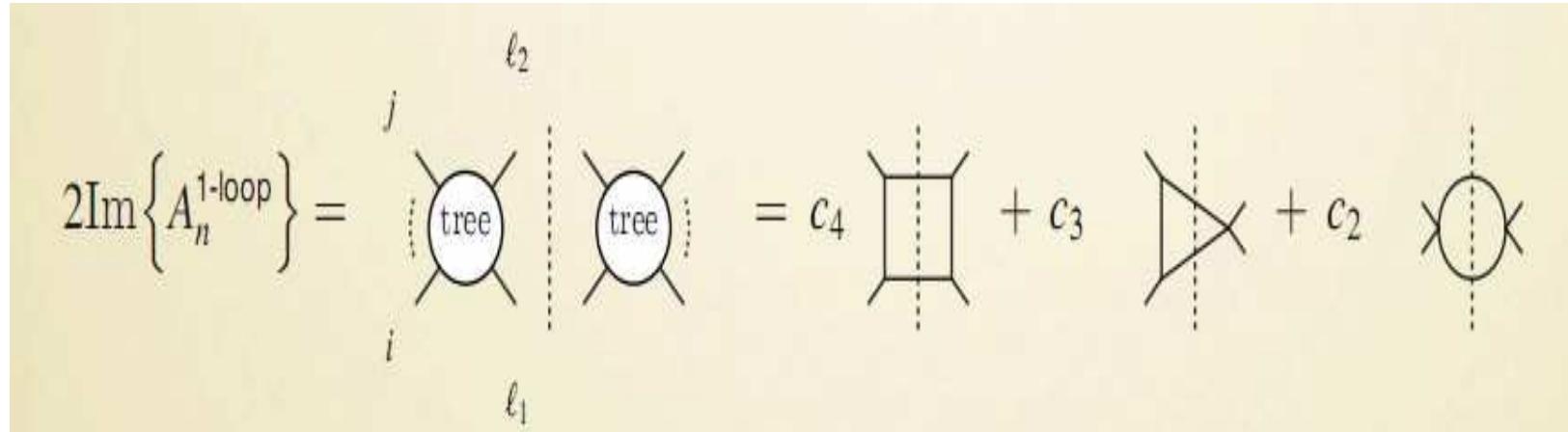
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what is unknown are the c_i : rational functions of the external momenta (and internal masses)

- Ordinary Cutkosky's cuts



▷ Match the cuts of any amplitude into the cuts of the master integrals, contribution from master integrals with different number of legs

Generalised unitarity

$$\begin{array}{c} \text{Diagram A} \\ \hline \end{array} = c_4 \begin{array}{c} \text{Diagram B} \\ \hline \end{array} + c_3 \begin{array}{c} \text{Diagram C} \\ \hline \end{array} + c_2 \begin{array}{c} \text{Diagram D} \\ \hline \end{array} + c_1 \begin{array}{c} \text{Diagram E} \\ \hline \end{array}$$

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$$\begin{array}{c} \text{Diagram A} \\ \hline \end{array} = c_4 \begin{array}{c} \text{Diagram B} \\ \hline \end{array}$$

Ideally start with the quadruple cut as it isolates c_4

quadruple cut implies products of 4 trees (evaluated at complex momenta)

Make your way up

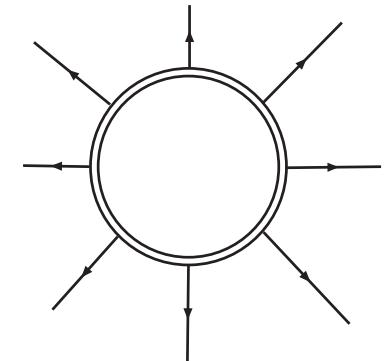
the more one cuts the simpler it gets but the more one loses also c_1, \dots

Reduction at the integrand level

Take the m -point one-loop sub-amplitude calculated in DR

$$\mathcal{A}_m = \int [d^n \bar{l}] \bar{A}(\bar{l})$$

$$\bar{A}(\bar{l}) = \frac{\bar{N}(\bar{l})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}, \quad \bar{D}_i = (\bar{l} + p_i)^2 - m_i^2, \quad p_0 \neq 0.$$



bar to denote objects living in $n = 4 + \epsilon$ dimensions.

$\bar{l}^2 = l^2 + \tilde{l}^2$, where \tilde{l}^2 is ϵ -dimensional and $(\tilde{l} \cdot l) = 0$.

The numerator function $\bar{N}(\bar{l})$ can be also split into a **4-dimensional** plus a ϵ -dimensional part
 $\bar{N}(\bar{l}) = N(l) + \tilde{N}(\tilde{l}^2, l, \epsilon)$.

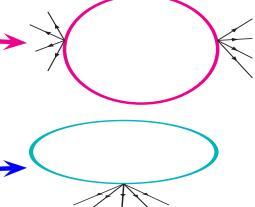
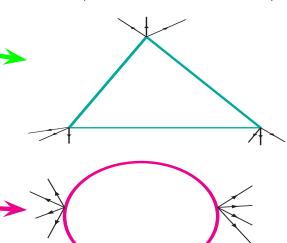
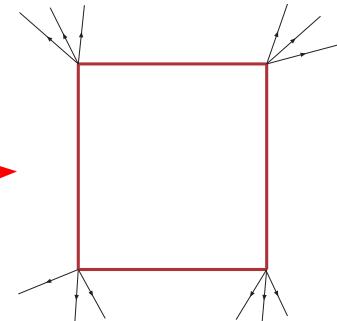
(mismatch between N and \bar{N} , D_i and \bar{D}_i source of rational terms)

OPP Method

$$\begin{aligned}
 N(l) = & \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(l; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
 + & \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(l; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
 + & \sum_{i_0 < i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(l; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
 + & \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(l; i_0)] \prod_{i \neq i_0}^{m-1} D_i \\
 + & \tilde{P}(l) \prod_i^{m-1} D_i .
 \end{aligned}$$

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 + & \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(l; i_0)] \prod_{i \neq i_0}^{m-1} D_i \\
 + & \tilde{P}(l) \prod_i^{m-1} D_i
 \end{aligned}$$



no pole

OPP

$$\begin{aligned}
 \int [d^n l] \bar{A}(\bar{l}) &= \int [d^n l] \frac{1}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \times \left(\right. \\
 &\quad \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(l; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
 &+ \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(l; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
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 &+ \sum_{i_0}^{m-1} [\textcolor{red}{a}(i_0) + \tilde{a}(l; i_0)] \prod_{i \neq i_0}^{m-1} D_i \\
 &+ \left. \tilde{P}(l) \prod_i^{m-1} D_i \right).
 \end{aligned}$$

Ossala Papadopoulos Pittau, OPP Method

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&\quad \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \left. \sum_{i_0}^{m-1} a(i_0) \prod_{i \neq i_0}^{m-1} D_i \right).
\end{aligned}$$

- The unknown coefficients d, c, b, a are computed from generating $N(l)$ a sufficient number of times (for different values of the loop momentum) and then **inverting the system of equations**.
- Need to be clever!, for $m = 6$ need a system of 56 equations! Not advisable to invert a 56×56 matrix
- Unitarity way: single out particular values of l so that $4, 3, 2, 1D_i$ vanishes simultaneously.
- l such that $D_i(l) = 0$ for $i = 0, \dots, m_i$
- Start with $4D_i$, ($m = 4$) (quadruple cut), etc...
- Algorithm can be carried at amplitude level. $N(l)$ generated from trees.

The Wonders and Fascinating Facets of $N = 4$ SYM

Dual to gravity/string theory on $AdS_5 \times S^5$. Very similar in IR to QCD (Magnea,...)

Planar (large N_c) theory is integrable, Yangian (Beisert, Drummond)

Strong-coupling limit and Wilson loops (Alday, Maldacena)

Planar amplitudes possess dual conformal invariance (Drummond, Henn, Korchemsky, Sokatchev)

Some planar amplitudes *known* to all orders in coupling (Bern, Dixon, Smirnov,...)

Grassmannian technology fss (Arkani-Hamed et al.)

Ideal framework for testing on-shell and related methods

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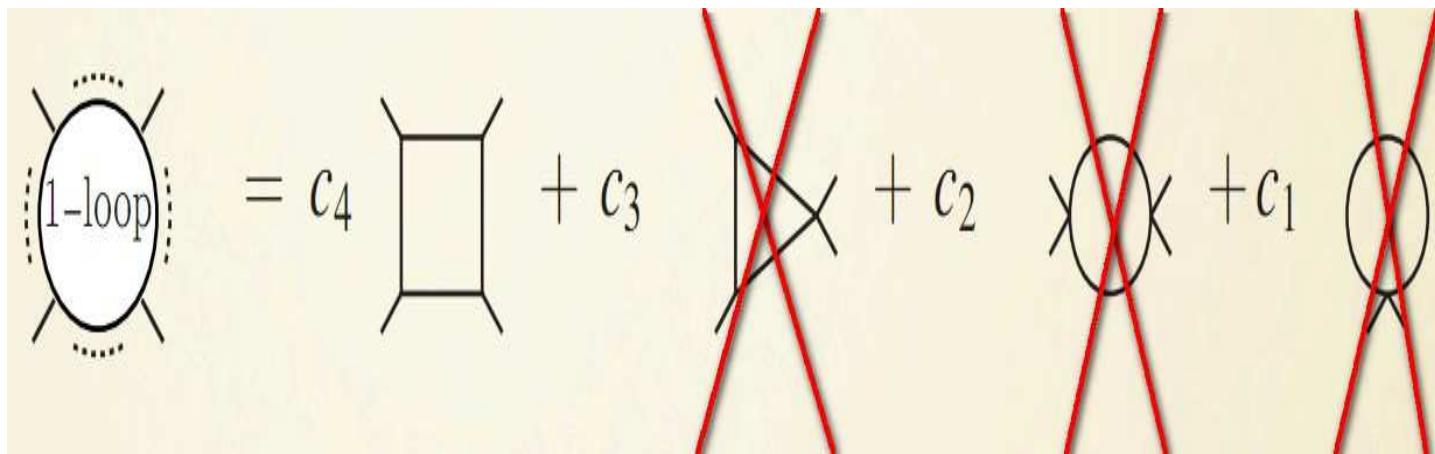
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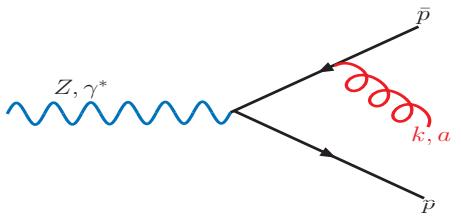
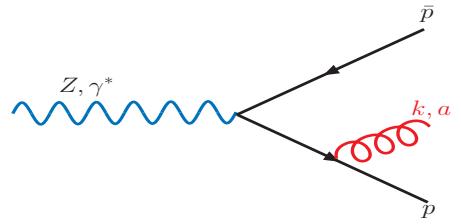
Yuri Dokhsitzer: “virtual SUSY” is helping QCD , QCD will pay back discovering “real” SUSY

Part B: Real Emission / Corrections, more trees

INFRARED/COLLINEAR DIVERGENCES

- Loop calculations give rise to infinities: Ultraviolet divergences that need renormalisation (almost trivial for SM, excluding widths..)
no so trivial for other models (SUSY, issue of $\tan \beta, \dots$)
- Collinear (small angle) and infrared soft (low momentum) divergences, compensation between soft/collinear radiation and virtual one-loop corrections (for properly defined observables)
infrared safe and either collinear safe or collinear factorisable (ask Gavin)
left-over collinear singularities are factorised into process independent structure and fragmentation functions

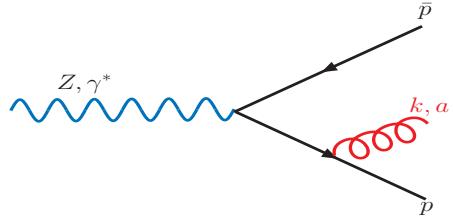
Compensation regularise with Dim Reg



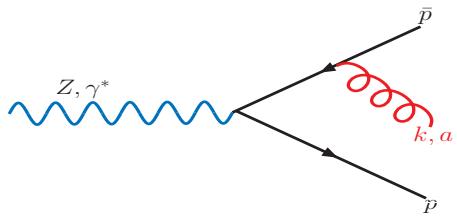
$$\sigma_{\text{real}} \sim \frac{C_F \alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$

For $1^* \rightarrow 2$, analytical result, integration easy over the one-gluon PS.

