Hadron-hadron interactions and the physics of neutron stars

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Nuclear Equation of State

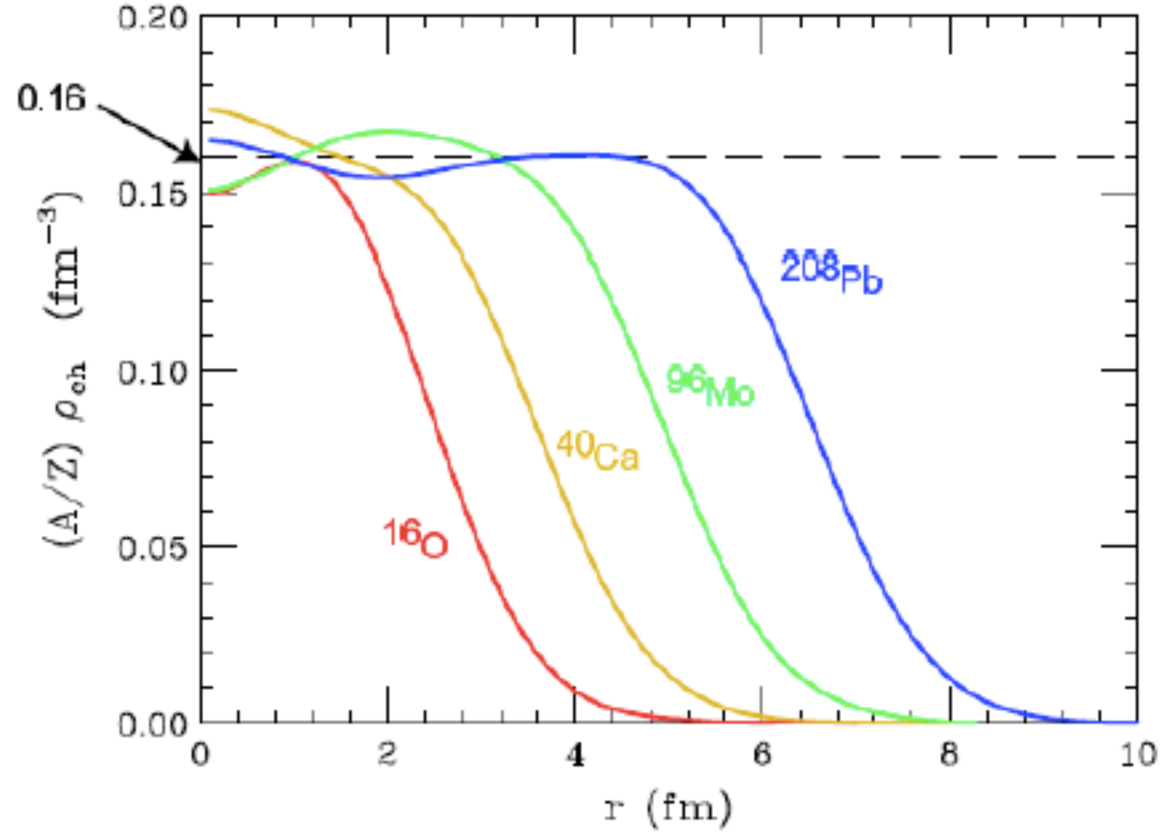
Experimental determination of the nuclear equation of state considering only nucleons

- Use nuclei to first asses the equation of state of normal nuclear matter and then extrapolate to higher densities
- Measure dense baryonic systems via heavy ion collisions at different energies and look for obervables able to constraint the EoS
- Start from nucleon-nucleon scattering and nuclear spectroscopy where the binding energy of nuclei are determined and constrained an effective field theory for many nucleon interactions that is then extrapolated to high densities (without any connection to experiment with large baryonic densities)
- In particular: how to measure three- or four-body forces ?

EOS from Nuclei

- We want to learn something about the equation of state around normal nuclear density
- Therefore we consider normal (well known) nuclei.
- Their density profile can be measured by electron scattering

$$-B(A,Z) = -16MeVA + a_s A^{2/3} + a_a \frac{(A-2Z)^2}{A} + a_c \frac{Z(Z-1)}{A^{1/3}} + ..$$



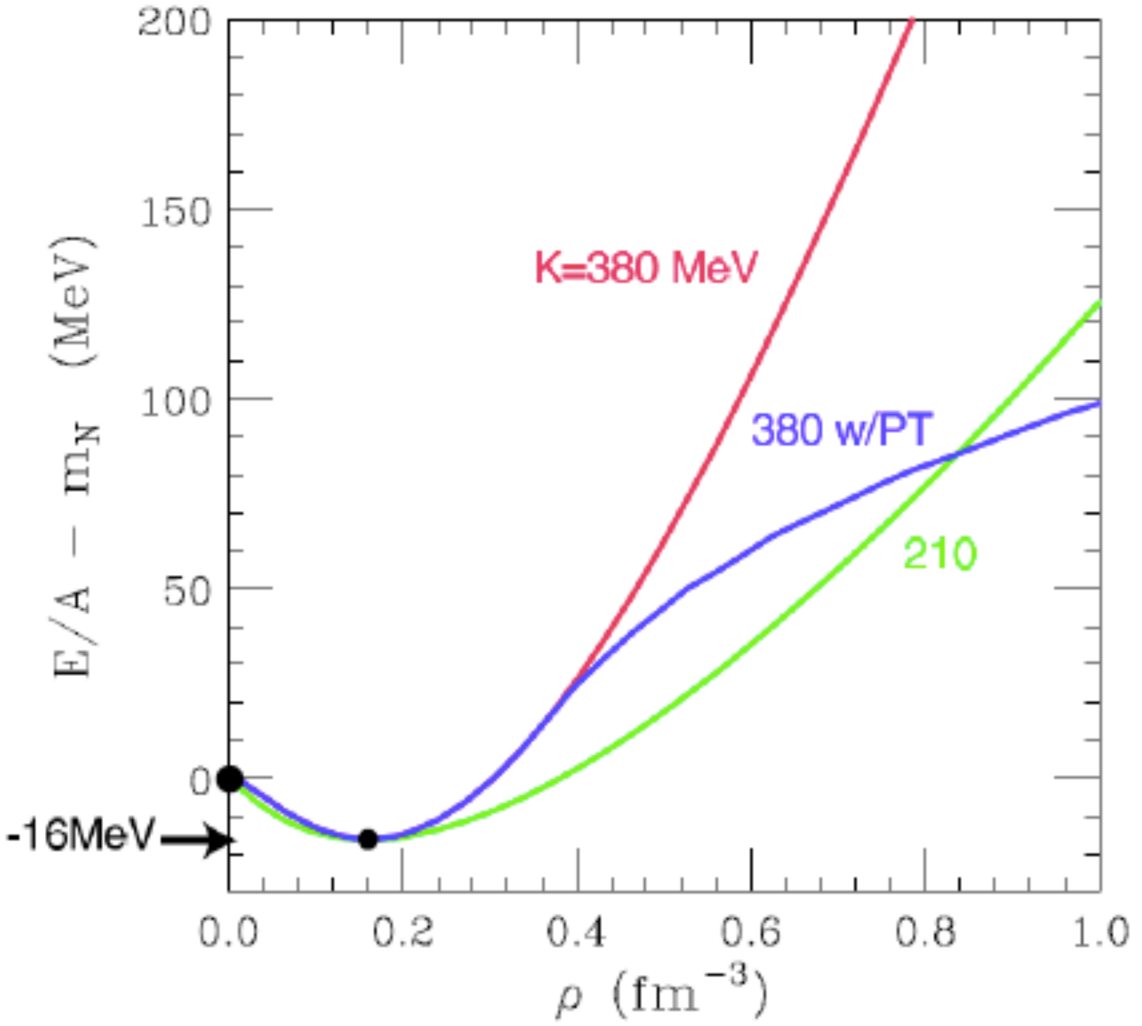
- All nuclei reach the same core density ρ_0≈**0.16fm^{-3}** ≈1/(6fm^3) with a binding energy of about **-16 MeV**
- At **ρ=0** the binding energy is **0 MeV** because nucleons are completely separated.

Incompressibility of the EOS

- With this we know already two points of the EoS:

- The incompressibility K quantifies how much energy per nucleon you need to compress matter.
- K is defined as the 2[nd] derivative of the energy E with respect to ρ.

$$K = 9\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right) = R^2 \frac{d^2}{dR^2}\left(\frac{E}{A}\right)$$

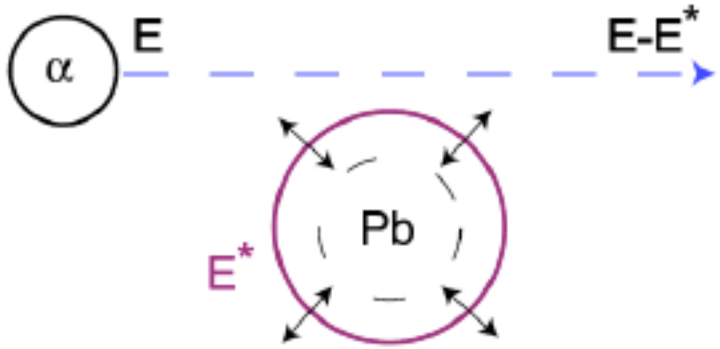


- Fitting a simple parabola through the two points would result in **K=290 MeV**.
- However the EoS could also be anything else than a parabola
- In General: EoS is called **soft if K<290 MeV**

  EoS is called **stiff if K>290 MeV**

Measure Incompressibility

- How to measure the incompressibility K?
- One can study the radial vibration of the nucleus!
- This can e.g. be done by shooting α particles on a nucleus.



- The excitation energy has the following form:

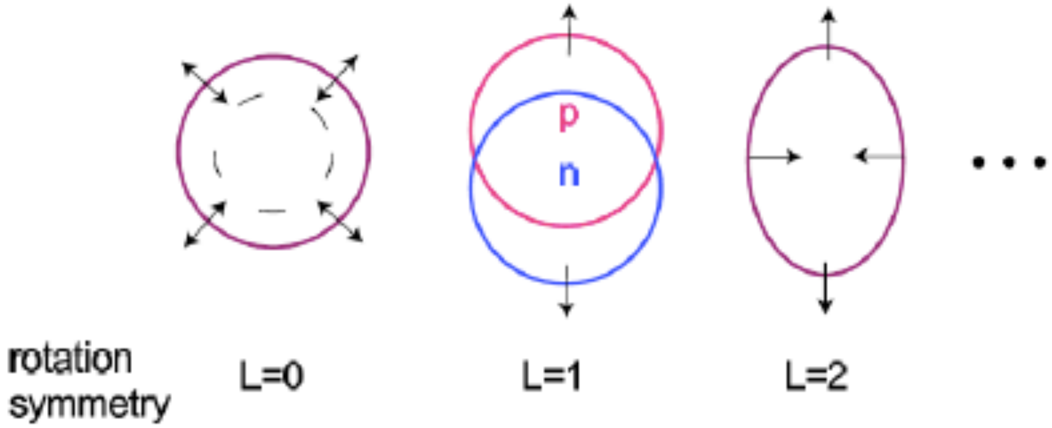
$$E_{tot} = \int d\vec{r}\rho \frac{m_N v^2}{2} + \frac{1}{2} AK(R-R_0)^2 = \frac{A m_N \langle r^2 \rangle_A \dot{R}^2}{2} + \frac{1}{2} AK(R-R_0)^2$$

$$\Rightarrow E^* = \hbar\omega = \hbar \sqrt{\frac{K}{m_N \langle r^2 \rangle_A}}$$

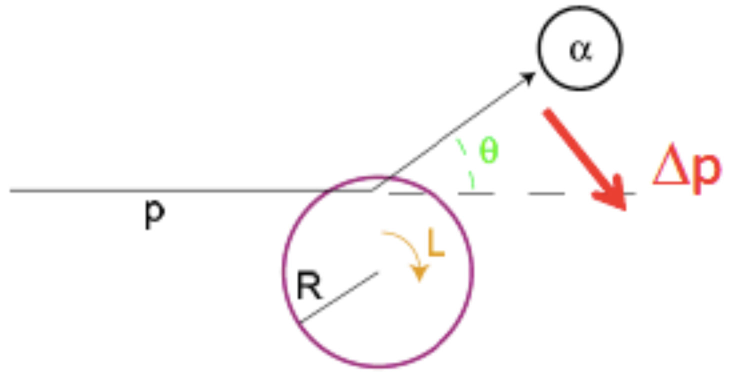
- Corrections are needed due to Coulomb interaction etc.
- Finally the K factor for infinite matter compared to K measured for nucleus: **$K_{inf}=3/2K_{nucl}$**

Measure Incompressibility

- Task for the measurement: Measure radial excitation, i.e. vibration angular momentum **L=0** (s-wave)!



- To isolate L=0 one has to measure at very forward angles! Why?



$$L < |p - p'| R = \Delta p R = p \sin(\Theta) R \approx p \Theta R$$

$$L < 1\hbar \quad \Rightarrow \quad \Theta < \frac{\hbar}{pR}$$

Measure Incompressibility

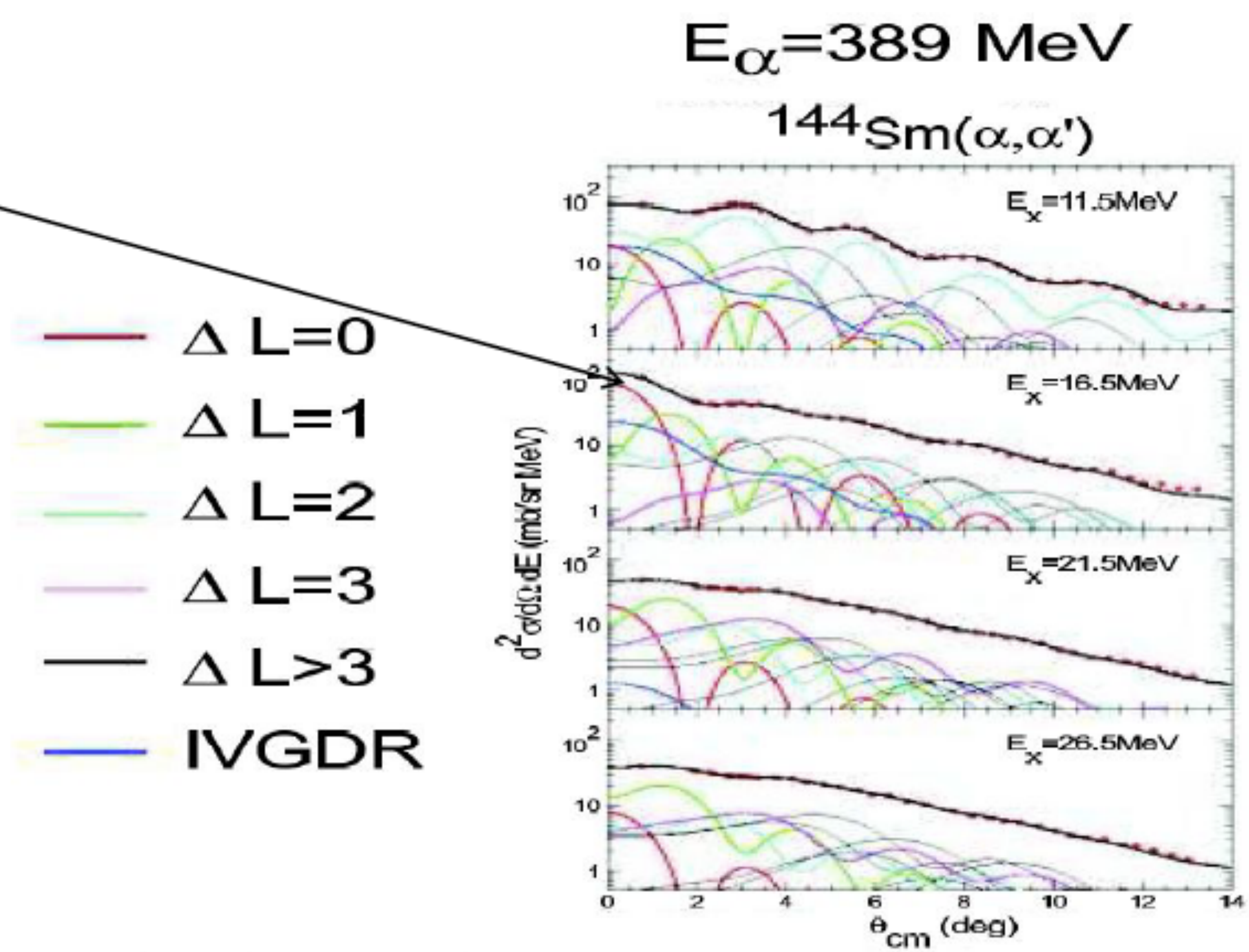
PRL 39, 19 (1977)

Results of measurement (Sm target):

