Hadron-Hadron interactions and physics of neutron stars

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Overview

- Equation of state of dense nuclear matter as possibly present inside neutron stars can can needs as input two- and three-body interactions.
- If we consider that neutron stars can contain nucleon and hyperons, it is hence necessary to study the hyperon-nucleon and hyperon-hyperon interactions.
- Today we learn about two-body scattering and femtoscopy at the LHC as tool to study two-body interactions including hyperons and nucleons.

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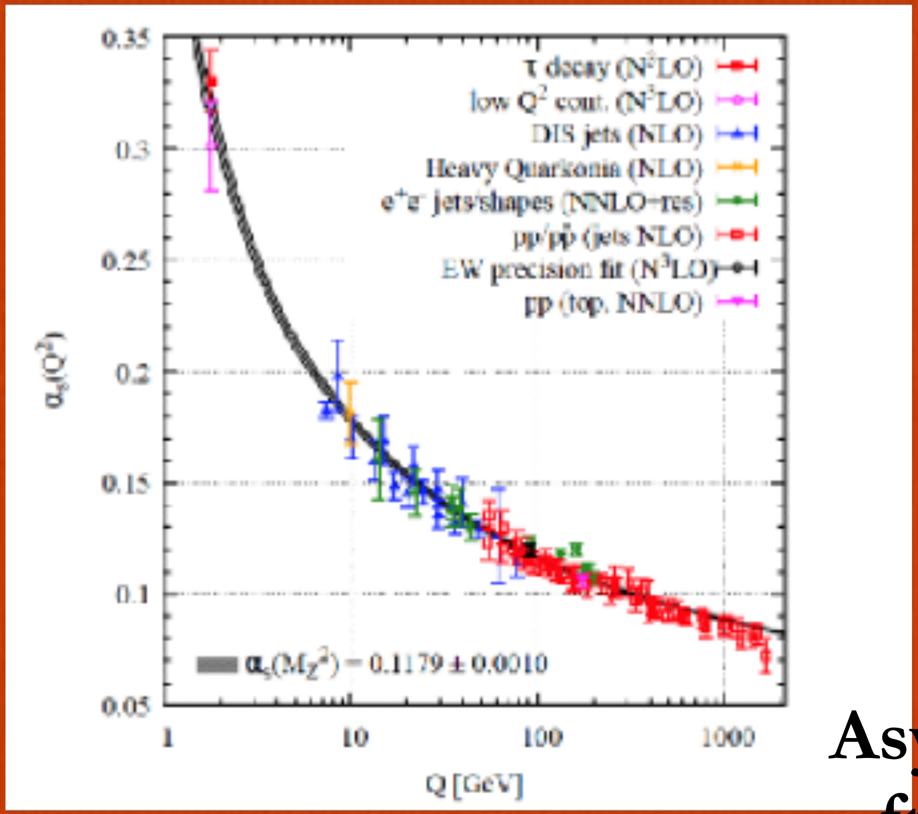
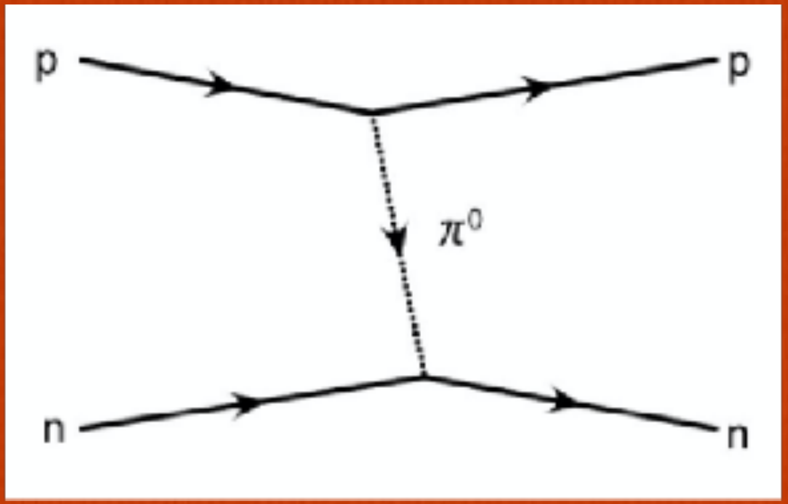
Residual strong interaction among hadrons

Confinement



Asymptotic freedom



Running coupling constant defines the boundaries of low-energy QCD

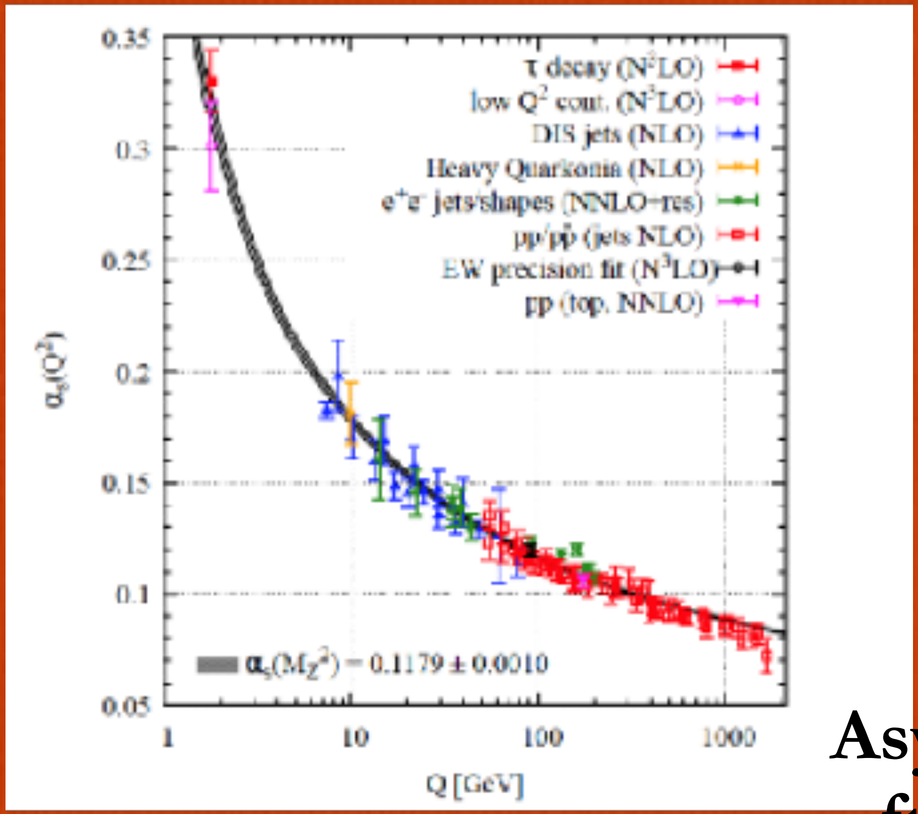
→ Q ~1 GeV, R ~ 1 fm

→ No perturbative methods are applicable

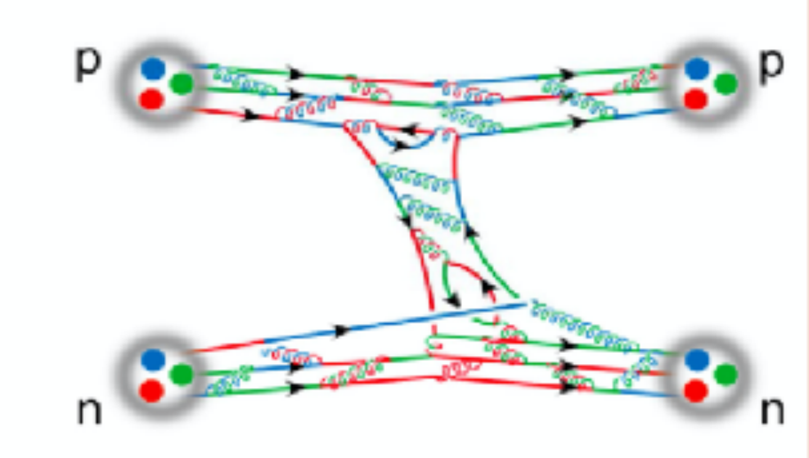
→ Effective theories with hadrons as degrees of freedom constrained to experimental data

Residual strong interaction among hadrons

Confinement



Asymptotic freedom



Running coupling constant defines the boundaries of low-energy QCD

→ Q ~1 GeV, R ~ 1 fm

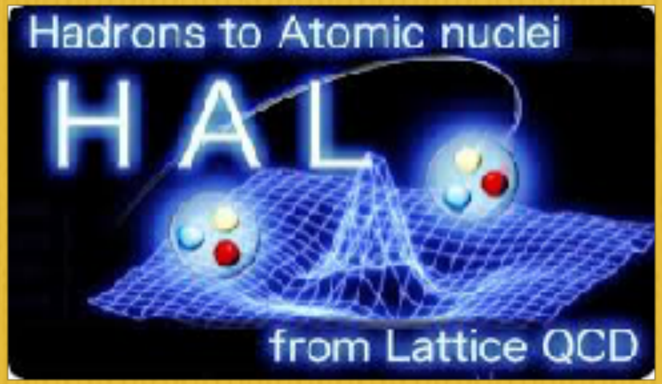
→ No perturbative methods are applicable

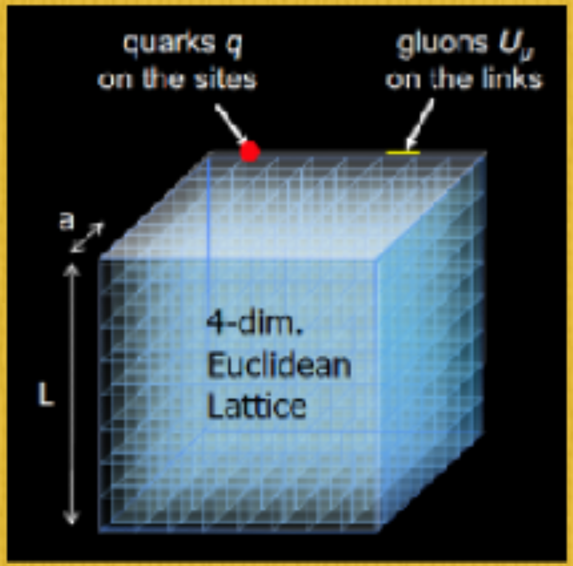
→ Effective theories with hadrons as degrees of freedom constrained to experimental data

→ Next Step: Understanding of the interaction starting from quark and gluons

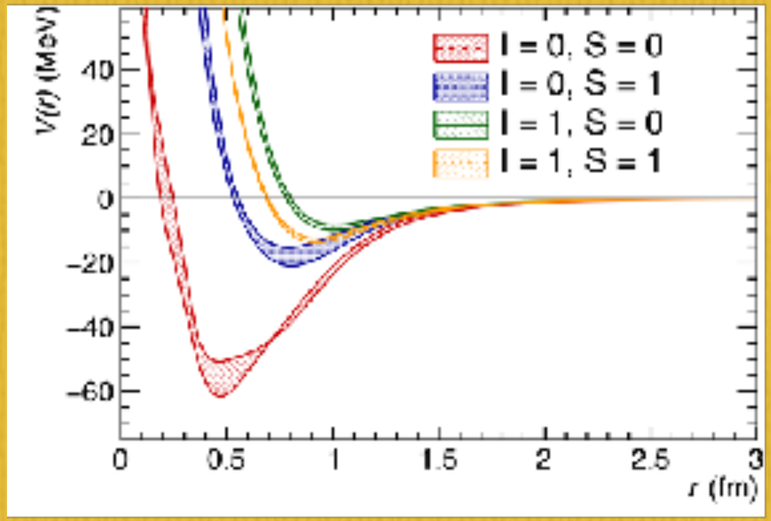
Lattice calculations for hyperons interactions

Numerical method to exrtact the hadron-hadron interactions starting from gluons and quarks as degrees of freedom





Local potentials for the Nucleon-Ξ interactions

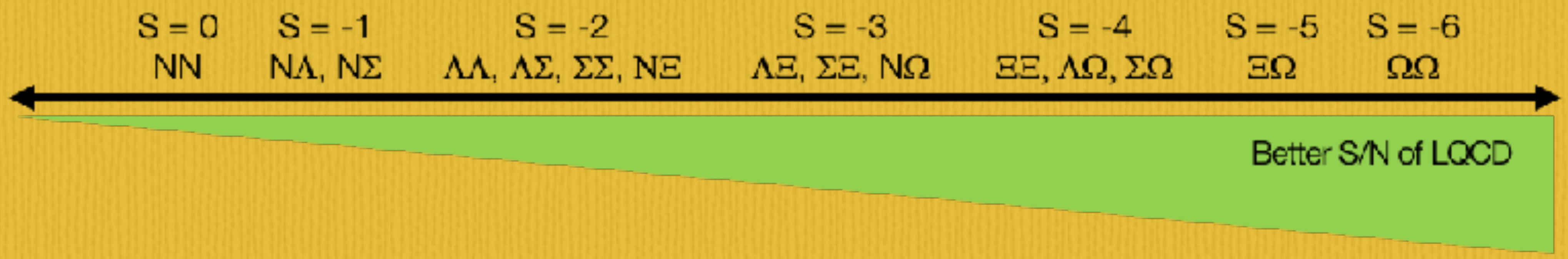


HAL QCD Coll., Nucl.Phys.A 998 (2020) 121737

T. Hatsuda, K. Sasaki et al.

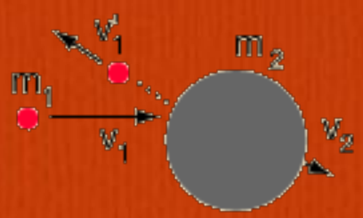
HAL QCD Coll., PLB 792 284-289 (2019)
HAL QCD Coll., Nucl.Phys.A 998 (2020) 121737
HAL QCD Coll. Phys. Rev.D 99 (2019) 1, 014514

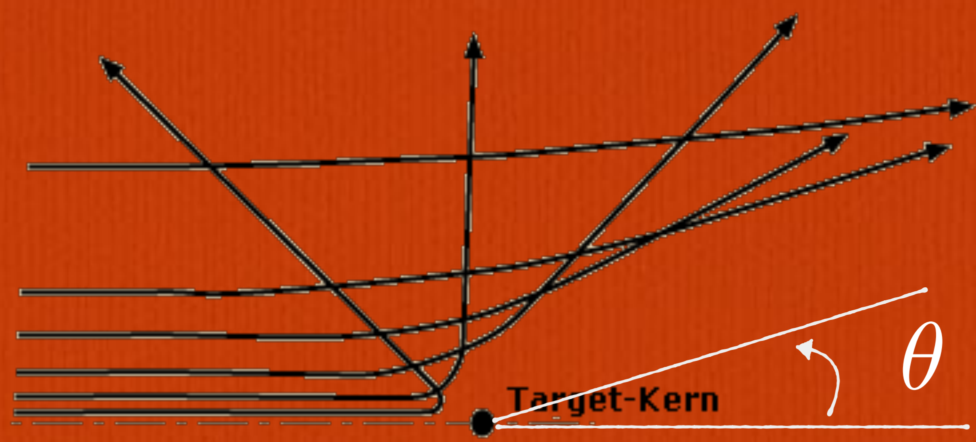
$$a = 0.085 \text{ fm}$$
$$L = 8.1 \text{ fm}$$
$$m_\pi = 146 \text{ MeV}/c^2$$
$$m_K = 525 \text{ MeV}/c^2$$



Scattering Data and Interaction Parameters

Scattering experiments -> Extraction fo the differential cross section





$$\longrightarrow \quad \frac{d\sigma}{d\Omega}$$

Expansion in partial waves:

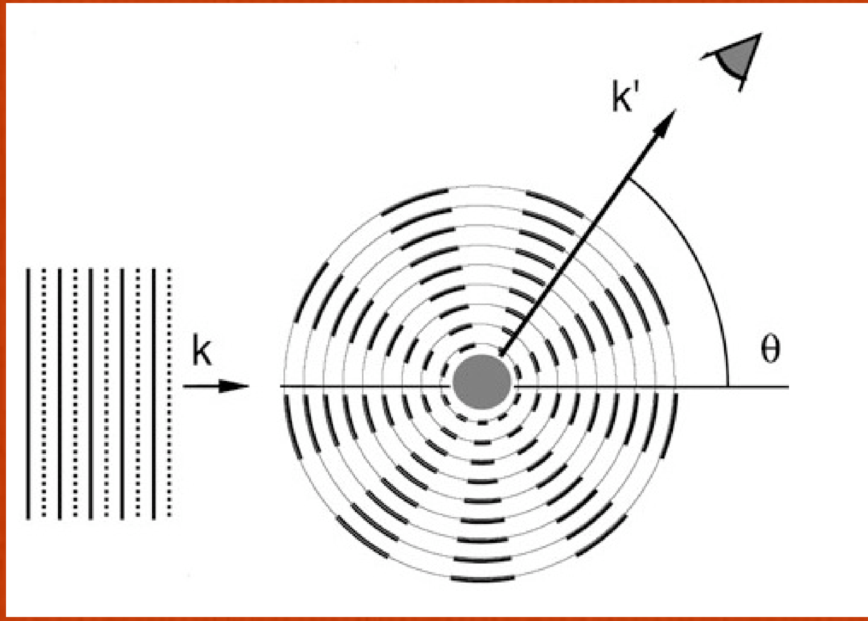
$$\sigma = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2(\delta_l).$$

What are these shifts?

Partial Wave Decomposition and Shifts

If we set, $\psi(\mathbf{r}) \simeq e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta)\frac{e^{ikr}}{r}$

$$f(\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)f_\ell(k)P_\ell(\cos\theta)$$



$$f_l(k) = \frac{e^{2i\delta_l(k)}-1}{2ik}$$

$\delta_\ell(k)$ **Phase Shifts**

$f(\theta)$ of the scattered wave clearly depends on the interacting potential between beam and target.
By measuring the scattering cross-section one can infer on the scattering parameters and determine the interaction

Determination of the phase shifts

How to determine δ_l ?

$$\psi(\mathbf{r}) = \sum_{\ell=0}^{\infty} R_\ell(r) P_\ell(\cos\theta)$$

Expansion in Legendre Polynomials for the wave function and the scattering amplitude f(θ)

$$\left[\partial_r^2 - U(r) + k^2\right] u(r) = 0$$

$$\begin{aligned} u(a_0) &= \sin(ka_0 + \delta_0) = \sin(ka_0)\cos\delta_0 + \cos(ka_0)\sin\delta_0 \\ &= \sin\delta_0\,[\cot\delta_0 \sin(ka_0) + \cos(kr)] \simeq \sin\delta_0\,[ka_0\cot\delta_0 + 1] \end{aligned}$$

$$a_0 = -\lim_{k\to 0} \frac{1}{k}\tan\delta_0(k).$$

For l=0

Scattering Length

$$a_0 = -\lim_{k\to 0} \frac{1}{k} \tan \delta_0(k).$$

l=0 -> s-wave only!!

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2} \sin^2 \delta_0(k) \overset{k\to 0}{\simeq} \frac{4\pi}{k^2} \frac{(ka_0)^2}{1+(ka_0)^2} \simeq 4\pi a_0^2$$

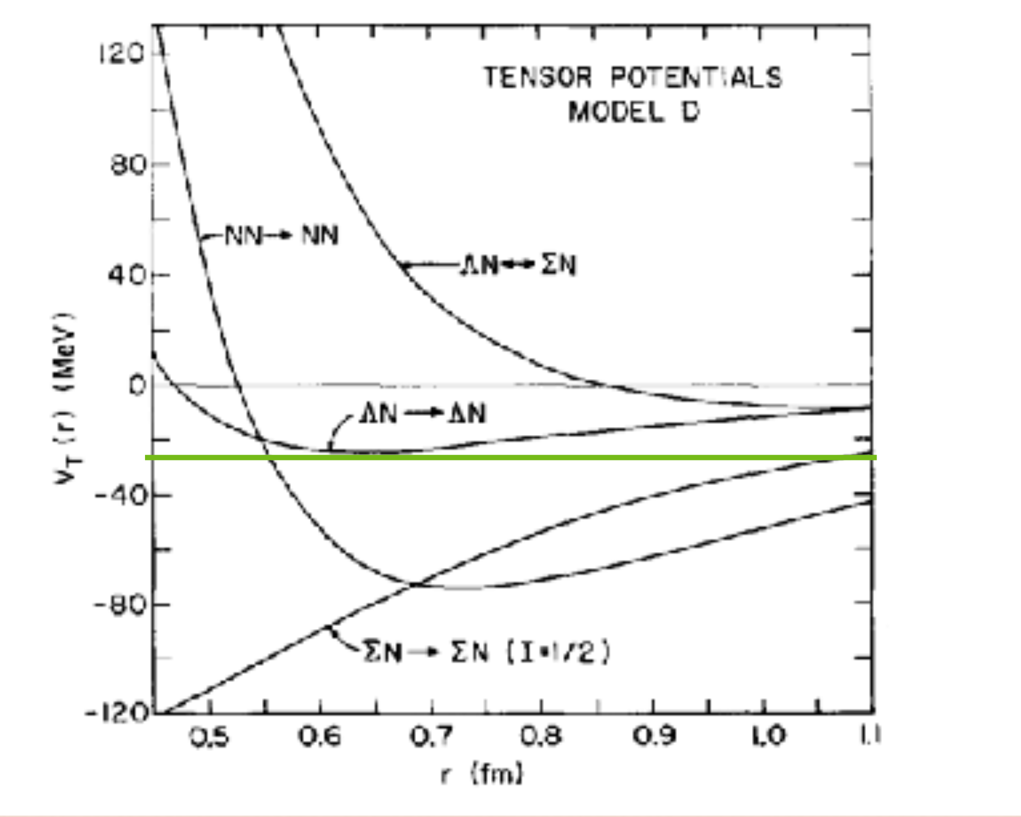
The scattering length charachterizes the **EFFECTIVE** size of the target

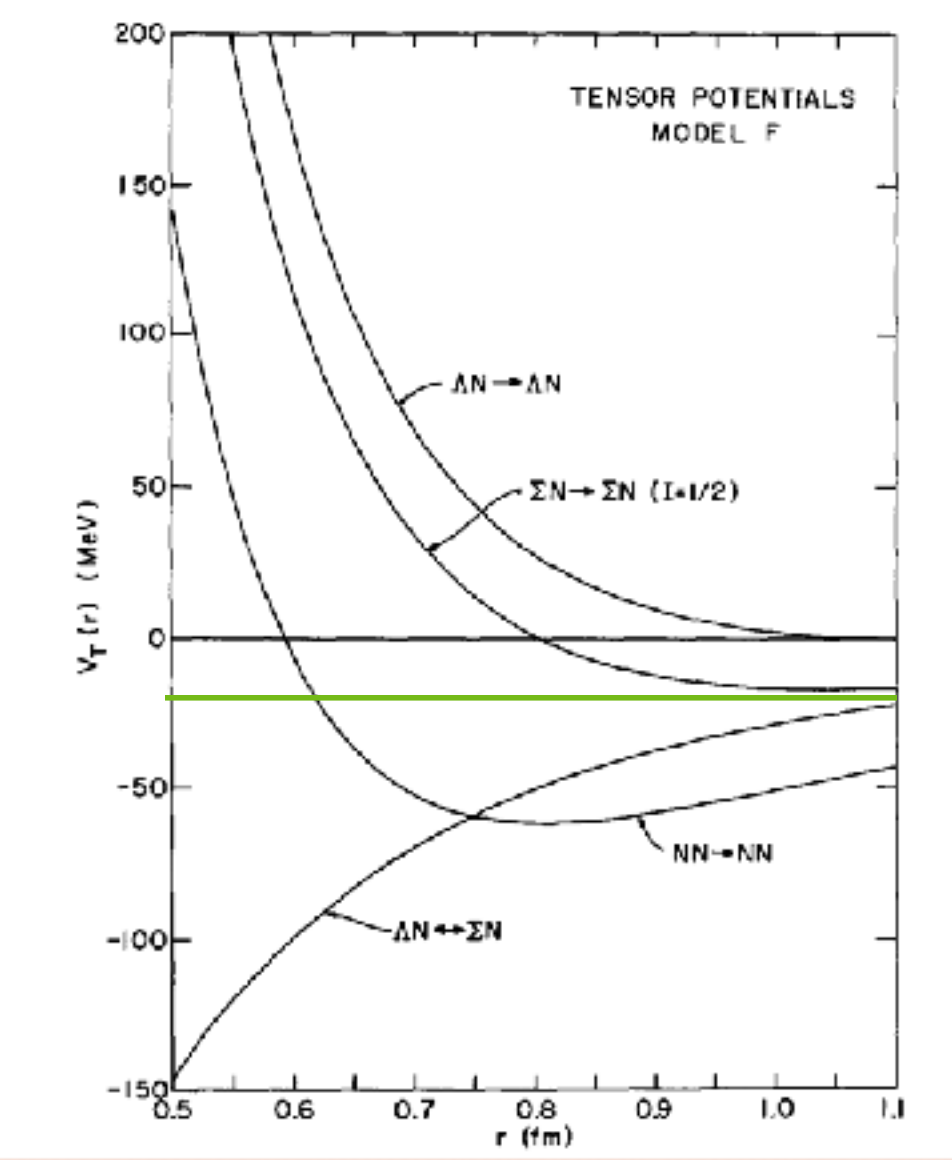
Effective range d_0 is used to define the range of the interaction

