

*Lecture 1:  
Quantum Entanglement,  
Quantum tomography applications in Collider Physics*

**Daniel Tapia Takaki**

**Work with  
John Ralston & John Martens**

**Indian-summer school in Prague**  
The Emauzy Abbey  
Prague, Czech Republic, June 24, 2022



# ENTANGLEMENT

- quantum
- quantum physics
- quantum computing
- wormhole
- teleportation
- love
- spacetime
- particle



entum-Entanglement ...  
infocan.com



entanglement link  
phys.org



Spatial overlap leads to useful quantum ...  
physicsworld.com



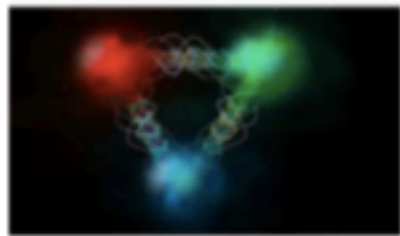
Quantum Entanglement ...  
scienceart.com



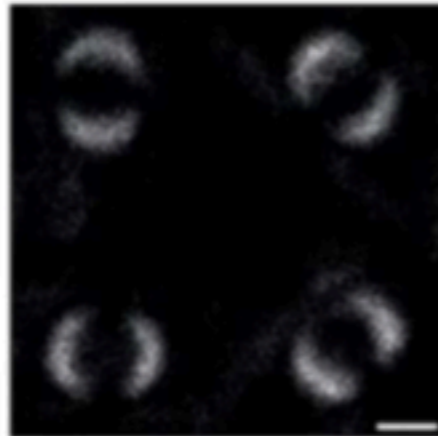
What is quantum entanglement? | Cosmos  
cosmosmagazine.com



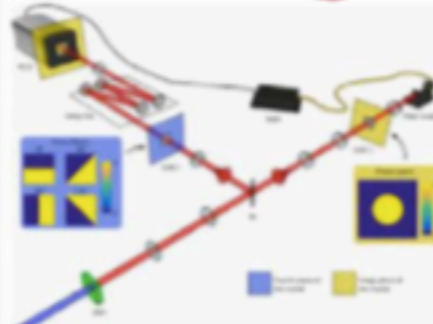
Quantum entanglement  
symmetrymagazine.org



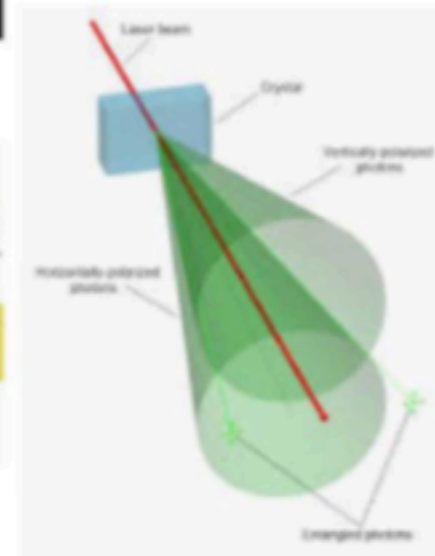
entum entanglement inside protons ...  
mce-news.org



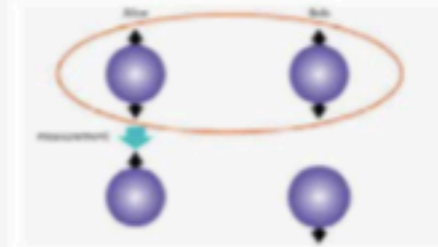
quantum entanglement ...  
businessinsider.com



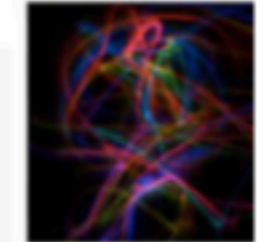
Quantum Entanglement ...  
extremetech.com



Quantum entanglement - Wikipedia  
en.wikipedia.org



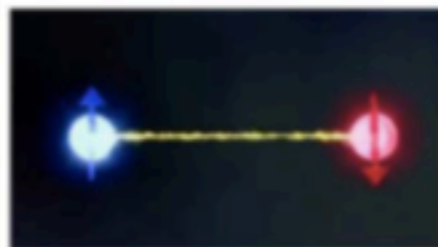
More evidence to support quantum theory ...  
sciencemag.org



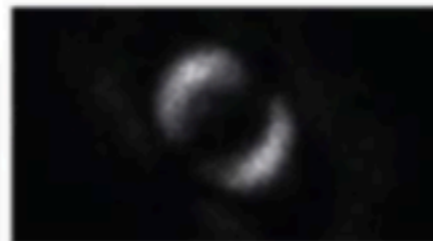
World Record For Quan  
evolving-science.com



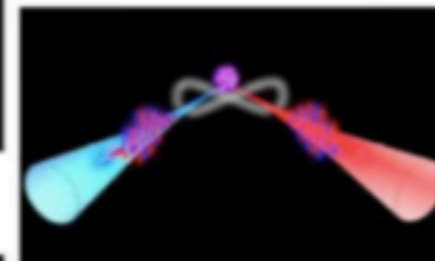
entum Entangled 10 Photon Pairs ...  
moodle.com



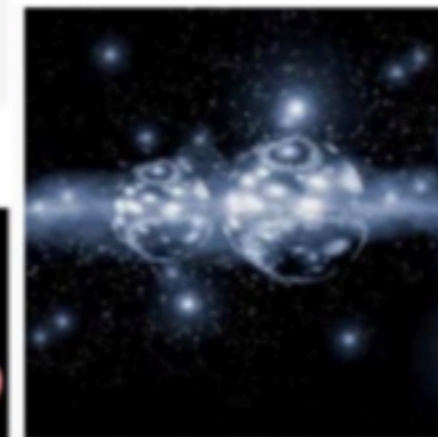
Quantum Reality: Space, Time, and ...  
worldsciencefestival.com



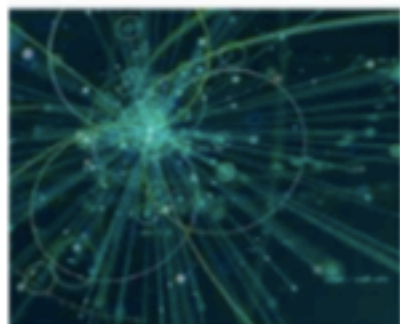
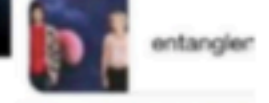
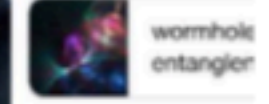
Quantum Entanglement ...  
extremetech.com



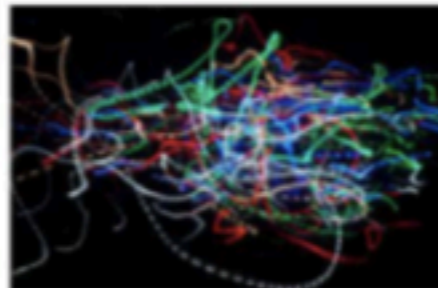
violation of Bell's inequality ...  
phys.org



What is quantum entanglement? | Cosmos  
cosmosmagazine.com



scists Entangle Particles Across ...  
mcsandronvally.com



Quantum entanglement mangles space ...  
news scientist.com

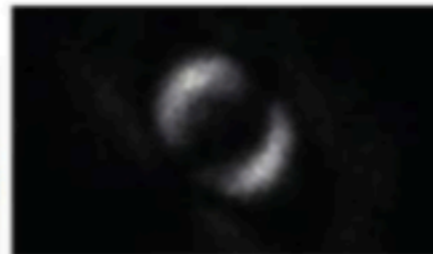
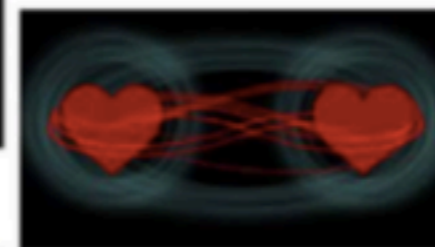
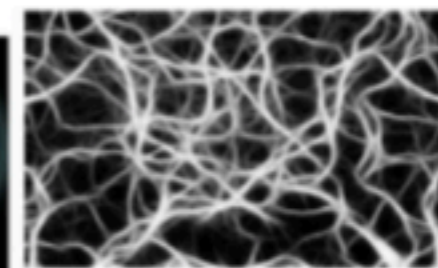


Image of quantum entanglement ...  
engadget.com



Love, quantum physics and 'entanglement'  
prl.org



Entanglement is an inevitable feature ...  
phys.org



Entanglement: Gravity's  
sciencenews.org



entum Entanglement | Brilliant Math ...  
brilliant.org

### Related searches

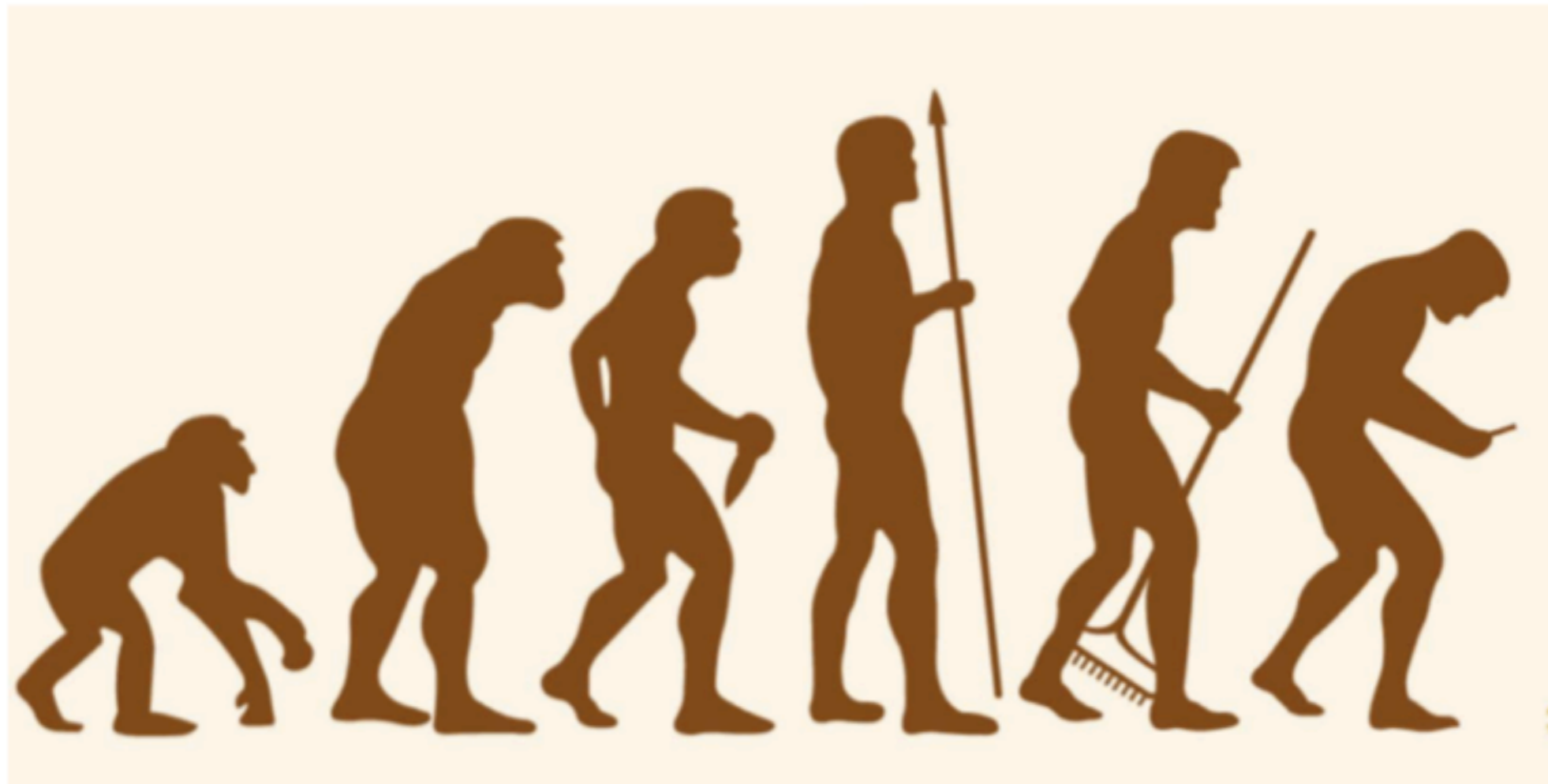


# THEORY HAS EVOLVED

COPENHAGEN WENT PAST ITS PULL-DATE

## WE NOW HAVE INTERNET COOKIES

INSTALLED IN YOUR BRAIN'S OPERATING SYSTEM



# PROBABILITY

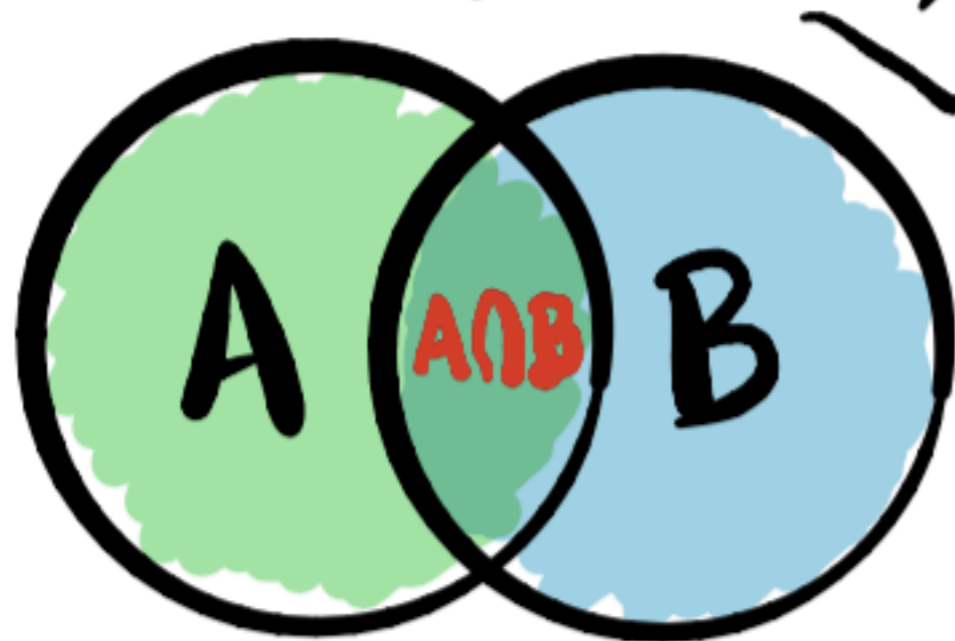
2 beautiful subject,  
try studying it!

Kolmogorov 1930's

- $P \geq 0$

- something else

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



UNION

INTERSECTION

## 1.2 THE STATISTICAL INTERPRETATION



But what exactly *is* this “wave function”, and what does it do for you once you’ve *got* it? After all, a particle, by its nature, is localized at a point, whereas the wave function (as its name suggests) is spread out in space (it’s a function of  $x$ , for any given time  $t$ ). How can such an object be said to describe the state of a *particle*? The answer is provided by Born’s **statistical interpretation** of the wave function, which says that  $|\Psi(x, t)|^2$  gives the probability of finding the particle at point  $x$ , at time  $t$ —or, more precisely,<sup>2</sup>



$$|\Psi(x, t)|^2 dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } x \text{ and } (x + dx), \text{ at time } t. \end{array} \right\} \quad [1.3]$$

For the wave function in Figure 1.2, you would be quite likely to find the particle in the vicinity of point  $A$ , and relatively unlikely to find it near point  $B$ .



The statistical interpretation introduces a kind of **indeterminacy** into quantum mechanics, for even if you know everything the theory has to tell you about the

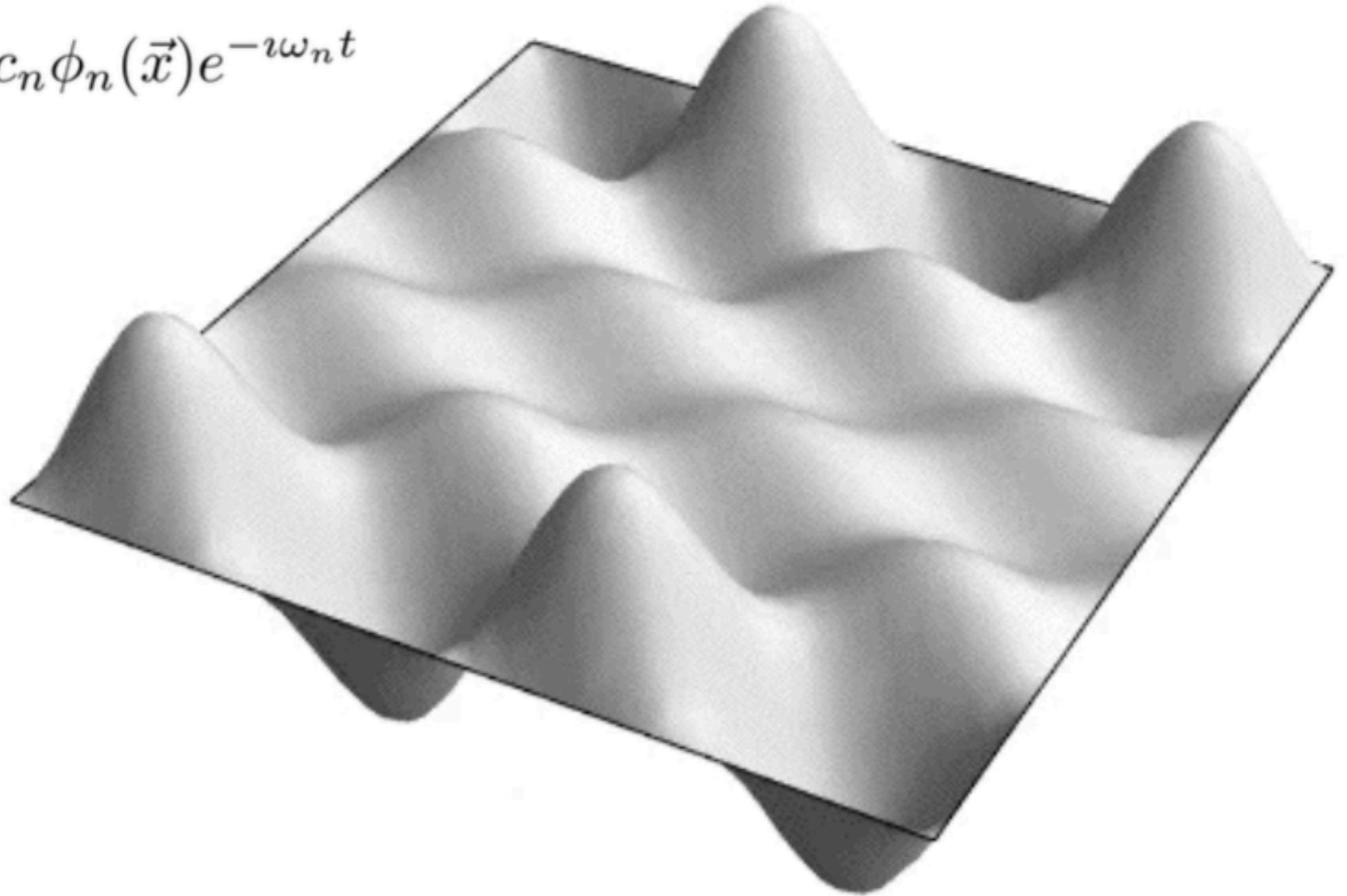
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<sup>2</sup>The wave function itself is complex, but  $|\Psi|^2 = \Psi^* \Psi$  (where  $\Psi^*$  is the complex conjugate of  $\Psi$ ) is real and nonnegative—as a probability, of course, must be.