Pronounced L=0 resonance at E*=15.5 MeV

$$K^{Sm} = \frac{E^{*2} m_N \langle r^2 \rangle_A}{\hbar^2} = 138 MeV$$

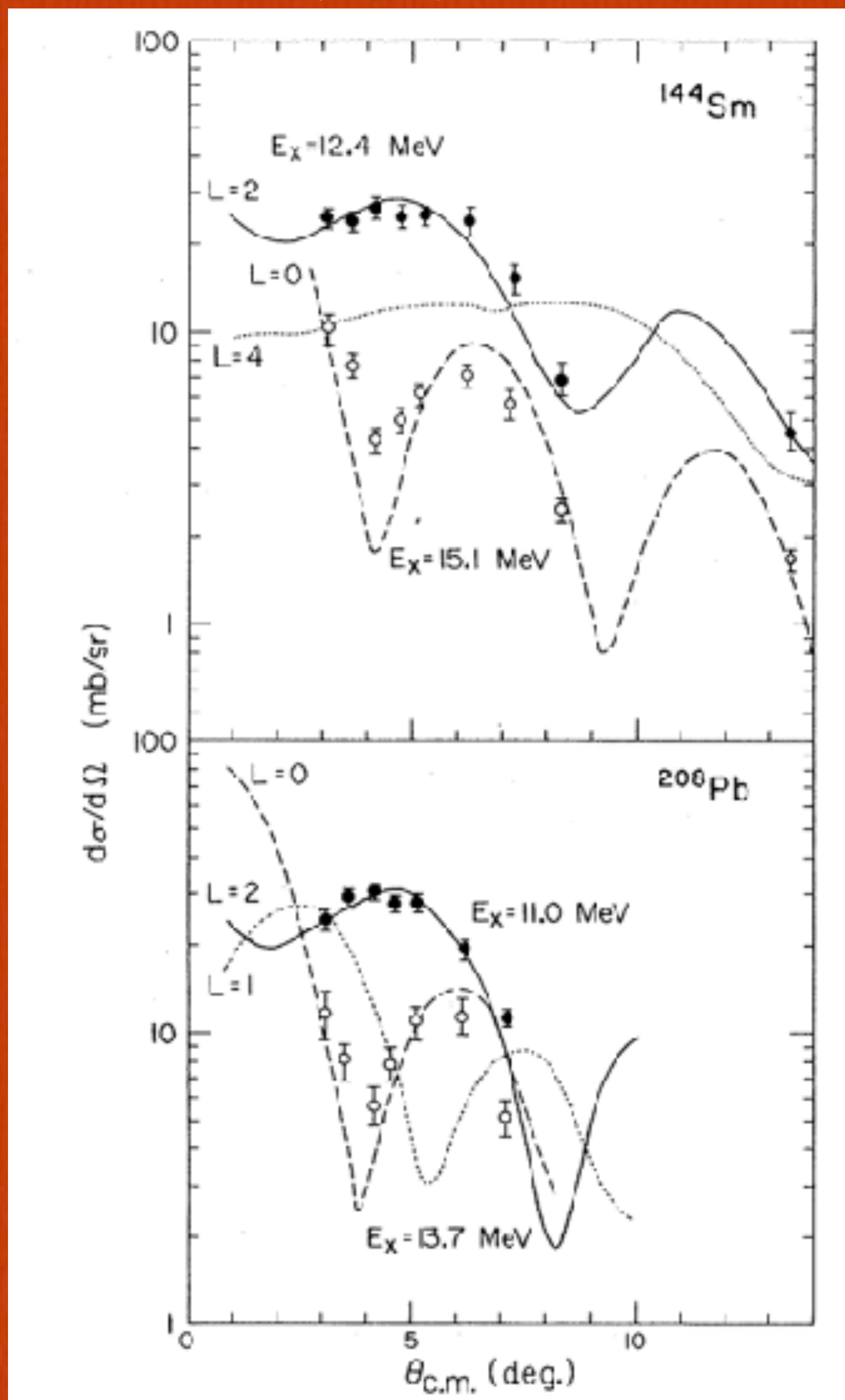
It follows:
$K = K^{sm}/0.67 = 210MeV$



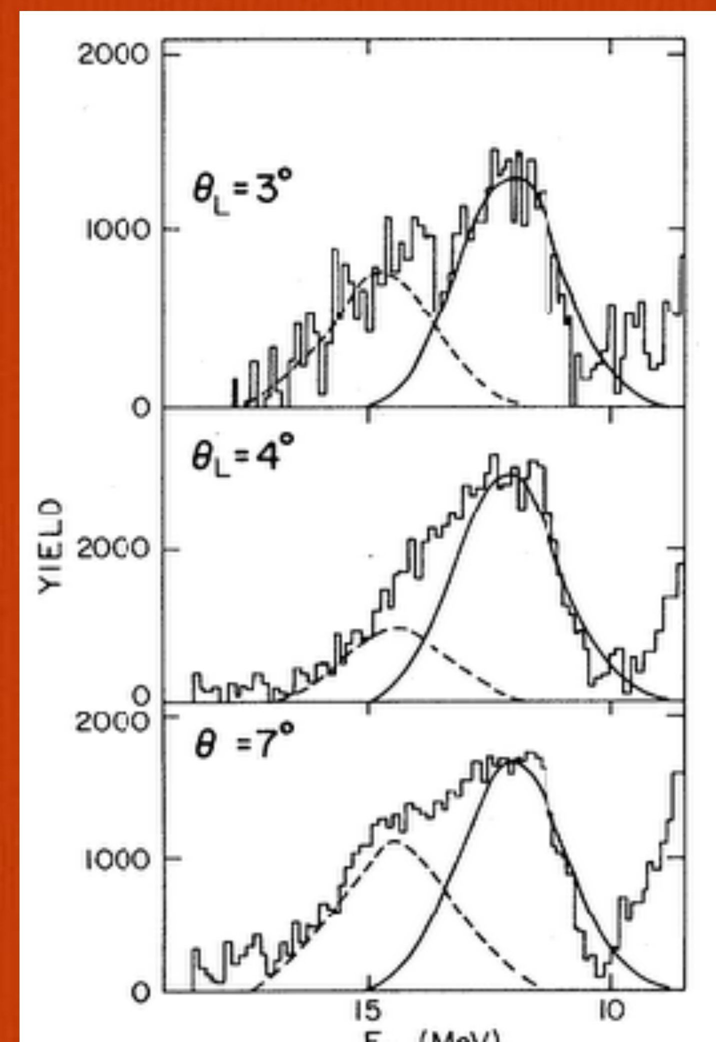
Compared with the value extracted before (290 MeV).
EoS seems to be **soft around ρ_0**!

Measure Incompressibility

PRL 39, 19 (1977)

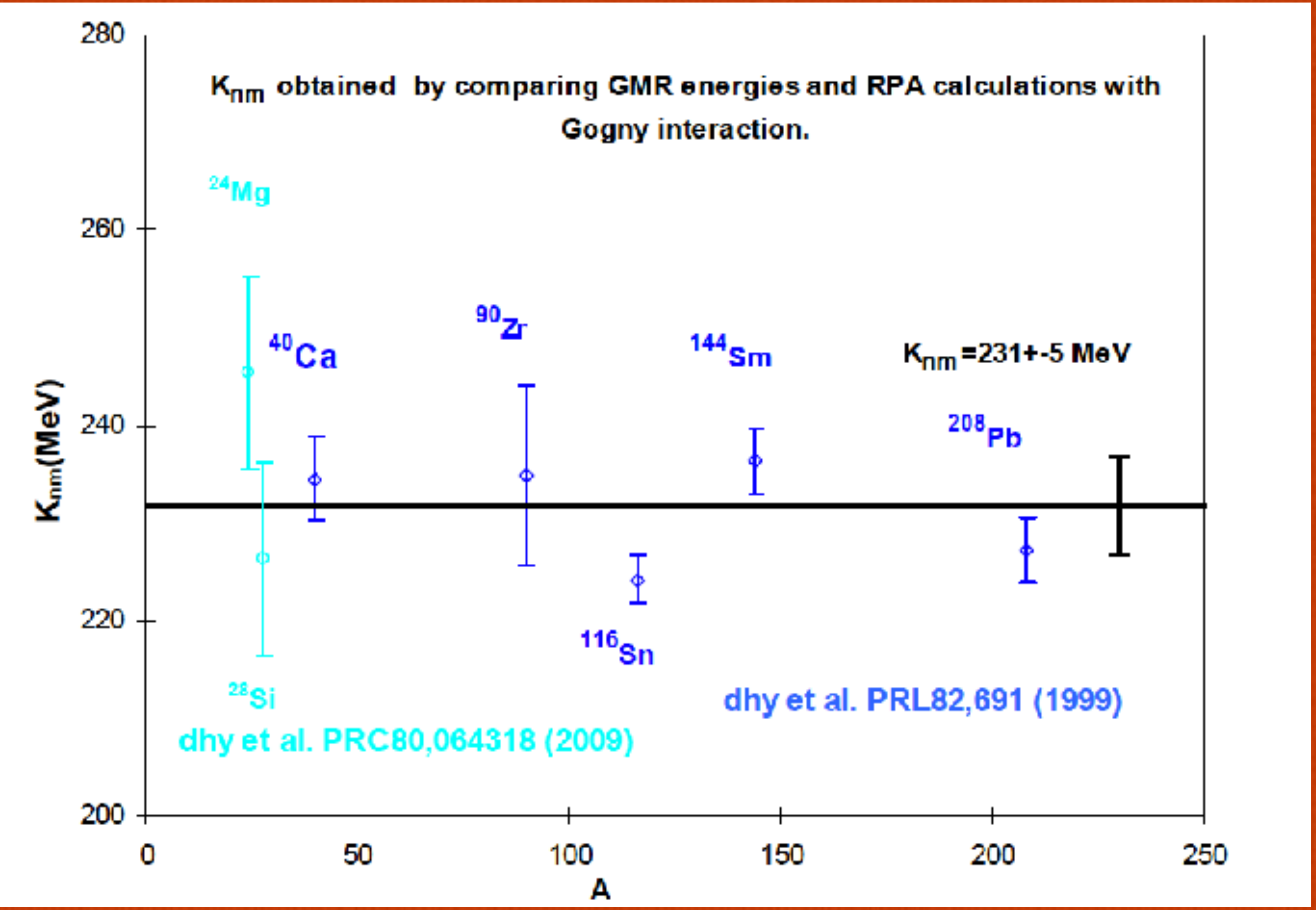


$^{144}Sm(\alpha, \alpha'),\ (E_\alpha = 96\,\mathrm{MeV})$



Simultaneous fit of angular distribution and energy spectrum

Measure Incompressibility



All measurements of giant monopoles speak for a rather soft nuclear equation of state at saturation density

Neutron Matter EoS

Starting point: EoS of cold nuclear matter

$$E(\rho,\delta) = E_0(\rho,\delta=0) + S(\rho)\delta^2, \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

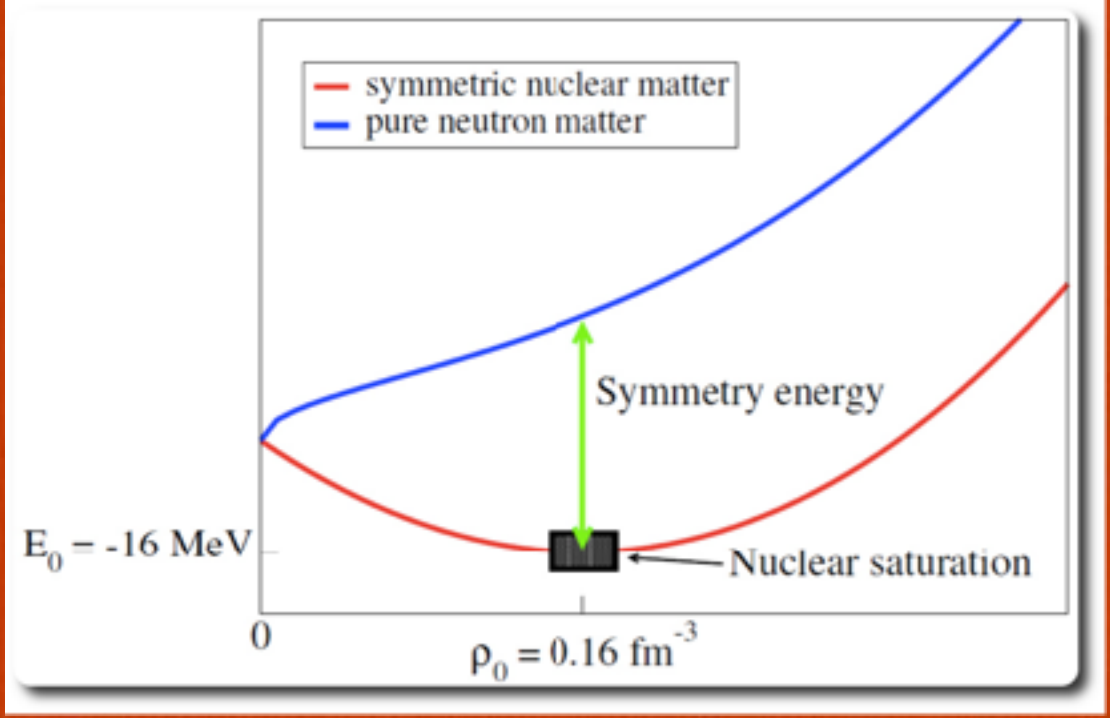
nuclear symmetry term

expansion around saturation density:

$$S(\rho) = S_0 - L\epsilon + K_{sym}\epsilon^2 + O[\epsilon^3] \quad \epsilon = \frac{\rho_0 - \rho}{3\rho_0}$$

slope parameter

curvature



$$L = 3\rho_0 \frac{dS(\rho)}{d\rho}|_{\rho_0} = [3/\rho_0]P_0$$

P_0 : Pressure in pure neutron matter

Experiments connected to the Symmetry Energy

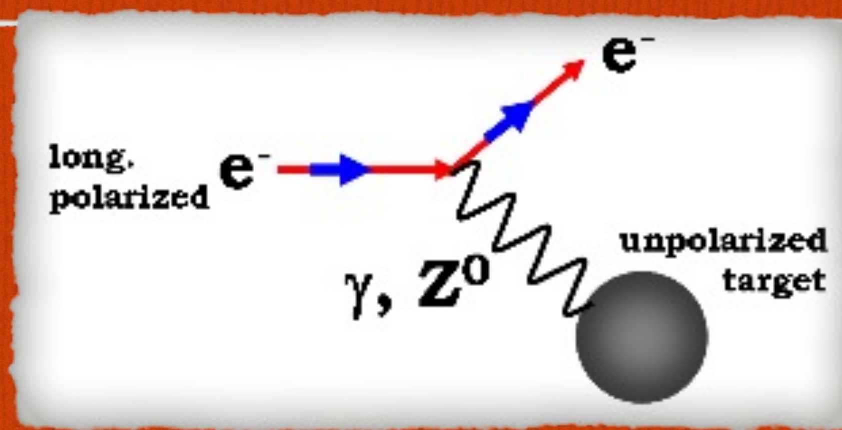
Saturation Density:

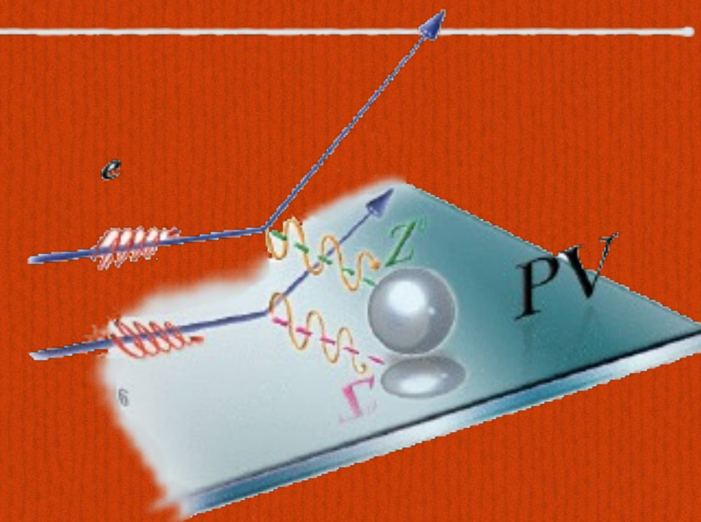
- P-Rex (JLab), M-Rex(Mainz)
- MSU Experiments

Beyond Saturation Density:

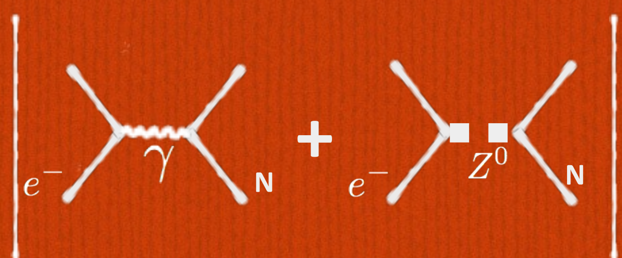
- ASY-EOS (Catania)
- HIC (for nuclear matter EOS)

Parity violation and neutron density

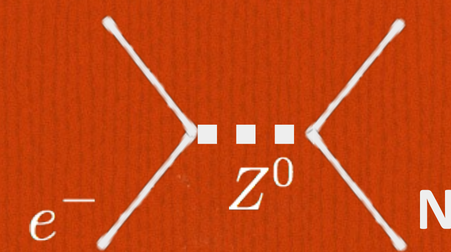




...since...