If we know U(r) we can solve the Schrödinger equation and determine the scattering parameters and compare this to the scattering data to see if it works :)
This is a simple way of treating the problem with a local potential that depends only from the distance between the two particles

Comparison of the N-N and Y-N Interactions

Progress in particle and nuclear Physics
Vol. 12, 1984 Pages 171-239





U(r) -> plug it in the Schrödinger equation -> solve it -> extract scattering parameters -> calculate cross-sections and compare to scattering data

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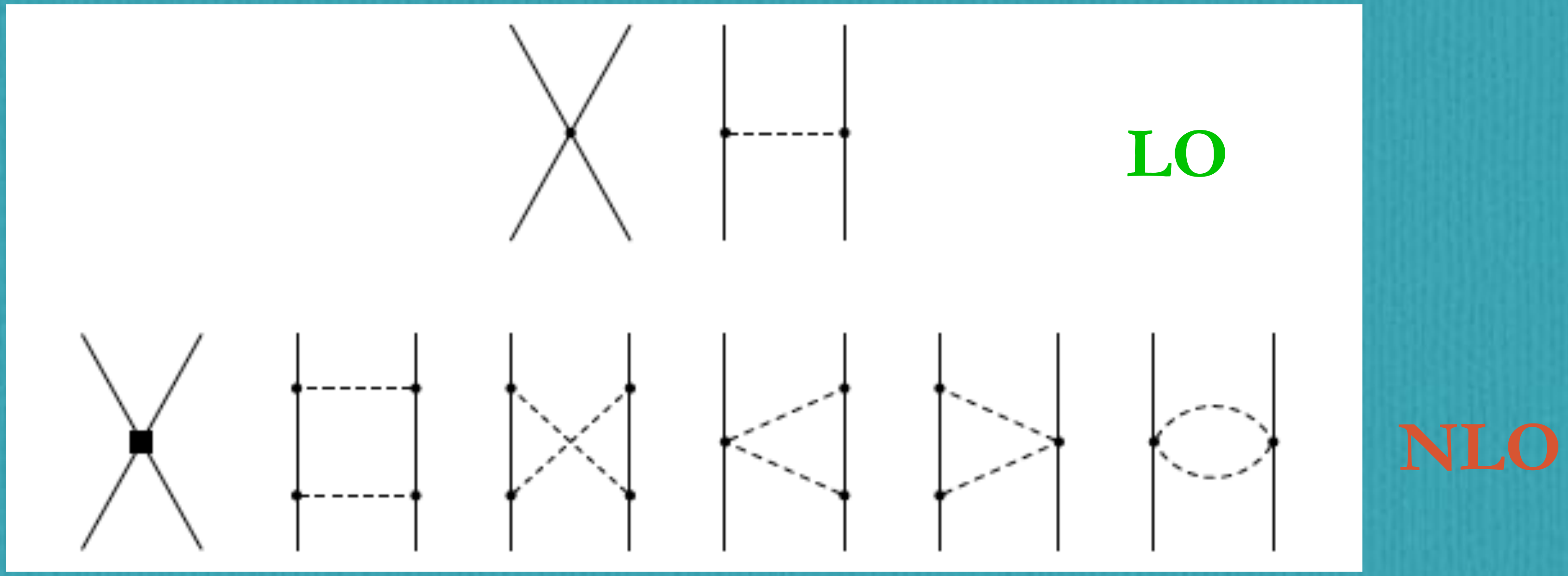
Calculation for Hyperon-Nucleon Scattering

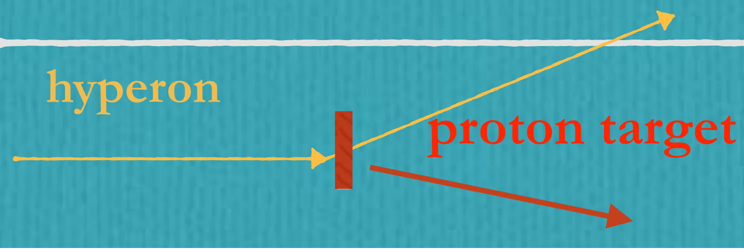
- Baryon-baryon interaction in SU(3) EFT à la Weinberg (1990)
- Advantages:
  - Power counting
  - systematic improvement by going to higher order
  - Possibility to derive two- and three baryon forces and external current operators in a consistent way

- degrees of freedom: baryon octet, pseudoscalar Goldstone boson octet
  - pseudoscalar-meson exchanges
  - contact terms – represent unresolved short-distance dynamics

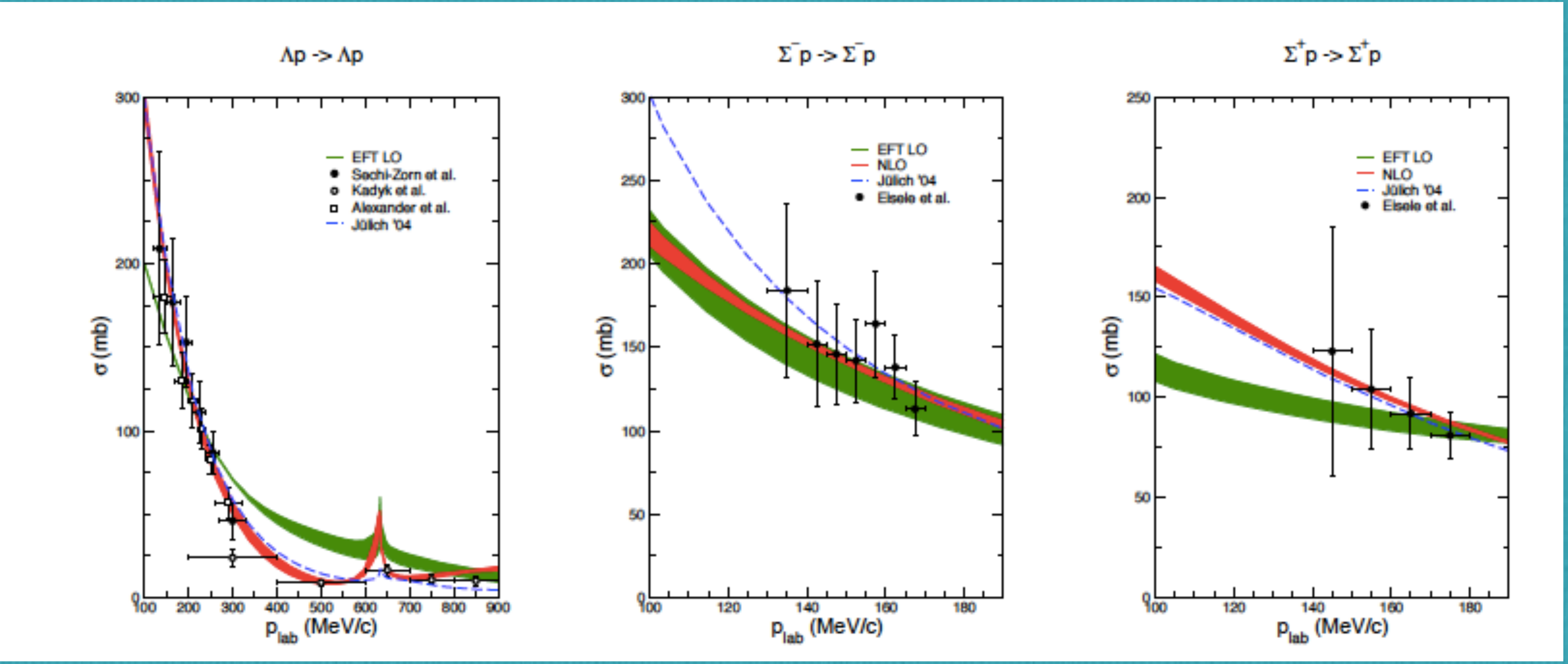


LO: H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244
NLO: J.H., N. Kaiser, U. Meißner, A. Nogga, S. Petschauer, W. Weise, NPA 915 (2013) 24

Hyperon-nucleon scattering results



LO: H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244
NLO: J.H., N. Kaiser, et al., NPA 915 (2013) 24
Jülich '04: J.H., U.-G. Meißner, PRC 72 (2005) 044005



LO

$$a^1S_0 = -1.91fm \quad d^1S_0 = 1.40fm$$
$$a^3S_1 = -1.23fm \quad d^3S_1 = 2.13fm$$

NLO

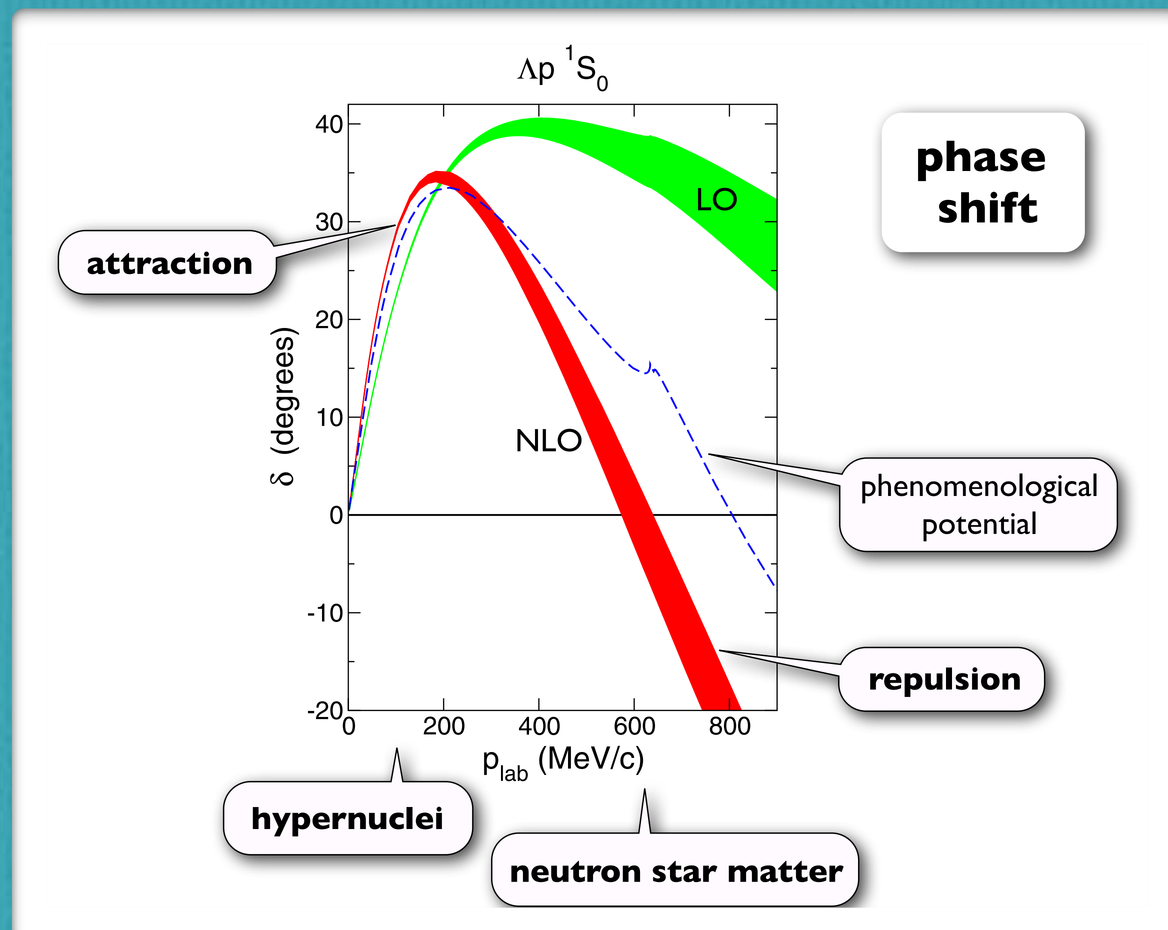
$$a^1S_0 = -2.91fm \quad d^1S_0 = 2.78fm$$
$$a^3S_1 = -1.54fm \quad d^3S_1 = 2.72fm$$

S= 1 S=0

$\mathcal{L}$

More about the interaction

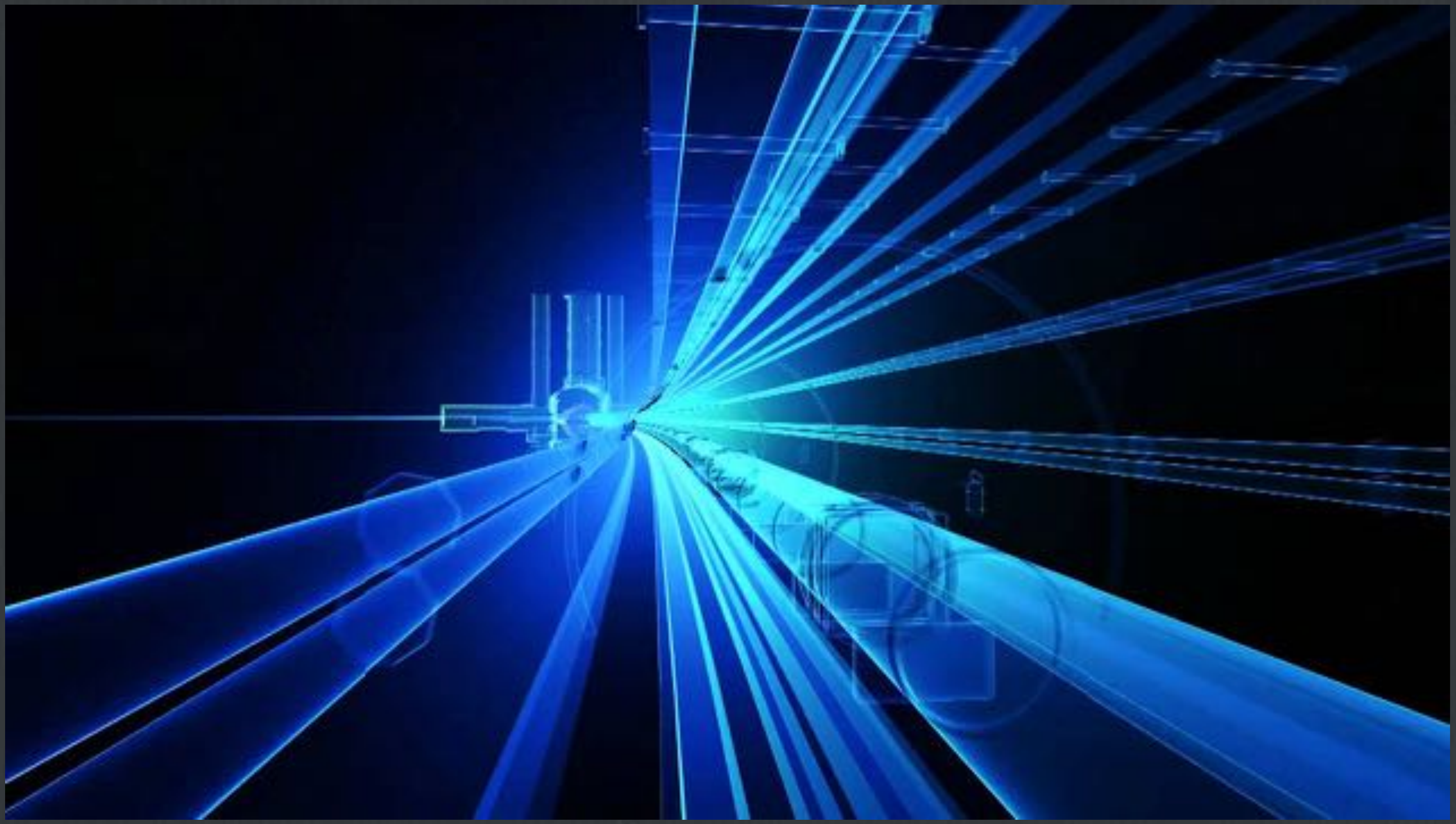
J. Haidenbauer, S. Petschauer et al.,
Nucl. Phys. A 915 (2013) 24



It all depends upon the Λ-N and Λ-NN interaction and whether or not it has a repulsive core
This repulsive core could stiffen again the EOS allowing for heavy neutron stars
Scattering data for hyperon-nucleon are very scarce!
Which other data can constrain the theory?

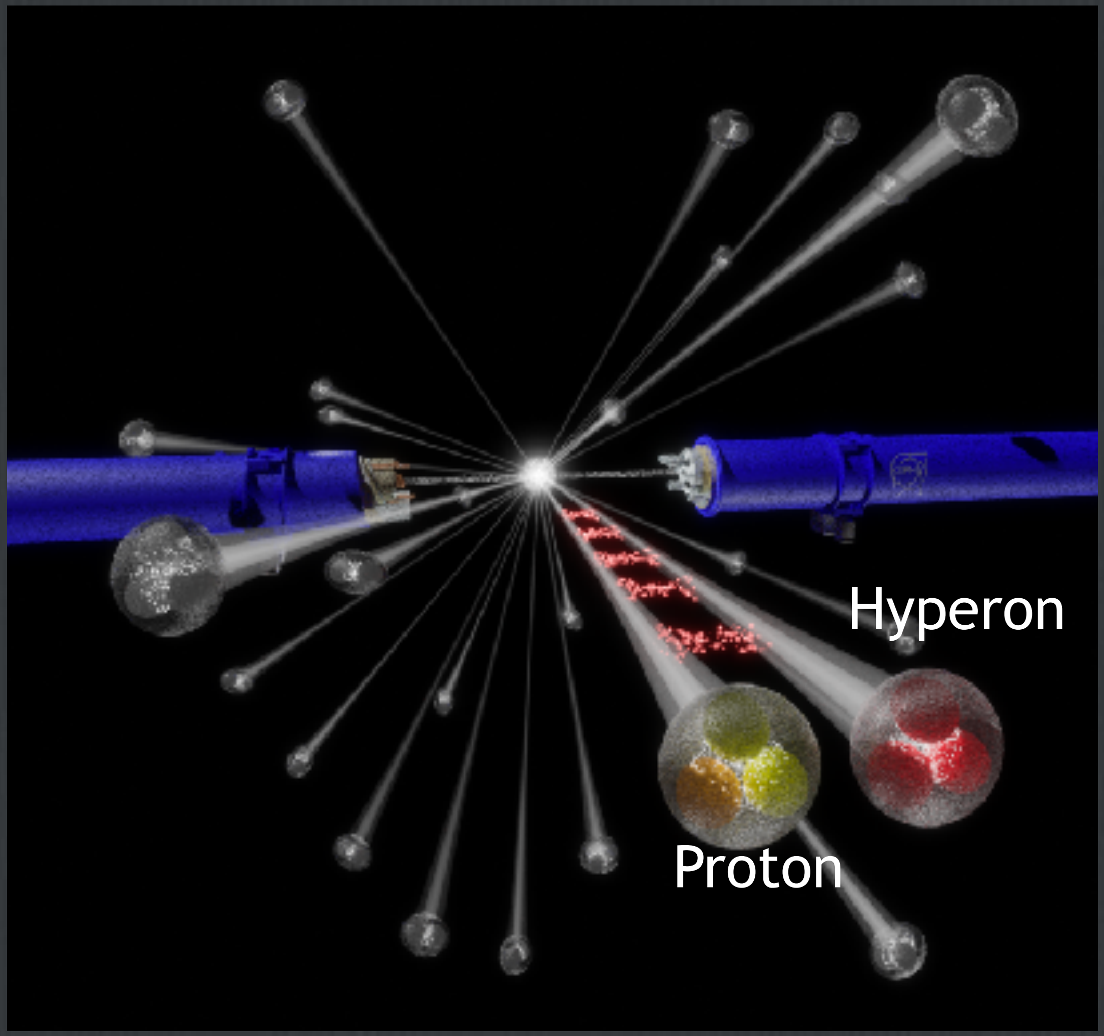
How can we solve this puzzle?

Large Hadron Collider LHC
The largest and fastest accelerator in the world



Particle production and decays

Courtesy D. Chinellato



The energy of the accelerated protons is (partly) converted into mass.

$$E = mc^2$$

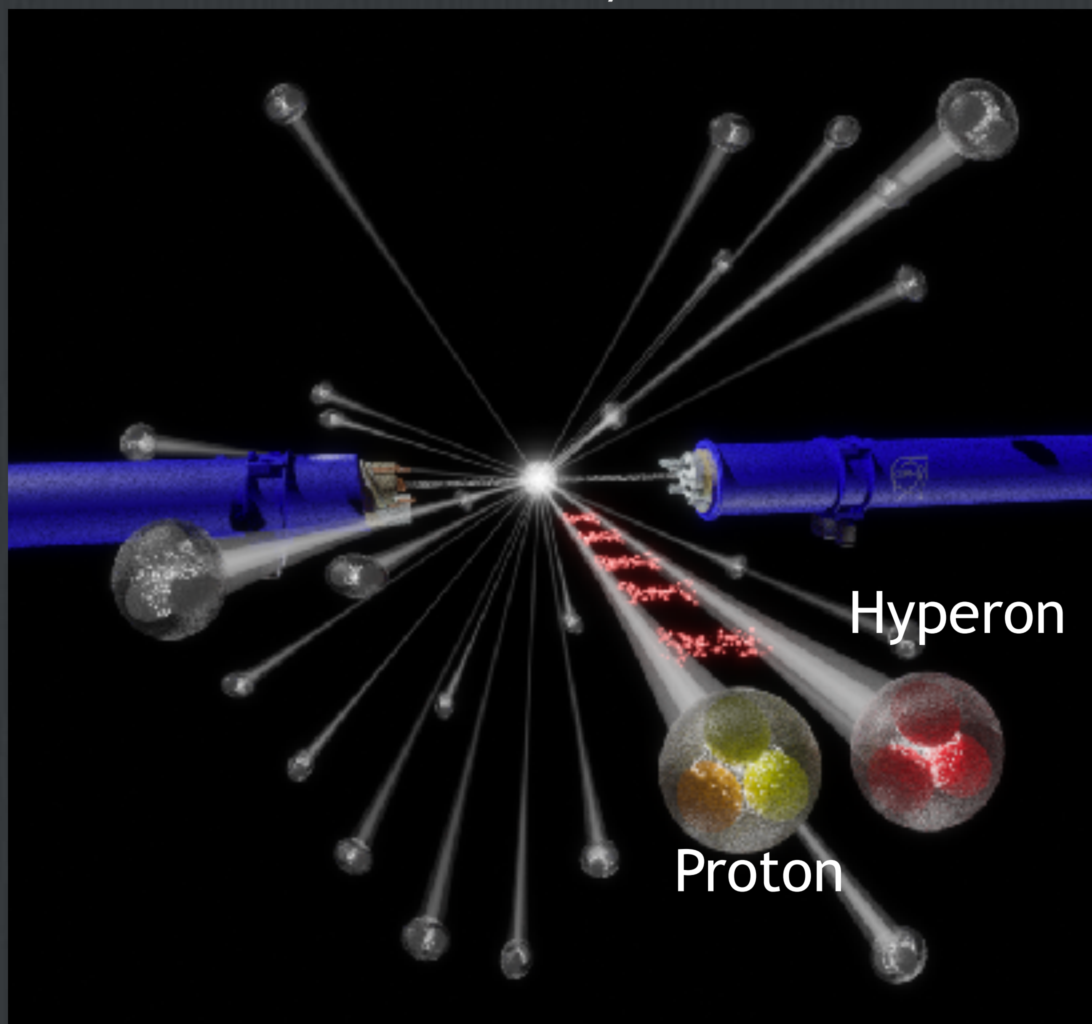
20-50 new particles are created from each collision.
Protons are stable, hyperons decay and the daughter particles are measured.

