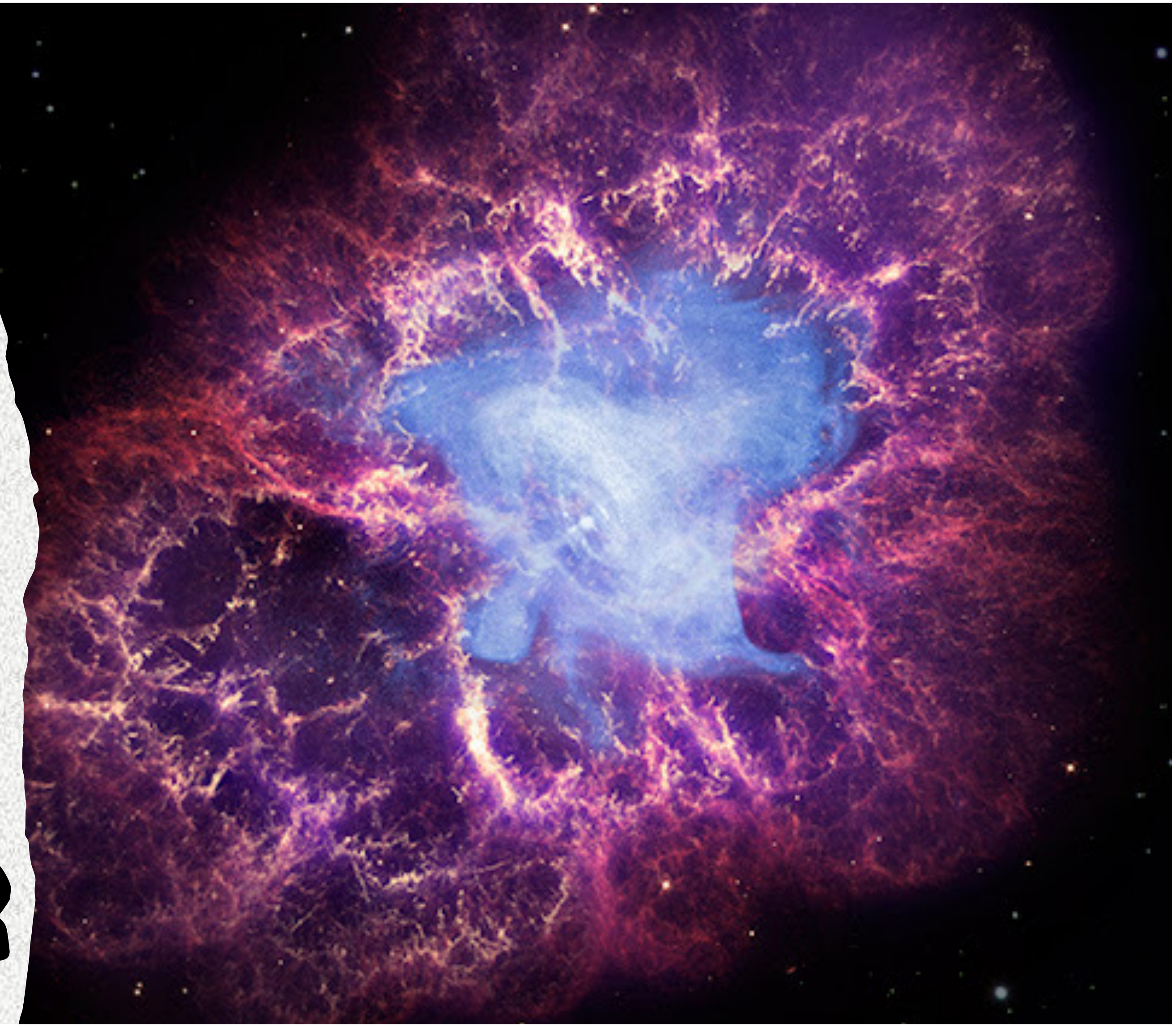
Neutron Stars & the Nuclear Equation of State

Isaac Vidaña, INFN Catania

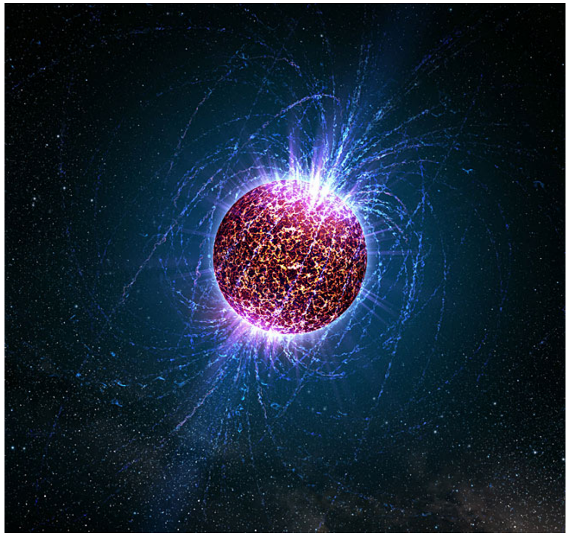
Indian-Summer School 2022
Prague, June 24th-26th



Lecture Program: Part 1

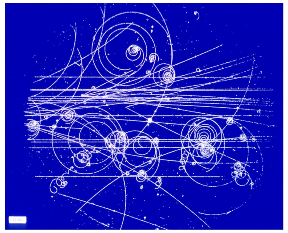
A short walk through the physics of neutron stars

What are Neutron Stars ?



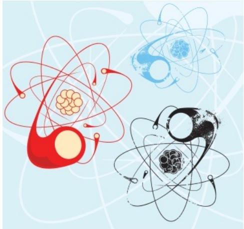
The answer depends on who you ask …

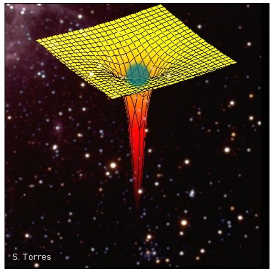
For astronomers are very little stars "visible" as radio pulsar or sources of X- and γ-rays.



For particle physicists are neutrino sources (when they born) and probably the only places in the Universe where (if it exists) deconfined quark matter may be abundant.

For nuclear physicists are the biggest nuclei of the Universe ($A \sim 10^{56}$-10^{57}, $R \sim 10$ km).



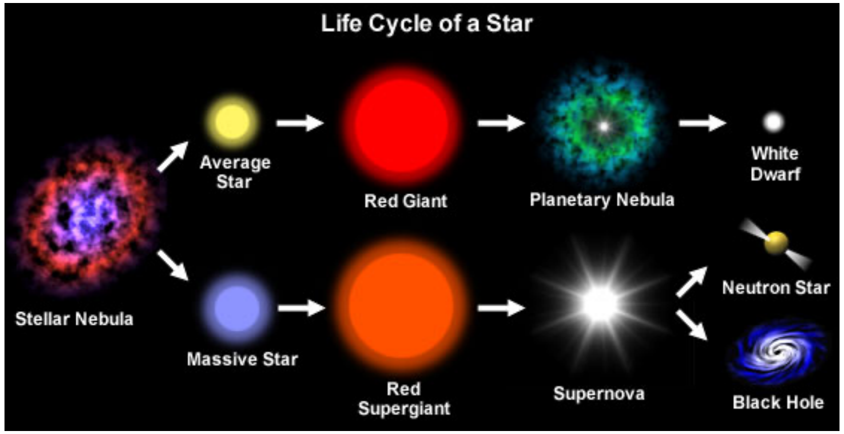


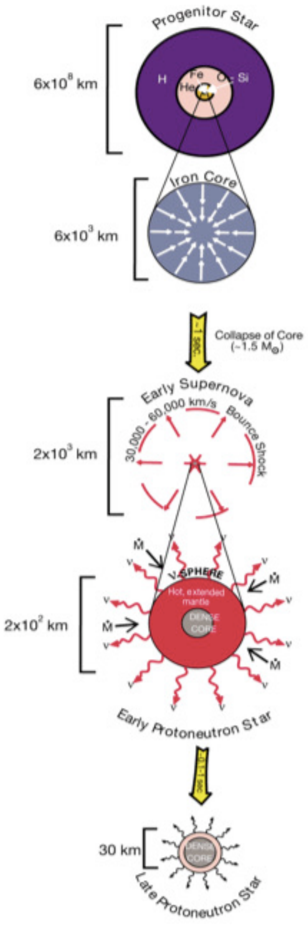
For cosmologists are “almost” black holes

For computational physicists are simple a nightmare

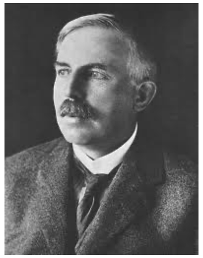
But everybody agrees that …

Neutron stars are a type of stellar compact remnant that can result from the gravitational collapse of a massive star ($8\,M_\odot < M < 25\,M_\odot$) during a Type II, Ib or Ic supernova event.



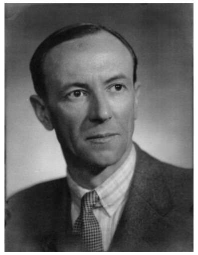


A bit of history & some pictures



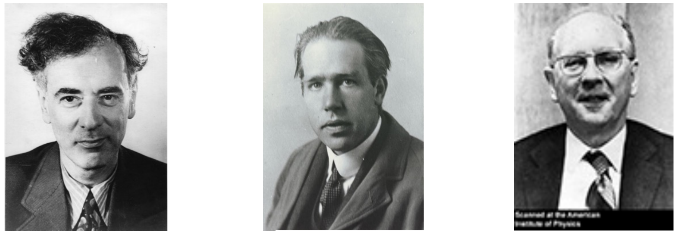
In 1920 Ernest Rutherford predicts the existence of the neutron

In 1932 James Chadwick discovers the neutron
(1935 Nobel Prize) 



In 1934 Walter Baade & Fritz Zwicky predict the existence of neutron stars and their formation in supernova events

Did Landau anticipate their existence in 1931 ?

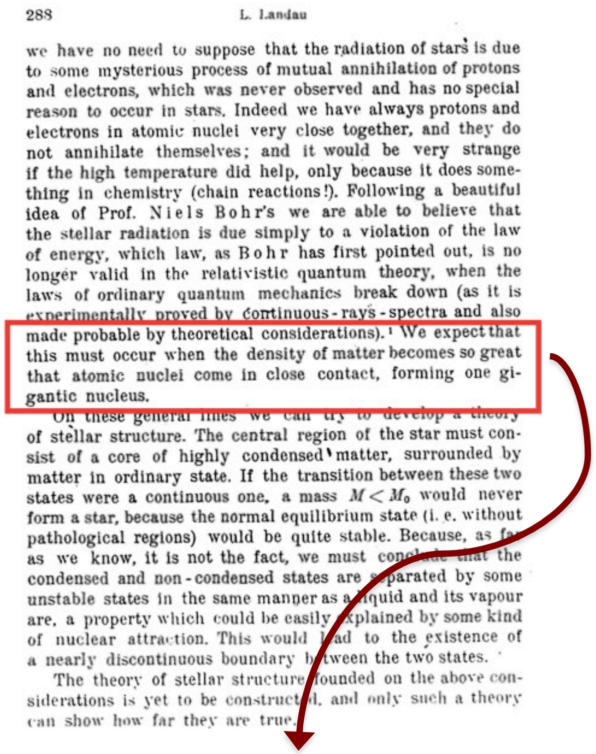


- ✓ February-March 1931 Landau, Bohr & Rosenfeld discuss in Copenhagen a paper by Landau (not published then) about the possible existence of very dense stars
- ✓ In February 1932 Landau publishes the article in a Russian journal that is completely unnoticed.

Source: G. Baym, P. Haensel, C. Petick & D. G. Yakovlev

- ✓ http://www.ift.uni.wroc.pl/ ~karp44/talks/yakovlev.pdf
- ✓ P. Haensel et al., *Neutron Stars 1. Equation of State & Structure* (2007)

Phys. Z. Sowjetunion 1, 285 (1932)



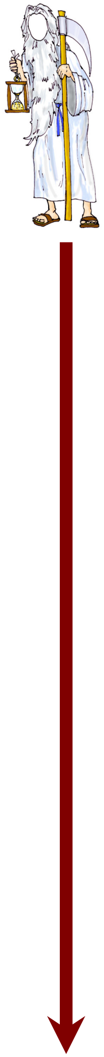
288 L. Landau

we have no need to suppose that the radiation of stars is due to some mysterious process of mutual annihilation of protons and electrons, which was never observed and has no special reason to occur in stars. Indeed we have always protons and electrons in atomic nuclei very close together, and they do not annihilate themselves; and it would be very strange if the high temperature did help, only because it does something in chemistry (chain reactions!). Following a beautiful idea of Prof. Niels Bohr's we are able to believe that the stellar radiation is due simply to a violation of the law of energy, which law, as Bohr has first pointed out, is no longer valid in the relativistic quantum theory, when the laws of ordinary quantum mechanics break down (as it is experimentally proved by continuous-rays-spectra and also made probable by theoretical considerations).[1] We expect that this must occur when the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus.