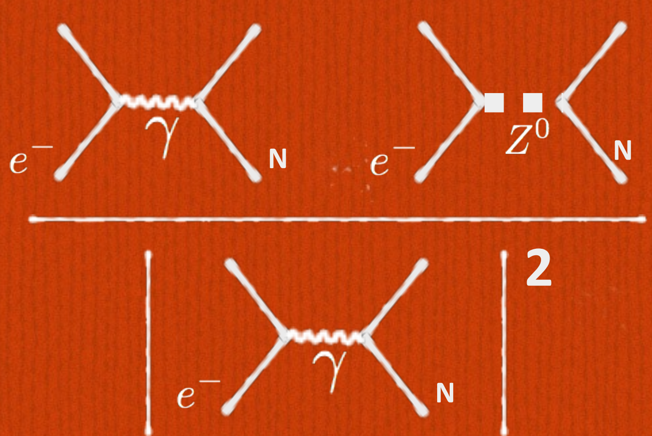
$\sigma \propto$ 

...to measure ...



....construct ....

$$A_{PV} = \frac{\left(\frac{d\sigma}{d\Omega}\right)_+ - \left(\frac{d\sigma}{d\Omega}\right)_-}{\left(\frac{d\sigma}{d\Omega}\right)_+ + \left(\frac{d\sigma}{d\Omega}\right)_-} \approx$$

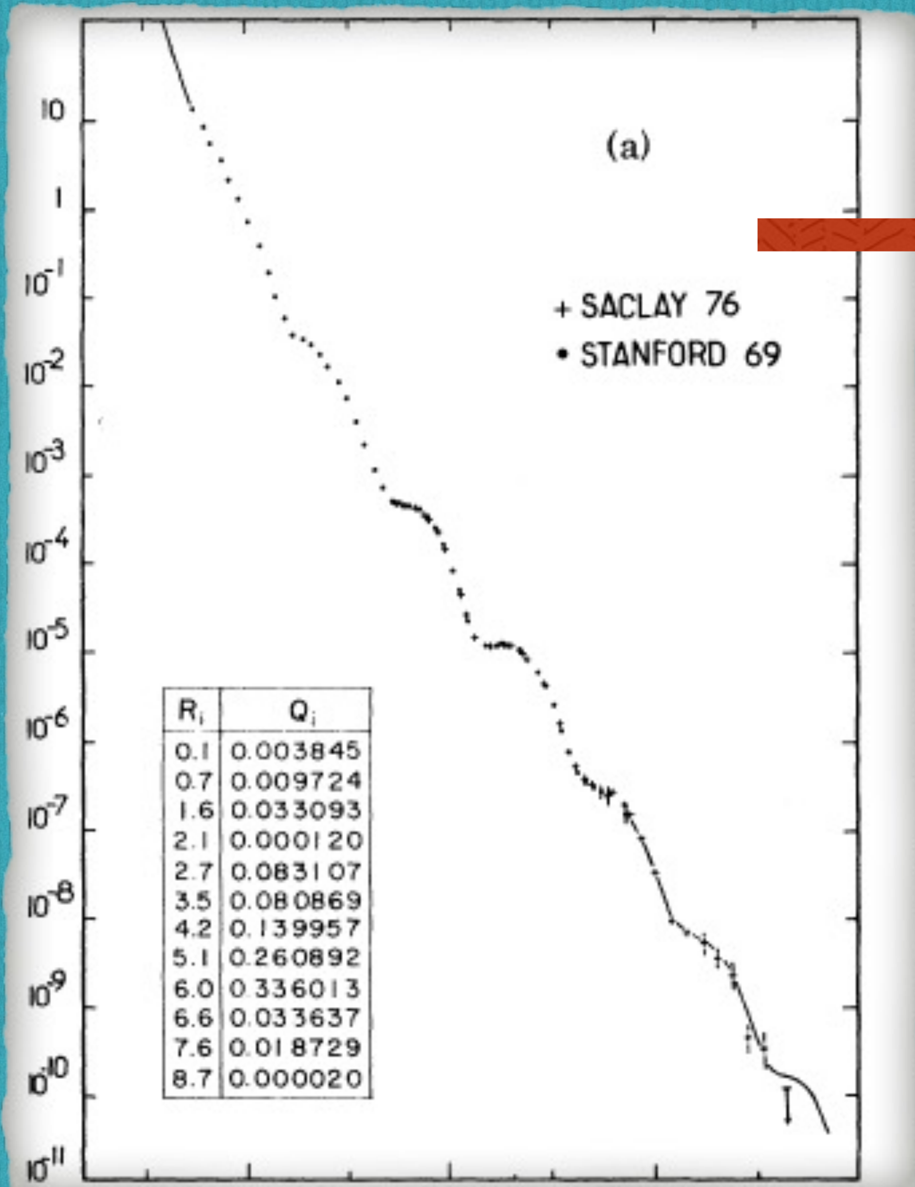


$$= \frac{G_F Q^2}{2\pi\alpha\sqrt{2}}\left[\underbrace{1-4\sin^2\theta_W}_{\approx 0} - \frac{F_n(Q^2)}{F_p(Q^2)}\right]$$

$$F_{n,p}(Q^2) = \frac{1}{4\pi}\int d^3r\; j_0(qr)\,\rho_{n,p}(r)$$

Matter and charge radii

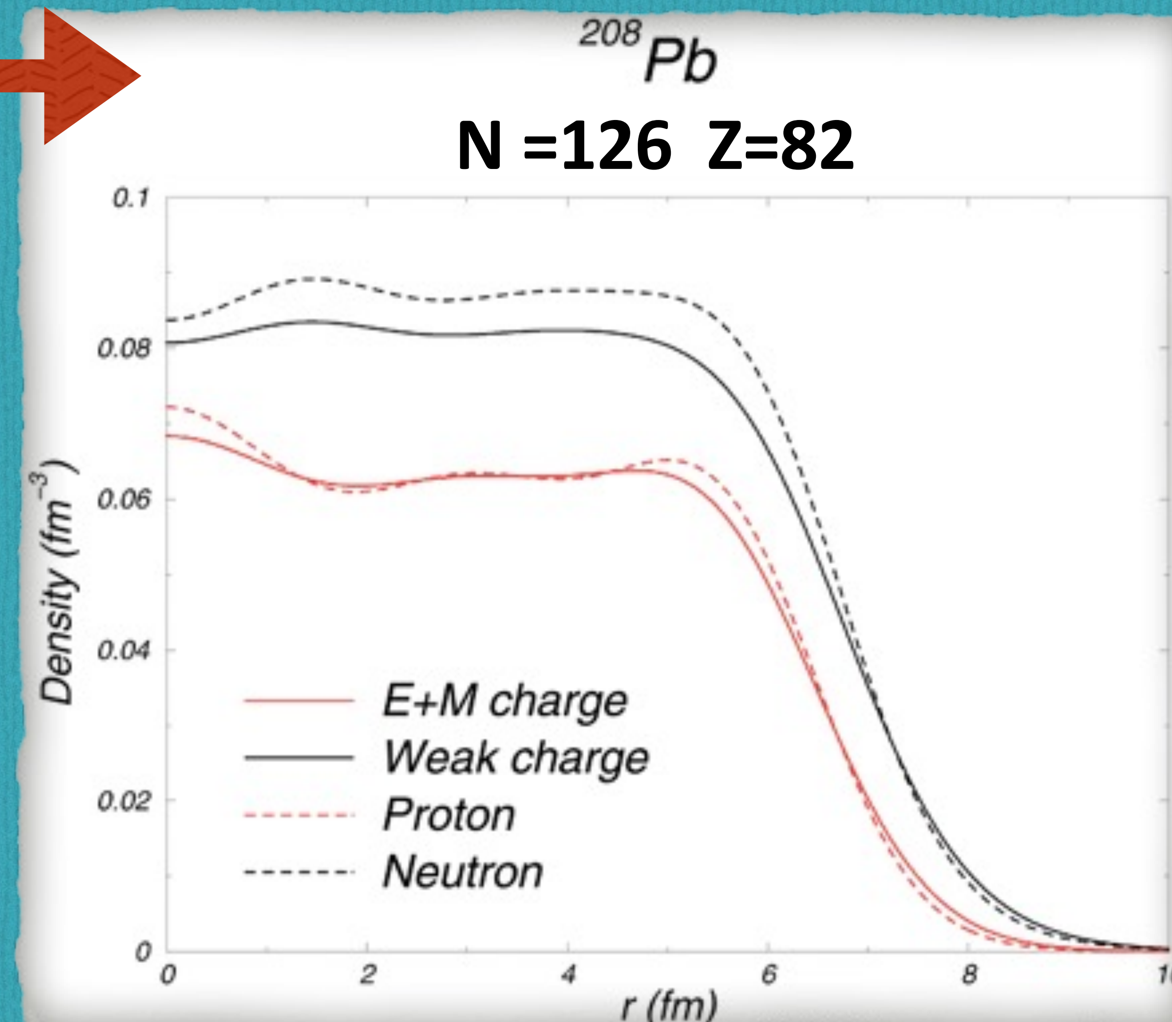
$\frac{d\sigma}{d\Omega}\left(\frac{mb}{str}\right)$



R_i	Q_i
0.1	0.003845
0.7	0.009724
1.6	0.033093
2.1	0.000120
2.7	0.083107
3.5	0.080869
4.2	0.139957
5.1	0.260892
6.0	0.336013
6.6	0.033637
7.6	0.018729
8.7	0.000020

$q(fm)^{-1}$

$$F_{n,p}(Q^2) = \frac{1}{4\pi}\int d^3r\ j_0(qr)\ \rho_{n,p}(r)$$



Pressure forces neutrons out against surface tension

P-REX results

Measured A_{PV}

Correct for Coulomb Distortions

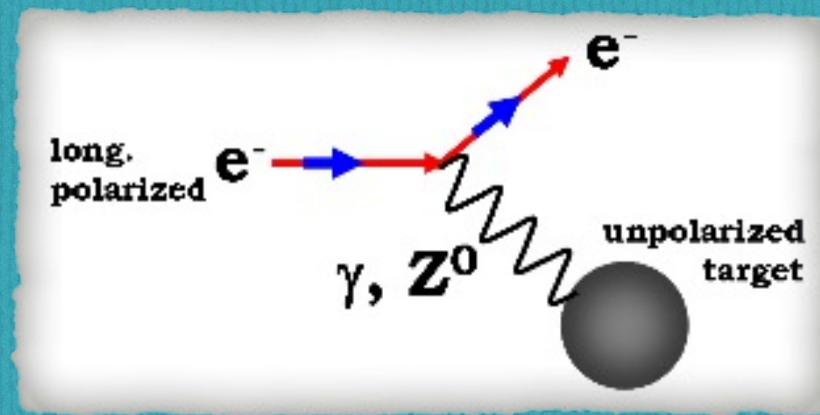
Weak density at one Q^2

Small corrections for G_E^n G_E^s MEC

Neutron density at one Q^2

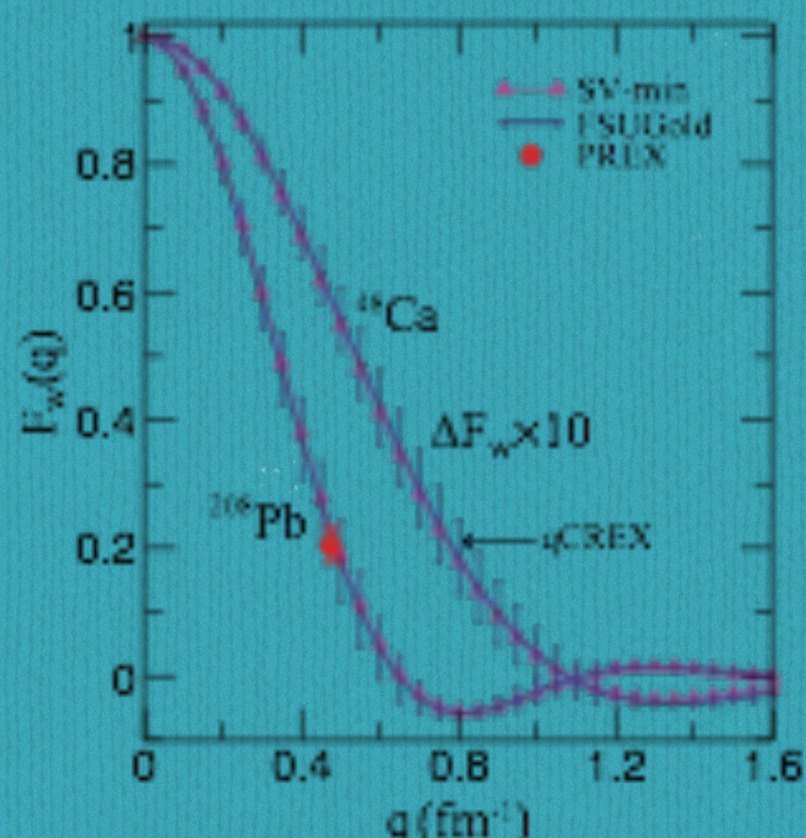
Assume surface thickness good to 25% (MFT)

R_n



Beam Energy = 1 GeV

$$A_{PV}^{Pb} = 656 \pm 60(stat) \pm 14(syst)ppb$$



$$R_n = 5.78^{+0.16}_{-0.18}$$

R_p measured with electron scattering

$$\delta r_{np} = R_n - R_p = 0.33^{+0.16}_{-0.18}$$

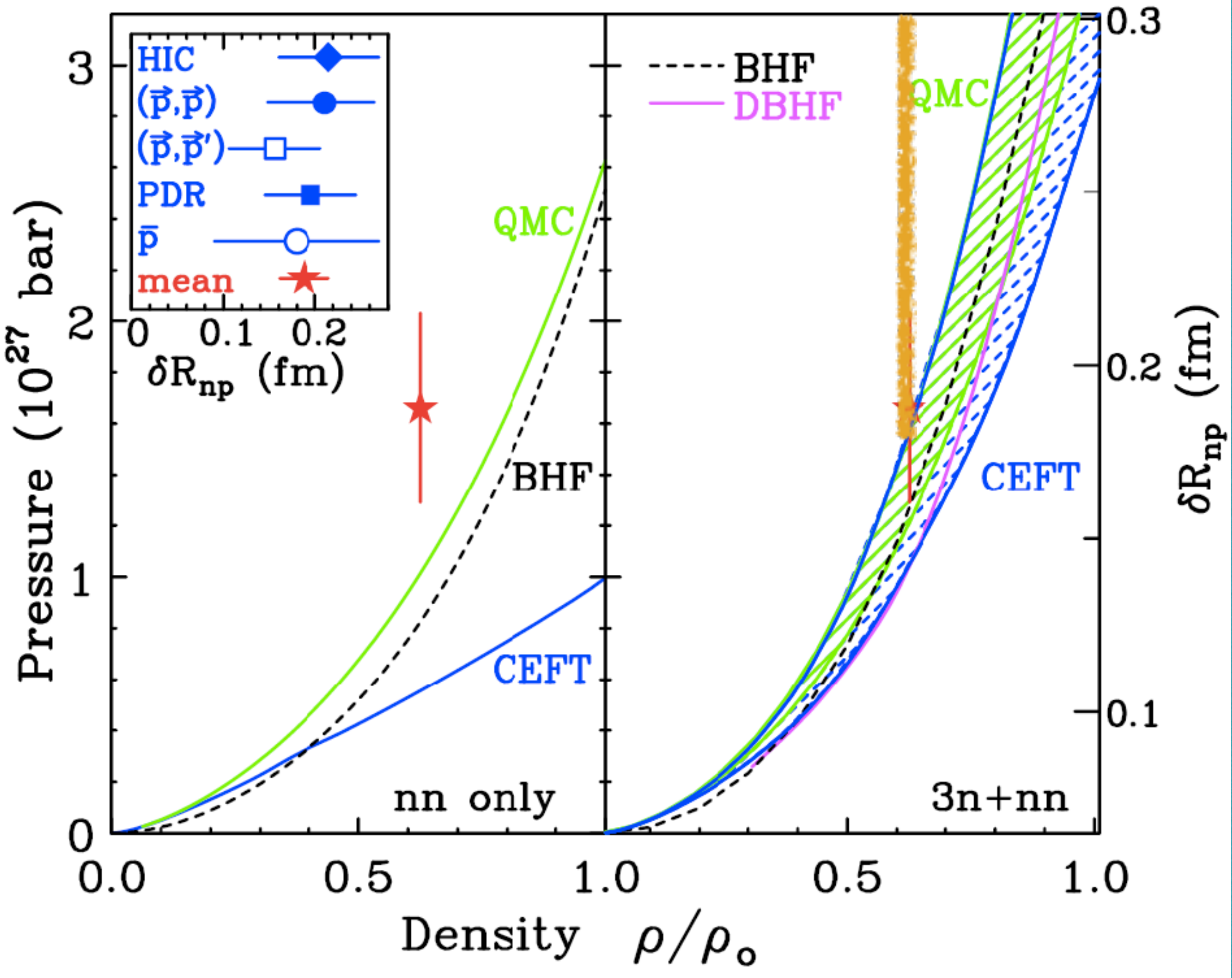
P-REX Coll., Phys. Rev. Lett. 108 (2012) 112502
C. J. Horowitz et al. Phys. Rev. C 85, 032501 (2012)
PHYSICAL REVIEW C88, 034325 (2013)