Compensation regularise with Dim Reg

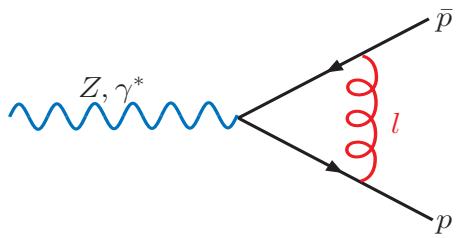


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For $1^* \rightarrow 2$, analytical result, integration easy over the one-gluon PS.

$$\int \frac{d^d l}{(2\pi)^d} \frac{N}{l^2(l-p)^2(l+\bar{p})^2}$$

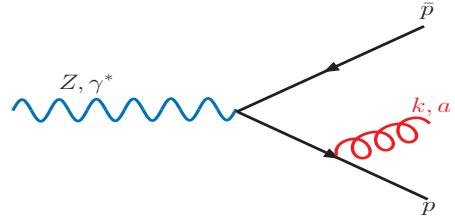


$l \rightarrow 0$ IR div, $l \rightarrow \infty$ UV

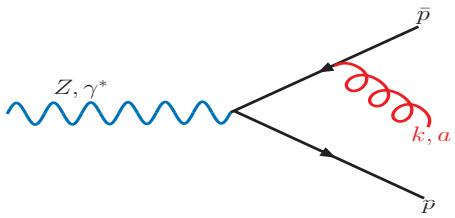
$l = xp, y\bar{p}$ Coll Div for any $x, y \rightarrow 0$ For $1^* \rightarrow 2$, analytical result, integration easy.

$$\sigma_{\text{virt}} \sim \frac{C_F \alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$

Compensation regularise with Dim Reg

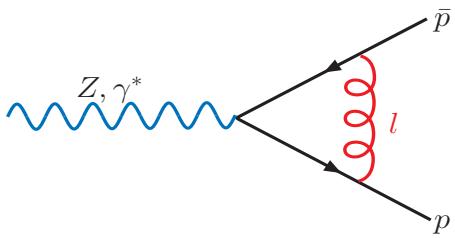


$$\sigma_{\text{real}} \sim \frac{C_F \alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$



For $1^* \rightarrow 2$, analytical result, integration easy over the one-gluon PS.

$$\int \frac{d^d l}{(2\pi)^d} \frac{N}{l^2(l-p)^2(l+\bar{p})^2}$$



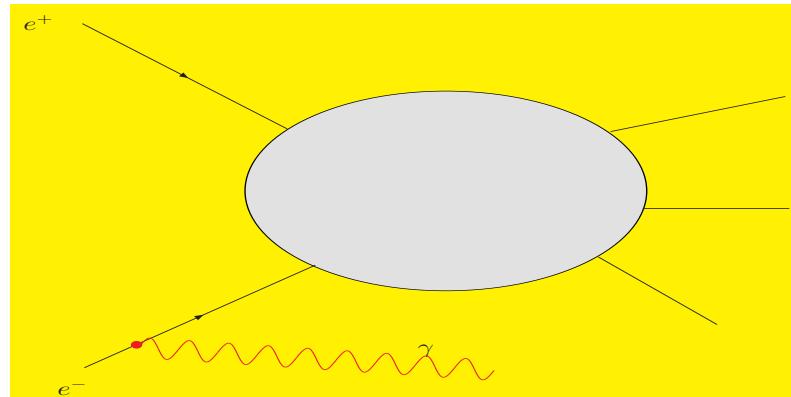
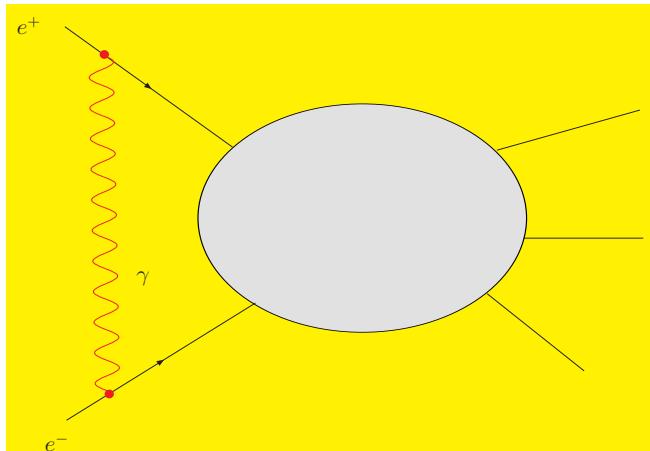
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$$\sigma_{\text{virt}} \sim \frac{C_F \alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right) \sigma_{\text{LO}}$$

$$\sigma_{NLO} = \left(1 + \frac{\alpha_s}{\pi} \right) \sigma_{LO}$$

INFRARED/COLLINEAR DIVERGENCES



must include bremsstrahlung

$$d\sigma_s(\lambda, E_\gamma < k_c) + d\sigma_H(\lambda, E_\gamma > k_c)$$

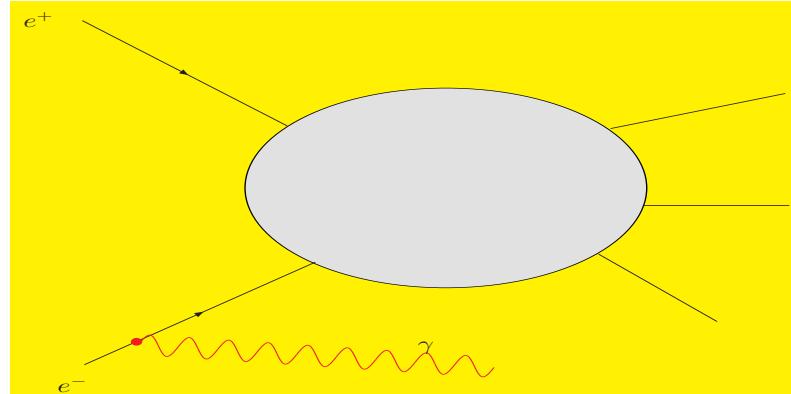
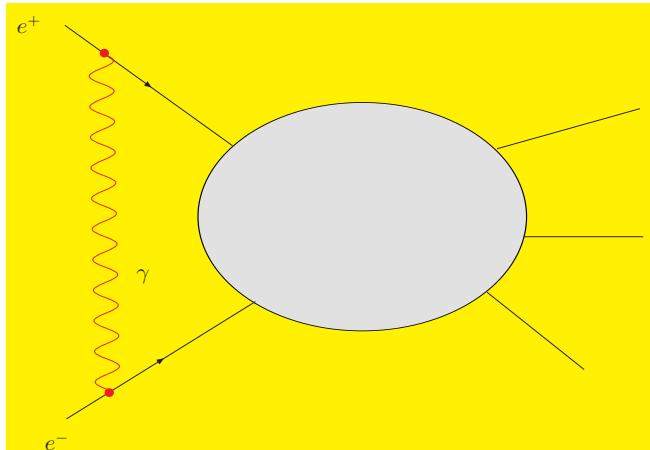
$d\sigma_s \rightarrow$ analytical: factorisation (automatised) ;

$d\sigma_H \rightarrow$ adaptive MC

$$\sigma_{O(\alpha)} = \underbrace{\int d\sigma_0 \left(1 + \delta_V^{EW} \right)}_{\sigma_0(1+\delta_W)} + \underbrace{\int d\sigma_0 \left(\delta_V^{QED}(\lambda) + \delta_S(\lambda, \mathbf{k}_c) \right)}_{\sigma_{V+S}^{QED}(\mathbf{k}_c)} + \underbrace{\int d\sigma_H(\mathbf{k}_c)}_{\sigma_H(\mathbf{k}_c)}.$$

strong cancellation, CPU time consuming for collinear parts in $\sigma_H(\mathbf{k}_c)$ and $\sigma_{V+S}^{QED}(\mathbf{k}_c)$

INFRARED/COLLINEAR DIVERGENCES



must include bremsstrahlung

$$d\sigma_s(\lambda, E_\gamma < k_c) + d\sigma_H(\lambda, E_\gamma > k_c)$$

$d\sigma_s \rightarrow$ analytical: factorisation (automatised) ;

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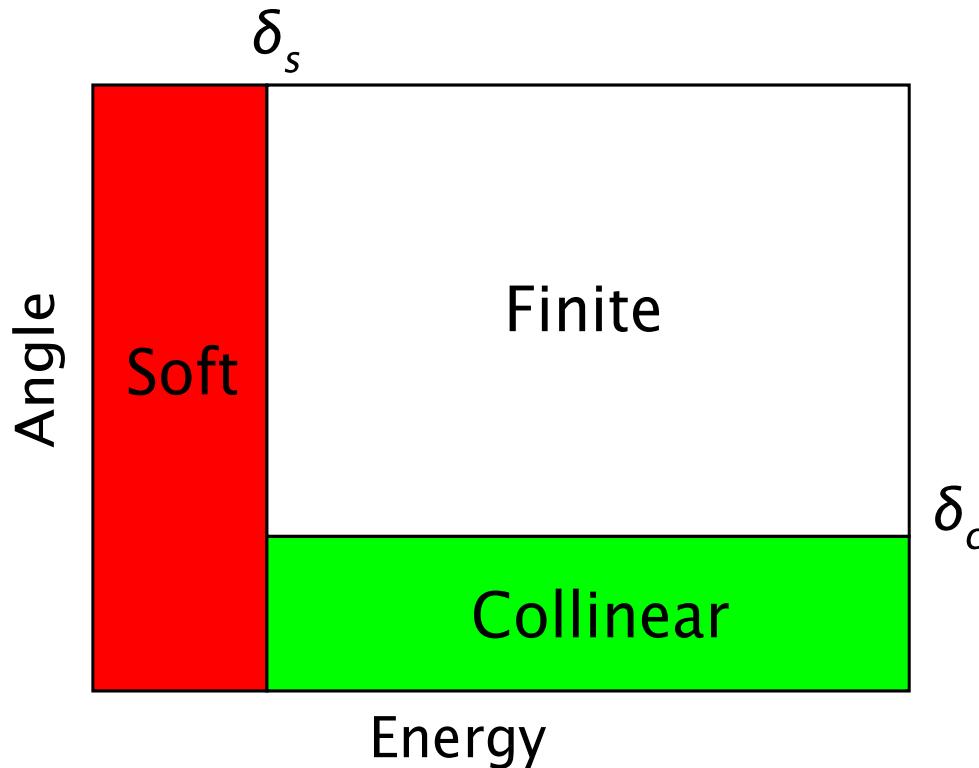
Dim Reg: $d\sigma_s$ part of $d\sigma_H \rightarrow$ Integrate in d -dim.