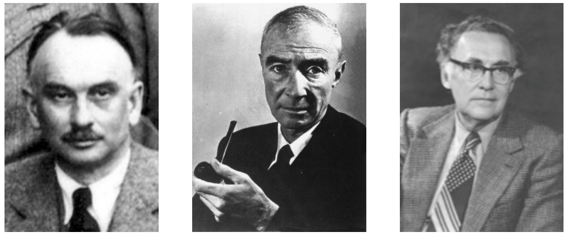
On these general lines we can try to develop a theory of stellar structure. The central region of the star must consist of a core of highly condensed matter, surrounded by matter in ordinary state. If the transition between these two states were a continuous one, a mass $M < M_0$ would never form a star, because the normal equilibrium state (i. e. without pathological regions) would be quite stable. Because, as far as we know, it is not the fact, we must conclude that the condensed and non-condensed states are separated by some unstable states in the same manner as a liquid and its vapour are, a property which could be easily explained by some kind of nuclear attraction. This would lead to the existence of a nearly discontinuous boundary between the two states.

The theory of stellar structure founded on the above considerations is yet to be constructed, and only such a theory can show how far they are true.

“We expect that this occur when the density of matter becomes so great that atomic nuclei come in contact, forming one gigantic nuclei”

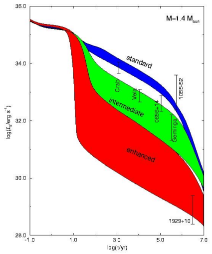


In 1939 Tolman, Oppenheimer & Volkoff obtain the equations that describe the structure of a static star with spherical symmetry in General Relativity (Chandrasekhar & von Neumann obtained them in 1934 but they did not published their work)



First "realistic" EoS of dense matter by Wheeler *et al.* in the 50s. In 1959 Cameron studies neutron star models with a Skyrme EoS finding $M_{max} \sim 2M_{\odot}$

In 1959 Migdal suggests superfluidity in neutron stars

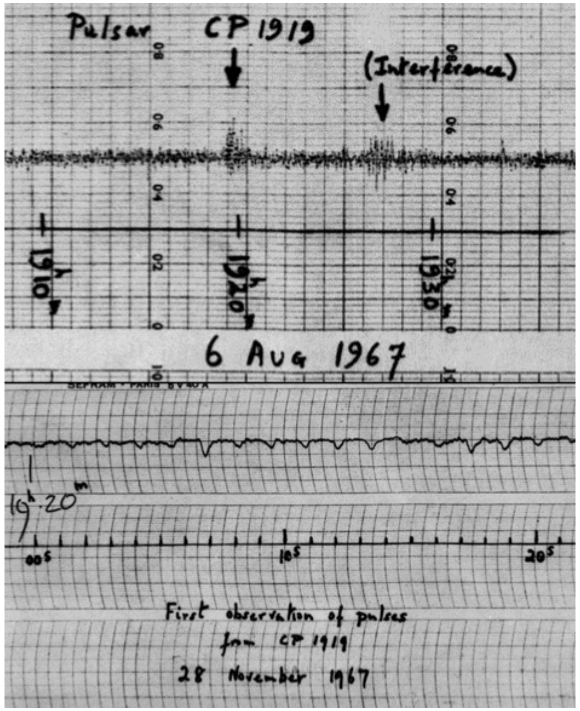


Theoretical efforts in the 60s focused on modeling neutron star cooling motivated by hope of detecting their thermal emission

Riccardo Giacconi starts in the 60s the first observations with X-ray telescopes on board of satellites discovering many X-ray sources (2002 Nobel Prize)

55 years of the discovery of the first radio pulsar

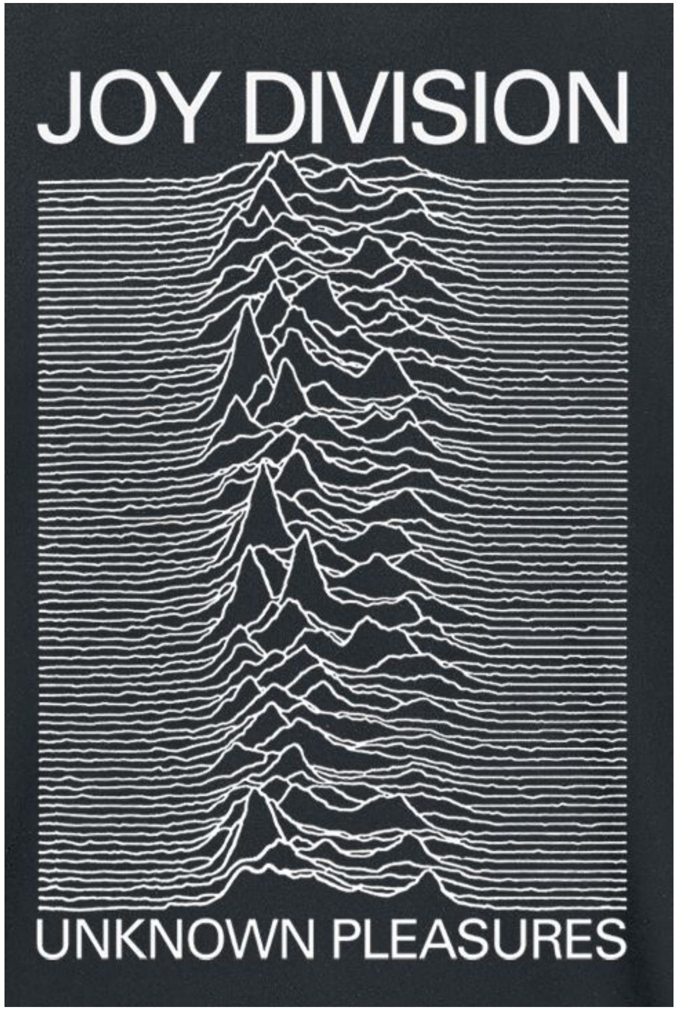
- ✧ radio pulsar at 81.5 MHz
- ✧ pulse period P=1.337 s



Most NS are observed as pulsars. In 1967 Jocelyn Bell & Anthony Hewish discover the first radio pulsar (PSR B1919+21), soon identified as a rotating neutron star (1974 Nobel Prize for Hewish but not for Jocelyn)

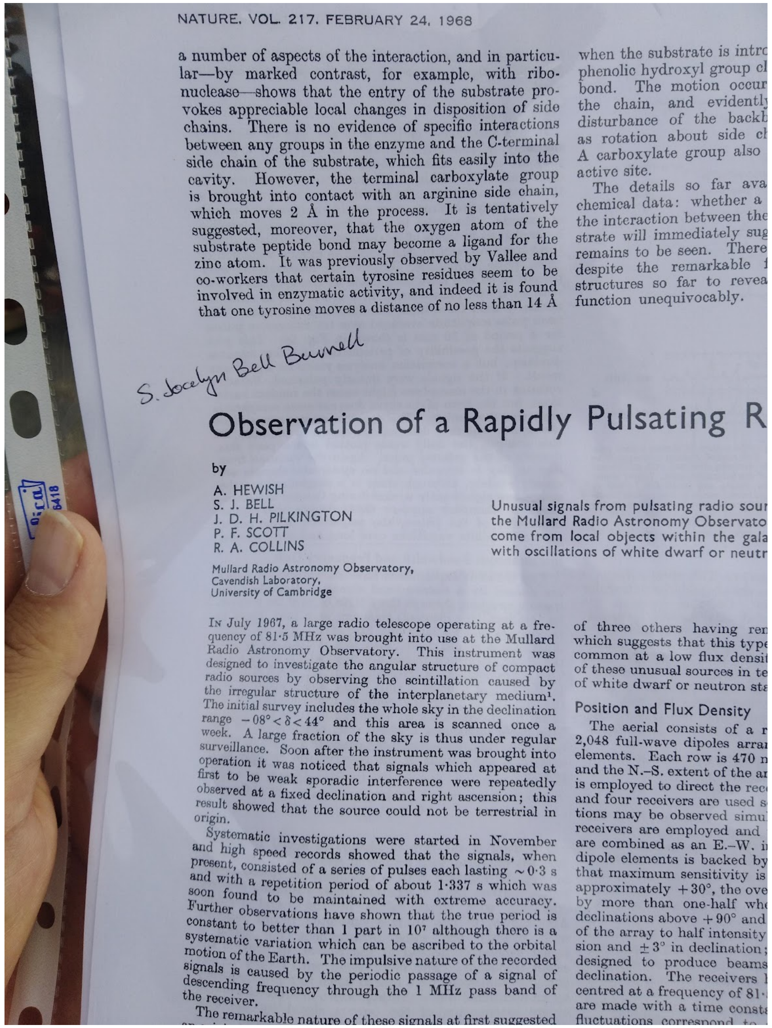
The discovery of the first pulsar had such important impact that even the English rock band Joy Division immortalize the signal of this source on the cover of its disc “Unknown Pleasures”

The image is the negative of the one published in the British Encyclopedia and shows a montage of the intensity of various pulses from PSR B1919+21



This is not really part of the story of neutron stars, but I am very proud to say that last week I met Jocelyn Bell in person 😊

And that she signed my copy of the paper where the observation of the first pulsar was published 😊



NATURE, VOL. 217, FEBRUARY 24, 1968

a number of aspects of the interaction, and in particular—by marked contrast, for example, with ribonuclease—shows that the entry of the substrate provokes appreciable local changes in disposition of side chains. There is no evidence of specific interactions between any groups in the enzyme and the C-terminal side chain of the substrate, which fits easily into the cavity. However, the terminal carboxylate group is brought into contact with an arginine side chain, which moves 2 Å in the process. It is tentatively suggested, moreover, that the oxygen atom of the substrate peptide bond may become a ligand for the zinc atom. It was previously observed by Vallee and co-workers that certain tyrosine residues seem to be involved in enzymatic activity, and indeed it is found that one tyrosine moves a distance of no less than 14 Å

when the substrate is intro
phenolic hydroxyl group cl
bond. The motion occur
the chain, and evidently
disturbance of the backb
as rotation about side ch
A carboxylate group also
active site.

The details so far ava
chemical data: whether a
the interaction between the
strate will immediately sug
remains to be seen. There
despite the remarkable f
structures so far to revea
function unequivocably.

S. Jocelyn Bell Burnell

Observation of a Rapidly Pulsating R

by

A. HEWISH
S. J. BELL
J. D. H. PILKINGTON
P. F. SCOTT
R. A. COLLINS

Mullard Radio Astronomy Observatory,
Cavendish Laboratory,
University of Cambridge

Unusual signals from pulsating radio sour
the Mullard Radio Astronomy Observato
come from local objects within the gala
with oscillations of white dwarf or neutr

In July 1967, a large radio telescope operating at a frequency of 81·5 MHz was brought into use at the Mullard Radio Astronomy Observatory. This instrument was designed to investigate the angular structure of compact radio sources by observing the scintillation caused by the irregular structure of the interplanetary medium[1]. The initial survey includes the whole sky in the declination range $-08° < \delta < 44°$ and this area is scanned once a week. A large fraction of the sky is thus under regular surveillance. Soon after the instrument was brought into operation it was noticed that signals which appeared at first to be weak sporadic interference were repeatedly observed at a fixed declination and right ascension; this result showed that the source could not be terrestrial in origin.

Systematic investigations were started in November and high speed records showed that the signals, when present, consisted of a series of pulses each lasting $\sim 0{\cdot}3$ s and with a repetition period of about 1·337 s which was soon found to be maintained with extreme accuracy. Further observations have shown that the true period is constant to better than 1 part in 10^7 although there is a systematic variation which can be ascribed to the orbital motion of the Earth. The impulsive nature of the recorded signals is caused by the periodic passage of a signal of descending frequency through the 1 MHz pass band of the receiver.