Neutron matter pressure

Density dependence over pressure for pure neutron matter

P-REX

$P = (1.41 \pm 0.41) \cdot 10^{-27} bar$



M.B. Tsang et al. Phys. Rev. C 86, 015803 (2012)

Calculations: Without 3N nuclear forces With 3N nuclear forces

Experimental value: average among all measurements

Experiments connected to the Symmetry Energy

Saturation Density:

- P-Rex (JLab), M-Rex(Mainz)
- MSU Experiments

Beyond Saturation Density:

- ASY-EOS (Catania)
- HIC (for nuclear matter EOS)

Symmetry energy at subnormal densities

Parametrisation of the symmetry energy used in transport models:

$$S(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

$$C_{s,k} = 25\,MeV \qquad S_0 = 30\,MeV$$

$$C_{s,p} = 35.2\,MeV$$

Tools:
measure neutron/proton yields
measure Isospin diffusion
in collisions involving ^{112}Sn and ^{124}Sn nuclei at kinetic energies of 50 MeV/A

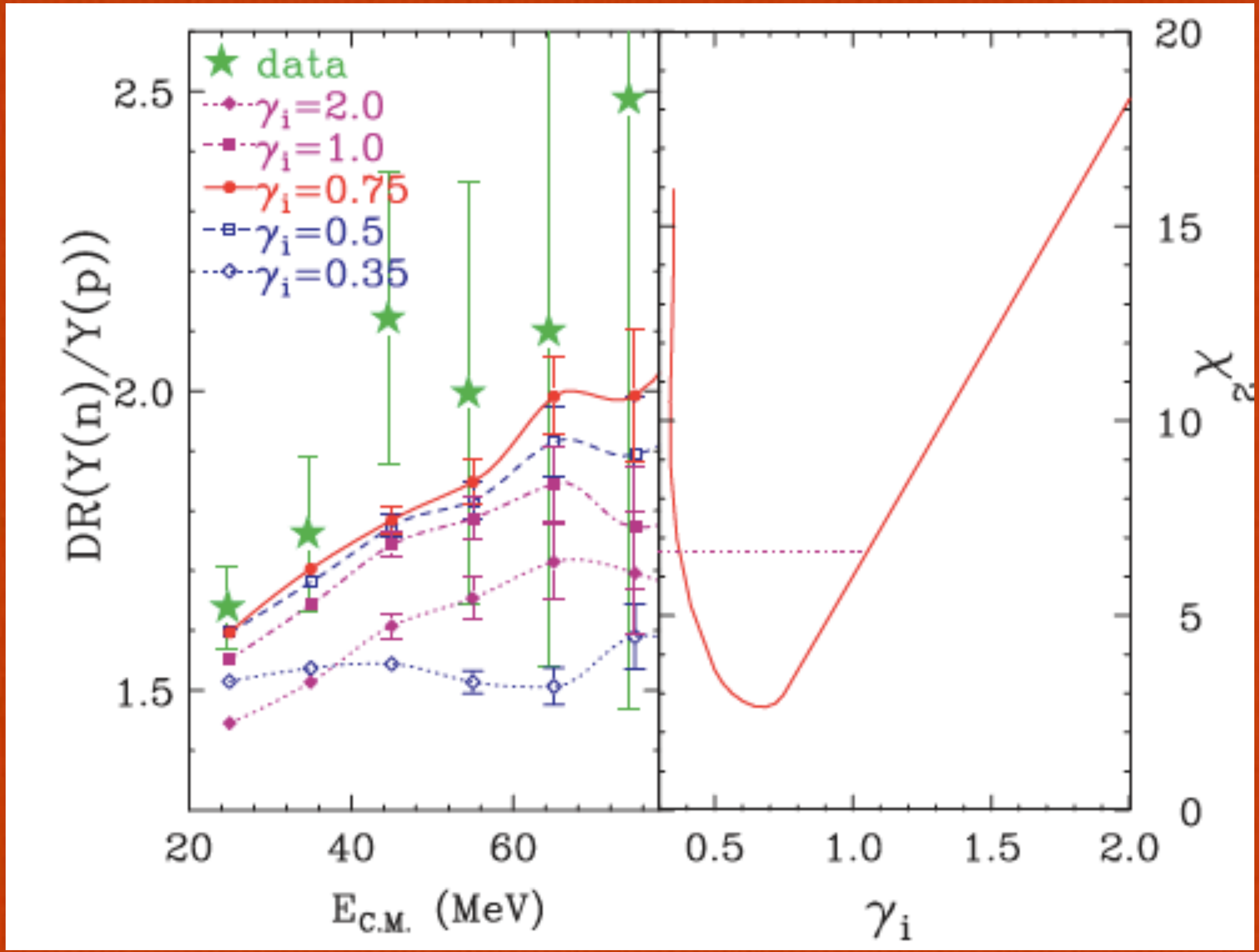
^{112}Sn: 62 neutrons 50 protons -> neutron poor
^{124}Sn: 74 neutron 50 protons -> neutron rich

The parameter γ is extracted by comparing the experimental data to transport models

Neutron to Proton Ratios

$$\mathrm{DR}\,(Y(n)/Y(p)) = R_{n/p}(A)/R_{n/p}(B) = \frac{dM_n(A)/dE_{\mathrm{c.m.}}}{dM_p(A)/dE_{\mathrm{c.m.}}} \cdot \frac{dM_p(B)/dE_{\mathrm{c.m.}}}{dM_n(B)/dE_{\mathrm{c.m.}}},$$

A= ^{124}Sn+ ^{124}Sn
B= ^{112}Sn + ^{112}Sn



PRL **102**, 122701 (2009)

$dM_n(A)/dE_{c.m.}$

Measured neutron energy spectrum for the colliding system A

DR> 1 since more neutron than protons
Increasing DR value as a function of energy: Neutron pushed because of the symmetry energy

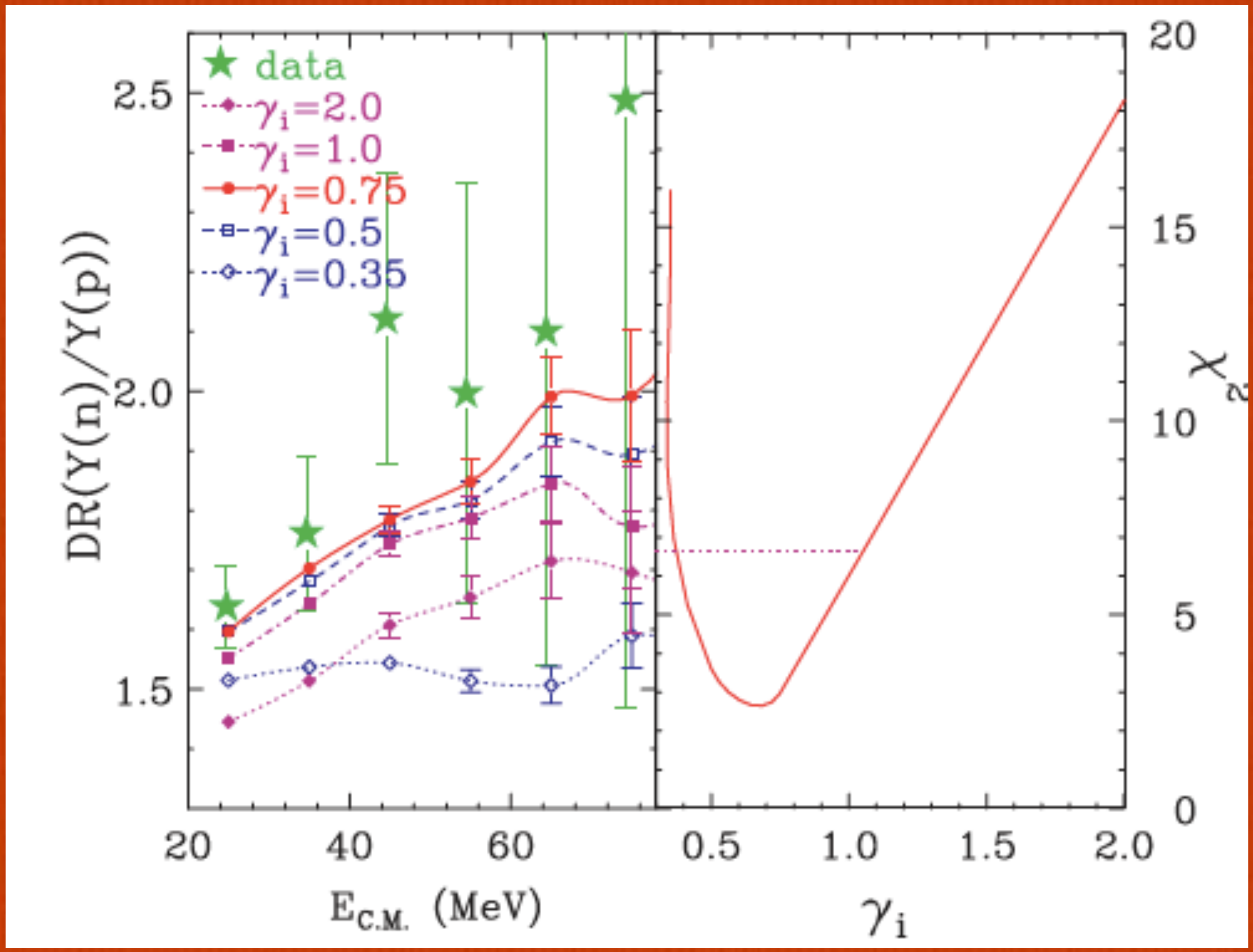
Neutron to Proton Ratios

$$\mathrm{DR}\,(Y(n)/Y(p)) = R_{n/p}(A)/R_{n/p}(B) = \frac{dM_n(A)/dE_{\mathrm{c.m.}}}{dM_p(A)/dE_{\mathrm{c.m.}}} \cdot \frac{dM_p(B)/dE_{\mathrm{c.m.}}}{dM_n(B)/dE_{\mathrm{c.m.}}},$$

A= ^{124}Sn+ ^{124}Sn

B= ^{112}Sn + ^{112}Sn

$$S(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$



Explanation: since for these esp.

$\rho < \rho_0$

the number of neutron is larger for smaller values of γ

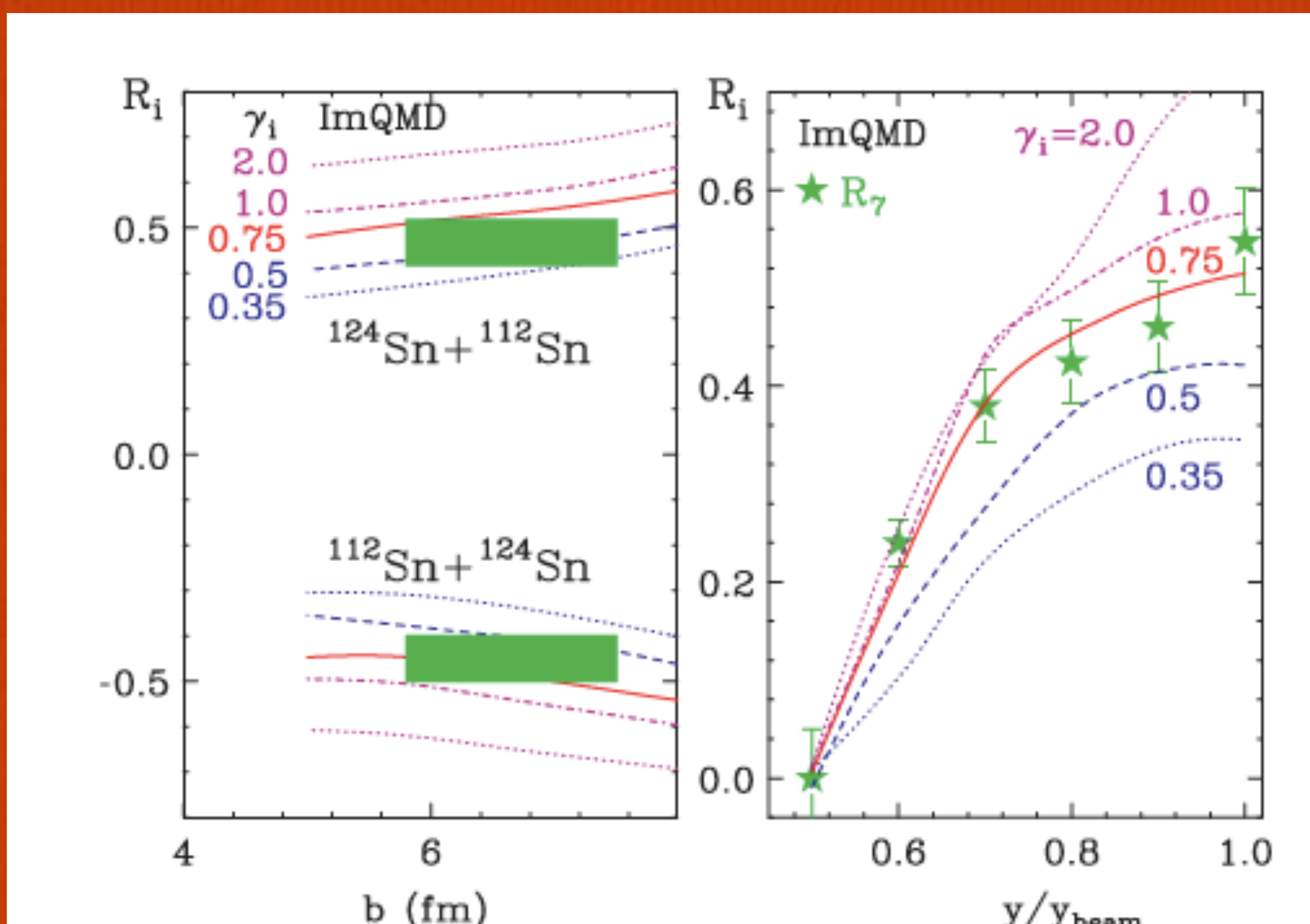
But for very small γ the system disintegrates so that the ratio goes towards the limit 1.2

PRL 102, 122701 (2009)

Isospin Diffusion

time

measured fragments



$$R_i(X) = 2\frac{X - (X_{A+A} + X_{B+B})/2}{X_{A+A} - X_{B+B}},$$

$$X = \frac{N - Z}{A}$$

PRL 102, 122701 (2009)