**HEY! wave function -> “Pure State” is actually exceptionally rare**

**The purpose of the wave function is to calculate frequencies and interactions**

$$\psi(\vec{x}, t) = \sum_n c_n \phi_n(\vec{x}) e^{-i\omega_n t}$$



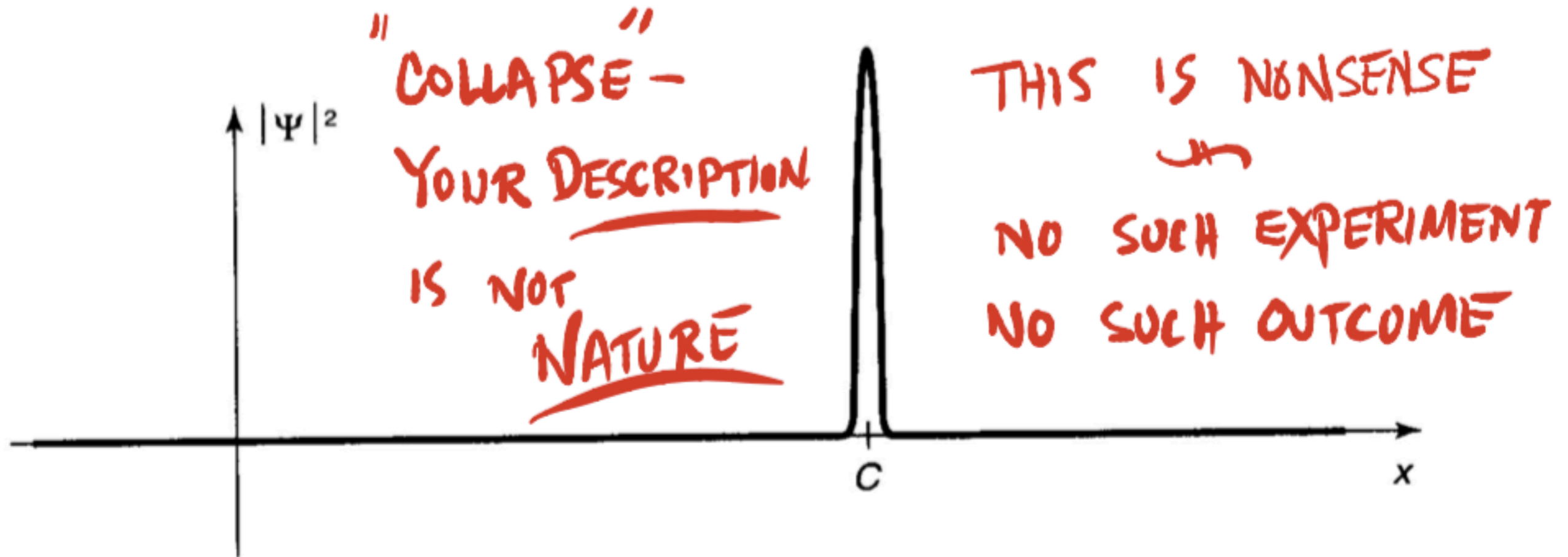


Figure 1.3: Collapse of the wave function: graph of  $|\Psi|^2$  immediately after a measurement has found the particle at point  $C$ .



# *Folks, this is malpractice...*

## 1.3 PROBABILITY

---

Because of the statistical interpretation, **probability** plays a central role in quantum mechanics, so I digress now for a brief discussion of the theory of probability. It is mainly a question of introducing some notation and terminology, and I shall do it in the context of a simple example.

Imagine a room containing 14 people, whose ages are as follows:



one person aged 14



one person aged 15



three people aged 16

two people aged 22

two people aged 24

five people aged 25.

If we let  $N(j)$  represent the number of people of age  $j$ , then

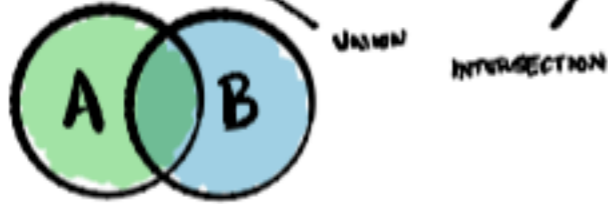
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**PROBABILITY**

a beautiful subject,  
try studying it!

Kolmogorov 1930's

- $P > 0$
- something else
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



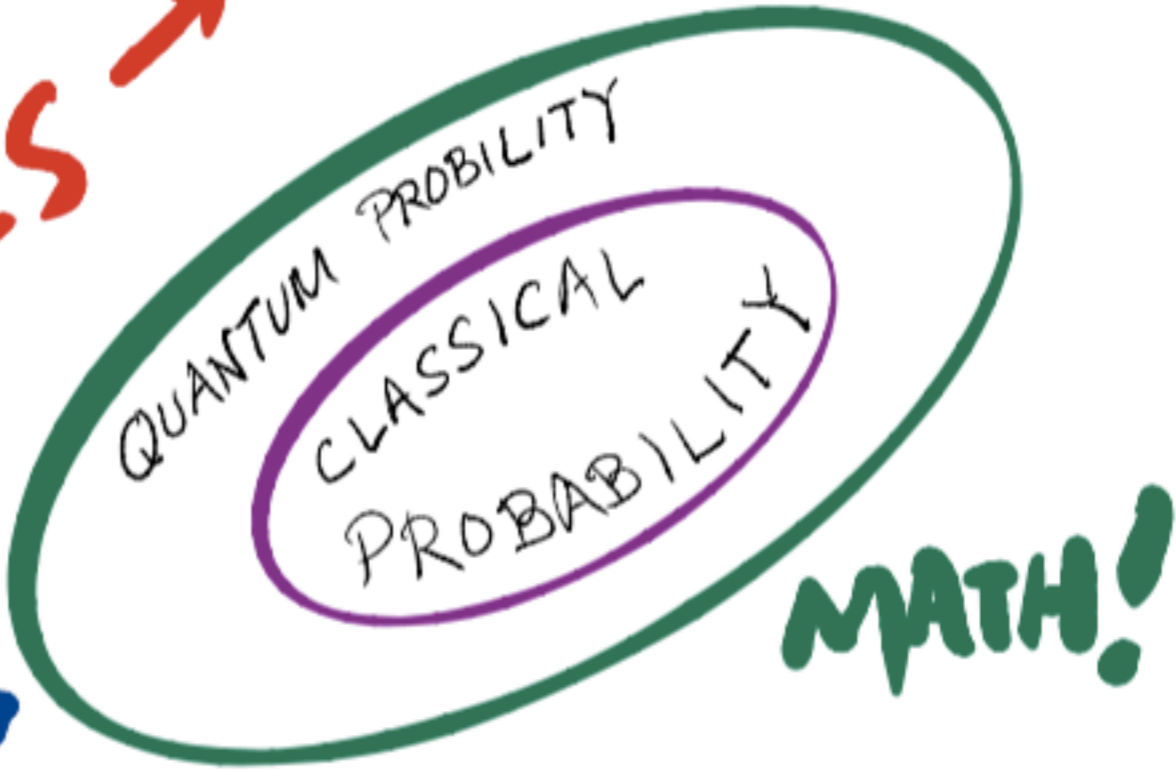
probability of finding the particle at point  $x$ , at time  $t=0$ :  
 $|\Psi(x, t)|^2$  gives the probability of finding the particle at point  $x$ , at time  $t=0$ :  
 precisely,<sup>2</sup> 


$$|\Psi(x, t)|^2 dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } x \text{ and } (x + dx), \text{ at time } t. \end{array} \right\}$$

THIS FAILS →

THIS FAILS →

THIS WORKS →

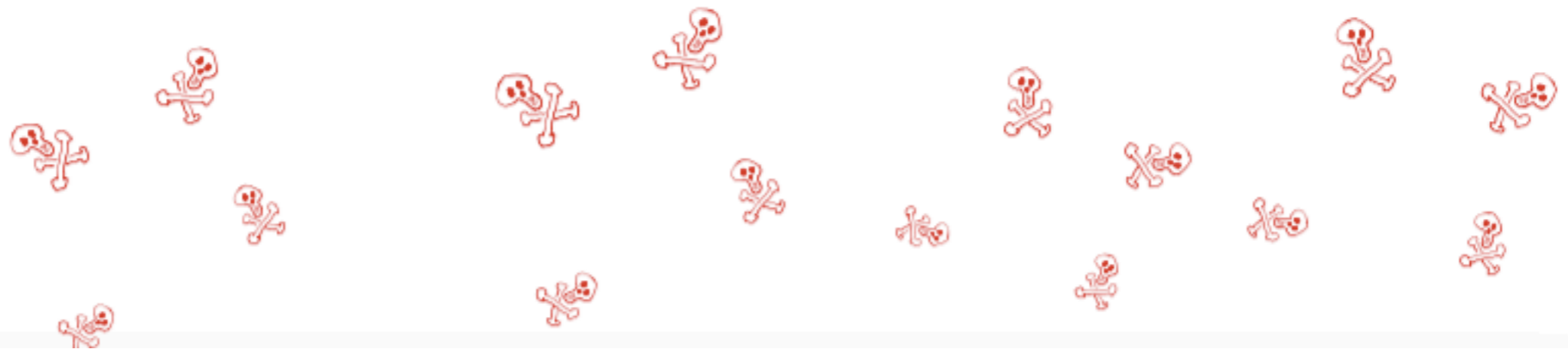




Unlike Newton's mechanics, or Maxwell's electrodynamics, or Einstein's relativity, quantum theory was not created—or even definitively packaged—by one individual, and it retains to this day some of the scars of its exhilarating but traumatic youth. There is no general consensus as to what its fundamental principles are, how it should be taught, or what it really “means.” Every competent physicist can “do” quantum mechanics, but the stories we tell ourselves about what we are doing are as various as the tales of Scheherazade, and almost as implausible. Richard Feynman (one of its greatest practitioners) remarked, “I think I can safely say that nobody understands quantum mechanics.”

Griffiths

**Add, from Griffiths: Niels Bohr said “If you are not confused by quantum mechanics, you have not studied it”.**



# QUANTUM MECHANICS DESCRIBES DATA

WITH NO NUMBO-JUMBO

$\langle \hat{A} \rangle, \langle \hat{B} \rangle, \langle \hat{C} \rangle =$  numbers from experiment

$|\psi\rangle =$  wave function = concise code

AND EXCEPTIONAL  
CASE OF MORE  
GENERAL DESCRIPTION

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = \psi_i^* \hat{A}_{ij} \psi_j = \hat{A}_{ij} \psi_j \psi_i^* = \text{tr}(\hat{A} |\psi\rangle\langle\psi|)$$

tr = TRACE

\* TRACE COMPUTES AN INNER PRODUCT

"HOW MUCH DOES PROBE A LOOK LIKE  $|\psi\rangle\langle\psi|$ "

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = \sum_i \psi_i^* \hat{A}_{ij} \psi_j = \sum_j \hat{A}_{ij} \psi_j \psi_i^* = \text{tr}(\hat{A} | \psi \rangle \langle \psi |)$$

\* TRACE COMPUTES AN INNER PRODUCT

# WAVE FUNCTIONS DESCRIBE SPECIAL CASES

Let  $| \psi \rangle \langle \psi | \rightarrow \sum_l | \psi_l \rangle \lambda_l \langle \psi_l |$

A MORE GENERAL  
KIND OF EXPERIMENTAL  
AVERAGE

$\rho$ , a "density matrix"

THE ONLY GENERAL DESCRIPTION  
OF A QUANTUM SYSTEM

$$\langle \hat{A} \rangle = \text{tr}(\hat{A} \rho)$$

"HOW MUCH  $\hat{A}$  LOOKS LIKE THE SYSTEM  $\rho$ "

# A theory of correlations

"STOKES MATRIX"  
1850's

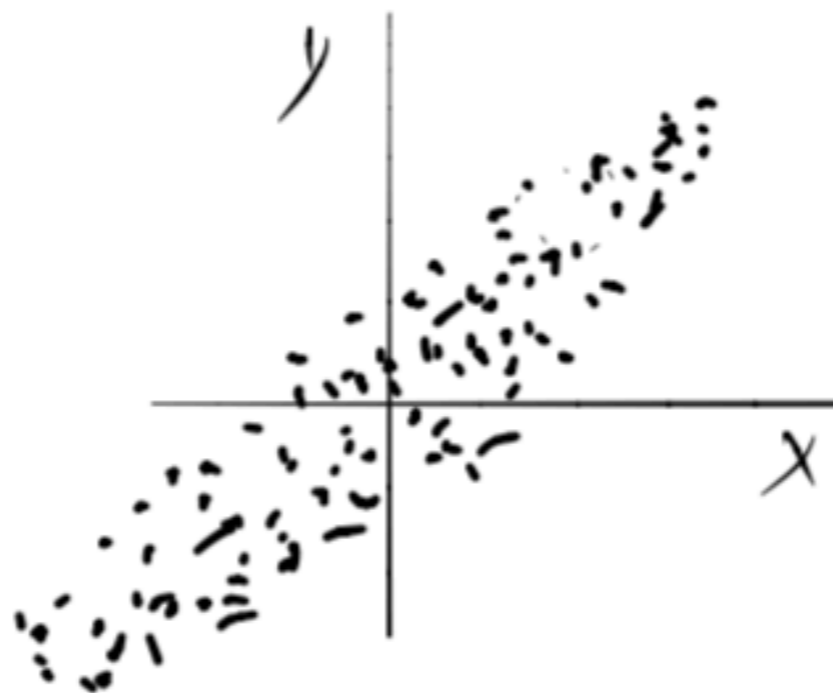
CARTESIAN ELECTRIC FIELD

AVERAGE OF YOUR DATA

$$\rho_{ij} = \langle E_i E_j^* \rangle$$

DENSITY MATRIX IN Q.M.

CORRELATION OF POLARIZATIONS  $i, j$



$$\langle x \rangle = 0$$
$$\langle y \rangle = 0$$
$$\langle xy \rangle = 0.7$$

# GENERIC DATA

$d_i^k$  ← EVENT  
 $d_i^k$  ← COMPONENT

QM  
TEXTBOOK

average over events

$$\rho_{ij} = \frac{1}{K} \sum_k d_i^k d_j^k$$

FACT:  $\rho = \rho^\dagger$

USE  $\sum_k d_i^k d_j^{k*}$  FOR COMPLEX

$$\rho |e^\alpha\rangle = \lambda_\alpha |e^\alpha\rangle$$
$$\lambda_\alpha > 0$$

EIGENVALUES  
OF  $\rho > 0$

"POSITIVE MATRIX"

# EMPIRICAL RULE:

NOBODY COULD  
POSSIBLY EXPLAIN  
A POSTULATE



$$\rho = \begin{pmatrix} P_1 & & \\ & P_2 & \\ & & P_3 \end{pmatrix}$$

$P_i > 0$      $P_1 + P_2 + P_3 = 1$

probability  
of  $|e^\alpha\rangle$ :

$$P_\alpha = \langle e^\alpha | \rho | e^\alpha \rangle$$

the BORN RULE  
of QUANTUM  
PROBABILITY

(we have an analytic proof, of course)



# WHERE IS THE BEGINNER'S BORN RULE?

$$P(|\psi\rangle) = \langle \psi | \rho | \psi \rangle$$

pure state !

CONSIDER SPECIAL CASE!  $\rho$  HAS ONE EIGENVECTOR  
 $\rho |\psi_0\rangle = |\psi_0\rangle; \quad \rho = |\psi_0\rangle\langle\psi_0|$

---

THEN

$$\langle \psi_{\text{ANY}} | \rho | \psi_{\text{ANY}} \rangle = \langle \psi_{\text{ANY}} | \psi_0 \rangle \langle \psi_0 | \psi_{\text{ANY}} \rangle$$
$$= |\langle \psi_{\text{ANY}} | \psi_0 \rangle|^2$$



# QUANTUM DYNAMICS

$$\dot{q}_j = \frac{\partial H}{\partial p_j} \quad \dot{p}_j = -\frac{\partial H}{\partial q_j}$$

CLASSICAL MECHANICS  
 $j = 1, \dots, N, \dots, \infty$

$$\psi_j = (q_j + i p_j) / \sqrt{2} \quad i = \sqrt{-1}$$

$$i \dot{\psi}_j = \left( i \frac{\partial H}{\partial p_j} + \frac{\partial H}{\partial q_j} \right) / \sqrt{2}$$

ALGEBRA... CHAIN RULE...  ...  ...  ALGEBRA... CHAIN RULE...

$$\rightarrow i \dot{\psi}_j = \hat{H}_{jk} \psi_k \quad \text{WHERE} \quad H(q,p) \rightarrow H(\psi, \psi^*) = \langle \psi | \hat{H} | \psi \rangle$$

**The Schroedinger Equation IS Hamilton's Equations in Complex NOTATION**



$h$  always  
cancels out...  
try it!