In general:
The trajectory, velocity and mass of each charged particle must be measured!!!

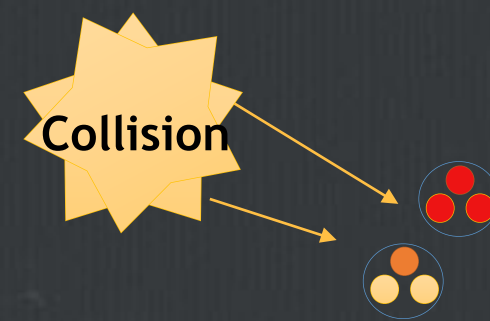
1000 'pictures' per second!

How can we measure the interaction?

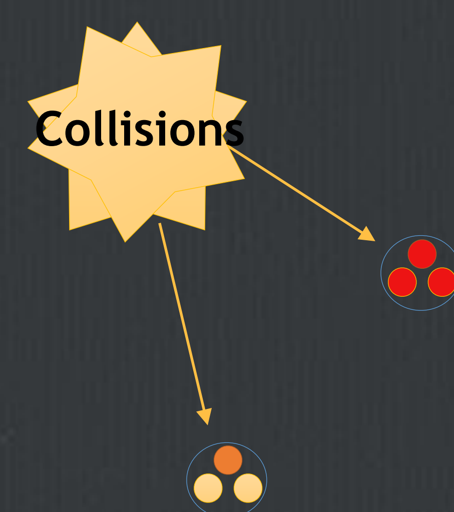
Courtesy D. Chinellato



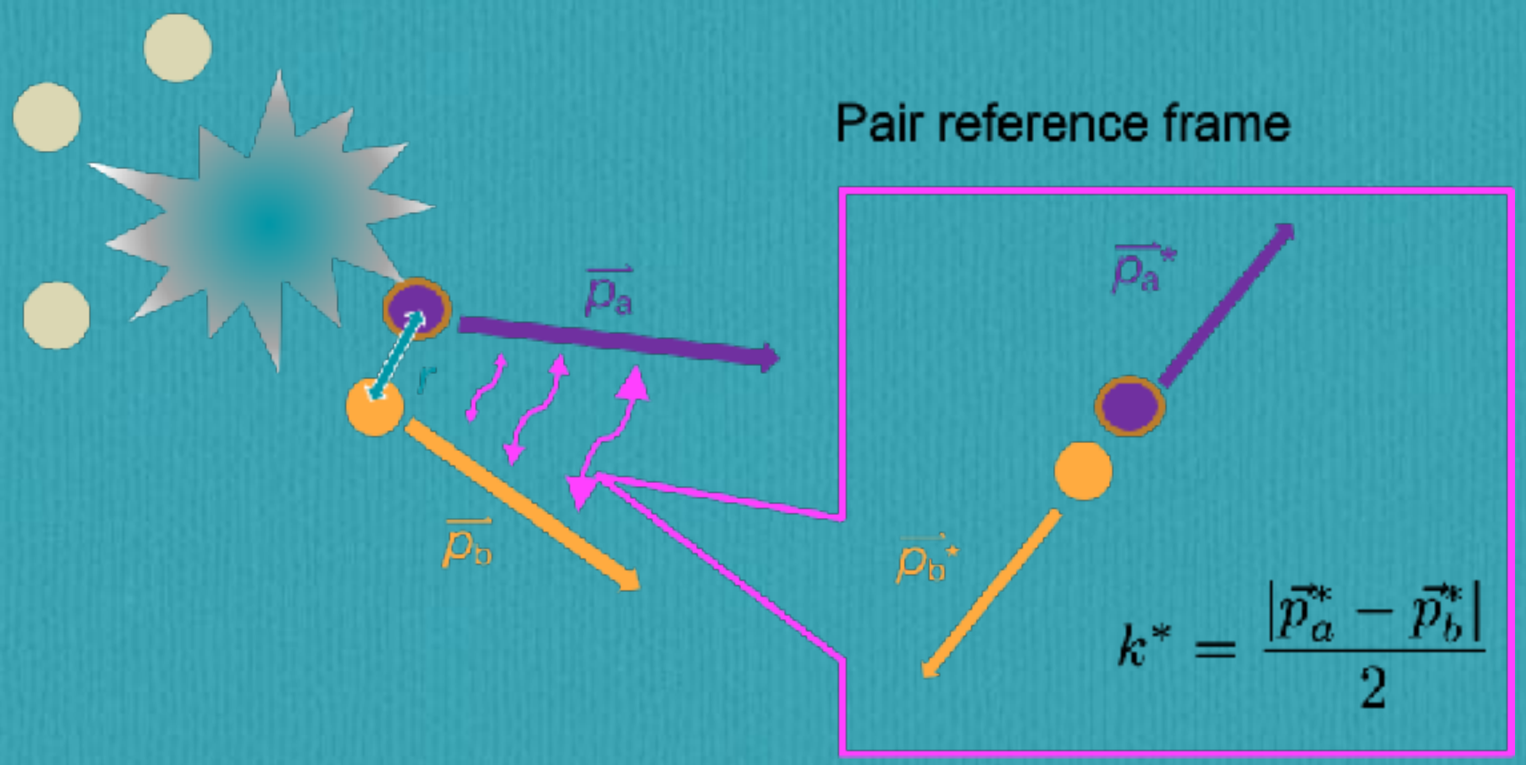
Attractive interaction



Repulsive interaction



Potentials and Correlation Functions



Schrödinger Equation:

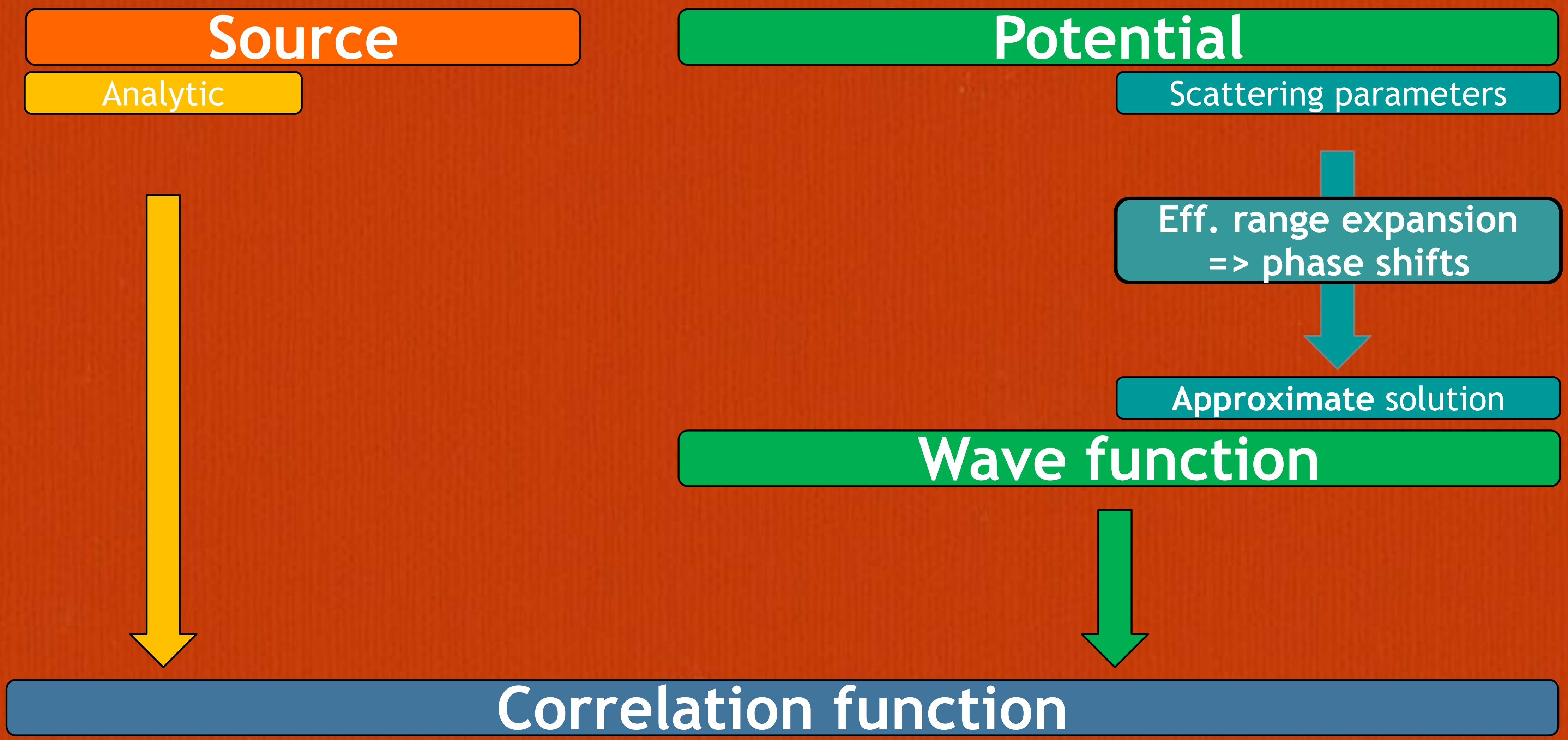
V(r) -> $|\psi(\vec{k}^*,\vec{r})|^2$ relative wave function for the pair

$$C(k^*) = \int S(r)\, |\psi(\vec{k}^*,\vec{r})|^2\, d^3r = \zeta(k^*) \cdot \frac{N_{same}(k^*)}{N_{mixed}(k^*)}$$

Emission source   Two-particle wave function

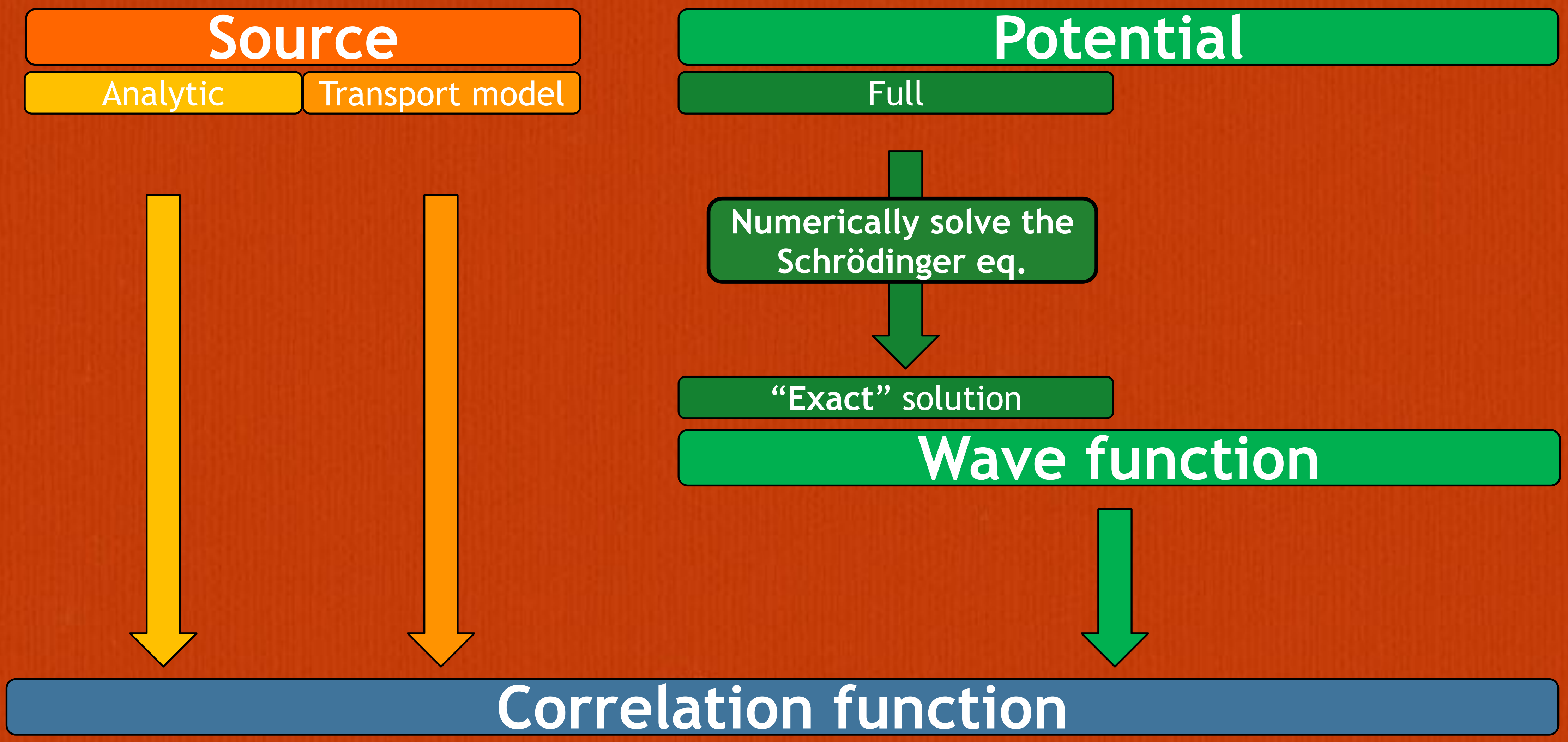
S. E. Koonin et al. PLB 70 (1977)

Lednicky


Source
Analytic
Potential
Scattering parameters
Eff. range expansion
=> phase shifts
Approximate solution
Wave function
Correlation function

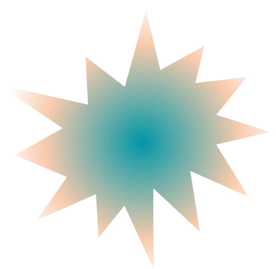
CATS (Correlation Analysis Tools using the Schrödinger equation)

D. Mihaylov, L. Fabbietti et al. EPJC 78 (2018)

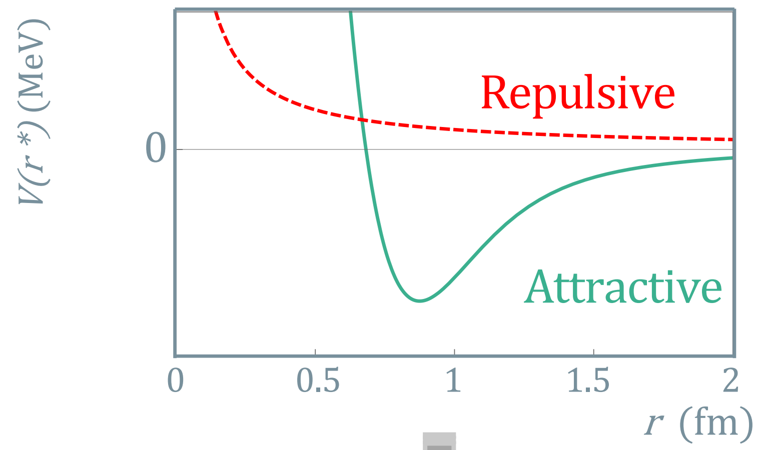


Potentials and Correlation Functions (CATS)

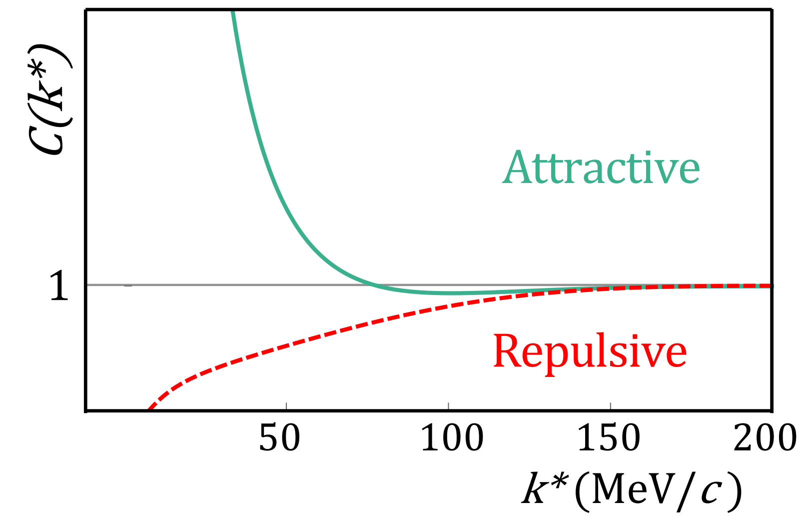
Source parametrisation



Interacting potential



Correlation function



Gaussian source

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot exp\left(-\frac{r^2}{4r_0^2}\right)$$

Schrödinger equation**

Two-particle wave function$|\Psi(k^*, r)|$

**CATS (Correlation Analysis Tool using the Schödinger equation)

D. Mihaylov et al. EPJC 78 (2018)

$$C(k^*) = \int S(r)\ \left|\psi(\vec{k}^*, \vec{r})\right|^2 d^3r$$

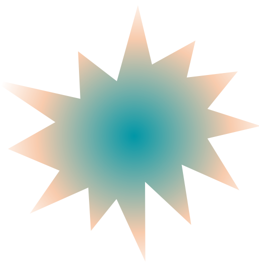
Emission source

Two-particle wave function

>1 if the interaction is attractive
= 1 if there is no interaction
<1 if the interaction is repulsive

Potentials and Correlation Functions (LL)

Source parametrisation

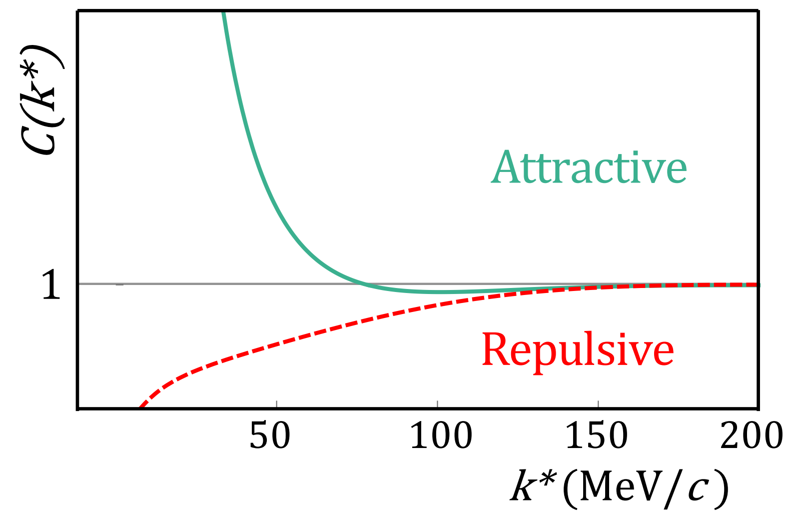


Scattering parameters**

S = spin state
d_0^S = effective range
f_0^S = scattering length

$$f(k^*)^S = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^*\right)^{-1}$$

Correlation function



Gaussian source

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot exp\left(-\frac{r^2}{4r_0^2}\right)$$

**R. Lednicky and V. L. Lyuboshits Sov. J. Nucl. Phys. 35 (1982)

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f(k^*)^S}{r_0}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0}\right) + \frac{2\Re f(k^*)^S}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{2\Im f(k^*)^S}{\sqrt{\pi} r_0} F_2(2k^* r_0\right.$$

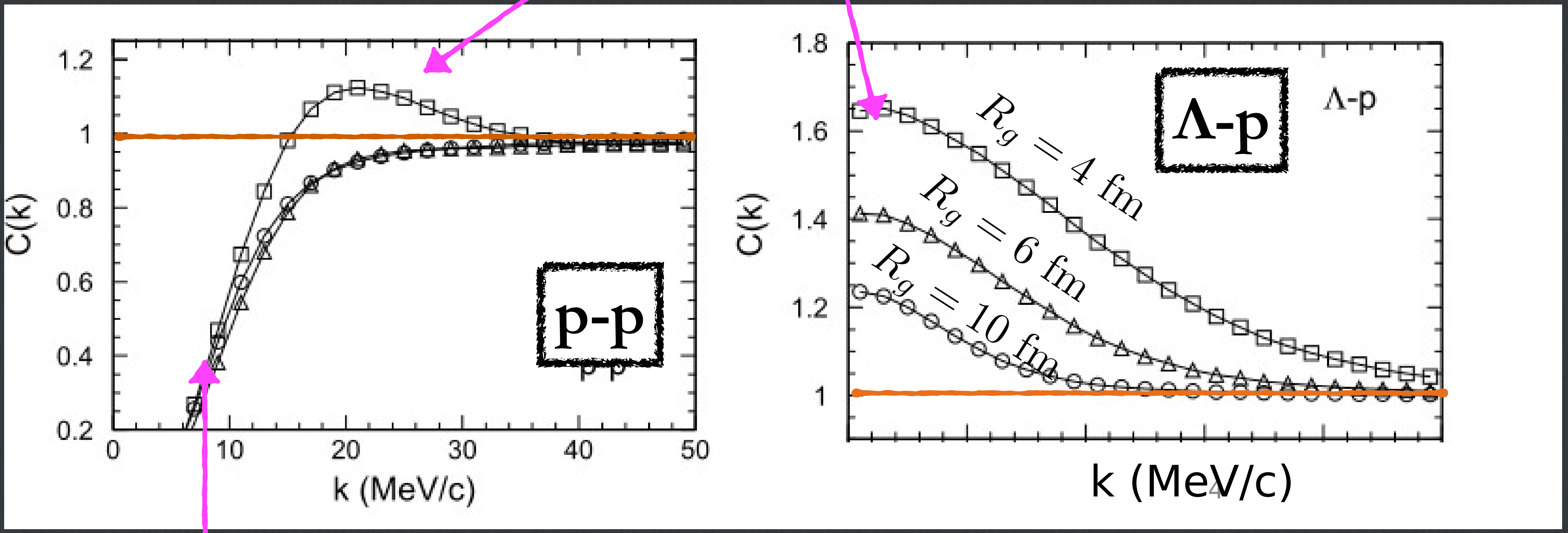
Based on effective range expansion, works well for large sources

Some correlations examples

Examples of Correlations from Calculations

F. Wang and S. Pratt, Phys. Rev. Lett. 83, 3138 (1999).

Strong Attraction C(k)>1



Coulomb Repulsion C(k)<1

Scattering parameters and Correlation Functions (LL model)

Lednicky-Lyuboshitz Sov. J. Nucl. Phys. A 35, 770 (1982)