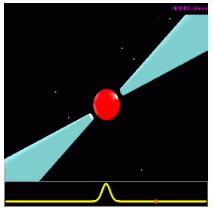
The remarkable nature of these signals at first suggested

of three others having ren
which suggests that this type
common at a low flux densit
of these unusual sources in te
of white dwarf or neutron sta

Position and Flux Density

The aerial consists of a r
2,048 full-wave dipoles arran
elements. Each row is 470 n
and the N.–S. extent of the ar
is employed to direct the rec
and four receivers are used s
tions may be observed simu
receivers are employed and
are combined as an E.–W. i
dipole elements is backed by
that maximum sensitivity is
approximately +30°, the ove
by more than one-half wh
declinations above +90° and
of the array to half intensity
sion and ±3° in declination;
designed to produce beams
declination. The receivers
centred at a frequency of 81·
are made with a time const
fluctuations correspond to

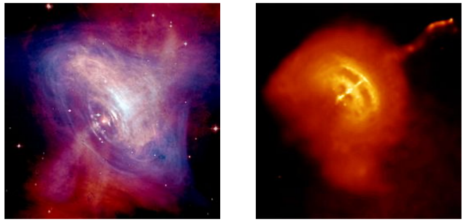
Also in 1967 Pacini shows that a rapidly rotating neutron star with a strong dipole magnetic field could power the Crab nebula



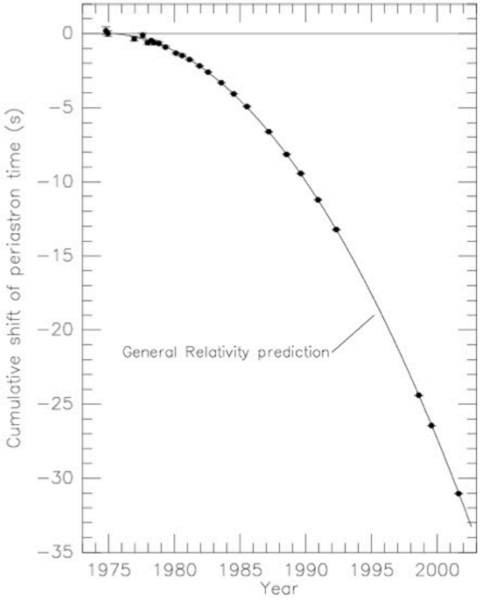
In 1968 Gold proposes that pulsars are strongly magnetized neutron stars radiating at expenses of their rotational energy

http://pulsar.ca.astro.it/pulsar/Figs

$$\dot{E}_{mag} = -\frac{2}{3c^3}\left|\ddot{\vec{\mu}}\right|^2$$



In 1968 the Crab & Velar pulsars are discovered in SNR confirming the prediction of Baade & Zwicky



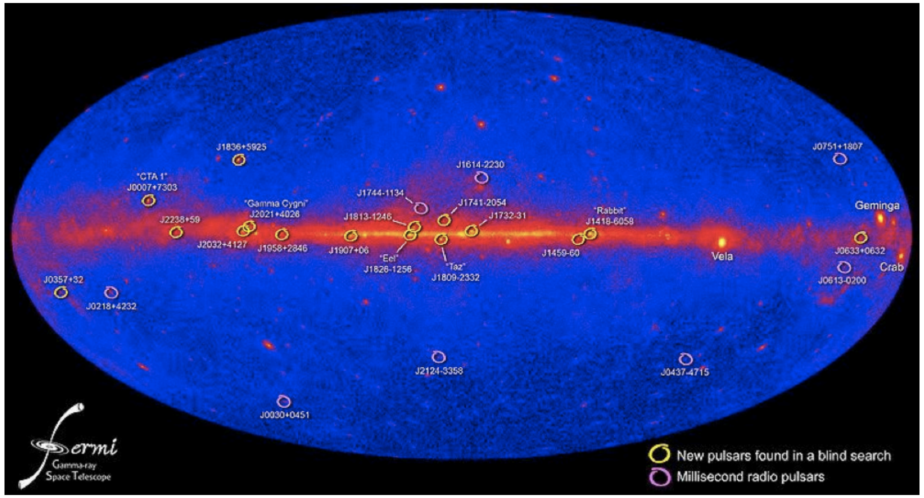
In 1974 R. A. Hulse & J. H. Taylor discover the first binary pulsar (1993 Nobel Prize)

80’s, 90’s and 2000’s: launch of satellites with X-ray (Einstein, ROSAT, ASCA, Chandra, XMM-Newton) and γ-ray (INTEGRAL, SWIFT, FERMI) telescopes

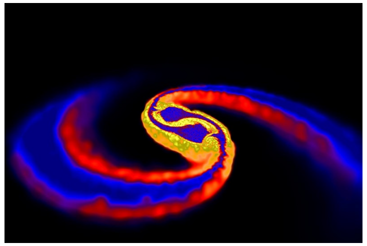
Nowadays more than 2000 pulsars are known (~ 1900 Radio PSRs (141 in binary systems), ~ 40 X-ray PSRs & ~ 60 γ-ray PSRs)

Observables

- Period (P, dP/dt)
- Masses
- Luminosity
- Temperature
- Magnetic Field
- Gravitational Waves (NS-NS, BH-NS mergers, NS oscillation modes)



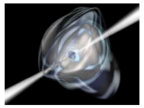
http://www.phys.ncku.edu.tw/~astrolab/mirrors/apod_e/ap090709.html

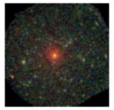


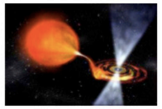
The 1001 Astrophysical Faces of Neutron Stars

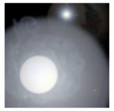
Neutron stars can be observed as

- ✧ isolated objects
- ✧ forming binary systems with other NS, ordinary stars or BH


Anomalous X-ray Pulsars

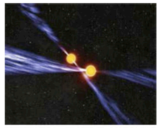

dim isolated neutron stars

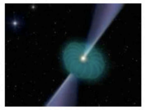

X-ray binaries

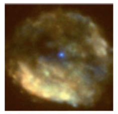

bursting pulsars


Soft Gamma Repeaters


pulsars


binary pulsars


Rotating Radio Transients


Compact Central Objects

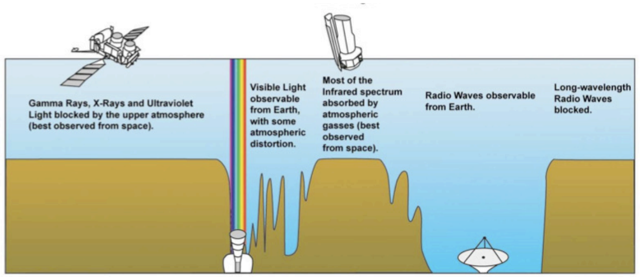

planets around pulsar

- ✧ **Isolated neutron stars**
  - ✓ Mostly detected as radio pulsars, X-ray pulsar or γ-ray pulsars
  - ✓ Radio-quite isolated neutron stars: CCOs & DINS
  - ✓ Soft gamma repeaters (SGRs) & Anomalous X-ray pulsars (AXPs)
- ✧ **Neutron stars in binary systems**
  - ✓ No mass exchange: NS behave as isolated objects
  - ✓ Mass exchange: observed as X-ray sources: X-ray pulsars, X-ray bursters or quasiperiodic X-ray oscillations. Classified as HMXRBs or LMXRBs depending on the mass of the companion or as persistent or transient sources according to the regularity or irregularity of their activity

Observation of Neutron Stars: Electromagnetic Signals

Neutron stars are observed in all bands of the electromagnetic spectrum

Their observation requires different types of ground-based & on-board telescopes



Radio:

Arecibo: d= 305 m

Green Banks: d= 100 m

Nançay : d ~ 94 m

Infrared & Optical

VLT

Ultraviolet & Optical

HST (Hubble)

Extreme ultraviolet, X- & γ-ray

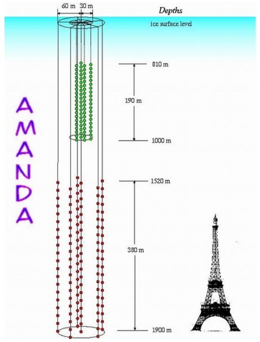
Chandra

Fermi

Observation of Neutron Stars: Neutrino Signals

Neutron stars are observed also through the detection of the neutrinos emitted during the supernova explosion that signals the birth of the star

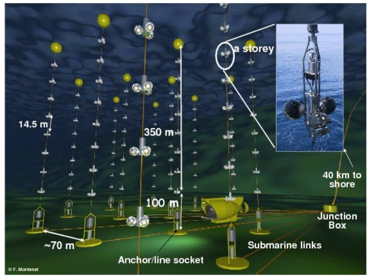
Under-ice telescoles



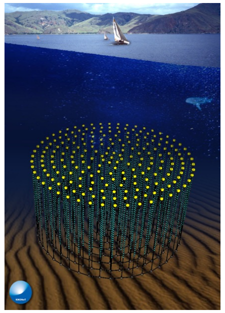
AMANDA

ICECUBE

Under-water telescopes



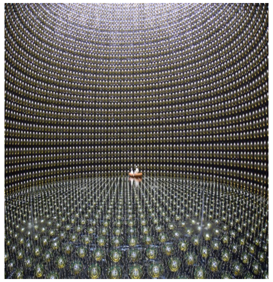
ANTARES



KM3NET

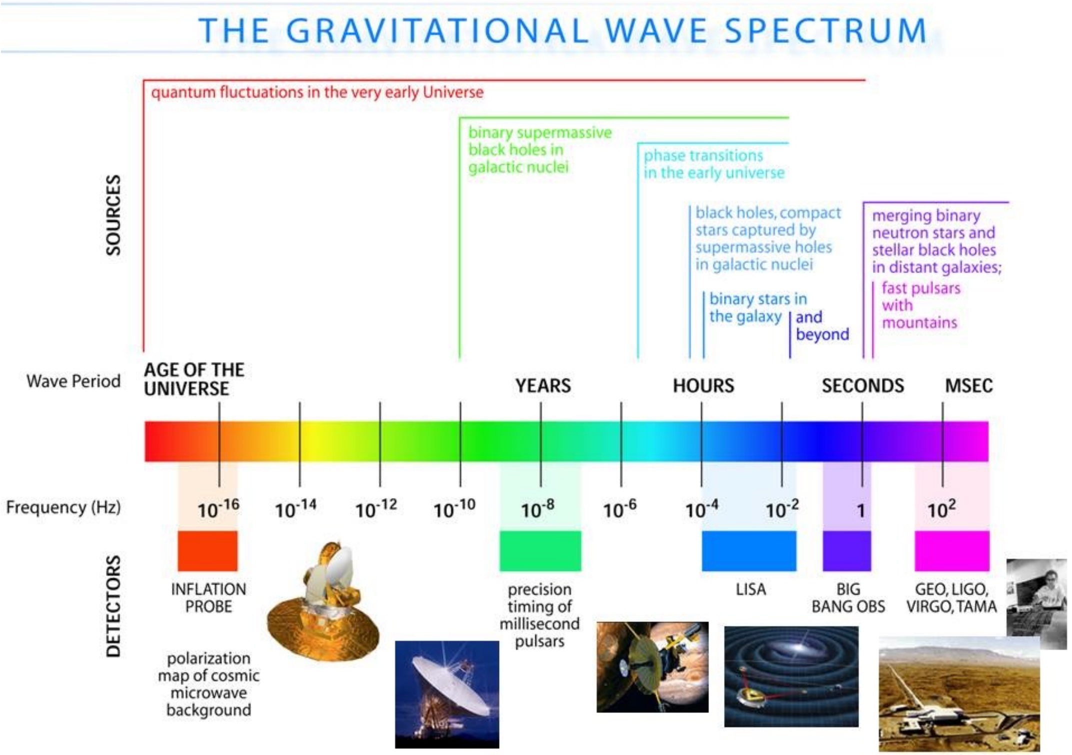
Under-ground telescopes

SNO

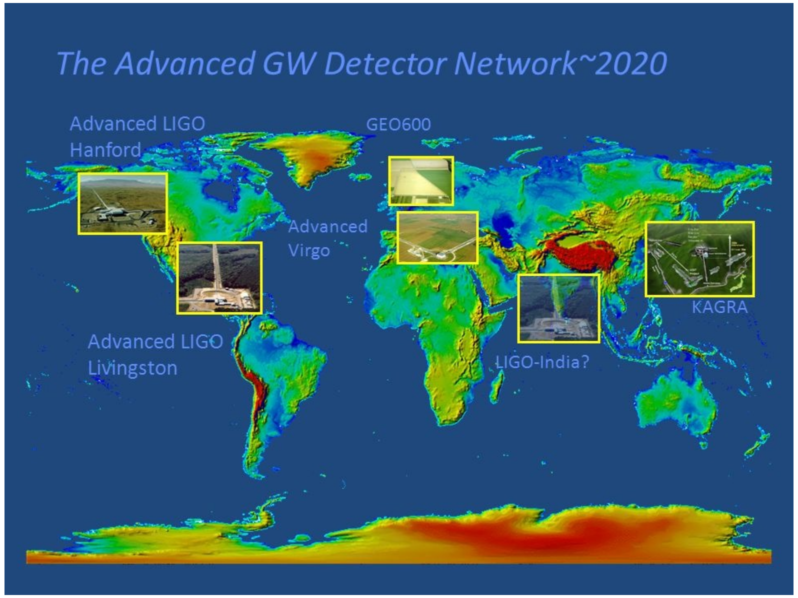


KAMIOKA

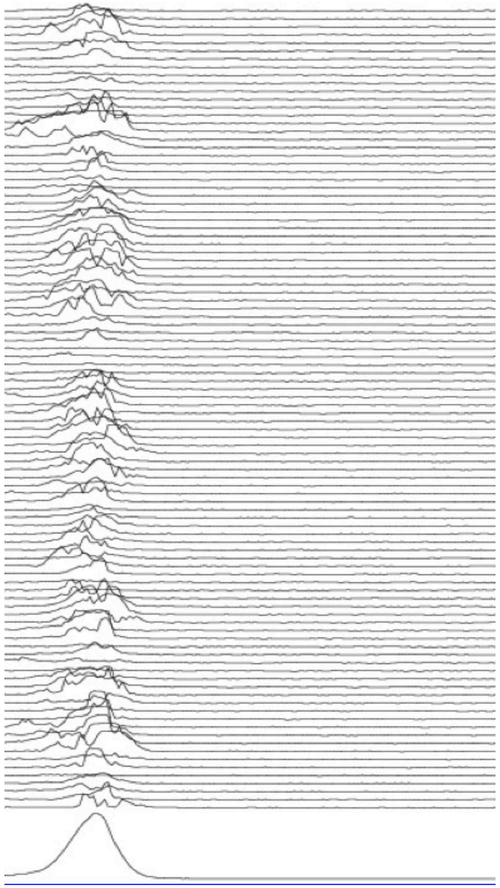
GW: A New Way of Observing Neutron Stars