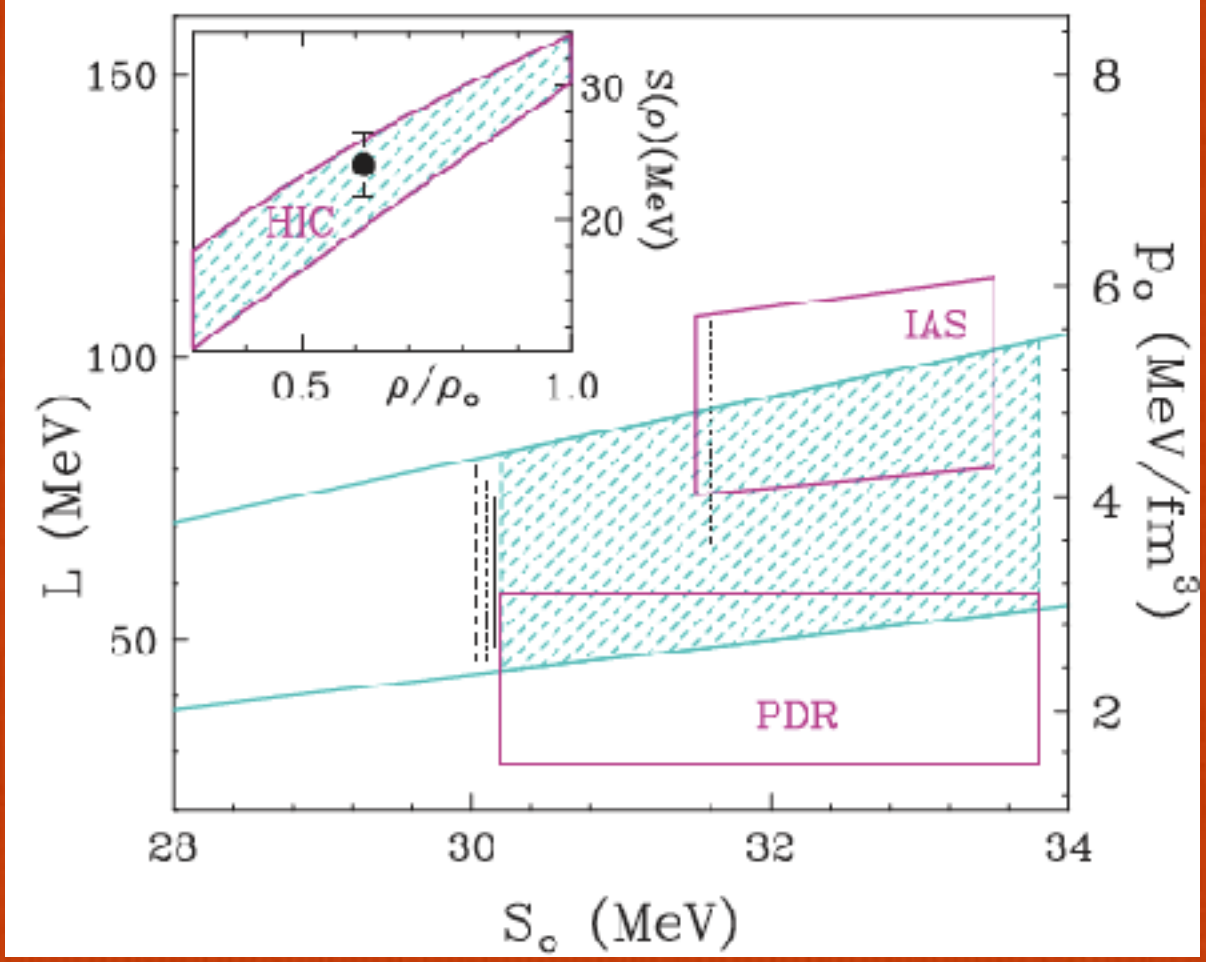
Symmetry Energy and Pressure

$$S(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

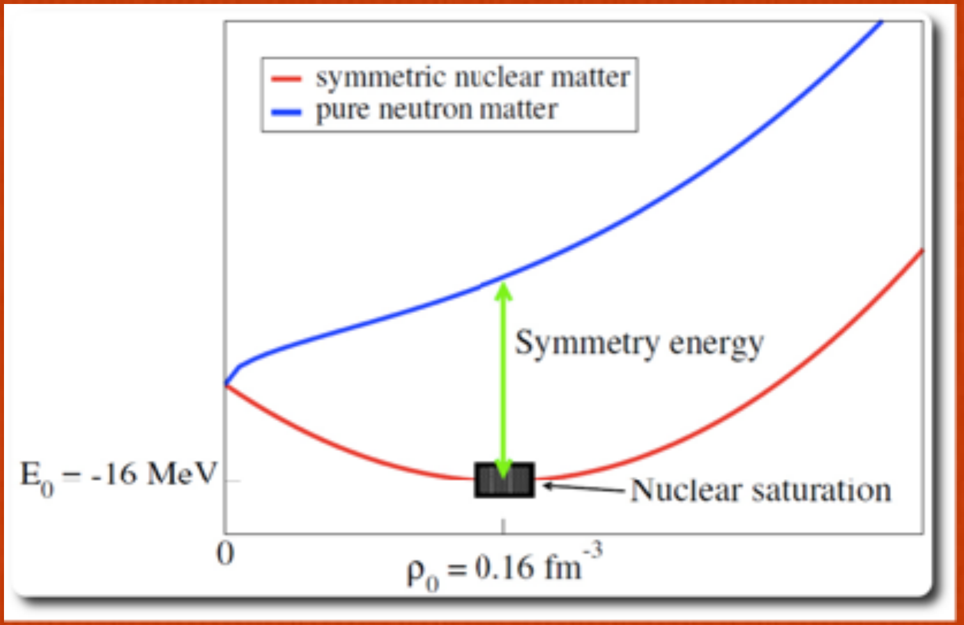
$\gamma_i \approx 0.75$

$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho - \rho_o}{\rho_o}\right) + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_o}{\rho_o}\right)^2 + \ldots$$

$$L = 3\rho_0 |dS(\rho)d\rho|_{\rho_0} = [3/p_0]_{\rho_0}$$



PRL 102, 122701 (2009)



P-REX

$$P = (1.41 \pm 0.41) \cdot 10^{-27} bar$$

$$\rightarrow P = 2.24 \pm 0.64 MeV/fm^3$$

Experiments connected to the Symmetry Energy

Saturation Density:

- P-Rex (JLab), M-Rex(Mainz)
- MSU Experiments

Beyond Saturation Density:

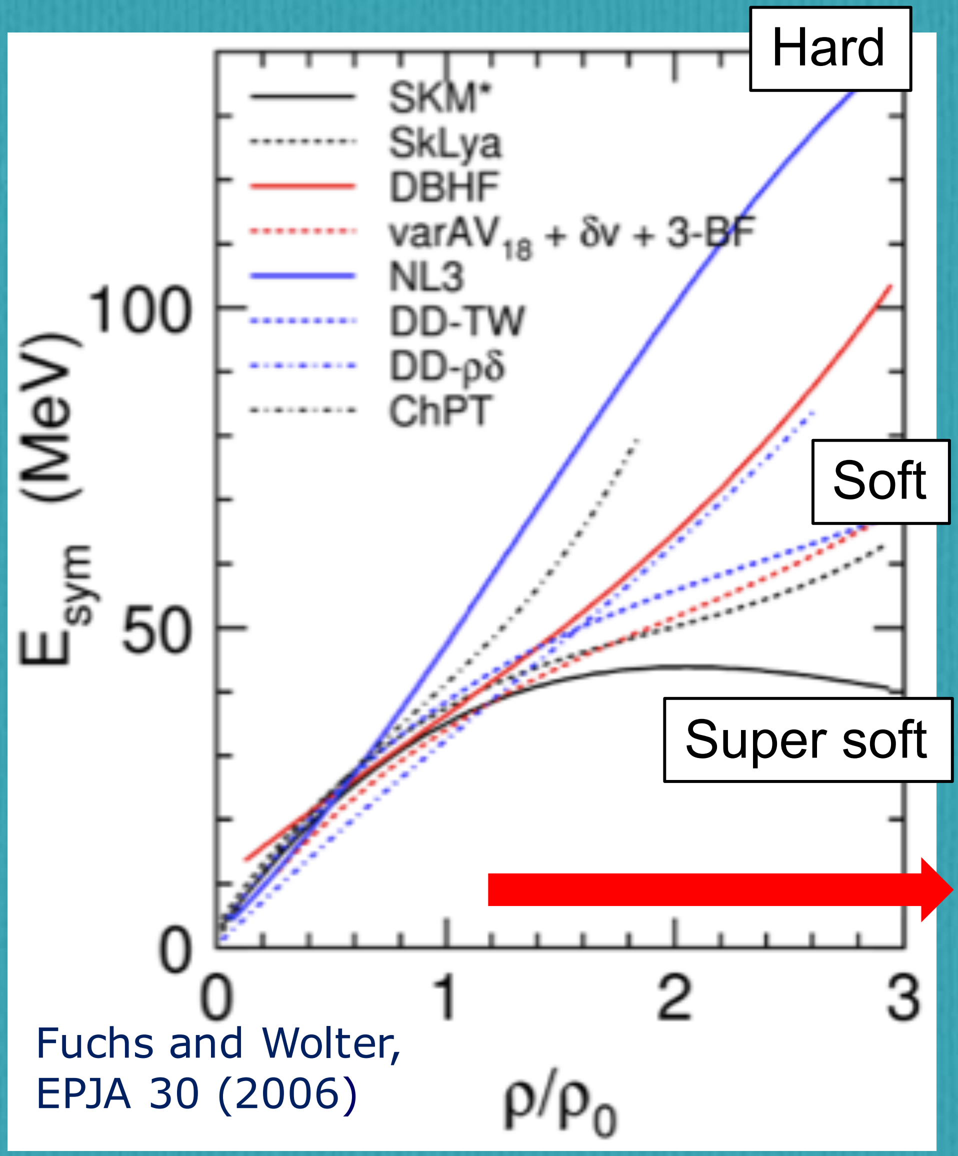
- ASY-EOS (Catania)
- HIC (for nuclear matter EOS)

Symmetry energy at supra-normal densities

Slope Parameter at ρ_0

$$L = 3\rho_0 \left| \frac{dE_{sym}(\rho)}{d\rho} \right|_{\rho_0}$$

Not constrained at large densities. Three-body forces unknown at large densities



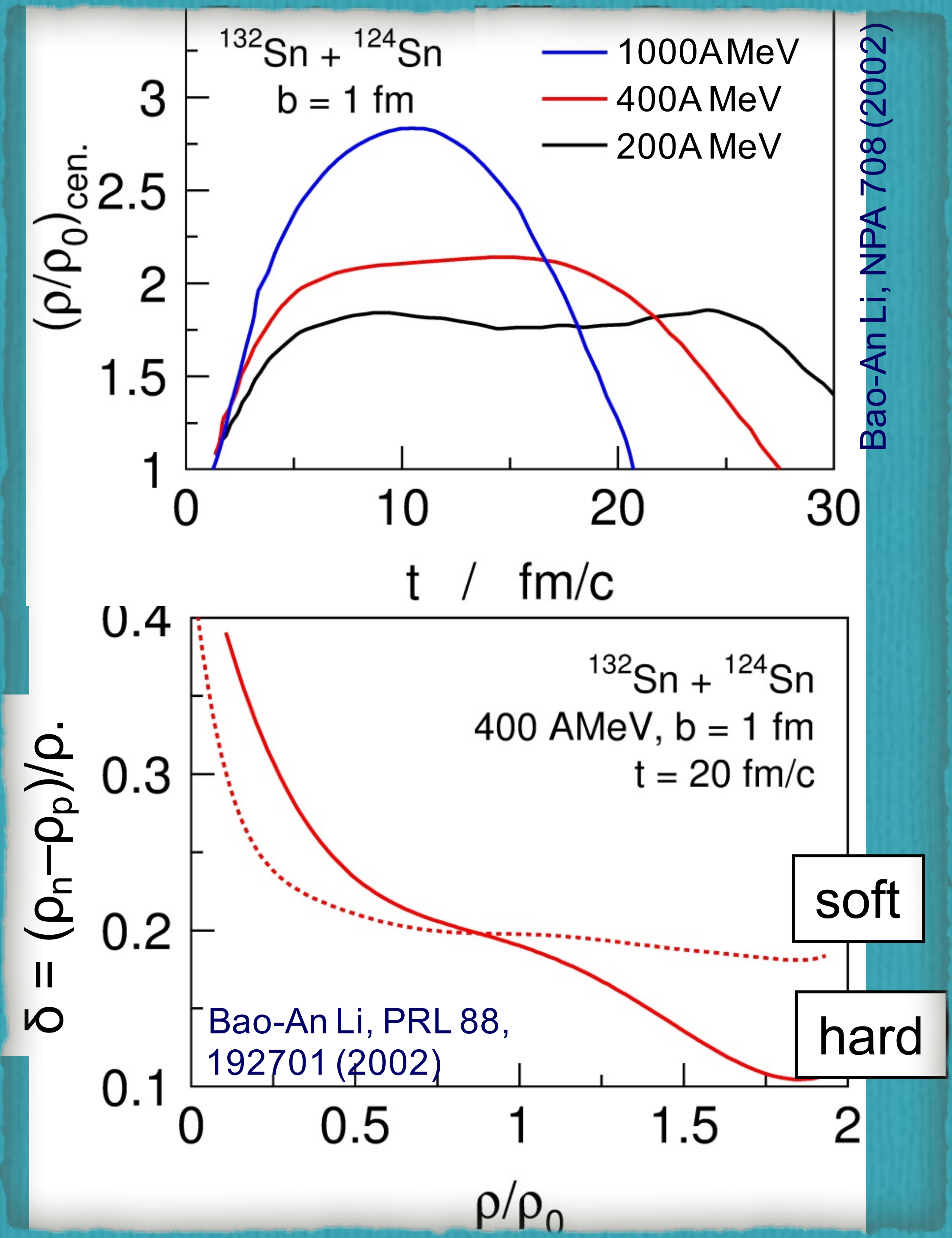
Fuchs and Wolter, EPJA 30 (2006)

Heavy Ion Reactions at Intermediate Energies

- reaching 2-3 ϱ_0
- non-equilibrated, dynamical system
- access to properties
    - microscopic transport models
- n/p ratio in dense phase depends on E_{sym}

Study observables sensitive to the difference in neutron/proton densities/potentials

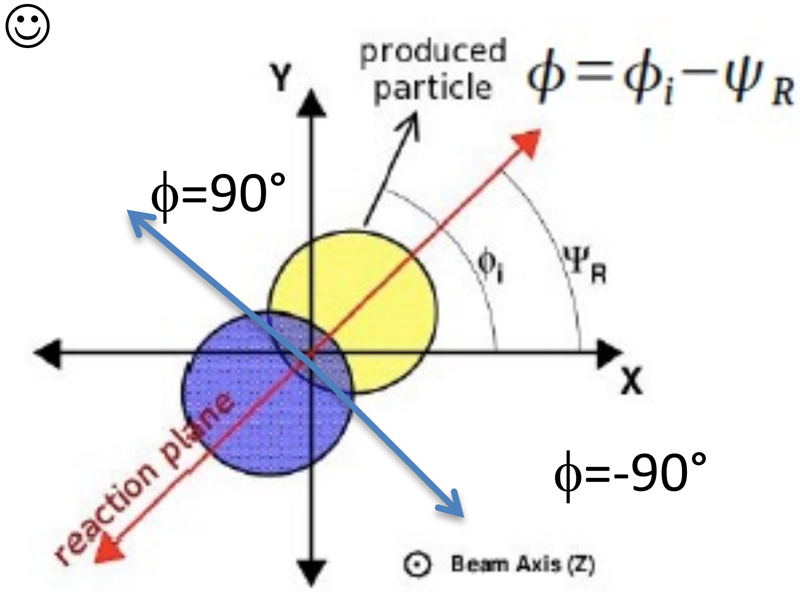
- n/p ratio and flows
- $t/^3He$ ratio and flows
- π^-/π^+ ratio, K^0/K^+ ratio ....