$$\sigma_{\mathcal{O}(\alpha)} = \underbrace{\int d\sigma_0 \left(1 + \delta_V^{EW} \right)}_{\sigma_0(1+\delta_W)} + \underbrace{\int d\sigma_0 \left(\delta_V^{QED}(\lambda) + \delta_S(\lambda, \mathbf{k}_c) \right)}_{\sigma_{V+S}^{QED}(\mathbf{k}_c)} + \underbrace{\int d\sigma_H(\mathbf{k}_c)}_{\sigma_H(\mathbf{k}_c)}.$$

Subtraction based on factorisation of collinear sing., more involved: dipoles, antennas,..but much more efficient (QCD)

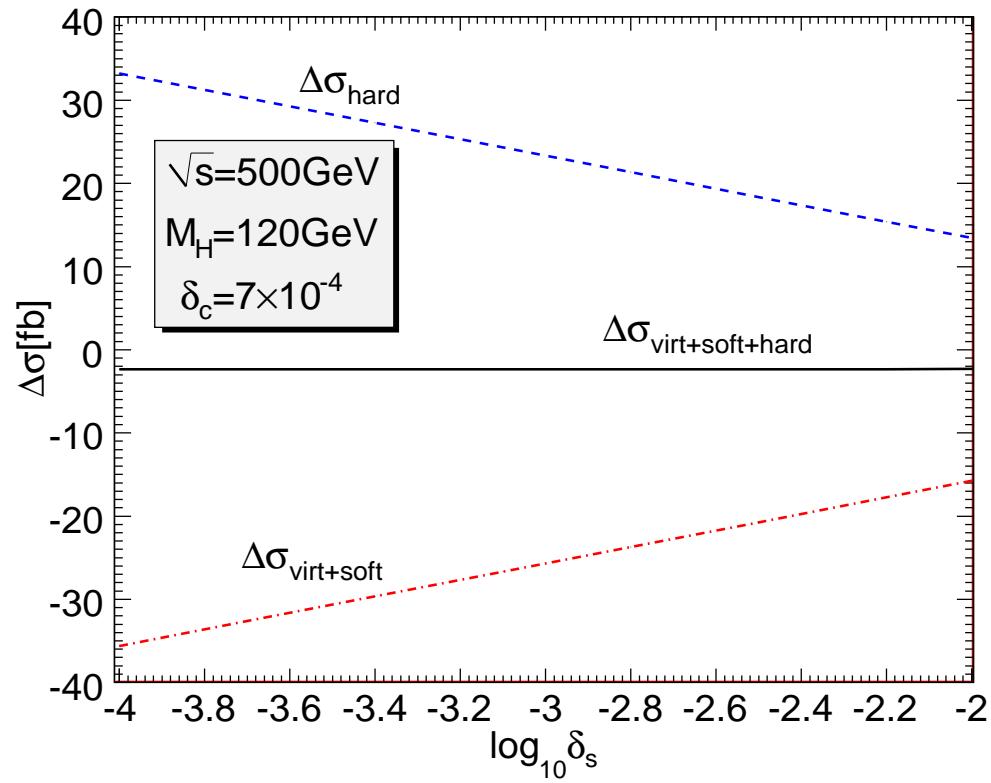
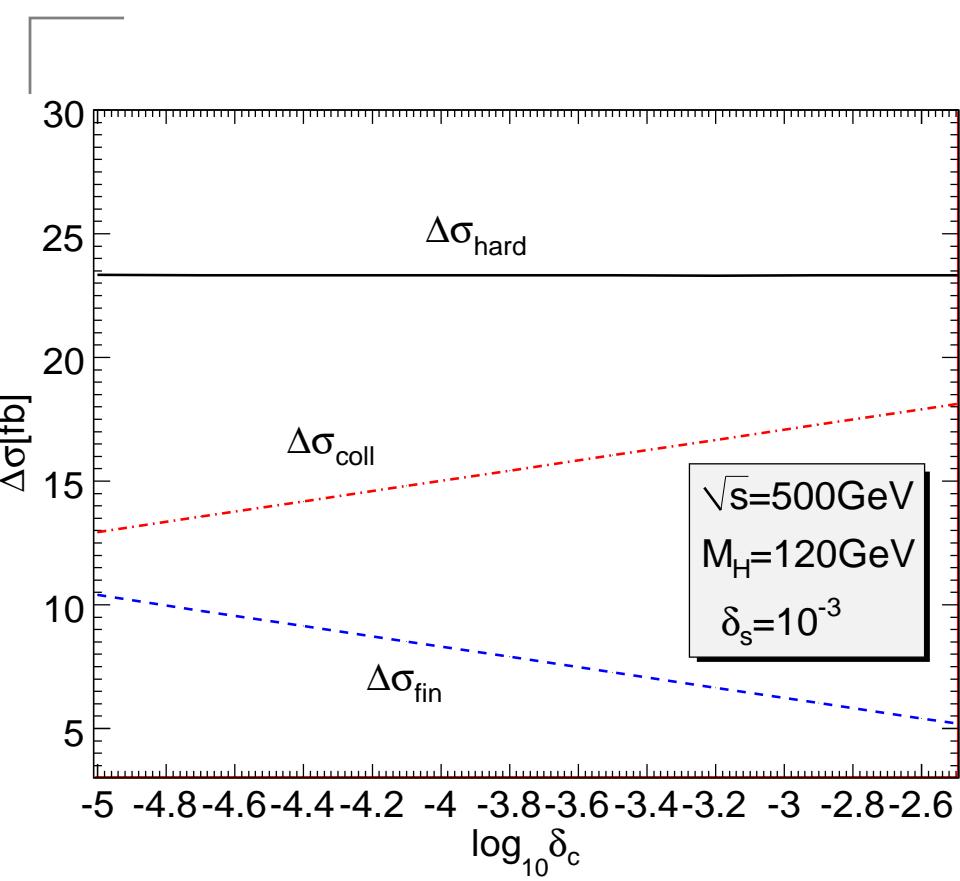
The Old not so good slicing method

recent example: NLO to $e^+e^- \rightarrow W^+W^-Z$ (Boudjema, Ninh Le Duc, Sun Hao, M. Weber, 2009)

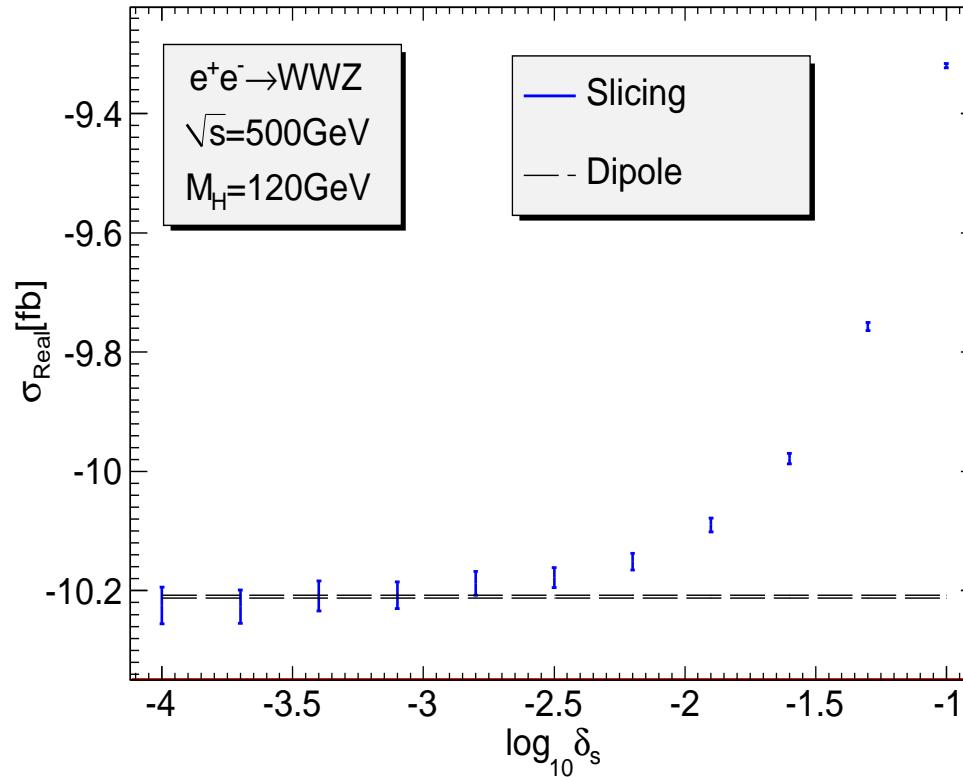


$$\begin{aligned}
 d\sigma^{e^+e^- \rightarrow W^+W^-Z} &= d\sigma_{virt}^{e^+e^- \rightarrow W^+W^-Z} + d\sigma_{real}^{e^+e^- \rightarrow W^+W^-Z\gamma}, \\
 d\sigma_{real}^{e^+e^- \rightarrow W^+W^-Z\gamma} &= d\sigma_{soft}^{e^+e^- \rightarrow W^+W^-Z\gamma}(\delta_s) + d\sigma_{hard}^{e^+e^- \rightarrow W^+W^-Z\gamma}(\delta_s), \\
 d\sigma_{hard}^{e^+e^- \rightarrow W^+W^-Z\gamma}(\delta_s) &= d\sigma_{coll}^{e^+e^- \rightarrow W^+W^-Z\gamma}(\delta_s, \delta_c) + d\sigma_{fin}^{e^+e^- \rightarrow W^+W^-Z\gamma}(\delta_s, \delta_c),
 \end{aligned}$$

The Old not so good slicing method: careful choice of matching/cuts



Slicing vs Dipole in $e^+e^- \rightarrow W^+W^-Z$

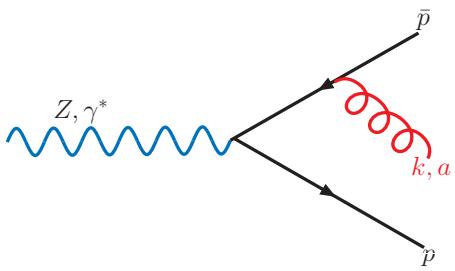
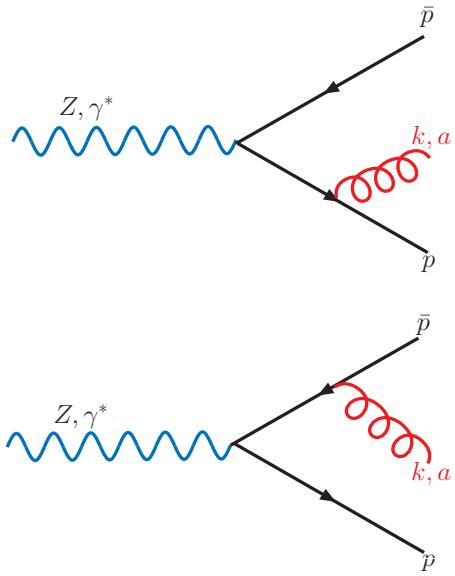


error in the dipole, thickness of line

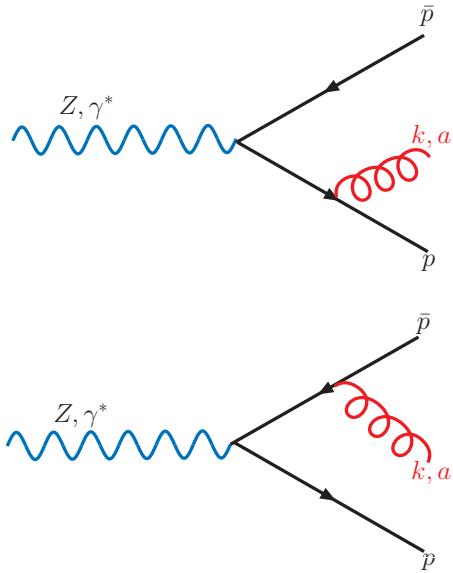
slicing 10-30 times slower for the same precision.

but even dipole takes longer than (optimised) virtual corrections (factor 2-3).