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{e^2}{4\pi\epsilon_0 r} \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi - \frac{e^2}{4\pi\epsilon_0 \hbar r} \psi$$

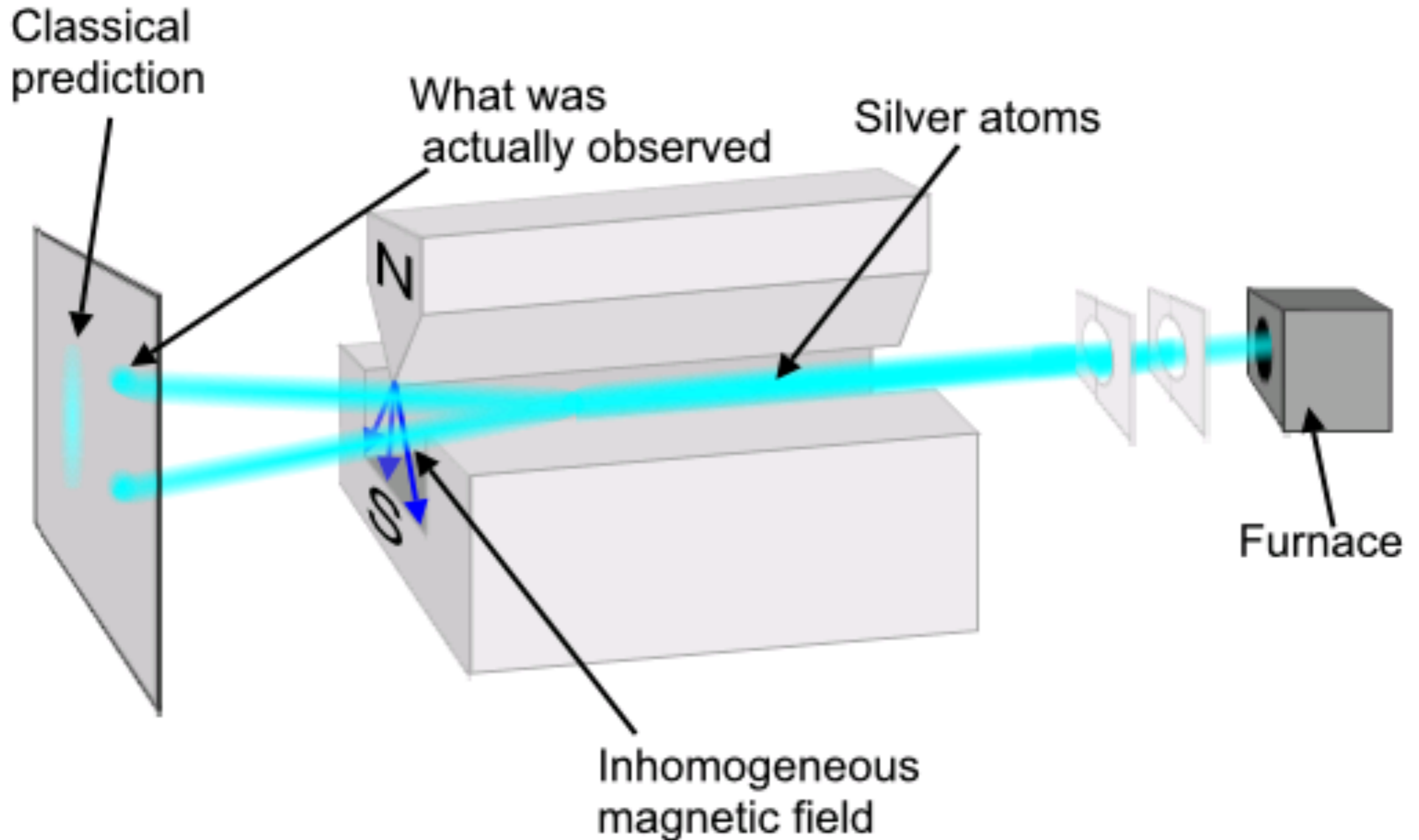
$$i \frac{\partial \psi}{\partial t} = -\frac{\hbar c^2}{2mc^2} \nabla^2 \psi - \frac{e^2 c}{4\pi\epsilon_0 \hbar c r} \psi$$

$$i \frac{\partial \psi}{\partial t} = -\underset{\uparrow}{c\lambda_c} \nabla^2 \psi - \frac{\alpha c}{r} \psi$$

COMPTON WAVELENGTH

$\frac{1}{137}$

# Stern-Gerlach Experiment with Silver Atoms (1922)



**§ 4. DOUBLE REFRACTION OF LIGHT EXPLAINED BY THE WAVE THEORY.**

By means of Iceland spar cut in the proper direction, double refraction is capable of easy illustration. Causing the beam which builds the image of our carbon-points to pass through the spar, the single image is instantly

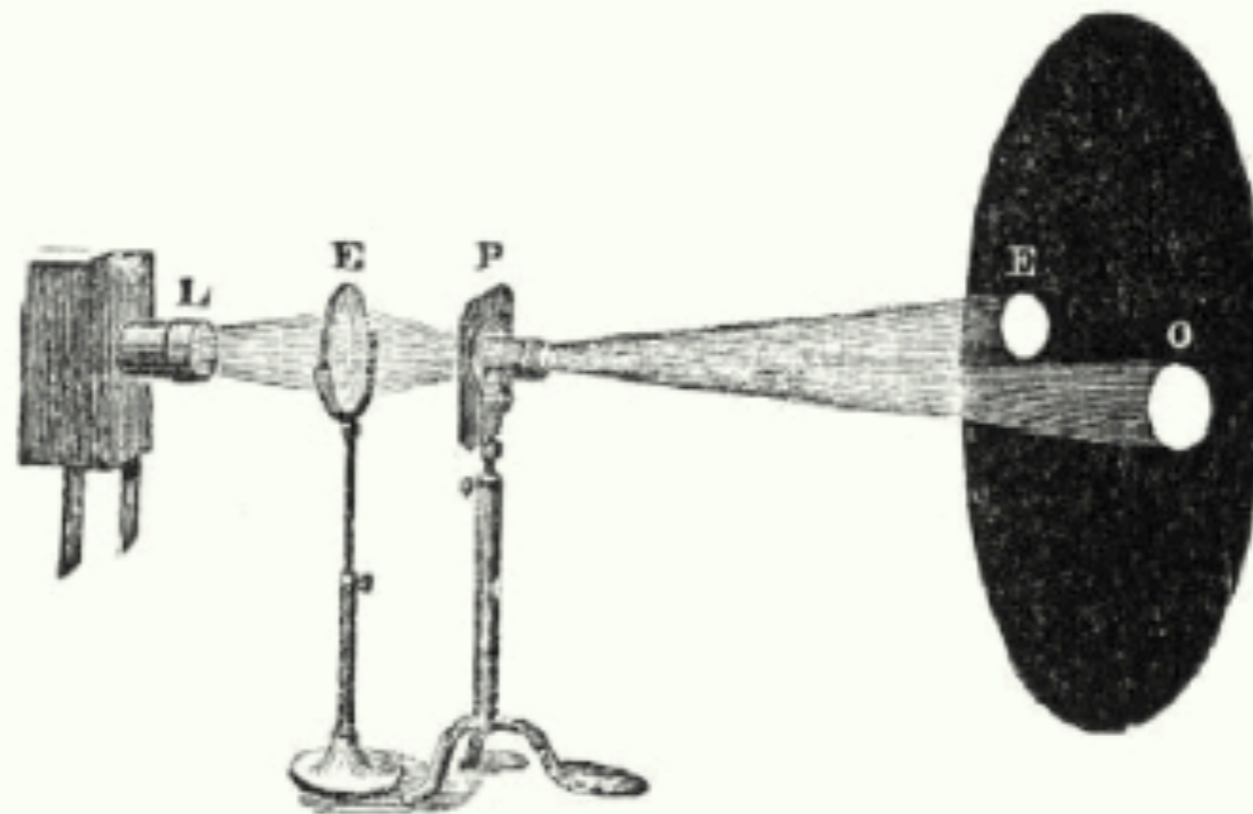


Fig. 26.

divided into two. Projecting (by the lens E, fig. 26) an image of the aperture (L) through which the light issues from the electric lamp, and introducing the spar (P), two luminous disks (E O) appear immediately upon the screen instead of one.

The Stern-Gerlach experiment discovered double refraction of matter waves. It had long been known for light

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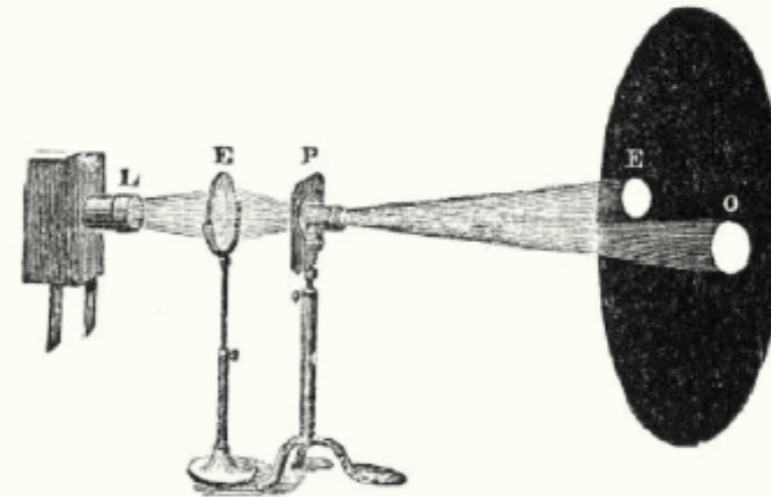


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The figure is from John Tyndall,

Six Lectures on Light

Delivered in the United States in **1872-1873**

# Spin was discovered by Zeeman

Nature 55, 297, 1897

*the thought occurred to me whether the period of the light emitted by a flame might be altered when the flame was acted upon by magnetic force. It has turned out that such an action really occurs. I introduced into an oxy-hydrogen flame placed between the poles of a Ruhmkorff's electromagnet, a filament of asbestos soaked in common salt. The light of the flame was examined with a Rowland's grating. Whenever the circuit was closed both D lines were seen to widen.*

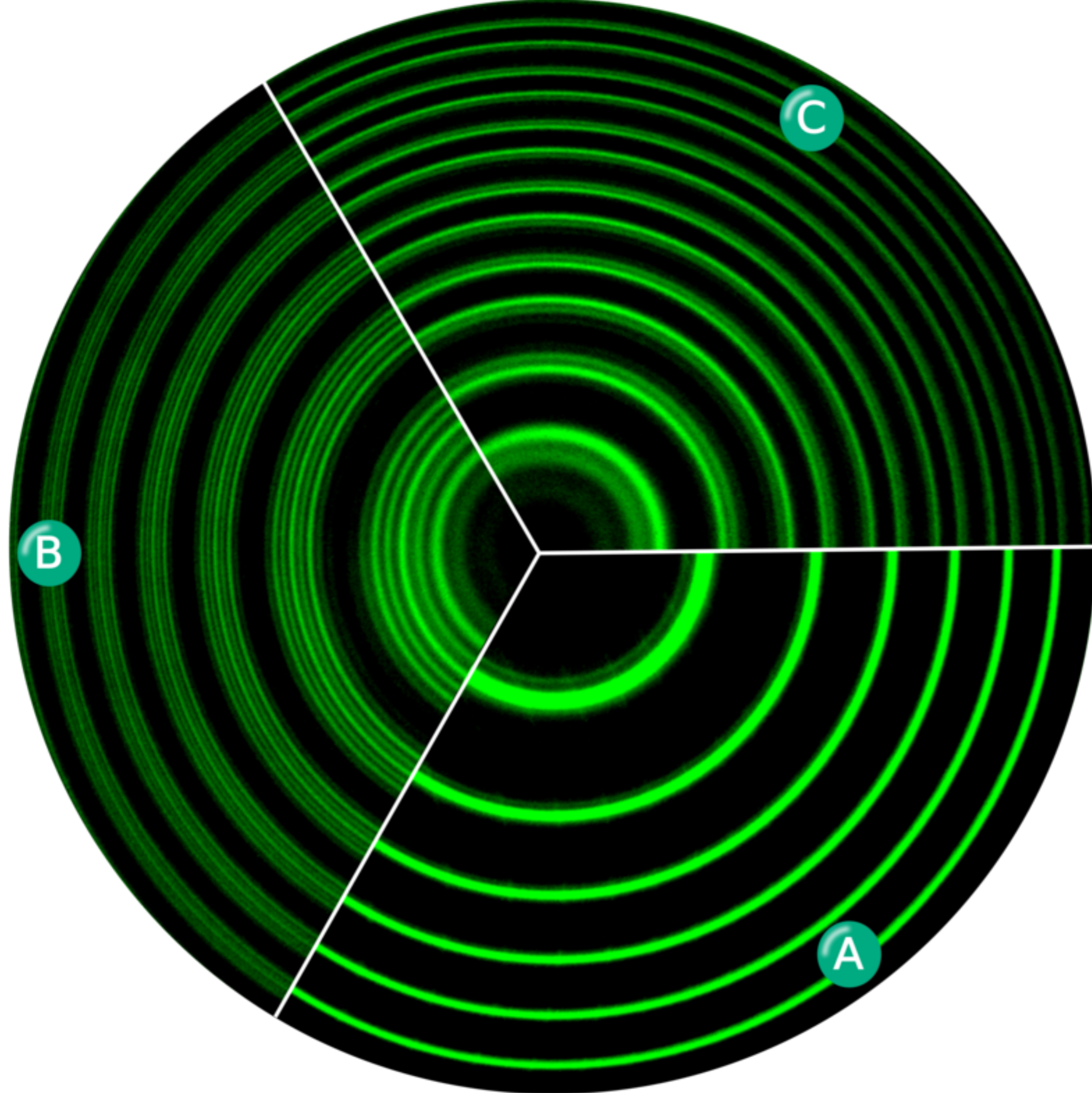


- Lorentz predicted that the light from the edges of the widened lines should be circularly polarized ...
- This *electrified* the imagination of the physicists !

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- This *electrified* the imagination of the physicists !

# Zeeman effect

- Zeeman later observed not just widening lines but the multiplication of its numbers



# Einstein and Bohr did not pay attention to Double Refraction !

## § 4. DOUBLE REFRACTION OF LIGHT EXPLAINED BY THE WAVE THEORY.

By means of Iceland spar cut in the proper direction, double refraction is capable of easy illustration. Causing the beam which builds the image of our carbon-points to pass through the spar, the single image is instantly

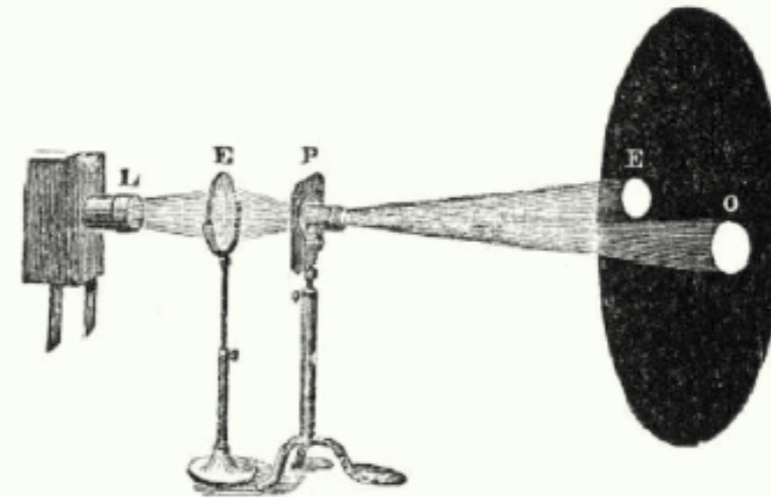


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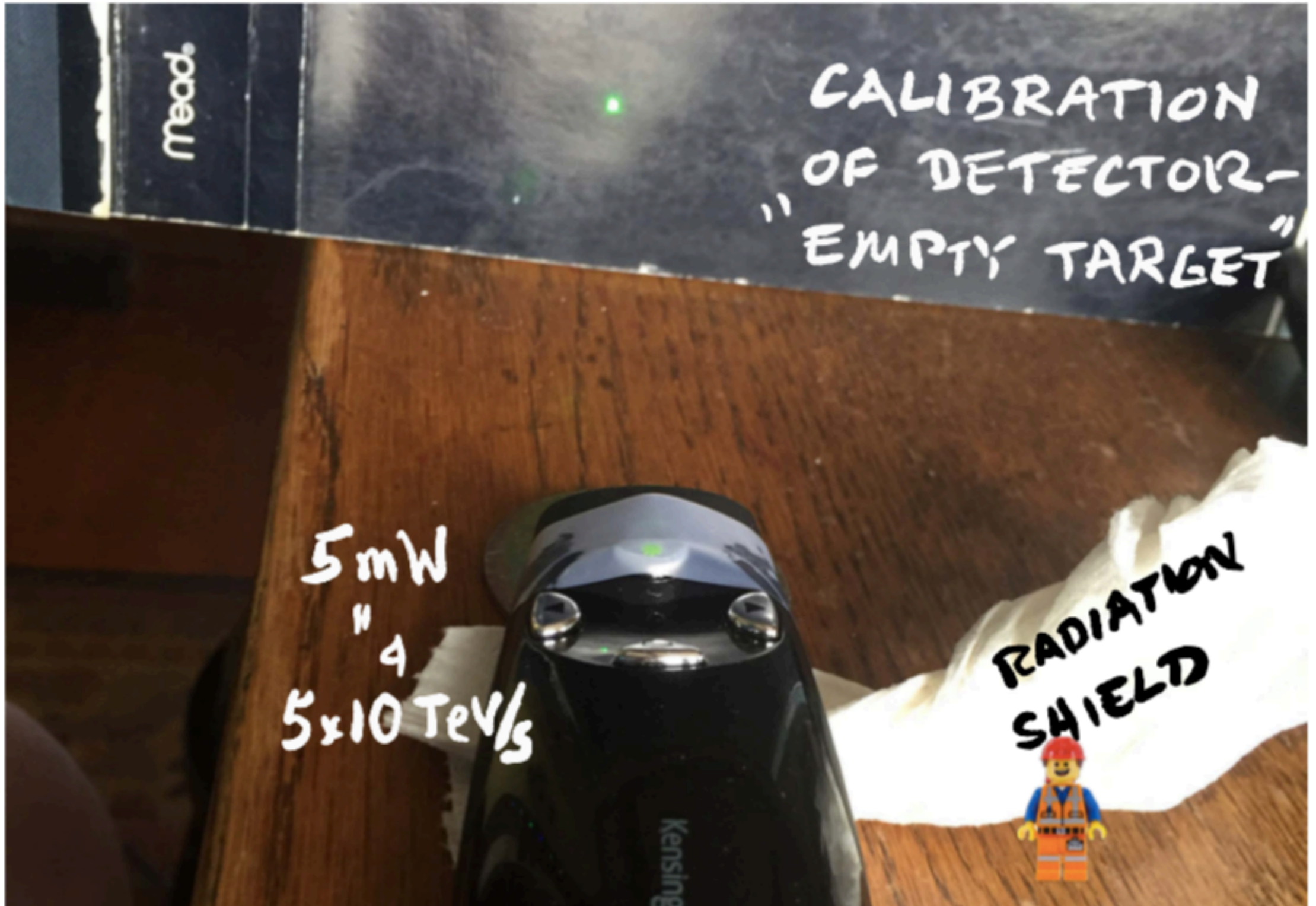
Delivered in the United States in **1872-1873**

# Why there are two spots in the SG experiment?

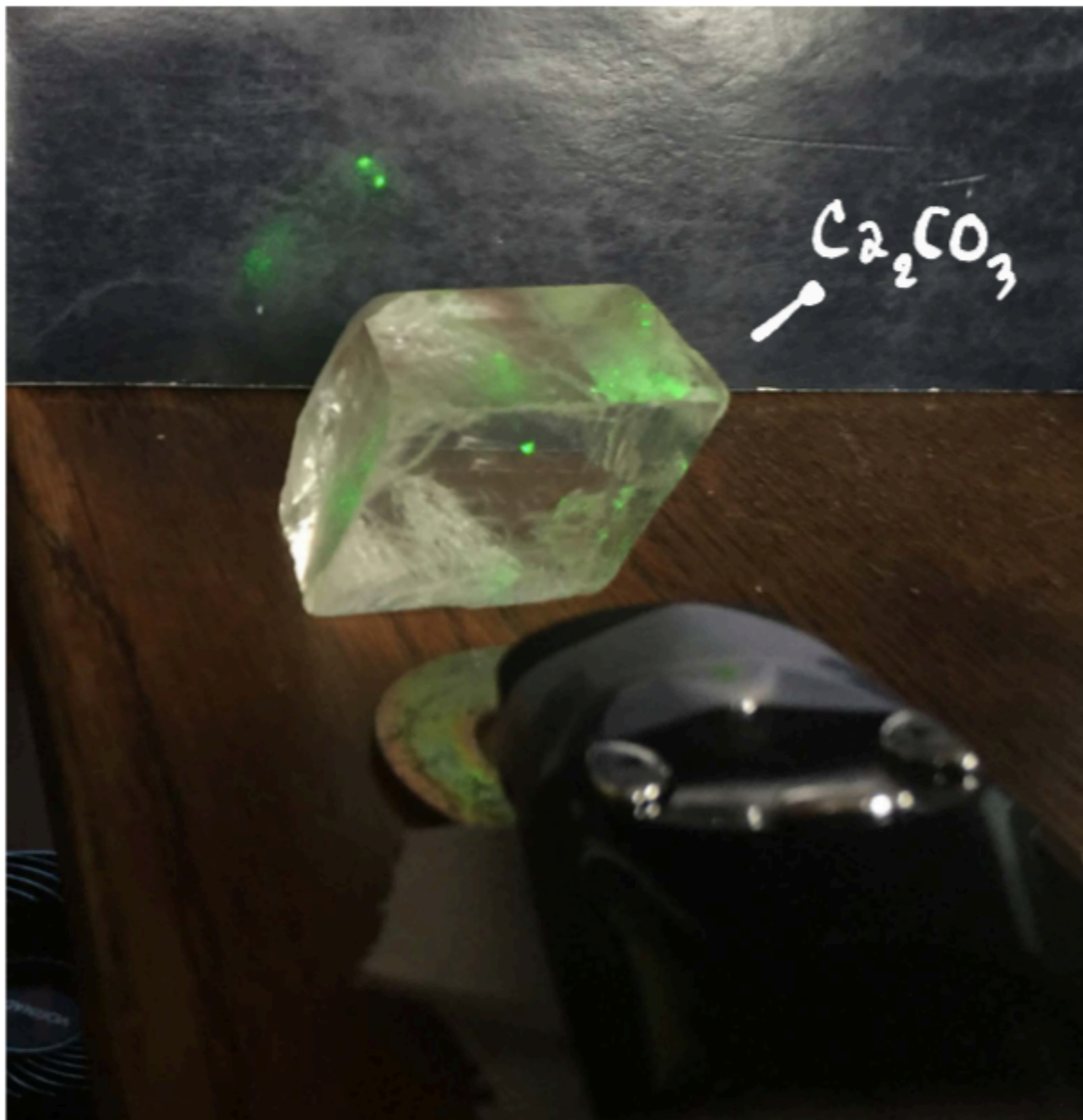
- Because the matter waves of the atoms of silver have two polarization!