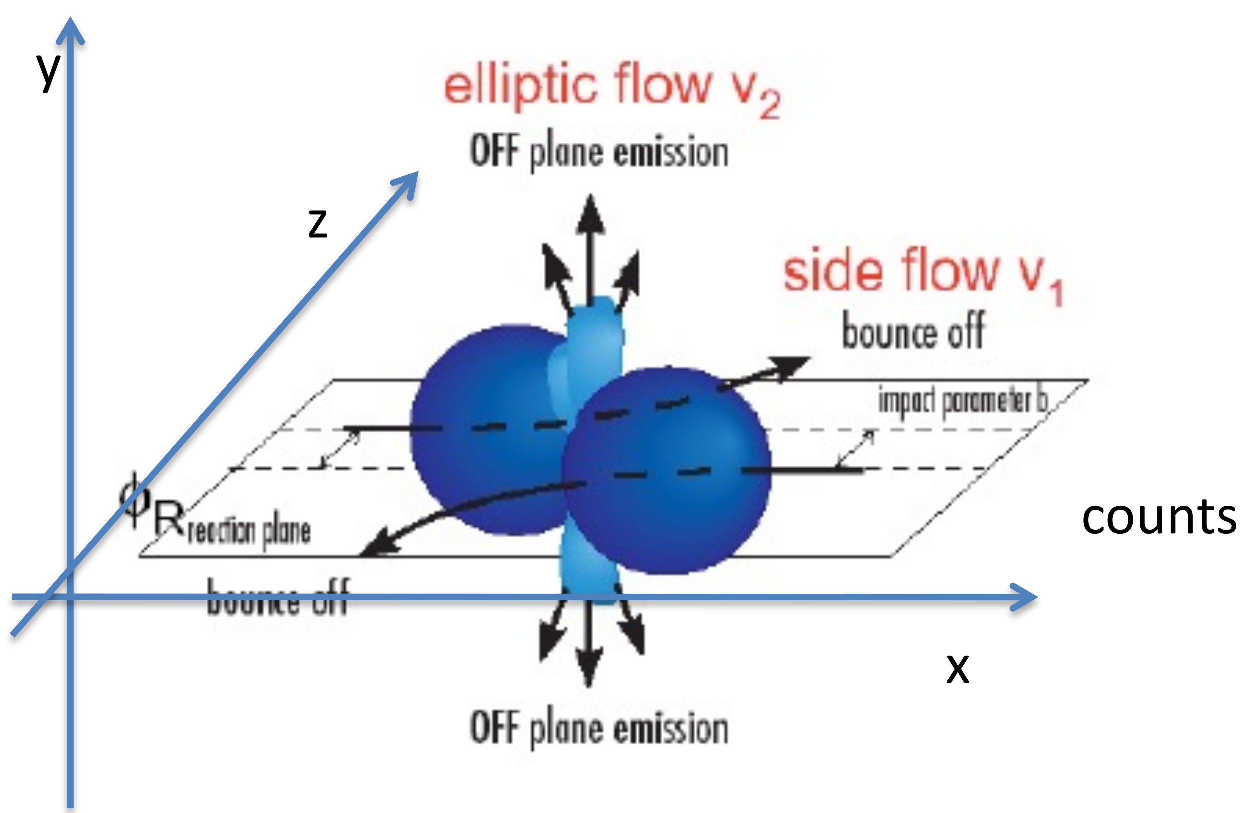


Definition of Flow

Again push and pull.. But respect to a reaction plane

Reaction plane: plane on which the reaction occurs ☺
Reacting nuclei trated like spheres



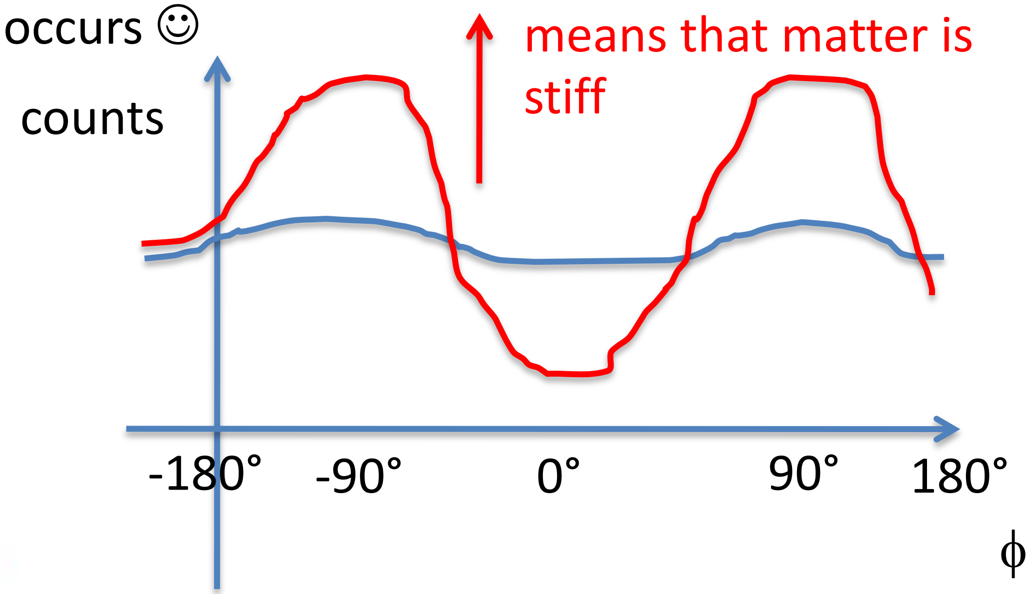


The strength of the interaction is extracted from the comparison with transport models

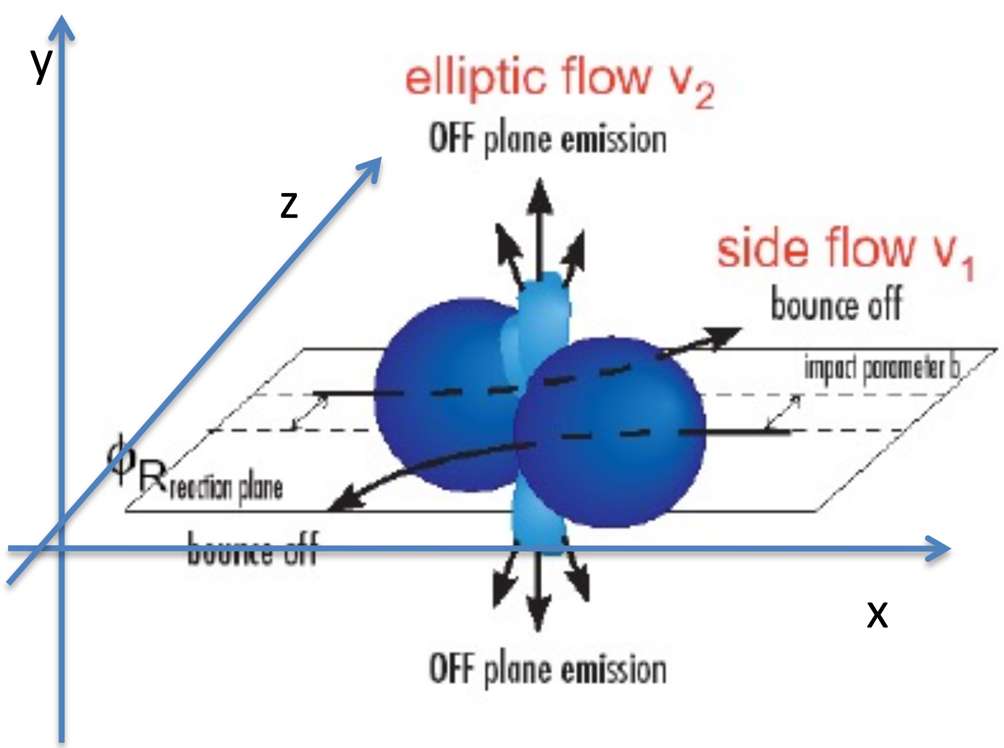
Flow in HIC at low (<GeV) energies

push and pull of particles.. But respect to a reaction plane

Reaction plane: plane on which the reaction occurs ☺
Reacting nuclei trated like spheres



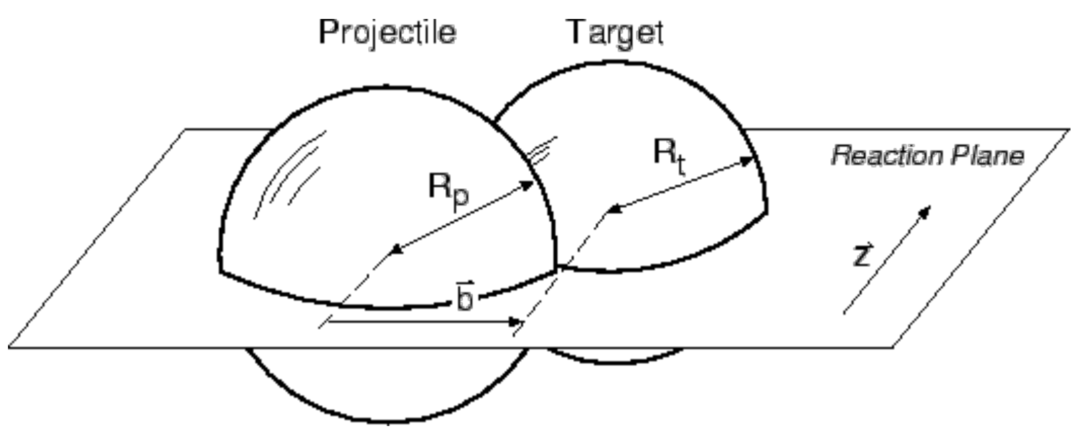
If matter is pushed out of plane becaue of the spectator presence it means that matter is stiff

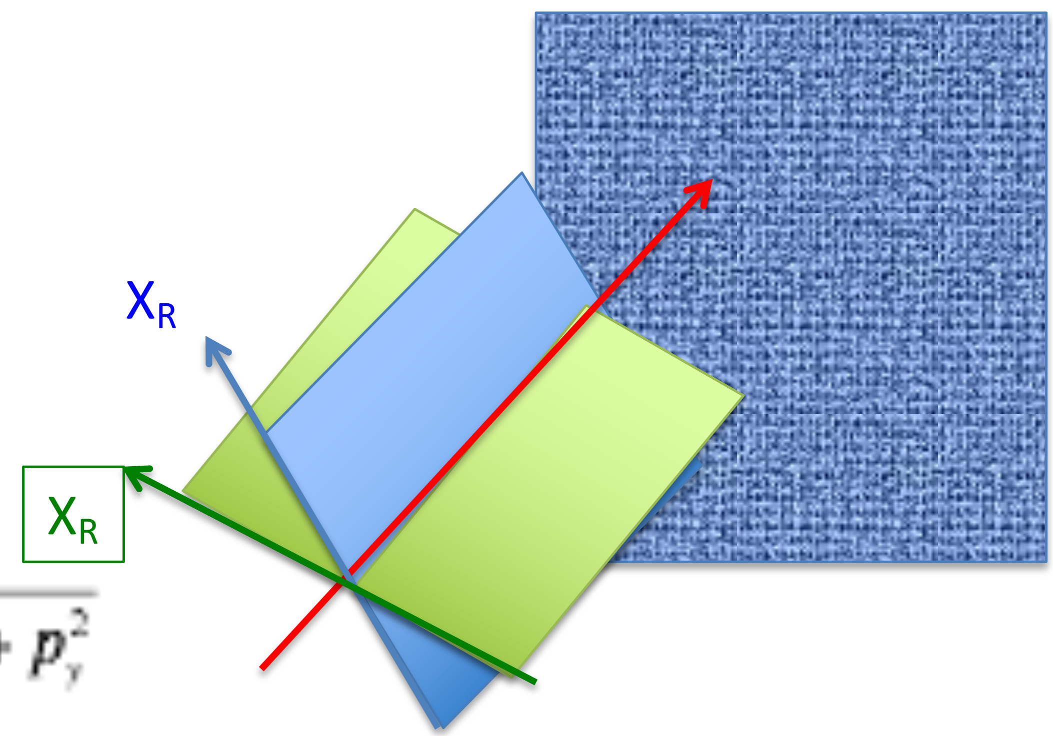


For us the question is whether neutrons are pushed out more than protons because of the symmetry energy!

The strength of the interaction is extracted from the comparison with transport models

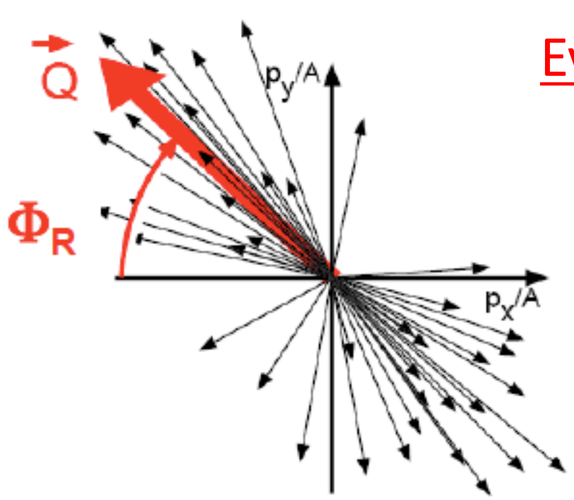
Reaction plane and angular variables





? Which is the angle ψ_R

Q= Sum of the P_t of all particles $p_T = \sqrt{p_x^2 + p_y^2}$



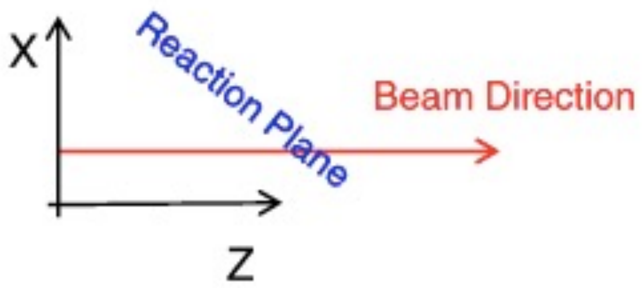
Event Plane!!

Beam Direction=z

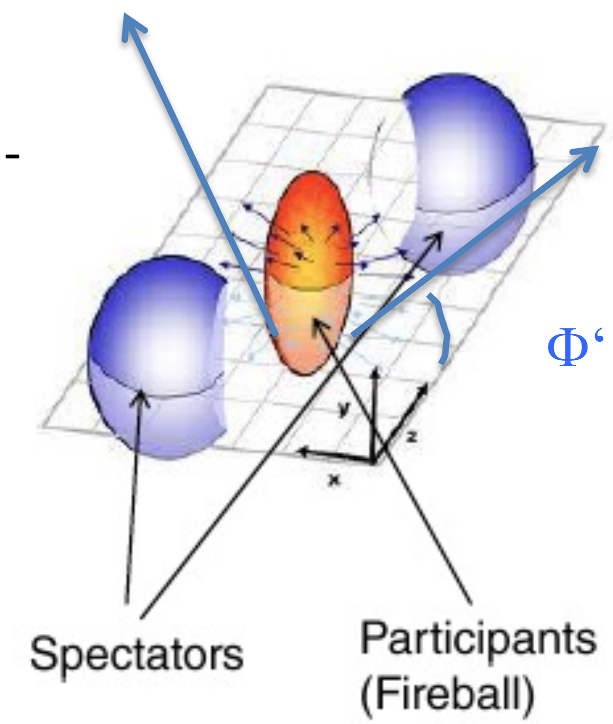
$\phi' = \phi - \Psi r$

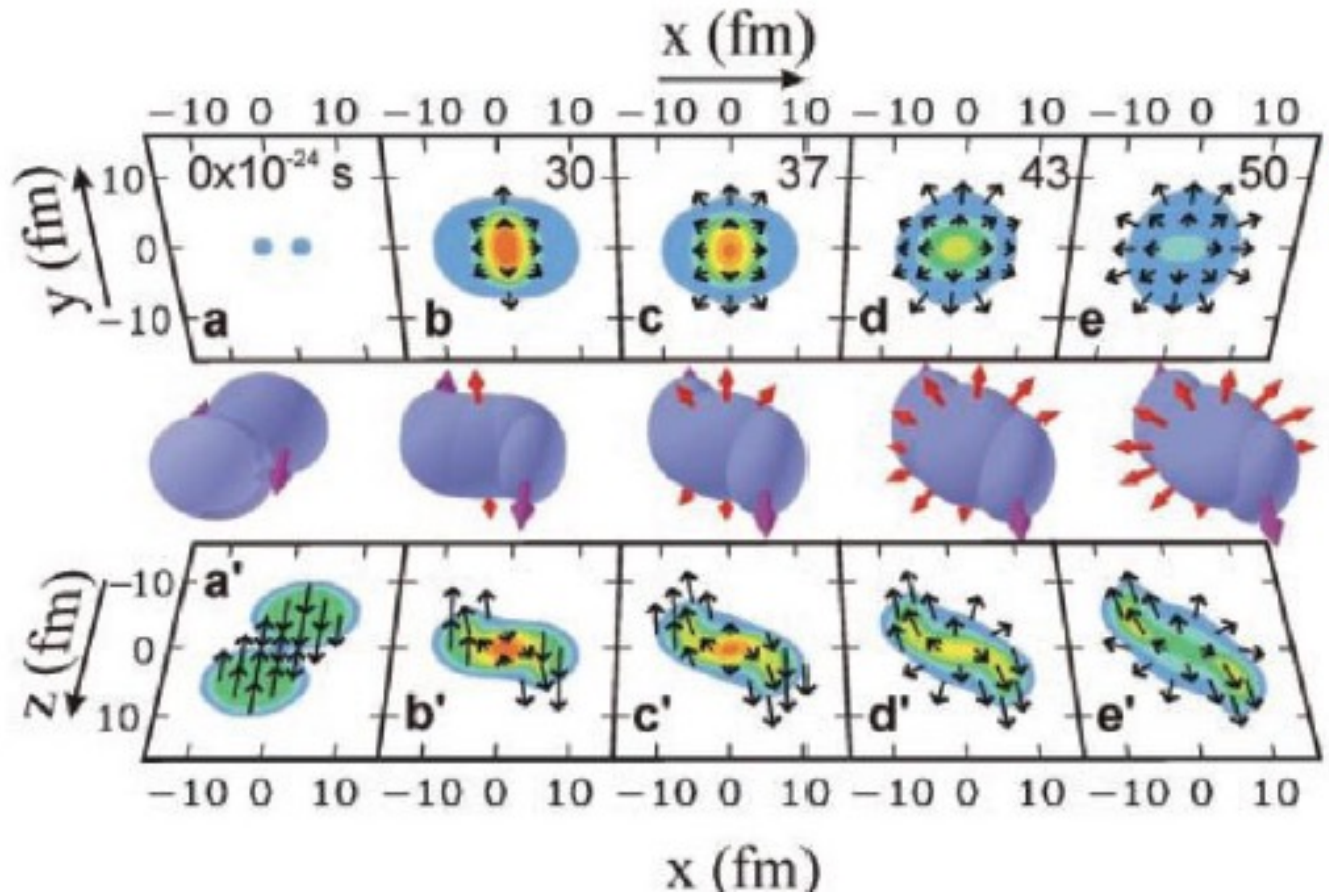
ϕ = Azimuthal Emission angle of one particle