GW Observatories



The Fingerprint of a Pulsar



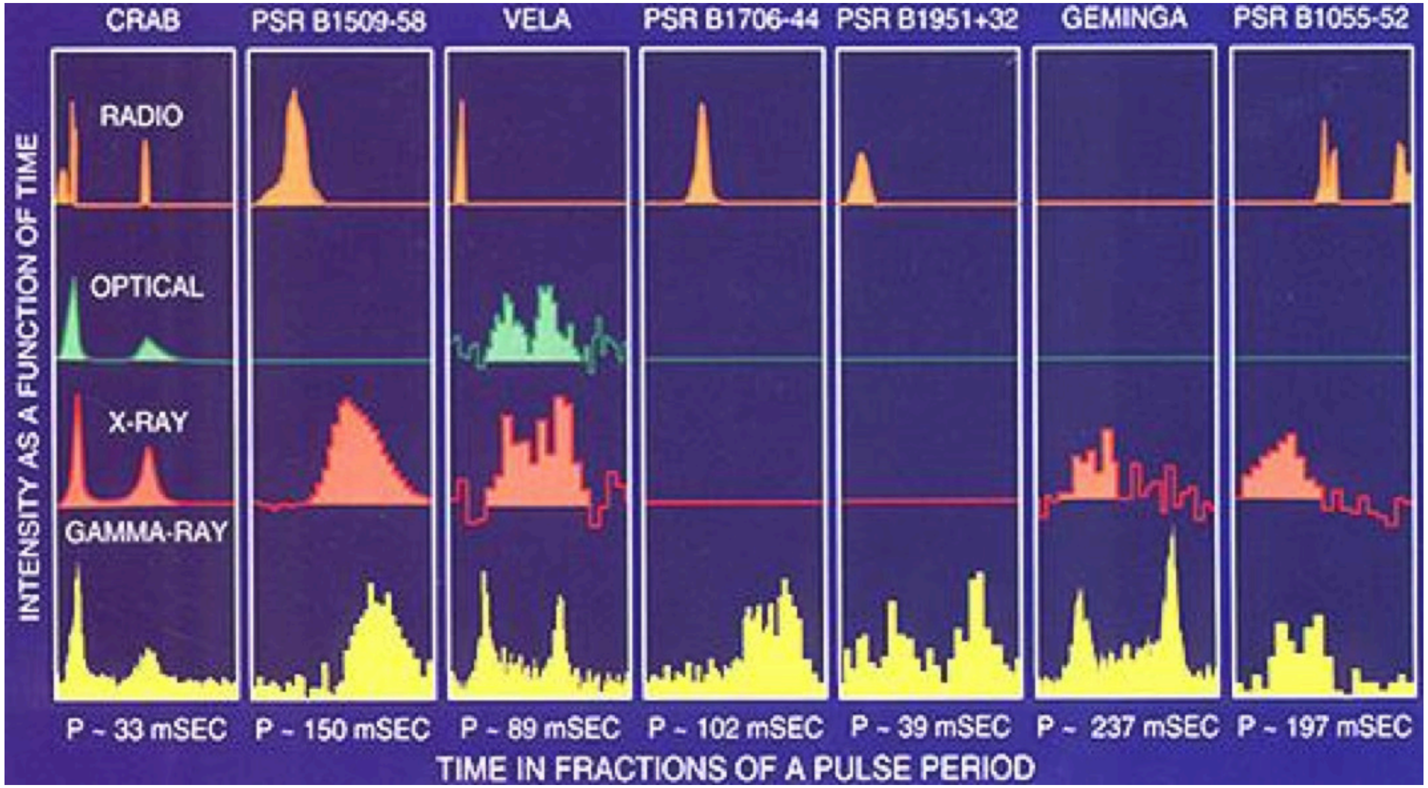
Individual pulses are very different. But the average over 100 or more pulses is extremely stable and specific of each pulsar

- Top: 100 single pulses from the pulsar PSR B0950+08 (P=0.253 s) showing the pulse-to-pulse variability in shape and intensity

- Bottom: Cumulative profile over 5 minutes (~ 1200 pulses)

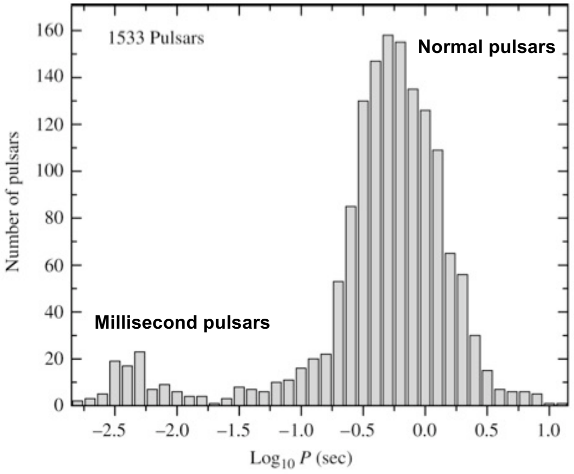
Observations taken with the Green Bank Telescope
(Stairs et al., 2003)

Pulsar shape at different wavelength



Pulsar Rotational Period

The distribution of the rotational period of pulsars shows two clear peaks that indicate the existence of two types of pulsars



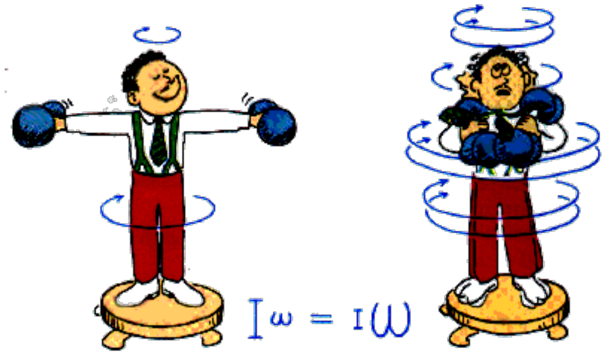
- normal pulsars with P ~ s
- millisecond pulsars with P ~ ms

Globular cluster Terzan 5

- First millisecond pulsar discovered in 1982 (Arecibo)
- Nowadays more than 200 millisecond pulsars are known
- PSR J1748-2446ad discovered in 2005 is until know the fastest one with P=1.39 ms (716 Hz)

Why Pulsars spin so fast ?

The simplest answers is: conservation of the angular momentum during the gravitational collapse of the iron core that will form the neutron star



If the initial iron core and the final neutron star are assumed to be rigid spheres with moment of inertia $I=(2/5)MR^2$

$$J_i = J_f \Rightarrow P_f = P_i\left(\frac{R_f}{R_i}\right)^2$$

Taking $P_i \sim 10^3$ s and $R_f/R_i \sim 10^{-2}$ one gets $P_f \sim 10^{-3}$ s

Maximum (Minimum) Rotational Frequency (Period) of a Neutron Star

Rotation of pulsars can be accurately measured. However, pulsars cannot spin arbitrarily fast. There is an absolute maximum (minimum) rotational frequency (period)

Centrifugal Force = Gravitational Force

⬇

Keplerian Frequency Ω_K

(EoS dependent)

Newtonian Gravity

$$P_{\min} = 2\pi\sqrt{\frac{R^3}{GM}} \approx 0.55\left(\frac{M_{sun}}{M}\right)^{1/2}\left(\frac{R}{10km}\right)^{3/2} ms$$

General Relativity

$$P_{\min} = 0.96\left(\frac{M_{sun}}{M}\right)^{1/2}\left(\frac{R}{10km}\right)^{3/2} ms$$

An observed frequency above the Ω_k predicted by a given EoS would rule out that model

Fasted pulsar known: PSR J1748-2446ad (P=1.39595482 ms)
cannot allow to put stringent constraints on existing EoS

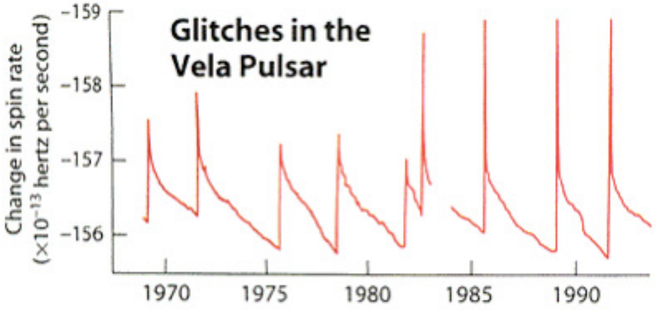
"... And that, Jimmy, is what we call 'centrifugal force'."

Pulsar glitches

Sometimes the period P of a pulsar decreases suddenly. These variations (glitches), although small, are observable

$$\frac{\Delta\Omega}{\Omega} \approx 10^{-9} - 10^{-5}$$

First glitches observed in the Crab & Vela pulsars. Nowadays we know more than 520 glitches in more than 180 pulsars



Vortex lines model:

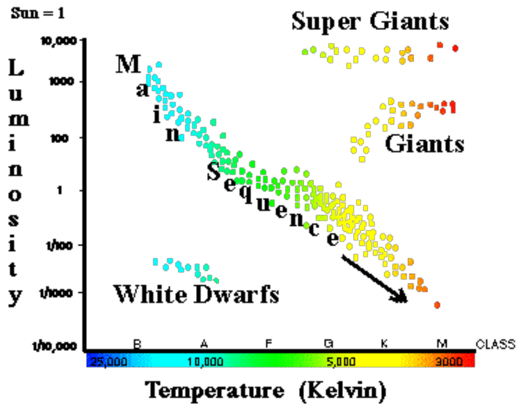
Glitches result from a sudden transfer of angular momentum from the neutron superfluid to the solid crust caused by the unpinning of many vortex lines or by the cracking of the crust to which vortex lines are pinned.

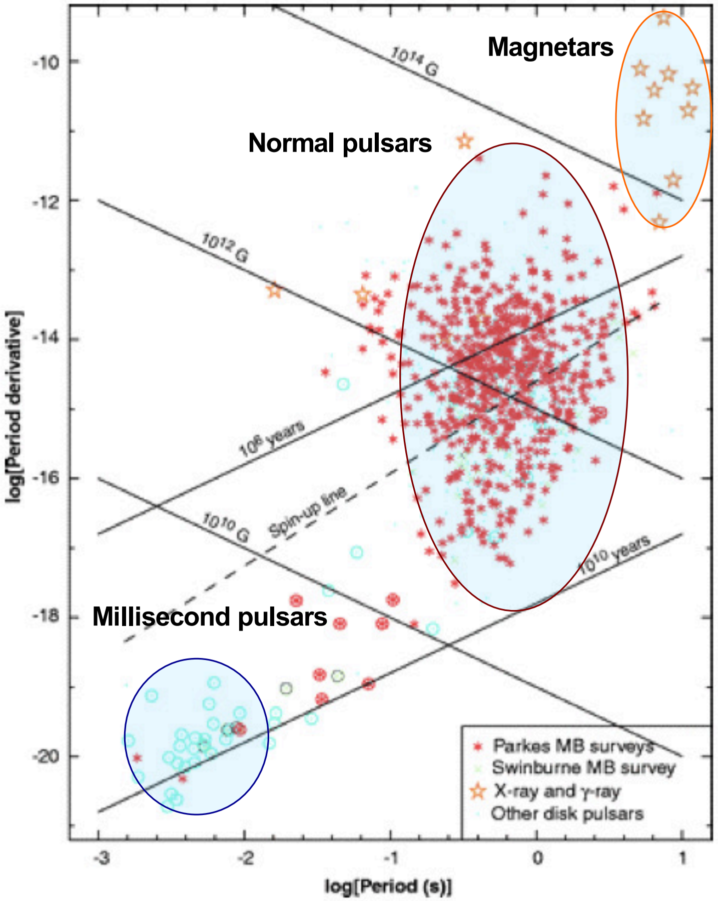
Other models include:

Starquakes between crust & core, magnetospheric instabilities or instabilitie in the moion of neutron superfluid

Pulsar distribution in the P-P plane

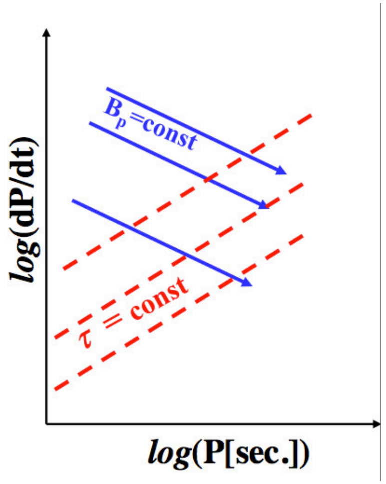
Pulsar equivalent of the Hertzprung-Russell diagram for ordinary stars





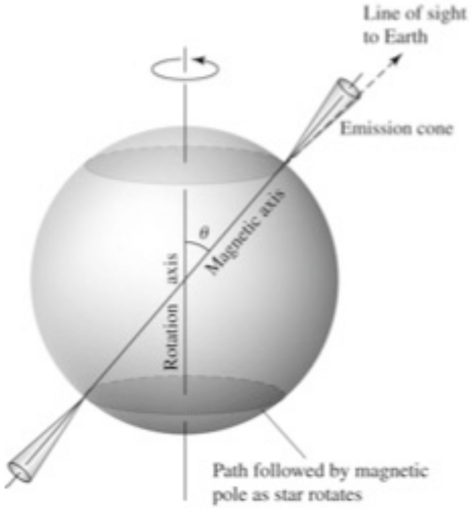
$$\log \dot{P} = \log\left[\frac{(2\pi)^2 R^6}{6c^3 I} B_p^2 \sin^2\alpha\right] - \log P$$

$$\log \dot{P} = \log P - \log(2\tau)$$



Basic Model of a Pulsar: Magnetic Dipole

Pulsar are believed to be highly magnetized rotating neutron stars radiating at the expenses of their rotational energy



$$\dot{E}_{mag} = -\frac{2}{3c^3}\left|\ddot{\vec{\mu}}\right|^2 = \dot{E}_{rot}$$