Let's  
do an  
experiment!

# GO TO A PARTICLE ACCELERATOR







RUN 1 2012-2014

OBSERVED 2 SPOTS

"PURELY QUANTUM  
MECHANICAL  
EFFECT"



Augustin Fresnel  
(1788-1827)

# Calcite crystal



# Entangled Theory

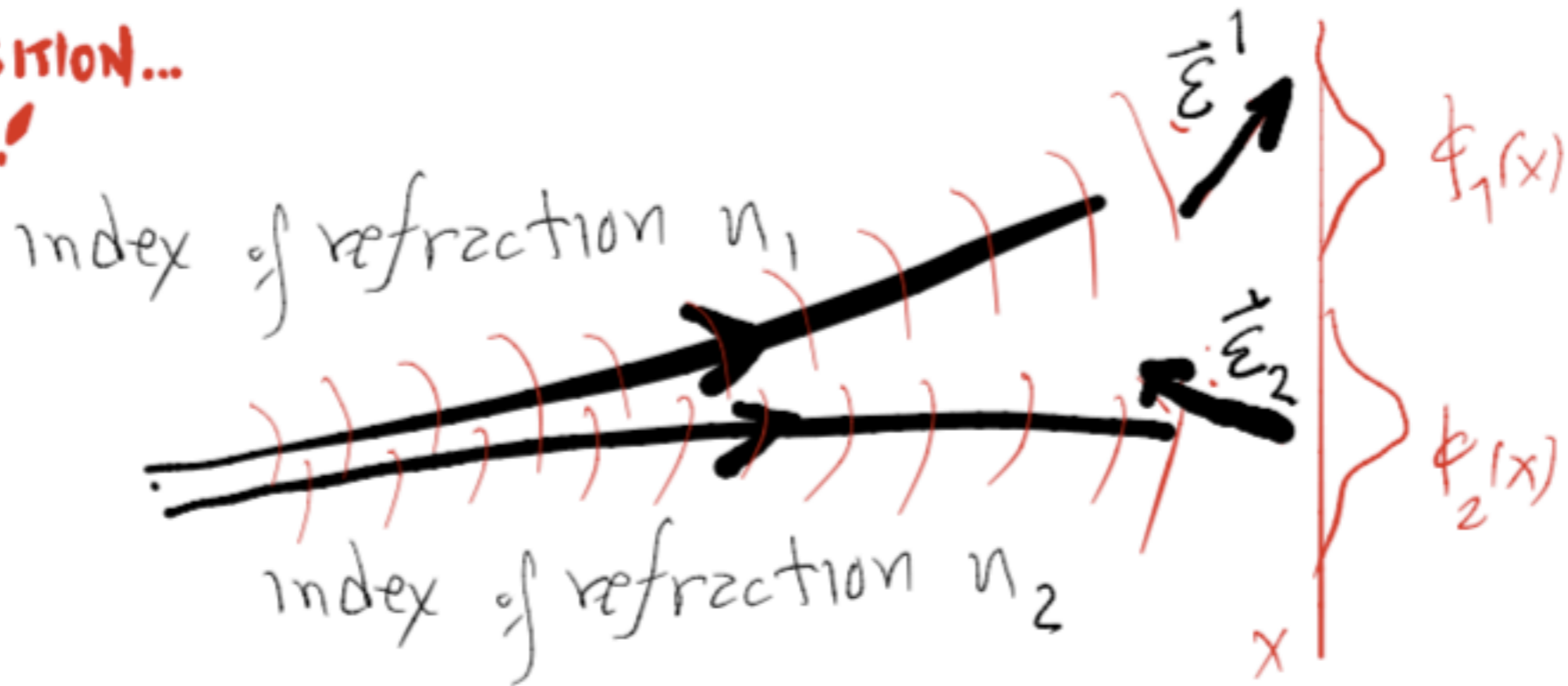
SINGULAR VALUE DECOMPOSITION... MATH!

$$E_2(x) = E_{2 \times 2} = \lambda^1 \epsilon_2^1 \phi^1(x) + \lambda^2 \epsilon_2^2 \phi^2(x)$$

$$\langle \epsilon^i | \epsilon^j \rangle = \langle \phi^i | \phi^j \rangle = \delta^{ij}$$



Augustin Fresnel (1788-1827)



STERN - GERLACH <sup>IS</sup> EXPLAINED

UP  
TO  
HERE...

ORDINARY CORRELATIONS  
REPRODUCE QUANTUM MECHANICS

IT AINT ALL THAT MYSTERIOUS

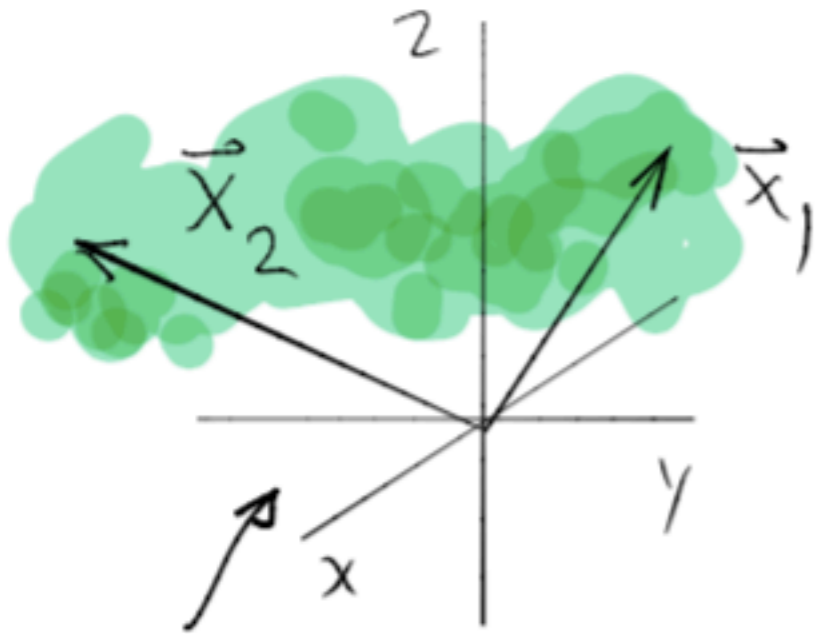
SHOW US

ENTANGLEMENT!



# BABY ENTANGLEMENT -

## WAVE FUNCTIONS



GENERIC GREEN STUFF  
NOT A GREEN FUNCTION

generic  $f(\vec{x}_1, \vec{x}_2) \neq f_1(\vec{x}_1) f_2(\vec{x}_2)$

"Entangled"

SCHOOLBOOKS

"Let  $\Psi(\vec{x}_1, \vec{x}_2) = \Psi_1(\vec{x}_1) \Psi_2(\vec{x}_2)$

"Not Entangled"

# **Applications of Quantum Tomography in Collider Physics**



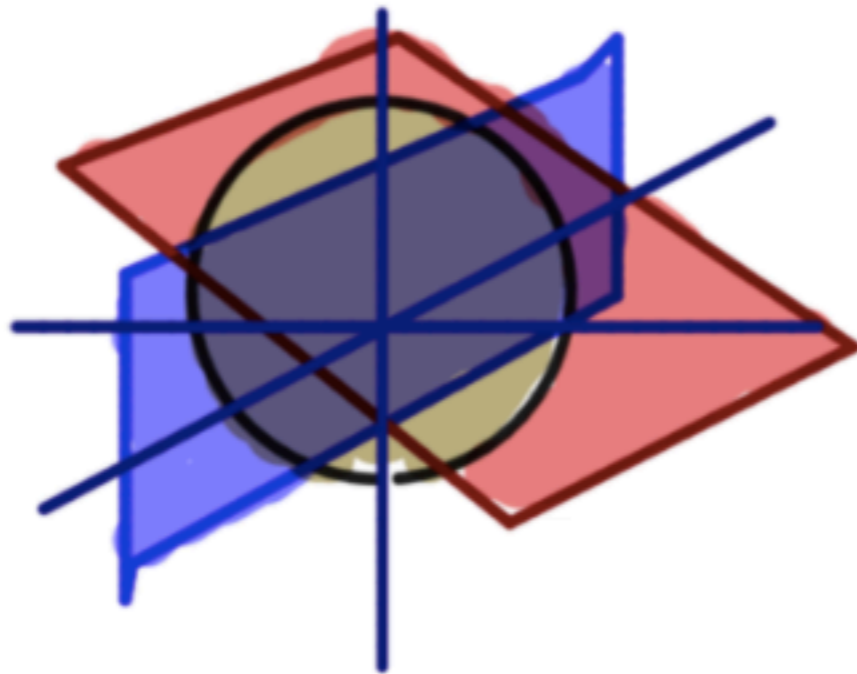
ALL YOU NEED  
TO KNOW

$$\text{rate} = d\sigma = \text{tr}(\rho_{\text{PROBE}} \rho_X)$$

ALSO: THIS IS AN INNER PRODUCT

# TOMOGRAPHY

reconstructs higher dimensional objects from lower dimensional projections



# QUANTUM TOMOGRAPHY

reconstructs density matrix or wave function from quantum observables

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$
$$\rightarrow \text{tr}(\rho \hat{A})$$

$$\rho \geq 0$$

positive exals

$$\rightarrow \rho = \rho^\dagger$$

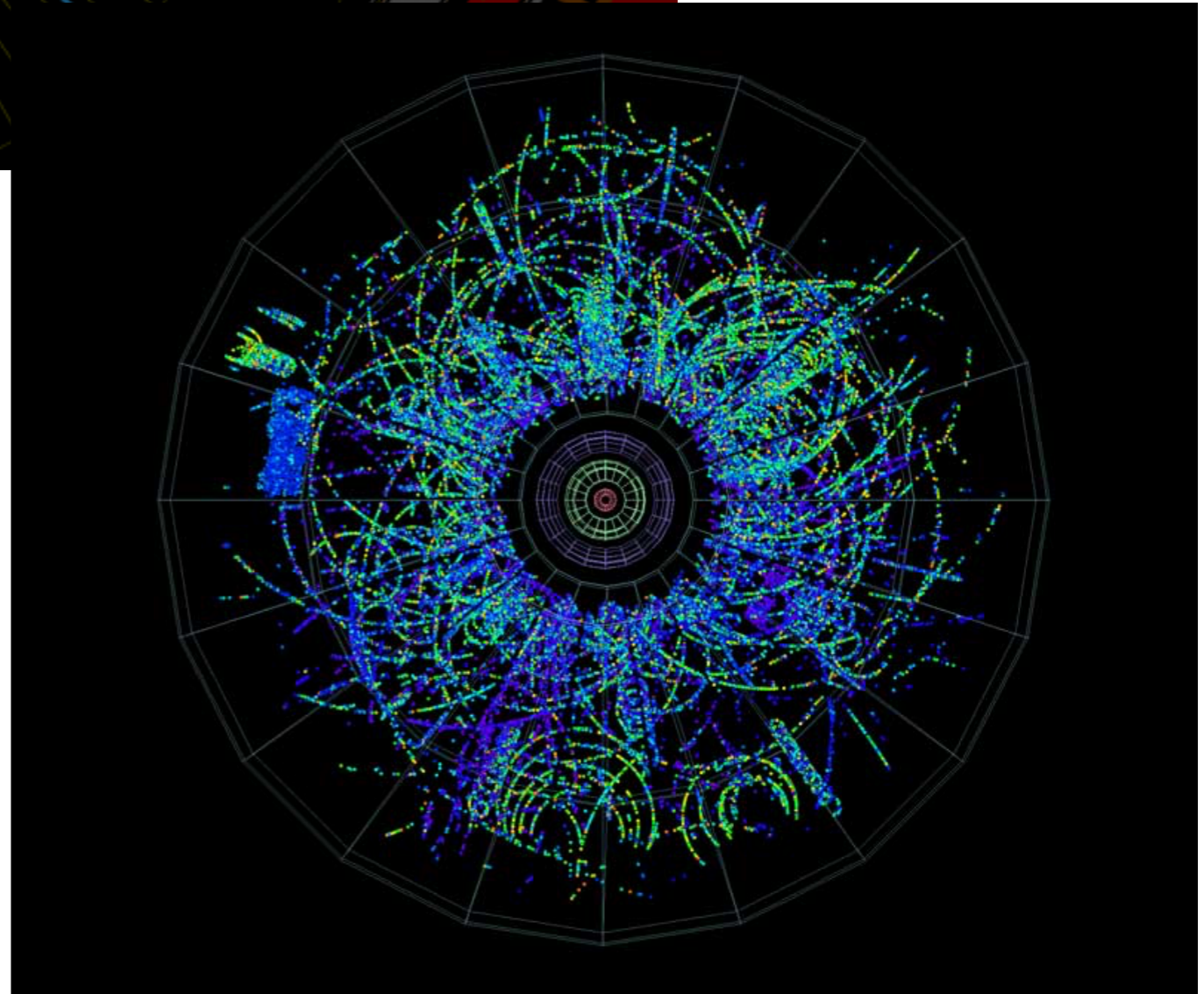
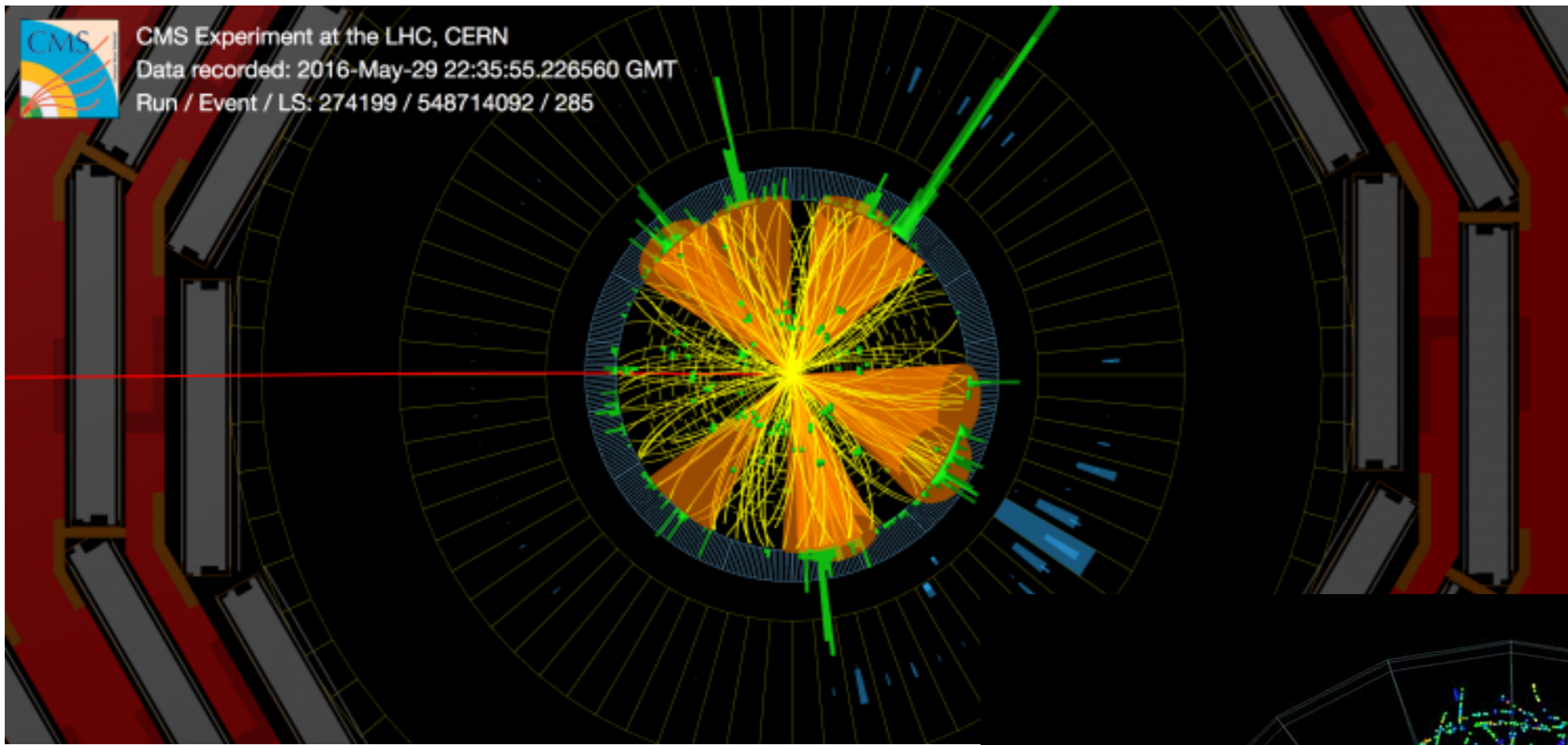




CMS Experiment at the LHC, CERN

Data recorded: 2016-May-29 22:35:55.226560 GMT

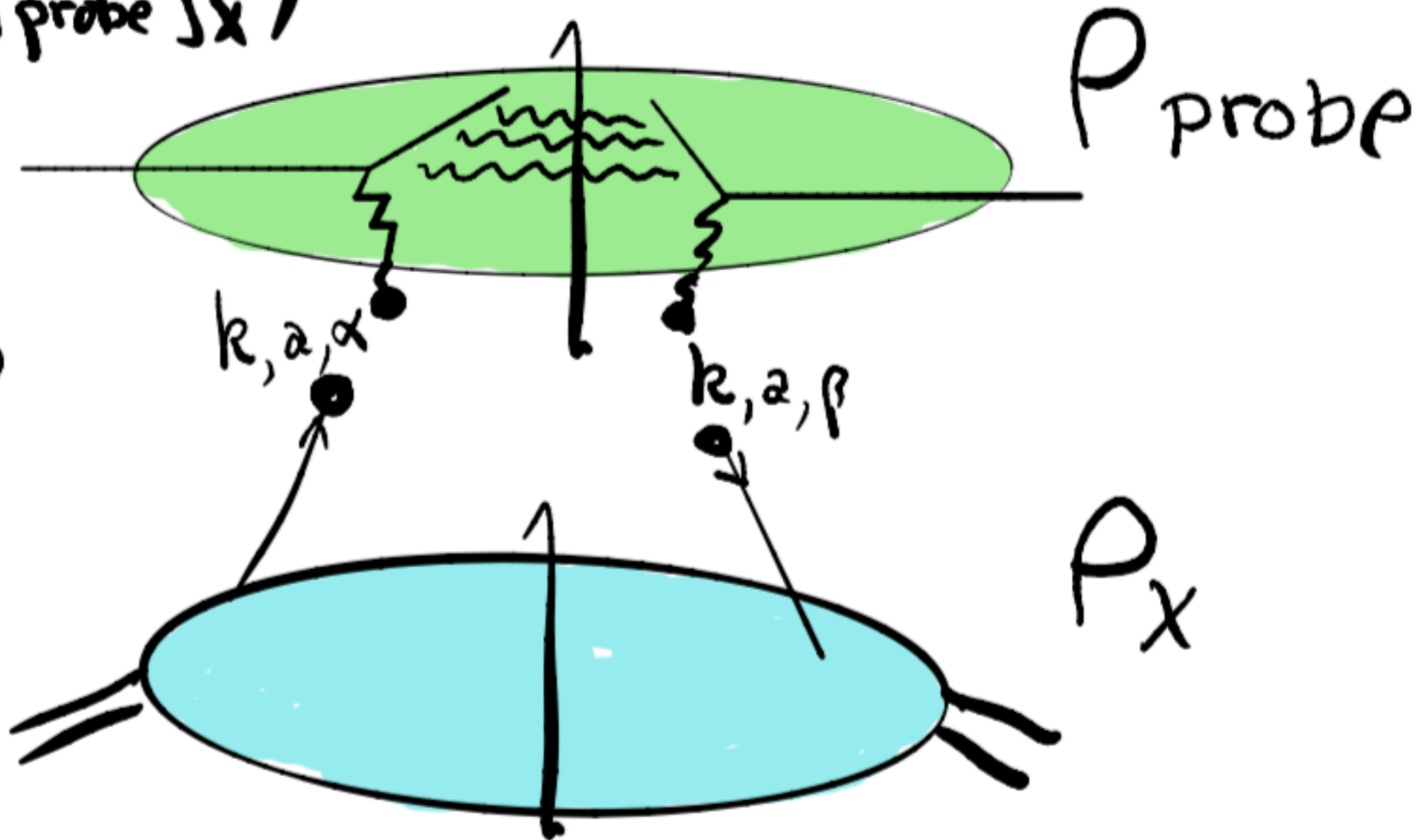
Run / Event / LS: 274199 / 548714092 / 285