Flow: sideward and elliptic



Different pressure gradient in in-plane and out-of-plane.
Expansion in in-plane





High Pressure Area in the collison zone:
Specator shade angular areas

Flow in low energy HIC

$$\frac{dN}{d(\phi-\phi_R)}(y,p_t)=\frac{N_0}{2\pi}\left(1+2\sum_{n\geq 1}v_n\cos n(\phi-\phi_R)\right)$$

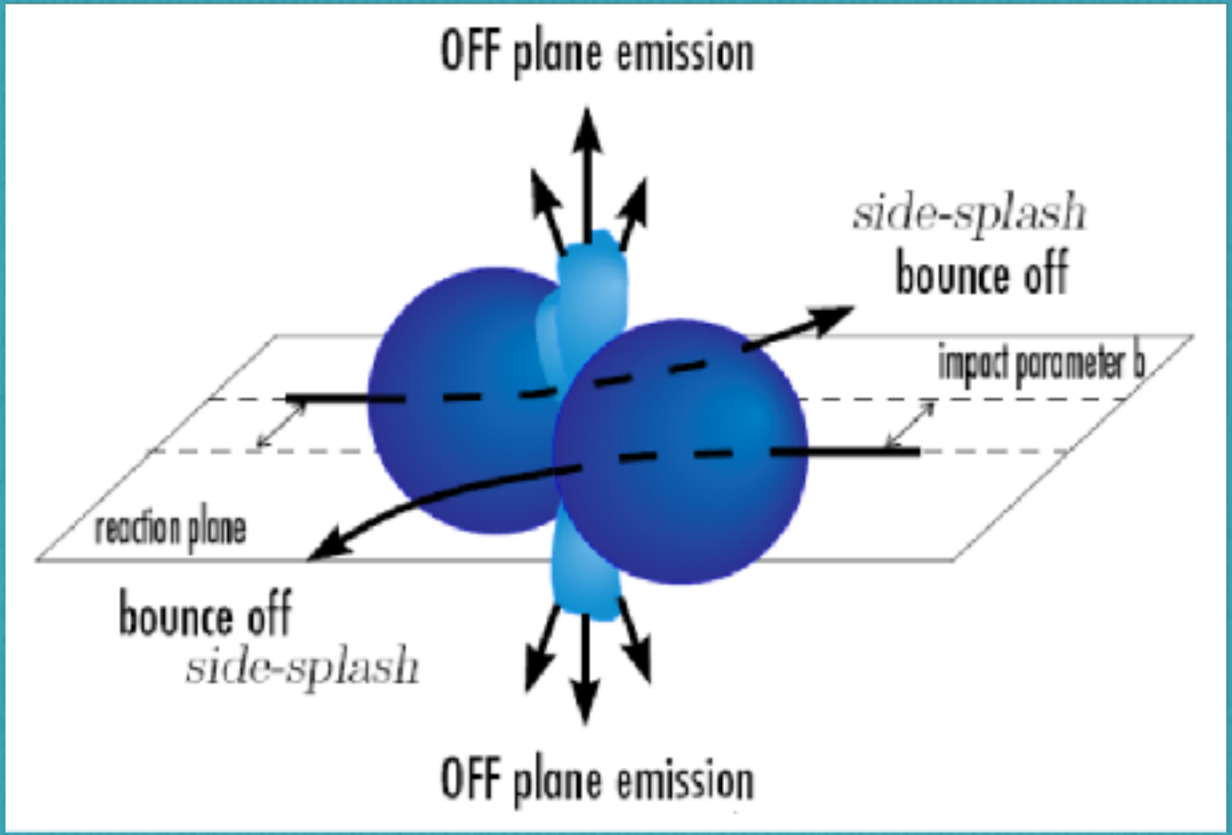
y = rapidity, pt = transverse momentum
ϕ_R = reaction plane orientation

$$V_1(y,p_t)\cdot\left\langle\frac{p_x}{p_t}\right\rangle$$

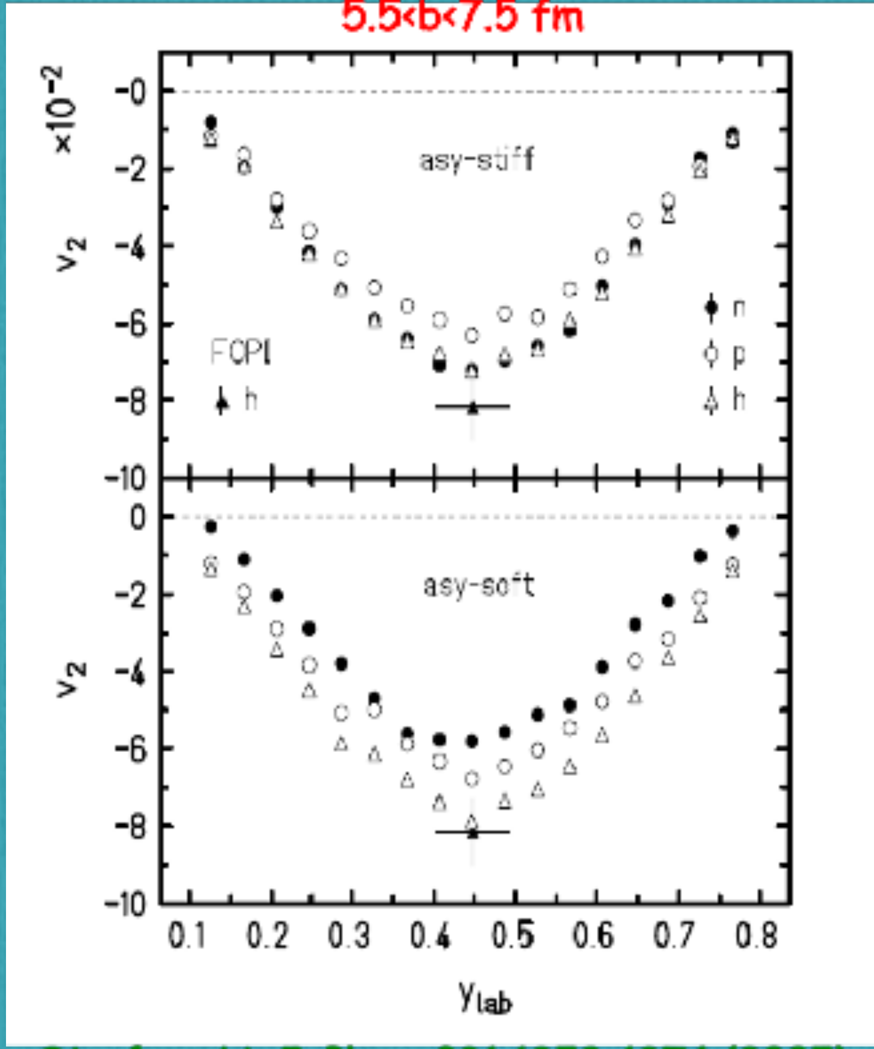
Transverse flow: provides information on the on the azimuthal anisotropy of the transverse nucleon emission

$$V_2(y,p_t)=\left\langle\frac{p_x^2-p_y^2}{p_t^2}\right\rangle$$

Elliptic flow: competition between in plane ($v_2>0$) and out-of-plane ejection ($v_2<0$)



UrQMD :Au+Au @ 400 AMeV
5.5<b<7.5 fm



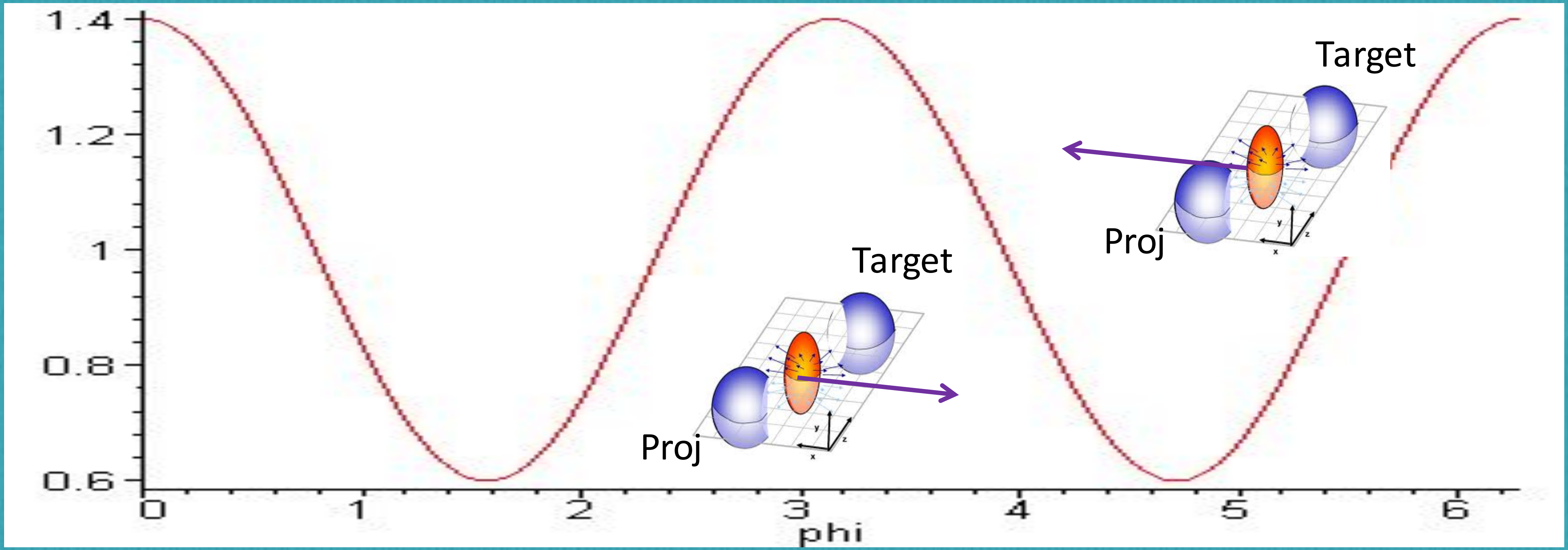
Qingfeng Li, J. Phys. G31 1359-1374 (2005)
P.Russotto et al., Phys. Lett. B 697 (2011)

Particle flow: v2

$$\frac{dN}{d(\phi - \psi_R)} \sim 1 + \sum_n 2v_n \cos[n(\phi - \psi_R)].$$

N=2 Eliptic flow

$v_2 = < \cos(2\phi) >$

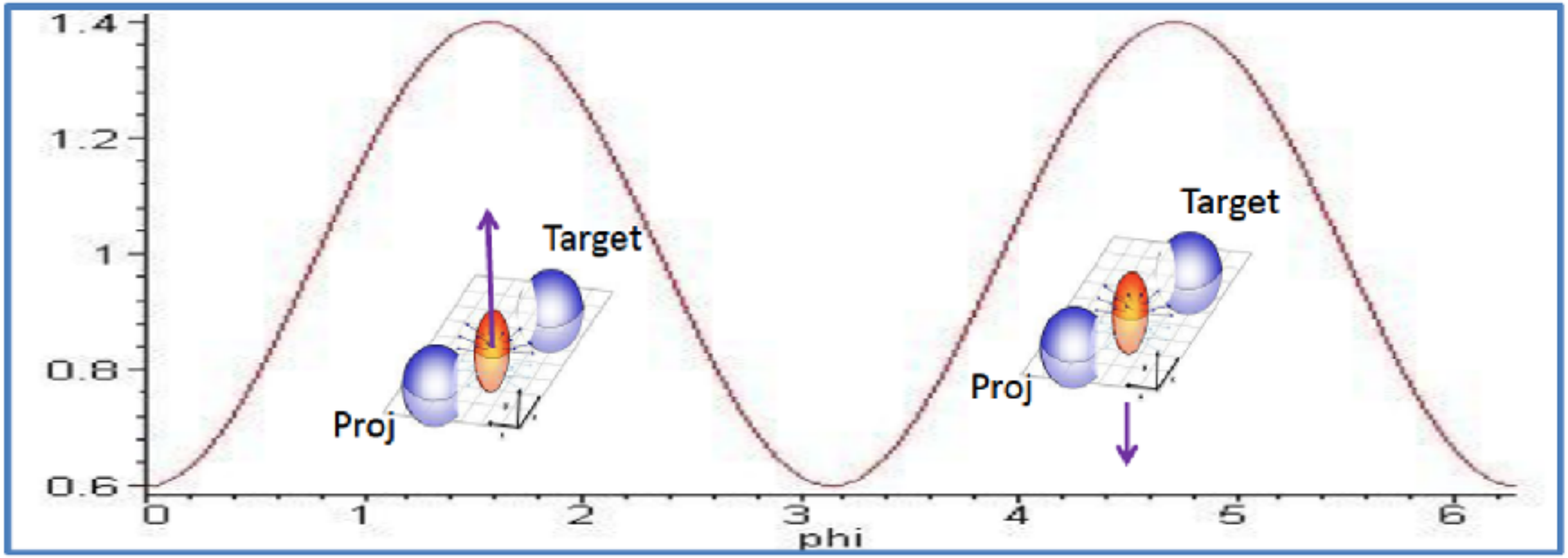


V2=0.2; Enhancement in the reaction plane

Particle flow: v2

$$\frac{dN}{d(\phi - \psi_R)} \sim 1 + \sum_n 2v_n \cos[n(\phi - \psi_R)].$$

N=2 Eliptic flow



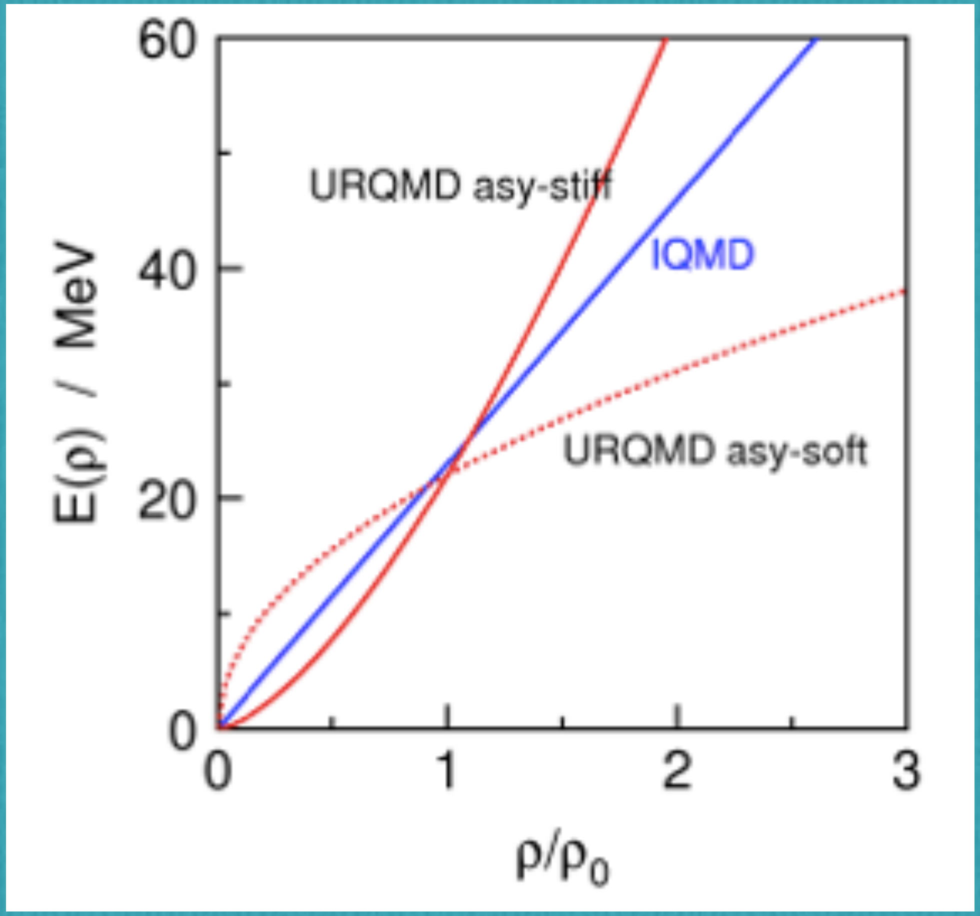
V2=-0.2; Enhancement out of reaction plane