Origin and justification of PS: soft and collinear divergencies

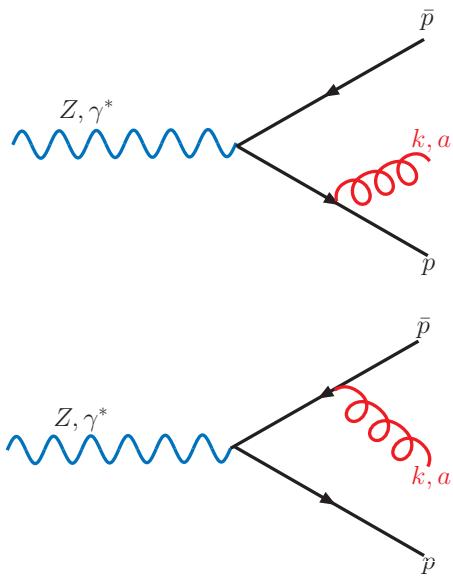


Origin and justification of PS: soft and collinear divergencies



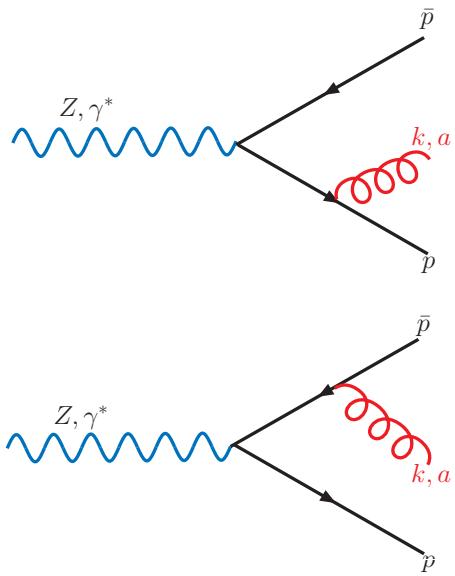
$$\begin{aligned}
 A_\mu &= \bar{u}(p) \not{e} (-ig_s t_a) \frac{-i}{\not{p} + \not{k}} \Gamma_\mu v(\bar{p}) \quad m_q = 0 \\
 &+ \bar{u}(p) \Gamma_\mu \frac{i}{\not{p} + \not{k}} (-ig_s t_a) \not{e} v(\bar{p}) \\
 &= -g_s \left(\frac{\bar{u}(p) \not{e} (\not{p} + \not{k}) \Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p) \Gamma_\mu (\not{p} + \not{k}) \not{e} v(\bar{p})}{2\bar{p}.k} \right) t_a \\
 2p.k &= 4E_g E_p \sin^2 \left(\frac{\theta_{pk}}{2} \right) \rightarrow 0 \text{ for } E_g \rightarrow 0, \quad \theta_{pk} \rightarrow 0
 \end{aligned}$$

Origin and justification of PS: soft and collinear divergencies

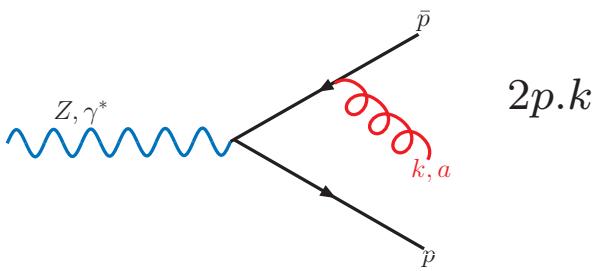


$$\begin{aligned}
 \mathcal{A}_\mu &= \bar{u}(p) \not{e} (-ig_s t_a) \frac{-i}{\not{p} + \not{k}} \Gamma_\mu v(\bar{p}) \quad m_q = 0 \\
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 &= -g_s \left(\frac{\bar{u}(p) \not{e} (\not{p} + \not{k}) \Gamma_\mu v(\bar{p})}{2p.k} - \frac{\bar{u}(p) \Gamma_\mu (\not{p} + \not{k}) \not{e} v(\bar{p})}{2\bar{p}.k} \right) t_a \\
 2p.k &= 4E_g E_p \sin^2\left(\frac{\theta_{pk}}{2}\right) \rightarrow 0 \text{ for } E_g \rightarrow 0, \theta_{pk} \rightarrow 0 \\
 \mathcal{A}_{\text{soft}}(k \rightarrow 0) &= -g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right) \mathcal{A}_0 \quad \text{diverges } k \rightarrow 0
 \end{aligned}$$

Origin and justification of PS: soft and collinear divergencies



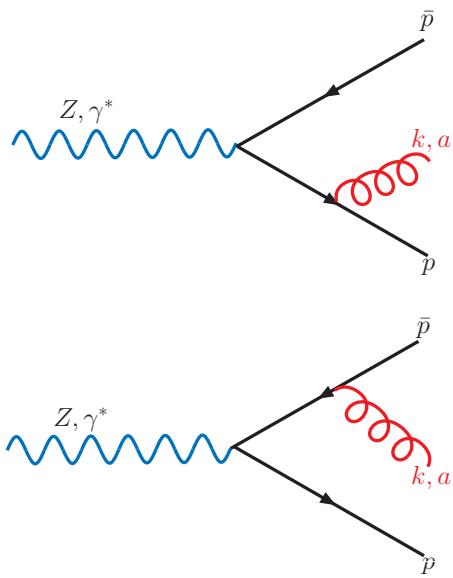
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 \end{aligned}$$



$$2p.k = 4E_g E_p \sin^2 \left(\frac{\theta_{pk}}{2} \right) \rightarrow 0 \text{ for } E_g \rightarrow 0, \theta_{pk} \rightarrow 0$$

$$A_{1g}(k \rightarrow 0) = \boxed{-g_s t_a \left(\frac{p.\epsilon}{p.k} - \frac{\bar{p}.\epsilon}{\bar{p}.k} \right)} A_{0g}$$

Origin and justification of PS: soft and collinear divergencies



$$\begin{aligned}
 A_\mu &= \bar{u}(p) \not{e} (-ig_s t_a) \frac{-i}{\not{p} + \not{k}} \Gamma_\mu v(\bar{p}) \quad m_q = 0 \\
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 \end{aligned}$$

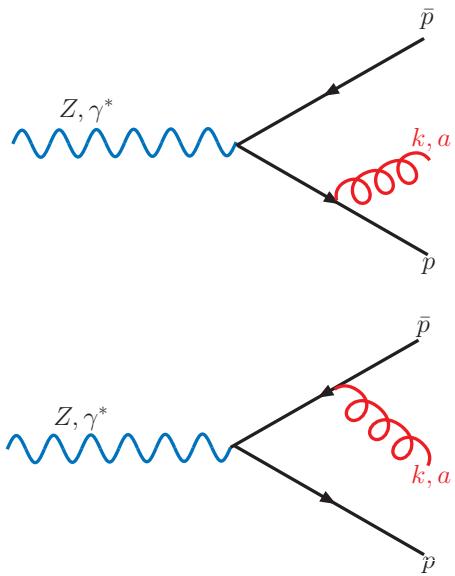
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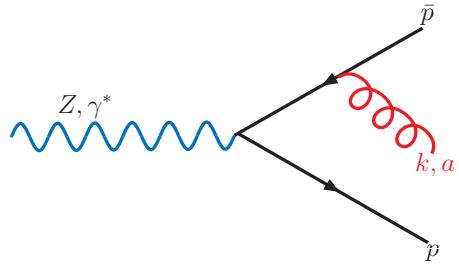
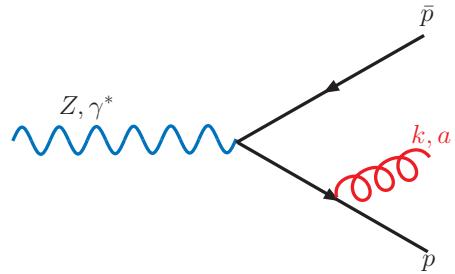
Universal Radiator Factor

We have **factorisation** of the soft emission (long distance) from the short distance i.e. the **hard process**

Squaring soft/collinear

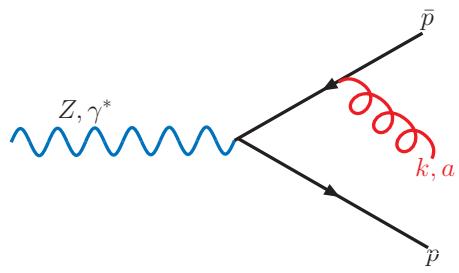
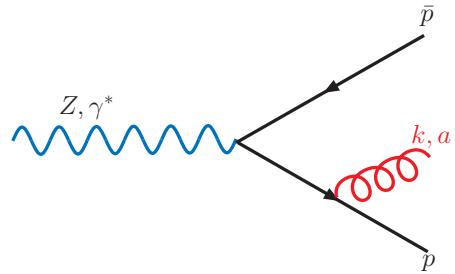


Squaring soft/collinear



$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) \mathcal{A}_{0g}$$

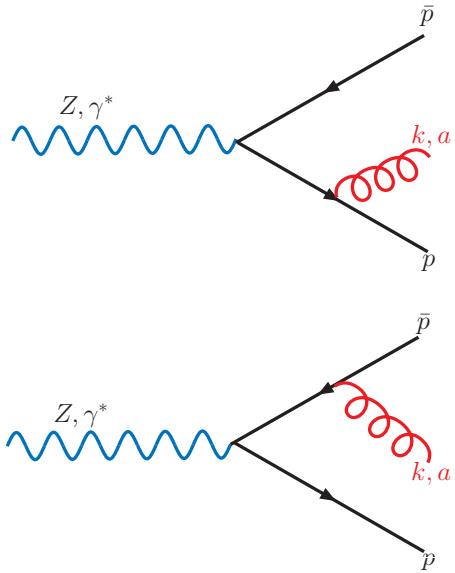
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$$|\mathcal{M}_{1g}|^2 = \sum_{a, pol.(\epsilon)} |\mathcal{A}_{1g}(k \rightarrow 0)|^2 = C_F g_s^2 \frac{2p \cdot \bar{p}}{p \cdot k \bar{p} \cdot k} |\mathcal{M}_{0g}|^2$$