EVERY CROSS SECTION

$$d\sigma = \text{tr}(\rho_{\text{probe}} \rho_X)$$

DIS



# QUANTUM TOMOGRAPHY

$$A = A^\dagger, \rho = \rho^\dagger$$

$$\langle A_\ell \rangle = \text{tr}(A_\ell^\dagger \rho) = \langle A | \rho \rangle$$

A PROJECTION, AN  
INNER PRODUCT  
OF OPERATORS

$$1 = \sum_\ell |A_\ell\rangle\langle A_\ell|$$

COMPLETENESS ;  
GENERATORS of  $U(N)$

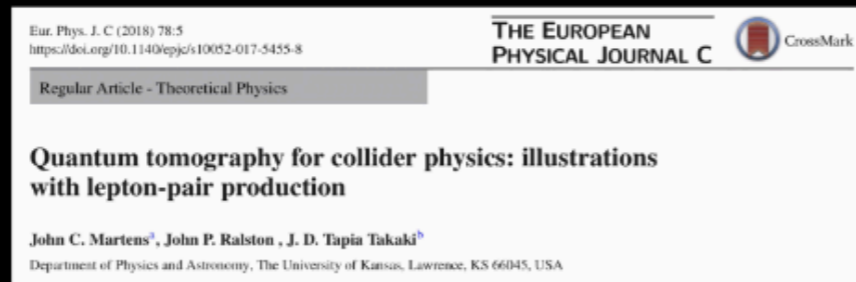
"OBSERVABLE"

$$\rho = \sum_\ell |A_\ell\rangle\langle A_\ell| \rho$$

EXPANSION

$$\rho = \sum_\ell |A_\ell\rangle\langle A_\ell|$$

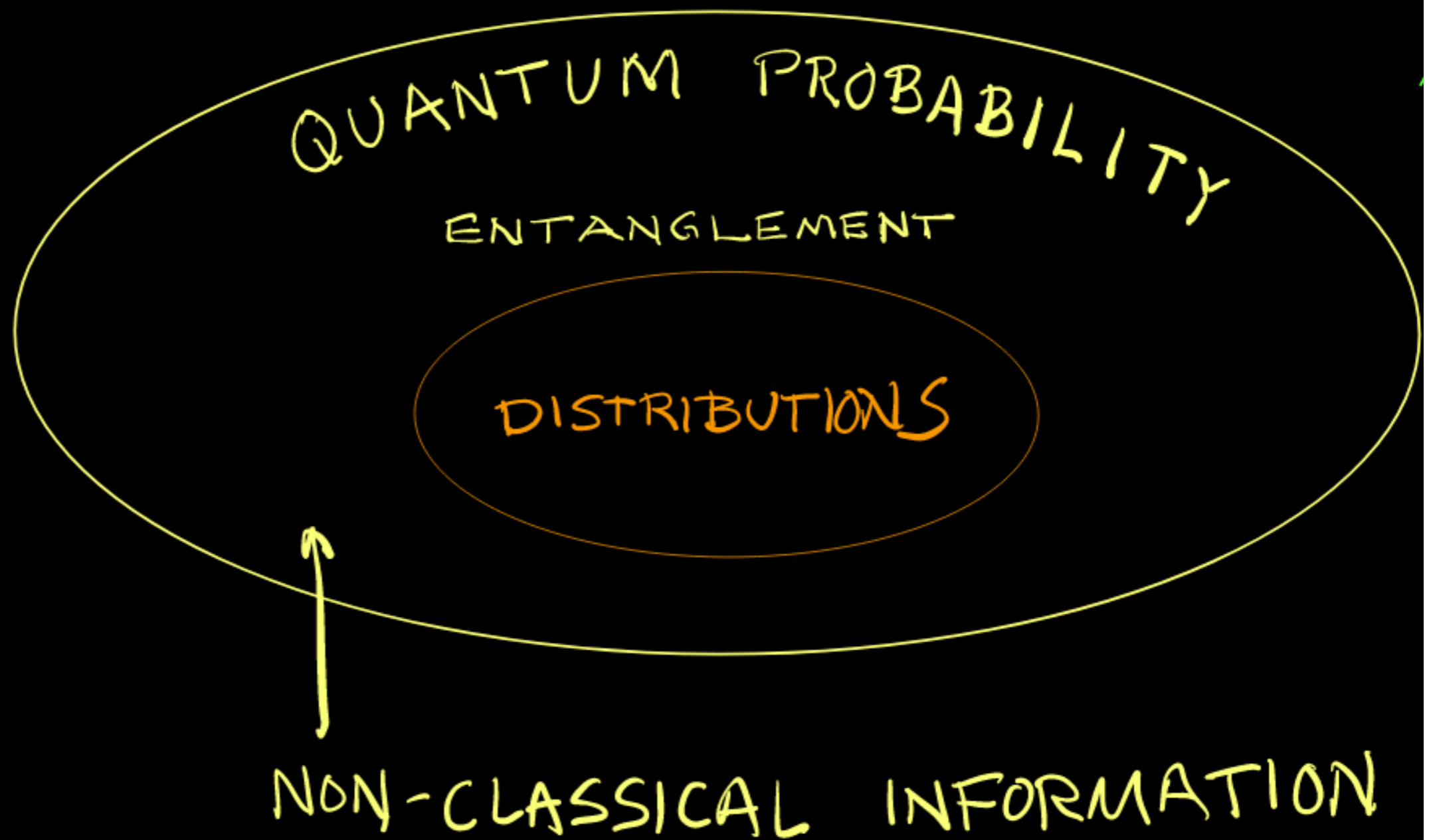
# QUANTUM TOMOGRAPHY



"OBSERVABLE"

Martens, Ralston, Tapia Takaki Eur. Phys. J. C78, 5, 2018

# QUANTUM INFORMATION FROM 4-MOMENTA



## THE DENSITY MATRIX

$$\rho = \rho^\dagger > 0 \quad \text{POSITIVE EIGENVALUES}$$

$$\langle \hat{A} \rangle = \text{tr}(\rho \hat{A})$$

PURE STATE IS EXCEPTIONAL

$$\rho \rightarrow |\psi\rangle\langle\psi|$$

$$\langle \hat{A} \rangle \Rightarrow \text{tr}(|\psi\rangle\langle\psi| \hat{A}) = \langle \psi | \hat{A} | \psi \rangle$$

TEXTBOOKS  
SHOW  
THE  
EXCEPTION

# PARAMETERIZING POSITIVITY

REAL  
NUMBERS

$$\rho = MM^\dagger$$

POSITIVE EIGENVALUES  
GUARANTEED

$$M = \begin{pmatrix} m_1 & m_4 + im_5 & m_6 + im_7 \\ 0 & m_2 & m_8 + im_9 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$d\sigma = \text{tr}(MM^\dagger \hat{A})$$

EXPERIMENTAL  
PROBE

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} (1 + m_3^2)$$

$$+ \frac{1}{4\pi} (1 - 3m_3^2 \cos^2\vartheta)$$

$$- \frac{1}{2} m_3 m_6 \sin 2\vartheta \cos\varphi$$

$$- \frac{1}{2} m_3 m_8 \sin 2\vartheta \sin\varphi$$

+ ... 2 JETS  
2 OF ANYTHING ...

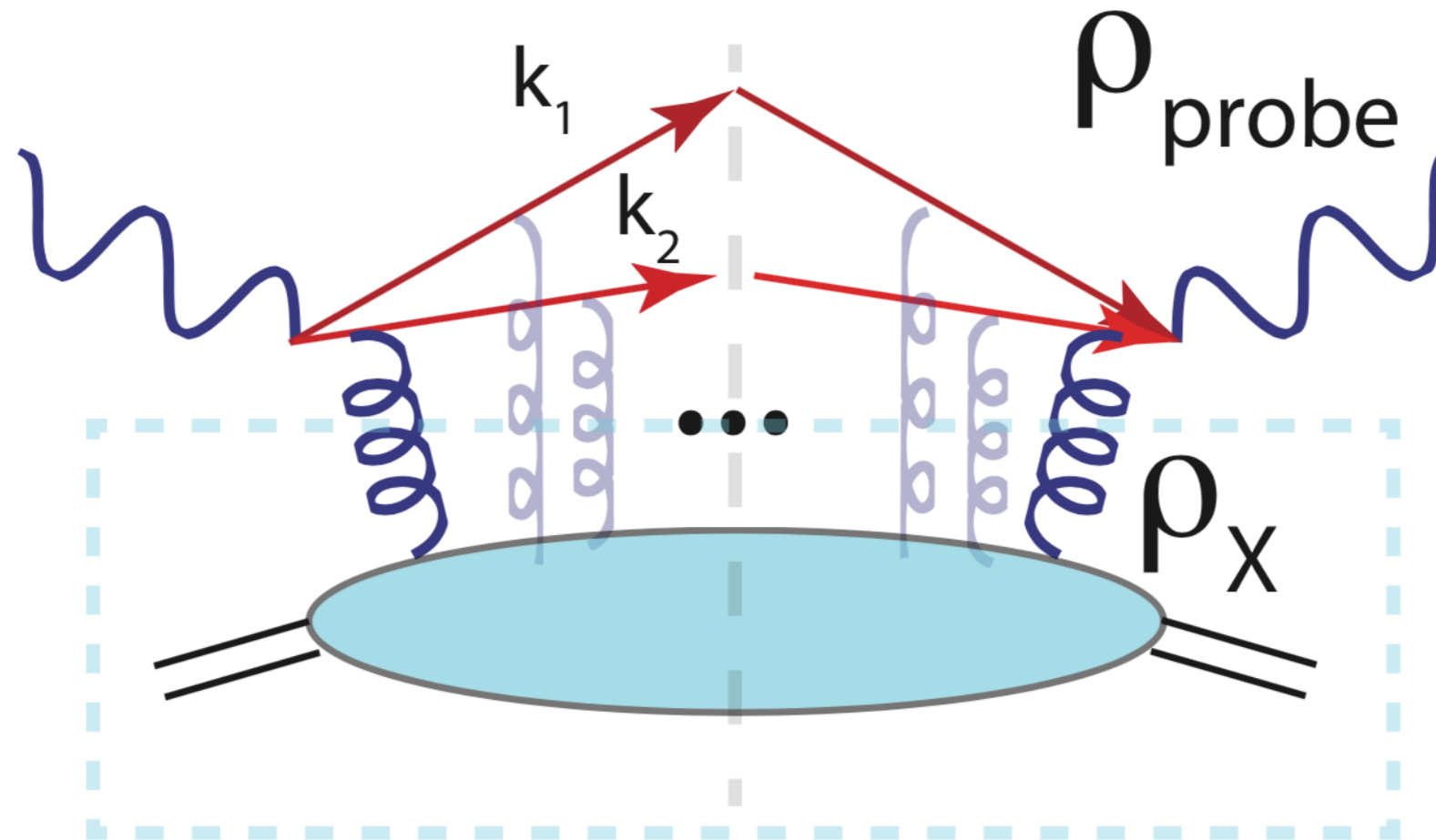
FIT  $m_l$  WITH LIKELIHOOD  
 $-1 < m_l < 1$

NO LOCAL MAXIMA

CONVEX FUNCTIONS

POSITIVITY IS EXACT

# No assumptions on perturbative theory nor one-photon exchange needed



**FIG. 1:** By analogy with deeply inelastic scattering, a dijet probe replaces the handle of the handbag diagram with a shoulder strap (red) defining new elements of the probe density matrix  $\rho_{probe}$ . Each orthogonal element of  $\rho_{probe}$  can extract a corresponding projection of the unknown system density matrix  $\rho_X$  inside the dashed box. Unlike the deeply inelastic structure functions no assumptions of perturbation theory or one-photon exchange need be made.



# Experimentally measure the density matrix

$$\frac{dN}{d \cos \theta d\phi} \sim \text{tr}(\rho_{probe} \rho_X)$$

$\rho_{probe}$  = known density matrix

$\rho_X$  = unknown density matrix

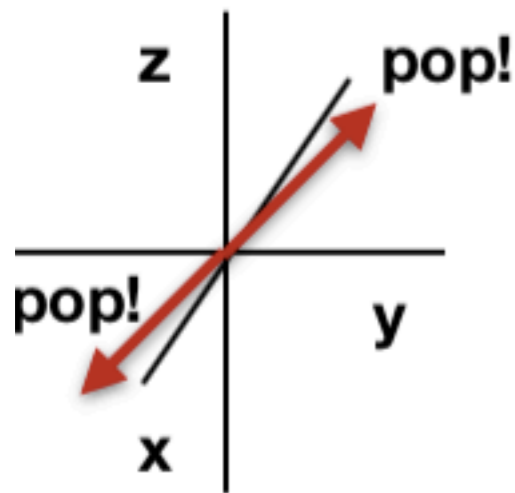
The notation does not look Lorentz invariant,  
but the quantities are

# ***Drell-Yan production in pp***

# EXAMPLE: Experimentally measure the polarization density matrix of a Z boson

Example:

*proton + proton  $\rightarrow$  Z + anything  $\rightarrow$   $\mu^+$  +  $\mu^-$  + anything*



$$\frac{dN}{d \cos \theta d\phi} \sim \text{tr}(\rho_{probe} \rho_X)$$

$\rho_{probe}$  = known density matrix

$\rho_X$  = unknown density matrix

A weighted sum over expectations makes a density matrix

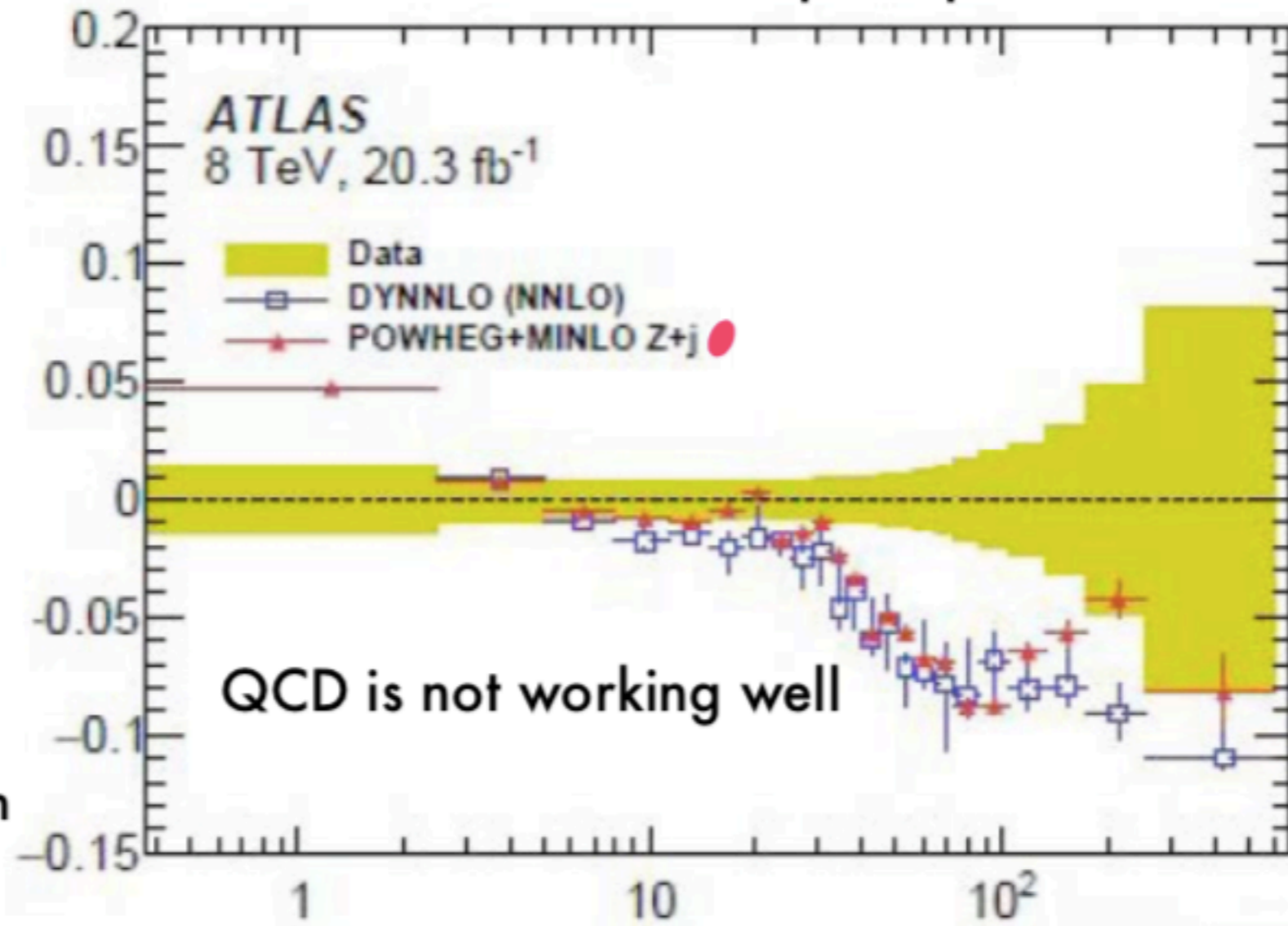
*everything in QM is a density matrix*

$proton + proton \rightarrow Z + anything \rightarrow \mu^+ + \mu^- + anything$

“lepton pairs”

$(A_0 - A_2)$

one of many terms in an traditional expansion of a certain angular distribution



$p_T$  of Z = our  $q_T$   $p_T$  [GeV]