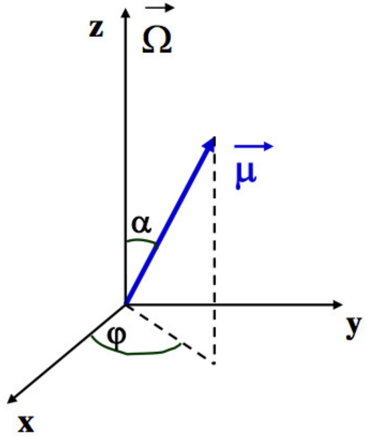
$\vec{\mu} \equiv$ Magnetic dipole moment

Pacini, Nature 216 (1967), 219 (1968)

Gold, Nature 218 (1968), 221 (1969)

Ostriker & Gunn, ApJ 157 (1969)

Basic Model of a Pulsar: Magnetic Dipole



$$\vec{\mu} = \mu \sin\alpha \cos\varphi \hat{e}_x + \mu \sin\alpha \sin\varphi \hat{e}_y + \mu \cos\alpha \hat{e}_z$$

supposing: α=const, μ=|$\vec{\mu}$|=const $\quad \Omega = \frac{d\varphi}{dt} = \dot{\varphi}$

$$\dot{\Omega}^2 << \Omega^4$$

$$\left|\ddot{\vec{\mu}}\right| = \mu^2 (\sin\alpha)^2 \left[\Omega^4 + \dot{\Omega}^2\right] \longrightarrow \left|\ddot{\vec{\mu}}\right| \approx \mu^2 (\sin\alpha)^2 \Omega^4$$

Therefore

$$\dot{E}_{mag} = -\frac{2}{3c^3} \mu^2 (\sin\alpha)^2 \Omega^4 = \dot{E}_{rot}$$

For a sphere with a pure dipole magnetic field

$$\mu = \frac{1}{2} B_P R^3$$

- ✓ B_p: magnetic field at the poles
- ✓ R: radius of the sphere

Basic Model of a Pulsar: Magnetic Dipole

Then $$\dot{E}_{mag} = -\frac{1}{6c^3} R^6 B_P^2 (\sin\alpha)^2 \Omega^4 = \dot{E}_{rot}$$

On the other hand $$E_{rot} = \frac{1}{2} I \Omega^2 \xrightarrow{\dot{I}=0} \dot{E}_{rot} = I\Omega\dot{\Omega}$$

One arrives to the PSR evolution differential equation

$$\dot{\Omega} = -K\Omega^3 \quad or \quad P\dot{P} = (2\pi)^2 K, \quad K = \frac{1}{6c^3}\frac{R^6}{I}(B_P \sin\alpha)^2$$

Allows to obtain the strength of the magnetic field in terms of observable quantities !!

More generally, one can write the PSR evolution differential equation as

$$\dot{\Omega} = -K\Omega^n \quad or \quad P^{n-2}\dot{P} = (2\pi)^{n-1}K, \quad K = \frac{1}{6c^3}\frac{R^6}{I}(B_P \sin\alpha)^2$$

with solution

$$\Omega(t) = \frac{\Omega_0}{\left[(n-1)K\Omega_0^{n-1}t+1\right]^{1/(n-1)}}, \quad P(t) = P_0\left[(n-1)K\Omega_0^{n-1}t+1\right]^{1/(n-1)}$$

Differenciating it assuming K=const, one obtains

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2}$$ braking index

n=3 within the magnetic dipole model

The three quantities P, $\dot{P}$ & $\ddot{P}$ have been measured for few PSRs

The Pulsar Age

The solution of the PSR evolution differential equation can be rewritten as

$$t = -\frac{1}{n-1}\frac{\Omega(t)}{\dot{\Omega}(t)}\left[1-\left(\frac{\Omega(t)}{\Omega_0}\right)^{n-1}\right]$$

or

$$t = \tau - \left[(n-1)K\Omega_0^{n-1}\right]^{-1}$$

“True” Pulsar Age

(valid under the assumption K=const.)

with

$$\tau = -\frac{1}{n-1}\frac{\Omega(t)}{\dot{\Omega}(t)} = \frac{1}{n-1}\frac{P(t)}{\dot{P}(t)} \xrightarrow{n=3} \tau = -\frac{1}{2}\frac{\Omega(t)}{\dot{\Omega}(t)} = \frac{1}{2}\frac{P(t)}{\dot{P}(t)}$$

Pulsar Dipole Age or Characteristic Pulsar Age

if $\Omega(t) << \Omega_0 \longrightarrow t \approx \tau$

(t=present time)

The measure of P and $\dot{P}$ gives the pulsar dipole age

Example: the age of the Crab Pulsar

SN explosion: 1054 AD

$P=0.0330847$ s, $\dot{P}=4.22765\times10^{-13}$ s/s

Braking index: $n=2.515 \pm 0.005$

$t_{Crab}=(2022-1054)$ yr = 968 yr, $\tau=1238$ yr (dipole age)

Assuming the validity of the pulsar dipole mode, using the previous equation for the true pulsar age we can infer the initial spin period of the Crab pulsar

$$P_0 = P\left(1-{t_{Crab}}/{\tau}\right)^{1/2} \cong 0.016s$$

But $n \neq 3$

Measured value of the braking index n

PSR	n	P (s)	P dot (10^{-15} s/s)	Dipole age (yr)
PSR B0531+21 (Crab)	2.512 +/- 0.005	0.03308	422.765	1238
PSR B0833-45 (Vela)	1.4 +/- 0.2	0.08933	125.008	11000
PSR B0540-69	2.839 +/- 0.005	0.1506	1536.5	1554
PSR B0540-69	2.01 +/- 0.02	0.0505	478.924	1672
PSR J1119-6127	2.91 +/- 0.05	0.40077	4021.782	1580

Deviations of braking index n from 3 probably due to:

- ✓ Torque on the pulsar from outflow particles
- ✓ Change with t of “constant” K, i.e., I(t), B(t), α(t)

Pulsar evolutionary path on the P-$\dot{\mathrm{P}}$ plane

Taking the logarithm of

$$P\dot{P} = (2\pi)^2 K, \quad K = \frac{1}{6c^3}\frac{R^6}{I}\left(B_P \sin\alpha\right)^2$$

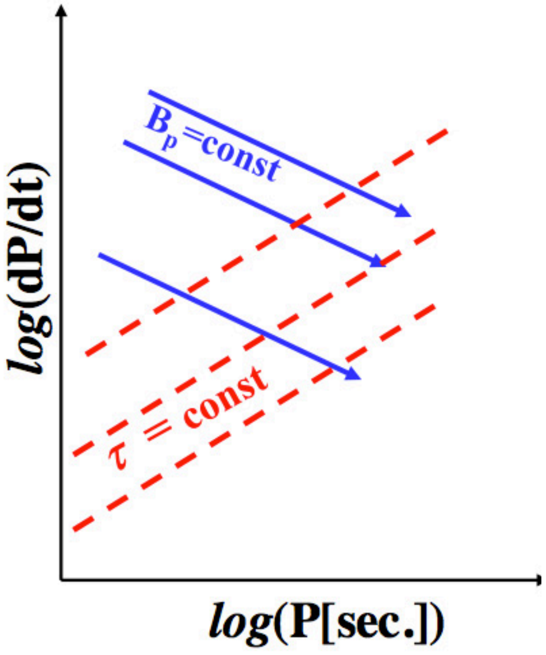
and

$$\tau = \frac{P}{2\dot{P}}$$

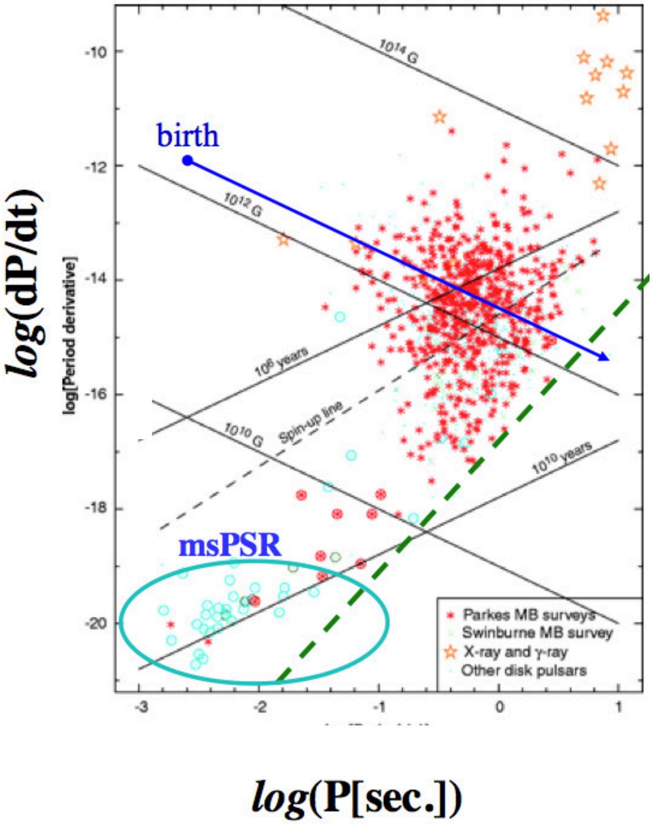
we get

$$\log\dot{P} = \log\left[\frac{(2\pi)^2 R^6}{6c^3 I} B_P^2 \sin^2\alpha\right] - \log P$$

$$\log\dot{P} = \log P - \log(2\tau)$$



Pulsar evolutionary path on the P-$\dot{P}$ plane



Millisecond pulsars are very old pulsars with dipole ages in the range 10^8-10^{10} yr

Millisecond pulsars are believed to result from the spin-up of a “slow” rotating NS through mass accretion (and angular momentum transfer) from a companion star in a binary stellar system

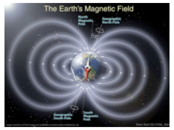
Magnetic Field of a Pulsar

Type of Pulsar	Surface magnetic field
Millisecond	$10^8 - 10^9$ G
Normal	10^{12} G
Magnetar	$10^{14} - 10^{15}$ G

Extremely high compared to …

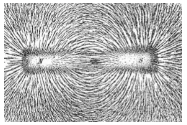
Earth

$0.3 - 0.5G$



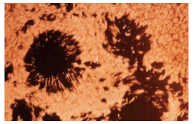
Magnet

$10^3 - 10^4 G$



Sun spots

$10^5 G$

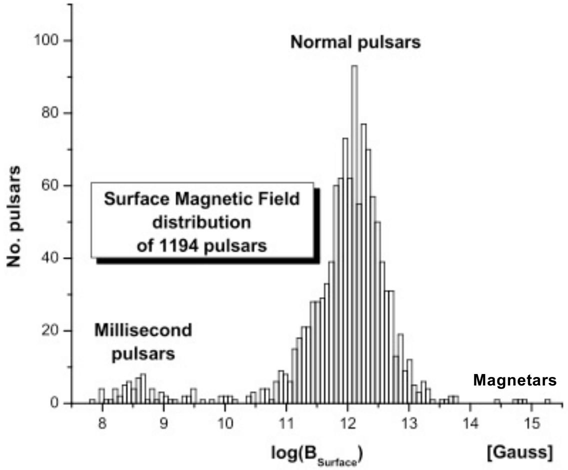


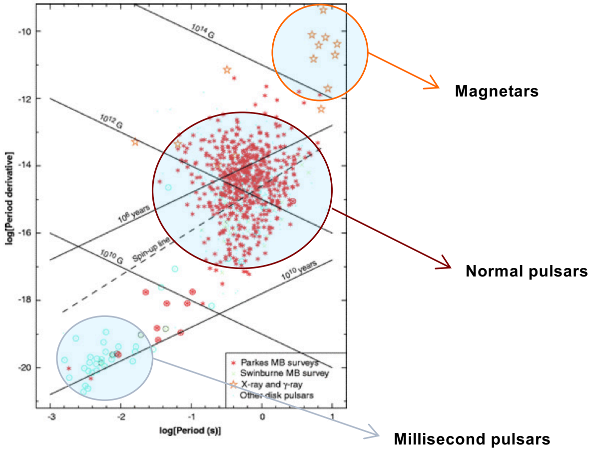
Largest continuous field in lab. (USA)

$4.5x10^5 G$

Largest magnetic pulse in lab. (Russia)

$2.8x10^7 G$





Where the NS magnetic field comes from ?