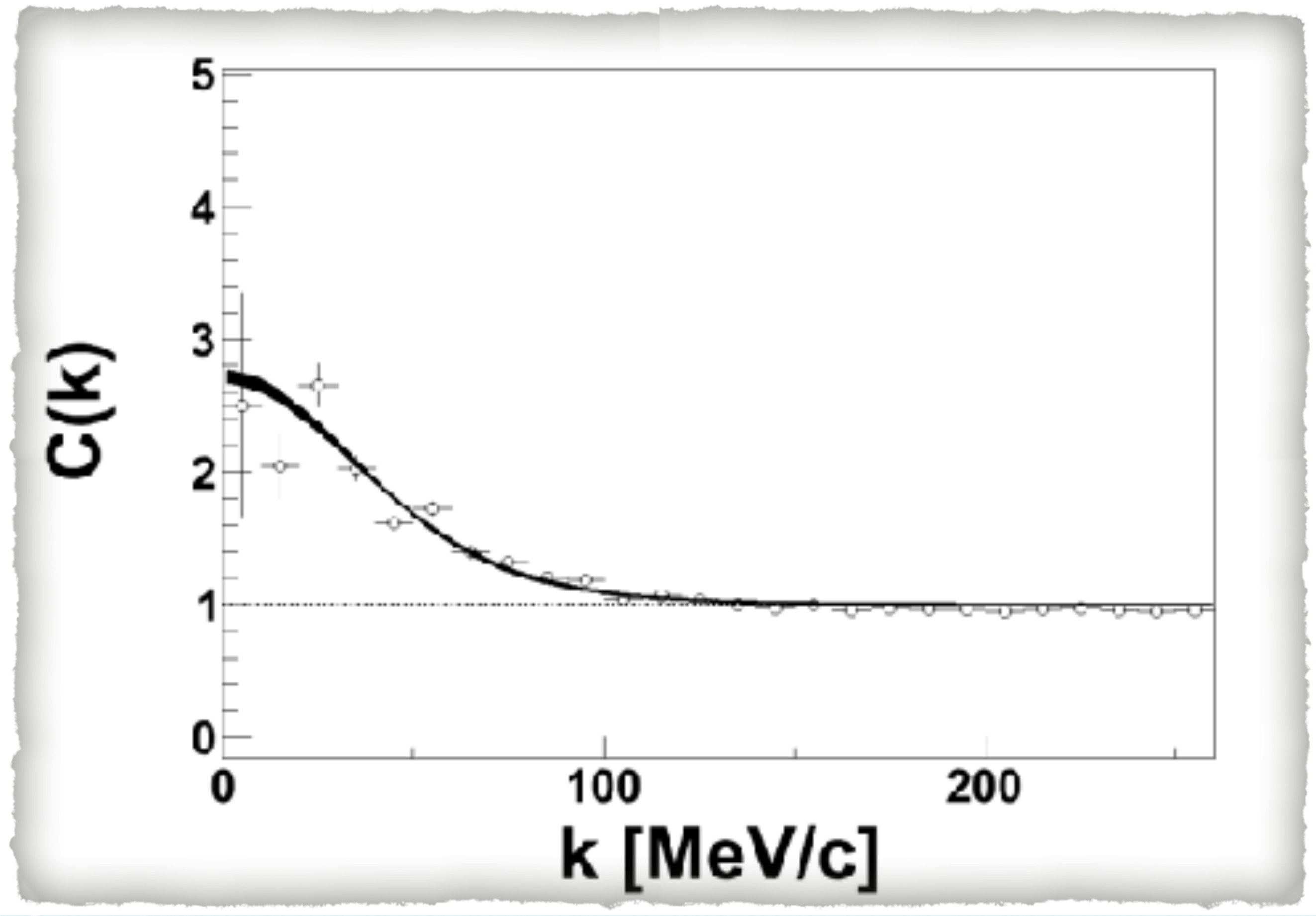
$$C(k) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k)}{r_0} \right|^2 \frac{2\mathcal{R} f^S(k)}{\sqrt{\pi} r_0} F_1(Qr_0) - \frac{\mathcal{I} f^S(k)}{r_0} F_2(Qr_0) \right]$$

Sum over all spin configurations

$$f^S(k) = \left(\frac{1}{a_0^S} + \frac{1}{2} d_0^S k^2 - ik \right)^{-1}$$

a_0^S = Scattering length

d_0^S = Scattering range



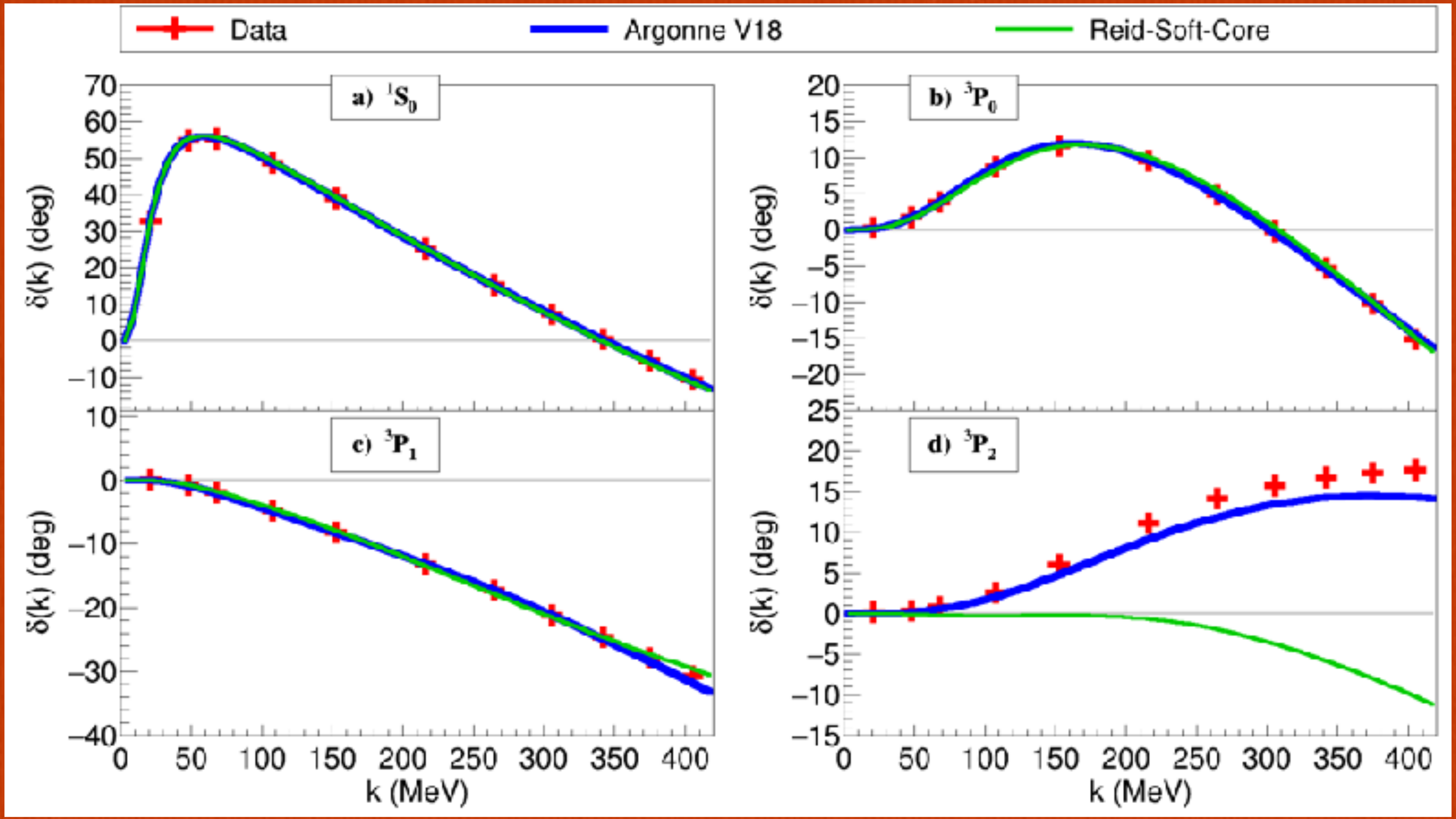
In this analytical formula the Source is assumed to be a Gaussian distribution with width-parameter r_0

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot exp \left(\frac{r^2}{4r_0^2} \right)$$

By **fitting** the measured correlation function one can extract the different parameters.

p-p Interaction

Potentials for the strong interactions tuned to scattering data of NN



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p-p Correlation

pp Pairs:
- Coulomb Interaction
- Strong Interaction (AV18)
- Quantum Statistics for Fermions

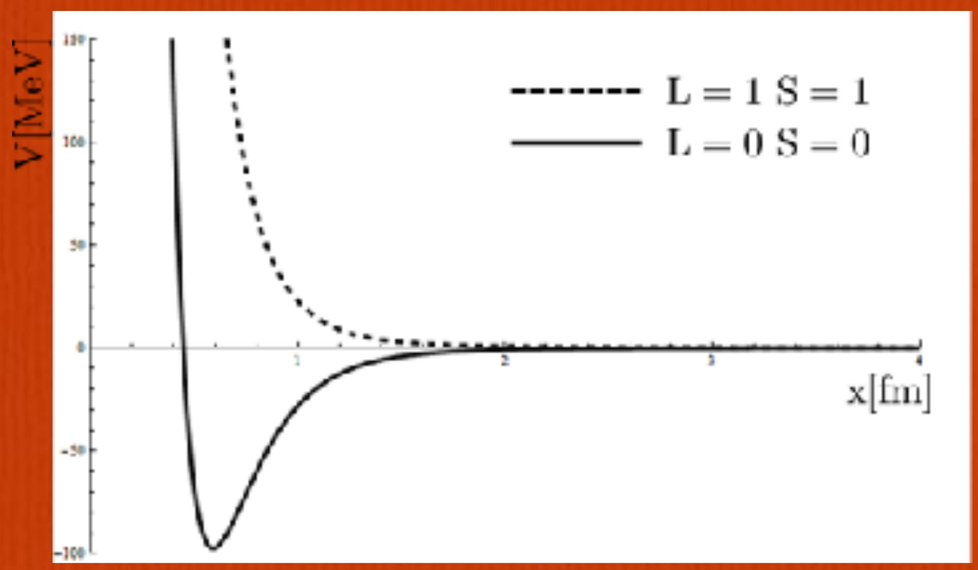
Koonin Fit Function -> Extraction of the Source Radius R_G

S. E. Koonin, Phys. Lett. B 70 (1977) 43
S. Pratt et al., Nucl. Phys. A 566 (1994) 103c

$$C(k) = \int dr^3 \phi_{\mathrm{rel}}^2(r,k) \exp\left(-\frac{r^2}{4R_G^2}\right)$$

ϕ_{rel} from Schroedinger Eq. with Coulomb and Strong interaction

p-p Strong Pot.



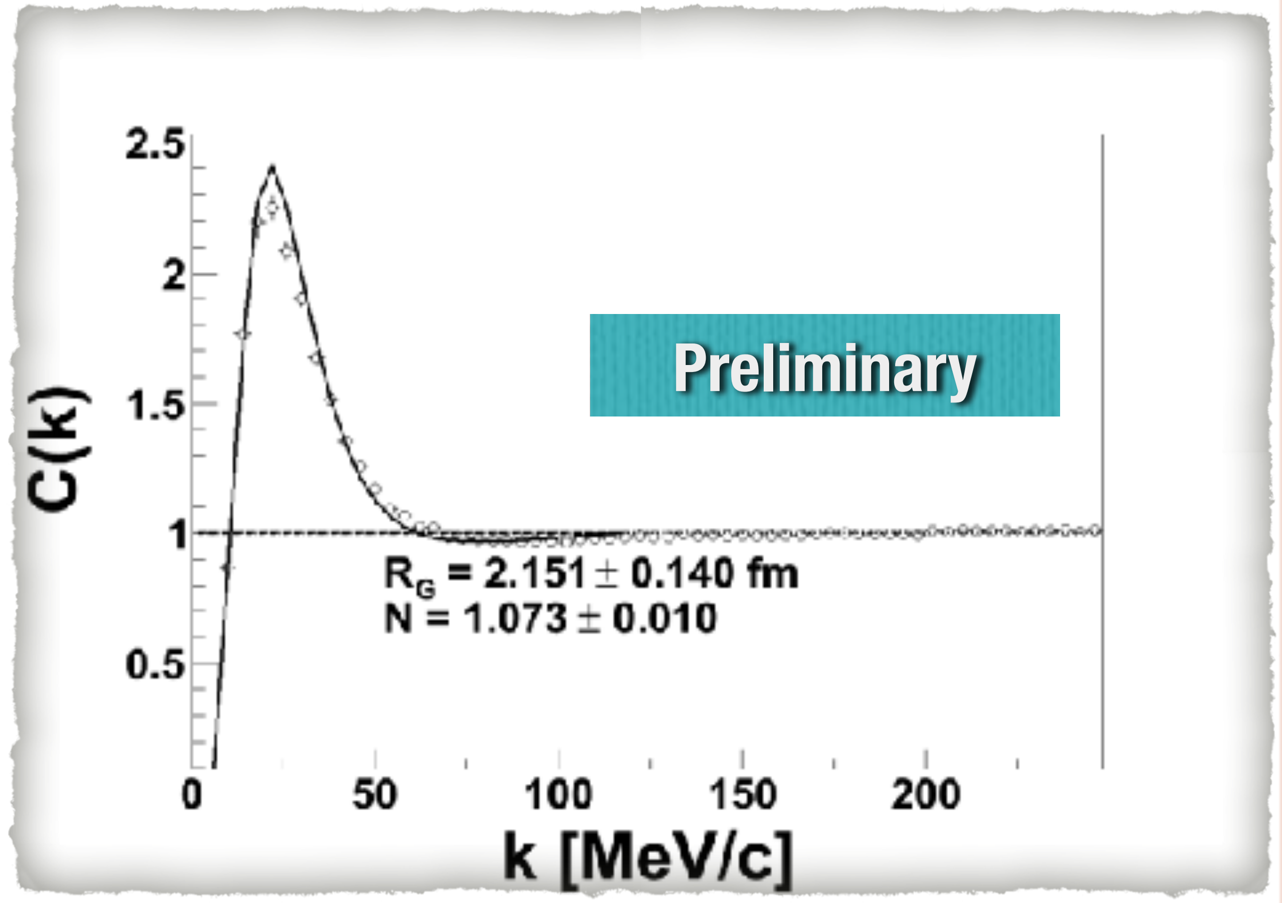
p-p Correlation

p+Nb, 3.5 GeV

Experimental Correlation after:
Close-Tracks rejection
Long-Range Correlation Correction via UrQMD

$$C(k) = \mathcal{N}\frac{N(\mathbf{p}_1,\mathbf{p}_2)_{\text{same}}}{N(\mathbf{p}_1,\mathbf{p}_2)_{\text{mixed}}} \qquad k = \frac{1}{2}|\mathbf{p}_1 - \mathbf{p}_2| \qquad \mathbf{p}_1 + \mathbf{p}_2 = 0$$

$$C(k) = \int dr^3 \phi^2_{\text{rel}}(r,k)\exp\left(-\frac{r^2}{4R_G^2}\right)$$



Example for p-p correlations

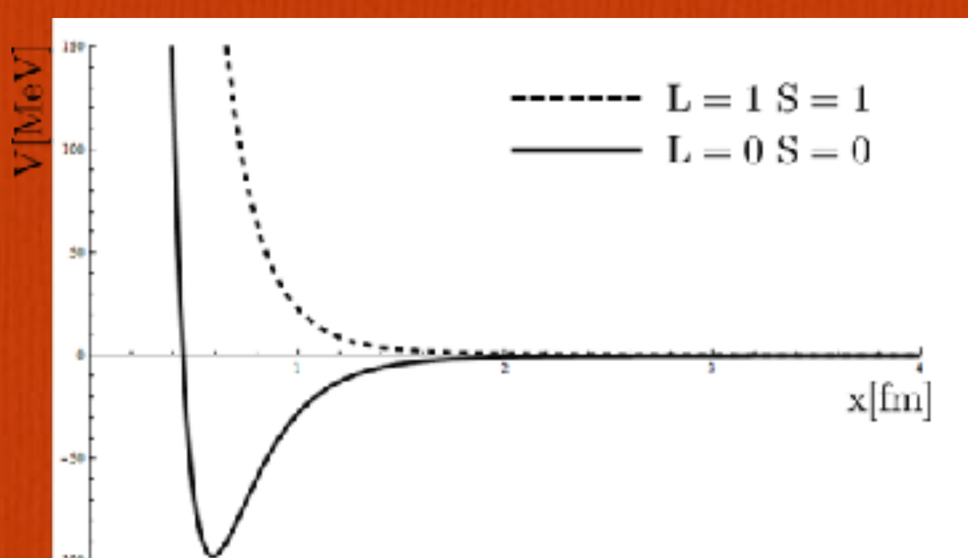
F. Wang, and S.Pratt, Phys. Rev. Lett. **83** (1999) 3138

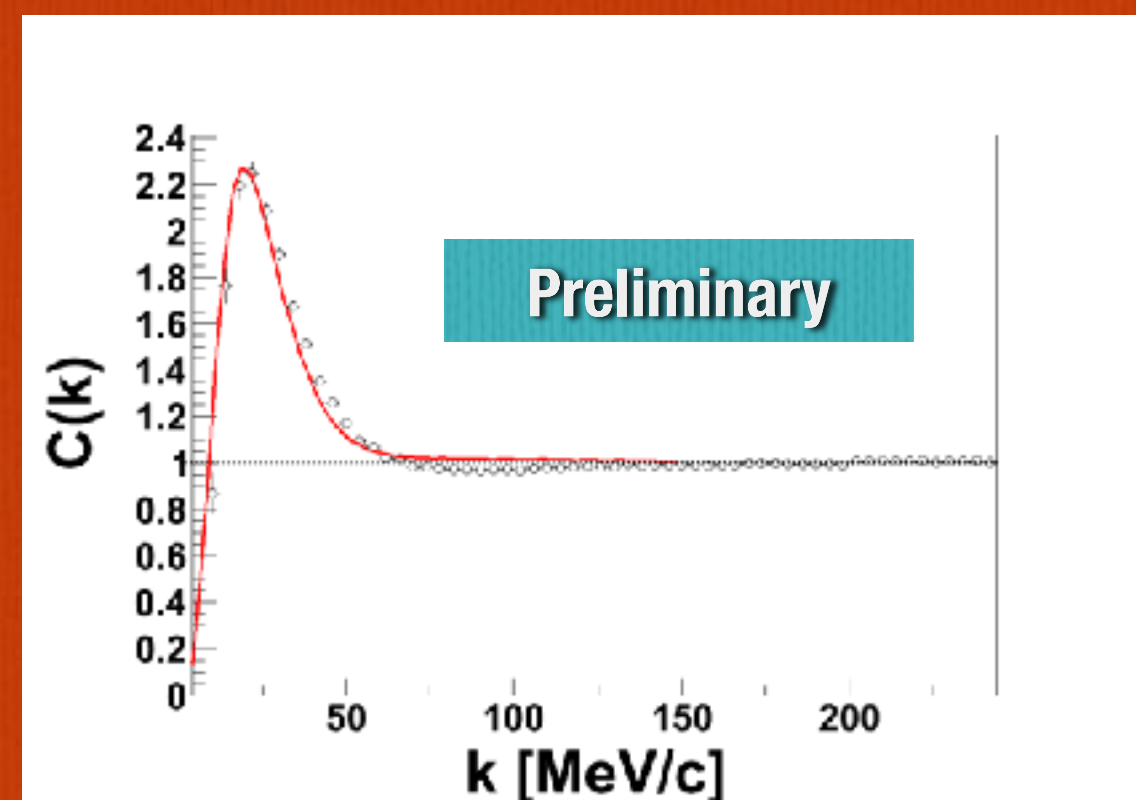
Simulation of the particle Production and Freeze-Out coordinates → After-Burner which includes the relevant Interactions → Comparison with the measured correlations

Excellent Agreement

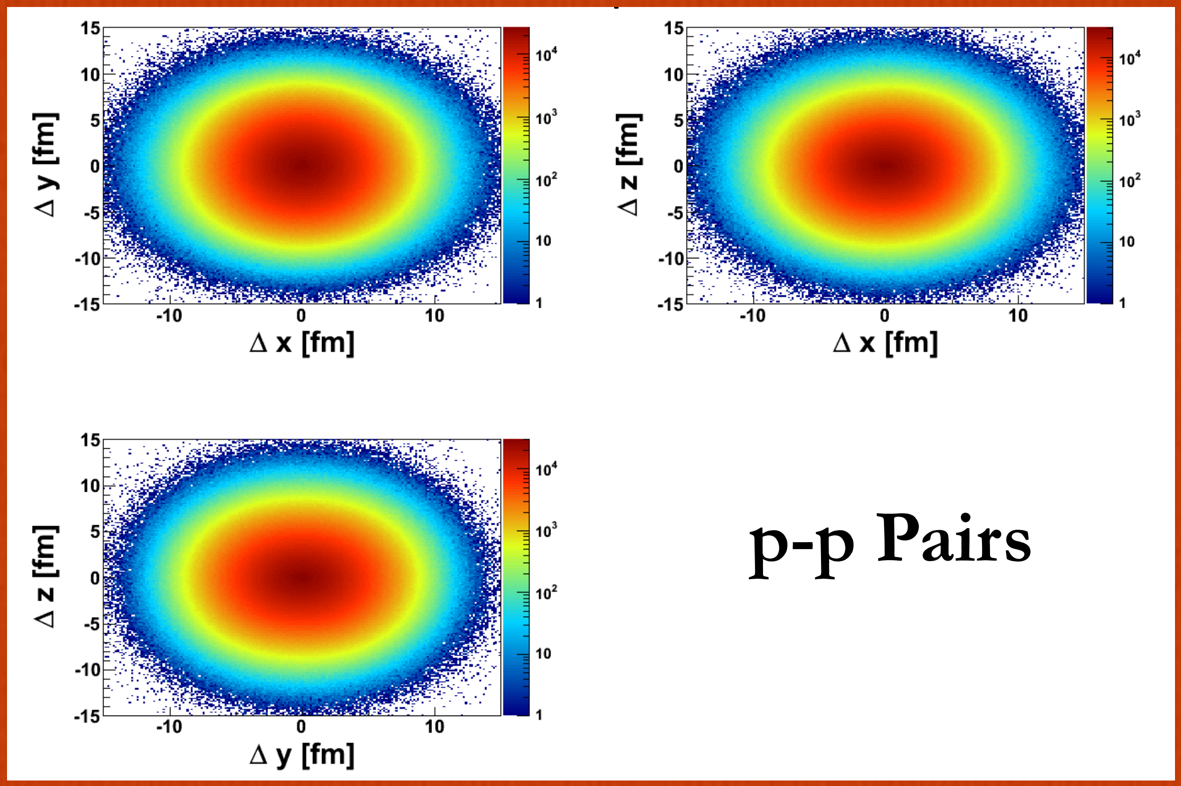
p+Nb reaction simulated in UrQMD + CRAB afterburner

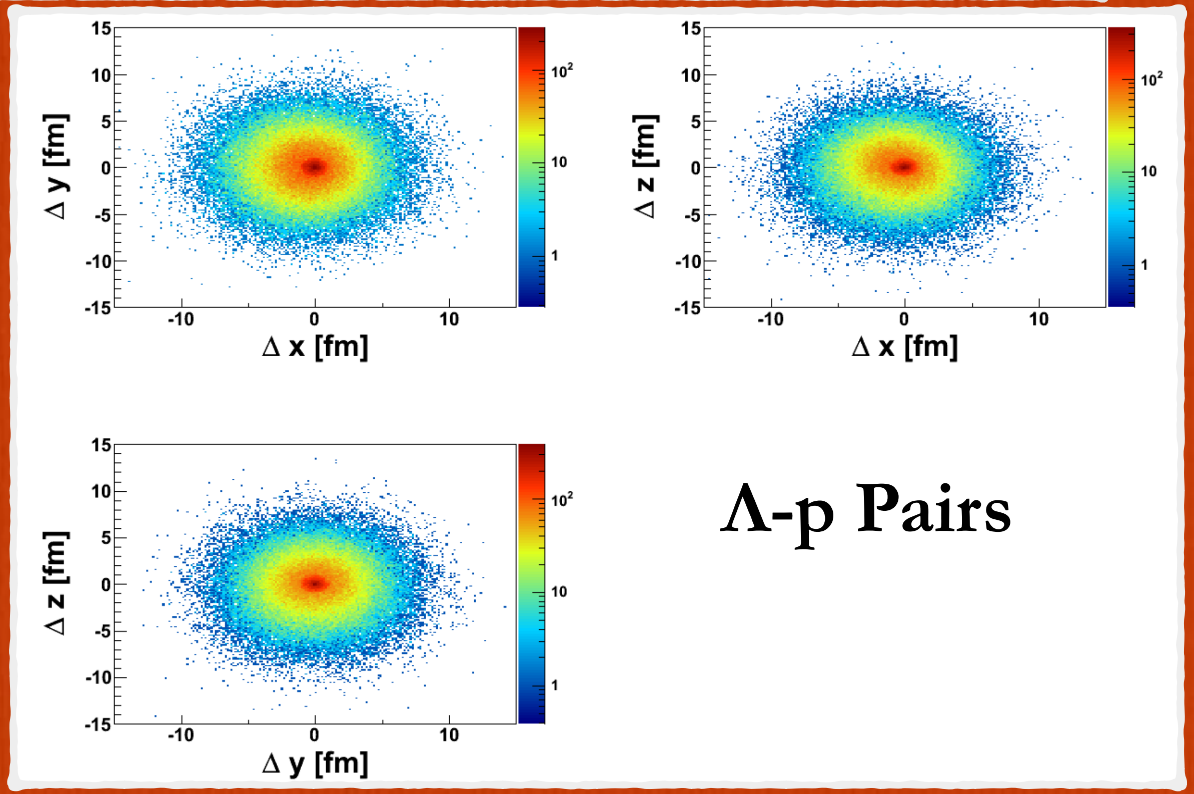
Coulomb +





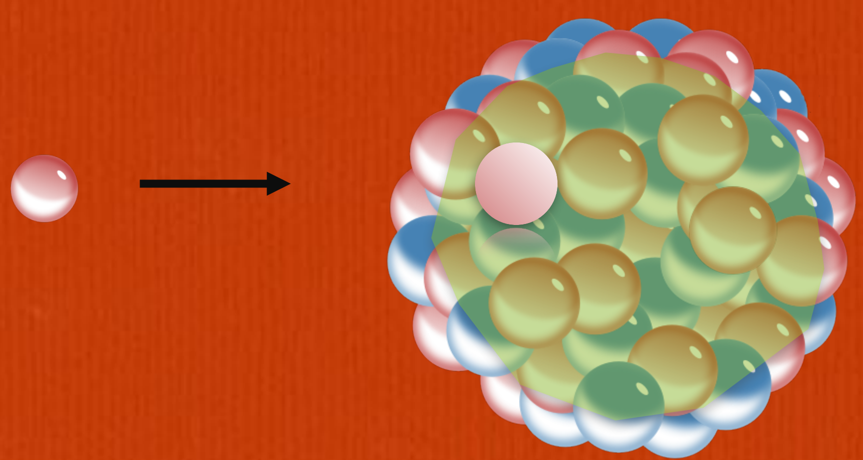
Source Determination at low energies via UrQMD