Squaring soft/collinear

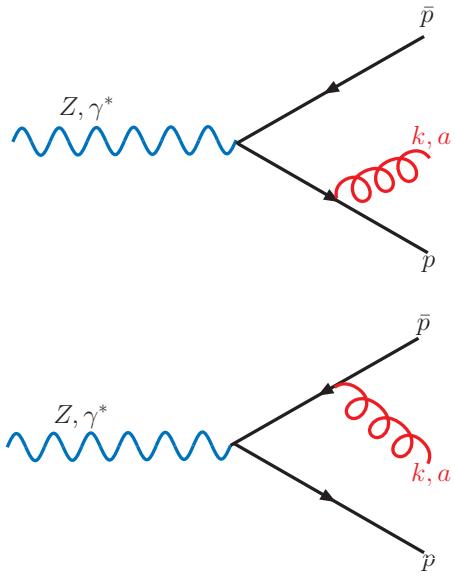


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Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p \cdot \bar{p}}{p \cdot k \bar{p} \cdot k}$$

Squaring soft/collinear



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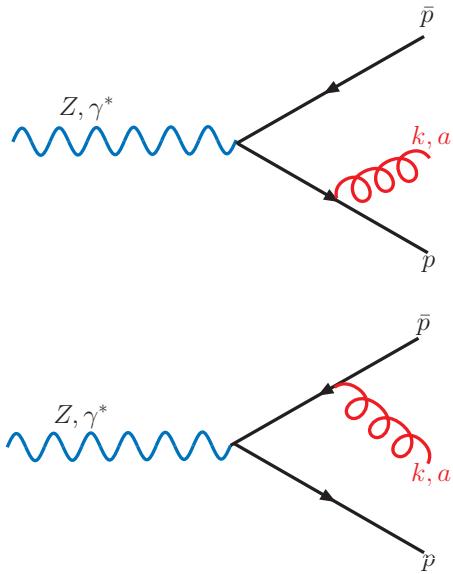
Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p \cdot \bar{p}}{p \cdot k \bar{p} \cdot k}$$

$$\theta = \theta_{\angle pk}, \quad \phi = \text{azimuth}$$

$$d\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin \theta} \frac{d\phi}{2\pi}$$

Squaring soft/collinear



$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) \mathcal{A}_{0g}$$

Phase Space

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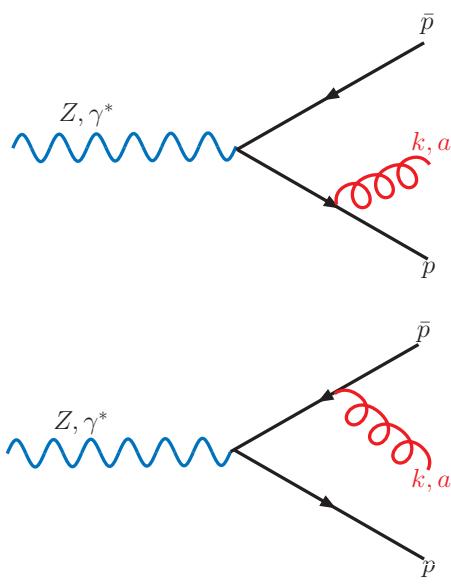
$$\theta = \theta_{\angle pk}, \phi = \text{azimuth}$$

$$d\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin \theta} \frac{d\phi}{2\pi}$$

- $d\mathcal{S}$ diverges for $\omega \rightarrow 0$, **Infrared divergence** (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \rightarrow 0$ and $\theta \rightarrow \pi$, **collinear divergence**

Squaring soft/collinear

$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) \mathcal{A}_{0g}$$



Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p \cdot \bar{p}}{p \cdot k \bar{p} \cdot k}$$

$$\theta = \theta_{\angle pk}, \phi = \text{azimuth}$$

$$d\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin \theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \rightarrow 1, \quad k \rightarrow 3$$

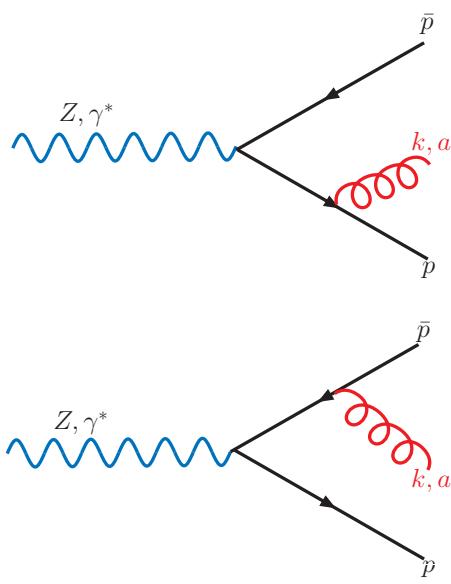
$$d\mathcal{S}_\phi = \frac{\alpha_s C_F}{2\pi} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2 \theta} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right) d\cos \theta dx_3$$

- $d\mathcal{S}$ diverges for $\omega \rightarrow 0$, Infrared divergence (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \rightarrow 0$ and $\theta \rightarrow \pi$, collinear divergence

Squaring soft/collinear

$$\mathcal{A}_{1g}(k \rightarrow 0) = -g_s t_a \left(\frac{p \cdot \epsilon}{p \cdot k} - \frac{\bar{p} \cdot \epsilon}{\bar{p} \cdot k} \right) \mathcal{A}_{0g}$$



Phase Space

$$|\mathcal{M}_{1g}|^2 d\Phi_{q\bar{q}g} = \left(|\mathcal{M}_{0g}|^2 d\Phi_{q\bar{q}} \right) d\mathcal{S}; \quad d\mathcal{S} \simeq \frac{d^3 \vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p \cdot \bar{p}}{p \cdot k \bar{p} \cdot k}$$

$$\theta = \theta_{\angle pk}, \phi = \text{azimuth}$$

$$d\mathcal{S} \simeq \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin \theta} \frac{d\phi}{2\pi}$$

$$x_i = 2E_i/E_{\text{tot}} \quad p \rightarrow 1, \quad k \rightarrow 3$$

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$$= \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2 \theta} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right) d\cos \theta dx_3$$

- $d\mathcal{S}$ diverges for $\omega \rightarrow 0$, **Infrared divergence** (needs virtual loop corrections, we'll say more if time permits)
- $d\mathcal{S}$ diverges for $\theta \rightarrow 0$ and $\theta \rightarrow \pi$, **collinear divergence**
- collinear divergence for $x_1 \rightarrow 1$ or $x_2 \rightarrow 1$ and Infrared divergence for $x_3 \rightarrow 0$

Splitting

$$\begin{aligned} dS_\phi &\simeq \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2 \theta} \frac{1 + (1 - x_3)^2}{x_3} \right) d\cos \theta dx_3 \\ \frac{2d\cos \theta}{\sin^2 \theta} &= \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \theta}{1 + \cos \theta} = \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \bar{\theta}}{1 - \cos \bar{\theta}} \sim \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for } \theta, \bar{\theta} \sim 0 \ll 1 \end{aligned}$$

Splitting

$$\begin{aligned}
 dS_\phi &\simeq \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2 \theta} \frac{1 + (1 - x_3)^2}{x_3} \right) d\cos \theta dx_3 \\
 \frac{2d\cos \theta}{\sin^2 \theta} &= \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \theta}{1 + \cos \theta} = \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \bar{\theta}}{1 - \cos \bar{\theta}} \sim \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for } \theta, \bar{\theta} \sim 0 \ll 1
 \end{aligned}$$

q and \bar{q} as independent emitters, notion of splitting as a probability

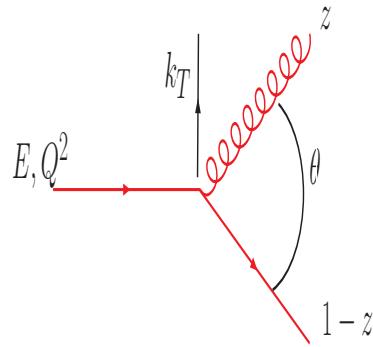
$$d\sigma_{1g} \sim d\sigma_{0g} \sum_{\substack{\bar{q} \rightarrow \bar{q}g \\ q \rightarrow qg}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \frac{1 + (1 - z)^2}{z} dz \quad (z \equiv x_3)$$

Splitting

$$\begin{aligned} dS_\phi &\simeq \frac{\alpha_s C_F}{2\pi} \left(\frac{2}{\sin^2 \theta} \frac{1 + (1 - x_3)^2}{x_3} \right) d\cos \theta dx_3 \\ \frac{2d\cos \theta}{\sin^2 \theta} &= \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \theta}{1 + \cos \theta} = \frac{d\cos \theta}{1 - \cos \theta} + \frac{d\cos \bar{\theta}}{1 - \cos \bar{\theta}} \sim \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2} \quad \text{for } \theta, \bar{\theta} \sim 0 \ll 1 \end{aligned}$$

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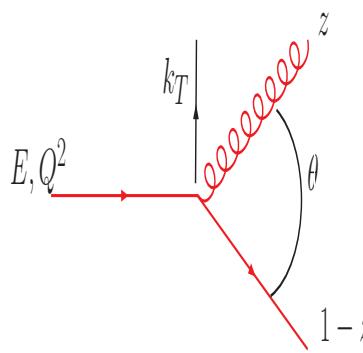


Splitting

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q and \bar{q} as independent emitters, notion of splitting as a probability

$$d\sigma_{1g} \sim d\sigma_{0g} \sum_{q \rightarrow qg}^{\bar{q} \rightarrow \bar{q}g} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \frac{1 + (1 - z)^2}{z} dz \quad (z \equiv x_3)$$



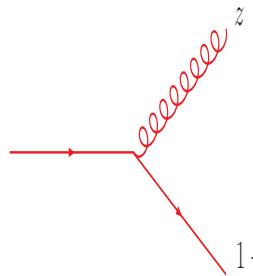
different choices of the evolution variables, equivalent in the collinear limit (diff. in practice/different codes)

$$\begin{aligned} Q^2 &= E^2 z (1 - z) \theta^2 & k_T^2 &= E^2 z^2 (1 - z)^2 \theta^2 \\ \frac{d\theta^2}{\theta^2} &= \frac{dQ^2}{Q^2} = \frac{dk_T^2}{k_T^2} \end{aligned}$$

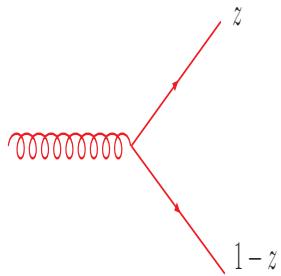
DGLAP

This generalises to different parton branching (gluon, quarks)

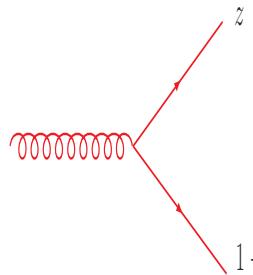
$$d\sigma_{bc} \sim d\sigma_a \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} P_{a \rightarrow bc}(z) dz$$



$$P_{gq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

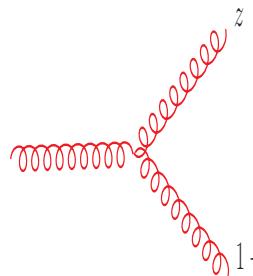


$$P_{qg}(z) = C_F \left(\frac{1+(1-z)^2}{z} \right)$$



$$P_{qg}(z) = T_R \left(z^2 + (1-z)^2 \right) \quad T_R = \frac{n_f}{2}$$

(divergences at $z = 0, 1$ dealt
with soft/virtual corr.)



$$P_{gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \quad C_A = 3 \quad (C_F = 4/3)$$

Gluons radiate the most

$P(z, \phi)$ can be defined for polarisation effects

- The singular parts of the QCD/QED matrix elements for real emission can be singled out in a general way through the factorisation of soft and collinear radiation
- Improved general purpose factorisation formulae have been derived: dipole factorisation formula.