# Bring us data: We'll give you a density matrix

*Example : events with 2 particles, or 2 jets plus anything else*

4-momenta  $k, k'$

total pair momentum  $Q = k + k'$

$$l^\mu = k^\mu - k'^\mu = \sqrt{Q^2}(0, \hat{\ell});$$

$$\hat{\ell} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).$$

pair rest frame  $Q^\mu = (\sqrt{Q^2}, \vec{0})$

$$P(Q, \ell | init) = P(\ell | Q, init)P(Q | init).$$

**Martens, Ralston, Tapia Takaki Eur. Phys. J. C78, 5, 2018**

# Everything is Lorentz Invariant and easy !

Define spatial axes  $X^\mu, Y^\mu, Z^\mu$  satisfying Lorentz invariant

$$Q \cdot X = Q \cdot Y = Q \cdot Z = 0. \quad (1)$$

The frame vectors being orthogonal implies

$$X \cdot Y = Y \cdot Z = X \cdot Z = 0$$

$$\begin{aligned} \tilde{Z}^\mu &= P_A^\mu Q \cdot P_B - P_B^\mu Q \cdot P_A; \\ \tilde{X}^\mu &= Q^\mu - P_A^\mu \frac{Q^2}{2Q \cdot P_A} - P_B^\mu \frac{Q^2}{2Q \cdot P_B}; \\ \tilde{Y}^\mu &= \epsilon^{\mu\nu\alpha\beta} P_{A\nu} P_{B\alpha} Q_\beta. \end{aligned}$$

*The first step in our quantum tomographic (QT) analysis expresses everything in a Lorentz-covariant fashion*

To analyze data for each event labeled  $J$ :

$$\text{Compute } Q_{(J)} = k_J + k'_J; \quad \ell_J = k_J - k'_J; \quad (X_J^\mu, Y_J^\mu, Z_J^\mu);$$

$$\vec{\ell}_{XYZ,J} = (X_J \cdot \ell_J, Y_J \cdot \ell_J, Z_J \cdot \ell_J);$$

$$\hat{\ell}_J = \ell_{XYZ,J} / \sqrt{-\ell_{XYZ,J} \cdot \ell_{XYZ,J}}.$$

**use lab momenta to compute invariants**

# Tomography builds higher dimensional structure from lower dimensional projections

probe operators  $G_\ell$

$$\text{tr}(G_\ell G_k) = \delta_{\ell k} \quad \text{orthonormal matrices}$$

observable:

$$\langle G_\ell \rangle = \text{tr}(G_\ell \rho_X)$$

$\rho_X =$  unknown system

reconstruction:

$$\rho_X = \sum_{\ell} \langle G_\ell \rangle G_\ell$$

**Completeness?** *It's complete for what it spans*

# The density matrix is observable

If and when  $\text{rank}=1$ ,

$$\rho|\psi\rangle = |\psi\rangle$$

defines  $|\psi\rangle$

Wave functions are observable, up to the undetermined phase of eigenstates



# Experimentally measure the density matrix

$$P(Q, \ell | init) = P(\ell | Q, init) P(Q | init).$$



$$\frac{dN}{d\Omega} = \frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \text{tr} (\rho(\ell) \rho(X)),$$

$$\rho(\ell) = \text{known density matrix} = \sum_{\ell} c_{\ell} G_{\ell}$$

$$\rho(X) = \text{unknown density matrix}$$

reconstruction:

$$\rho_X = \sum_{\ell} \langle G_{\ell} \rangle G_{\ell}$$

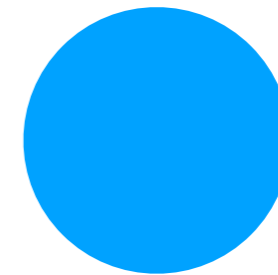
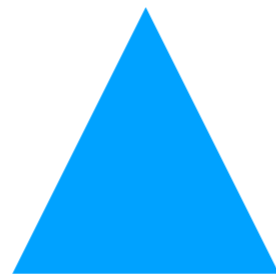
# The Mirror trick

3 spin 1 tensors

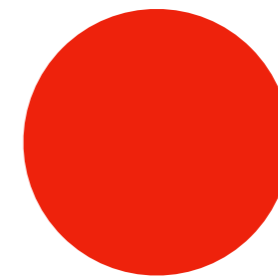
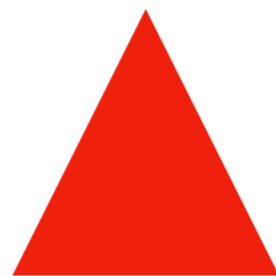
5 spin 2 tensors

Probe:  $\rho_{ij}(\ell) = \frac{1}{3}\delta_{ij} + b\hat{\ell} \cdot \vec{J}_{ij} + aU_{ij}(\hat{\ell});$  where  $U_{ij}(\hat{\ell}) = \frac{\delta_{ij}}{3} - \hat{\ell}_i\hat{\ell}_j = U_{ji}(\ell); \text{tr}(U(\ell)) = 0;$   
 (1)

System:  $\rho_{ij}(X) = \frac{1}{3}\delta_{ij} + \frac{1}{2}\vec{S} \cdot \vec{J}_{ij} + U_{ij}(X);$  where  $U(X) = U^T(X); \text{tr}(U(X)) = 0.$



probe



system

$$\langle \triangle | \square \rangle = 0, \text{ etc.}$$

DATA

KNOWN; 8 TERMS

$$\frac{d\sigma}{d\Omega} = \text{tr}(\rho \rho_x)$$

GET 8 TERMS

3x3  $\rho = \rho^\dagger$

$$= \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12}^\dagger & \dots & \dots \\ \rho_{13} & \dots & \dots \end{pmatrix} \rightarrow \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{pmatrix}$$

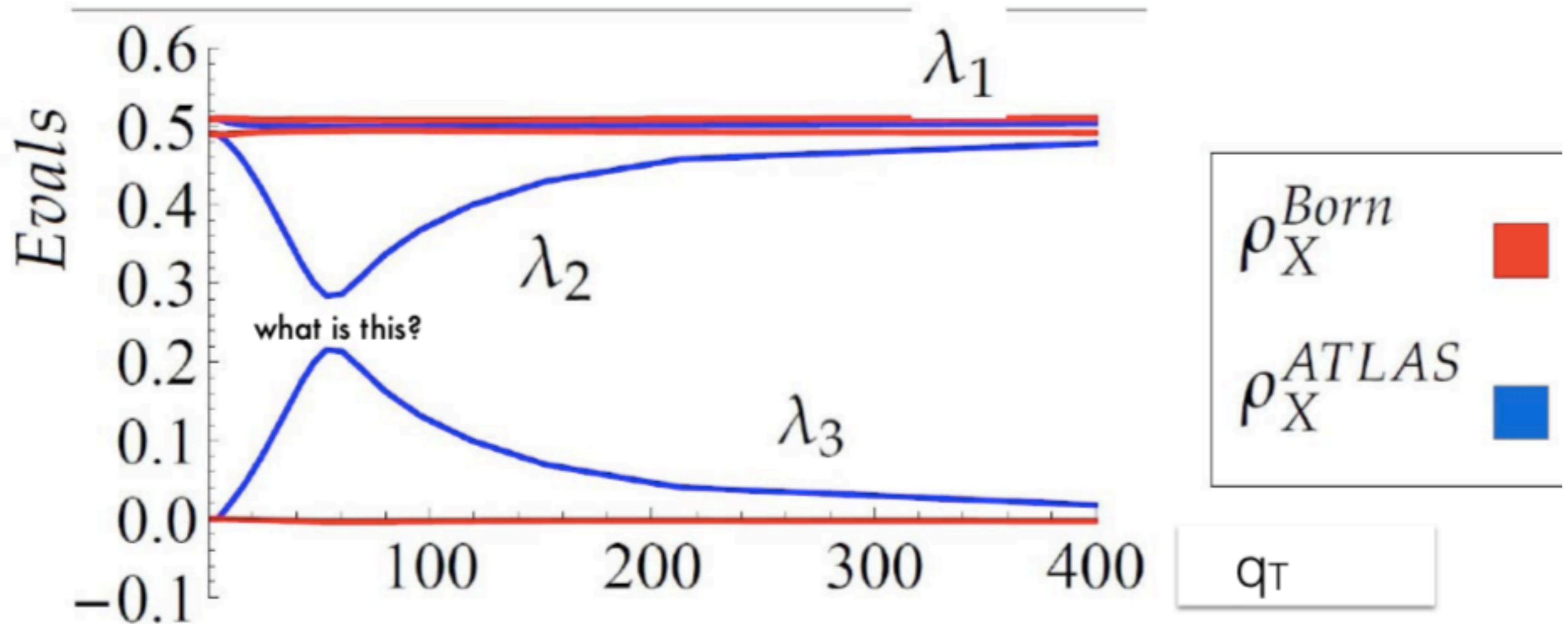
EIGENVALUES  
ARE  
INVARIANTS

READ THE PAPER. MATHEMATICA CODE  
ONLINE PROCESS  $10^9$  EVENTS = FEW SEC  
"CONVEX OPTIMIZATION"

# The density matrix eigenvalues are strange

y-integrated data. arXiv: 1606.00689

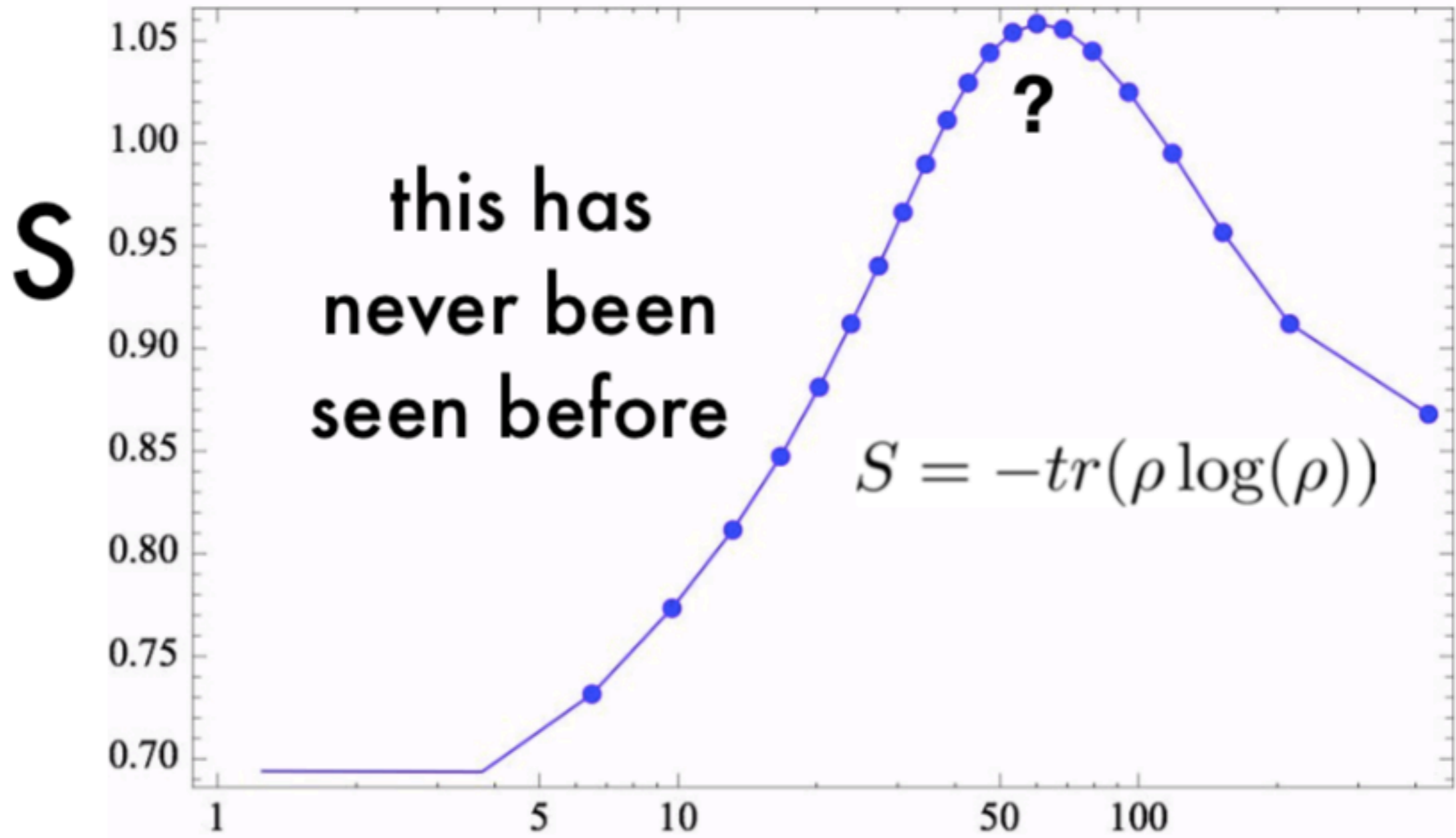
Evals  $\lambda_j$  of  $\rho_X^{ATLAS}$  (blue) are very different from evals from Born-level physics (red).



there is no precedent for the resonance-like bump

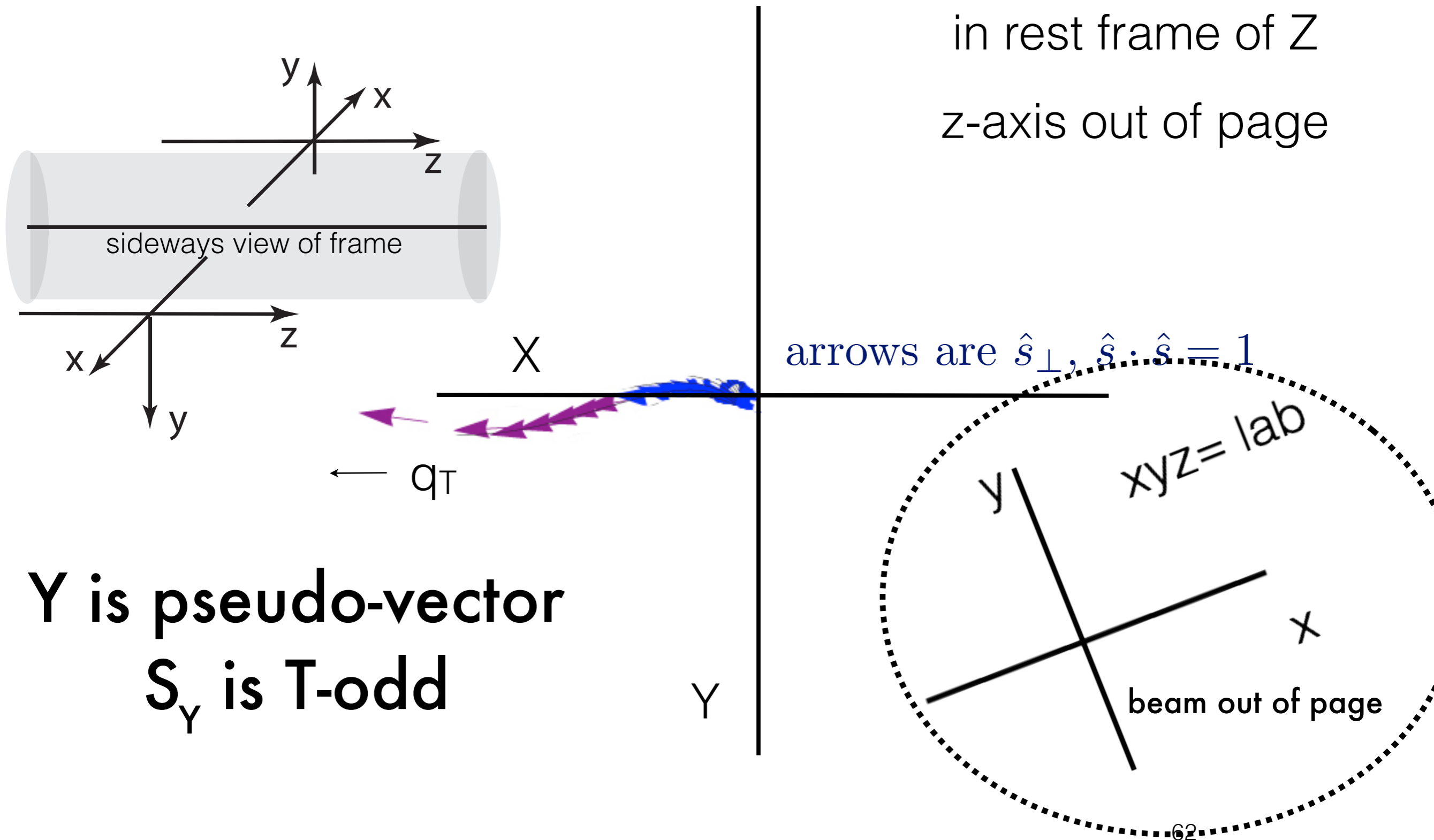
# The entanglement entropy is strange

entropy v  $p_T$



$p_T$  (same as  $q_T$ ) of pair in GeV units

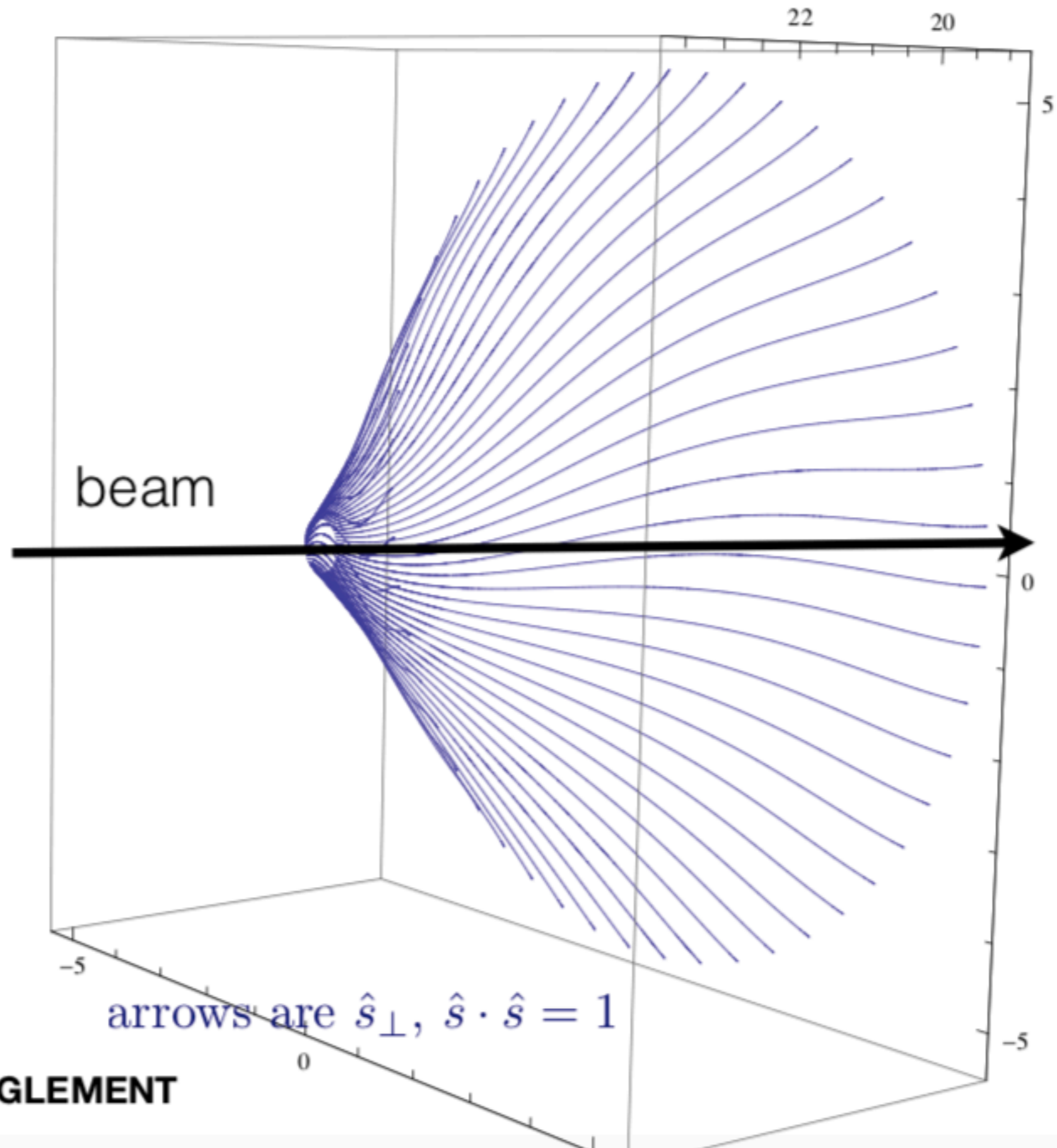
# Unexpected discovery in spin parameters of the Z