A satisfactory answer does not exist yet. Several possibilities have been considered:

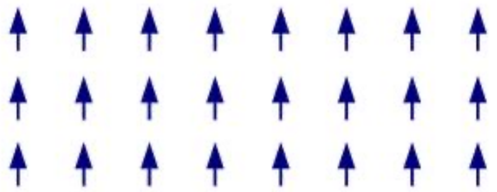
✧ Conservation of the magnetic flux during the gravitational collapse of the iron core

$$\phi_i = \phi_f \Rightarrow B_f = B_i\left(\frac{R_i}{R_f}\right)^2$$

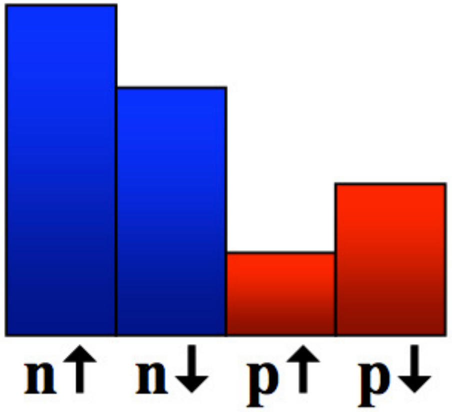
For a progenitor star with $B_i \sim 10^2$ G & $R_i \sim 10^6$ km we have $B_f \sim 10^{12}$ G

✧ Electric currents flowing in the highly conductive NS interior

✧ Spontaneous transition to a ferromagnetic state due to the nuclear interaction



Spin-polarized Isospin Asymmetric Nuclear Matter



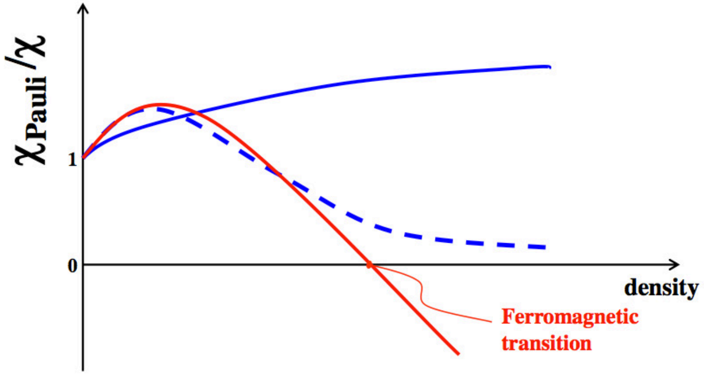
✧ Densities & Asymmetries

- ✓ $\rho_n = \rho_{n\uparrow} + \rho_{n\downarrow}, \quad \rho_p = \rho_{p\uparrow} + \rho_{p\downarrow}$
- ✓ $\rho = \rho_n + \rho_p, \quad \beta = \dfrac{\rho_n - \rho_p}{\rho}$
- ✓ $S_n = \dfrac{\rho_{n\uparrow} - \rho_{n\downarrow}}{\rho_n}, \quad S_p = \dfrac{\rho_{p\uparrow} - \rho_{p\downarrow}}{\rho_p}$

✧ Magnetic Susceptibility

$$\frac{1}{\chi_{ij}} = \frac{\rho}{\mu_i \rho_i \mu_j \rho_j} \frac{\partial^2 (E/A)}{\partial S_i \partial S_j}$$

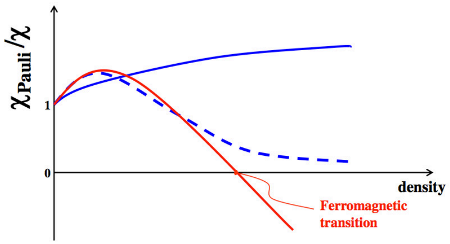
Stability against
spin
fluctuations if $\chi > 0$



Ferromagnetic Transition

Considered by many authors with contradictory results:

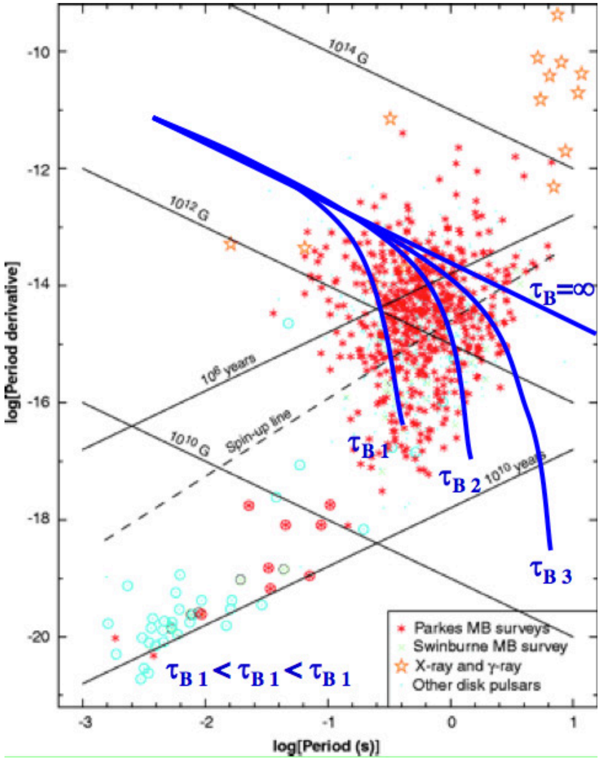
Year	Autor/Model	Ferromagnetic Transition ?
1969	Brownell, Callaway, Rice (hard sphere gas)	Yes, $k_F>2.3$ fm^{-1}
1969	Clark & Chao	No
1970	Ostgard	Yes, $k_F>4.1$ fm^{-1}
1972	Pandharipande et al., (variational)	No
1975	Backman, Kallaman, Haensel (BHF)	No
1984	Vidaurre (Skyrme)	Yes, $k_F>1.7$-2.0 fm^{-1}
1991	S. Marcos et al., (DBHF)	No
2001	Fantoni et at. (AFDMC)	No
2002/2005	I.V., et al. (BHF)	No
2005/2006	I.V. et al., (Skyrme,Gogny)	Yes, $k_F>2$-3.4 fm^{-1}
2007-2011	F. Sammarruca (DBHF)	No



- ✧ Calculations based on phenomenological interactions (e.g., Skyrme, Gogny) predict the transition to occur at (1-4)ρ_0

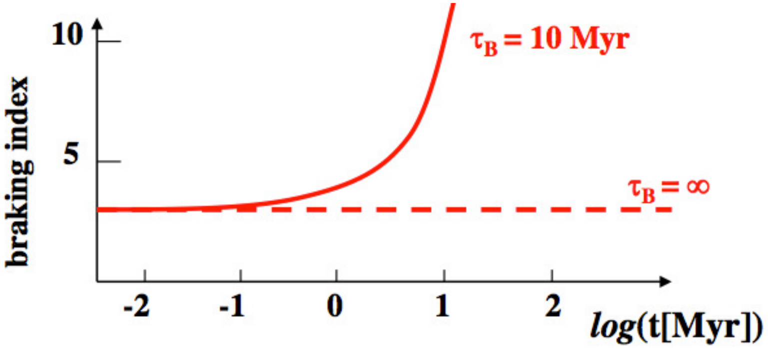
- ✧ Calculations based on realistic NN & NNN forces (e.g., Monte Carlo, BHF, DBHF, LOCV) exclude such a transition

Magnetic field decay in Neutron Stars



There are strong theoretical & observational arguments indicating the decay of NS magnetic field (Ostriker & Gunn 1969)

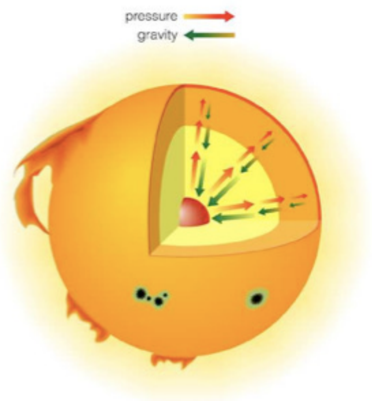
$$B(t) = B_\infty + \left[B_0 - B_\infty\right]\exp\left[-t/\tau_B\right]$$



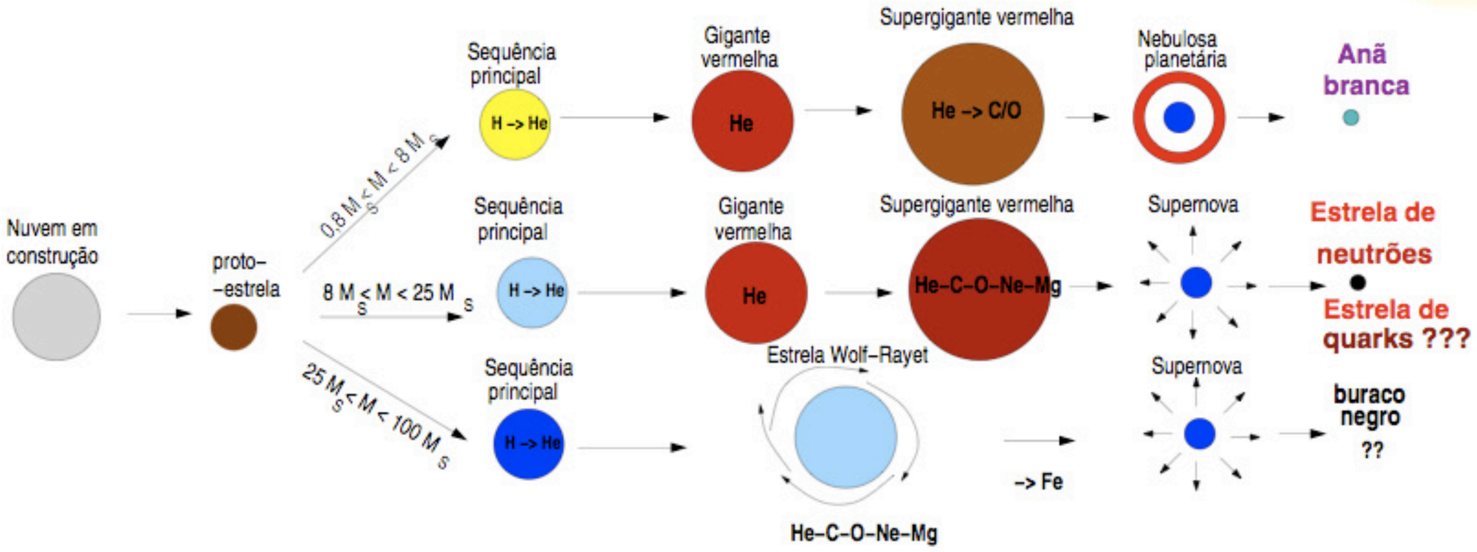
$$n(t) = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{3c^3 I\dot{B}}{R^6 B^3 \left(\sin\alpha\right)^2 \Omega^2}$$

Birth of a Compact Star

A bit of stellar evolution



Life of a star is due to the hydrostatic equilibrium of the thermal pressure of the gas (or radiation pressure in the case of very massive stars) and the gravitational force

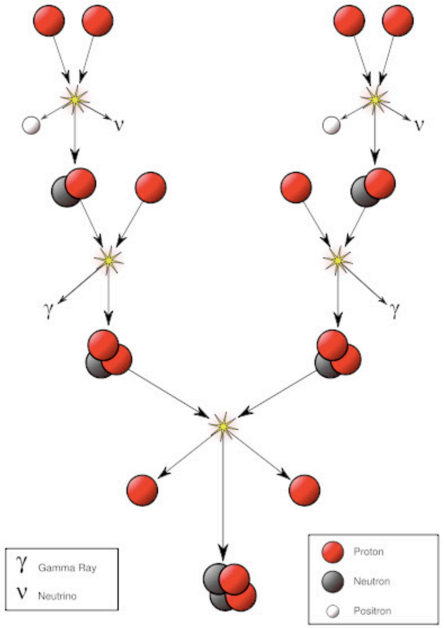


Some Fusion Mechanisms

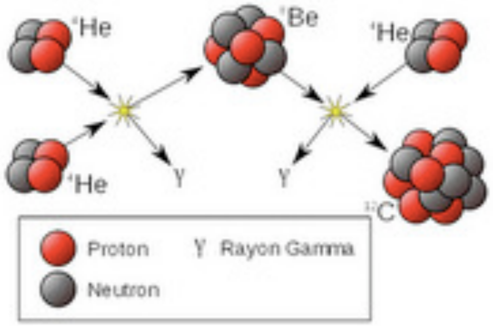
- ✧ Proton-proton Chain

$$H\left(H,e^{+}\nu_{e}\right)D\left(H,\gamma\right)^{3}He$$

$$^{3}He\left(^{3}He,2p\right)^{4}He$$

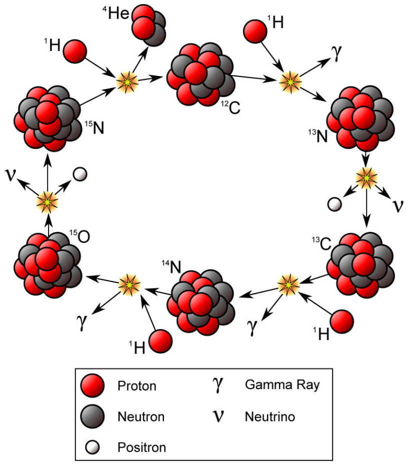


- ✧ Triple-alpha Process



$$^{4}He\left(^{4}He,\gamma\right)^{8}Be\left(^{4}He,\gamma\right)^{12}C$$

CNO Cycle



$$^{12}C(H,\gamma)^{13}N\left(e^{+}\nu_{e}\right)^{13}C(H,\gamma)^{14}N(H,\gamma)^{15}O\left(e^{+}\nu_{e}\right)^{15}N\left(H,^{4}He\right)^{12}C$$