Strategy

$$\begin{aligned}\sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B, \\ \sigma_{ab}^{NLO} &= \underbrace{\int_{m+1} d^{(d)} \sigma_{ab}^R}_{\text{div if } d=4} + \underbrace{\int_m d^{(d)} \sigma_{ab}^V}_{\text{div if } d=4}. \\ &\qquad\qquad\qquad \text{sum is finite}\end{aligned}$$

for $2 \rightarrow m$ (Born, V) and $2 \rightarrow m + 1$ (real)

Factorisation and dipoles

$$\sigma_{ab}^{NLO} = \underbrace{\int_{m+1} \left(d^{(d)} \sigma_{ab}^R - d^{(d)} \sigma_{ab}^A \right)}_{finite} + \underbrace{\int_m \left(d^{(d)} \sigma_{ab}^V + d^{(d)} \sigma_{ab}^A \right)}_{finite}.$$

sum is finite

$$\begin{aligned} \sigma_{ab}^{NLO} &= \int_{m+1} \left(d^{(4)} \sigma_{ab}^R - d^{(4)} \sigma_{ab}^A \right) \\ &+ \underbrace{\int_m \left(\int_{loop} d^{(d)} \sigma_{ab}^V + \int_1 d^{(d)} \sigma_{ab}^A \right)_{\varepsilon=0}}_{finite}. \end{aligned}$$

Factorisation and dipoles: properties of subtraction term(s)

- For a given process $d\sigma^A$ independent of the particular jet observable
- $d\sigma^A$ has to match the singularities of $d\sigma^R$ in d dimension
- form convenient for Monte Carlo integration routines
- has to be analytically integrable in d -dimension over the single parton PS

$$d^{(d)}\sigma_{ab}^A = \sum_{\text{dipoles}} d^{(4)}\sigma_{ab}^B \times d^{(d)}V_{\text{dipole}}.$$

- The singularities in the real emission, either soft or collinear factorise and are universal
- i.e **Process Independent**
- these universal terms are known, if we subtract their contribution from the full real emission terms, the obtained contribution has no singularity and could therefore **be integrated numerically over all of phase space**
- the singularities in the real emission **compensate** those in the virtual emission, this assumes we are using the same regularisation scheme

Factorisation: Summary

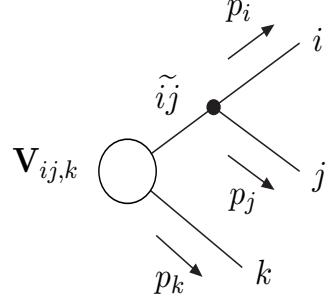
When dealing with multiparticle final states,
integration over phase space can only be performed **numerically**
these observation are very important

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integration over phase space can only be performed **numerically**
these observation are very important

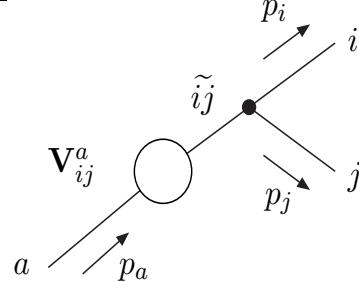
- Calculate the real terms and subtract the **soft/collinear counterterms**
- One can then integrate in 4-dimension **numerically**
- add these counterterms ***analytically*** to the virtual contribution (**properly UV renormalised**) to obtain a diff cross section that is soft/coll finite and that can be integrated **numerically**
- some massaging to do, can be automated: $dPS_{LO+1} \rightarrow dPS_{LO} \times dPS_{\text{gluon}}$ (boosts,...)
- there can be a lot of emitters/dipoles!

Dipoles la Catani-Seymour

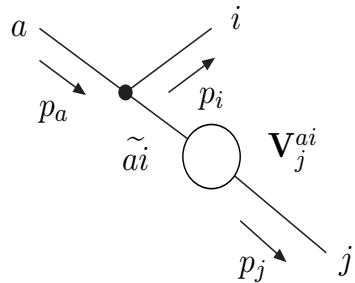
$\mathcal{D}_{ij,k}$:



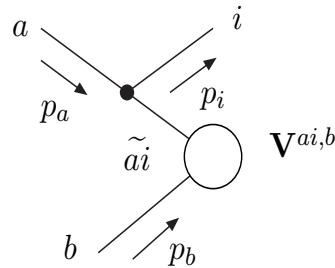
\mathcal{D}_{ij}^a :



\mathcal{D}_j^{ai} :



$\mathcal{D}^{ai,b}$:



Dipoles: final-state emitter with final-state spectator ($\mathcal{D}_{ij,k}$), final-state emitter with initial-state spectator (\mathcal{D}_{ij}^a), initial-state emitter with final-state spectator (\mathcal{D}_j^{ai}) and initial-state emitter with initial-state spectator ($\mathcal{D}^{ai,b}$).

Physics!!!

(Gleisberg, Hoeche, Krauss, Schöbenheuer,
Schumann, Siegert, Winter)

NLO with *BlackHat+Sherpa*

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[\int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

(S. Catani, M.H. Seymour, 1997)

(T. Gleisberg, F. Krauss, 2007)

(a glance to NLO automation!)

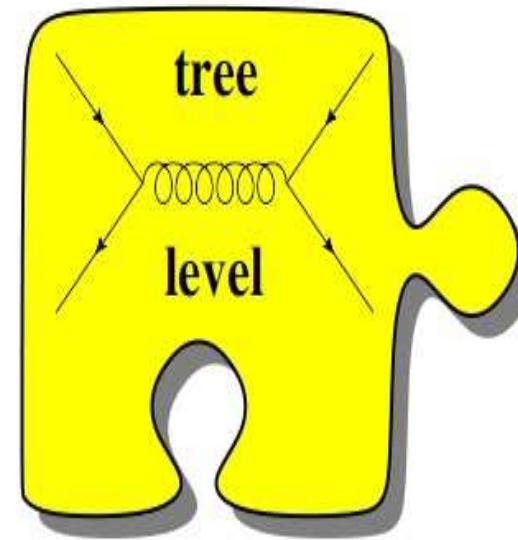


+



High level of automatisation, public

- ALPGEN (Mangano et al.)
- CalcHEP
(Pukhov,Belyaev,Christensen)
- CompHEP (Boos et al.)
- Grace (Yuasa et al.)
- HELAS / PHEGAS (Papadopoulos et al.)
- MADGRAPH / MADEVENT
(Maltoni,Stelzer)
- O'Mega / WHIZARD
(Kilian,Moretti,Ohl,Reuter)
- SHERPA / Amegic (Krauss,Kuhn)



virtual Corrections:

- **Feynmanians**

FeynArts/FormCalc (

Hahn,Perez-Victoria,v.Oldenborgh

Grace-loop (Shimzu et al.

Golem (Binoth et al,)

SloopS (Boudjema et al.)

Many Process Specific
few)

- **Unitaritarians/cuts** (at amplitude level
so)

HELAC-1LOOP+CutTools

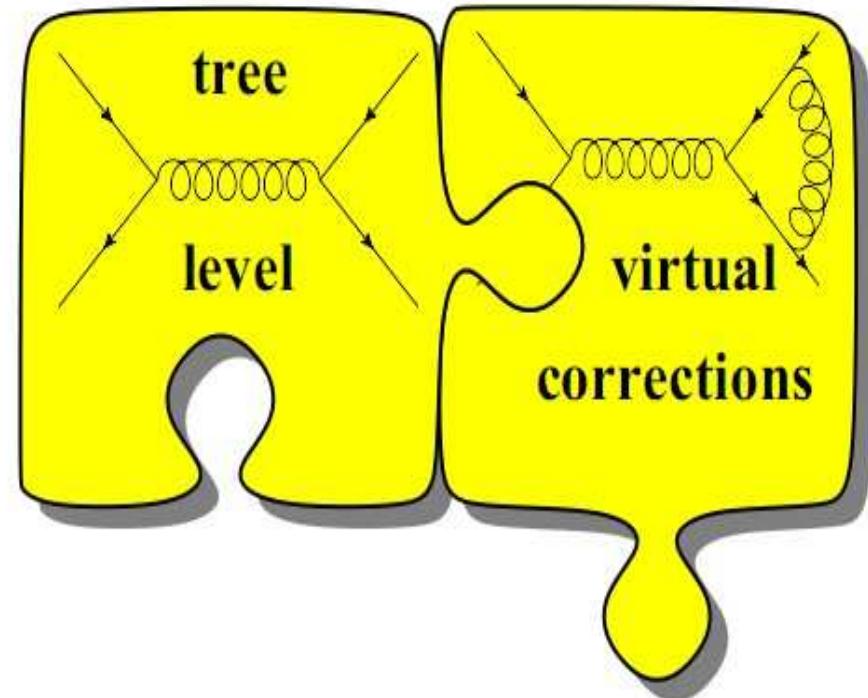
(v.Hameren,Ossola,Papadopoulos

BlackHat (Berger et al.)