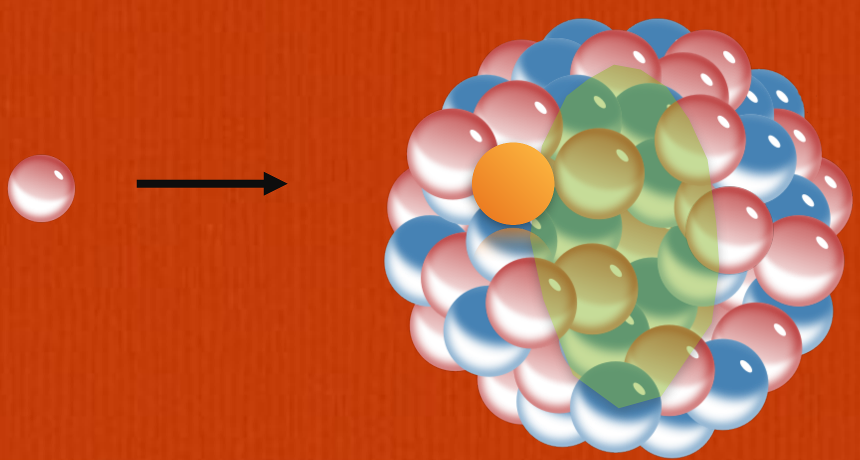


Λ-p source: 1.24 times smaller than p-p source (from UrQMD)

p-scattering in the nucleus

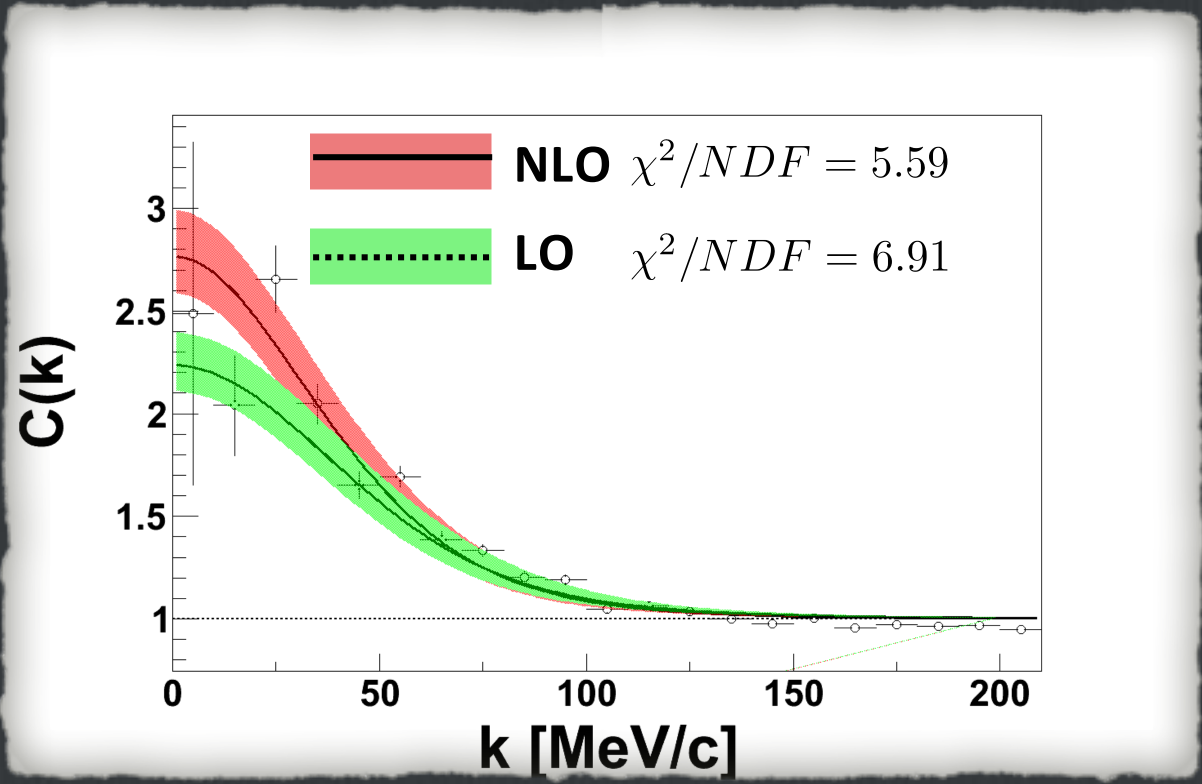


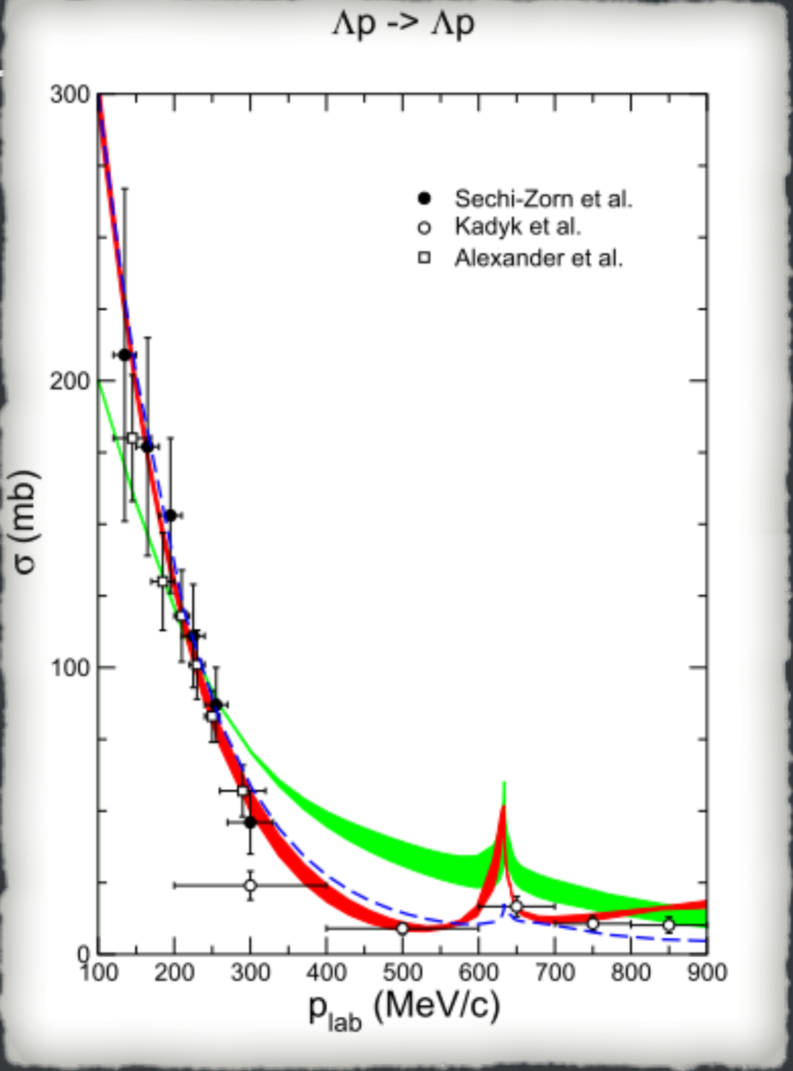
Λ-scattering in the nucleus



Λ-p Correlation in p+Nb collisions at 3.5 GeV

J. Adamczewski-Musch et al.,[HADES coll.] Phys. Rev. C. 94 (2016).





$$C(k) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k)}{R_G^{\Lambda p}} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} R_G^{\Lambda p}}\right) + 2\frac{\mathcal{R} f^S(k)}{\sqrt{\pi} R_G^{\Lambda p}} F_1(Q R_G^{\Lambda p}) - \frac{\mathcal{I} f^S(k)}{R_G^{\Lambda p}} F_2(Q R_G^{\Lambda p}) \right]$$

LO

$a^1 S_0 = -1.91 fm \quad d^1 S_0 = 1.40 fm$

$a^3 S_1 = -1.23 fm \quad d^3 S_1 = 2.13 fm$

NLO

$a^1 S_0 = -2.91 fm \quad d^1 S_0 = 2.78 fm$

$a^3 S_1 = -1.54 fm \quad d^3 S_1 = 2.72 fm$

a < 0 means attraction!

ALICE data

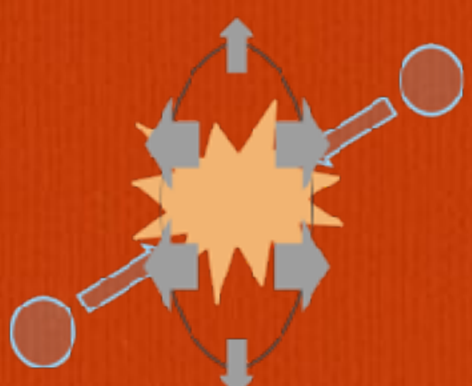
p+p 13 TeV High Multiplicity trigger, RUN 2 ~1000 Millions Events

For such collisions it is possible to

1) Model an universal Source for all hadrons!
2) Produce much larger yields of even the rarest hyperons!

Source

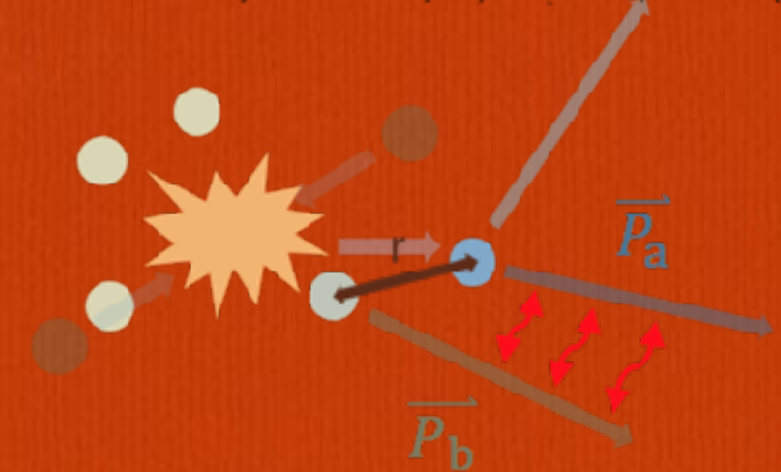
Elliptic flow



Radial flow

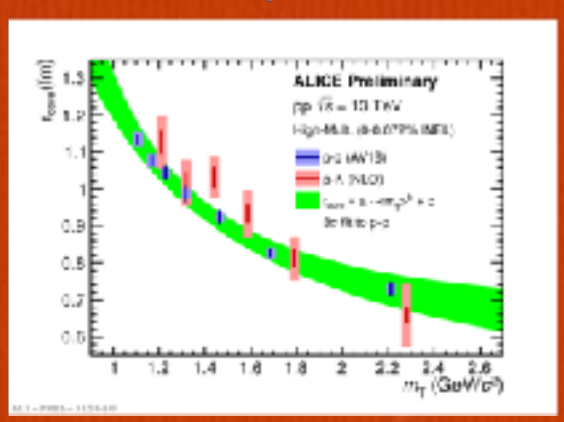
Strong decays of broad resonances

U. A. Wiedemann, U. W. Heinz, Phys.Rept. 319, 145-230 (1999)



Collective effects -> m_T scaling

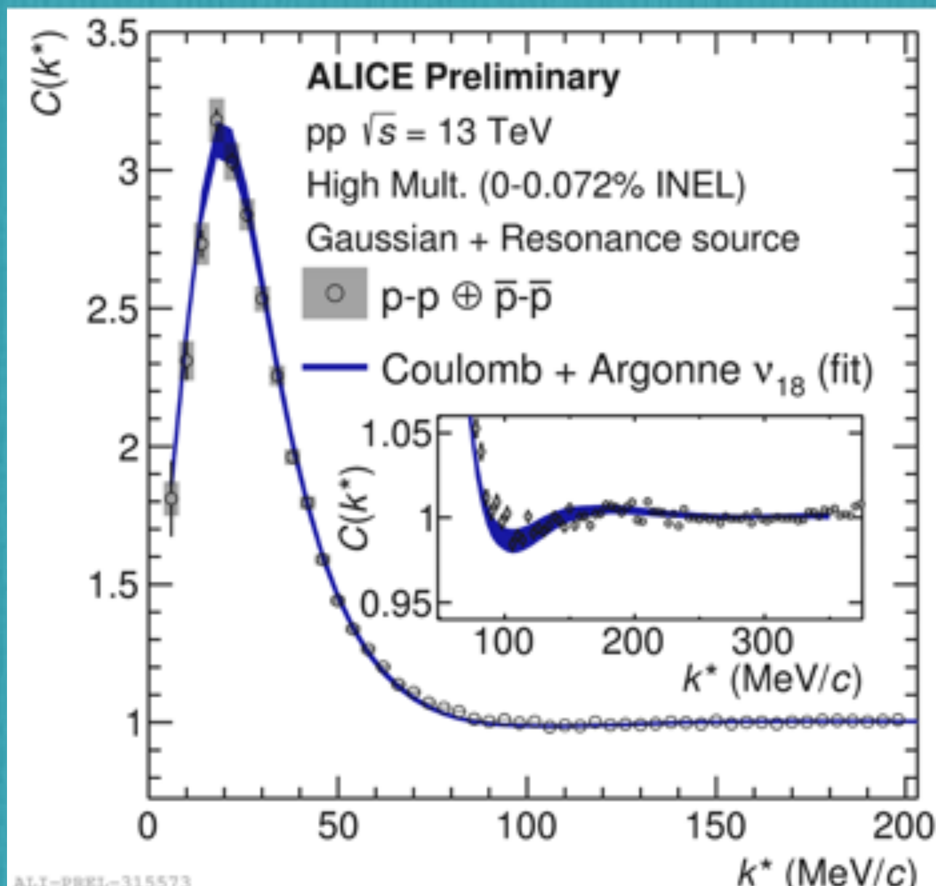
UNIVERSAL!!

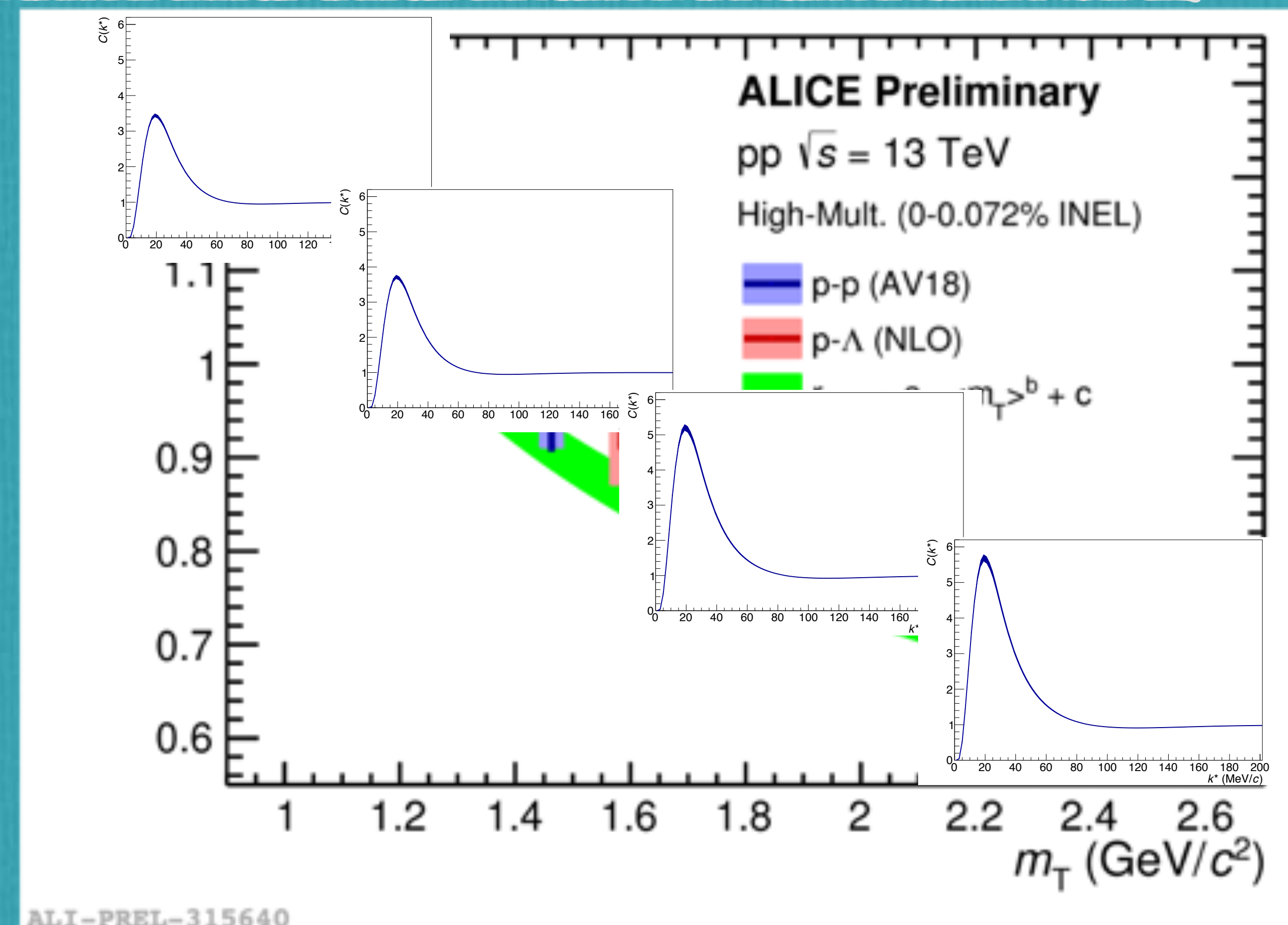


$\otimes$

'Tail' in the source distribution due to the specific strong resonance contribution for each pair of interest.

Source determination using p-p correlations

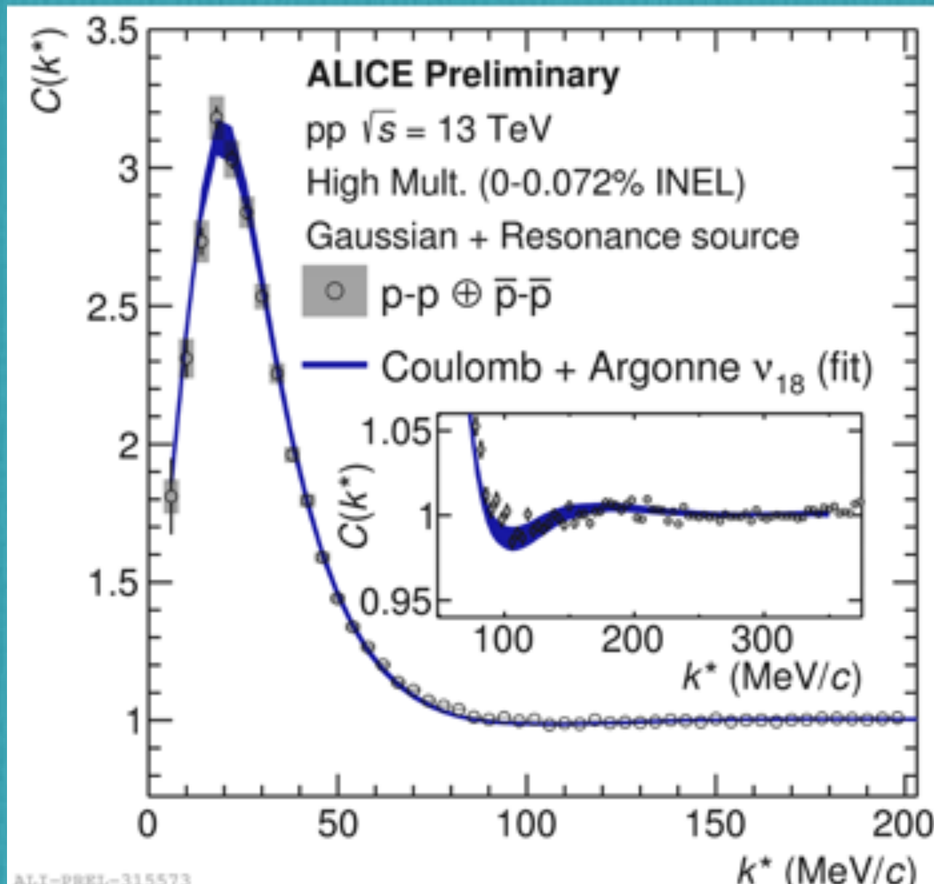


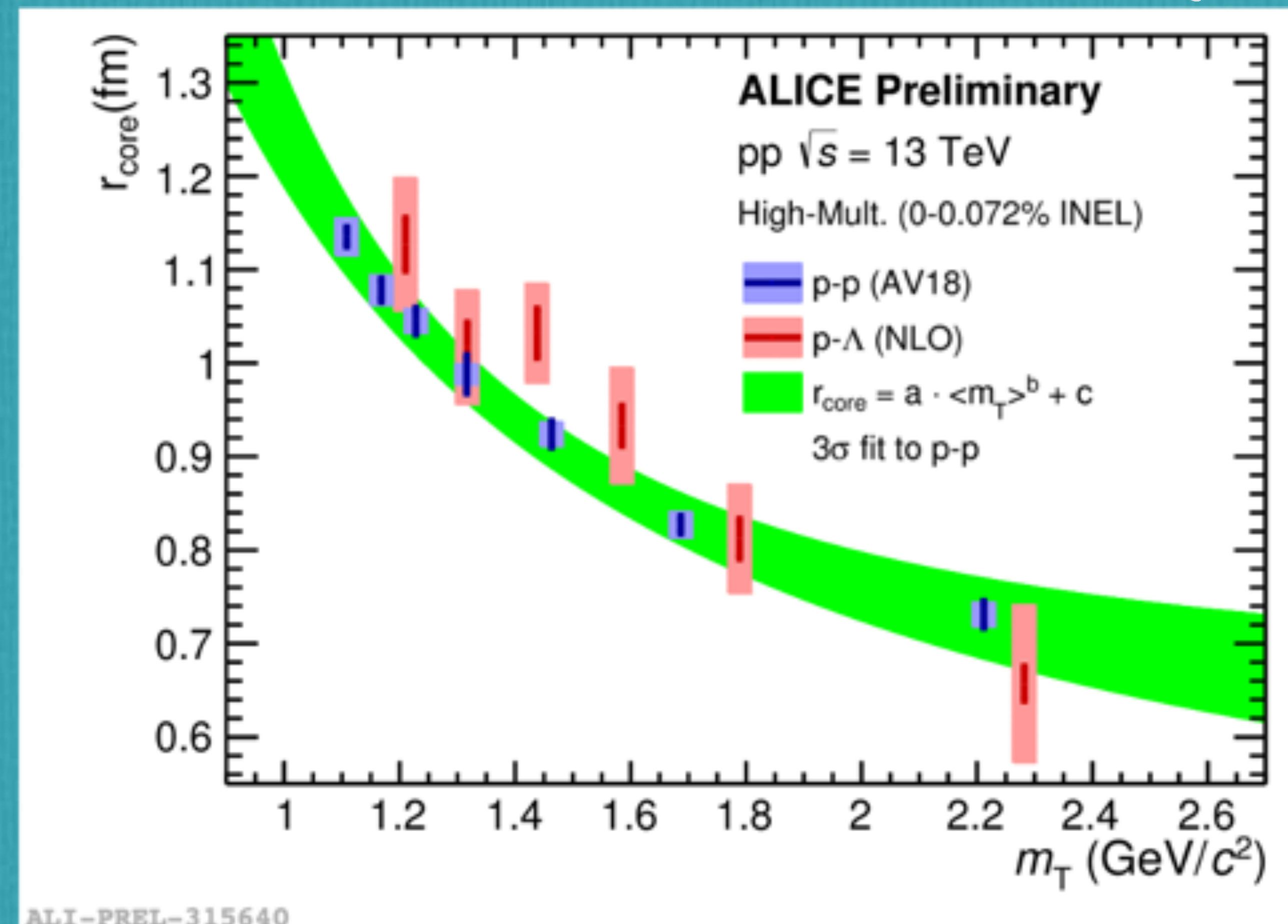


$$C(k) = \int dr^3 \phi^2_{\rm rel}(r,k) \exp\left(-\frac{r^2}{4R_G^2}\right)\frac{1}{s}e^{-r/s}$$