Other Processes

$$^{12}C+^{4}He\rightarrow^{16}O+\gamma$$

$$^{12}C+^{12}C\rightarrow^{20}Na+^{4}He$$

$$^{12}C+^{12}C\rightarrow^{23}Mg+n$$

$$^{12}C+^{12}C\rightarrow^{24}Mg+\gamma$$

$$^{16}O+^{16}O\rightarrow^{32}S+\gamma$$

$$^{16}O+^{16}O\rightarrow^{31}P+H$$

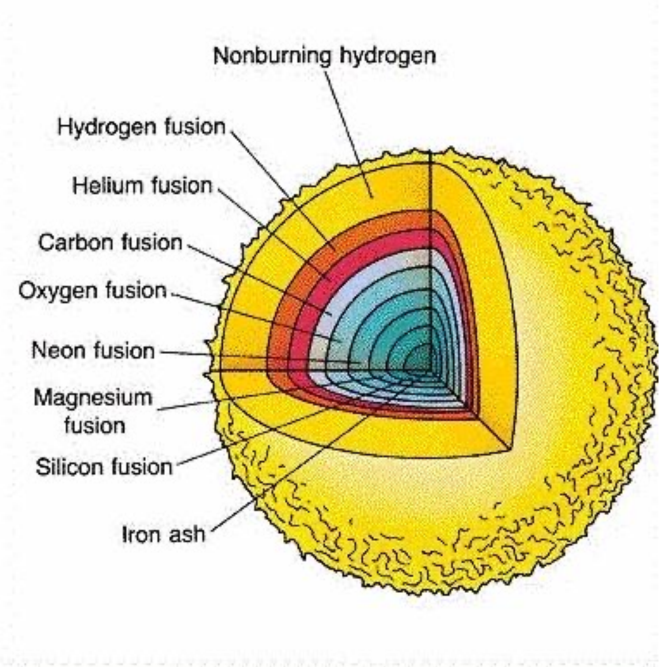
$$^{16}O+^{16}O\rightarrow^{31}S+n$$

$$^{16}O+^{16}O\rightarrow^{28}Si+^{4}He$$

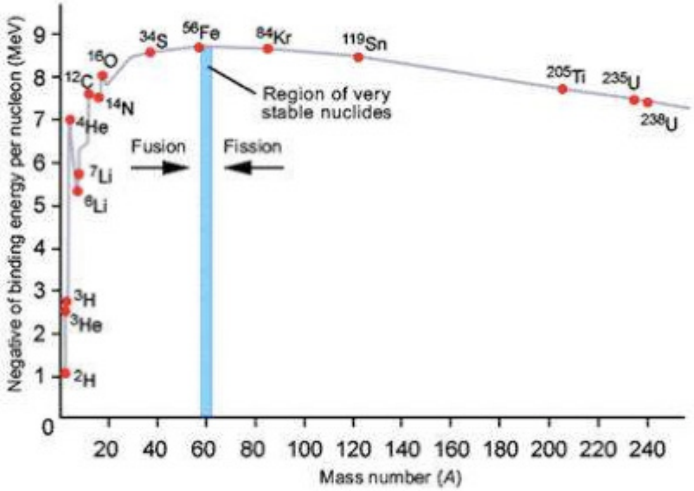
$$^{16}O+^{16}O\rightarrow^{24}Mg+2^{4}He$$

The Onion Structure

In the more massive stars ($M > 8\text{-}10\ M_{\odot}$) fusion reactions continue leading to the formation of more and more heavy elements (C, O, Ne, …) till iron, the most stable element , is formed

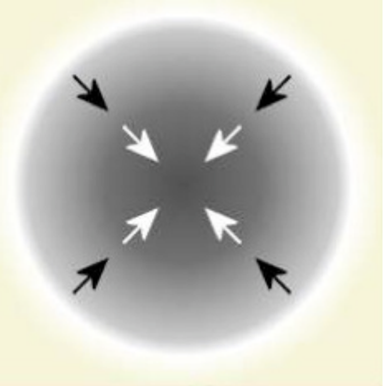


End of Nucleosynthesis & Gravitational Collapse

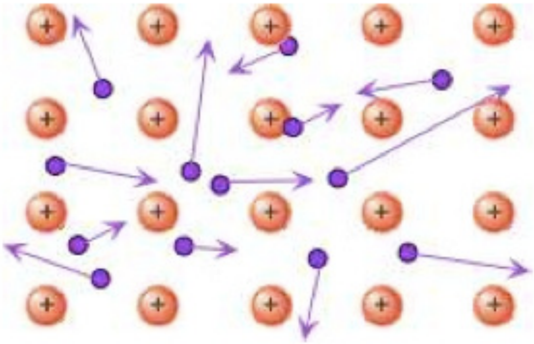


With the formation of Fe & other nuclei of the Fe family nuclear fusion stops & the radiation pressure is reduced

The Fe core cannot support the gravitational attraction and collapses. Matter is compressed to densities more and more high



Pauli stops the collapse

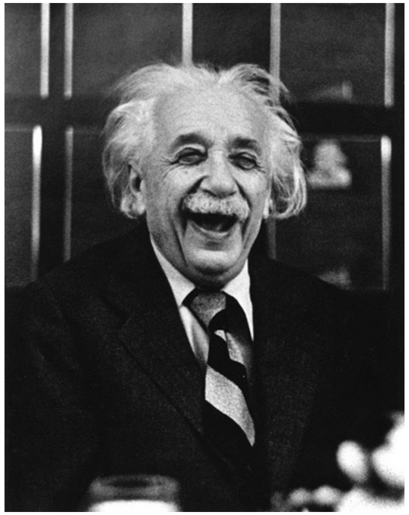


When density reach values ~ 10^{14} g/cm^{3} atoms ionize completely and the electrons form a degenerate Fermi gas

But electrons being fermions cannot occupy the same quantum state

➔ The degeneracy pressure of the electrons $P_e(\rho)$ stops the gravitational collapse (R. H. Fowler (1926))

Relativity & Maximum Mass



Quantum degeneracy ➔ finite electron velocity even at T=0

Electrons become ultra-relativistic for $\rho > 10^6$ g/cm^3
➔ their velocity cannot be larger than c

➔ Electrons CANNOT support an arbitrary high mass
➔ A maximum mass should exist

The Chandrasekhar Mass Limit

The total energy of an ultra-relativistic white dwarf can be written as

$$E = \frac{A - B}{R} + CR \quad \text{with} \quad A \sim \hbar c \left(\frac{M}{m_p} \right)^{4/3}, \quad B \sim GM^2, \quad C \sim \frac{m_e^2 c^3}{\hbar} \left(\frac{M}{m_p} \right)^{2/3}$$

The equilibrium radius R_{eq} (obtained by minimizing E) only exists for $A - B > 0$

$$\longrightarrow \quad M < M_{Chand} \sim \left(\frac{\hbar c}{G} \right)^{3/2} \left(\frac{1}{m_p} \right)^2$$

Chandrasekhar Mass Limit

What happens if M> $M_{Chandrasekhar}$?

In this case gravitational collapse continues

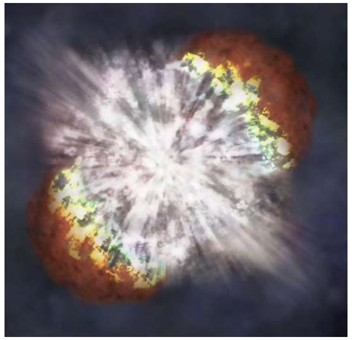
- ✓ Density increases & electron gas becomes more and more degenerate ⟶ electron capture by protons make matter more & more neutron rich

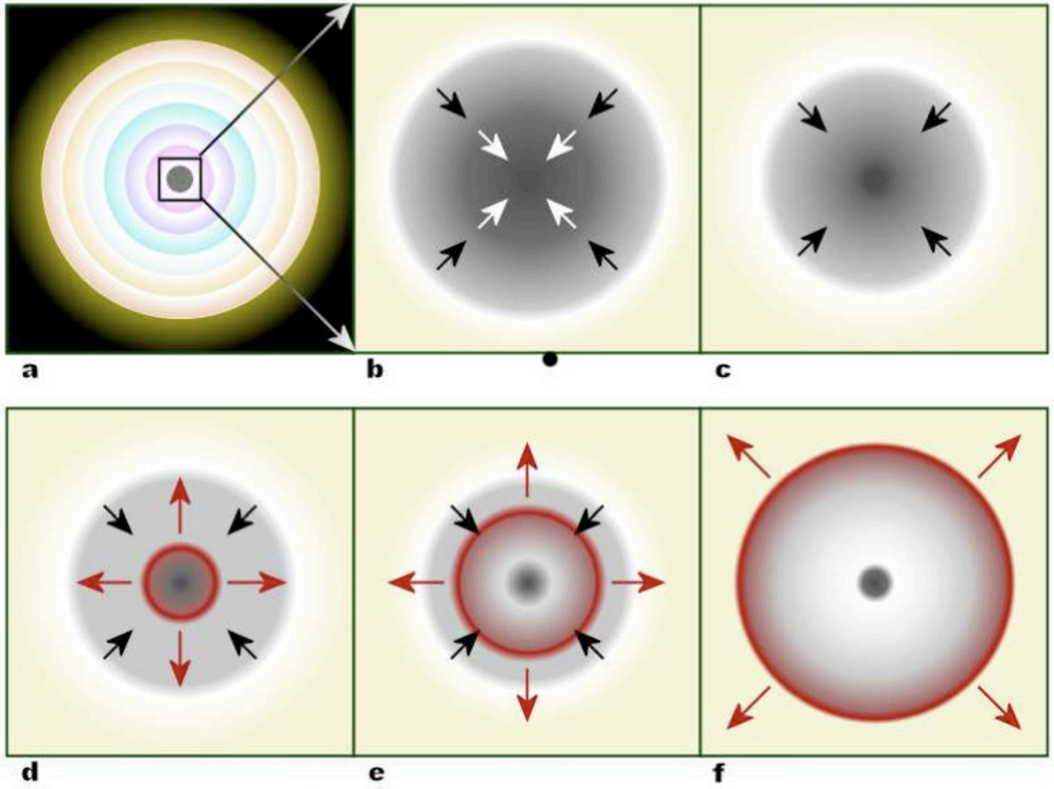
$$e^- + p \rightarrow n + \nu_e$$

- ✓ The core of the star is effectively transformed into a “neutron gas”

- ✓ If the degeneracy pressure of neutrons can (cannot) stop the collapse a NS (BH) is formed

Supernova Explosion in a Nutshell





- Instability
- Collapse

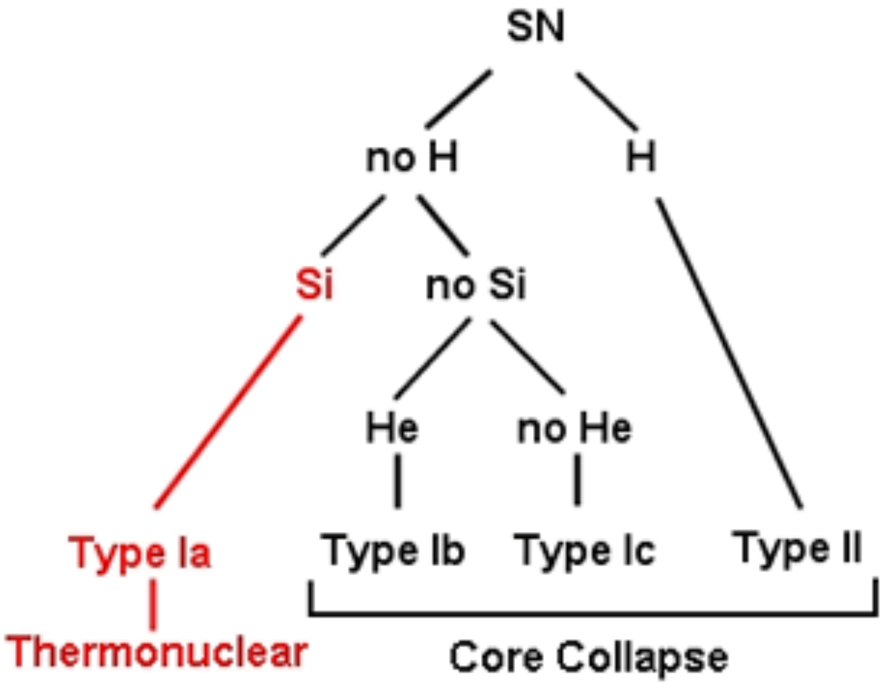
  0.1 s
  velocity ~ 0.1 c
  pressure ~10^6 tm/cm^3

- Shock wave
- SN explosion

emission of the external layers &
formation of the Compact Star

Supernovae Classification

SN are classified based on the presence or the absence of certain elements in their optical spectrum



Some historical supernovae

SN 185: In 185 A.D. Chinese astronomers observe the appearance in the sky of a new “shinning star” that will shine during 8 months.

Commodus (161-192 A. D.) waste public money in games and spectacles, participating as gladiator in more than 1000 fights

SN 393: In 393 A.D. Chinese astronomers observe again the appearance of a “shinning star” in the the constellation of Scorpio

Olympic Games are declared illegal by Theodosius I (379-195 D. D.)