Rocket (Ellis, Melnikov,
Zanderighi)

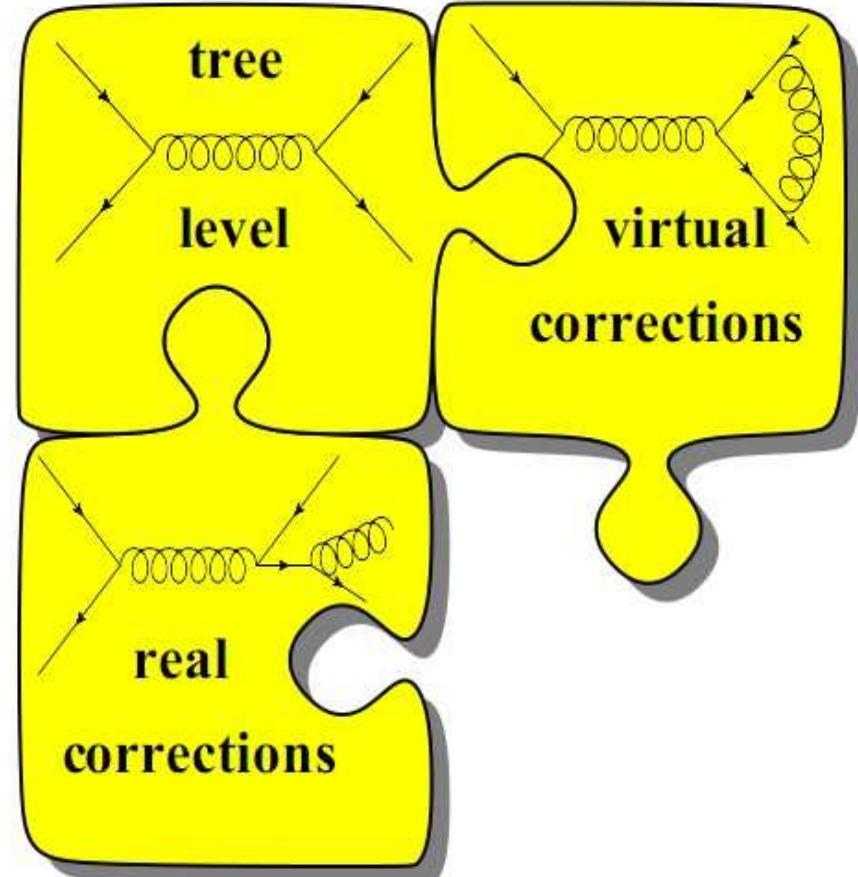
Generalized Color-Dress

Unitarity (Giele,Kunszt,Winter

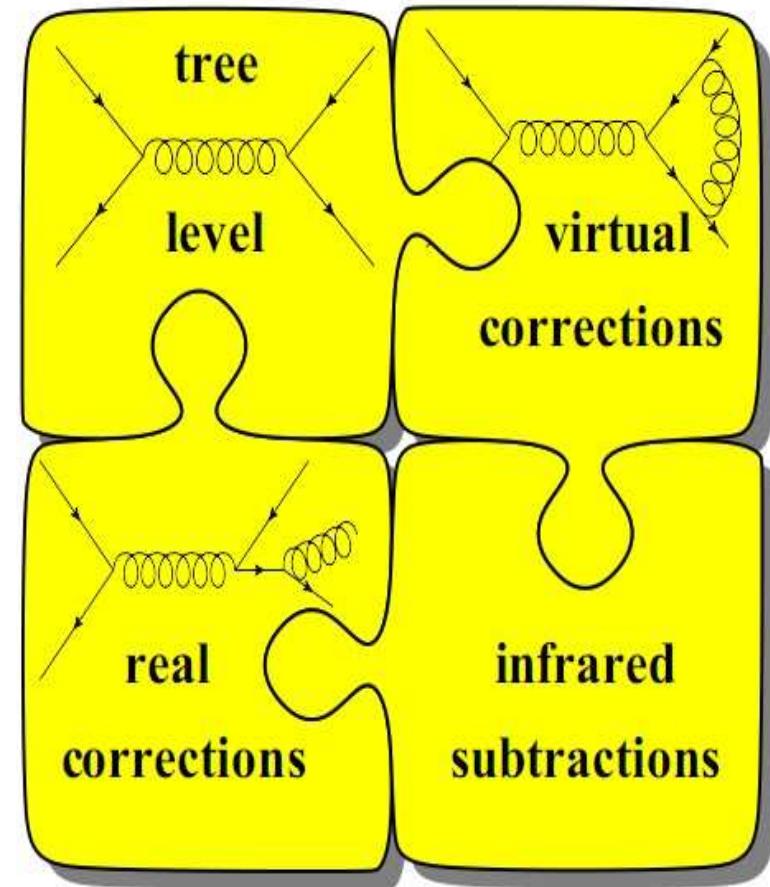


Putting the pieces together

Radiation, in principle same as tree-level



- Catani-Seymour dipoles
 - AutoDipole (Hasegawa, Moch, Uwer)
 - HELAC-DPOLE (Czakon, Papadopoulos, Worek)
 - MadDipole (Frederix, Gehrmann, Greiner)
 - Sherpa (Gleisberg, Krauss)
 - TevJet (Seymour, Tevlin)
- MadFKS (Frederix, Frixione, Maltoni, Stelzer)



2 Higgs phenomenology

- Carlo
- B. Mele
- Sally
- Susanne
- Dieter
- Rekhi
- Gianpiero
- Laura
- John
- Silvana

3. New H₀ calculations

- Markus
- Giacinto
- Matthew
- Stefano
- Stefan
- Kai
- Joe
- D.
- S.
- G.

wishlist

- Fazlollah
- Dieten
- Gabriele
- Vittorio
- Rishabh
- Nicolas
- Stefano
- Stefan
- Stefan(w)

4. NLO techniques

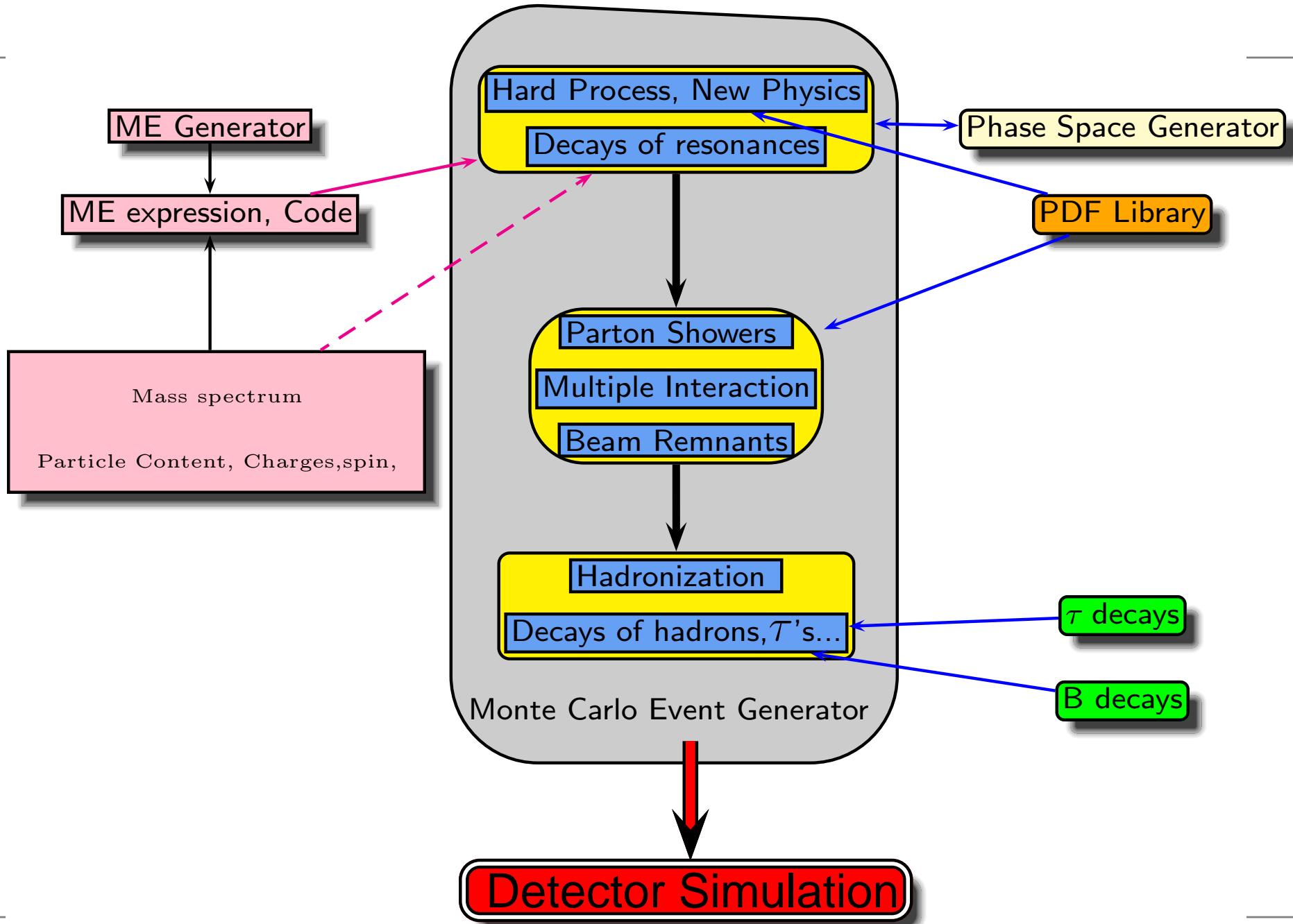
- Standardization/Automatic
- Fazlollah
- Dieten
- Gabriele
- Vittorio
- Frank-Peter
- Due Niki
- Marcos
- Stefanas
- Grigoriyan
- Sven
- Tongju
- Daniel
- Rishabh
- Nicolas

5. NLO parton shower

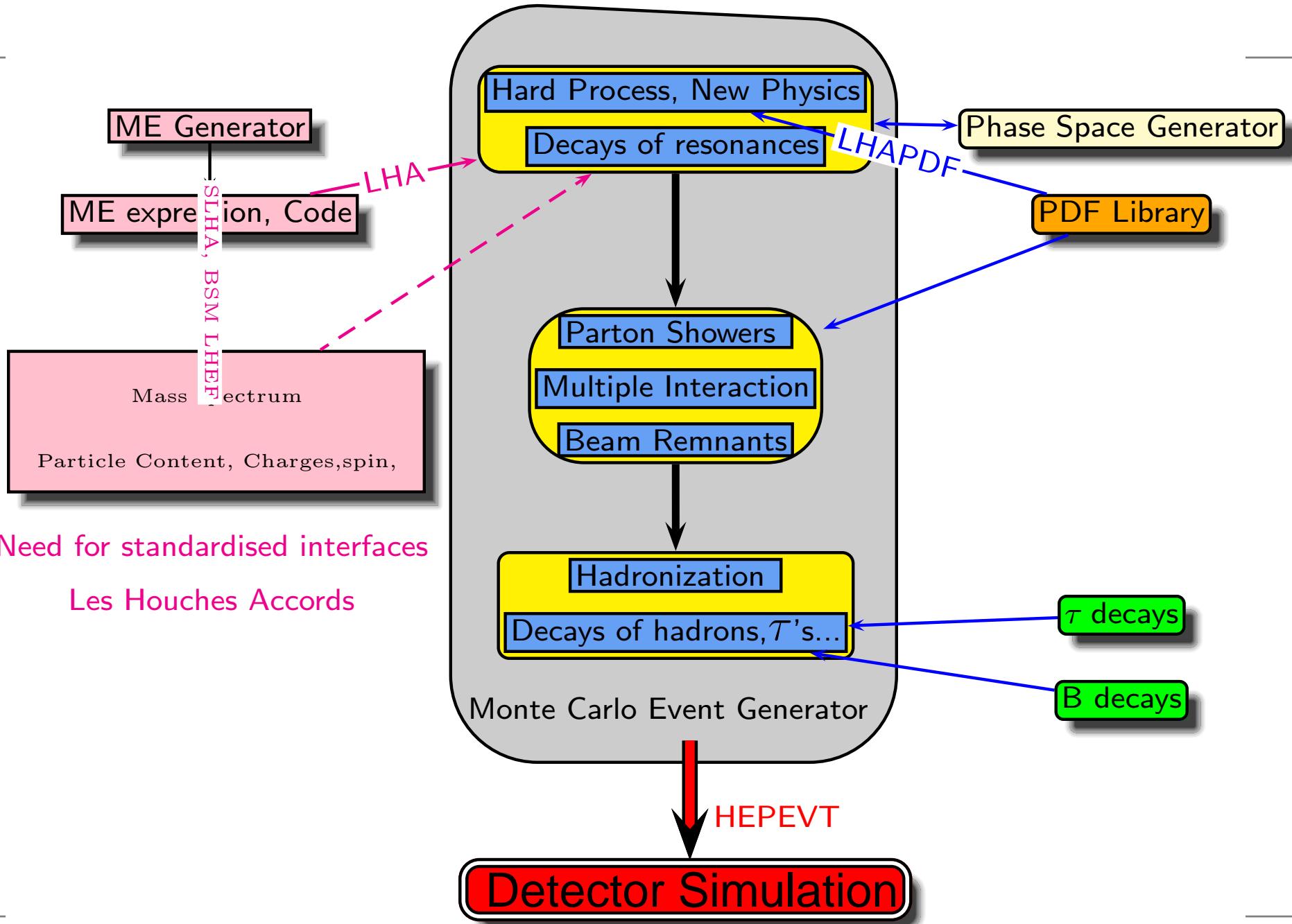
- higher wL MC
- Carl
- Sven
- Stefan
- Isabella
- Stefan
- Ruth
- Thomas (R)
- Maria Vittoria
- Due Niki
- Marcos