3D  
holography  
of the  
Z spin,  
lab frame

$(q_x, q_y, q_z)$

2% of Z's are  
polarized  
pure state  
spinning  
as shown



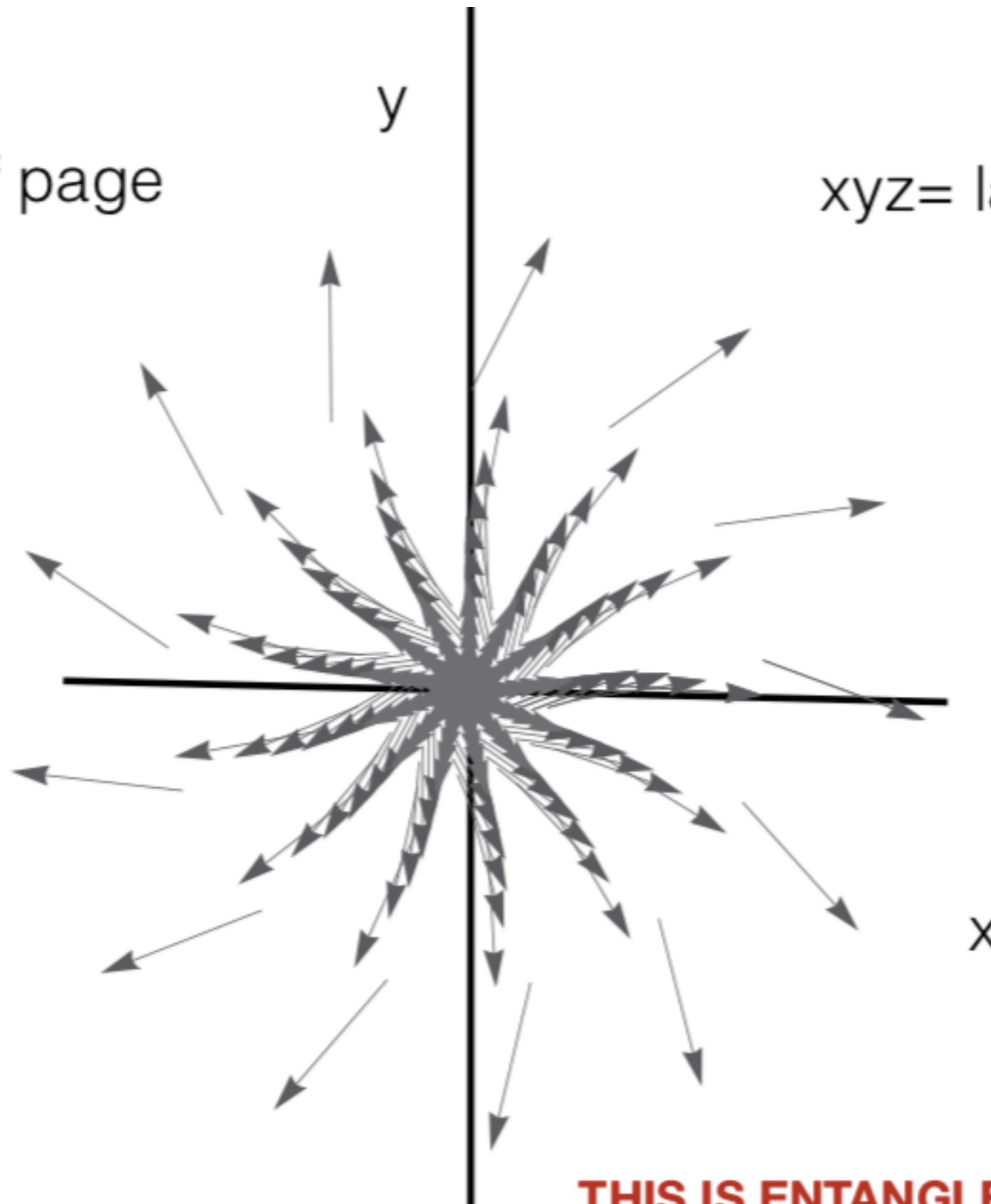
arrows are  $\hat{s}_\perp$ ,  $\hat{s} \cdot \hat{s} = 1$

**THIS IS ENTANGLEMENT**

beam-axis out of page

xyz= lab

2% of Z's  
are  
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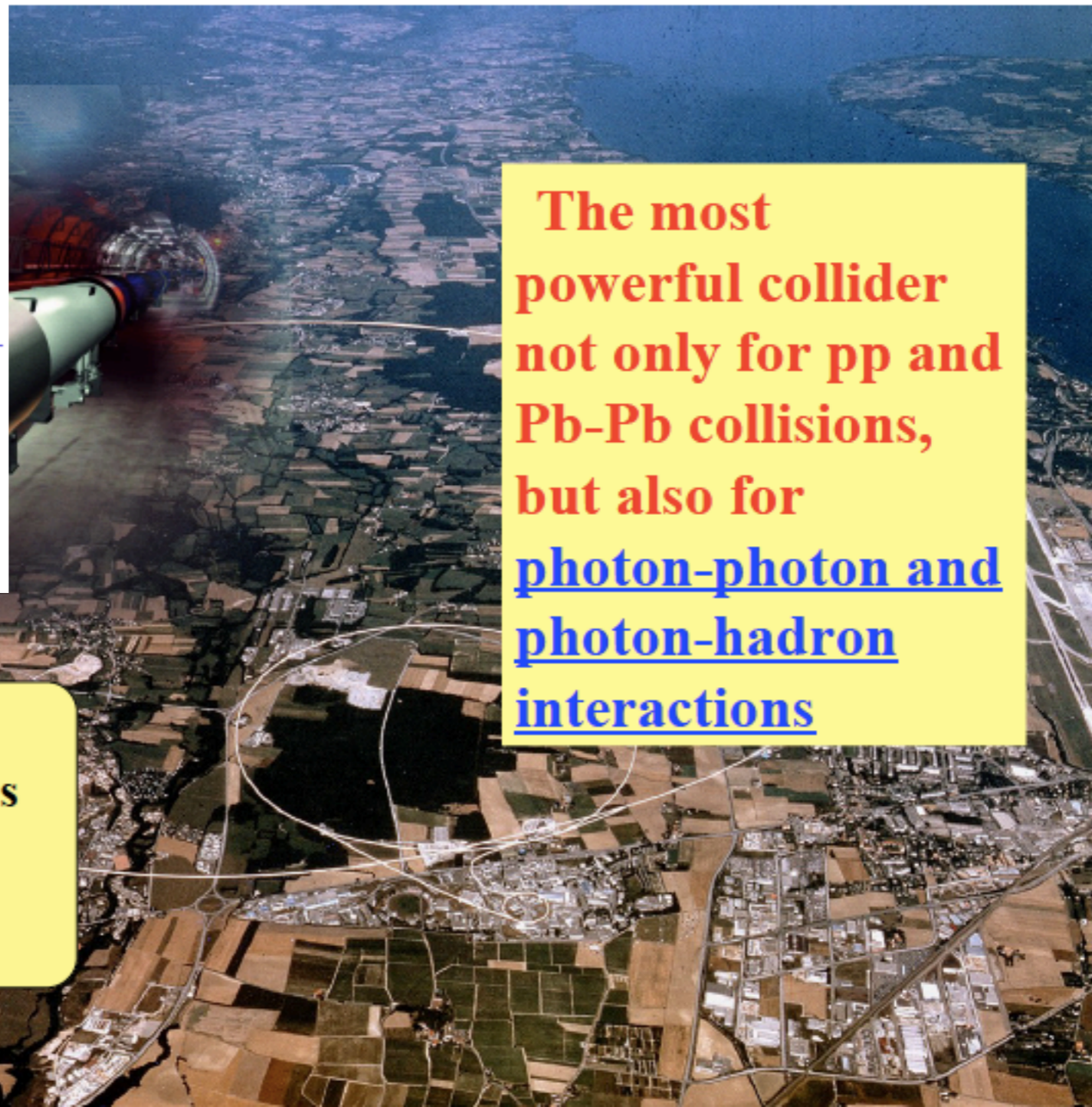
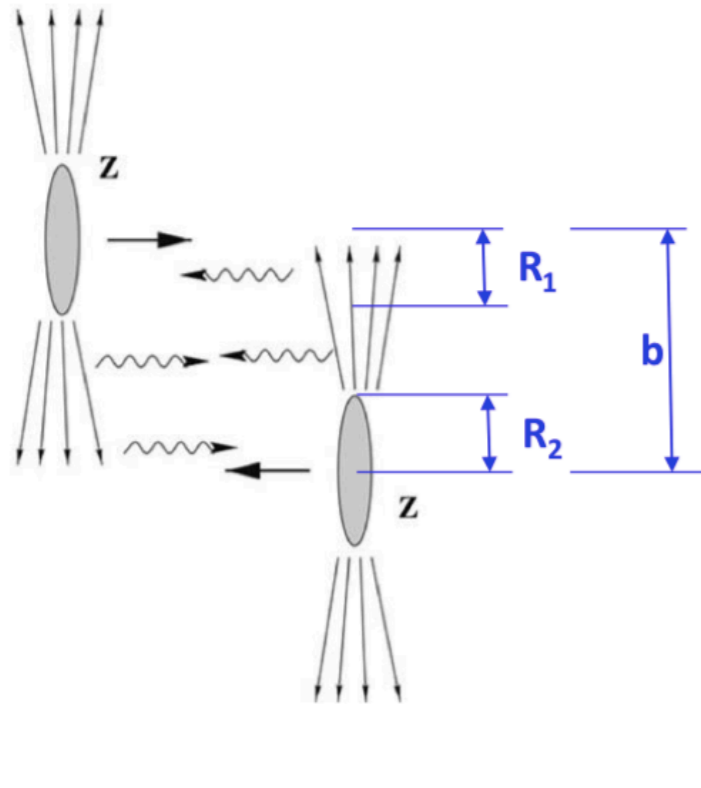
**THIS IS ENTANGLEMENT**

arrows are  $\hat{s}_\perp$ ,  $\hat{s} \cdot \hat{s} = 1$



*Another example*

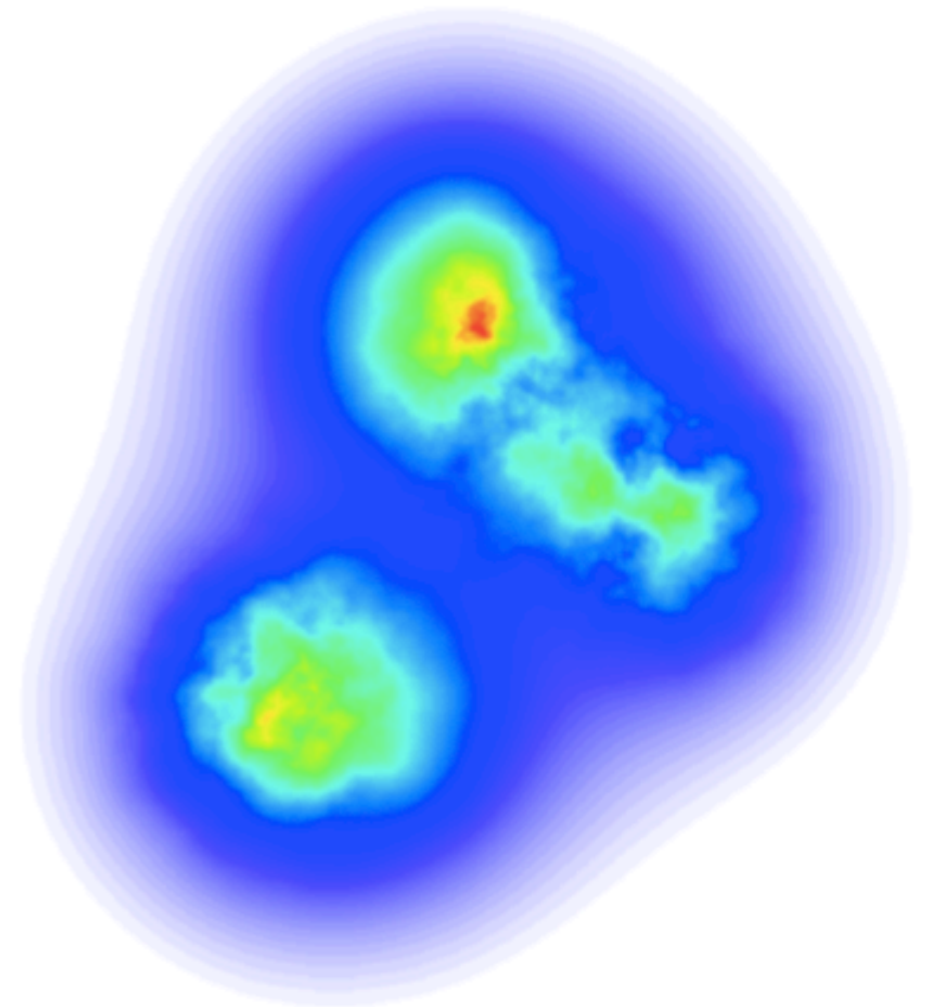
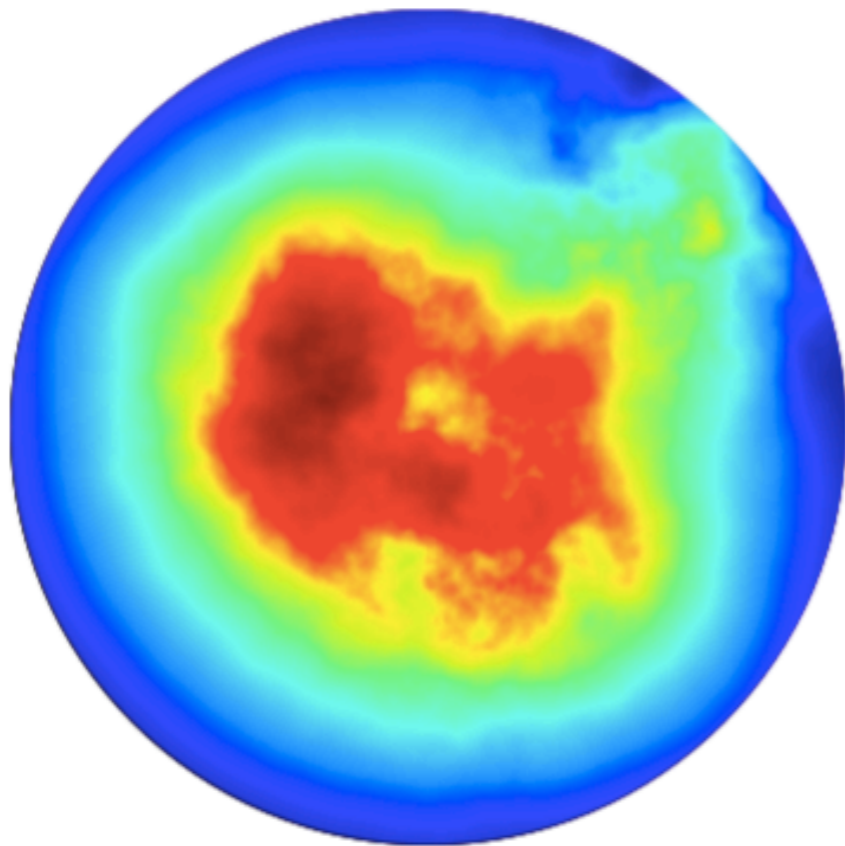
# Photon-photon, photon-p, photon-A collider



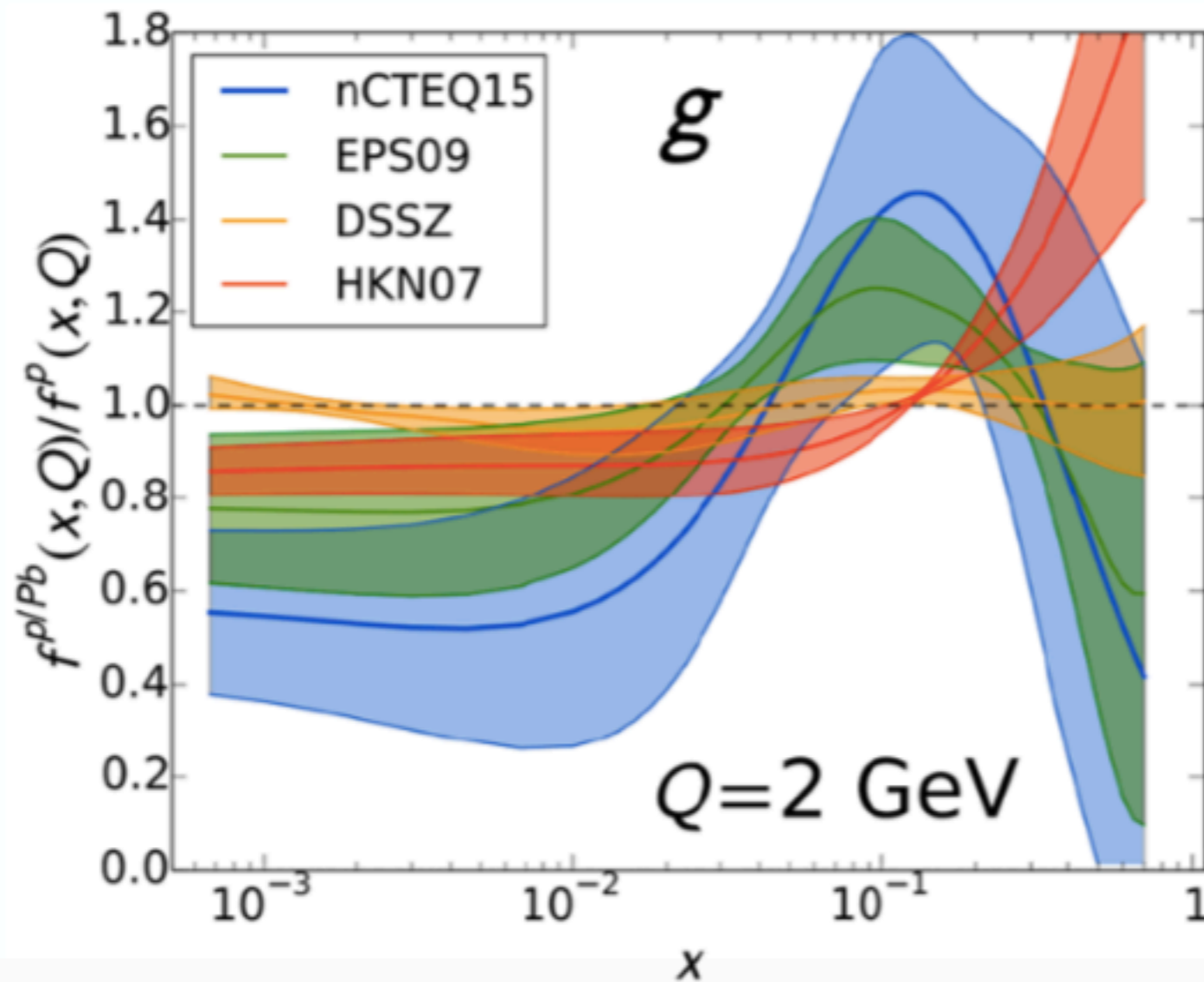
**The most powerful collider not only for pp and Pb-Pb collisions, but also for photon-photon and photon-hadron interactions**

**UPC physics at LHC**

What does the proton look like?