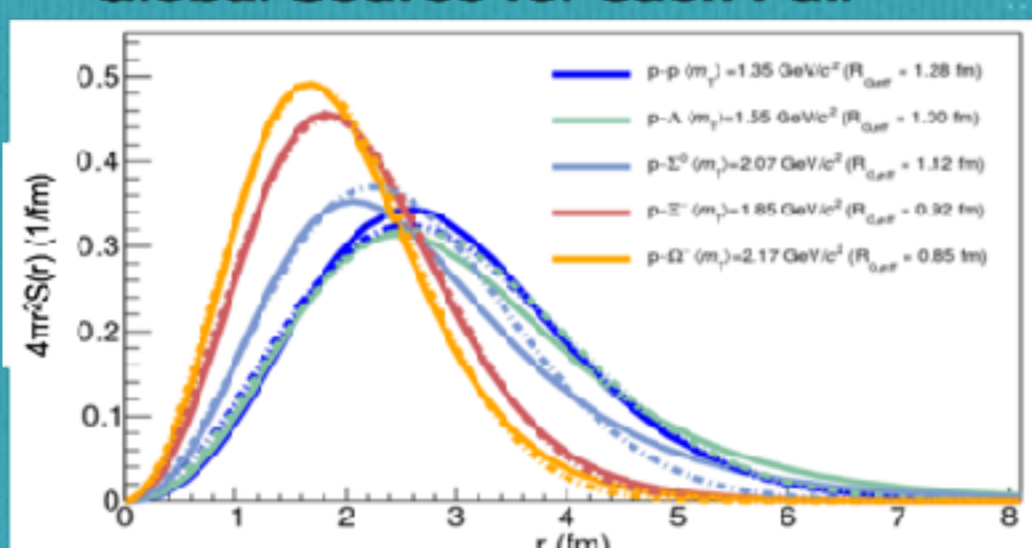
$s = \beta\gamma\tau_{res}$ for the pertinent Ensamble of resonances decaying into protons via strong decay

Source determination using p-p correlations





Global Source for each Pair



Pair	r_{Core} [fm]	r_{Eff} [fm]
pp	0.96	1.28
pΛ	0.88	1.3
pΣ^0	0.75	1.12
pΞ$^-$	0.8	0.92
pΩ−	0.73	0.85

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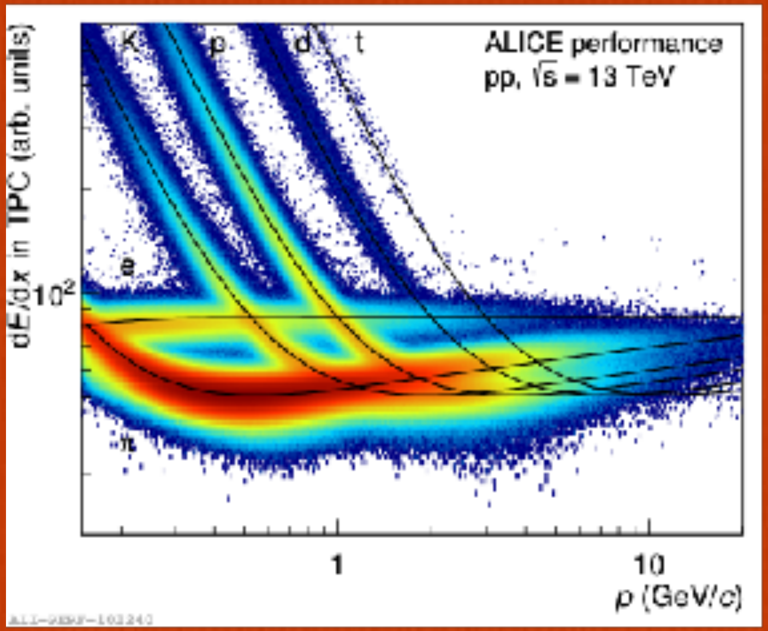
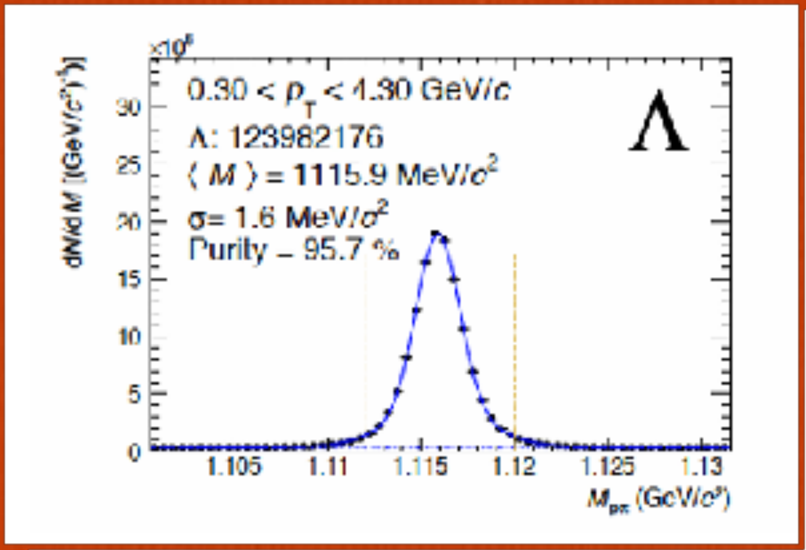
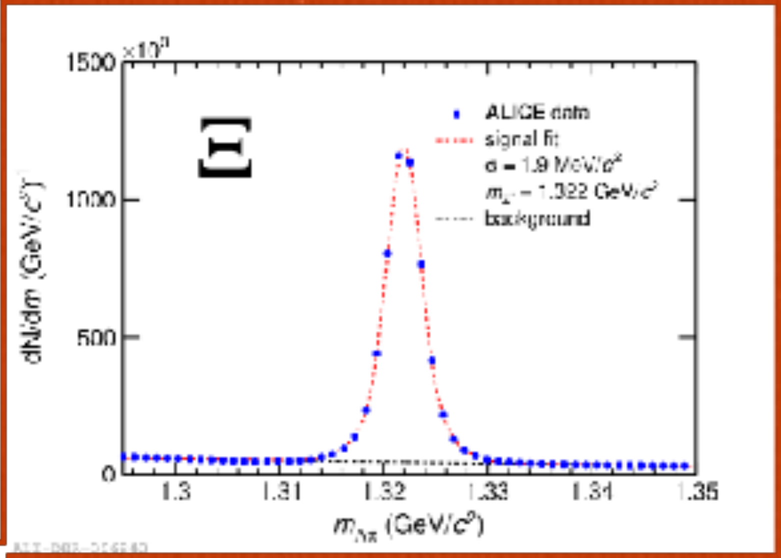
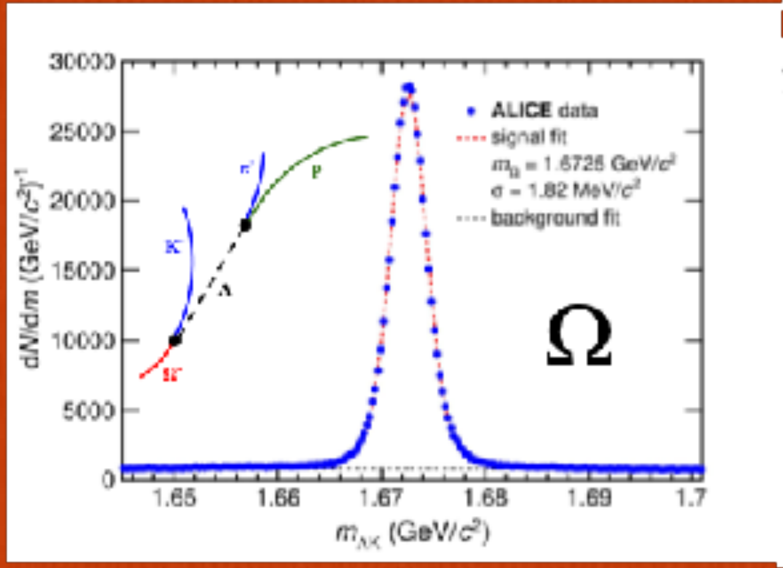
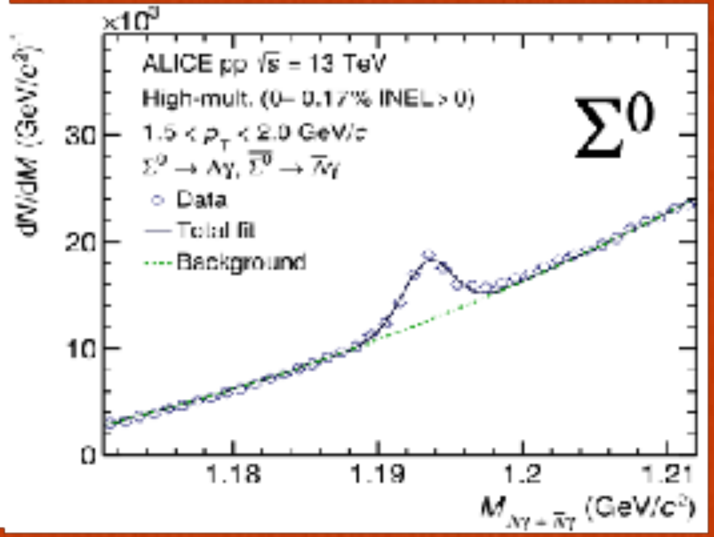
Considerations about Hyperons statistics











$$\Lambda \rightarrow p\pi^-$$

$$\Xi^- \rightarrow \Lambda\pi^-$$

$$\Omega^- \rightarrow \Lambda K^-$$

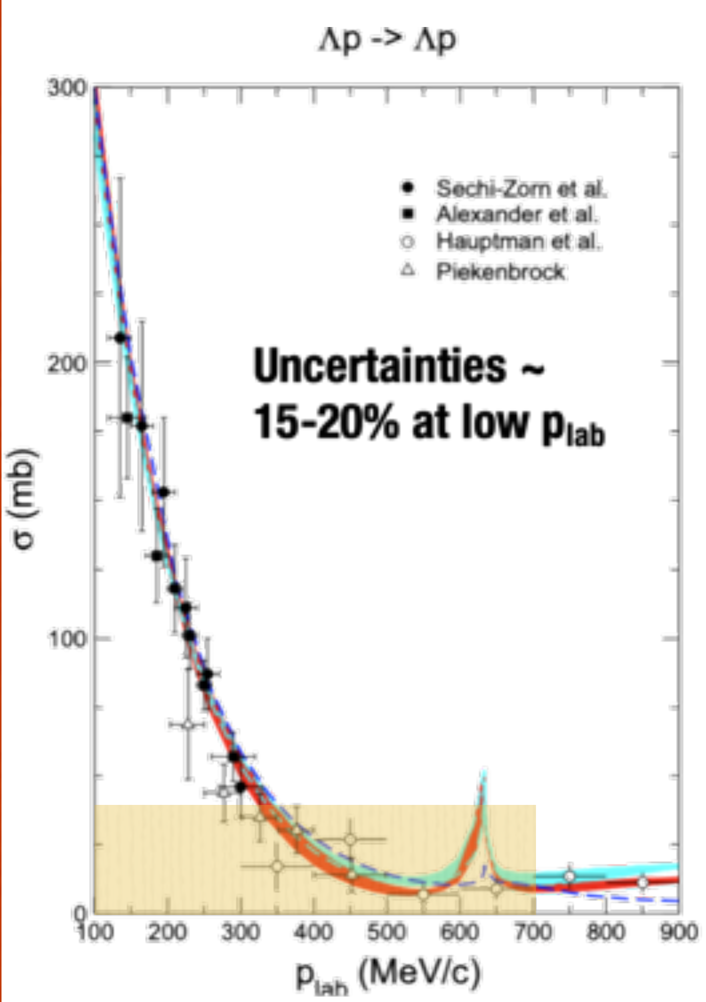
$$\Sigma^0 \rightarrow \Lambda\gamma$$

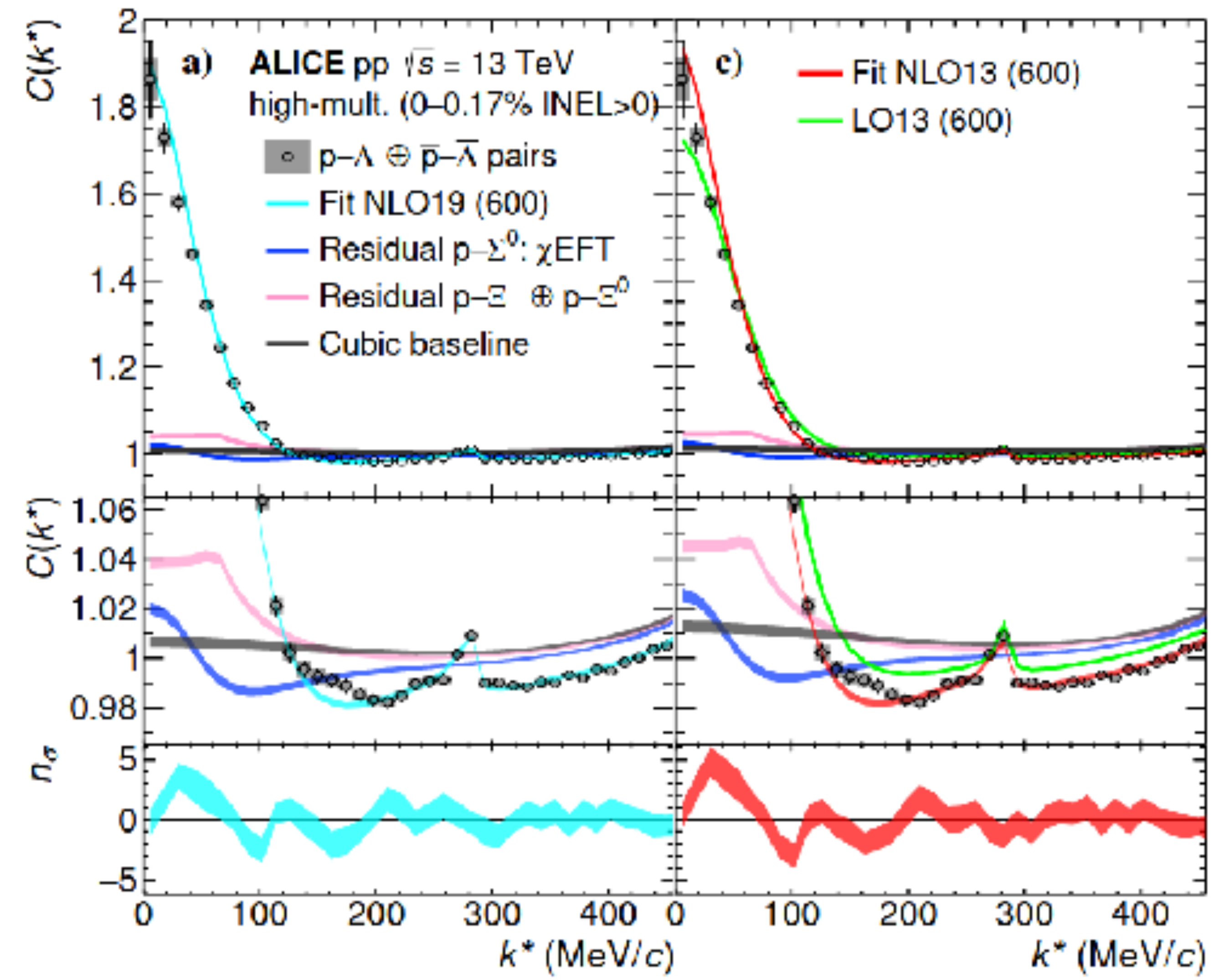
Λ-p Interaction

Scattering Data

ALICE femtoscopy data

arXiv:2104.04427





NLO13: J.Haidenbauer, N.Kaiser et al., NPA 915, 24 (2013)

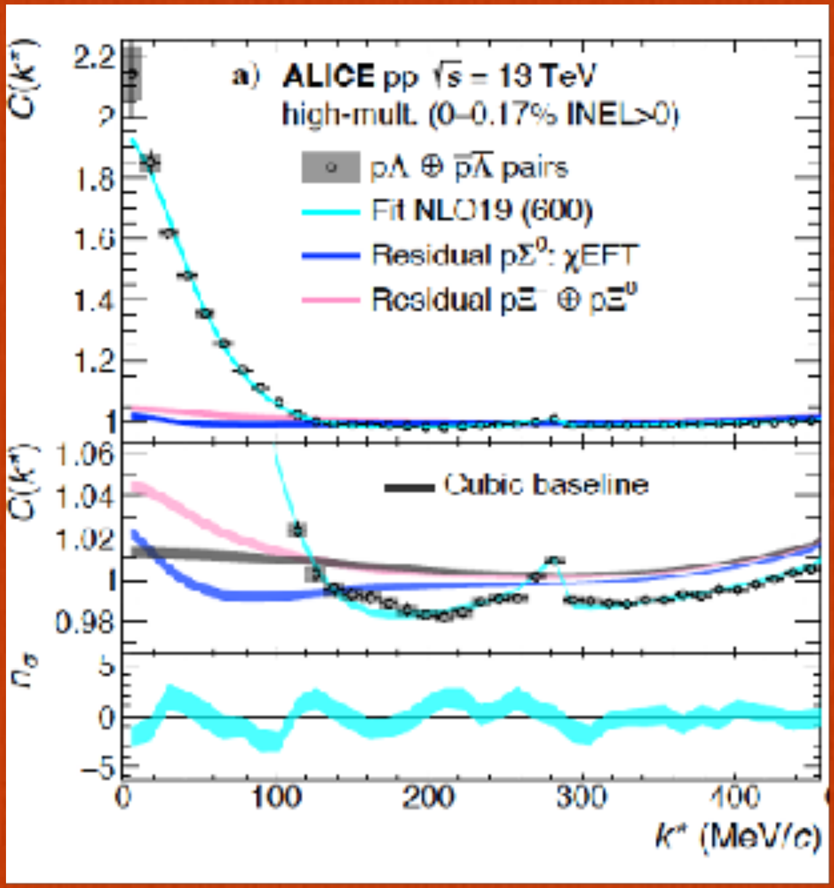
NLO19: J.Haidenbauer, U. Meißner, Eur.Phys.J.A 56 (2020)

$p_{lab} = 2 \cdot k^*$

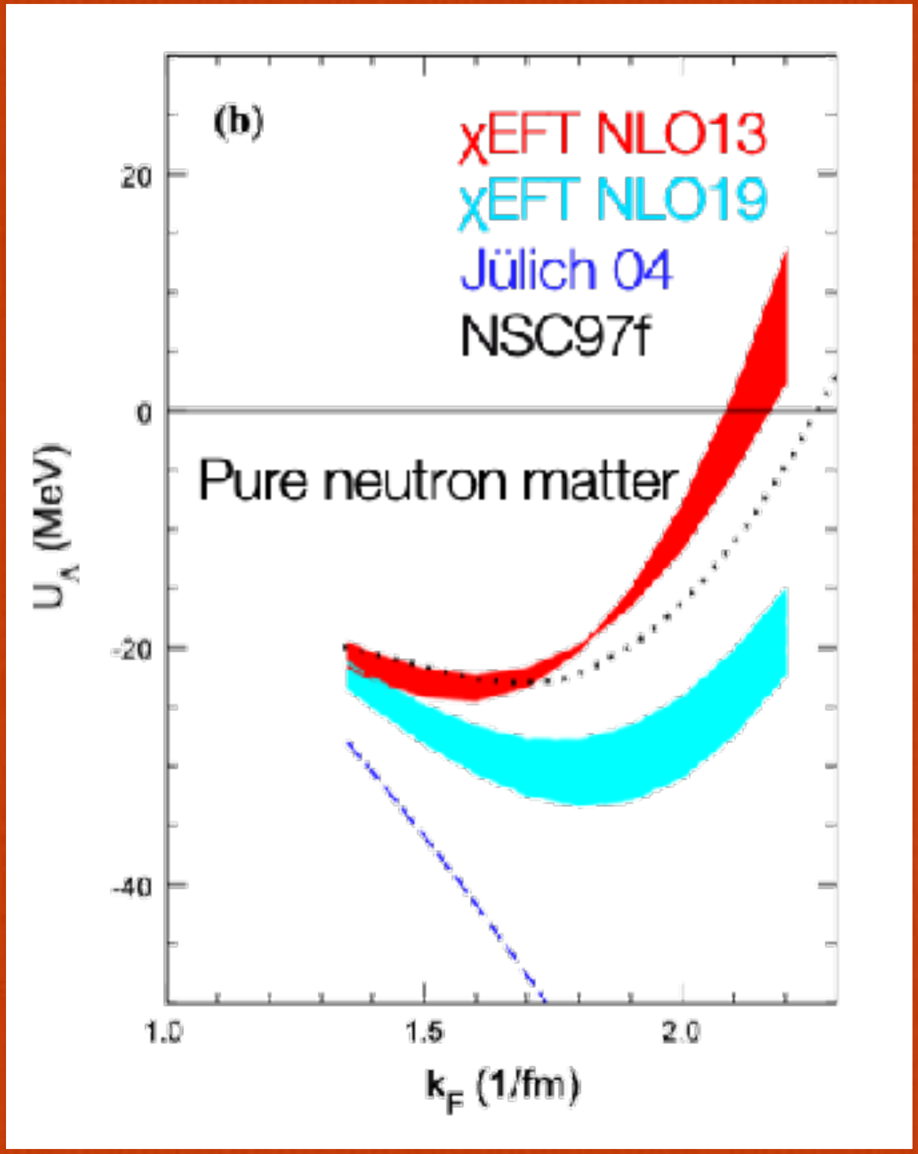
New Data: Factor 20-25 improvement in the statistics !