09.06.2009

Putting all together



Putting all together, Les Houches Accords



Binoth LHA

- hadronic cross section and partonic subprocesses

$$\begin{aligned}\sigma_{had}(p_1, p_2) &= \sum_{a,b} \int dx_1 f_{a/H_1}(x_1, \mu_F^2) \int dx_2 f_{b/H_2}(x_2, \mu_F^2) \\ &\times \left[d\sigma_{ab}^{\text{LO}}(x_1 p_1, x_2 p_2; \mu_R^2) + d\sigma_{ab}^{\text{NLO}}(x_1 p_1, x_2 p_2; \mu_R^2, \mu_F^2) \right],\end{aligned}$$



$$\begin{aligned}\sigma_{ab}^{\text{LO}} &= \int_m d\sigma_{ab}^B, \\ \sigma_{ab}^{\text{NLO}} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V + \int_m d\sigma_{ab}^C(\mu_F^2, \text{F.S.}) .\end{aligned}$$

- for $2 \rightarrow m$ (Born, V) and $2 \rightarrow m + 1$ (real)

$$d\sigma_{ab}^V = d\text{LIPS}(\{k_j\}) \mathcal{I}(\{k_j\}).$$

- Take DR after renormalisation

$$\mathcal{I}(\{k_j\}, \text{R.S.}, \mu_R^2, \alpha_S(\mu_R^2), \alpha, \dots) = C(\epsilon) \left(\frac{A_2}{\epsilon^2} + \frac{A_1}{\epsilon} + A_0 \right).$$

The goal of the interface is to facilitate the transfer of information between
one-loop programs, OLP
and programs which provide
tree amplitude information and incorporate methods to
perform the integration over the phase space:
Monte Carlo tool (MC).

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Monte Carlo tool (MC).

Interaction works in 2 phases

- initialisation exchange of basic information: availability of sub-processes, input parameters, schemes,...
- Run-time MC asks OLP for one-loop contributions at points in PS. Finite part may be split (more efficient integration, sampling)

contracts

OLP	Input	Output
Initialisation	Model parameters: $\alpha(0), \alpha_s(M_Z), \dots, m_t, m_b, \dots, CKM$ values	confirm values
	Schemes: UV-renormalisation / IR-factorisation	confirm schemes
	Operational information: colour/helicity treatment, approximations, etc.	confirm options

contracts

OLP	Input	Output
Initialisation	Model parameters: $\alpha(0), \alpha_s(M_Z), \dots, m_t, m_b, \dots, \text{CKM values}$	confirm values
	Schemes: UV-renormalisation / IR-factorisation	confirm schemes
	Operational information: colour/helicity treatment, approximations, etc.	confirm options
Run-time	Events: $(E, p_x, p_y, p_z, M)_{j=1, \dots, m+2}, \mu, \alpha_s(\mu_R)$	$(A_2, A_1, A_0, \text{Born} ^2)$ optional information



INITIALIZATION

PP → ZZ
ME
FILE

PHASE

Loop

RUN PHASE

Urbano-Veltman

$$\rho(1-\varepsilon) \\ (4\pi)^2$$

$$\begin{matrix} 21 & 21 & 6 & -6 \\ 3 & -3 & 6 & -6 \end{matrix}$$

DETAILED?

the order file

Example: Here is an example of an order file for the partonic $2 \rightarrow 3$ processes, $gg \rightarrow t\bar{t}g$, $q\bar{q} \rightarrow t\bar{t}g$ and $qg \rightarrow t\bar{t}q$, needed for the evaluation of $pp \rightarrow t\bar{t} + \text{jet}$

```
# example order file

MatrixElementSquareType CHsummed
IRregularisation CDR
OperationMode LeadingColour
ModelFile ModelInLHFormat.slh
SubdivideSubprocess yes
AlphasPower 3
CorrectionType QCD

# g g  -> t tbar g
21 21 -> 6 -6 21
# u ubar -> t tbar g
2 -2 -> 6 -6 21
# u g   -> t tbar u
2 21 -> 6 -6  2
```

The contract file

Example:

```
# example contract file
# contract produced by OLP, OLP authors, citation policy

MatrixElementSquareType CHsummed | OK
IRregularisation CDR | OK
OperationMode LeadingColour | OK
ModelFile ModelFileInLHFormat.slh | OK
SubdivideSubprocess yes | OK
CorrectionType QCD | OK

# g g -> t tbar g
21 21 -> 6 -6 21 | 2 13 35 # 2 channels: cut-constructable,&
& rational part
# u ubar -> t tbar g
2 -2 -> 6 -6 21 | 1 29
# u g -> t tbar u
2 21 -> 6 -6 2 | 3 8 23 57 # 3 channels: leading,&
& subleading, subsubleading colour
```

The contract file

Example:

```
# example contract file
# contract produced by OLP, OLP authors, citation policy

MatrixElementSquareType CHsummed | Error: unsupported flag
# CHaveraged is supported
IRregularisation          DRED      | Error: unsupported flag
# CDR, tHV are supported
OperationMode           LeadingColour | Error: unsupported flag
# see OLP Documentation
ModelFile      FavouriteModel.slh | Error: file not found
# Modelfile is called: SM.slh
SubdivideSubprocess yes           | Error: unsupported flag
# no is supported
CorrectionType          EW        | Error: unsupported flag
# QCD is supported
MyWayOfDoingThings true          | Error: unknown option

# g g -> t tbar g
21 21 -> 6 -6 21    | Error: massive quarks not supported
# u ubar -> t tbar g
2 -2 -> 6 -6 21    | Error: process not available
# u g -> t tbar u
2 21 ->> 6 -6 2     | Error: check syntax
```

In Memoriam

Dedicated to Thomas Binoth
(Les Houches 2009)



Warning ! Used by Thomas often at LH09

“NLO tools are not DAUs...” (Stefan Dittmaier)

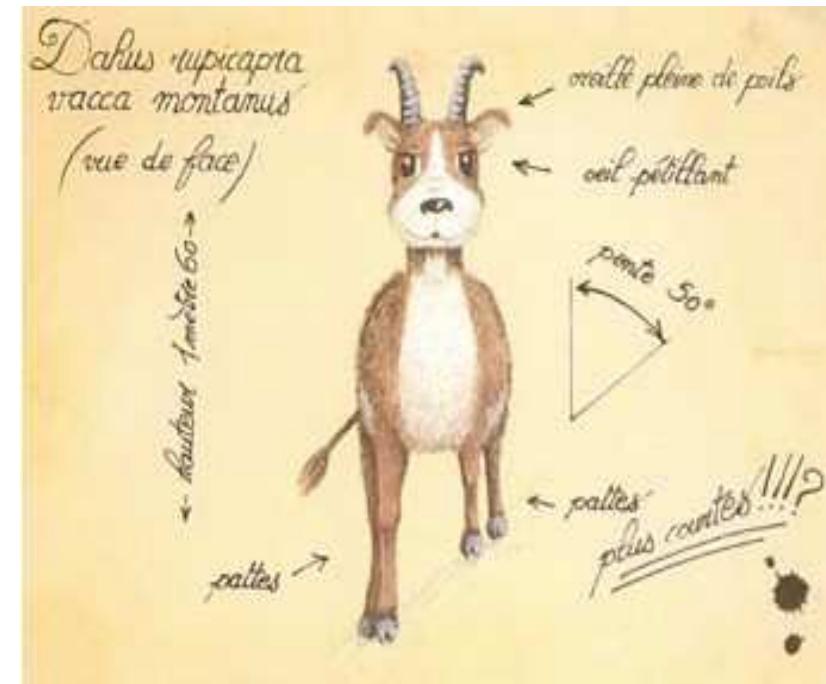
DAU= dümmst anzumehmender user = most imaginable
ignorant user

Warning ! Used by Thomas often at LH09

“NLO tools are not DAUs...” (Stefan Dittmaier)

DAU= dümmst anzumehmender user = most imaginable
ignorant user

A Dahu, quoi..! as I told him for a multi-leg alpine (?) animal...



Chuss Thomas!



Les Houches 2011

ÉCOLE DE PHYSIQUE - LES HOCHES



Workshop
PHYSICS at TeV COLLIDERS
les Houches, France, May 30 - June 17, 2011

International Organizing Committee

- N. ARKANI-HAMED (Princeton, USA)
- E. BOOS (Moscow State Univ., Russia)
- S. CATANI (Florence, Italy)
- S. DAWSON (Brookhaven National Laboratory, USA)
- L. DIXON (SLAC, USA)
- G. GIUDICE (CERN, Switzerland)
- R. GODBOLE (IISc, Bangalore, India)
- J.-F. GRIVAZ (LAL Orsay, France)
- W. HOLLIK (MPI, München, Germany)
- J. HUSTON (Michigan State University, USA)
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AIM AND FORMAT

This Workshop is the seventh in a series whose aim is to bring together theorists and experimentalists working on the physics of the LHC and the Tevatron Collider. In 2011 we will finally have data from the LHC and, therefore, particular attention will be given to (1) the progress in new techniques for the simulation of Standard Model processes and (2) the latest developments concerning new mechanisms of electroweak symmetry breaking and other physics beyond the Standard Model. The Workshop will address how best to exploit new data from the colliders as well as how to prepare for future data. Another issue will address the physics potential of a higher energy LHC. These activities will be conducted in close connection with the development and improvement of tools, in particular, of Monte-Carlo event generators. The meeting in Les Houches is the culmination of this year-long Workshop.

Les Houches is a village located in the Chamonix valley, in the French Alps. Established in 1951, the Physics School is situated at 1150 m above sea level in natural surroundings, with breathtaking views on the Mont-Blanc range. Les Houches Physics School is affiliated with the Université Joseph Fourier Grenoble I (UJF). It is a joint interuniversity facility of UJF and Grenoble-INP, and is supported by the UJF, the Centre National de la Recherche Scientifique (CNRS) and the Direction des Sciences de la Matière du Commissariat à l'Energie Atomique (CEA/DSM).
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