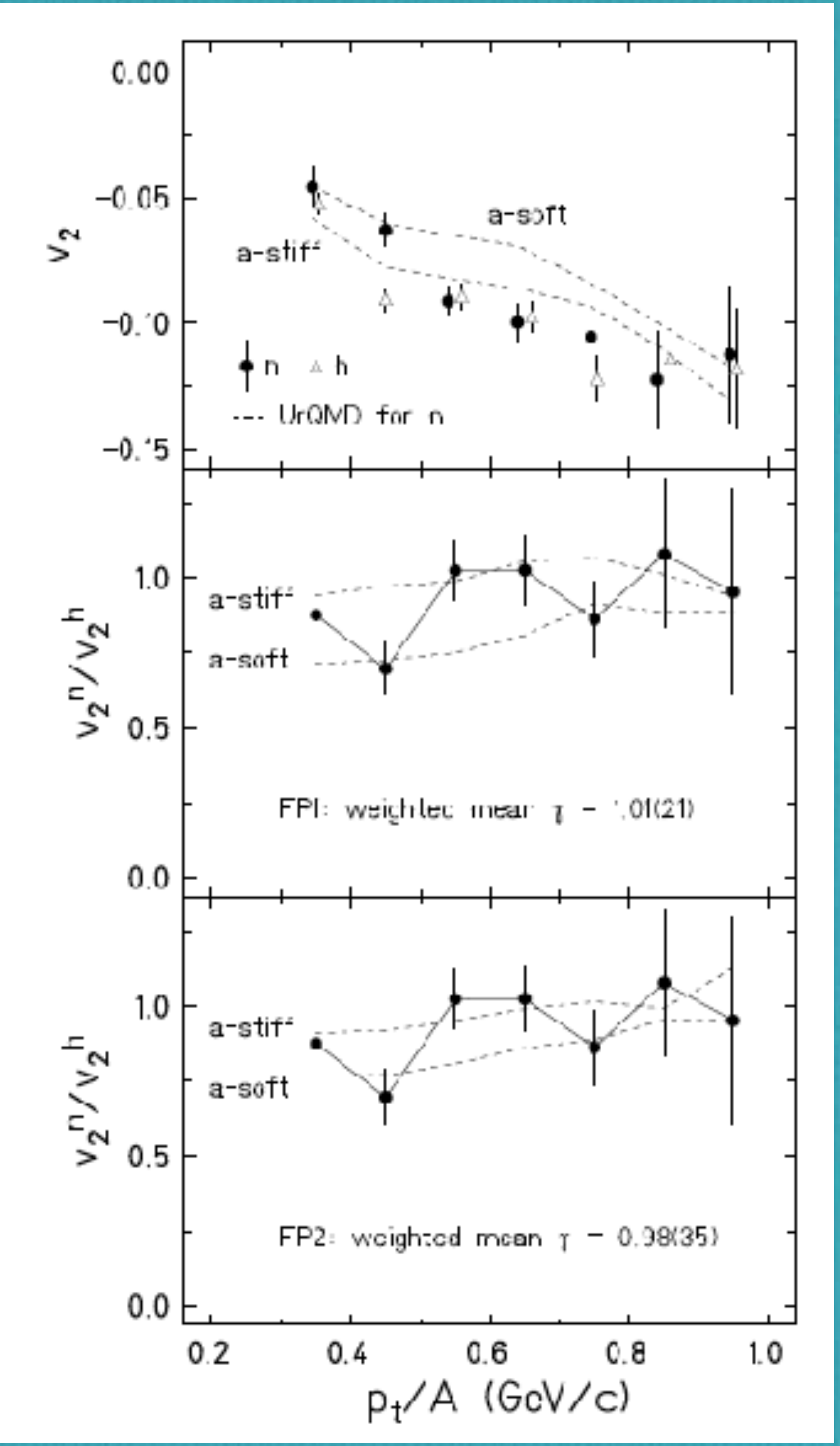
Results compared to Models

P. Rusotto et al PLB, 267 (2011)

Comparison to models, used parametrization of E_{SYM} for densities beyond saturation density

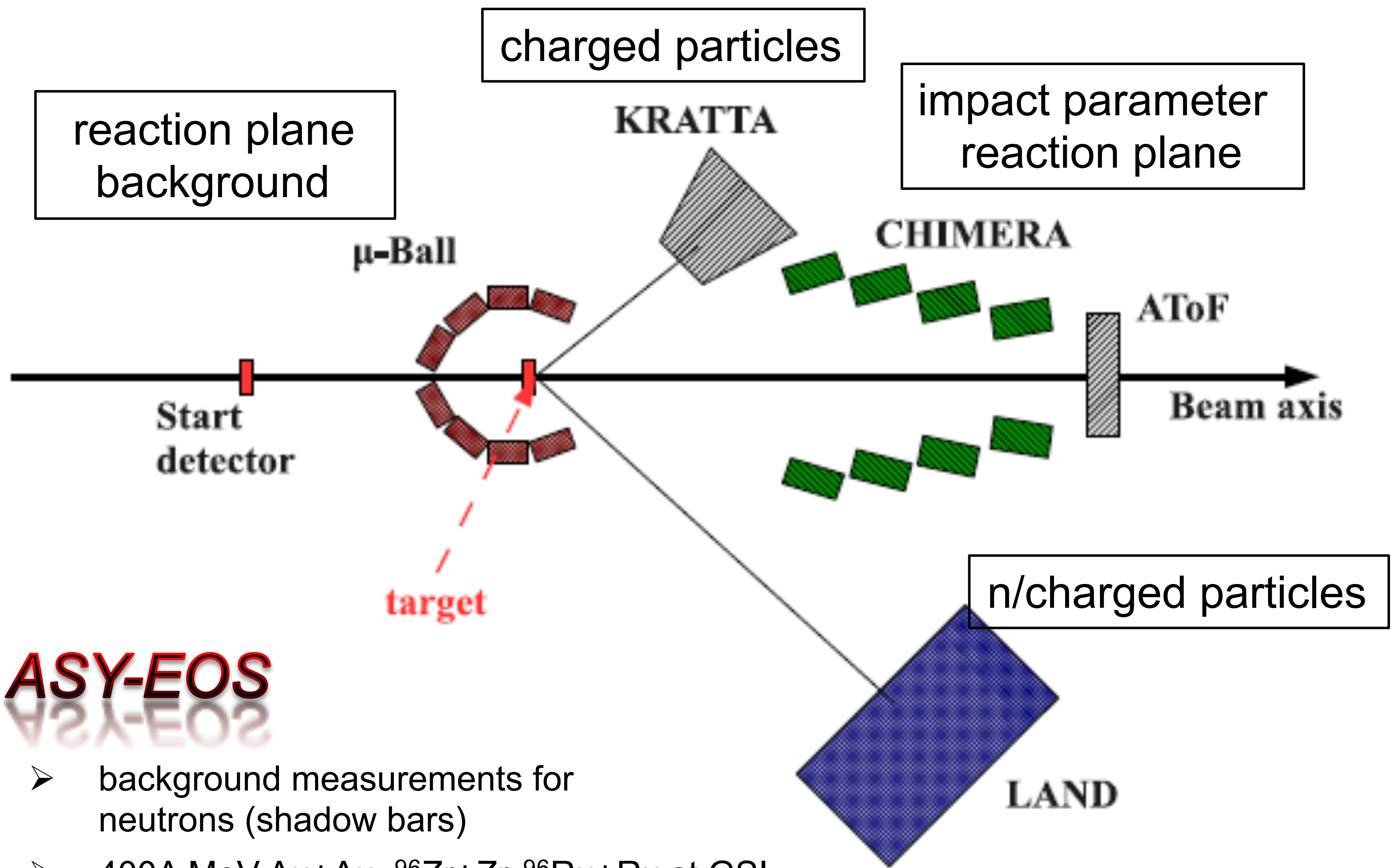
$$E_{SYM} = E^{pot}_{SYM} + E^{KIN}_{SYM} = 22MeV \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma} + 12MeV \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3}$$





$$\gamma = 0.9 \pm 0.4$$

ASY-EOS Experiment



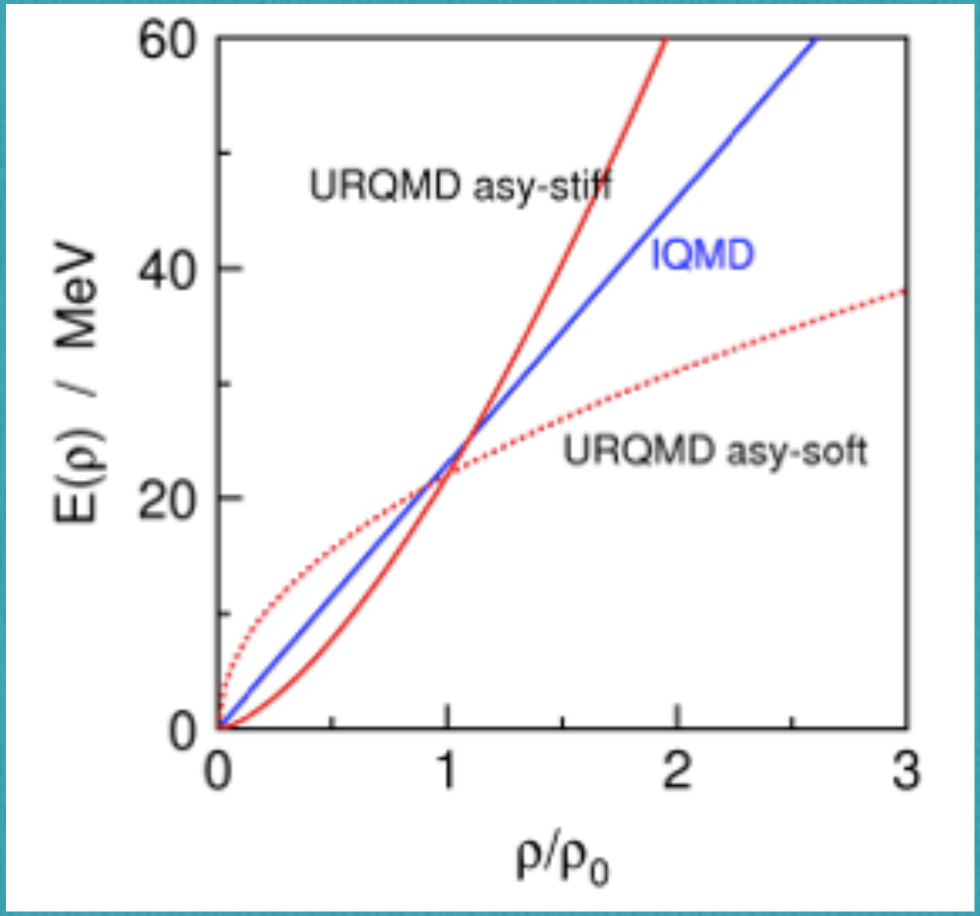
- background measurements for neutrons (shadow bars)
- 400A MeV Au+Au, ^{96}Zr+Zr, ^{96}Ru+Ru at GSI

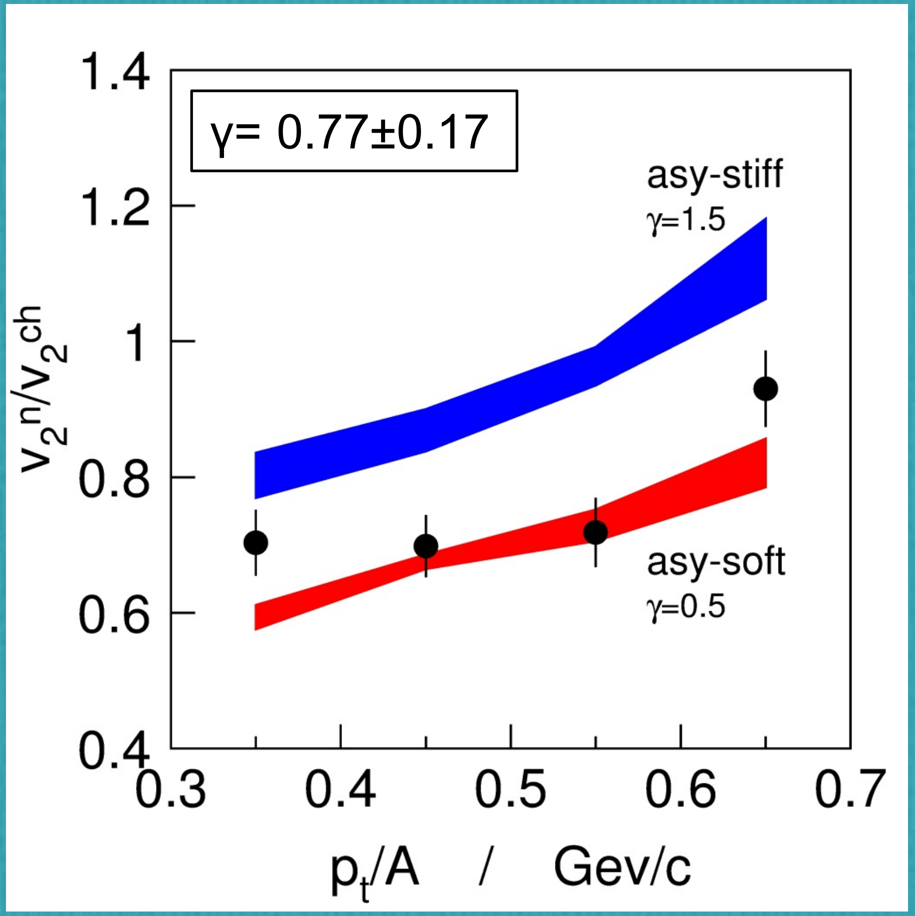
Results compared to Models

courtesy Y. Leifels

Comparison to models, used parametrization of E_{SYM} for densities beyond saturation density

$$E_{SYM} = E_{SYM}^{pot} + E_{SYM}^{KIN} = 22MeV \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma} + 12MeV \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3}$$





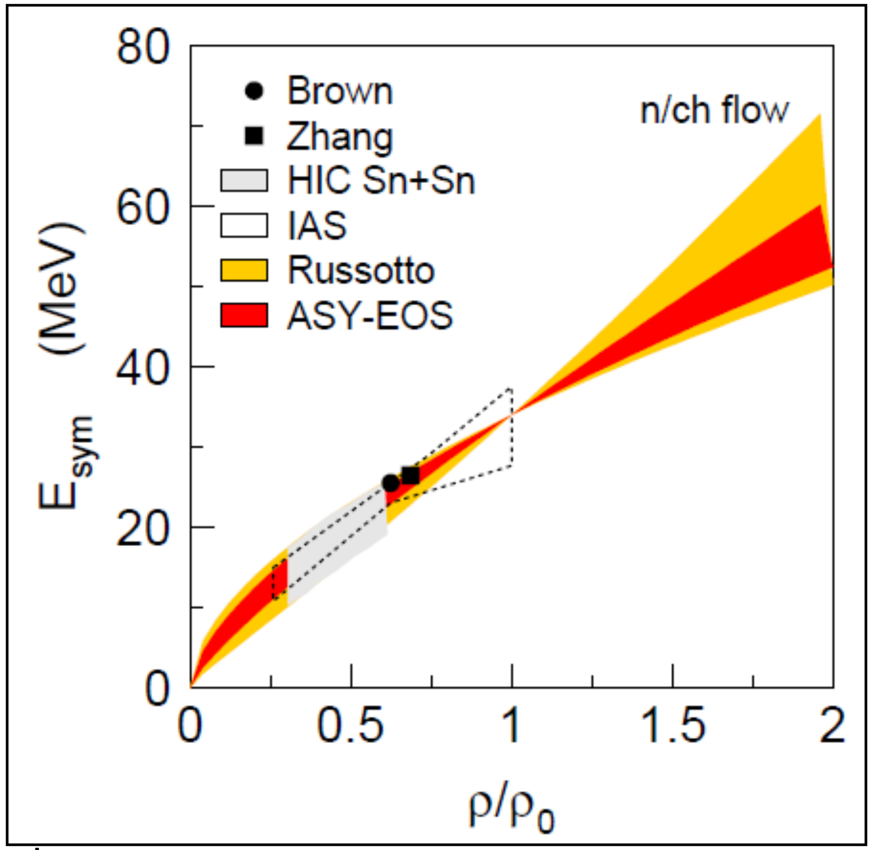
Current Situation

courtesy Y. Leifels

Only limited experimental data available at supra-normal densities

- FOPI: π^-/π^+, t/^{3}He (Au+Au) Reisdorf NPA 781 (2007)
- FOPI+LAND: n+H/p,Russotto PLB 267 (2010)
- ASY-EOS: n/charged particles

- n/p differential elliptic flow leads to *robust* constraints for the density dependence of the symmetry energy.
- π^-/π^+ (E_{beam}) promising but sophisticated treatment of pion interaction in medium needed.
- for t/^{3}He observables: models have to account for clusterization
- K^0/K^+ ratio sensitive to high densities



Horrowitz et al. JPhG 41 (2014)
Brown: arXiv:1308.3664
Zhang: PLB 726 (2013)

Summary

Several different experiments (mostly very model dependent) suggest that the Equation of state of nuclear matter up to $2\rho_0$ is rather soft

This is not so well in agreement with stiff nuclear EoS used for neutron stars

Additionally: if hyperons are added the EoS might become even softer

What is the way of the future

1) Measure three-body forces including hyperons via hypernuclei and correlations
2) Continue on experiments at low energies to study dense baryonic matter, although the studies are very model dependent !
3) Consider the possibility of axion fiels within neutron stars able to stiffen the EoS