Clear evidence of the $\Sigma N - \Lambda N$ cusp

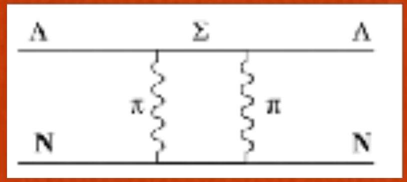
Implication for dense nuclear matter



Single Particle Potential U_Λ



* ΣN coupling strength deeply affects the behaviour of Λ at finite density



* Relevance for EoS in NS and for connection to role of ΛNN three-body interaction
* Updated NLO19 with weaker coupling strength in NΛ-NΣ leading to more attractive U_Λ at large densities and to softer EoS

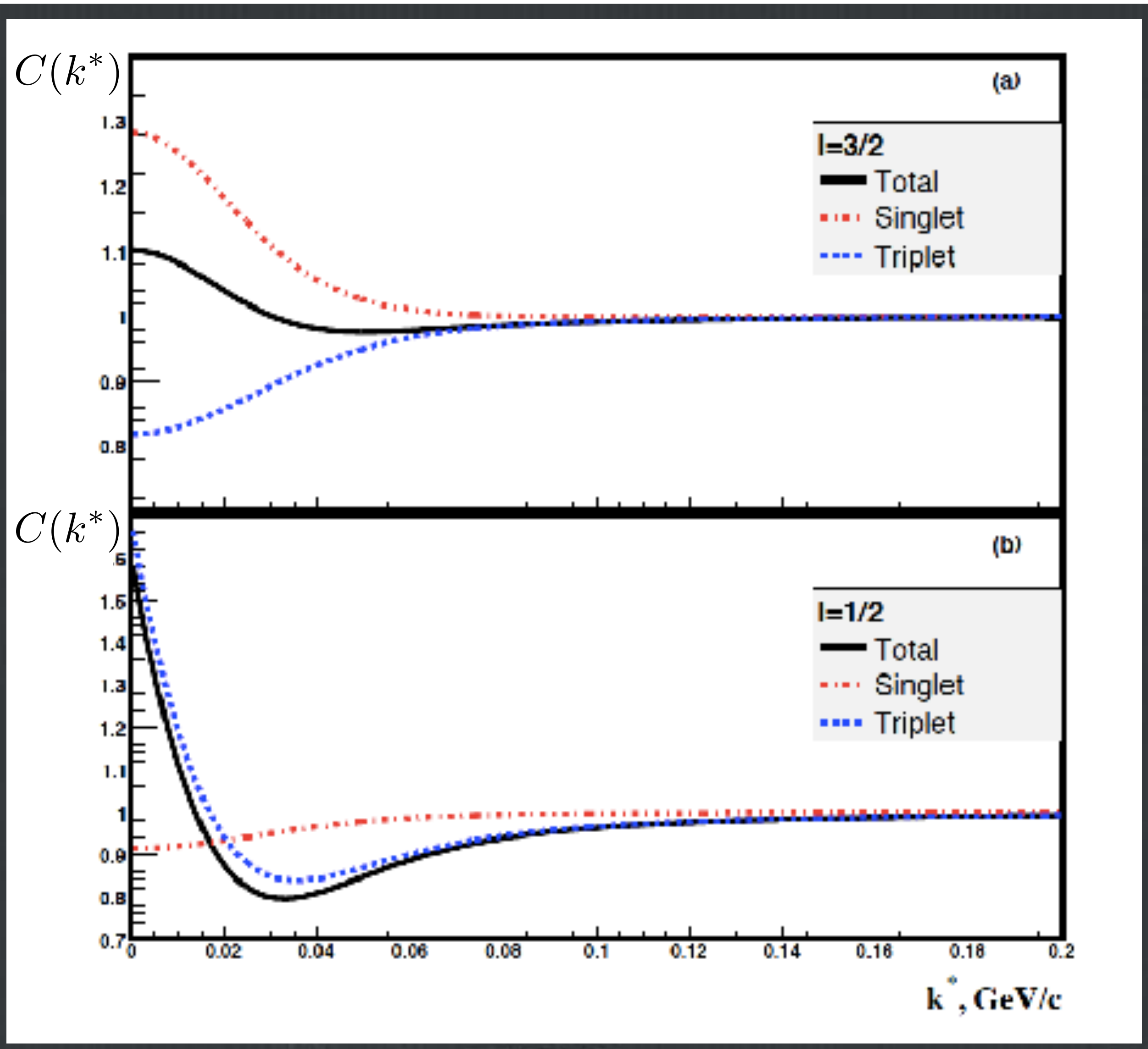
$\Sigma^0 - p$ Interaction

$\Sigma^0 \rightarrow \Lambda + \gamma$

$E_\gamma \approx 80 MeV$

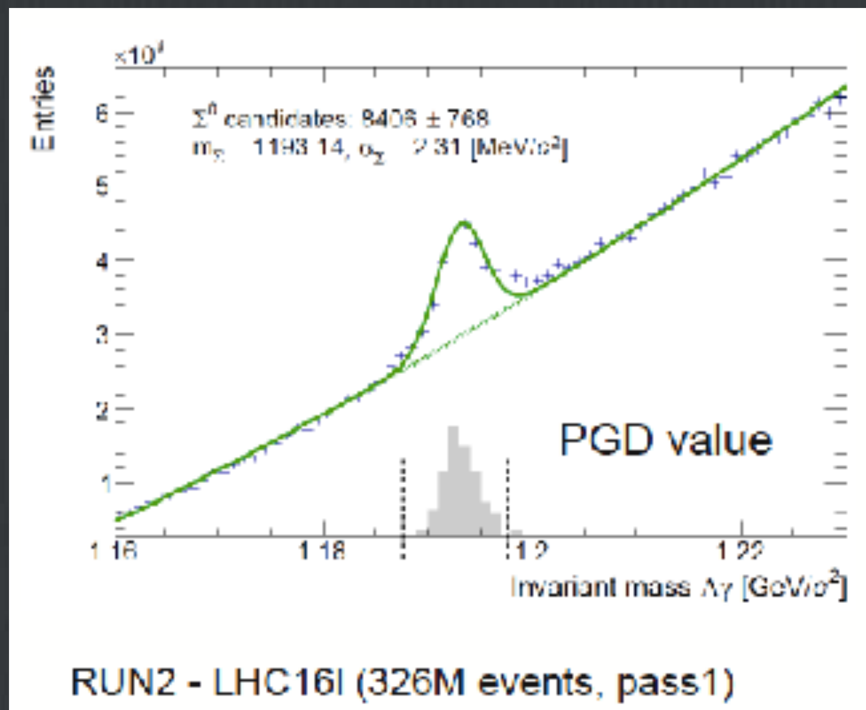
Interaction moderately attractive for I=1/2 but repulsive for I=3/2

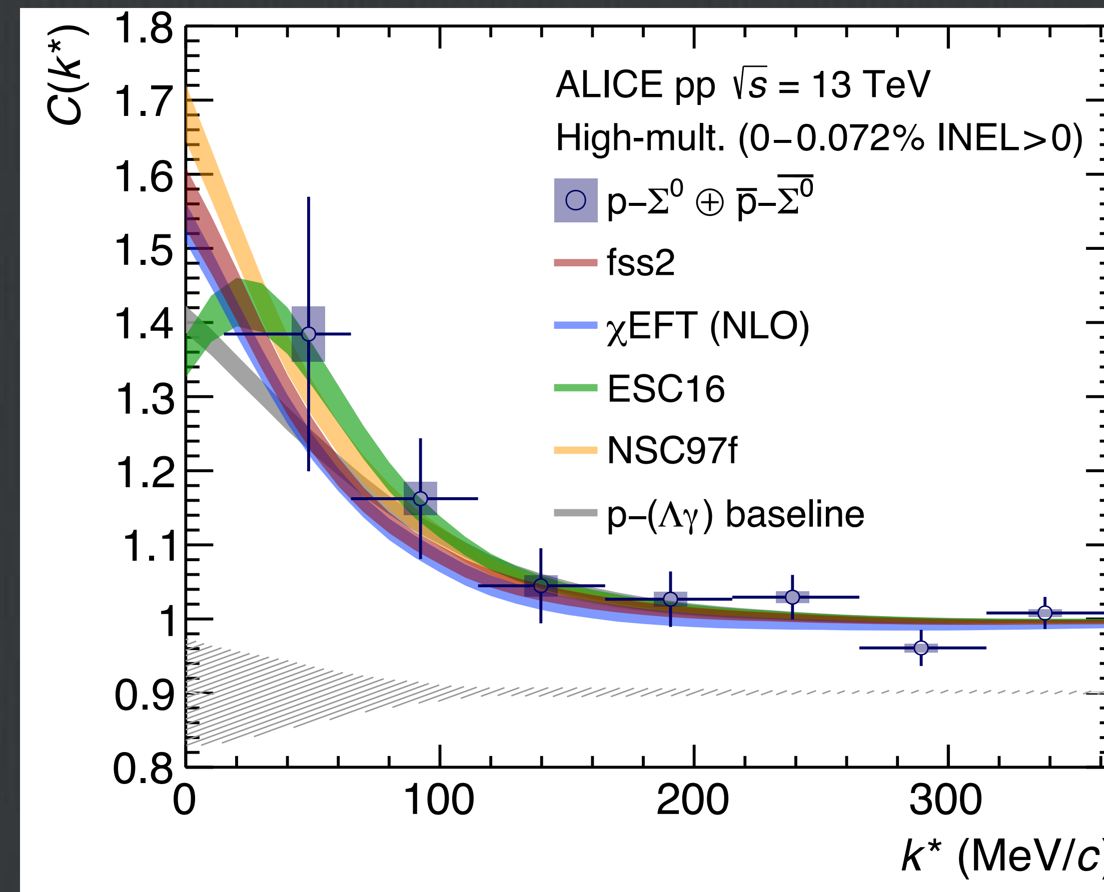
Isopin I	$a_I^{S=0}$ [fm]	$a_I^{S=1}$ [fm]	$d_I^{S=0}$ [fm]	$d_I^{S=1}$ [fm]
1/2	-1.1	-1.1+i4.3	-1.5	-2.2-i2.4
3/2	2.51	-0.73	4.92	-1.22



$\Sigma^0 - p$ Interaction

ALICE coll., PLB 805 (2020) 135419

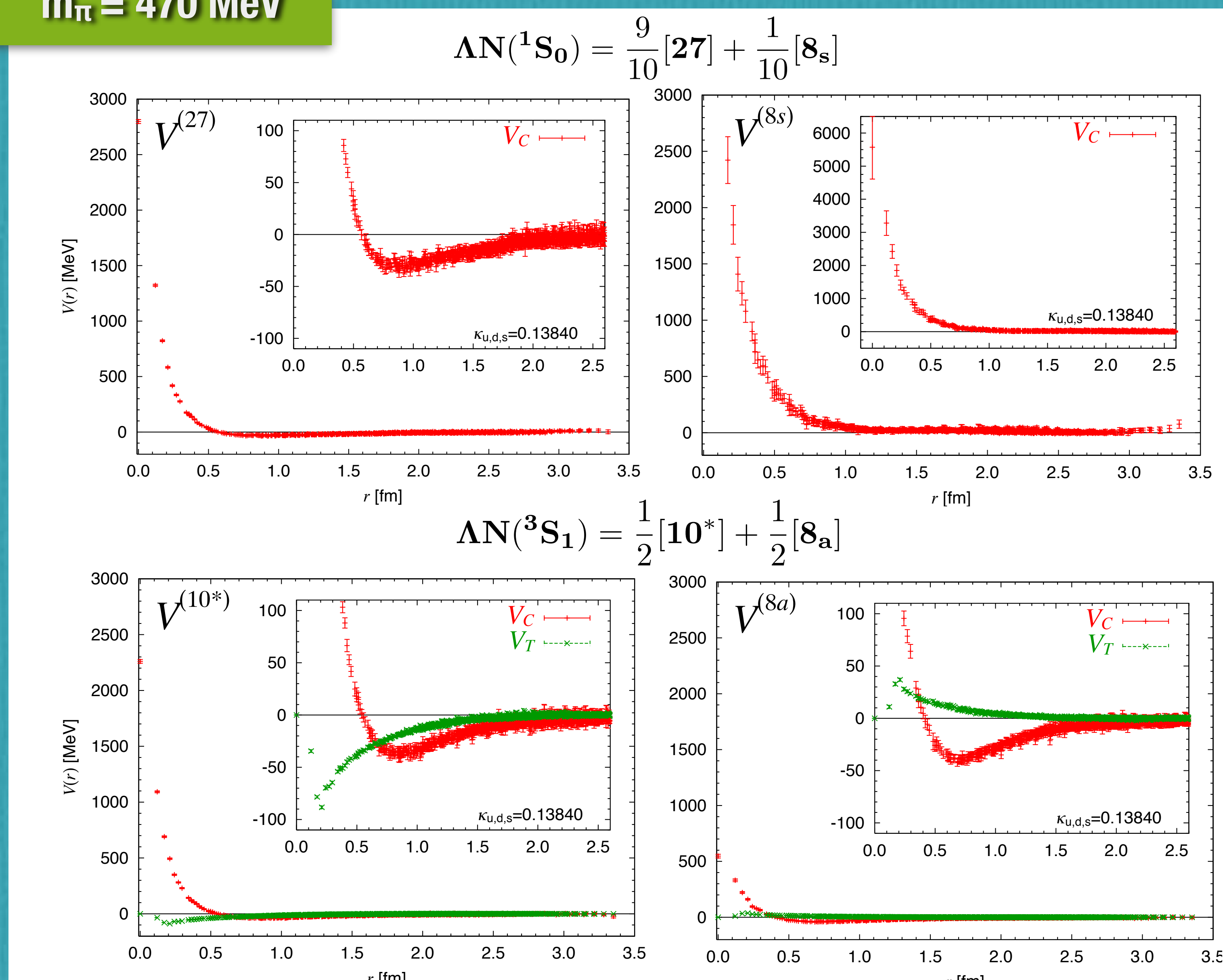




- Very challenging measurement via the difficult electromagnetic decay $\Sigma^0 \to \Lambda\,\gamma$
- Data can not distinguish between different models but the interaction should be rather **shallow**

Lattice Potentail for $\Lambda - N$

$m_\pi = 470$ MeV

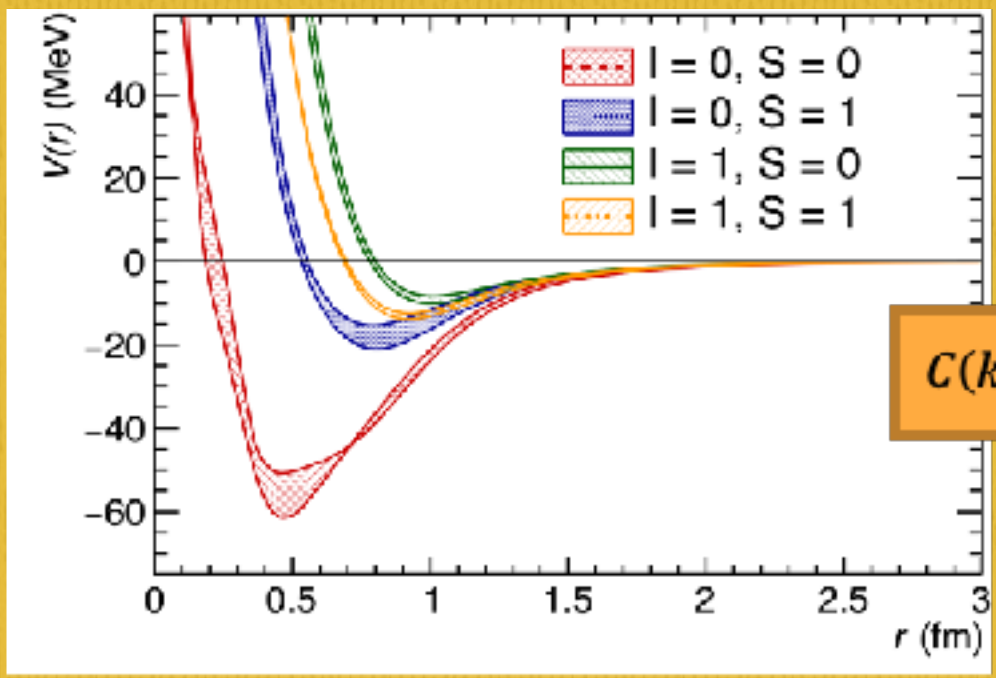


T. Inoue et al. Nucl. Phys. A881 (2012) 28

$p - \Xi^-$ Interaction

Interaction of p–Ξ⁻ pairs in four Isospin (I = 0,1) and Spin (S=0,1) states

Lattice Potential



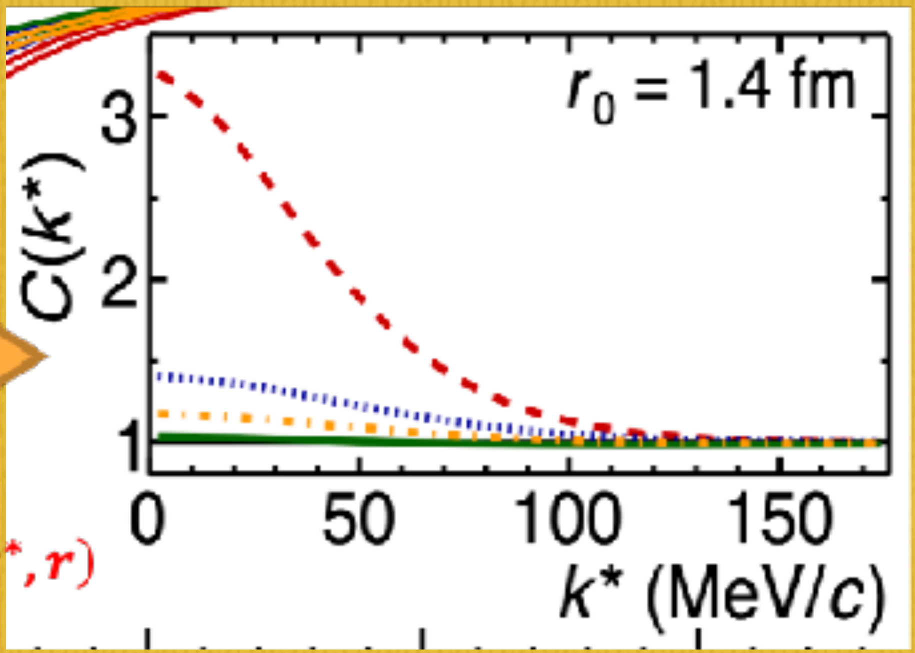
HAL QCD Coll., Nucl.Phys.A 998 (2020) 121737

r_{eff}= 1.4 fm

$$C(k^*) = \int S(\mathbf{r})\, |\psi(k^*, \mathbf{r})|\, d^3r$$

$$\hat{\mathcal{H}} \cdot \psi(k^*, \mathbf{r}) = \mathrm{E} \cdot \psi(k^*, \mathbf{r})$$

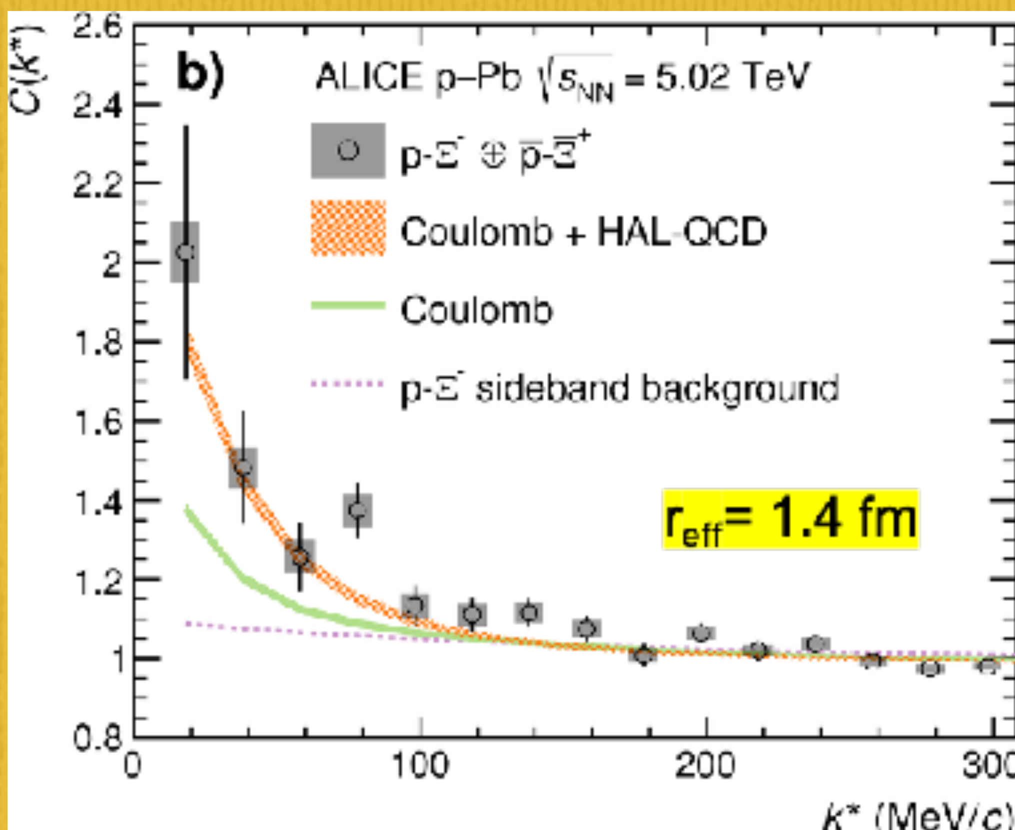
Predicted correlation function



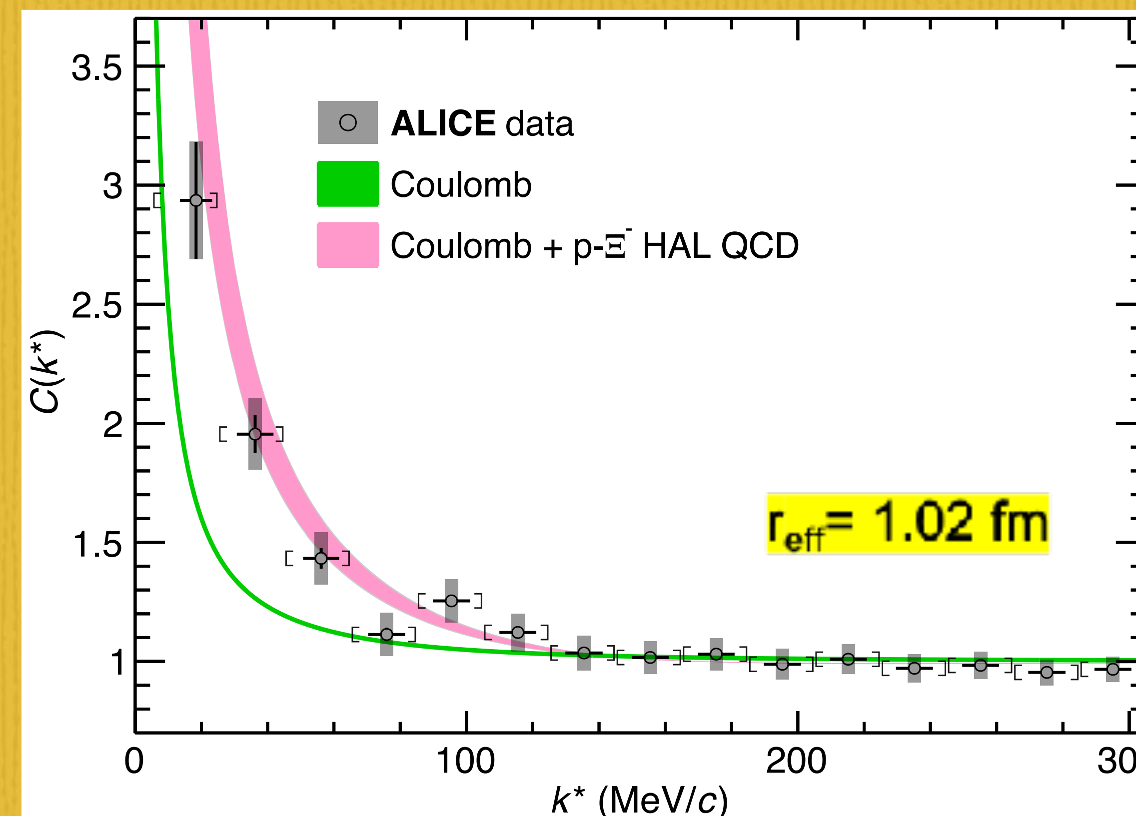
$$C_{p-\Xi^-} = \frac{1}{8} C_{N-\Xi}\,(I=0, S=0) + \frac{3}{8} C_{N-\Xi}\,(I=0, S=1) + \frac{1}{8} C_{N-\Xi}\,(I=1, S=0) + \frac{3}{8} C_{N-\Xi}\,(I=1, S=1).$$