SN 1006: In 1006 a new “shinning star” is observed by Chinese, Japanese and Arabian astronomers at the South of the constellation of Lupus. The Egyptian astronomer Ali ib Ridwan estimates that the shine of this star is ¼ that of the moon

The holmgang (a kind of Scandinavian duel) is declared declared illegal in Iceland

SN 1181: In 1181 Chinese and Japanese astronomers observe a “shinning star” in the constelation of Casiopea. The radio source 3C58 is probably the remnant of this SN

Pope Alexander III by means of bull “Regis Aeterni” gives the Jubilee to the cathedral of Santiago de Compostela

The supernova of 1054 SN 1054

On July 4th 1054 Chinese astronomers observe a very “shinning star” in the constellation of Taurus. The star, observed also by Japanese and Arabian astronomers, is visible during daylight for 23 days and disappears of the sky after 2 years.

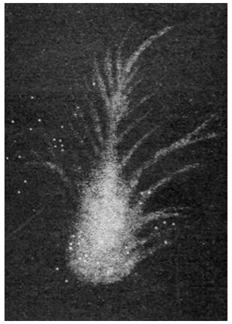
East-West Schism: separation of the Catholic & Orthodox churches

A petroglyph in Chaco Canyon suggest the possibility that this “star” was also observed by the Anasazi Native American tribe

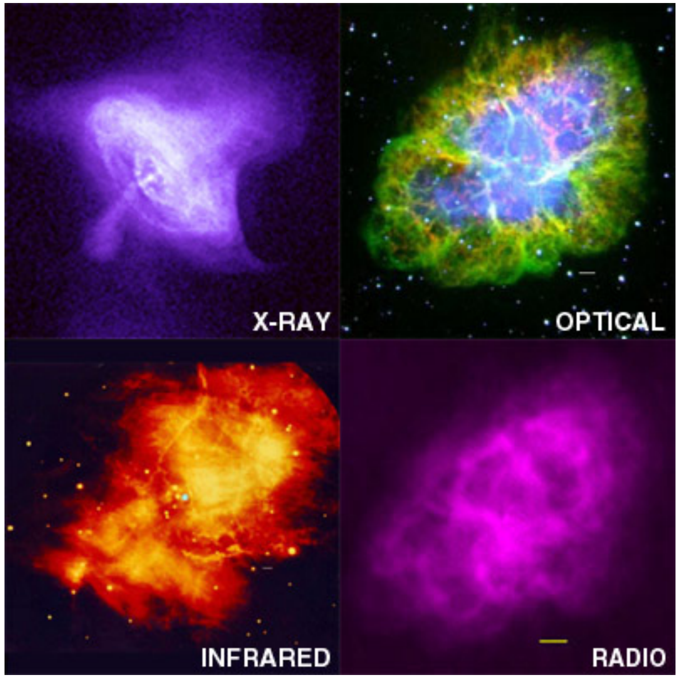
The Crab Nebula

The “shinning star” observed by the Chinese astronomers in 1054 was re-discovered in 1731 by the British astronomer John Bevis and is the first object that appears in the catalogue of Charles Messier published in 1781

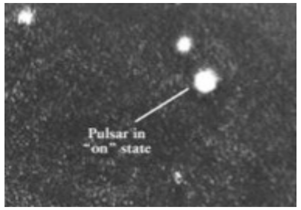
William Parson, Lord Rosse, named for the first time Crab Nebula to the object M1 of Messier’s catalogue due to its filamentous structure

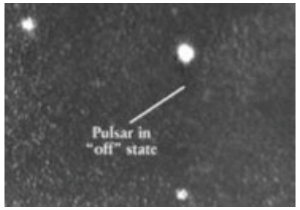


The Crab Nebula Today

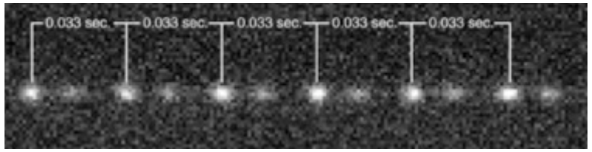


X-ray: Chandra Satellite
Optical: Palomar
Infrared: Kreck
Radio: VLA



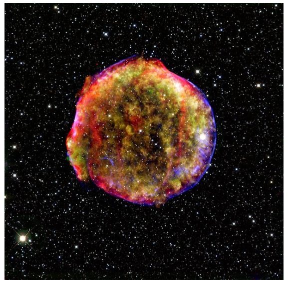


The pulsar PSR B0531+21 of the Crab Nebula was discovered in 1968 by the astronomers of the Green Bank observatory (USA). Its period is only 33 miliseconds

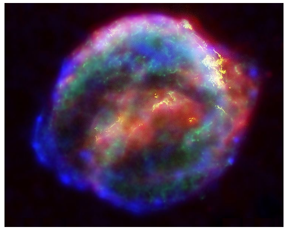


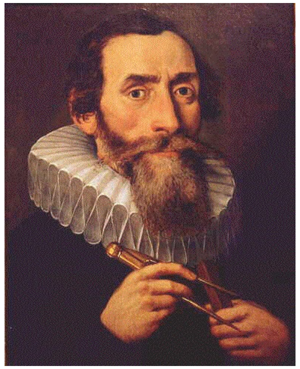
SN 1572 o Tycho’s Supernova

Observed on November 11th 1572 in the constellation of Casiopea by Tycho Brahe and other astronomers. Visible at eye till 1574. Used by Tycho Brahe to refute the Aristoteliam dogma of the sky inmutability



(you can visit his burial place in the Church of Tyn here in Prague)





(lived also in Prague for sometime)

SN 1604 o Kepler’s Supernova

Observed (probably in Prague) on November 9th 1604 in the constellation of Ophiucus by Kepler. Last SN observed in our galaxy till today

Here in Prague you can find several places that testimony the presence of both astronomers

(Tyn Church)

(Tycho's gravestone)

(Kepler lived in Karlova 4)

(Gymnázium Jana Keplera,
Praha 6)

Estimation of Neutron Star Mass & Radius

Imagine that a neutron star is:

- ✓ a sphere of uniform density
- ✓ made only of neutrons
- ✓ in addition to the nuclear force neutrons feel also the gravity

<u>Idea</u>: use Bethe-Weizsäker semi-empirical mass formula including the gravitational force

$$B(Z,A) = a_v A - a_s A^{2/3} - a_{coul}\frac{Z^2}{A^{1/3}} - a_{sim}\frac{(Z-N)^2}{A} + \delta a_p A^{-1/2}$$

Only Neutrons (Z=0) + Gravitational Energy (sphere with M=Nm$_n$ & R)

$$B(Z=0, A=N) = a_v N - a_s N^{2/3} - a_{sim} N + \delta a_p N^{-1/2} + \tfrac{3}{5}\frac{G(Nm_n)^2}{R}$$

Since N > $N^{2/3}$ & $N^{-1/2}$

$$B(Z=0, A=N) \approx \left(a_v - a_{sim}\right) N + \frac{3}{5}\frac{Gm_n^2}{r_0} N^{5/3}$$

$$R = r_0 N^{1/3} = 1.15 \times 10^{-15} N^{1/3}\, m$$

The minimum number of neutrons needed to bound gravitationally is obtained imposing

B > 0

The condition B > 0 tell us that:

$$\left(a_v - a_{sim}\right)N + \frac{3}{5}\frac{Gm_n^2}{r_0}N^{5/3} > 0 \Rightarrow N > \left(\frac{5}{3}\frac{\left(a_{sim} - a_v\right)r_0}{Gm_n^2}\right)^{3/2}$$

Using the values:

$$a_v = 16\,MeV, \quad a_{sim} = 30\,MeV, \quad G = 6.707 \times 10^{-39}\hbar c\left(\frac{c^4}{GeV^2}\right), \; m_n = 0.939\frac{GeV}{c^2}$$

We finally arrive to:

$$N \sim 10^{56} - 10^{57} \qquad M \sim 1 \;\; M_\odot \qquad R \sim 10\,km$$

Which gives an average density of:

$$\rho \sim 10^{14} - 10^{15}\, g/cm^3$$

$$N \sim 10^{56} - 10^{57}$$

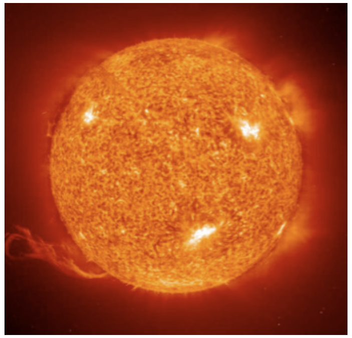
$$M \sim 1 \;\; M_{\odot}$$

A neutron star
is a kind of GIANT ATOMIC
NUCLEUS
in which particles are
gravitationally bound

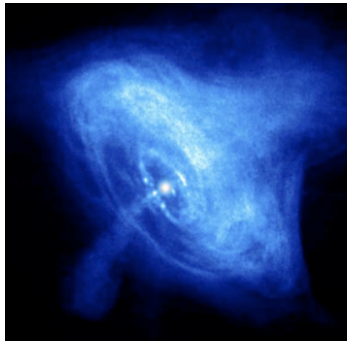
$$R \sim 10\,km$$

$$\rho \sim 10^{14} - 10^{15}\, g/cm^{3}$$

A neutron star
has a mass similar to that
of the Sun, but with a
radius about
70.000 smaller !!!



Radius: ~ 700.000 km
Mass: 1.989x10^{30} kg



Radius ~ 10 km
Mass ~ 1.989x10^{30} kg

Prague vs Neutron Star

- Prague

$$M_{prague} \sim \left(1.26\times10^{6}\right)\times 75 \quad kg$$
$$\sim 4.77\times10^{-23} M_{\odot}$$

$$A_{prague} \sim 496 \quad km^{2}, \quad H_{residents} \sim 1.75m$$

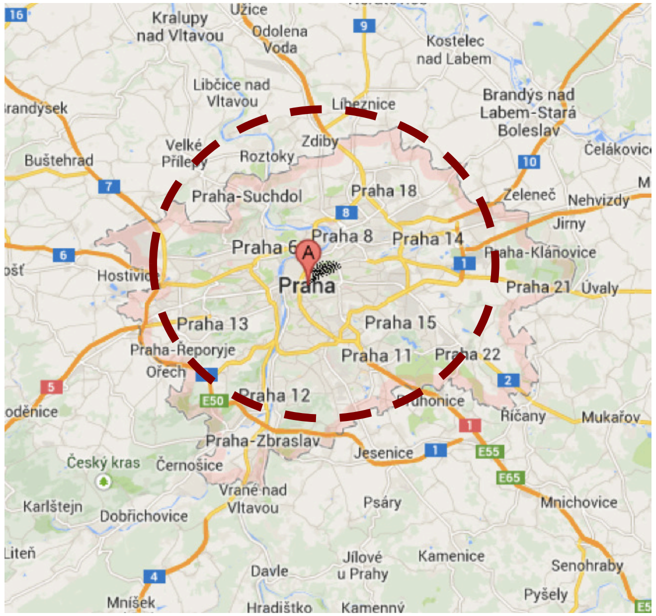
$$\rho_{residents} \sim 1.08\times10^{-4} \quad g/cm^{3}$$

- Neutron Star

$$M_{NS} \sim 1M_{\odot}$$

$$R_{NS} \sim 10 \quad km$$

$$\rho_{NS} \sim 10^{14} - 10^{15} \quad g/cm^{3}$$



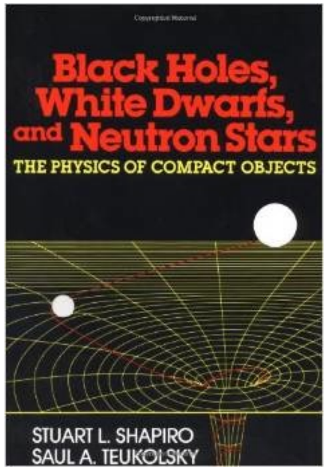
$$\pi R_{NS}^{2} \sim 300 \quad km^{2}$$

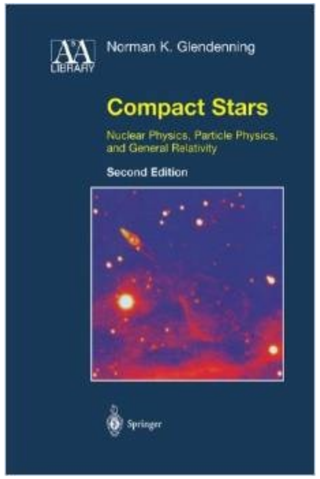
The take away message of this lesson

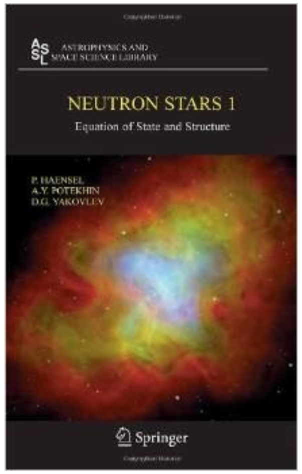
Neutron stars are excellent observatories to test fundamental properties of matter under extreme conditions and offer an interesting interplay between nuclear processes and astrophysical observables

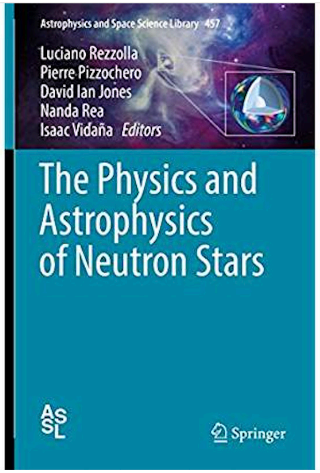
For further reading

Four excellent monographs on this topic for interested readers are:









- You for your time & attention
- The organizers for their invitation & support