# Nuclear gluon density

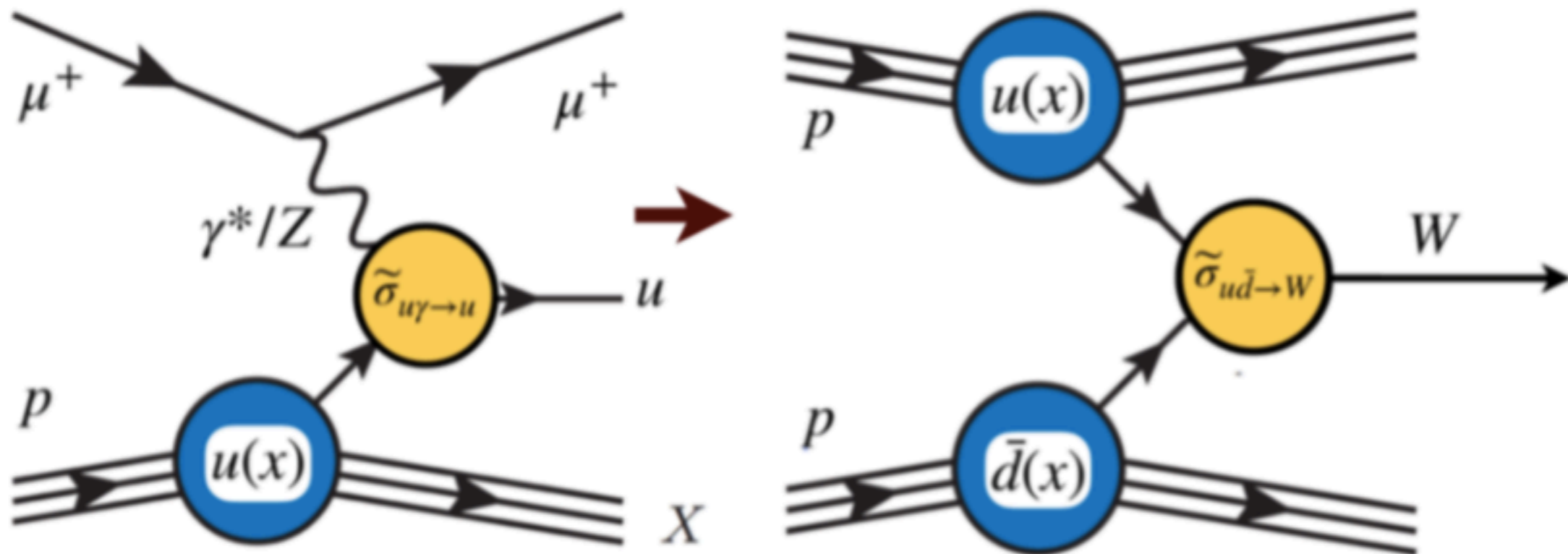


*UPC studies provide the best information the community will get for the next 10 years before, the EIC turns on*

# The Global QCD analysis paradigm

QCD factorisation theorems: **PDF universality**

$$\sigma_{lp \rightarrow \mu X} = \tilde{\sigma}_{u\gamma \rightarrow u} \otimes u(x) \quad \rightarrow \quad \sigma_{pp \rightarrow W} = \tilde{\sigma}_{ud \rightarrow W} \otimes u(x) \otimes \bar{d}(x)$$

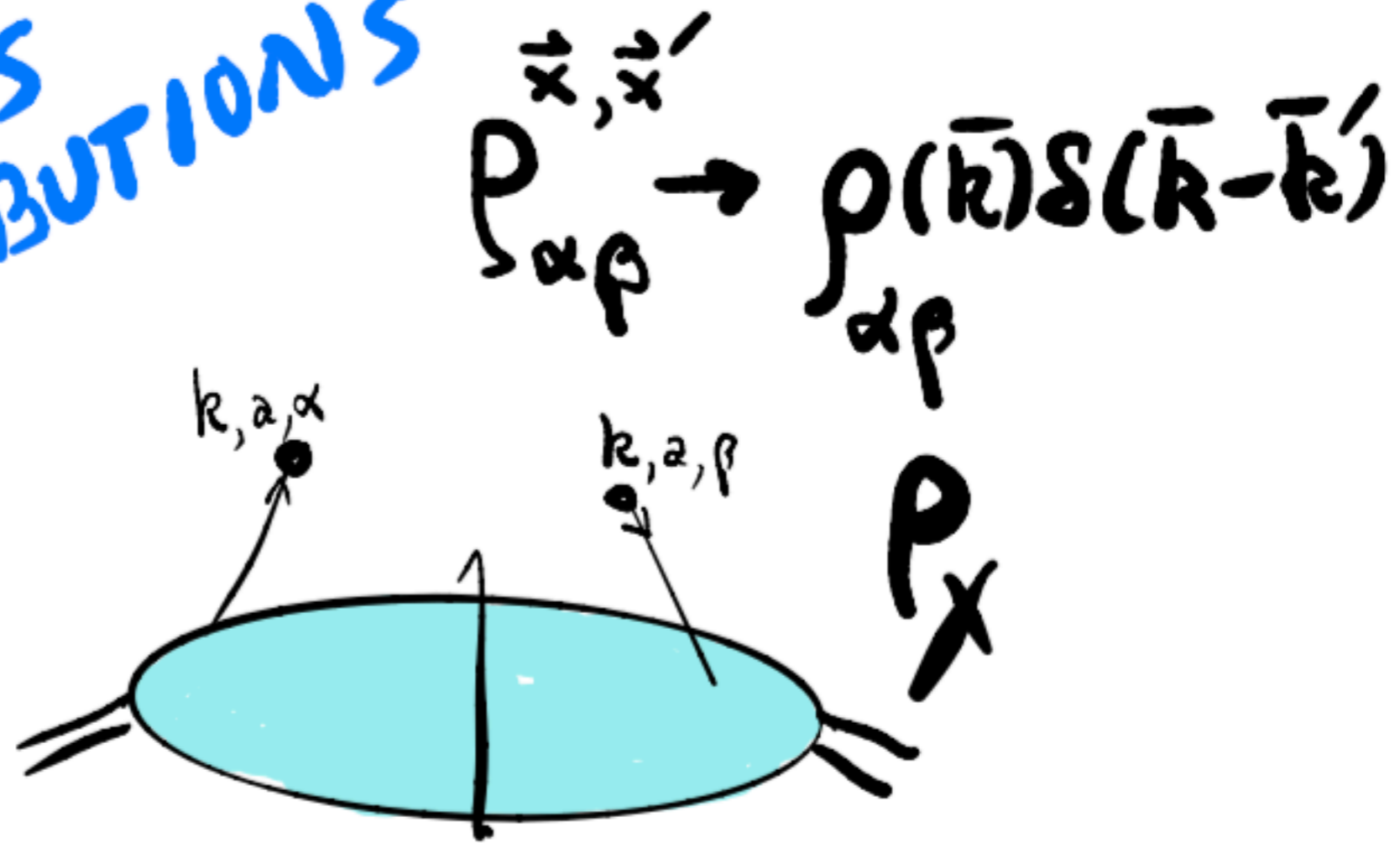


Determine PDFs from **deep-inelastic scattering...**

... and use them to compute predictions for **proton-proton collisions**

From J. Rojo. DIS 2019

**PARTON DISTRIBUTIONS  
AIN'T DISTRIBUTIONS**



**PARTON "DISTRIBUTIONS" ARE DENSITY MATRICES**

DIAGONAL IN MOMENTUM, NOT POLARIZATION

# Begin with Some Results

“dijets” means  
2 LHC jets, each  
made of many particles  
plus everything else not measured

histograms show a  
Lorentz-invariant angular  
distribution of jet1 v jet 2  
measuring a density matrix

*raw data processed,  
bypassing 600 pages  
of theory papers*

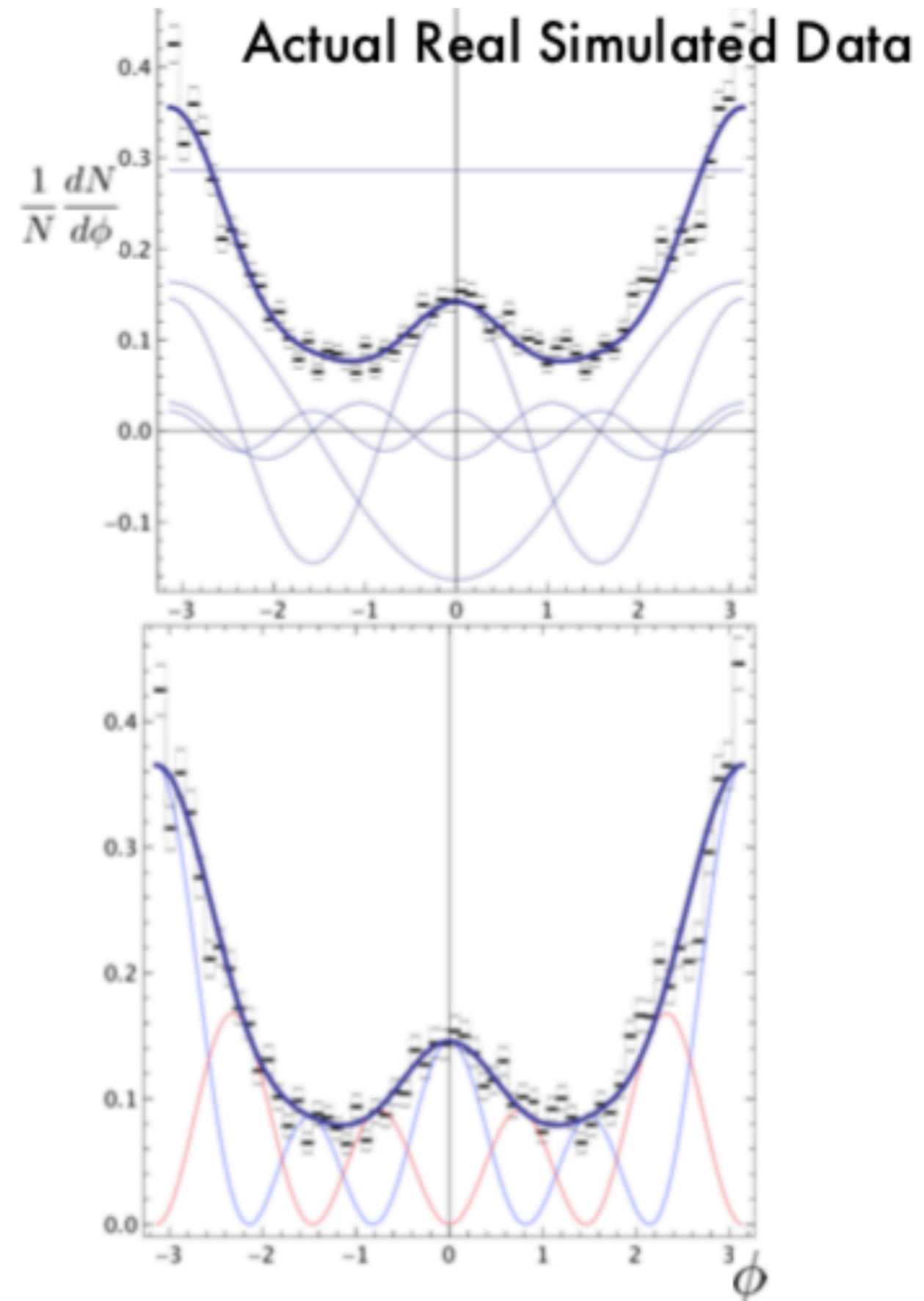


FIG. 4: Top: Maximum likelihood fit, with the contributions of  $\cos m\phi$  for  $m = 0 - 4$ . Bottom: weighted distributions defined by  $f_+(\phi) = \text{Re}(\psi)^2$  (blue) and  $f_-(\phi) = \text{Im}(\psi)^2$  (red), coming from the eigenstates of the rank two density matrix.

**It's time  
to wind this up...**





KIDS  
LEARN  
WHAT  
YOU  
TEACH  
THEM



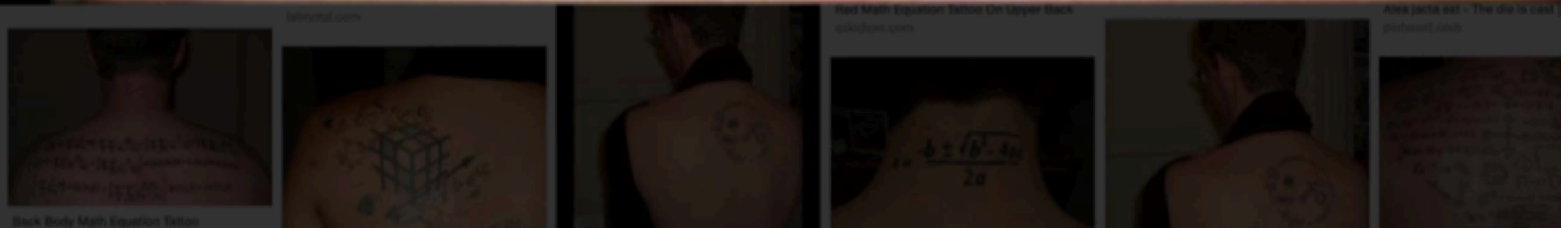
The image shows a search engine results page for the query "atoms first grade". The search bar at the top contains the text "atoms first grade" and a magnifying glass icon. Below the search bar are filters for "ALL", "SHOPPING", "IMAGES", "VIDEOS", "NEWS", "MAPS", "BOOKS", "FLIGHTS", and "PERSONAL". There are also filters for "Label", "DF", "ID", and "Product" with a color-coded bar. The search results are categorized into "worksheet", "science experiment", "charged atoms", "atomic theory", "balancing equations", "science worksheets", "matter", and "elementary". The results include various educational resources such as "Elements 1-20", "STEM for Kids candy molecules", "paper plate atom", "Elements and atoms", "Introduction to the atom", "ATOMS LESSON PLAN BUNDLE", and "Mastering CHEMISTRY".

20  
YEARS  
of  
NONSENSE  
NEVER  
CORRECTED



Don't  
Do  
This!

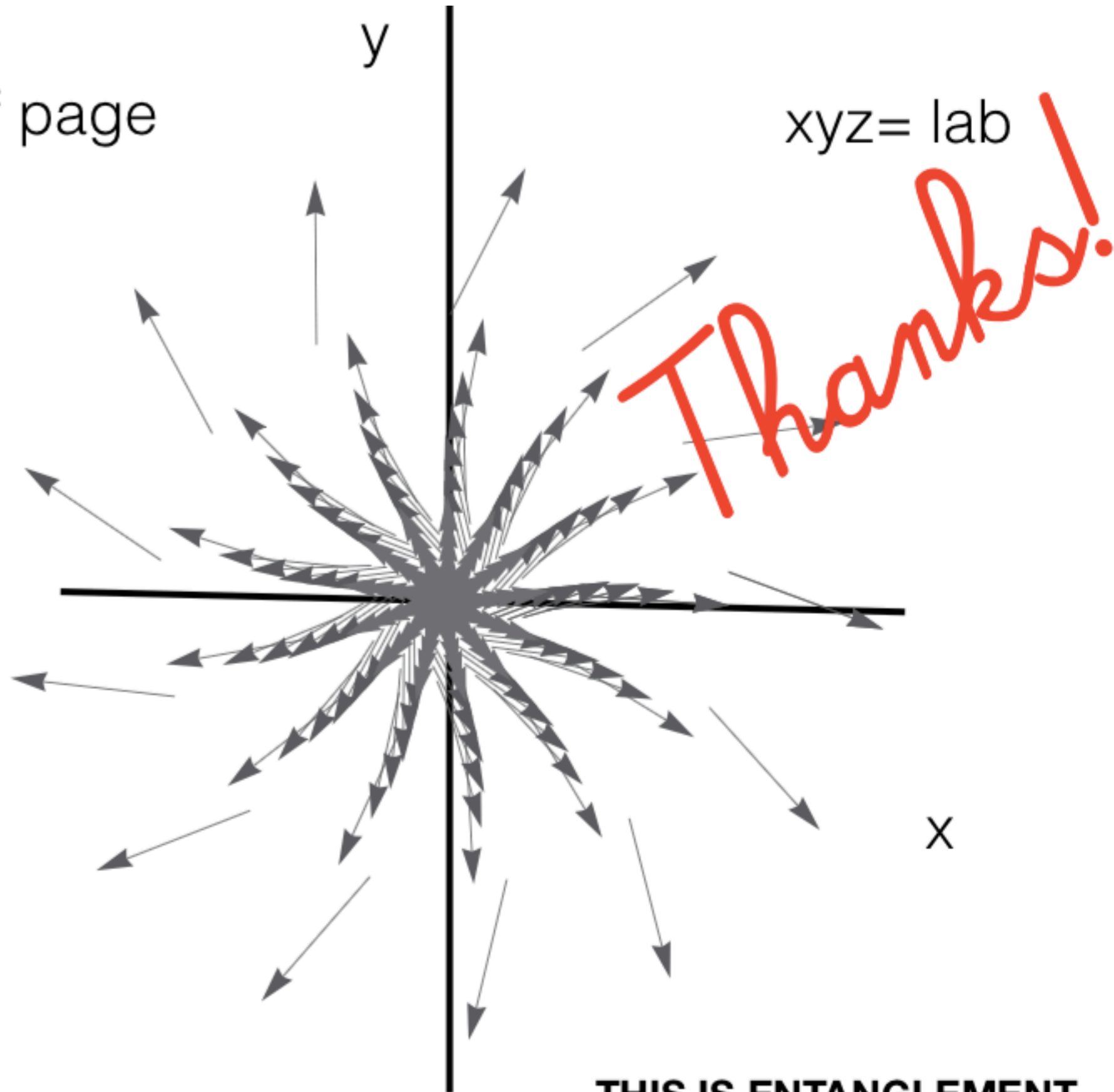
$$\begin{aligned}
 &-\frac{1}{2} \sum_i \nabla_i^2 - \sum_a \frac{1}{2m_a} \nabla_a^2 - \sum_i \sum_a \frac{Z_a}{|r_i - r_a|} + \frac{1}{2} \sum_i \sum_{j \neq i} \frac{1}{|r_i - r_j|} + \frac{1}{2} \sum_a \sum_{\beta \neq a} \frac{Z_a Z_\beta}{|r_a - r_\beta|} \\
 &\left[ -\frac{1}{2} \sum_i \nabla_i^2 - \sum_a \sum_b \frac{Z_a}{|r_a - r_b|} + \frac{1}{2} \sum_i \sum_{j \neq i} \frac{1}{|r_i - r_j|} \right] \Psi(\{r_i\}, \{r_a\}) = \mathcal{E}_0(\{r_a\}) \Psi(\{r_i\}, \{r_a\}) \\
 &\left[ -\sum_i \frac{1}{2m_i} \nabla_i^2 + \mathcal{E}_0(\{r_a\}) + \frac{1}{2} \sum_\beta \sum_{\gamma \neq \beta} \frac{Z_\beta Z_\gamma}{|r_\beta - r_\gamma|} \right] \Phi(\{r_a\}) = \mathcal{E} \Phi(\{r_a\}).
 \end{aligned}$$



beam-axis out of page

xyz= lab

2% of Z's  
are  
polarized  
pure state  
spinning  
as shown



arrows are  $\hat{s}_\perp$ ,  $\hat{s} \cdot \hat{s} = 1$

**THIS IS ENTANGLEMENT**