$p - \Xi^-$ Interaction

ALICE Coll, Phys. Rev. Lett 123, (2019) 112002



ALICE Coll. *Nature* **588**, 232–238 (2020)



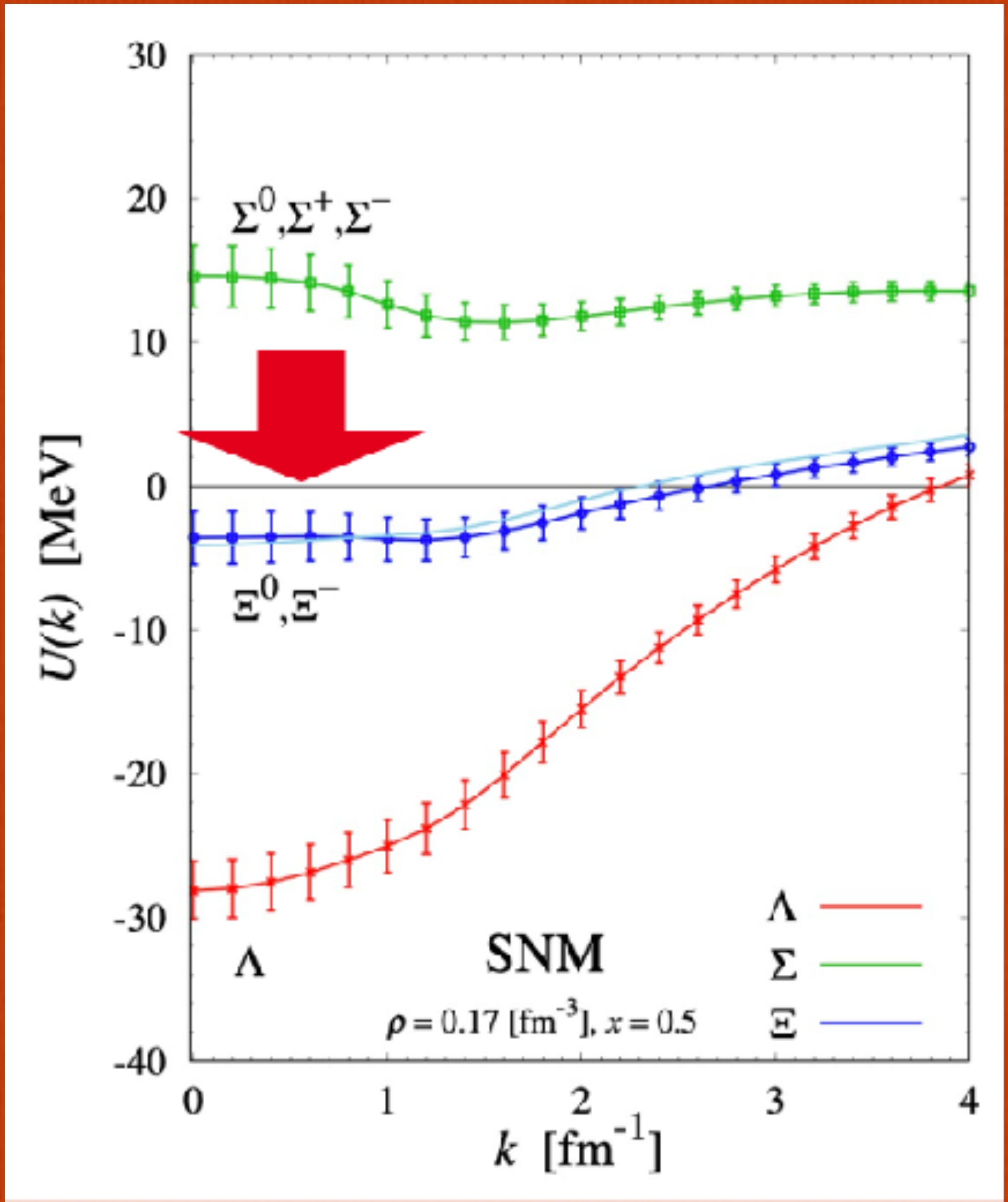
Observation of the strong interaction beyond Coulomb

Agreement with LQCD calculations confirmed in pp and p-Pb colliding systems

Consequences for Neutron Stars

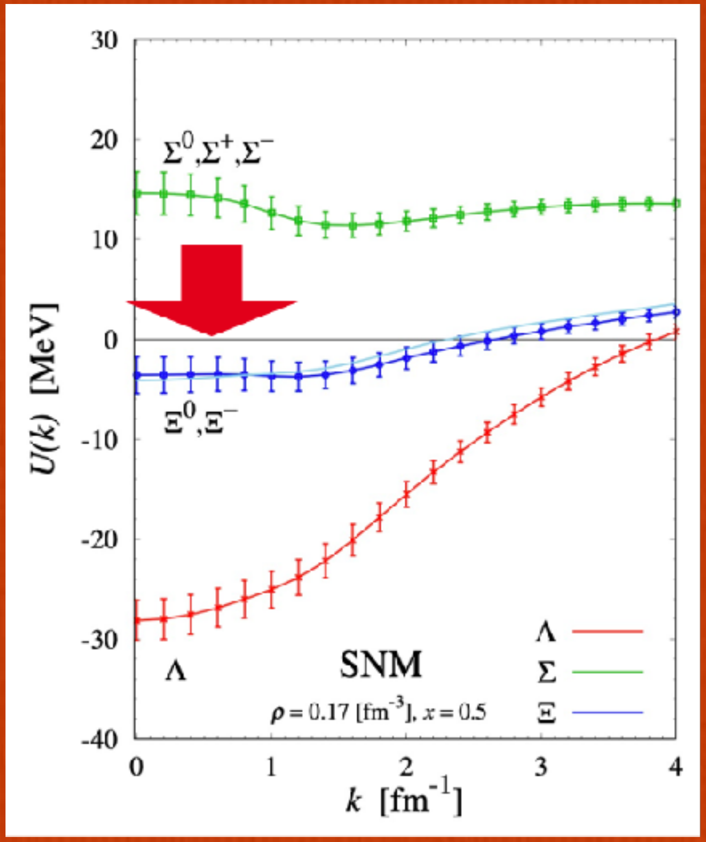
Attractive pΞ^- interaction lead to slightly attractive single particle potential in symmetric nuclear matter (SNM) and slight repulsion in neutron rich matter. (Isospin symmetries)

→ Ξ^- appears at larger densities in NS!

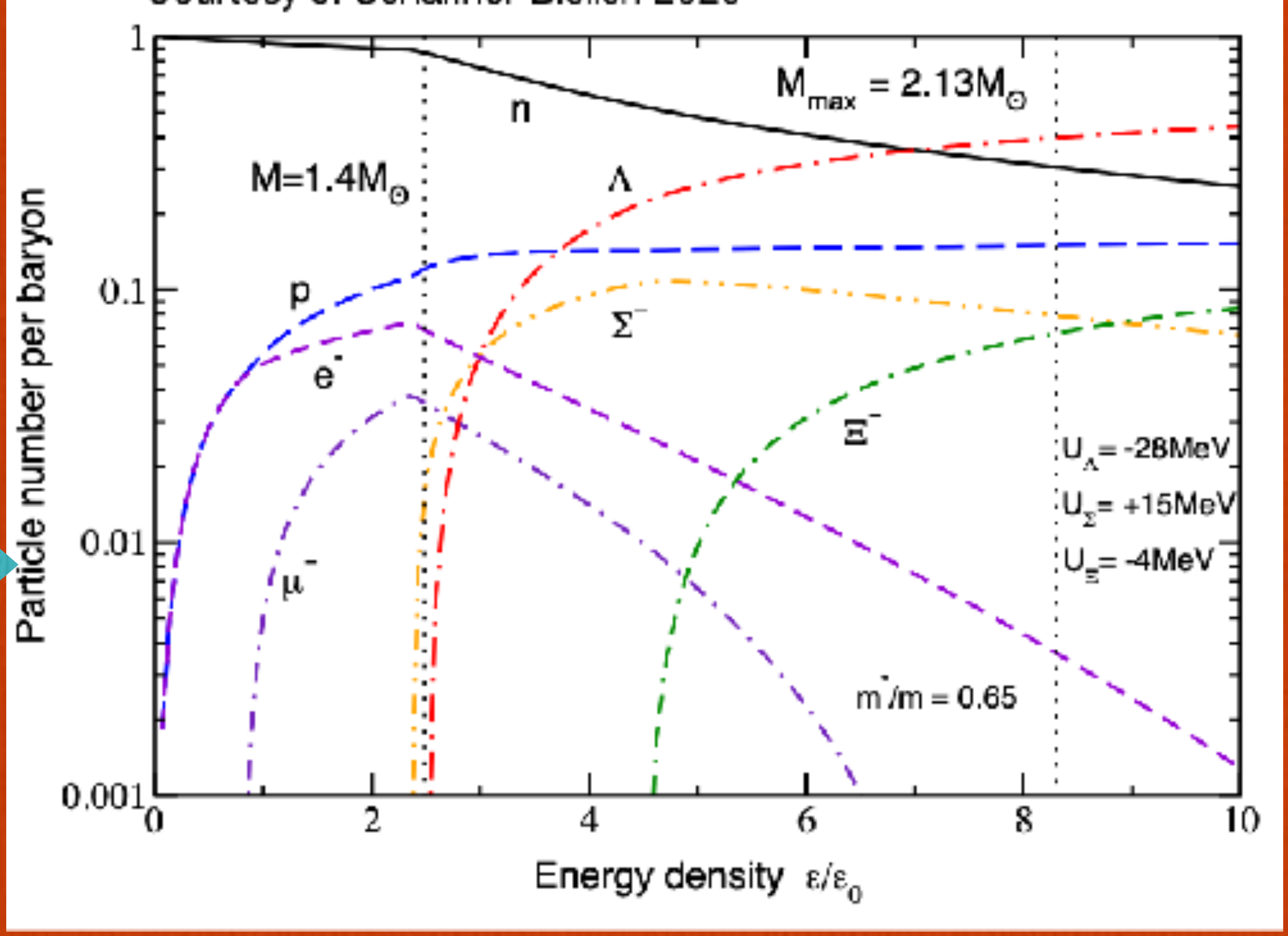


Consequences for Neutron Stars

Updated RMF Model with single particle ptential consistent with the femtoscopy measurements



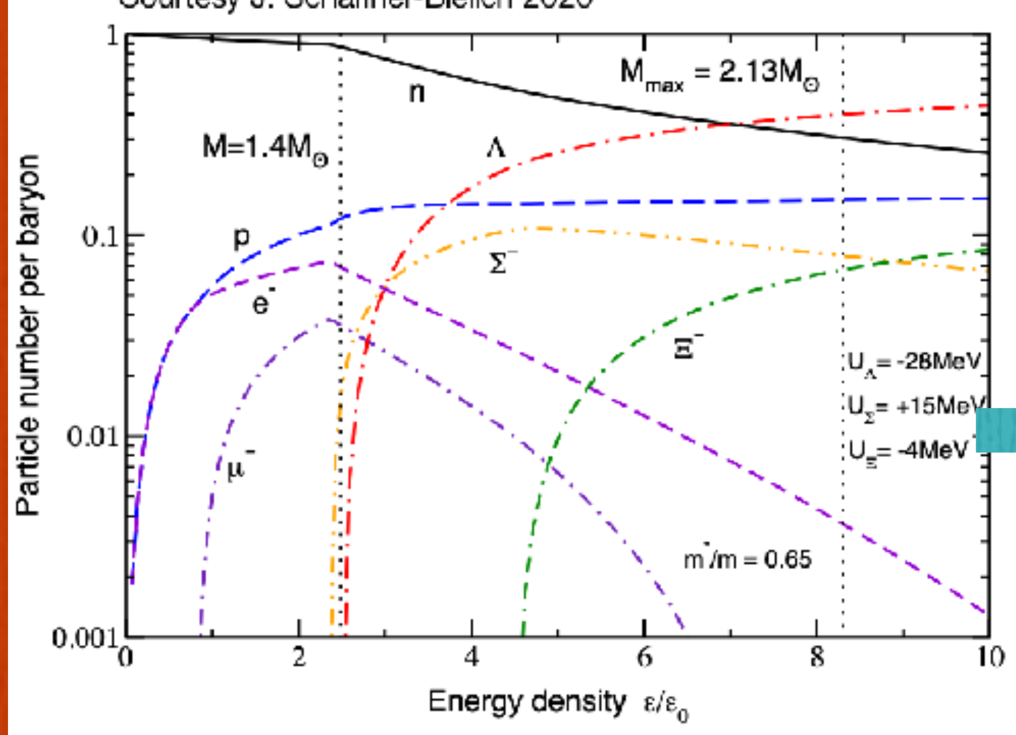
Courtesy J. Schaffner-Bielich 2020



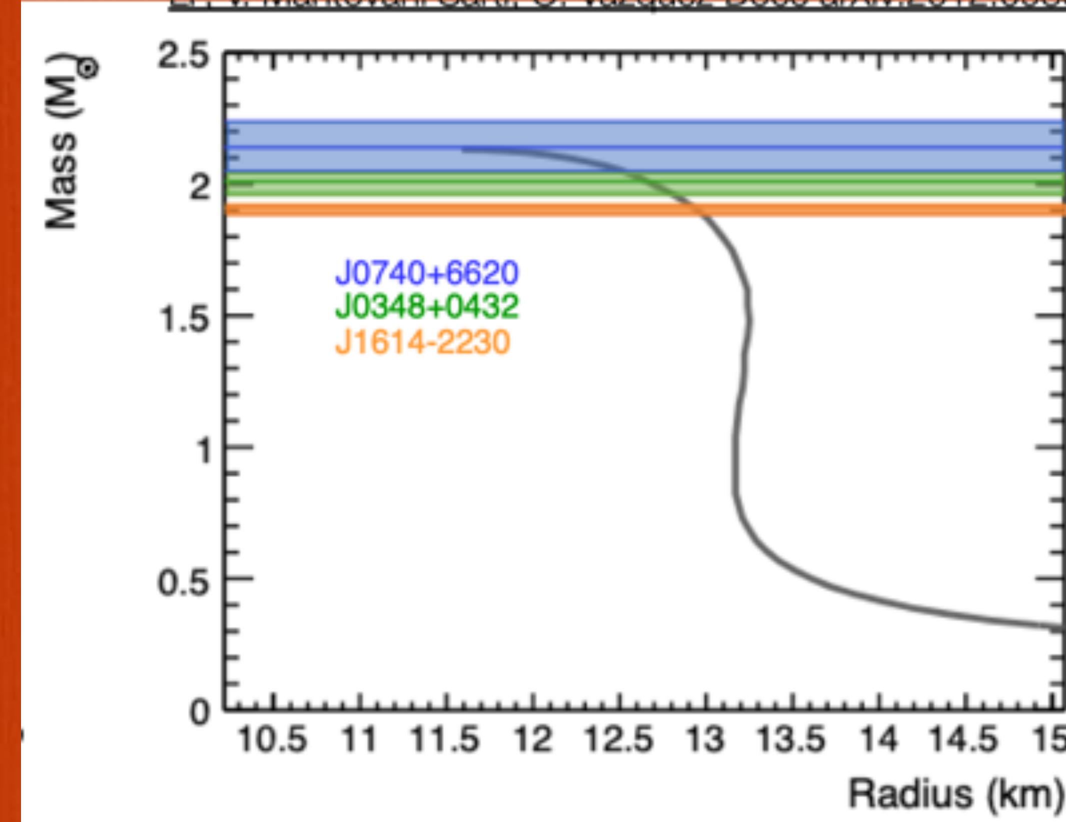
Attractive pΞ^- interaction lead to slightly attractive single particle potential in symmetric nuclear matter (SNM) and slight repulsion in neutron rich matter. Ξ^- appears at larger densities in NS!

Consequences for Neutron Stars

Courtesy J. Schaffner-Bielich 2020

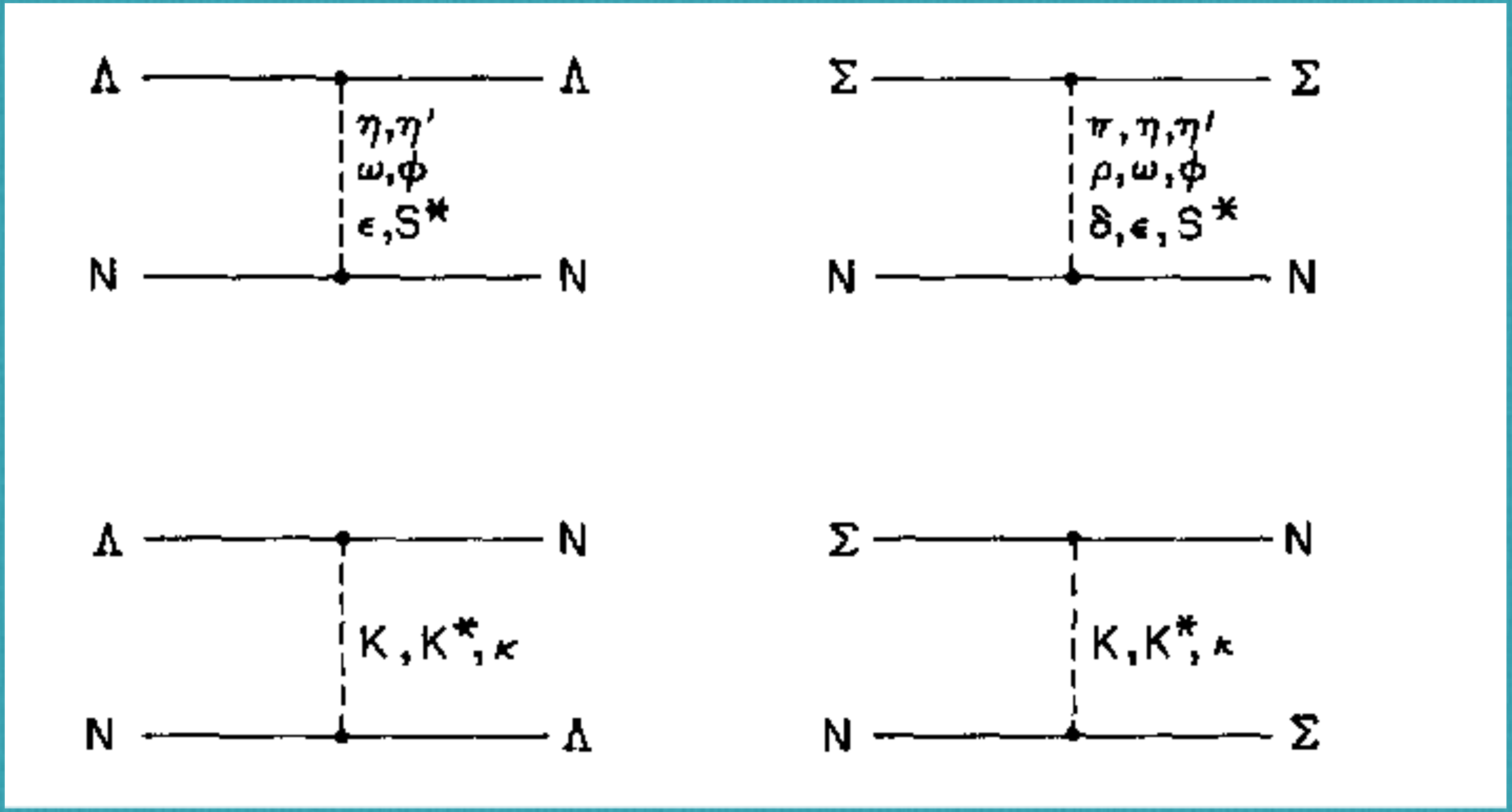


L.F. V. Mantovani Sarti, O. Vazquez Doce arXiv:2012.09806



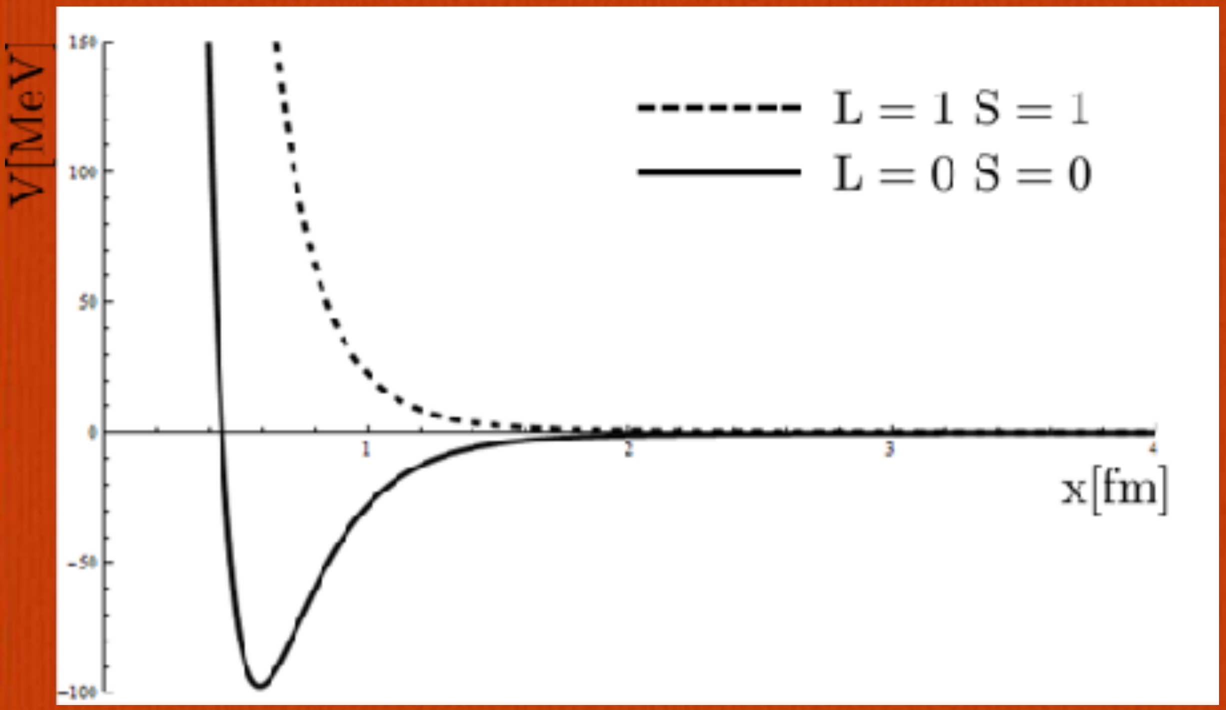
Attractive $p\Xi^-$ interaction lead to slightly attractive single particle potential in symmetric nuclear matter (SNM) and slight repulsion in neutron rich matter. Ξ^- appears at larger densities in NS!

Which are the building blocks of the interaction?



Example of local Potentials

Nucleon-Nucleon Potential



Similar to the NN potential:
attractive for large distances and with a repulsive core

Hyperon-Nucleon Potential

Repulsive Core

$$V_{\Lambda p} = V_C + \left(\bar{V} - \frac{1}{4}V_\sigma \sigma_\Lambda \cdot \sigma_p\right) T_\pi^2$$

$$T_\pi = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x} \left(1 - e^{-cr^2}\right)$$

$$V_C = W_C \left[1 + exp\left(\frac{r-R}{d}\right)\right]^{-1}$$